Effect of interactions on the topological expression for the chiral separation effect

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$\Lambda/\bar{\Lambda}$-polarization from Chiral Effects in Heavy Ion Collisions

- CSE for free fermions [Vilenkin (1980), Metlitski & Zhitnitsky (2005)]

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- CVE (and CSE) in HIC [A.Sorin, O.Teryaev, Phys.Rev.C95 011902 (2017)] as spin polarization

$$\langle \Pi_0^{\Lambda} \rangle = \langle \frac{m_\Lambda}{N_\Lambda p_y} \rangle \, Q_5^S = \langle \frac{m_\Lambda}{N_\Lambda p_y} \rangle \, \frac{N_c}{2\pi^2} \int d^3x \mu_s^2(x) \omega_0(x)$$

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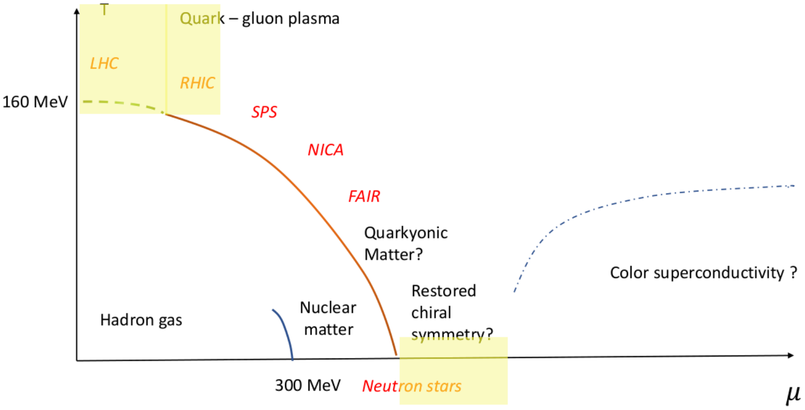
- CSE susceptibility in Lattice QCD points from [Brandt et al. arXiv:2212.02148]

$$C_{\mathrm{CSE}}(T) = (N_c H \sum_f q_f^2)^{-1} \left. \frac{dJ_z^5}{d\mu} \right|_{\mu=0}$$

Hot and Dense QCD as a model for QGP



CSE in Cold Dense QCD

- Local renormalized axial current density (using Wigner-Weyl fomralism [Suleymanov & Zubkov Nucl.Phys.B938 171 (2019)])

$$\mathrm{j}^5_k(x) = -\int_{\mathcal{M}} \frac{d^D p}{(2\pi)^D} \mathrm{tr}\ \left[\gamma^5 \mathrm{G}_W(x,p) \partial_{p_k} \mathrm{Q}_W(x,p)\right]$$

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- As the linear response to F_{ij} at $T = 0$

$$\bar{J}^5_k(x) = \sigma_{ijk} F_{ij} \delta\mu$$

$$\sigma_{ijk} = -\frac{1}{2\mathrm{V}} \int_{\Sigma_3} \frac{d^3 p}{(2\pi)^4} \int d^3 x \mathrm{tr} \left(\gamma^5 \left[\mathrm{G}^{(0)}_W \star (\partial_{p_{[i}} \mathrm{Q}^{(0)}_W) \star \mathrm{G}^{(0)}_W \times \right.\right.$$
$$\left.\left. \times \star (\partial_{p_{j]}} \mathrm{Q}^{(0)}_W) \star \mathrm{G}^{(0)}_W \right] \partial_{p_k} \mathrm{Q}^{(0)}_W \right)$$

CSE in Cold Dense QCD

- As the linear response to μ and F_{ij} at $T = 0$

$$J^5_k(x) = \sigma_{ijk} F_{ij} \delta\mu$$

$$\sigma_{ijk} = -\frac{1}{2V} \int_{\Sigma_3} \frac{d^3p}{(2\pi)^4} \int d^3x \mathrm{tr} \left(\gamma^5 \left[\mathrm{G}^{(0)}_W \star (\partial_{p_{[i}} \mathrm{Q}^{(0)}_W) \star \mathrm{G}^{(0)}_W \times \right.\right.$$
$$\left.\left. \times \star (\partial_{p_{j]}} \mathrm{Q}^{(0)}_W) \star \mathrm{G}^{(0)}_W \right] \partial_{p_k} \mathrm{Q}^{(0)}_W \right)$$

where Weyl symbols and Moyal product are

$$Q_W(x,p) \equiv \int_{\mathcal{M}} dq e^{ixq} \langle p + q/2 | \hat{Q} | p - q/2 \rangle, \quad \star = e^{\frac{i}{2} \left(\overleftarrow{\partial}_x \overrightarrow{\partial_p} - \overleftarrow{\partial_p} \overrightarrow{\partial}_x \right)}$$

CSE conductivity as a topological invariant

The conductivity

$$\sigma_{ijk} = -\epsilon_{ijk}\sigma_{CSE}/2, \;\; \sigma_{CSE} = \frac{\mathcal{N}}{2\pi^2}$$

is a topological invariant if G (anti)commutes with γ_5

$$\mathcal{N} = \frac{1}{48\pi^2 \mathrm{V}} \int_{\Sigma_3} \int d^3x \mathrm{tr} \left[\gamma^5 G_W \star dQ_W \star G_W \wedge \star dQ_W \star G_W \star \wedge dQ_W\right]$$

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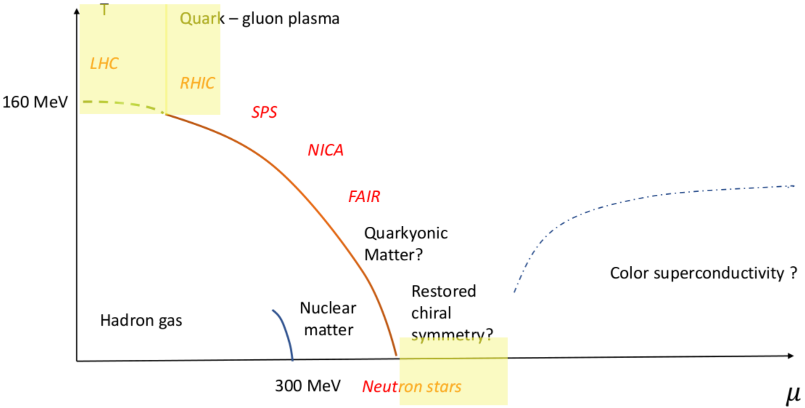
- σ_{CSE} at $T = 0$ is immutable and renorm-invariant in Chiral-symmetric phase

CSE in Hot QCD with the Field Correlator Method



Field Correlators in QCD

Background Perturbtaion Theory: $A = B + a$

$$\langle \ldots \rangle = \int \mathcal{D}A \ldots = \int \mathcal{D}B\mathcal{D}a \ldots = \langle\langle \ldots \rangle_a \rangle_B \,,$$

$(\ldots)$ — a gauge-invariant(!) operator

$$\frac{g^2}{N_c}\langle \mathrm{tr}\ E_i(0)\Phi E_j(u)\Phi^\dagger\rangle_B = \delta_{ij}\left[D^E(u) + D_1^E(u) + u_4^2\frac{\partial D_1^E}{\partial u^2}\right] + u_i u_j \frac{\partial D_1^E}{\partial u^2},$$

$$\frac{g^2}{N_c}\langle \mathrm{tr}\ H_i(0)\Phi H_j(u)\Phi^\dagger\rangle_B = \delta_{ij}\left[D^H(u) + D_1^H(u) + \vec{u}^2\frac{\partial D_1^H}{\partial \vec{u}^2}\right] - u_i u_j \frac{\partial D_1^H}{\partial u^2},$$

$$\frac{g^2}{N_c}\langle \mathrm{tr}\ H_i(0)\Phi E_j(u)\Phi^\dagger\rangle_B = \epsilon_{ijk}\, u_4 u_k \frac{\partial D_1^{EH}}{\partial u^2}.$$

QCD: $\sigma = 0.18\ \mathrm{GeV}^2, \quad \lambda \sim 1\ \mathrm{GeV}^{-1}; \quad \sigma\lambda^2 \ll 1$

$\alpha_s(Q^2 \to 0) \approx 0.6$

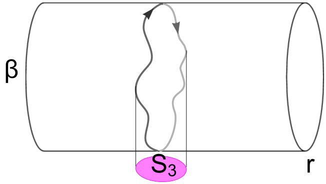
[Simonov Phys. Rev. D 99, 056012 (2019)]

CSE in Hot QGP

$$
\begin{aligned}
\langle j^5_\mu(x)\rangle &= \langle q\mathrm{tr}\ _{c,D}\gamma_5\gamma_\mu S^{(\mathrm{reg})}(x,x)\rangle \\
&= \langle q\mathrm{tr}\ _{c,D}\gamma_5\gamma_\mu(\not{D}(a,B,\mathcal{A})+m)^{-1}_{xx}\rangle_{a,B} \\
&\approx \langle q\mathrm{tr}\ _{c,D}\gamma_5\gamma_\mu(-\not{D}(B,\mathcal{A})+m)_x \times \\
&\quad \times \int_0^{+\infty} ds\ \xi(s)\ (\overline{\mathcal{D}^4 z})^s_{xx} e^{-m^2 s - K} \times \\
&\quad\quad \times P_F P_B \exp\left(ig\oint B\cdot dz + iq\oint \mathcal{A}\cdot dz + \right. \\
&\quad\quad\quad \left. + \int_0^s d\tau\sigma^{\rho\sigma}(gF_{\rho\sigma}(z,z_0)+q\mathcal{F}_{\rho\sigma})\right)\rangle_B
\end{aligned}
$$

CSE in Hot QCD with the Field Correlator Method

- Single Line Approximation for a quark [Orlovsky & Simonov Phys.Rev.D89, 054012 (2014)]



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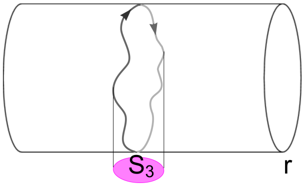

- Polyakov Line (numerical – from Lattice) [Simonov, Trusov Phys.Lett.B650, 36 (2007)]

$$\langle \mathrm{tr}\ L(0)L^{\dagger}(r)\rangle \sim \exp(-\beta V_1(r, T, \mu)),$$

$$L = \exp(-\beta V_1(r \to +\infty, T, \mu)/2), \quad V_1 \approx V_1(T > T_c)$$

$$V_1(T > T_c) = \frac{175\ \mathrm{MeV}}{1.35\ T/T_c - 1}, \qquad T_c = 160\ \mathrm{MeV}.$$

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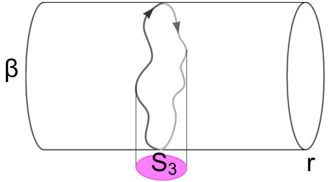

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- Color-Magnetic Confinement [Agasian, Simonov Phys.Lett.B639, 82 (2006)]

$$M^2 = m^2 + c_D^2 \sigma_{H,f}(T)/4 + \Delta m^2_{\text{npSE}}, \quad \sigma_{H,f}(T) \approx c_\sigma^2 g^4(T) T^2$$

$$g^{-2}(T) = 2b_0 \log \frac{T}{T_c L_\sigma}, \quad (4\pi)^2 b_0 = \frac{11}{3} N_c - \frac{2}{3} N_f$$

Single Line Approximation

$$
\begin{aligned}
\langle N_c^{-1}\mathrm{tr}\,_c W[z^{(n)}]\rangle_B &\approx \Big\langle N_c^{-1}\mathrm{tr}\,_c P\exp ig\int \vec{B}(\vec{z}^{(n)}, z_4^{(n)})\cdot d\vec{z}^{(n)}(\tau)\Big\rangle_B \times \\
&\times \Big\langle N_c^{-1}\mathrm{tr}\,_c P\exp ig\int B_4(\vec{z}^{(n)}, z_4^{(n)}) dz_4^{(n)}(\tau)\Big\rangle_B \\
&\approx \exp(-\sigma_{H,f} S_3[\vec{z}(\tau)])\; L^{(n)},\; L^{(n)} \approx L^{|n|},
\end{aligned}
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Single Line Approximation

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$$\approx \exp(-\sigma_{H,f} S_3[\vec{z}(\tau)])\; L^{(n)},\;\; L^{(n)} \approx L^{|n|},$$

$$\int (\mathcal{D}z_4)^s_{0,n\beta} e^{-K_4} \langle N_c^{-1}\mathrm{tr}\,{}_c P_B \exp\left(ig\int B_4 dz_4 + iq\int \mathcal{A}_4 dz_4\right)\rangle_B \approx$$

$$\approx \frac{e^{-\frac{n^2\beta^2}{4s}}}{\sqrt{4\pi s}} L^{|n|} \exp(\mu n\beta).$$

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$$\int (\mathcal{D}^3\vec{z})^s_{\vec{x},\vec{x}} e^{-K_3 - m^2 s} \exp(-\sigma_{H,f} S_3[\vec{z}(\tau)]) \sim \frac{e^{-M^2 s}}{(4\pi s)^{3/2}},$$

CSE with Non-perturbative correction in Hot QGP

$$\langle j_z^5 \rangle \approx \frac{N_c q^2 H}{2\pi^2} I_0, \quad C_{\mathrm{CSE}}(T) = \left.\frac{dI_0}{d\mu}\right|_{\mu=0},$$

$$I_0 = -\int_0^{+\infty} \frac{p^2 dp}{\sqrt{M^2+p^2}} \left(f'(\sqrt{p^2+M^2} + V_1/2 - \mu) - (\mu \to -\mu) \right).$$

c.f. free fermions [Metlitski & Zhitnitsky (2005)]

$$I_0^{\mathrm{free}} = -\int_0^{+\infty} \frac{p^2 dp}{\sqrt{m^2+p^2}} \left(f'(\sqrt{p^2+m^2} - \mu) - (\mu \to -\mu) \right)$$

Comparison to Lattice QCD

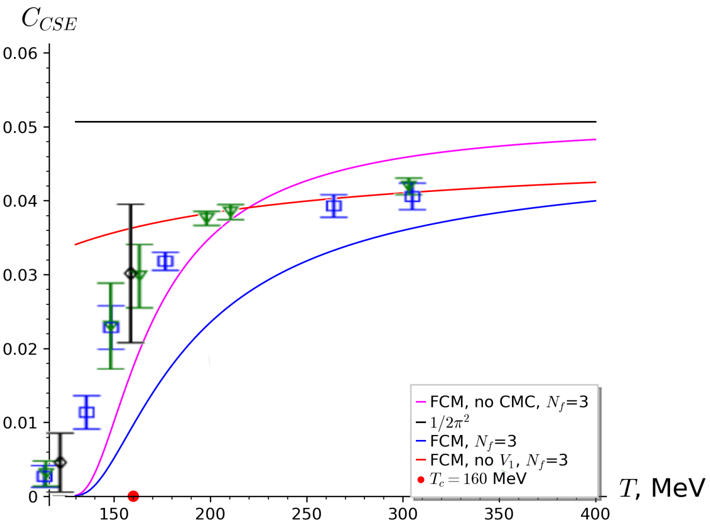

Figure: points from [Brandt et al. arXiv:2212.02148]

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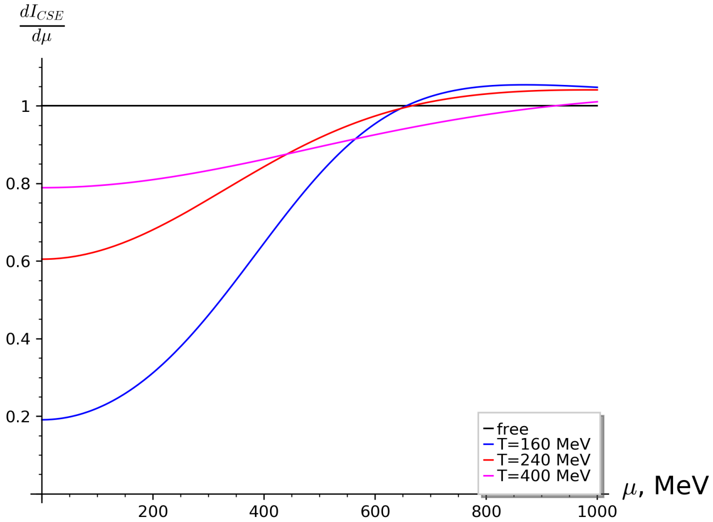

Results summary

QCD Chiral-symmetric phase at $T = 0$

- $\sigma_{CSE} = \frac{\mathcal{N}}{2\pi^2}$ — immutable and renorm-invariant (topological invariant)

Hot QCD $T > T_c$

- LO expression in α_s for σ_{CSE} with non-perturbative corrections
- σ_{CSE} tends to the free limit at large temperatures and densities