Vacuum Decay and New Instantons

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Based on the papers:

JHEP 07 (2022) 147, 2206.13994 [hep-th]
Phys.Lett. B827 (2022) 136951, 2111.13928 [hep-th]
JCAP 10 (2021) 049, 2105.01996 [hep-th]
JCAP 10 (2021) 066, 2104.12661 [hep-th]
Fortsch. Phys. 69 (2021) 2, 2000100, 2009.12445 [hep-th]
Fortsch. Phys. 69 (2021) 2, 2000101, 2009.12444 [hep-th]

XII International Conference on New Frontiers in Physics (ICNFP 2023)
OAC, Crete, 10 July 2023

“Find the beginning of everything, and you will understand much.”

Kozma Prutkov

Citations iNSPIRE hep
January 11, 2023

Bubbles in Metastable Vacuum
I. Yu. Kobzarev, L. B. Okun, M. B. Voloshin
Sov.J.Nucl.Phys. 20 (1975) 644-646, *Yad.Fiz.* 20 (1974) 1229-1234
428 citations (iNSPIRE hep)

The Fate of the False Vacuum. 1. Semiclassical Theory
Sidney R. Coleman
Phys.Rev.D 15 (1977) 2929-2936, *Phys.Rev.D* 16 (1977) 1248 (erratum)
2364 citations (iNSPIRE hep)

The Fate of the False Vacuum. 2. First Quantum Corrections
Curtis G. Callan, Jr., Sidney R. Coleman
Phys.Rev.D 16 (1977) 1762-1768
1526 citations (iNSPIRE hep)

Action Minima Among Solutions to a Class of Euclidean Scalar Field Equations
Sidney R. Coleman, V. Glaser, Andre Martin
Commun.Math.Phys. 58 (1978) 211-221
311 citations (iNSPIRE hep)

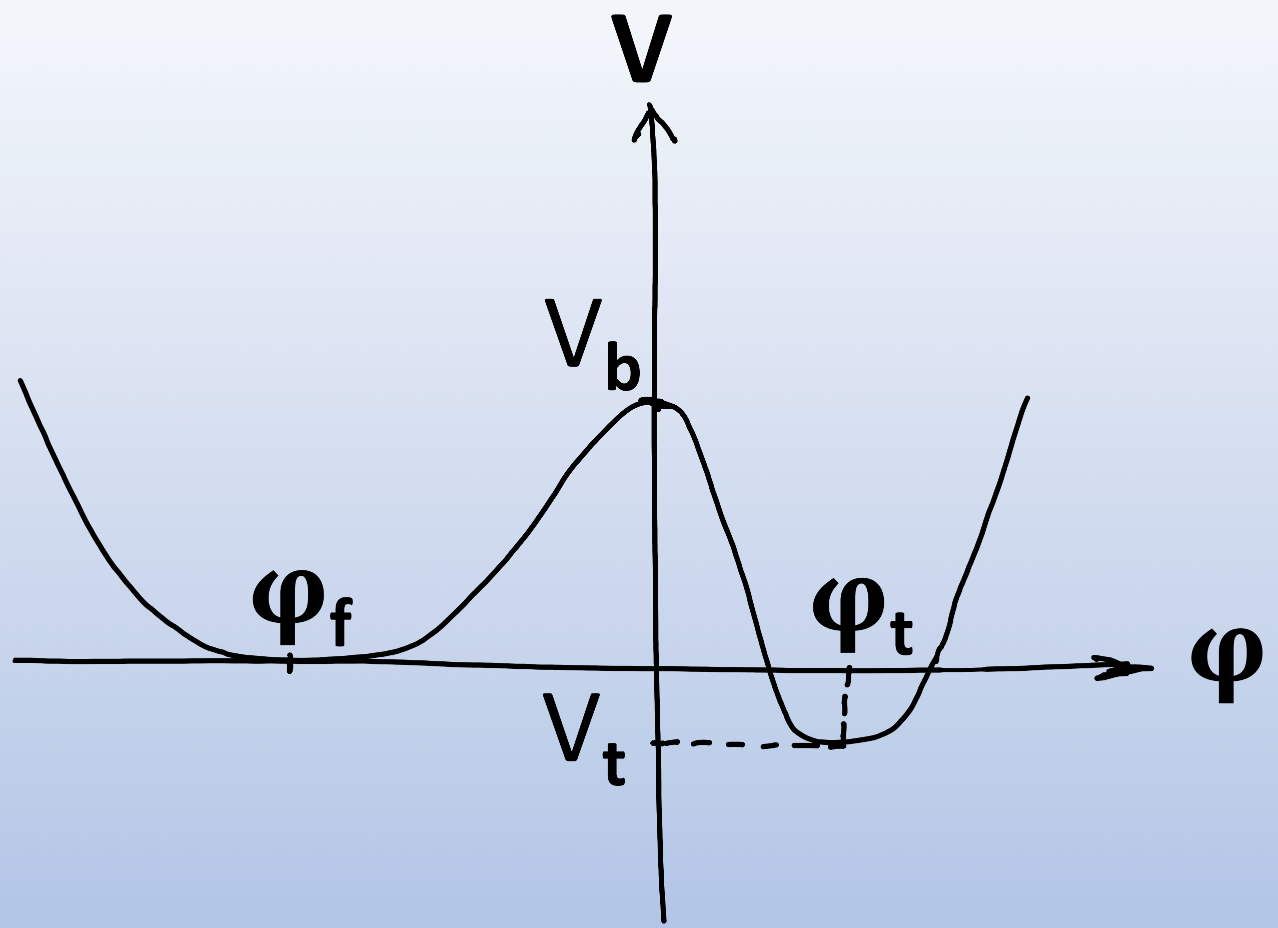
Gravitational Effects on and of Vacuum Decay

Sidney R. Coleman and Frank De Luccia

Phys.Rev.D 21 (1980) 3305

1610 citations (iNSPIRE hep)

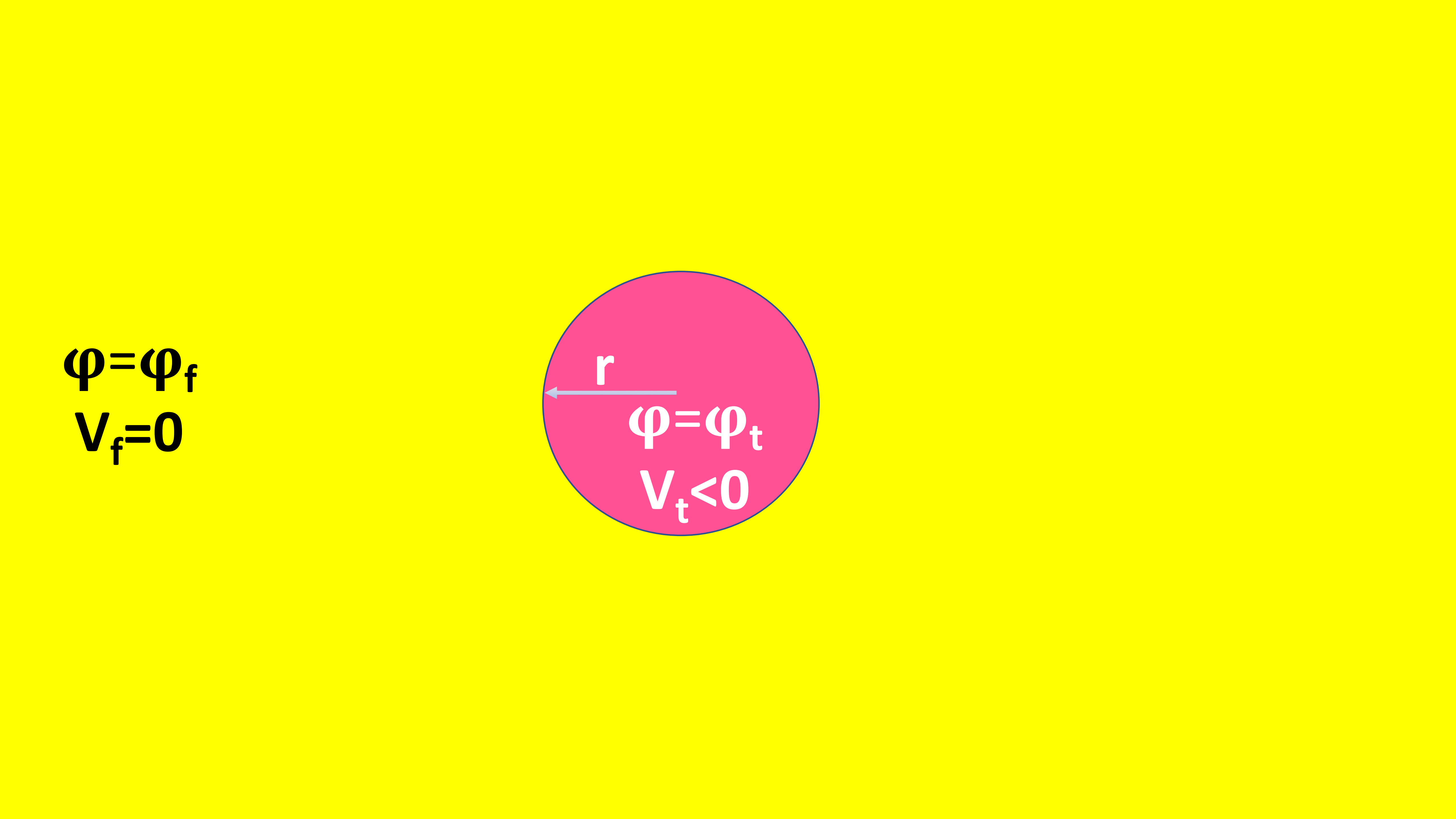
Citations iNSPIRE hep
January 11, 2023



$$V_f = 0, \quad V_b \gg |V_t|$$

$\varphi=\varphi_f$

$V_f=0$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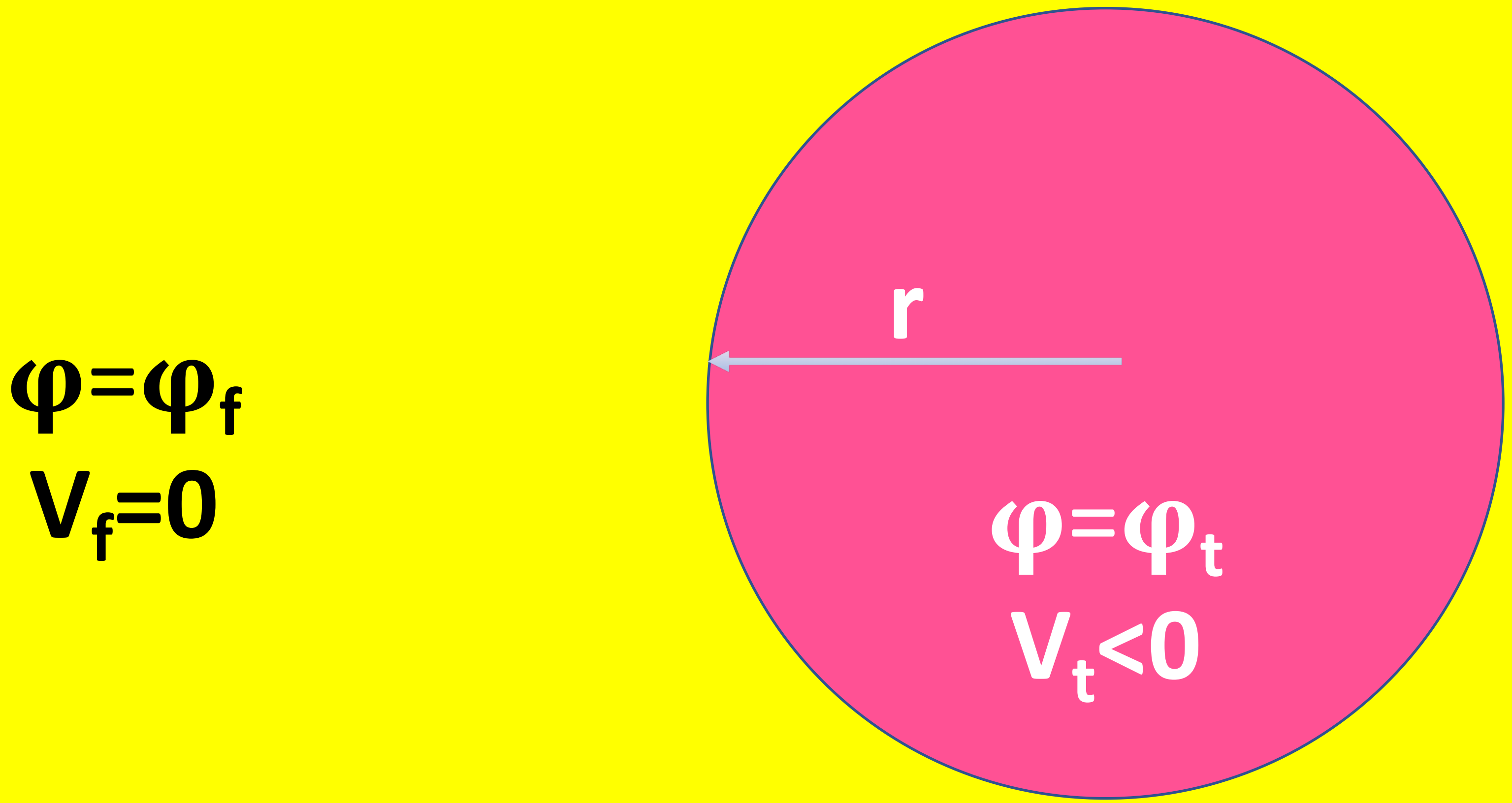
Vf=0
r
φ=φt
Vt<0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Vf=0
r
φ=φt
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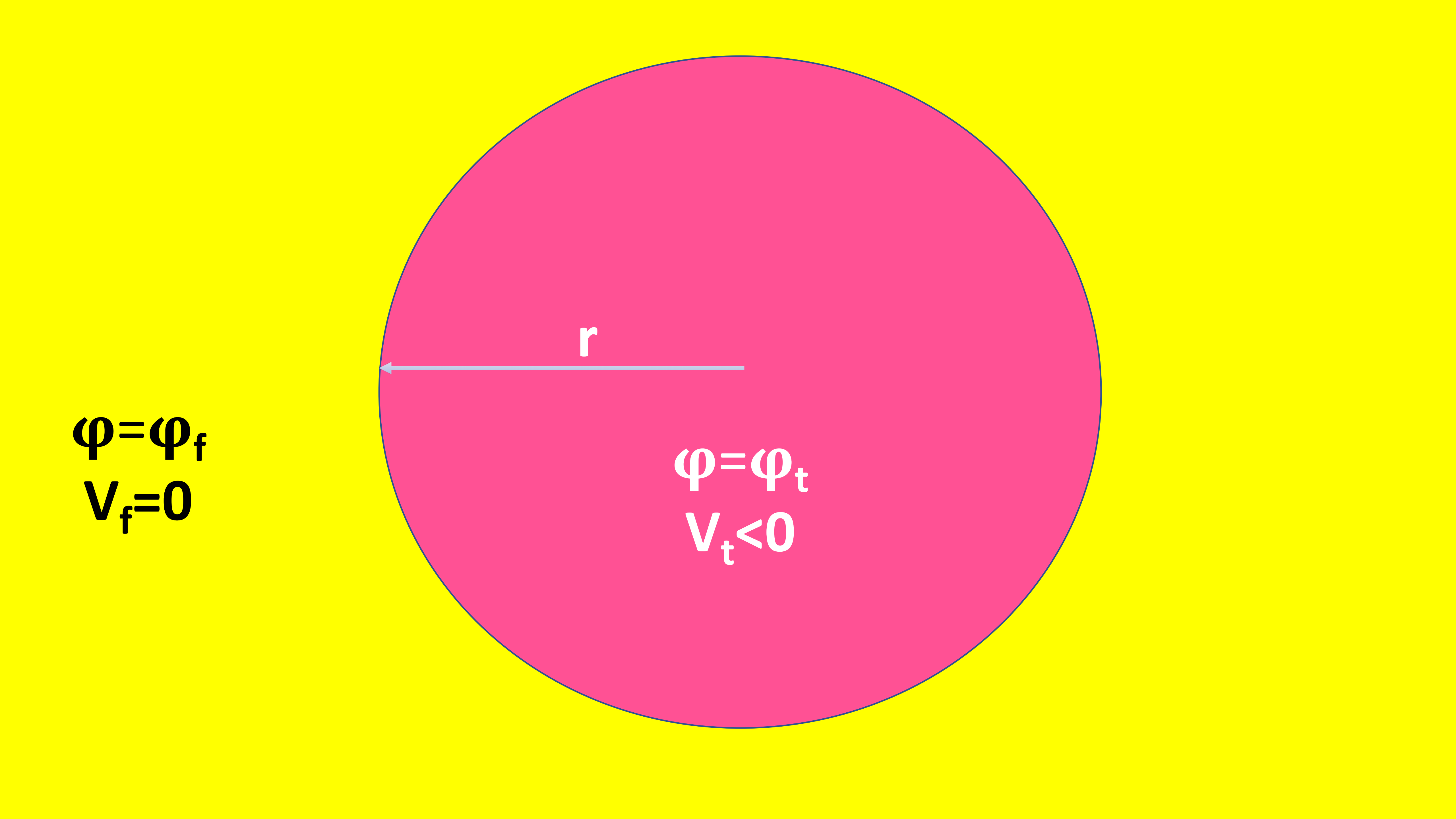
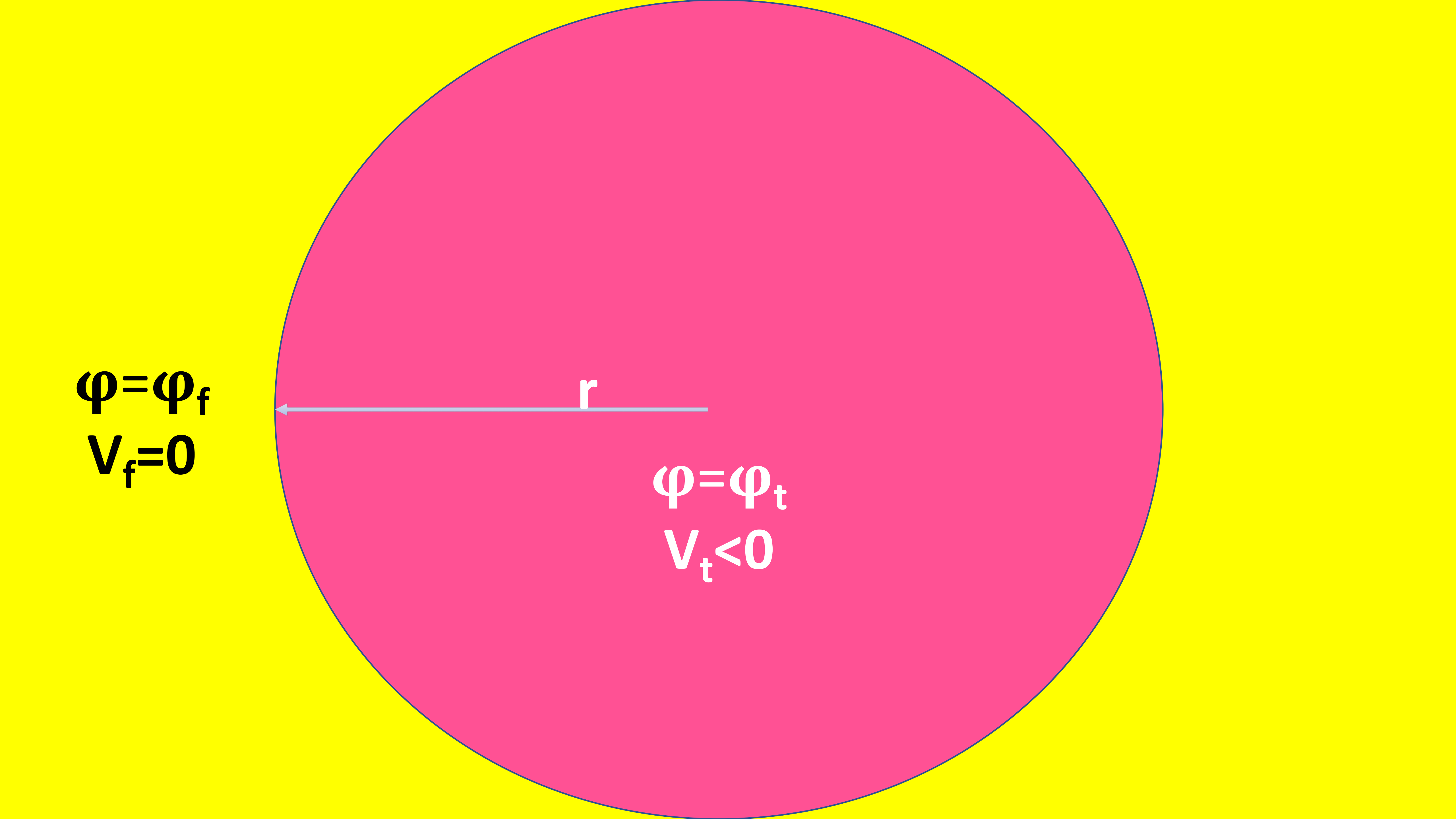

r
φ=φf
Vf=0
φ=φt
Vt<0



φ=φf
Vf=0
r
φ=φt
Vt<0

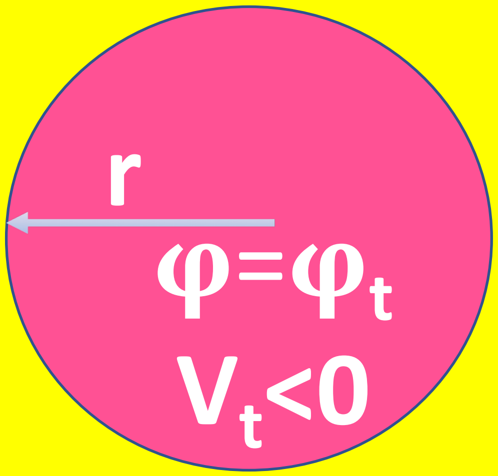
$$\varphi = \varphi_t$$

$$V_t < 0$$

Heuristic approach

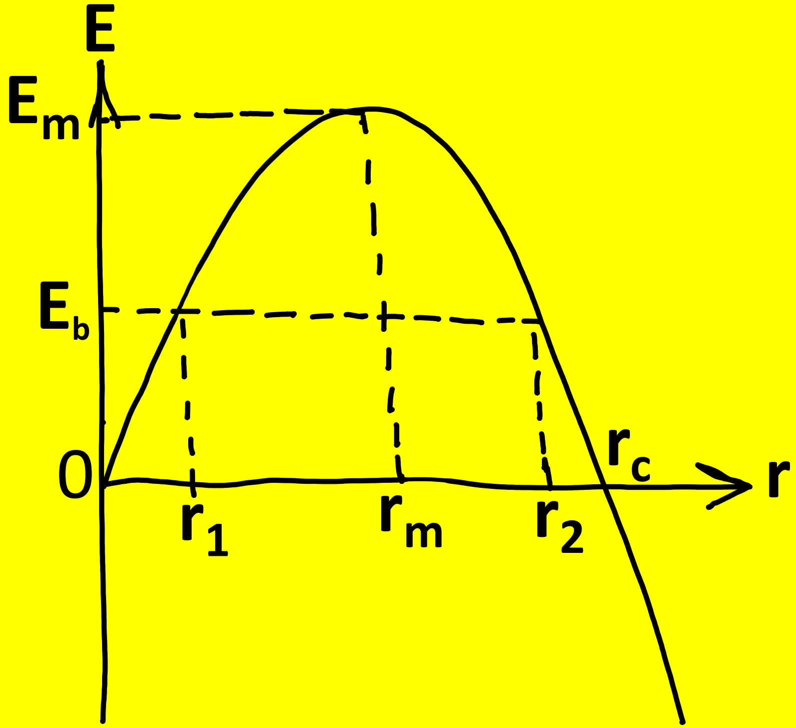
$\varphi = \varphi_f$

$V_f = 0$



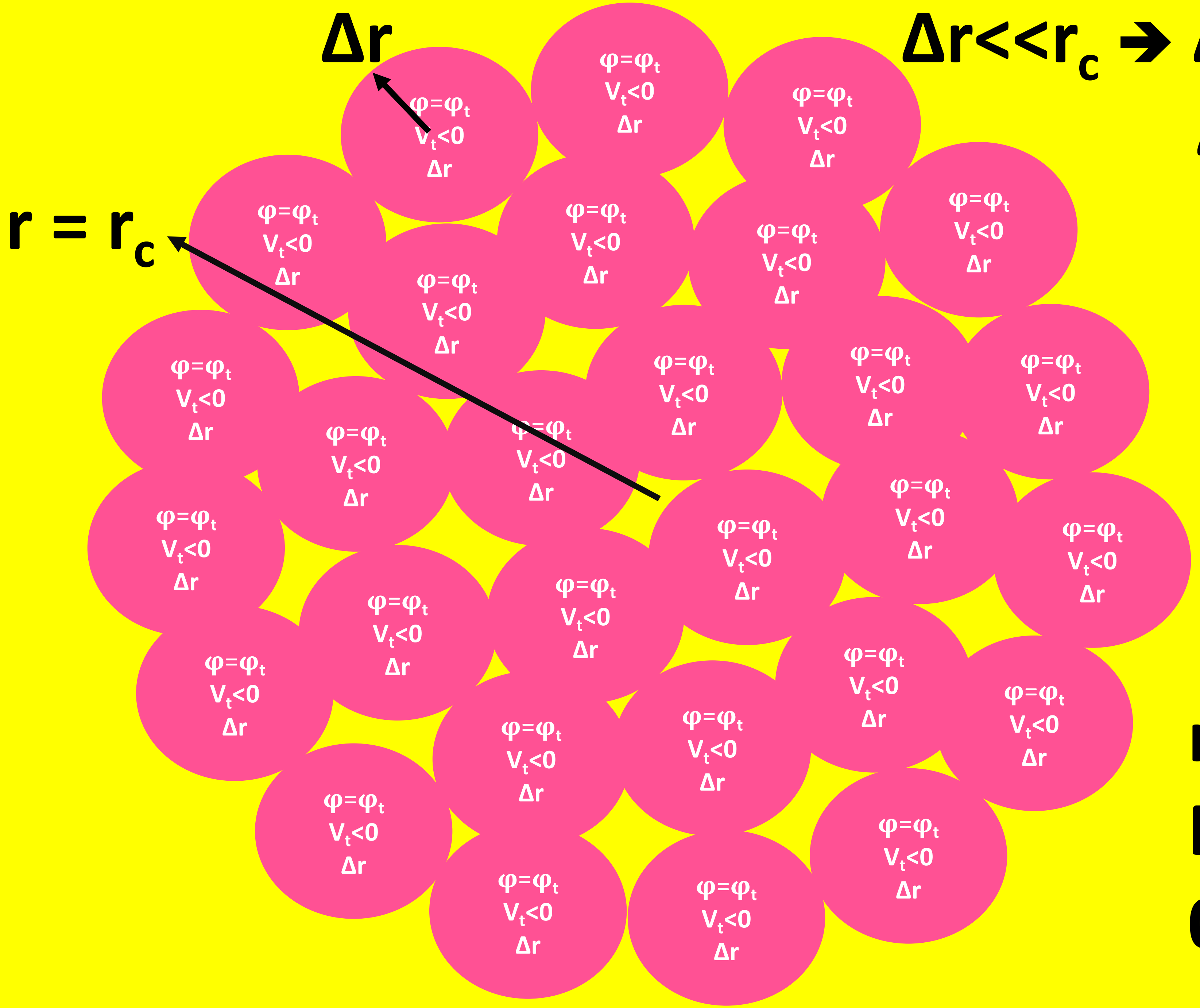
$$E_b \sim r^3 V_t + r^2 \sigma$$

$$E_b \sim 0 \longrightarrow r \sim r_c \equiv -\sigma / V_t$$



$\varphi=\varphi_f$

$V_f=0$



$\Delta r \ll r_c \rightarrow \Delta E \sim \sigma\, \Delta r^2$

$\Delta r\, \Delta E \sim 1$

$\Delta r \sim 1/\sigma^{1/3}$

$r_c \sim \sigma/|V_t|$

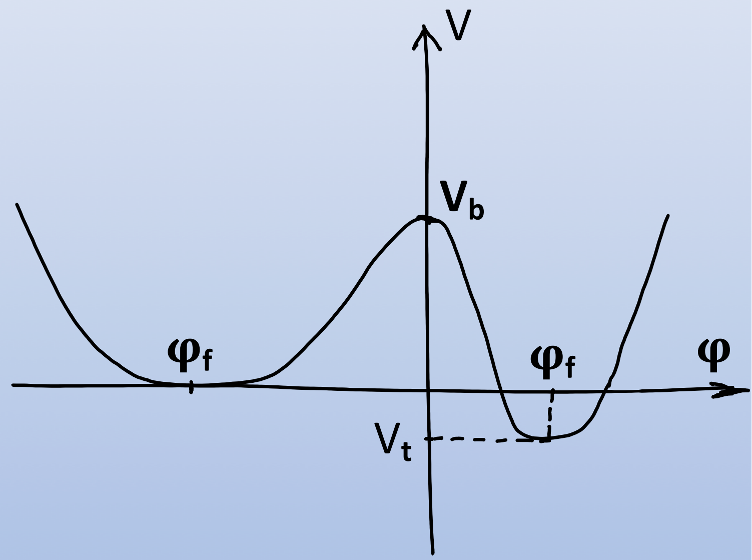
$n \sim r_c^{\,3}/\Delta r^3$

$P \sim p^n$

$0 < p \equiv e^{-a} < 1$

$$n \sim \sigma^4/|V_t|^3 >> 1$$

$$P \sim \exp(-a\, \sigma^4/|V_t|^3) << 1$$



In the thin-wall approximation:

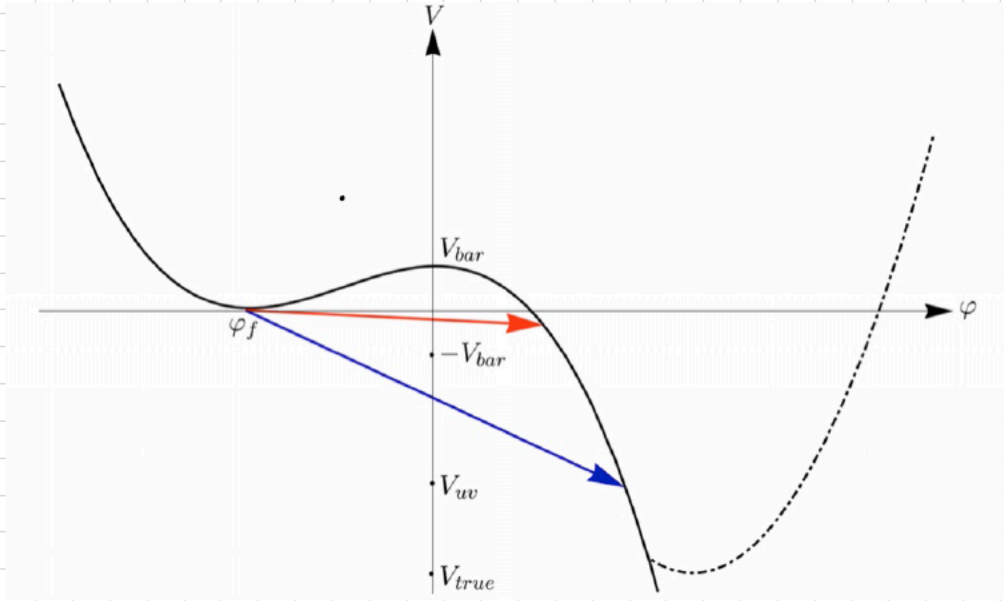
$$\mathbf{a = 27\,\pi/2}$$

$$V_b \gg |V_t|$$

I.Yu. Kobzarev, L.B. Okun, M.B. Voloshin

Sov.J.Nucl.Phys. 20 (1975) 644-646, *Yad.Fiz.* 20 (1974) 1229-1234

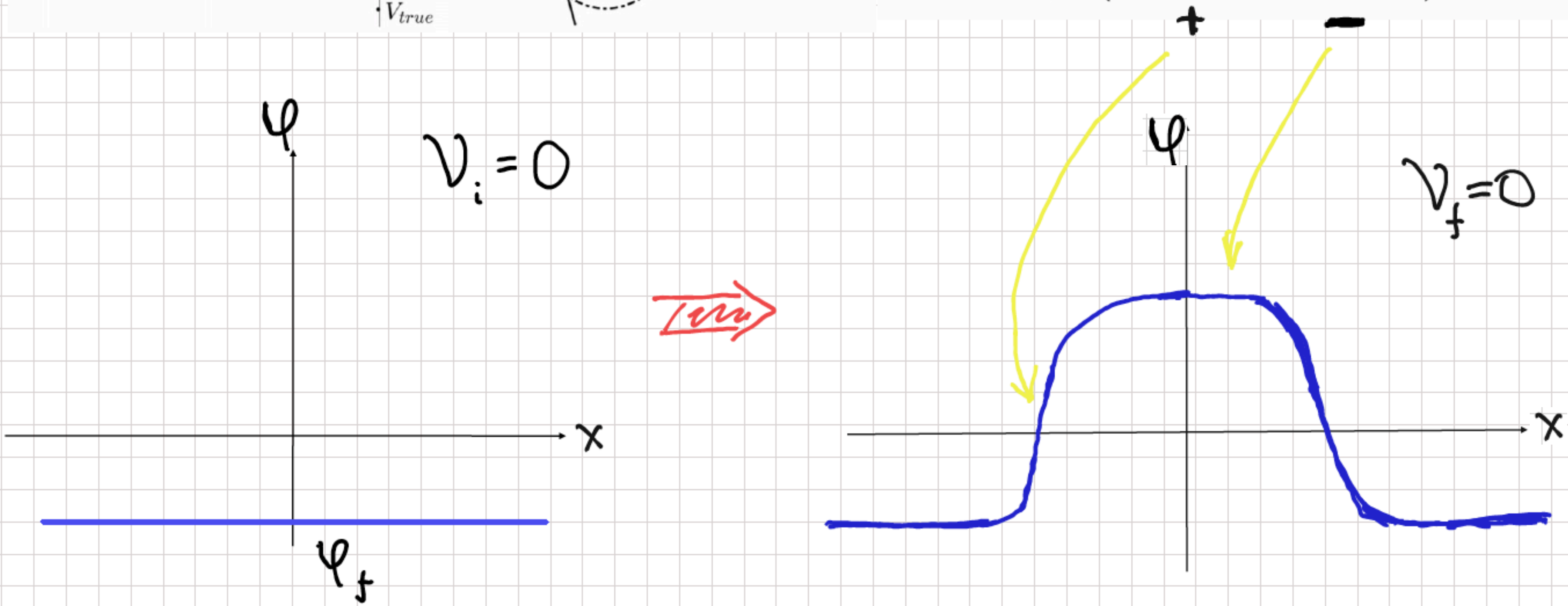
The false vacuum decay



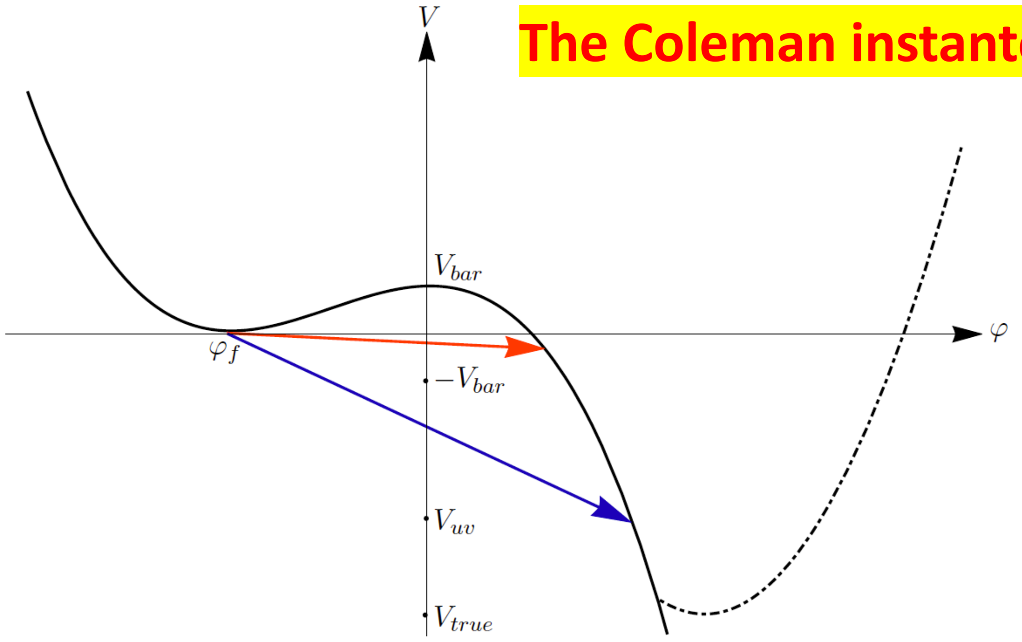
$$S = \int (\mathcal{K} - \mathcal{V})dt\,,$$

$$\mathcal{K} \equiv \frac{1}{2}\int (\partial_t \varphi_{\mathbf{x}})^2 d^3x\,,$$

$$\mathcal{V} \equiv \int \left(\frac{1}{2}(\partial_i \varphi_{\mathbf{x}})^2 + V(\varphi_{\mathbf{x}})\right) d^3x\,.$$



The Coleman instanton (*Phys.Rev.D* 15 (1977) 2929): quasiclassical approximation



$$S_E = \int \left(\frac{1}{2} (\partial_\tau \varphi)^2 + \frac{1}{2} (\partial_i \varphi)^2 + V(\varphi) \right) d^3x d\tau$$

$$\partial_\tau^2 \varphi + \Delta\varphi - V' = 0 \,,$$

$$\varphi(\tau \to -\infty, \mathbf{x}) = \varphi_f$$

$$\varphi(\tau = 0, \mathbf{x}) \text{ with } \partial\varphi/\partial\tau = 0 \text{ and } \mathcal{V}(\varphi(\mathbf{x})) = 0$$

The false vacuum decay rate: $\Gamma \simeq \varrho_0^{-4} \exp(-S_E)$

$O(4)$-invariant solution: φ depends only on $\varrho = \sqrt{\tau^2 + \mathbf{x}^2}$

$$S_E = 2\pi^2 \int_0^{+\infty} d\varrho\, \varrho^3 \left(\frac{1}{2} \dot{\varphi}^2 + V(\varphi) \right)$$

$$\ddot{\varphi}(\varrho) + \frac{3}{\varrho} \dot{\varphi}(\varrho) - V' = 0$$

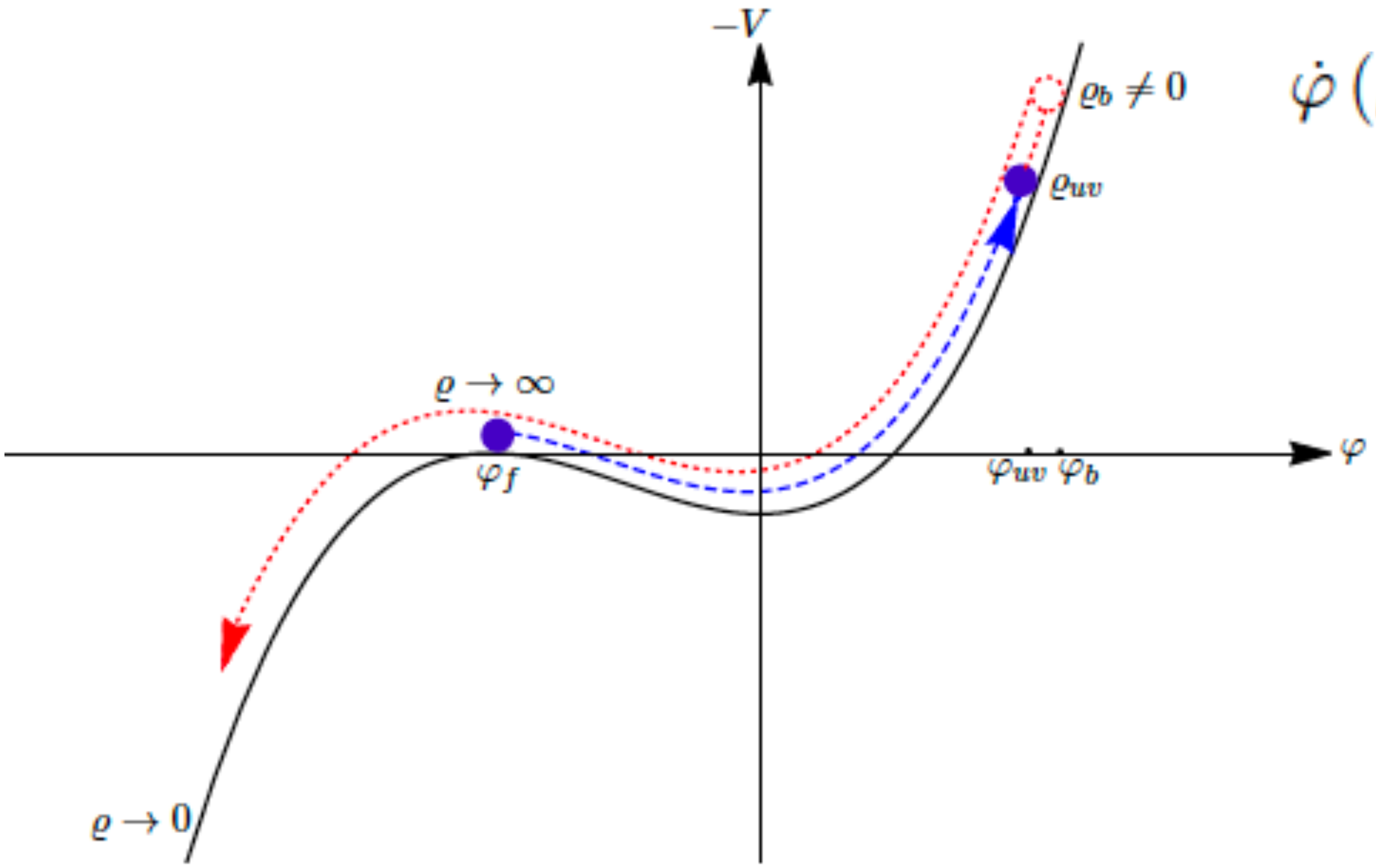
with boundary conditions:

$$\varphi(\varrho \to \infty) = \varphi_f$$
$$\dot{\varphi}(\varrho = 0) = 0$$

$$\ddot{\varphi}(\varrho) + \frac{3}{\varrho}\dot{\varphi}(\varrho) - V' = 0$$

$$\varphi\left(\varrho \to \infty\right) = \varphi_f$$

$$\dot{\varphi}(\varrho = 0) = 0$$

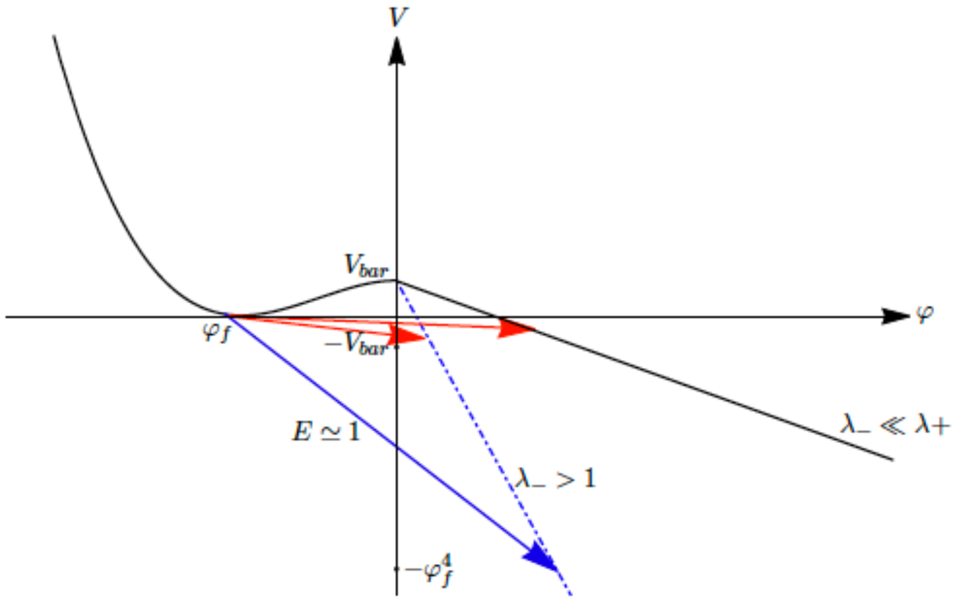


$$\dot{\varphi}\left(\varrho_b\right) = 0$$

The two puzzles with the Coleman instanton

1. The too fast false vacuum decay

V.F. Mukhanov, E. Rabinovici, and A.S.S., Fortsch. Phys. 69 (2021) 2, 2000100 [arXiv:2009.12445]



$$V(\varphi) = \begin{cases} \lambda_- \varphi_0^3 \varphi + \frac{\lambda_+}{4}\varphi_0^4 & \text{for } \varphi < 0\,, \\ \frac{\lambda_+}{4}(\varphi - \varphi_0)^4 & \text{for } \varphi > 0\,. \end{cases}$$

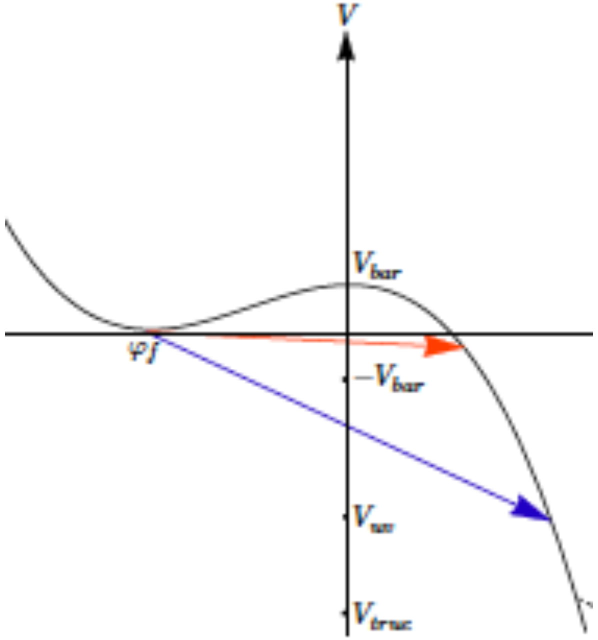
$\lambda_- \gg 1$ (*zero size instanton problem*)

$\varrho_0 << 1$, $S_E << 1$, $\Gamma >> 1$

The quasiclassical approximation is not trustable!

2. There is no Coleman instanton at all

V.F. Mukhanov, E. Rabinovici and A.S.S., Fortsch. Phys. 69 (2021) 2, 2000101 [arXiv:2009.12444]



$$V(\varphi) = \begin{cases} \frac{\lambda_+}{4}(\varphi - \varphi_0)^4 & \text{for } \quad \varphi > \beta\,\varphi_0 \\ -\frac{\lambda_-}{4}(\varphi^4 - \beta^3\varphi_0^4) & \text{for } \quad \varphi < \beta\,\varphi_0\,, \end{cases}$$

$$\beta = \frac{\lambda_+^{1/3}}{\lambda_+^{1/3} + \lambda_-^{1/3}}\,,$$

The nonlocal integrals of motion

$$E(\alpha) = \varrho^{\frac{4}{\alpha-2}}\left(\frac{1}{2}\varrho^2\dot{\varphi}^2 + \frac{2}{\alpha-2}\varrho\varphi\dot{\varphi} - \varrho^2 V - \frac{2(\alpha-4)}{(\alpha-2)^2}\varphi^2\right)$$
$$+\frac{2}{\alpha-2}\int_0^{\varrho} d\overline{\varrho}\,\overline{\varrho}^{\frac{6-\alpha}{\alpha-2}}\left[(\alpha-4)\left(\overline{\varrho}\dot{\varphi}+\frac{2}{\alpha-2}\varphi\right)^2 + \overline{\varrho}^2(\alpha V - \varphi V')\right],$$

V.F. Mukhanov and A.S.S.,
Phys.Lett. B827 (2022) 136951, 2111.13928 [hep-th]

$$\frac{dE}{d\varrho} = \varrho^{\frac{\alpha+2}{\alpha-2}}\left(\ddot{\varphi}(\varrho) + \frac{3}{\varrho}\dot{\varphi}(\varrho) - V'\right)\left(\varrho\dot{\varphi} + \frac{2}{\alpha-2}\varphi\right)$$

Theorem 1. (S R. Coleman, V. Glaser, A. Martin, *Commun.Math.Phys.* 58 (1978) 211-221)
The Coleman instanton exists in D-dimensions, if for the continuous differentiable potential V (φ) with a local minimum at φ = 0 there exist positive numbers a, b, α and β, such that

$$\beta < \alpha < 2D/(D-2)$$

$$V(\varphi) \geq a\,|\varphi|^{\beta} - b\,|\varphi|^{\alpha}$$

Theorem 2. (V.F. Mukhanov and A.S.S., *Phys.Lett.* B827 (2022) 136951, 2111.13928 [hep-th])
If potential V (φ) has a maximum at φ = 0 and has no any local minima at positive φ (unbounded from below) and satisfies the inequality

$$V(\varphi) < a\,|\varphi|^{\beta} - b\,|\varphi|^{\alpha}$$

then the Coleman instanton, which is supposed to describe the decay of the false vacuum at the absolute local minimum at $\varphi_f < 0$, does not exist regardless of the form of the potential at negative φ.

There exists a broader class of unbounded potentials for which the instantons with the Coleman boundary conditions do not exist: for any unbounded potential, which for positive φ can be represented as

$$V(\varphi) = -\varphi^{\alpha} \int^{\varphi} d\bar{\varphi}\, v'_{\alpha}(\bar{\varphi}), \qquad \alpha \geq \frac{2D}{D-2}, \qquad v'_{\alpha} \geq 0,$$

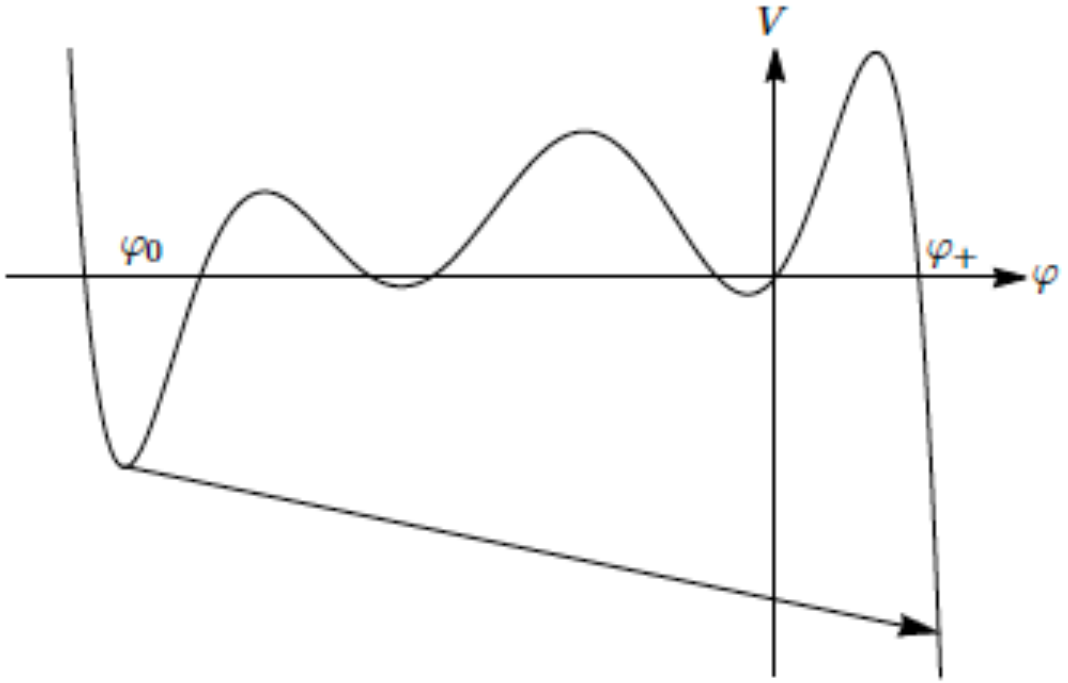
the Coleman instanton does not describe the decay of the deepest false vacuum at $\varphi_f < 0$ regardless of the form of the potential for negative values of φ.

The Coleman instantons do not exist: examples of the potentials

$$V(\varphi) = -\varphi^{\alpha}\int^{\varphi} d\bar{\varphi}\, v'_{\alpha}(\bar{\varphi})\,, \quad v'_{\alpha} = a\,\varphi^{-\alpha}\prod_{i=1}^{\alpha-2(m+1)}(\varphi-\lambda_i)\prod_{j=1}^{m}\left((\varphi+\beta_j)^2+\gamma_j^2\right)$$, where a > 0, $\gamma_j\,, \lambda_i \le 0$, $\alpha \ge 4$

$$V(\varphi) = \Lambda\varphi^4 + a\left(\varphi^3+\beta\,\varphi^2+\frac{\beta^2+\gamma^2}{3}\varphi\right)$$, Λ < 0 ;

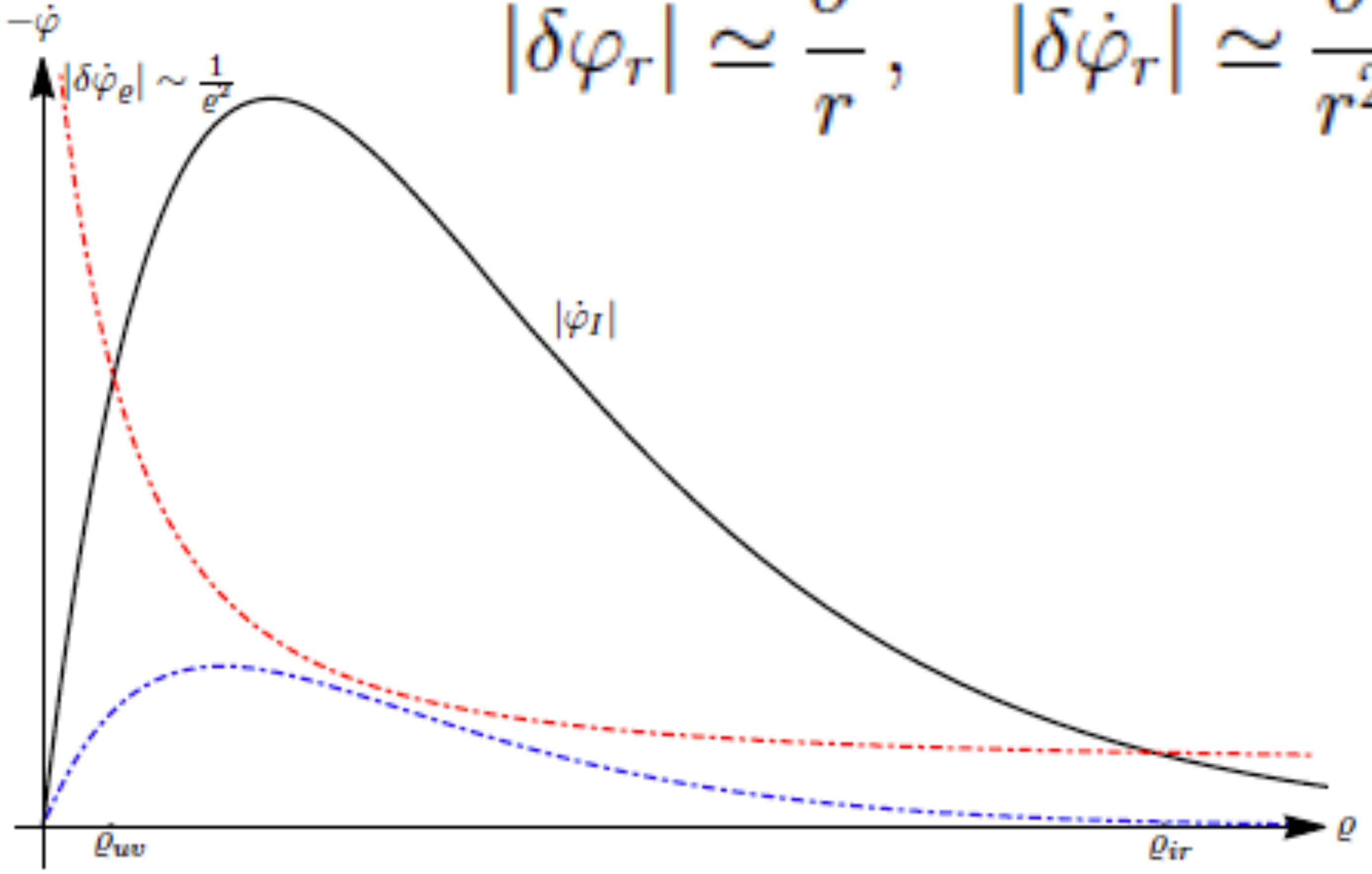
$$V = -\frac{\lambda}{4}\varphi^4 \ln\left(\frac{\varphi}{\mu}\right)$$, λ > 0



V.F. Mukhanov and A.S.S., JCAP 10 (2021) 066, [arXiv:2104.12661]

The resolution of the Coleman instanton puzzles: quantum fluctuations

$$|\delta\varphi_r| \simeq \frac{\sigma}{r}, \quad |\delta\dot{\varphi}_r| \simeq \frac{\sigma}{r^2}$$



ϱ_{uv} and ϱ_{ir} are two different solutions of the equation: $\dot{\varphi}_I(\varrho) \simeq \frac{\sigma}{\varrho^2}$

New instantons with a quantum core

$$\ddot{\varphi} + \frac{D-1}{\varrho}\,\dot{\varphi} - V' = 0,$$

$$\varphi\,(\varrho \to \infty) = \varphi_f\,,$$
$$\dot{\varphi}\,(\varrho = \varrho_b) = 0\,,$$

ϱ_b is an arbitrary new free parameter

$$|\dot{\varphi}_{\rm uv}| = \frac{\sigma\,(D-2)}{2}\;\varrho_{\rm uv}^{-\frac{D}{2}}\,, \quad |\dot{\varphi}_{\rm ir}| = \frac{\sigma\,(D-2)}{2}\;\varrho_{\rm ir}^{-\frac{D}{2}}$$

$$S_I = \frac{2\,\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)}\left(\int_{\varrho_{uv}}^{\varrho_{ir}} d\varrho\,\varrho^{D-1}\left(\frac{1}{2}\,\dot{\varphi}^2 + V(\varphi)\right) + \frac{\varrho_{uv}^{D}}{D}\,V_{uv}\right)$$

The false vacuum decay rate: $\Gamma \simeq \varrho_0^{-D}\,\exp\left(-S_I\right)$, $\varphi(\varrho_0) = 0$

With the precision allowed by the uncertainty relatior $\varrho_{\rm uv}\,\mathcal{V} \simeq O(1)$ the potential energy

$$\mathcal{V} = \frac{2\,\pi^{\frac{D-1}{2}}}{\Gamma\left(\frac{D-1}{2}\right)}\left(\int_{\varrho_{uv}}^{\varrho_{ir}} d\varrho\,\varrho^{D-2}\left(\frac{1}{2}\,\dot{\varphi}^2 + V(\varphi)\right) + \frac{\varrho_{uv}^{D-1}}{D-1}\,V_{uv}\right),$$

vanishes and the bubble with the quantum core emerges from under the barrier.

The friction dominated new instantons

$$\ddot{\varphi} + \frac{D-1}{\varrho}\,\dot{\varphi} \simeq 0$$

$$\varphi(\varrho) = \varphi_f + \frac{E}{(D-2)^2\,|\varphi_f|\,\varrho^{D-2}} \quad \rightarrow \quad \varrho(\varphi) = \left(\frac{E}{(D-2)^2\,|\varphi_f|\,(\varphi-\varphi_f)}\right)^{\frac{1}{D-2}}$$

E is the parameter, which can be expressed in terms of ϱ_b and vice versa.

$$V_{\rm fr}(\varphi) \equiv \frac{D-1}{\varrho}|\dot{\varphi}| \equiv -\frac{1}{2}\,(D-2)^{\frac{2D}{D-2}}\left(\frac{|\varphi_f|}{E}\right)^{\frac{2}{D-2}}(\varphi-\varphi_f)^{\frac{2(D-1)}{D-2}} \leq 0$$

$$\ddot{\varphi} + \frac{D-1}{\varrho}\,\dot{\varphi} \simeq 0 \quad \rightarrow \quad \frac{1}{2}\,\dot{\varphi}(\varrho)^2 + V_{\rm fr}(\varphi) = 0\,.$$

$$\ddot{\varphi} + \frac{D-1}{\varrho}\dot{\varphi} - V' = 0 \rightarrow \ddot{\varphi} + U'_{\rm eff}(\varphi) = 0\,, \quad \rightarrow \quad \frac{1}{2}\dot{\varphi}^2 + U_{\rm eff} = 0$$

$$U_{\rm eff} = V_{\rm fr} - V$$

The value of the scalar field at which its velocity vanishes satisftes: $U_{\rm eff}(\varphi) = 0 \rightarrow V_b \equiv V(\varphi_b) \simeq V_{\rm fr}(\varphi_b)$

If we assume that $\dot{\varphi}$ at the location of the maximum of the potential $V(\varphi = 0) = V_{\rm bar}$ is determined by the friction term, then

$$|V_{\rm fr}(\varphi = 0)| \gg V_{\rm bar}\,, \quad \rightarrow \quad 1 \ll E \ll (D-2)^D |\varphi_f|^D\ (2V_{\rm bar})^{\frac{2-D}{2}}$$

$$|V_{\rm fr}(\varphi_b)| \geq |V_{\rm fr}(0)| \quad \rightarrow \quad |V_b| \gg V_{\rm bar}\ !$$

Thus, the tunnelling depth is much larger than the height of the potential barrier, which corresponds to the thick-wall instantons.

The friction dominated new instantons: the thick-wall approximation

$$S_{\text{I}} = \frac{\alpha \pi^{\frac{D}{2}} E \left(\varphi_b - \varphi_f\right)}{\Gamma\left(\frac{D+2}{2}\right)(D-2)\left|\varphi_f\right|} = \frac{\alpha\,(D-2)^{D-1}\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D+2}{2}\right)} \frac{\left(\varphi_b + \left|\varphi_f\right|\right)^D}{\left|2V_b\right|^{\frac{D-2}{2}}},$$

$$\varrho_0 = \left(\frac{E}{(D-2)^2\,\varphi_f^2}\right)^{\frac{1}{D-2}} = \frac{(D-2)}{\sqrt{|2V_b|}} \left(1 + \frac{\varphi_b}{|\varphi_f|}\right)^{\frac{D-1}{D-2}} |\varphi_f|$$

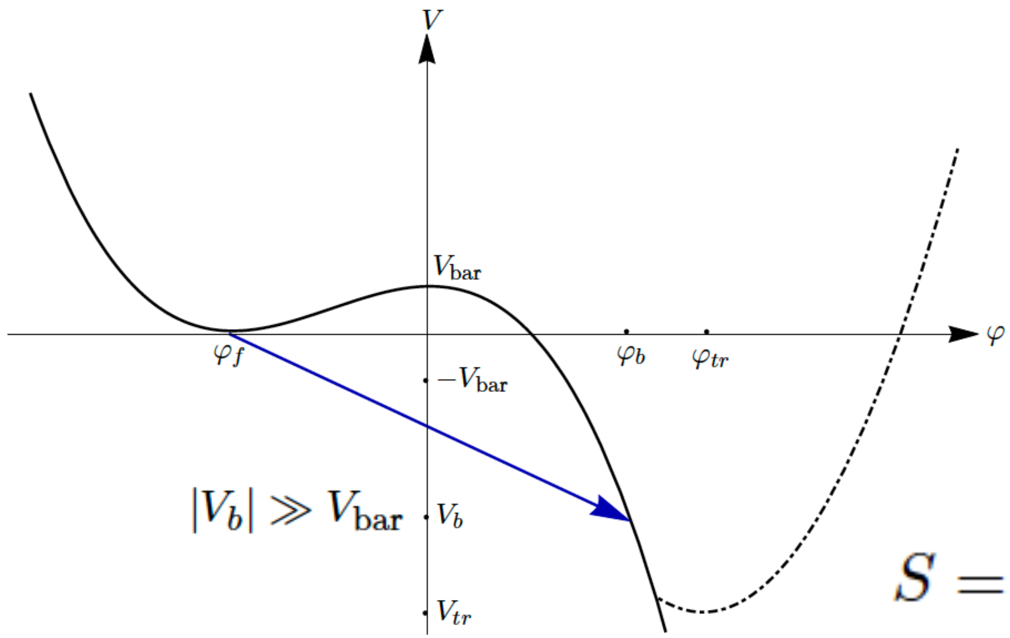
$$V\left(\varphi_b\right) = V_{\text{fr}}\left(\varphi_b\right)$$

$$V\left(\varphi_b^{\max}\right) \simeq -\frac{1}{2}(D-2)^{\frac{2D}{D-2}} \left(1 + \frac{\varphi_b^{\max}}{|\varphi_f|}\right)^{\frac{2(D-1)}{D-2}} |\varphi_f|^{\frac{2D}{D-2}}$$

The condition under which the thick-wall approximation is applicable:

$$\left(1 + \frac{\varphi_b}{|\varphi_f|}\right)^{\frac{2(D-1)}{D-2}} V_{\text{bar}} \ll |V_b| \ll \frac{1}{2}(D-2)^{\frac{2D}{D-2}} \left(1 + \frac{\varphi_b}{|\varphi_f|}\right)^{\frac{2(D-1)}{D-2}} |\varphi_f|^{\frac{2D}{D-2}}$$

Example for D=4



$$V(\varphi) = \begin{cases} \frac{\lambda_+}{4}(\varphi - \varphi_f)^4 & \text{for } \varphi < \beta\varphi_f \\ -\frac{\lambda_-}{n}\varphi_f^4\left(\frac{\varphi}{\varphi_f}\right)^n + V_{bar} & \text{for } \varphi > \beta\varphi_f \end{cases}$$

$$\beta \equiv \frac{\lambda_+^{\frac{1}{3}}}{\lambda_+^{\frac{1}{3}} + \lambda_-^{\frac{1}{3}}}, \quad V_{\rm bar} \equiv \frac{\lambda_-}{4}\beta^3\varphi_f^4$$

$$S = \frac{8\pi^2\alpha}{\lambda_-}\left(1 + \frac{|\varphi_f|}{\varphi_b}\right)^4, \quad \varrho_0 = \sqrt{\frac{8}{\lambda_-}}\left(1 + \frac{\varphi_b}{|\varphi_f|}\right)^{3/2}\frac{|\varphi_f|}{\varphi_b^2}$$

α =1 for $\varphi_b \ll |\varphi_f|$ and α =1/3 for $\varphi_b \gg |\varphi_f|$.

$$V\left(\varphi_b^{\max}\right) = \left(\frac{64}{\lambda_-}\right)^3 \varphi_f^4$$

$$\left(1 + \frac{\varphi_b}{|\varphi_f|}\right)^3 V_{\rm bar} \ll \frac{1}{4}\lambda_-\varphi_b^4 \ll 8\left(1 + \frac{\varphi_b}{|\varphi_f|}\right)^3 |\varphi_f|^4$$

or

$$1 \ll E \ll (D-2)^D|\varphi_f|^D\,(2V_{\rm bar})^{\frac{2-D}{2}}$$

$$V(\varphi_b) = V_{\rm fr}(\varphi_b) \quad \rightarrow \quad \left(1 + \frac{|\varphi_f|}{\varphi_b}\right)^3 \frac{|\varphi_f|}{\varphi_b} = \frac{\lambda_- E}{32}$$

Conclusions

The Coleman boundary condition $\dot{\varphi}(\varrho = 0) = 0$ for O(4) instantons must be abandoned due to quantum fluctuations which induce UV-cutoff determined by the instanton parameters.

This cutoff regularizes the original singular solutions, thus there is an infinite class of new nonsingular instantons with a quantum core which contribute to a false vacuum decay.

For potentials unbounded from below or having a true vacuum with a depth exceeding the barrier height, the new instantons, which provide the tunneling, are dominated by the friction term in the instanton equation and the corresponding true-vacuum bubbles have thick walls.

Then, one can replace the non-autonomous instanton equation by the autonomous completely solvable equation, which is a good approximation for the original one, and there exist the general formulae for the false-vacuum decay rate for arbitrary potentials in any number of dimensions.

Thank you for attention!