A Simple Model of 1/f Fluctuations from Amplitude Modulation and Demodulation

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- Origin of 1/f fluctuations is **the beat of waves with accumulating frequencies (Amplitude Modulation, AM)**

- The frequency accumulation spontaneously arises from **synchronization, resonance, and infrared divergence, etc.**

- Any **demodulation (DM) process** is inevitable for the appearance of 1/f fluctuations: this provides us with variety of 1/f fl.

1/f fluctuation = 1/f noise = pink noise = PSD power -0.5 to -1.5

https://www.nature.com/articles/s41598-023-34816-2

1. 1/f fluctuation

◆ First observation of 1/f fl. at 1925

- The power spectrum (PSD) of the **voltage squared fluctuation** $\overline{V^2}$(obs.) **in the vacuum tube behaves 1/f on the low frequency side.** (f: frequency)
- JB Johnson, Phys. Rev. 26 (1925) 71.
- $P(\omega) = \left|\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt\right|^2$
- Or discrete version FFT, is calculated.

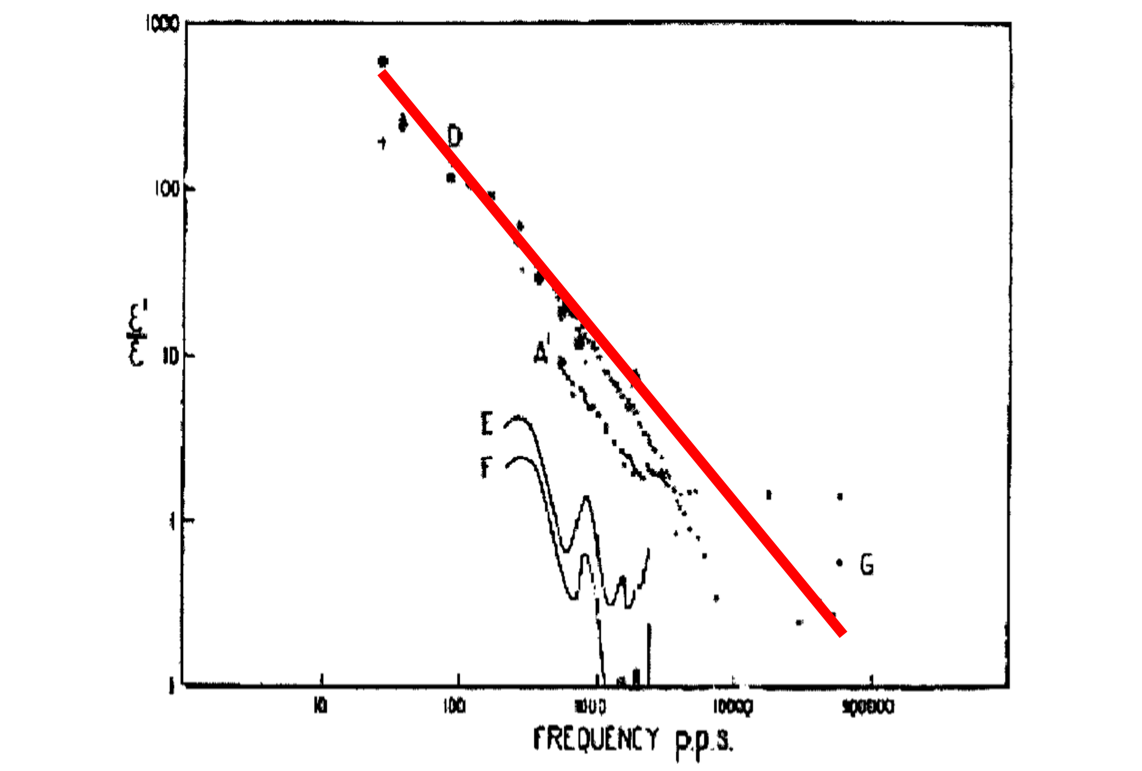
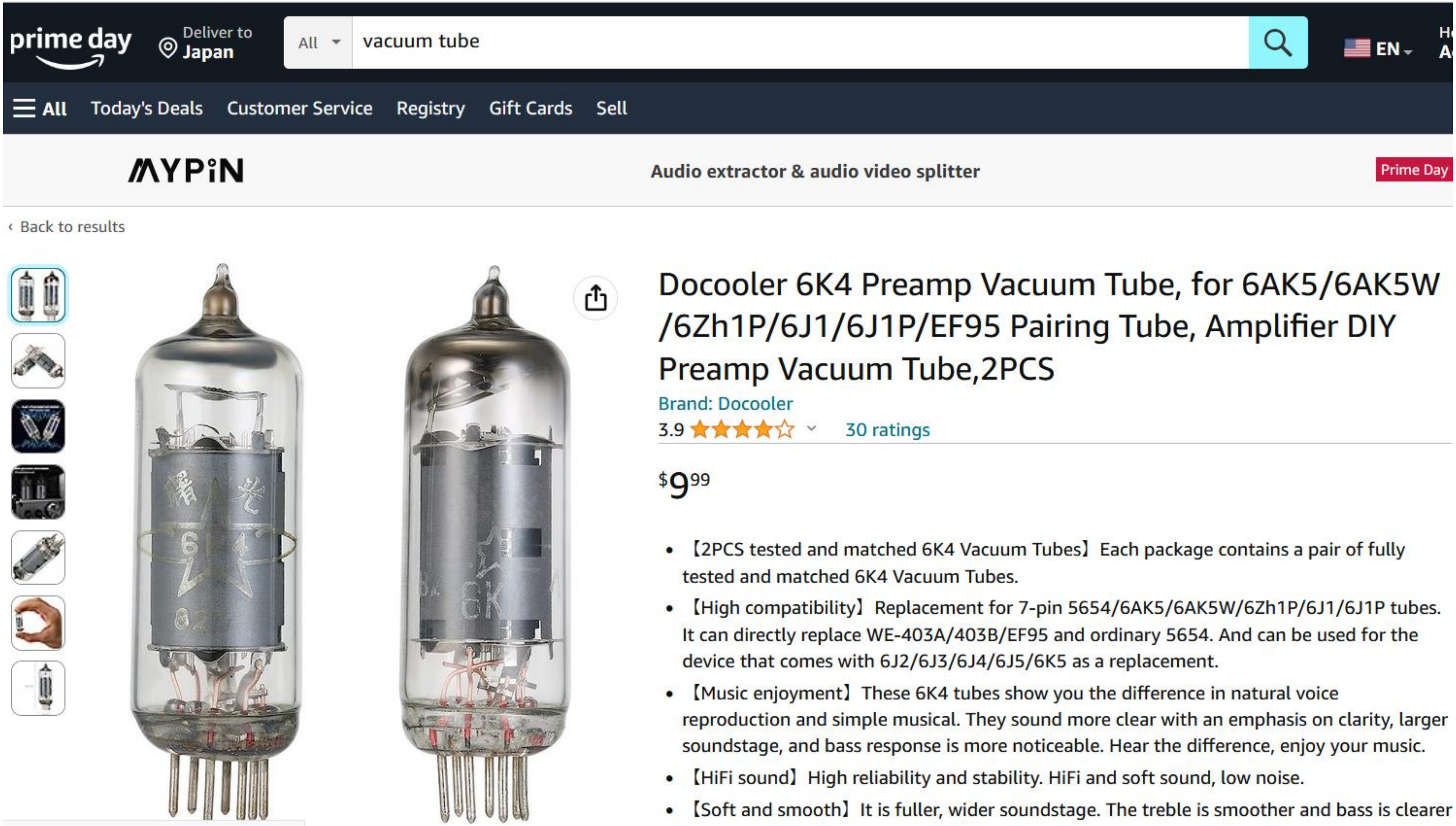
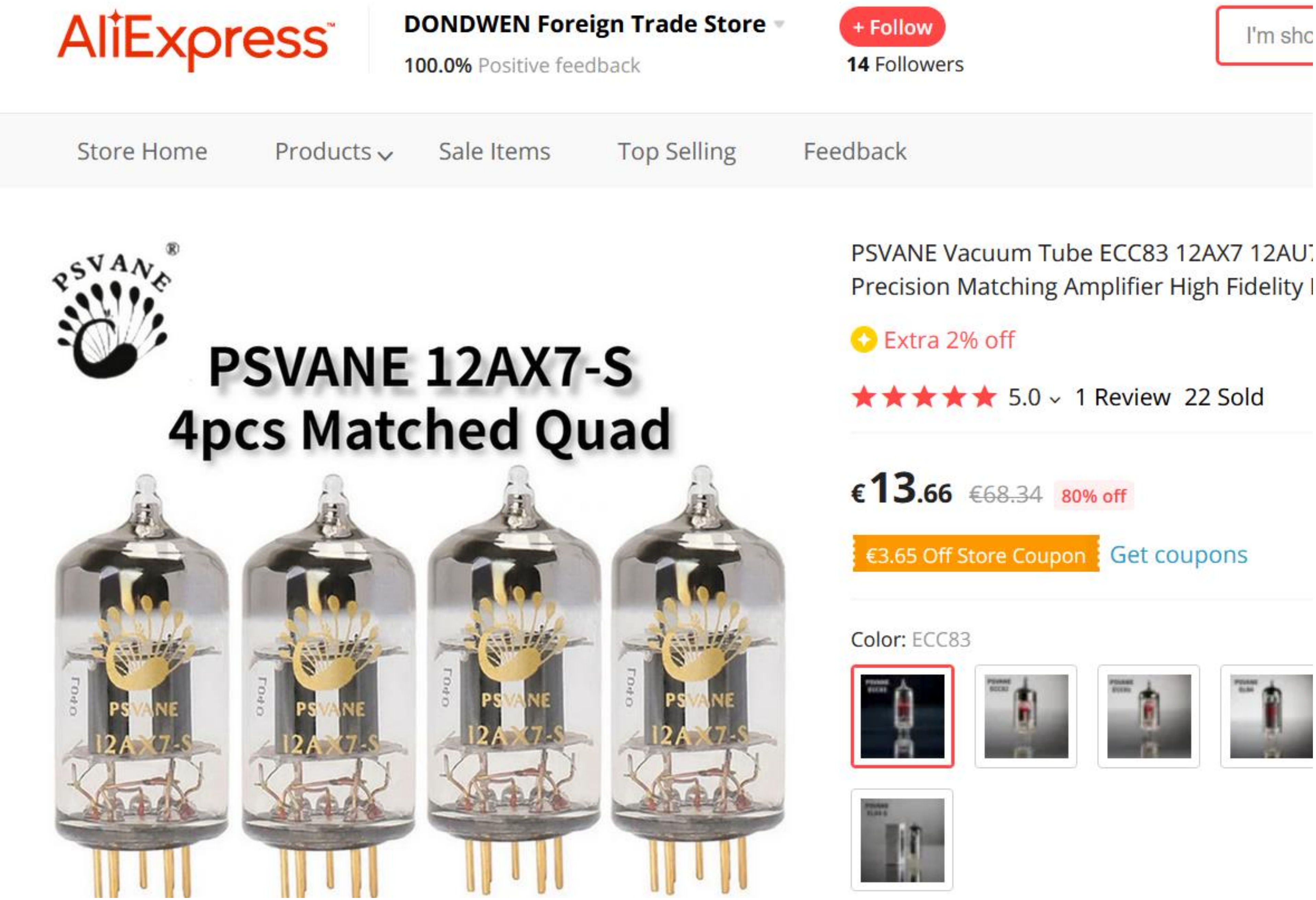


Fig. 6. Frequency variation for tube No. 2, coated filament; same data as in Fig. 4 plotted to a frequency scale; curves E and F give Hartmann's results for 2 m-a. and 20 m-a.; points G were obtained with less steady measuring circuit.

You can buy vacuum tubes from Amazon:



You can also buy them from AliExpress:



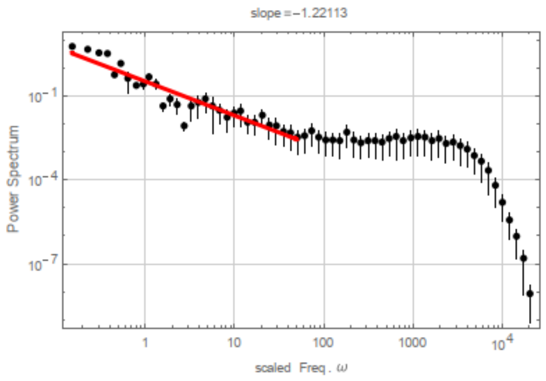
2. Universality and various theories

- 1/f fluctuation (pink noise) exists **everywhere** in nature

Semiconductors, thin metals, potential fluctuations in bio-membranes, current in electrolytes, crystal oscillators, high-stability frequency standard oscillators, oscillation frequency fluctuations, ultra-long-term temperature fluctuations, flow fluctuations on highways, magnitude of symphonies, Variations in the rotation speed of the Earth, variations in the intensity of cosmic rays, heart rate, postural control, MEG and EEG (brain), …,…

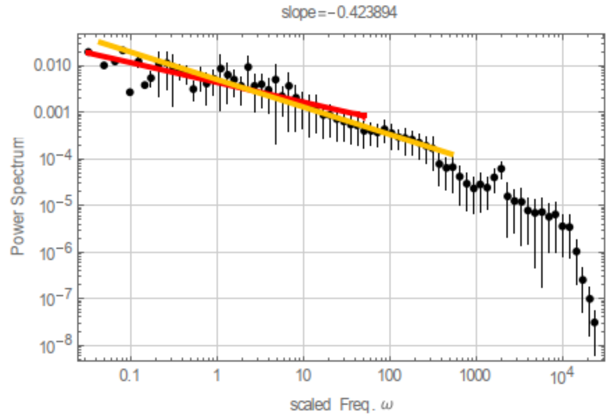
Infrared divergence of the primordial density fluctuations

◆チャイコフスキー 弦楽セレナード_小澤征爾 サイトウキネン



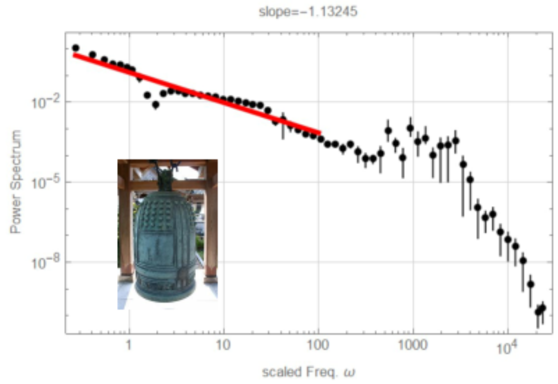
◆水琴窟京都宝泉院左筒

5

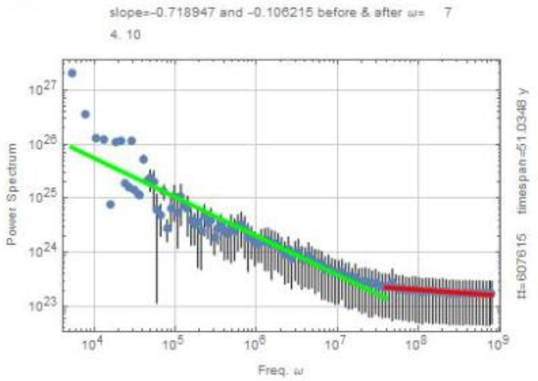


◆ 圓光寺（京都）の鐘の音

6

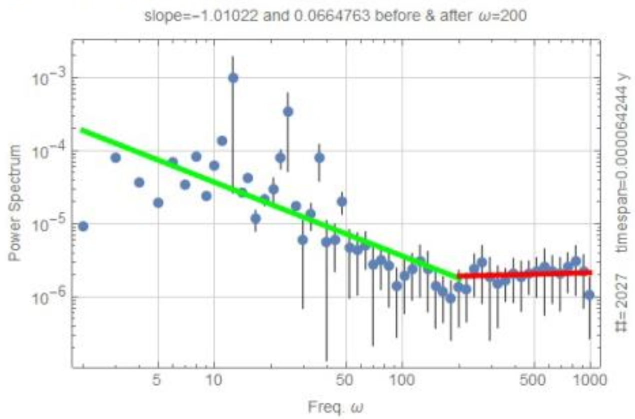


◆50年間の世界の地震時系列の PS



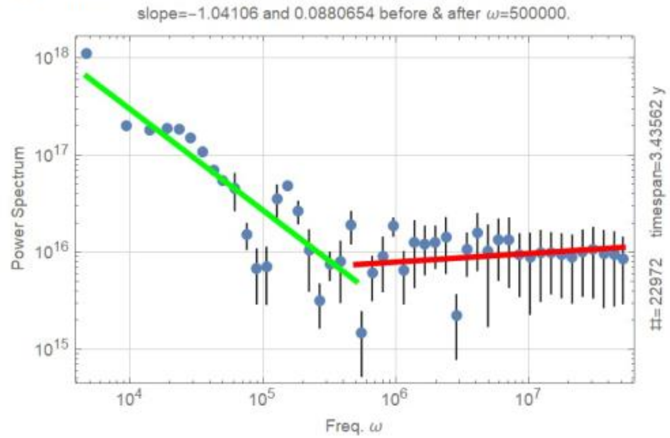
◆太陽フレア時系列

8

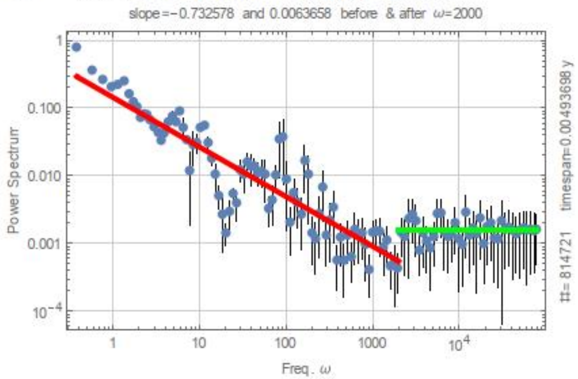


◆変光星 RT カリーナ

9



◆ Mira のライトカーブに対する PSD



◆ NO3 -アイスコア 南極 Motizuki2022

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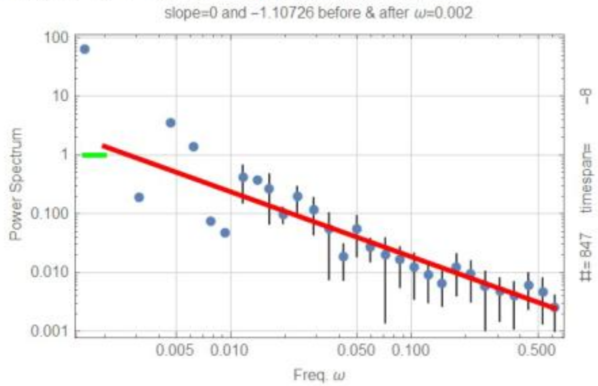
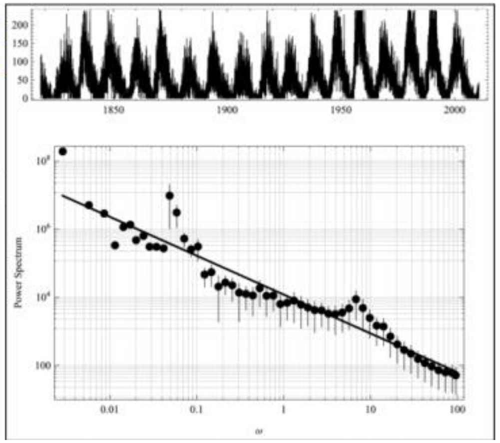


◆太陽黒点 結合スピンモデル中道

12

での 2 つのピークの上にインデックス -1.1 を持つべき乗則 ω≈ 0.05 および ω≒7。



◆ **Many theories** have been proposed. But, no decisive theory.

representative theories and limitations:

1. Flicker noise theory: **fluctuating resistance**
   J. B. Johnson1948.
2. **Surface defect** scattering theory:
   D. E. McCumber1969.
3. **Two-level systems** theory
   P. W. Anderson1992.
4. **Charge trapping** theory: trapping and de-trapping
   M. B. Weissman1988.
5. **Fractal geometry** theory:
   B. B. Mandelbrot1982.
6. **Burst noise** theory:
   E. A. Ash and G. Nicholls 1982.
7. **Self-organized criticality** theory:
   P. Bak1996.

8. Power-law **tunneling** model:
D. C. Mattis and M. L. Glasser1998.

Further:

Diffusion Limited Aggregation (DLA) Theory
Fractional Calculus Theory
Power-Law Noise Theory
Entropic Noise Theory
Quantum Tunneling Theory
⋯ and many more

However, none of them are universal explanations.

➡ We would like to propose a **universal simple physics.**

3. Some Important Hints of 1/f fl.

1. **waves** are often involved: sound waves, electric currents, air fluids,
   → 1/f fl. from wave interference?
2. **small system**: A very low frequency signal is emitted from a small system. In an extreme example [Liu2013] , the 1/f fluctuations generated from the 2.5 nm layer in the semiconductor film
   →1/f fl. may not be primary fluctuations.
3. **apparent long memory:**
   small semiconductors 1/f up to 10^{-7}Hz [Dukelov1974] voltage 1/f fl. semiconductor up to $10^{-6.3}$Hz [Caloyannides1974] …
   →The Wiener-Khinchin theorem has nothing to do with memory

$$S(\omega) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dt \langle x(t)x(t-\tau)\rangle e^{-2\pi i\omega\tau}$$

4. **nothing to do with dissipation**

Conservative Hamiltonian Mean Field (HMF) models also shows 1/f noise [Yamaguchi 2017]

→ Fluctuation dissipation theorem may not hold $\left\langle \delta x^2 \right\rangle \propto RkT$

5. **Data squared.**..

the original signal shows NO 1/f fl. but,

Square of it shows 1/f fl.

For music [Voss1977]... Loudness PSD

for HMF [Yamaguchi 2017]… M^2

➡ **Wave beat may cause 1/f noise**

4. Origin of 1/f Fl.: Amplitude Modulation

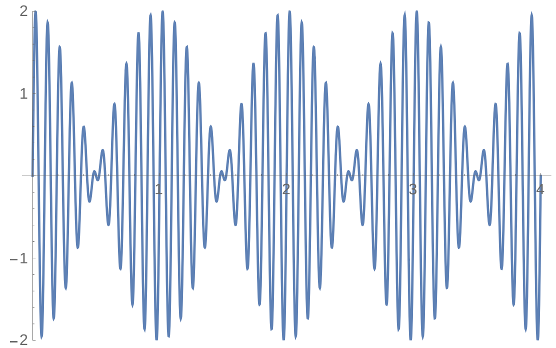
a) Amplitude modulation (AM) by waves with accumulating frequencies

b) Demodulation (DM) $\Rightarrow$ 1/f fluctuation

- **AM** :

$$\phi = \sin(\omega t + \lambda t) + \sin(\omega t + \lambda t)$$ … beat

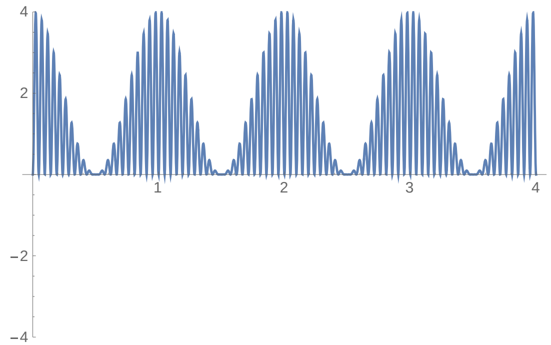
PSD of ϕ: two peaks in high freq.



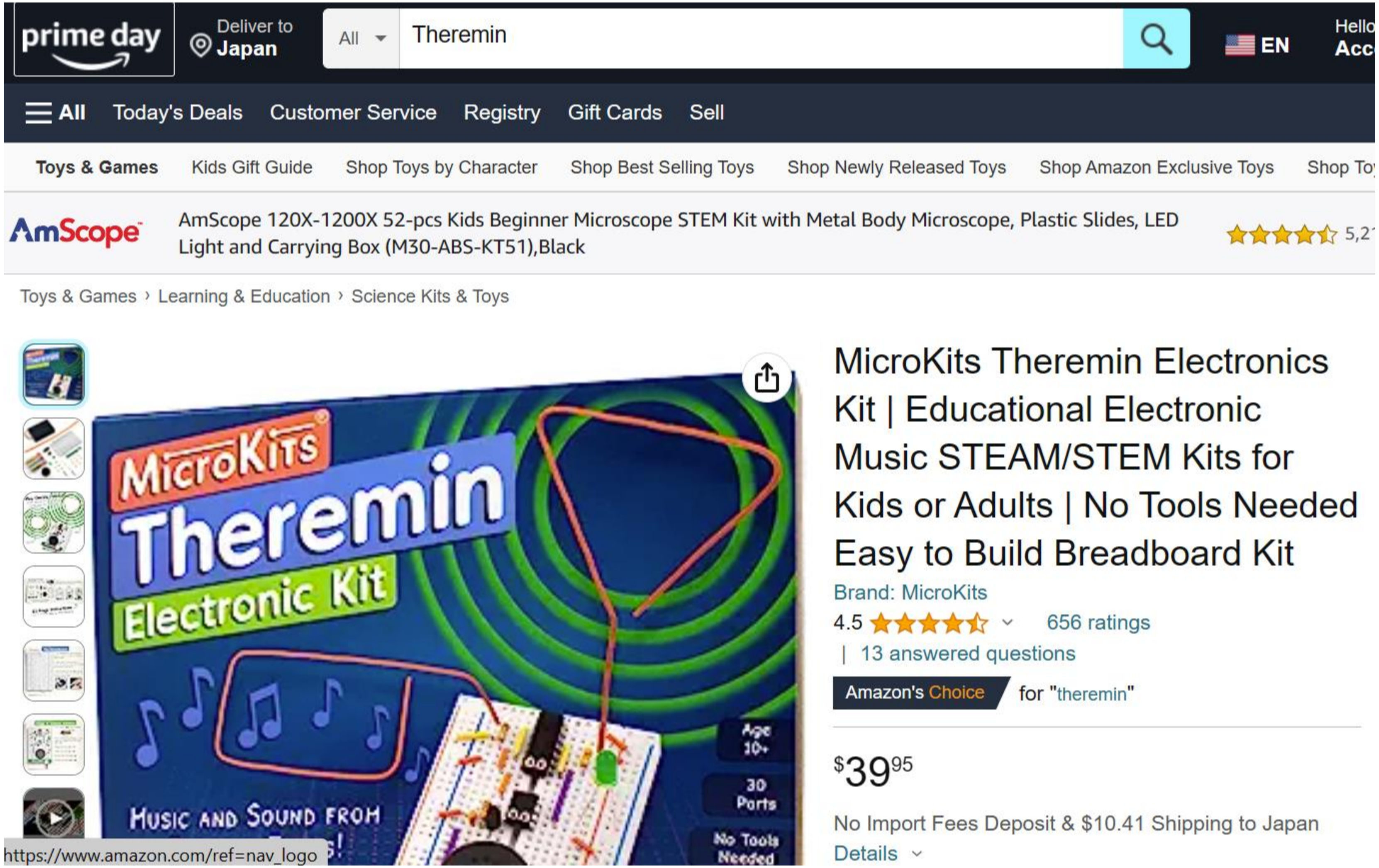
- **DM** : ex. data squared

PSD of ϕ^2: $\pm 2\lambda$ in low-freq.

A beat appears in PSD…1/f fluctuation



◆ Example of AM in music: Theremin



Theremine.mp4

high freq. 1000kHz + 999.560kHz → 440Hz (audible sound)

→ yields arbitrary low freq. signal with No memory, No dissipation.

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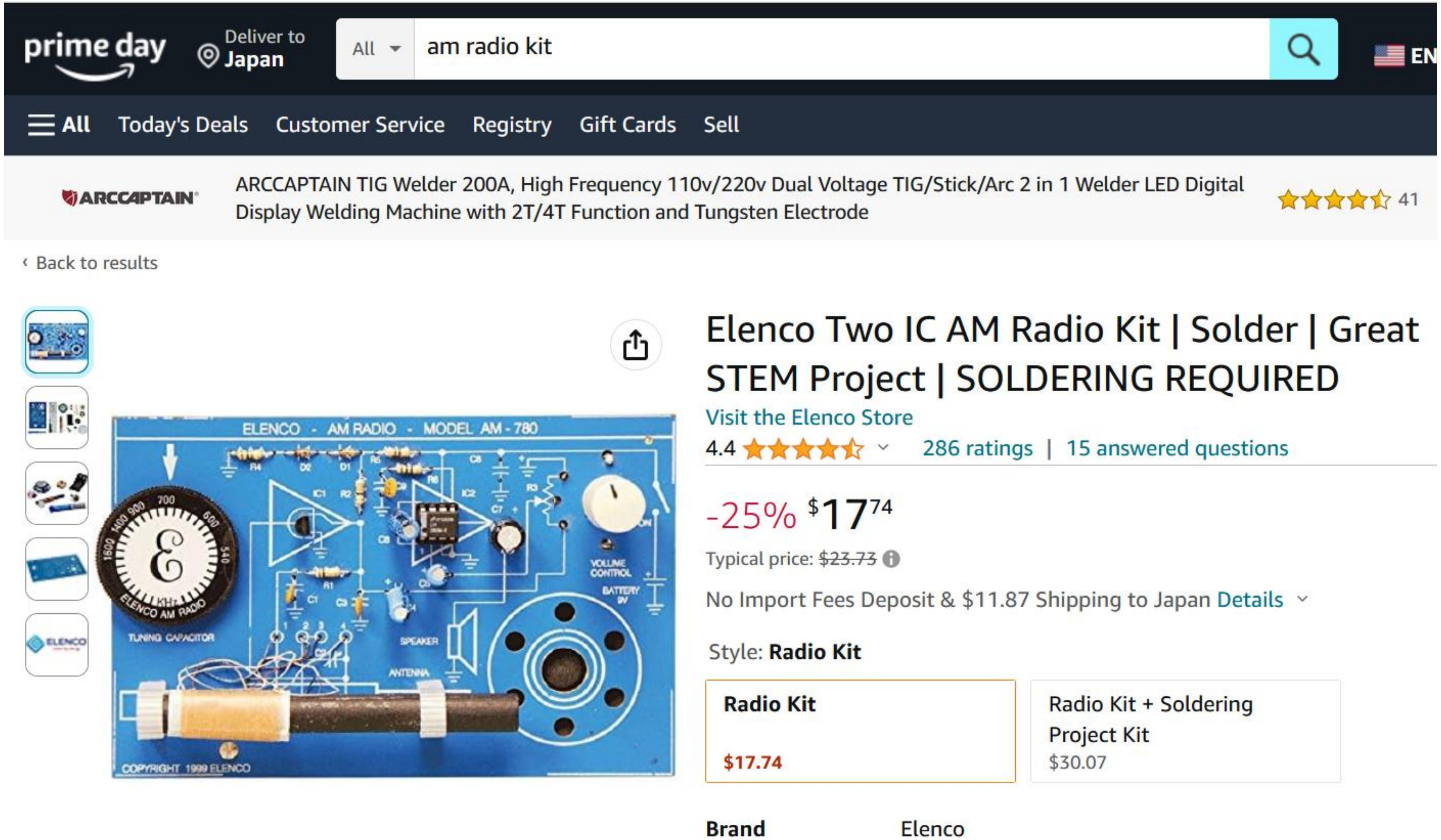
Quantity:

– 1 + Only 4 left

Ships to Japan

◆ another Example of AM: AM radio



Audible sound signal is encoded on the wave 526.5 kHz to 1606.5 kHz

→AM yields arbitrary low freq. signal: No memory, No dissipation.

5. Diversity sources of 1/f fluctuations

Many AM methods

a) Synchronization
b) resonance
c) infrared divergence

Many DM methods

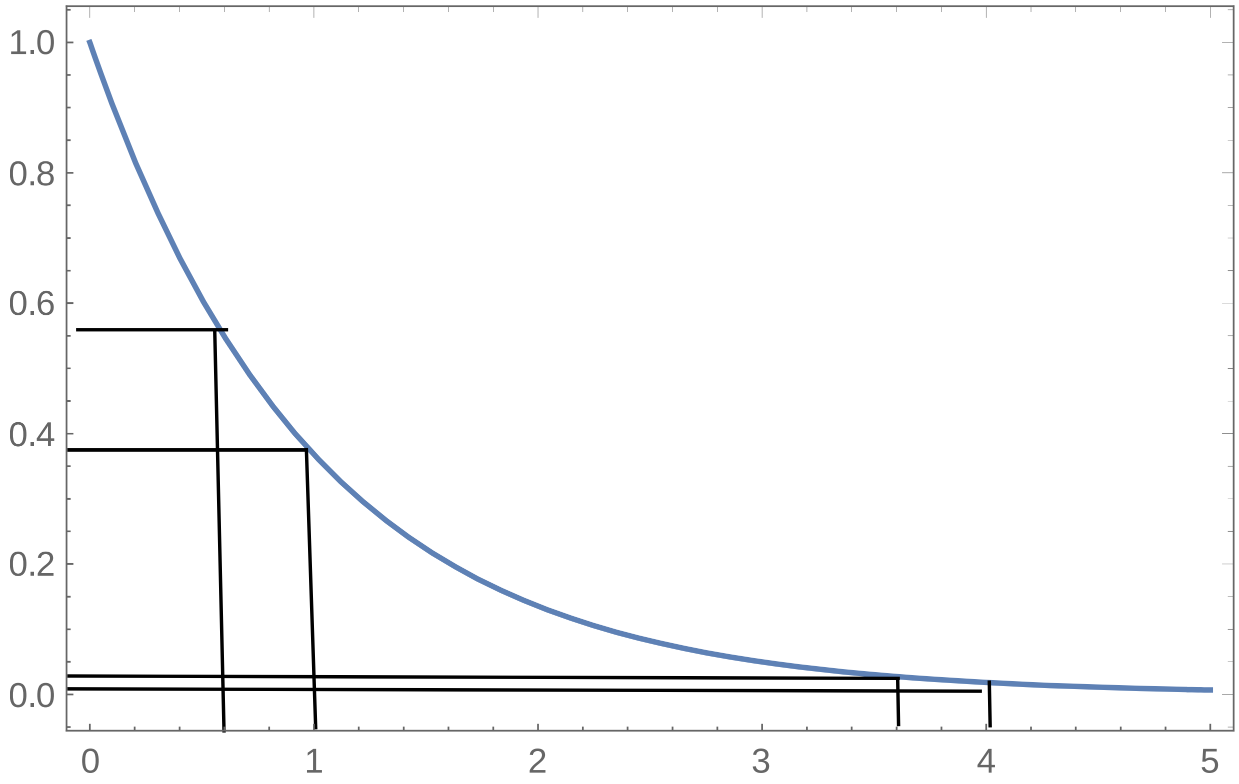
a) **Operational** DM of Data Processing:
Square/Absolute Value of Data

b) **Intrinsic** DM associated with the system:
nerve firing, fault rupture, magnetic reconnection. . .

6. AM from synchronized waves

Synchronized multiple waves accumulate their frequencies and create a signal in the low frequency range:

a) Exponential accumulation…the typical synchronization



Frequency distribution:

$$\omega = e^{-\lambda t}$$

$$P(\omega)d\omega = P(t)dt$$

Freq. distribution $P(\omega) = P(t)\,|d\omega/dt|^{-1} = p(\lambda\omega)^{-1} \propto \omega^{-1}$

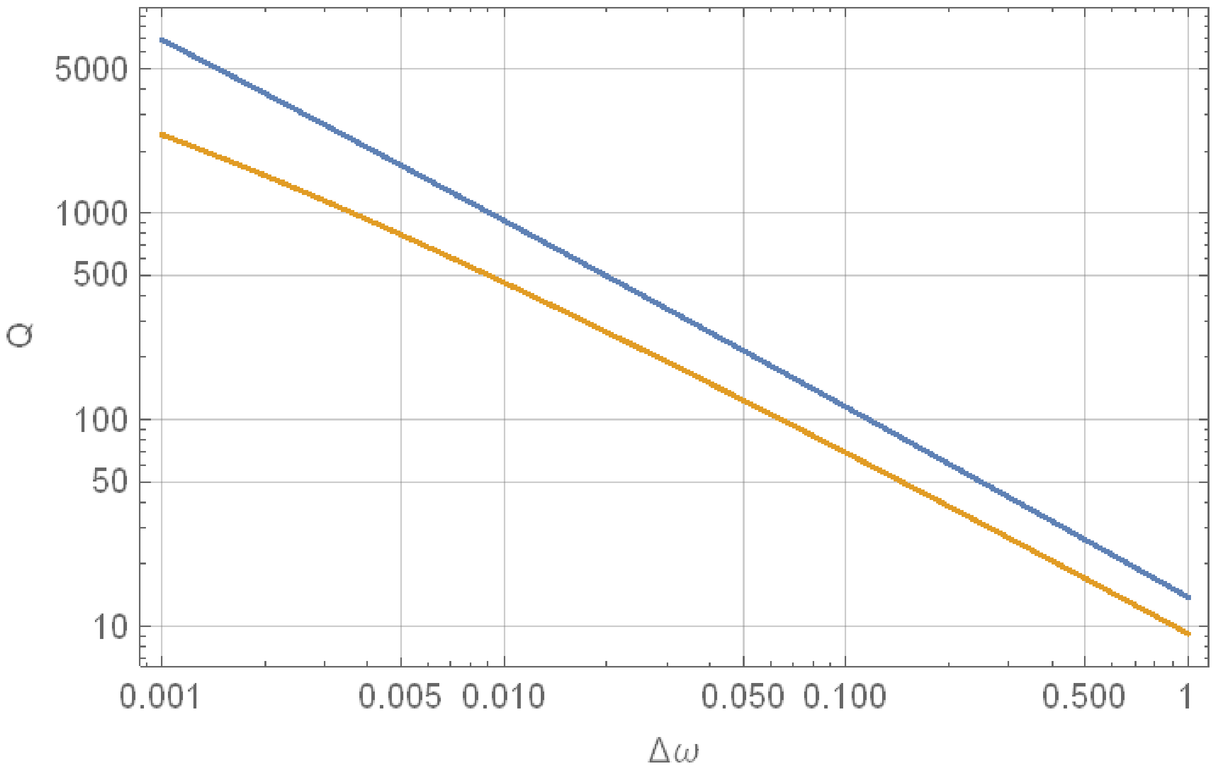
p:const.

Freq. difference distribution:

$$\omega_1 < \Delta\omega < \omega_2$$

$$Q(\Delta\omega) = \int_{\omega_1}^{\omega_2} d\omega P(\omega+\Delta\omega)P(\omega)$$

$$= \frac{p^2}{\lambda^2\Delta\omega}\ln\left(\frac{\omega_2(\omega_1+\Delta\omega)}{\omega_1(\omega_2+\Delta\omega)}\right)$$



$p \to 1, \lambda \to 1,$

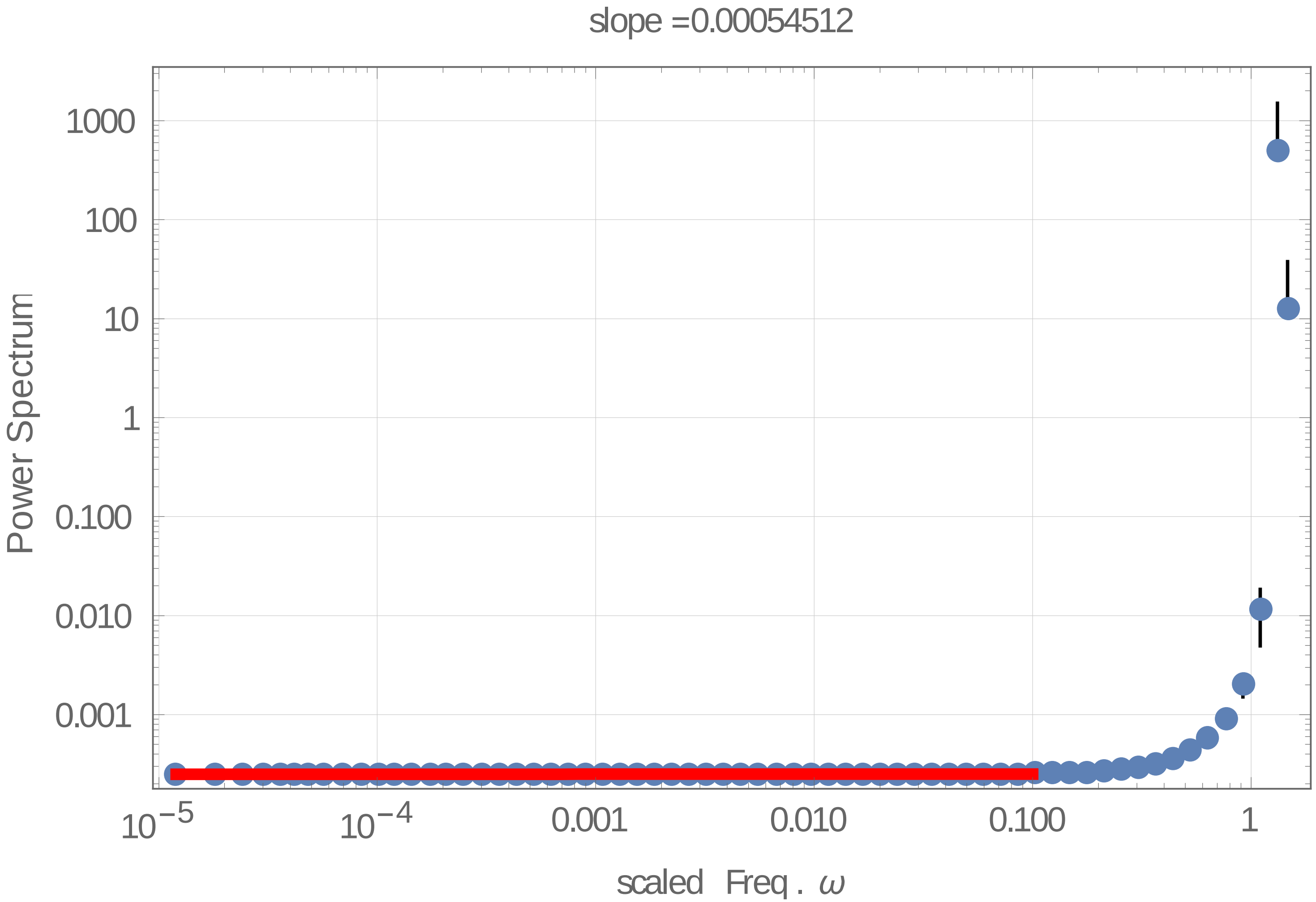
$\omega 1 \to 10^{-4}, 10^{-6}$

$\omega 2 \to 10^{5}$

→almost 1/f

PSD of φ the original signal

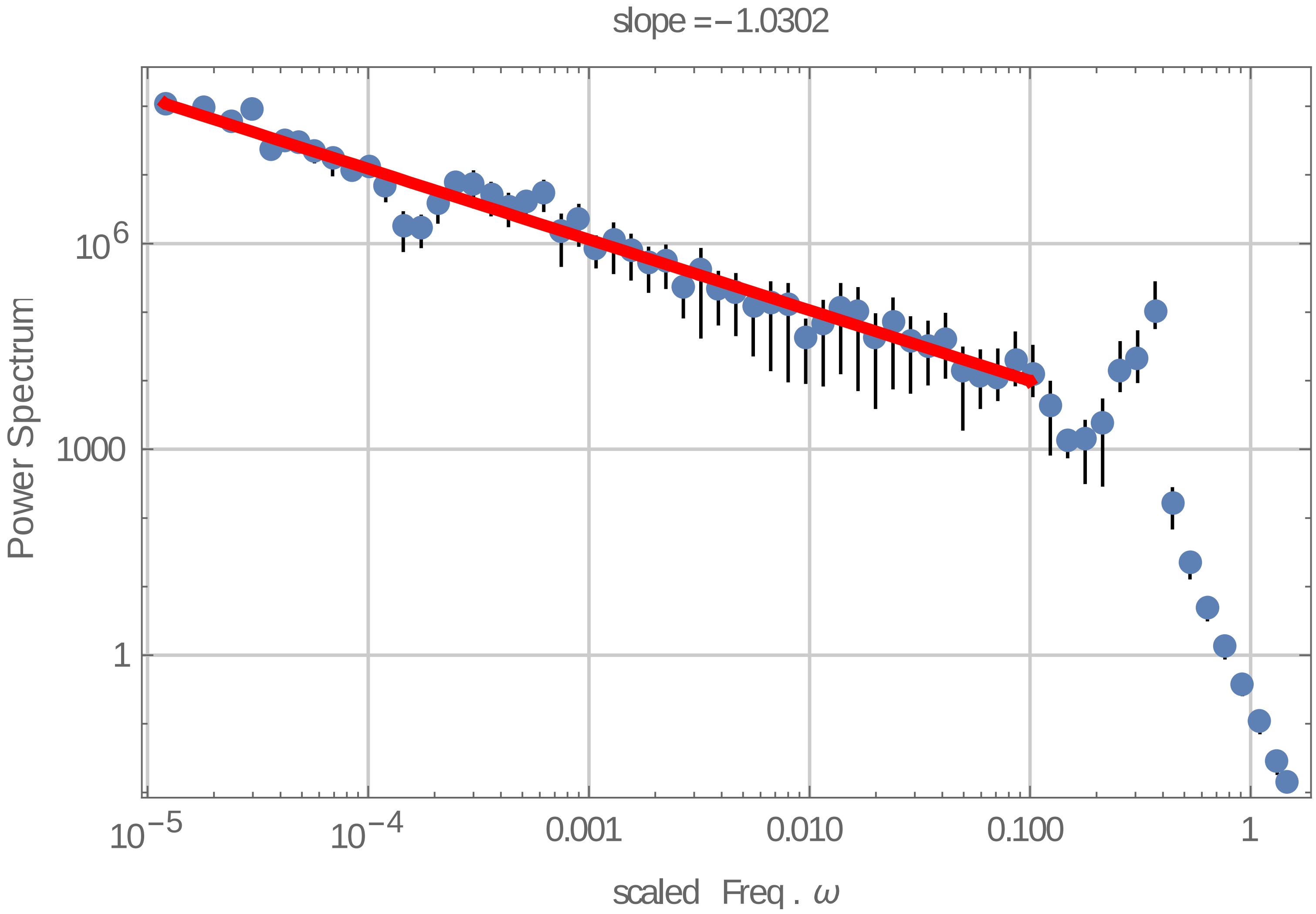
$\phi = \sum \mathrm{Sin}[2\pi\,\omega(1 + 0.1e^{-r})t]$ $r = random[0,10]$ $\omega = 440$



Shows no 1/f

$PSD\ of\ \phi^2$ data squared

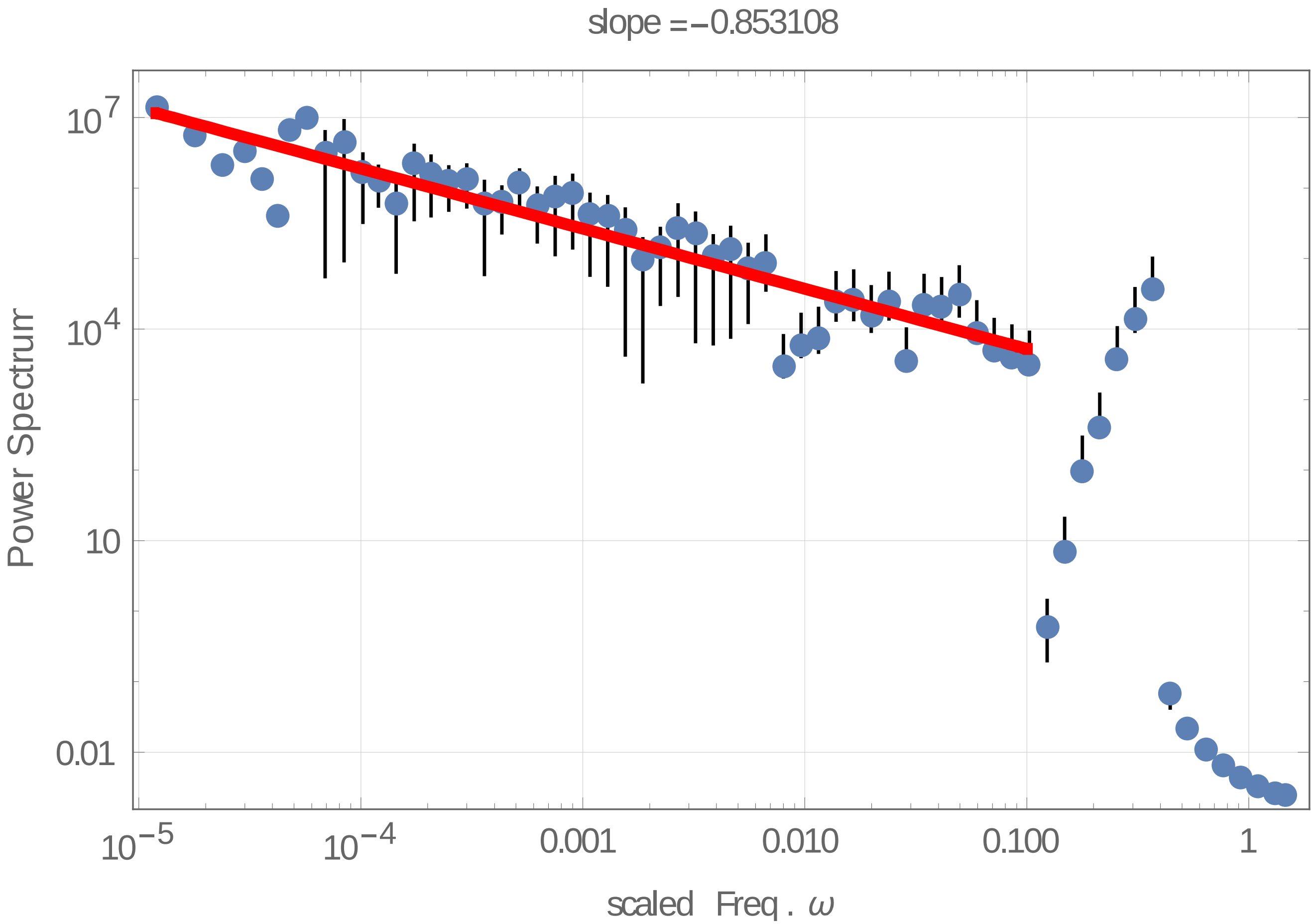
$$\phi = \sum \mathrm{Sin}[2\pi\,\omega(1+0.1e^{-r})t] \quad r = random[0,10]\ \omega = 440$$



→almost 1/f

PSD of ϕ^2 with random phase still yields 1/f fl.

$$\phi = \sum \mathrm{Sin}[2\pi\, \omega(1 + 0.1e^{-r})t + r]$$



$\omega = 440$

$r = random[0,10]$

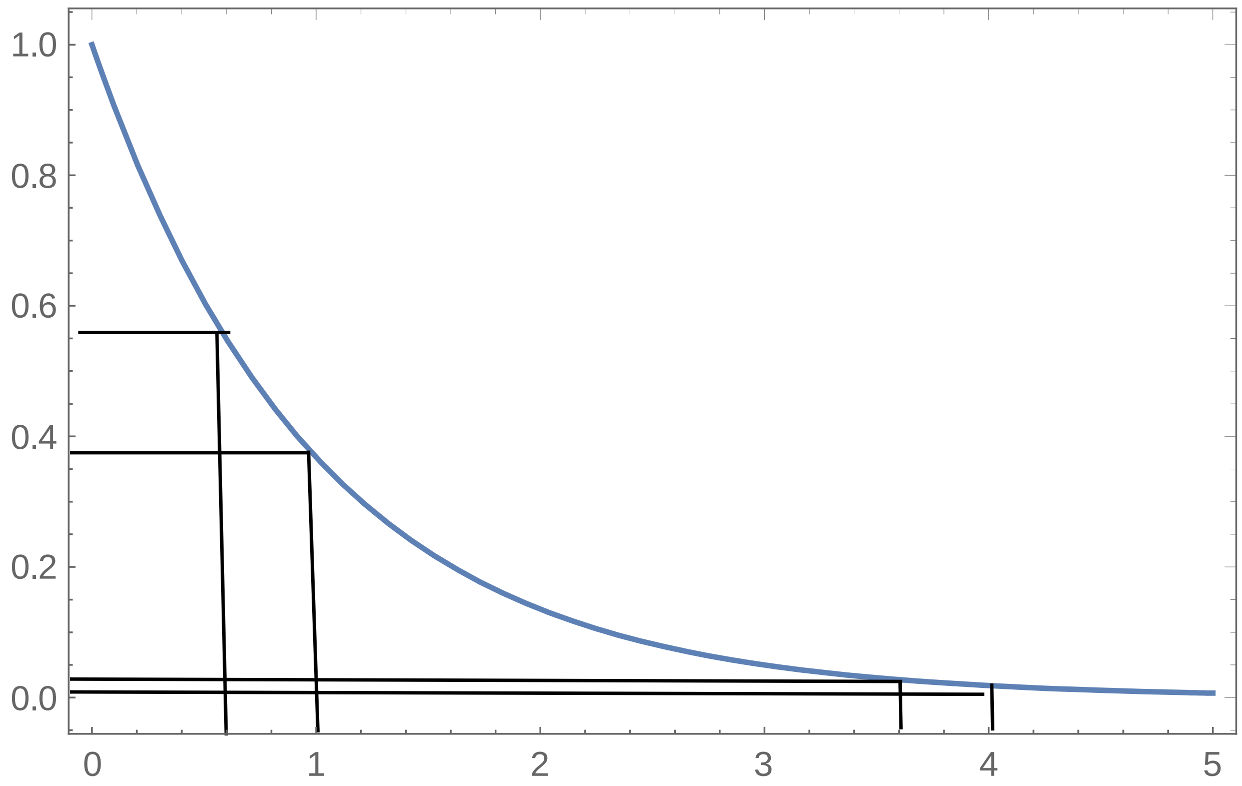
$\omega = 440$

→**1/f fl. is robust**, but the exponent is slightly smaller

b) Power law accumulation:

Frequency distribution: $\omega = t^{-\alpha} \quad P(\omega)d\omega = P(t)dt$

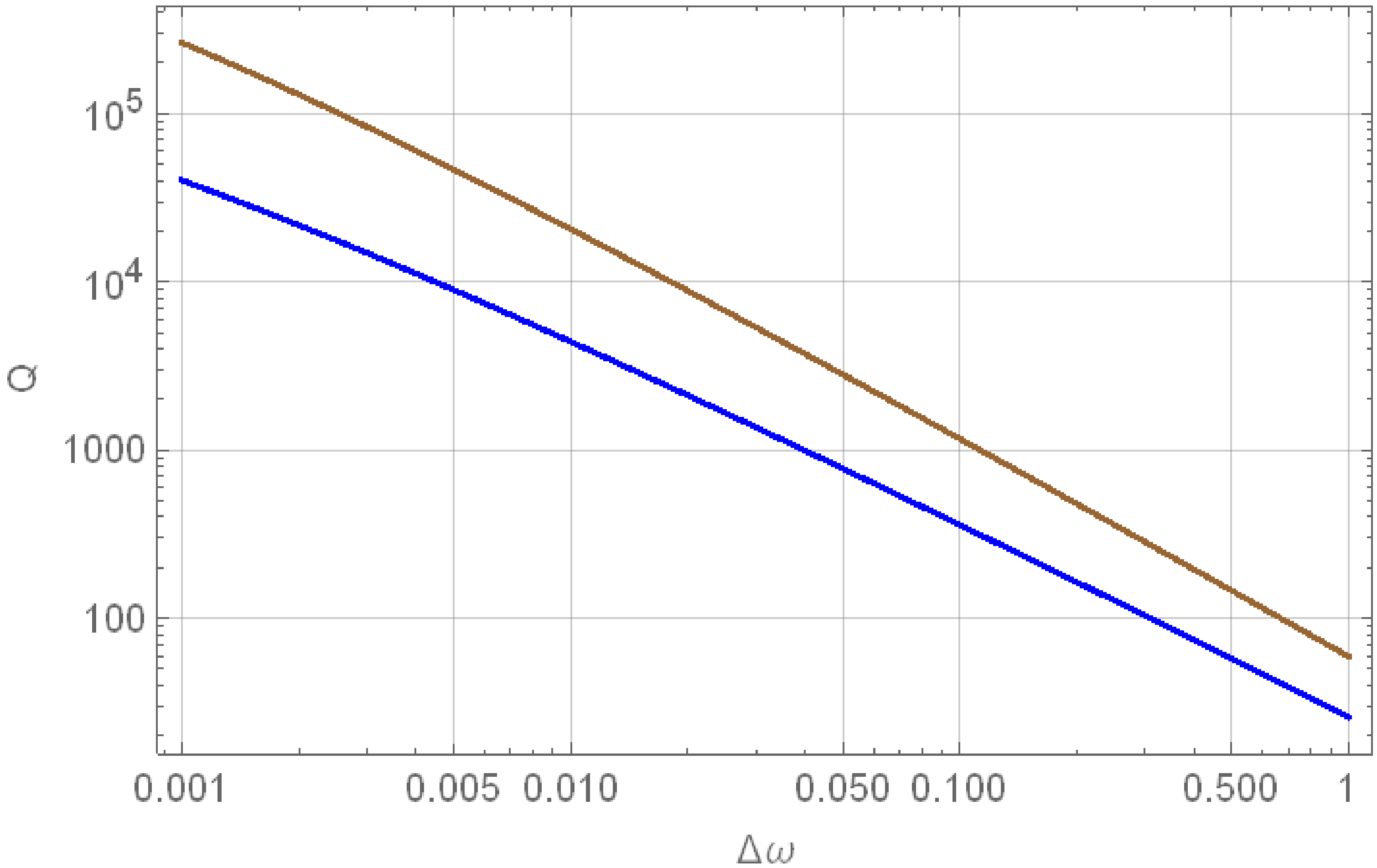
$$P(\omega) = \underset{p\text{:const.}}{P(t)} \, |d\omega / dt|^{-1} = \underset{c}{p\alpha^{-1}} \, \omega^{\overbrace{-\left(1-\frac{1}{\alpha}\right)}_{\beta}}$$



Frequency difference distribution: $\omega_1 < \Delta\omega < \omega_2$

$$Q(\Delta\omega) = \int_{\omega_1}^{\omega_2} d\omega P(\omega + \Delta\omega) P(\omega)$$

$$= \frac{[c^2(\omega 2^{1-\beta}(\Delta\omega + \omega 2)^{1-\beta} \; {}_2F_1\left(1,2-2\beta;2-\beta;-\frac{\omega 2}{\Delta\omega}\right) - \omega 1^{1-\beta}(\Delta\omega + \omega 1)^{1-\beta} \; {}_2F_1\left(1,2-2\beta;2-\beta;-\frac{\omega 1}{\Delta\omega}\right))]}{(1-\beta)\Delta\omega}$$



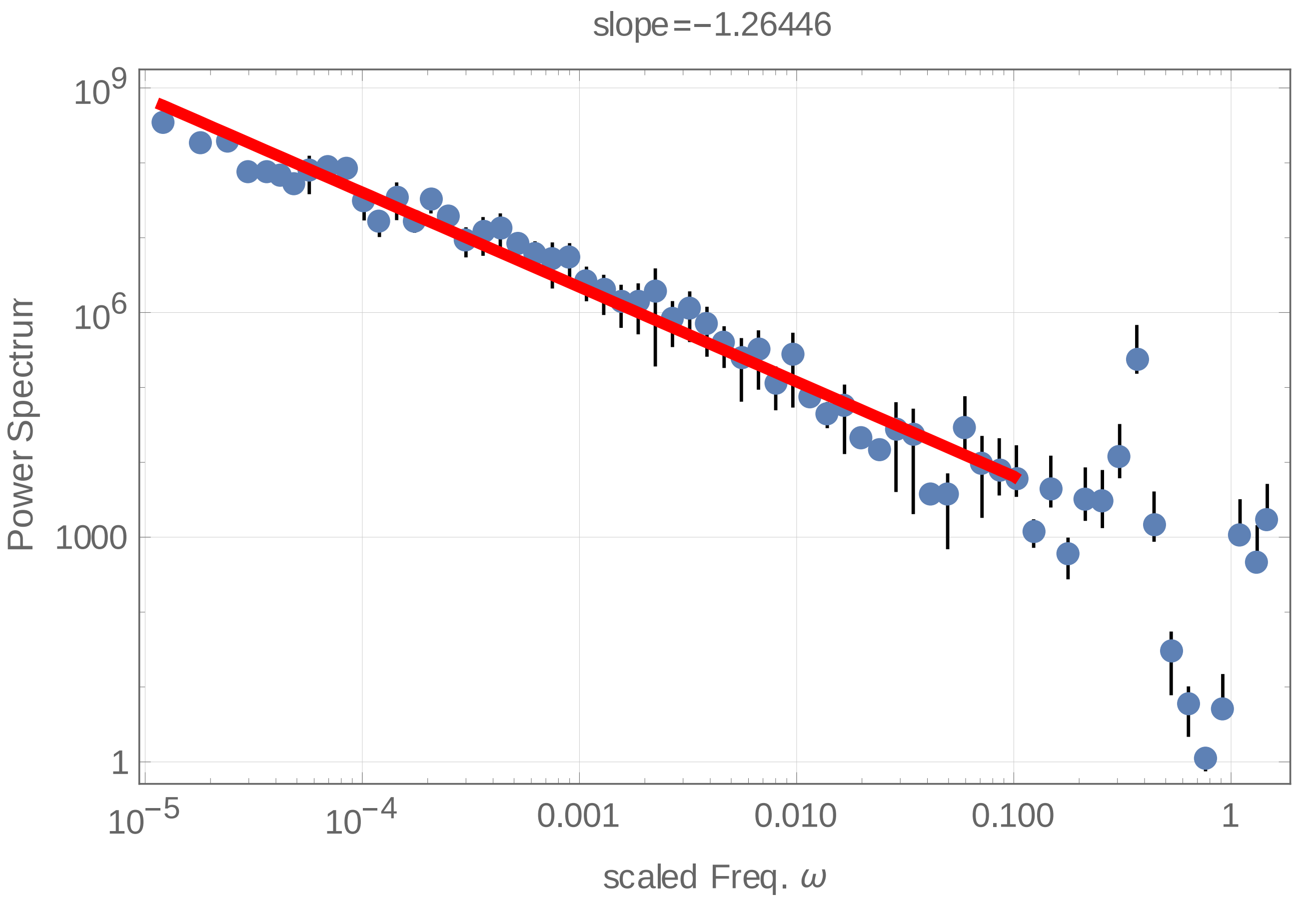
$c \rightarrow 1, \beta \rightarrow$
1.33, 1.2

$\omega 1 \rightarrow 10^{-4}$

$\omega 2 \rightarrow 10^{5}$

→almost 1/f

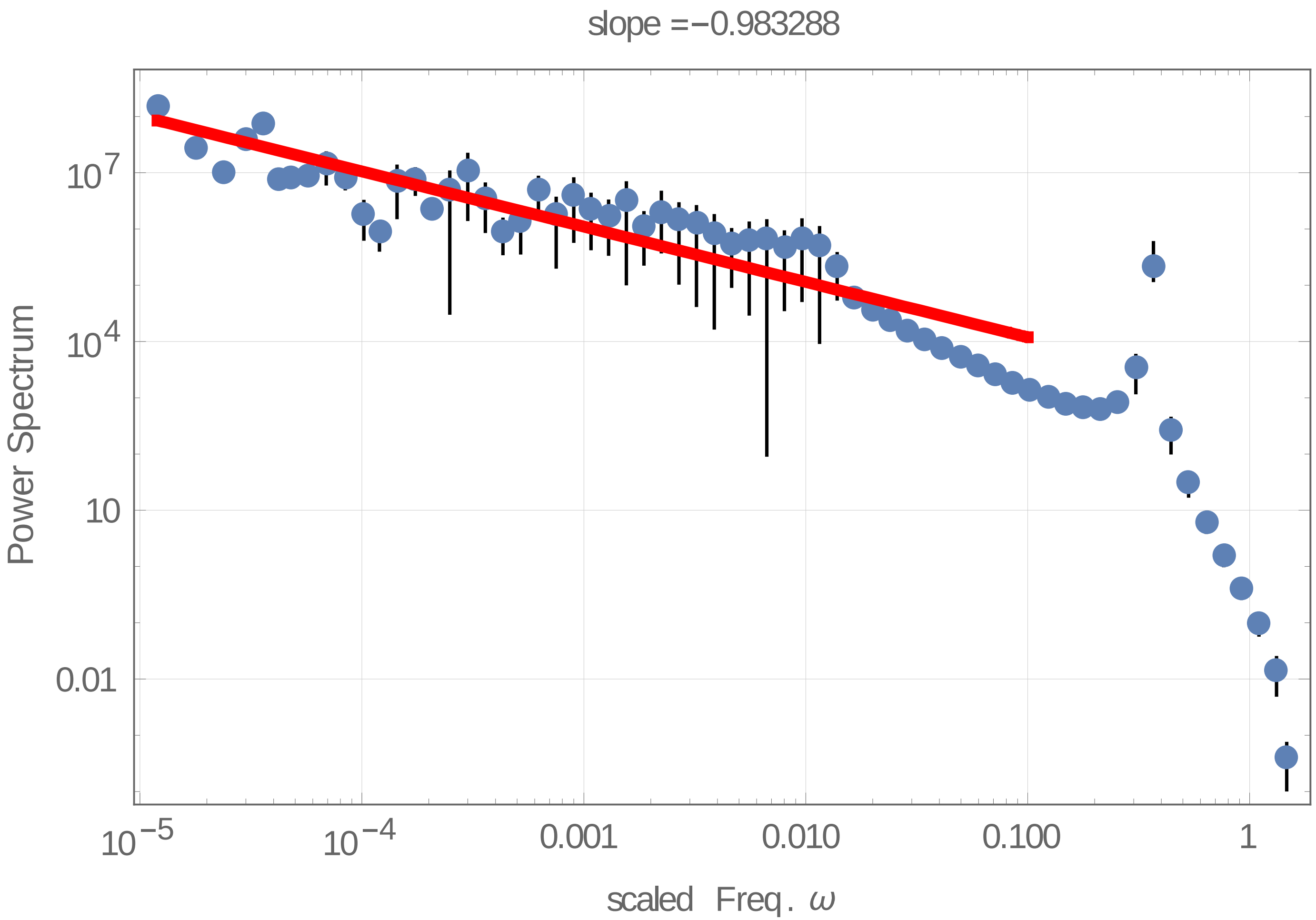
PSD of ϕ^2 (power -3)



$$\phi = \sum \mathrm{Sin}[2\pi\,\omega(1 + 0.3r^{-3})t]$$

$$r = random[0,20]\ \omega = 440$$

$PSD\ of\ \phi^2$ (power 3)



$$\phi = \sum \text{Sin}[2\pi\ \omega(1 + 0.01r^3)t]$$

$$r = random[0,1]\ \omega = 440$$

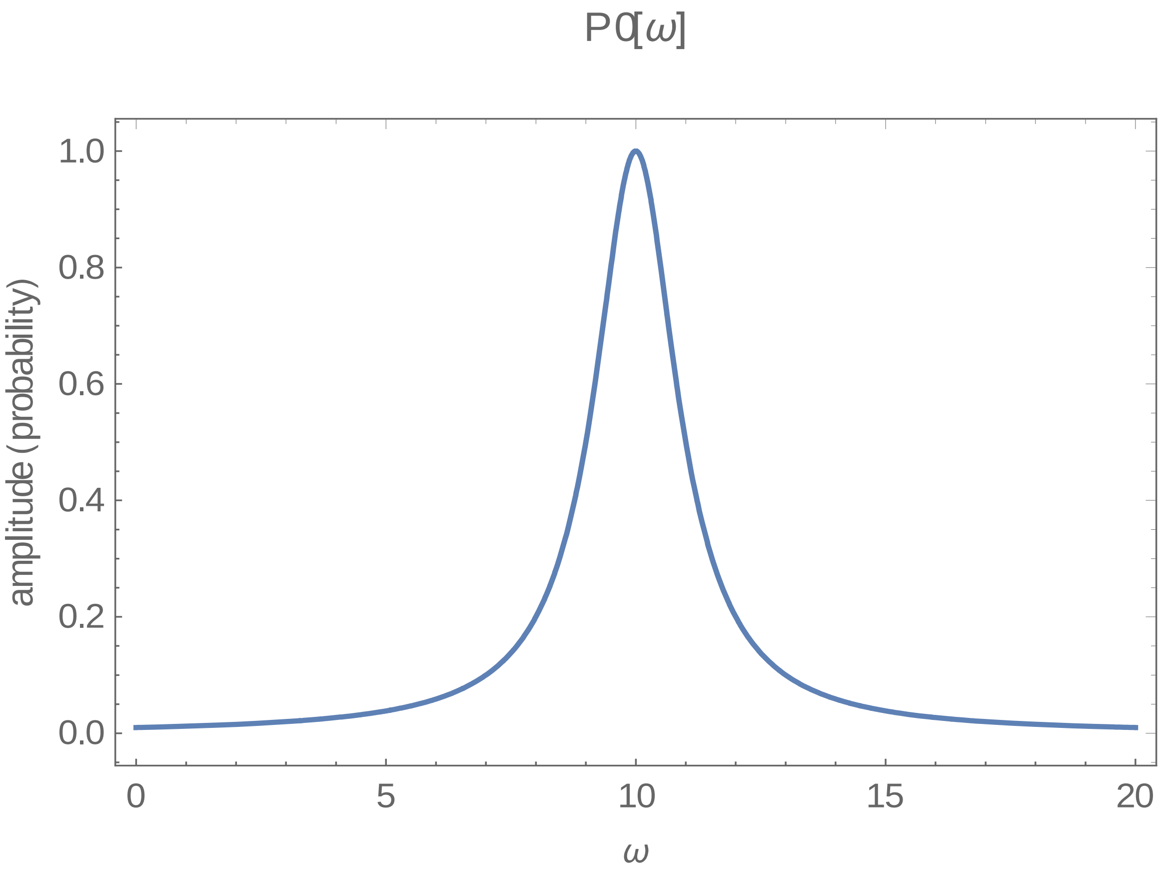
c) A dynamic synchronization 1
Coupled **spin model** for Earth's magnetic field, Solar magnetic field

d) A dynamic synchronous 2
Kuramoto model for Music

. . .

7. AM from resonance

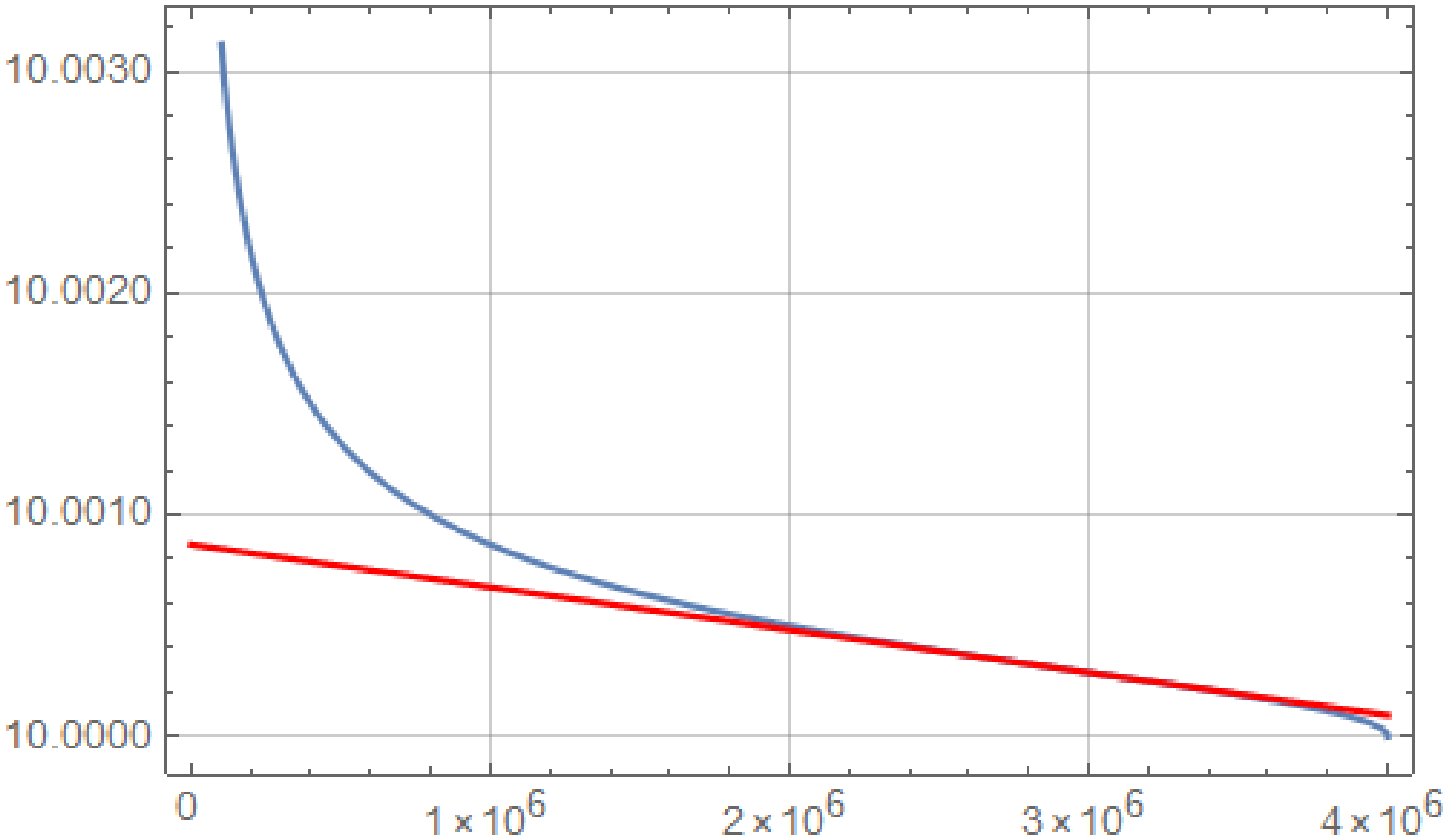
- Crystals resonate with finite width around a fiducial frequency ω_0
- Close frequency modes beat each other to yield low-freq. signal.
- A typical Cauchy (Bright-Wigner) distribution $f(\omega)$ is



$$R(\omega) = \frac{1}{(\kappa/2)^2 + (\omega - \Omega)^2}$$

$$\omega = R^{-1}[t] = \frac{\sqrt{-t\left(\kappa^2 t - 4\right)}}{2t} + \Omega$$

- Log-linear plots $\omega = f^{-1}$ and exponential approximations

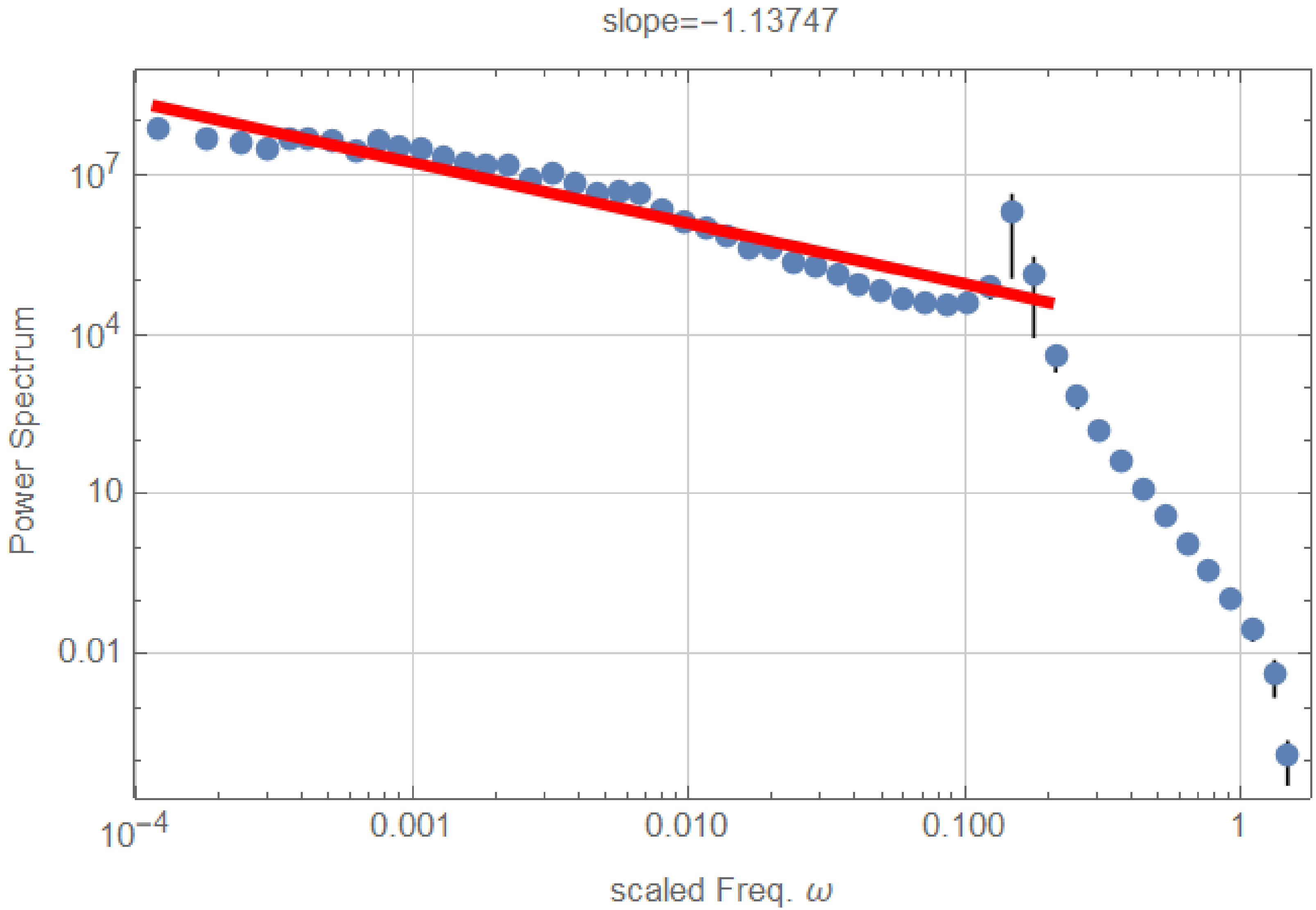


→Nearly straight line: Behaves as 1/f in the middle region.

$\boldsymbol{PSD\ of\ \phi^2}$

$\phi = \mathrm{Sum}[\mathrm{Sin}[2\pi\omega(1 + 0.04\tan(\mathrm{rd})t]$

$\mathrm{rd} := \mathrm{RandomReal}[\{-1,1\}], \omega \to 440$

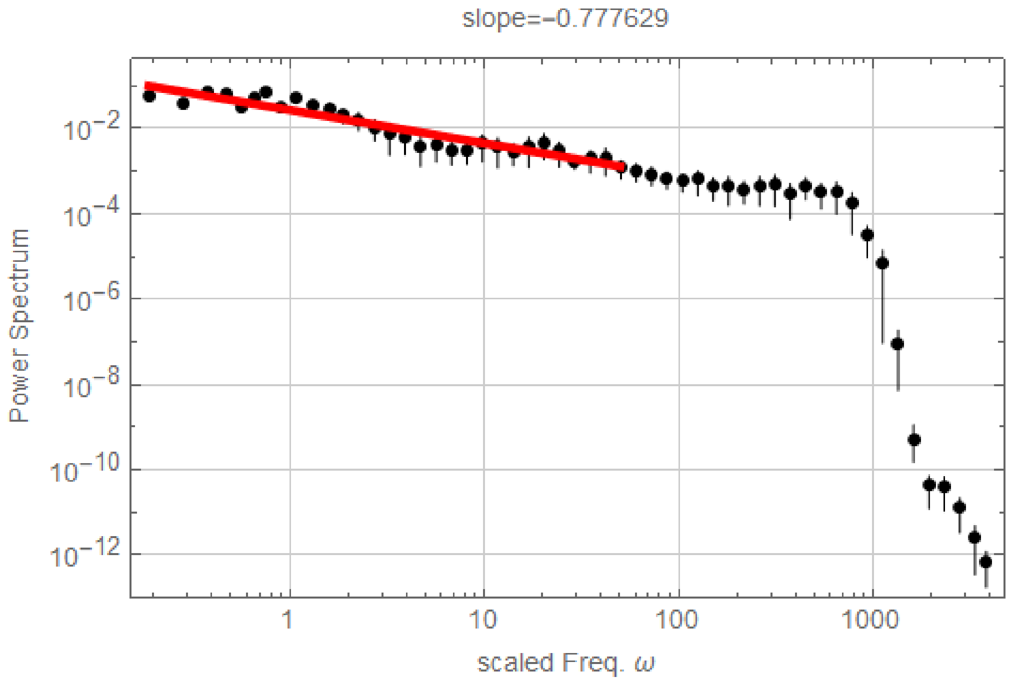


Example 1) unexpected sound …Is it an iceberg icequake 

https://en.wikipedia.org/wiki/List_of_unexplained_sounds

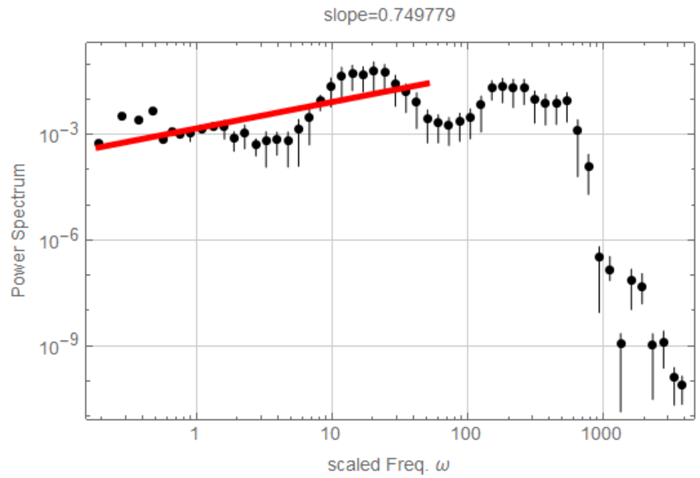
PSD of signal **squared**

→ Nearly 1/f fluctuation



PSD of **original** signal

→almost flat



Example 2)

	AM(Resonance)	DM
- **Earthquake**…	Earth Free Oscillation	fault rupture
- **Solar flare**…	Sun's five minutes osc.	magnetic reconnection
- **The hustle and bustle of a city**… reflection from buildings		data squared
- **Suikinkutsu** (Kyoto Hosen-in Temple)… Underground pottery jar		data squared

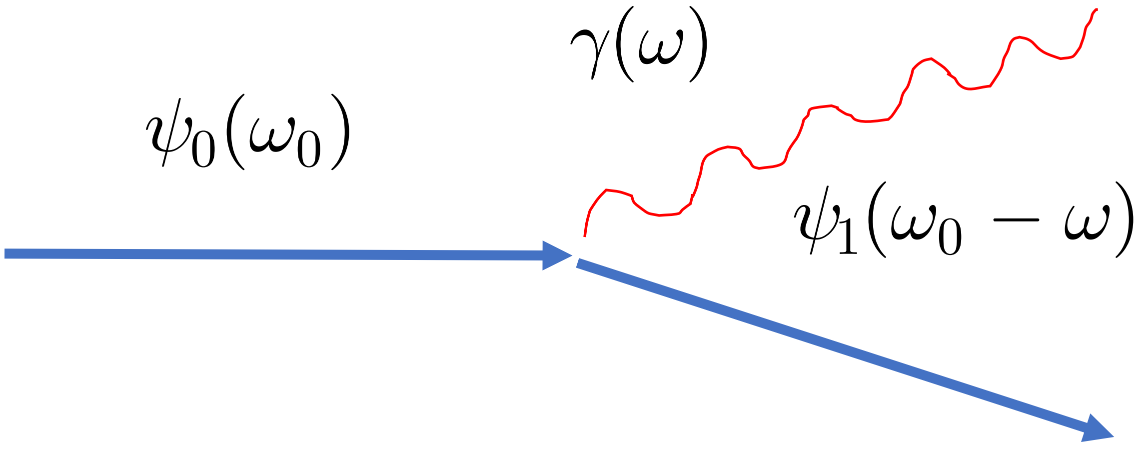
https://arxiv.org/abs/2307.03192

8. AM from infrared divergence (IR div)

- Electric currents in semiconductors often exhibit 1/f fluctuations.
- PH Handel (1975) proposed a quantum origin of 1/f fluctuations.
- However, his theory is denied for some reasons (1986, 1987)

  Kiss, Heszler, Nieuwenhuizen, Frenkel, van Kampen,…
- Crucial problem: $|\psi\rangle = \alpha|\psi_0\rangle|0\rangle + \beta|\psi_1\rangle|1\rangle$, both terms are orthogonal

  → no **quantum current beat** and no 1/f fluctuations



- If so, 1/f fluctuations should be a **classical phenomenon**.

-In semiconductors, mfp $\approx$ 10nm $\approx$several tens of lattice size.
Within this size, electrons form a **wave packet**:

$$\psi(x,t) = e^{i(k_0 x - \omega_0 t)} \int \phi(k_0 + k') \exp\left[ik'(x - v_g t)\right] dk'$$

- then, 10^{10} packets exist in a sample of $1\mathrm{mm}^3$ semiconductor.
- these packets are scattered by impurities with the **photon emission**.
- The packet behaves as

$$\psi''(t) = -\psi(t) + \xi(t)\psi(t) - \kappa\psi'(t) - \lambda\psi(t)^3$$

where $\xi(t)$ reflects the back reaction of photon emision

$$dN = \frac{d\mathscr{E}}{k^0} = e^2 \left| \frac{\varepsilon \cdot p^i}{k \cdot p^i} - \frac{\varepsilon \cdot p^f}{k \cdot p^f} \right|^2 \frac{d^3 k}{2(2\pi)^3 k^0}$$ with probability $\propto \frac{1}{\omega}$…IR div.

➡ These frequency modulated (FM) wave packets beat with each other and form 1/f fl. as in the case of synchronization:

$$Q(\Delta\omega) = \int_{\omega_1}^{\omega_2} d\omega P(\omega + \Delta\omega) P(\omega) \approx -p^2 \ln[\Delta\omega] (\lambda^2 \Delta\omega)^{-1}$$

$$\psi''(t) = -\psi(t) + \xi\,(t)\psi(t) - \kappa\psi'(t) - \lambda\psi(t)^3$$

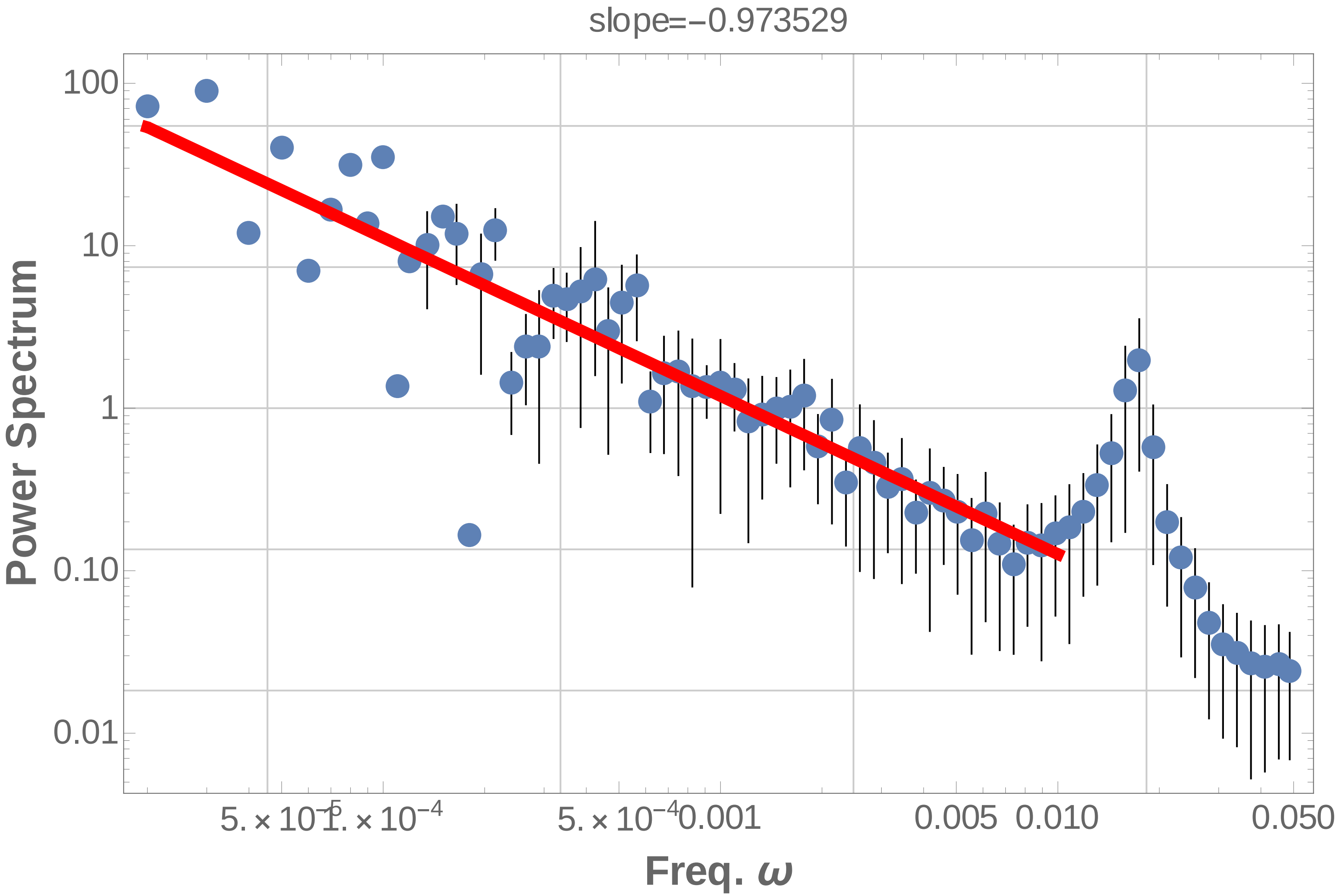
with $\kappa = 0.001, \lambda = 0,\ \xi$: back reaction of γ em.

PSD of $\psi(t)^2$

SEC2.wav

SEC1.wav

→ almost 1/f



◆ 1/f fluctuation **inherits to surroundings**. [DM]

Current fluctuation f^{-1}

→ Vacuum tubes/semiconductors/thin-metals f^{-1}

[piecewise root mean square]

→ Potential fluctuations of biological membranes f^{-1}

→ nervous system f^{-1} [firing of nerve cells]

→ heart beat f^{-1}

→ MEG and EEG (brain) f^{-1}

→ Posture control (sway) f^{-1}

→ Electrolyte f^{-1}

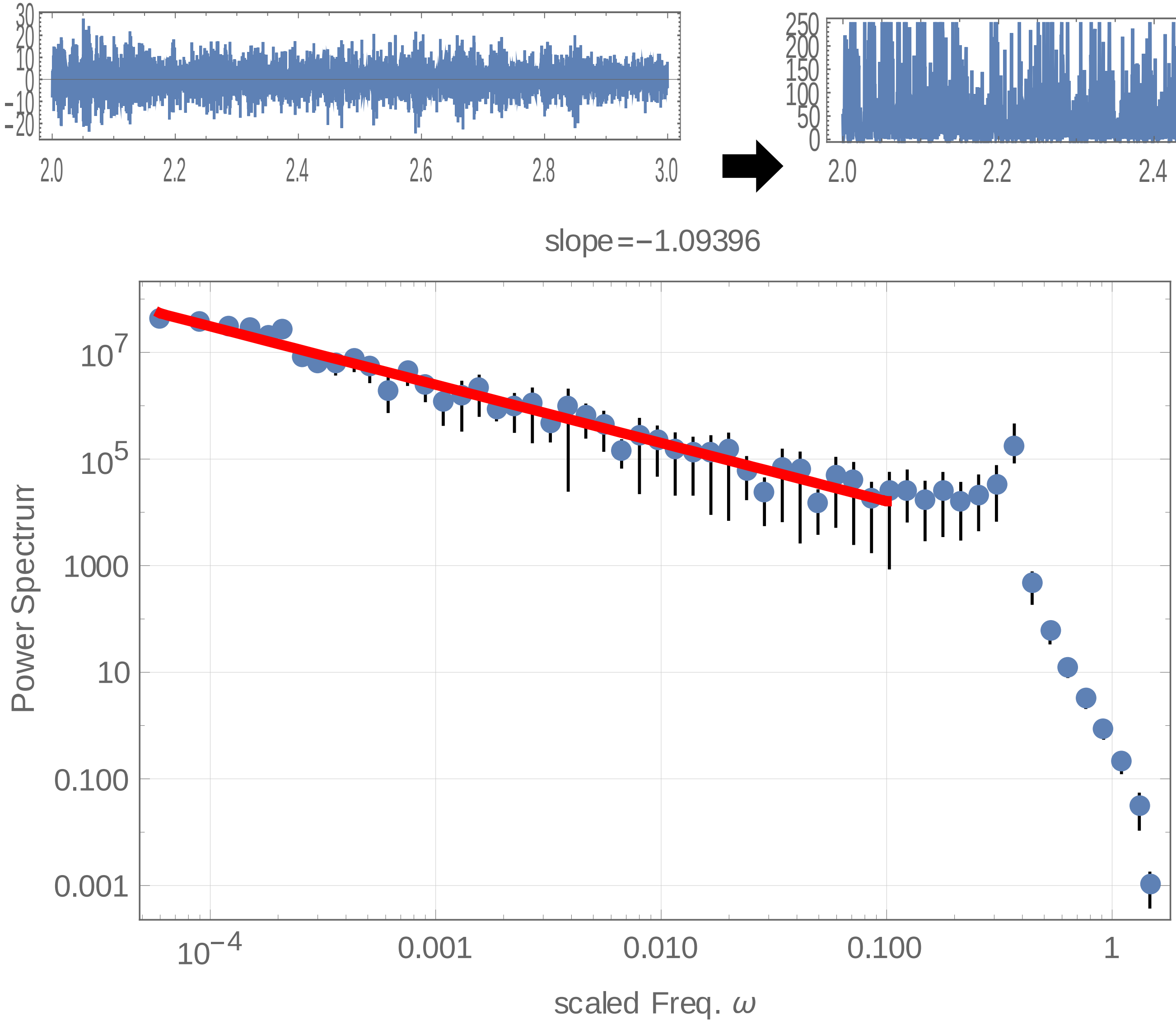
9. Discussions and AM/DM Verifications

Previous "Some Important hints"

1. **Waves** behind: otherwise, 1/f is not AM/DM.
2. Small system: if small-limit or frequency-limit, 1/f is not AM/DM.
3. long-time **memory is illusion**: if real memory, 1/f is not AM/DM.
4. **No relation to dissipation**: if fl.-diss. thm. holds, 1/f is not AM/DM.
5. **Data squared**: if bare ϕ shows 1/f fl. it is not AM/DM

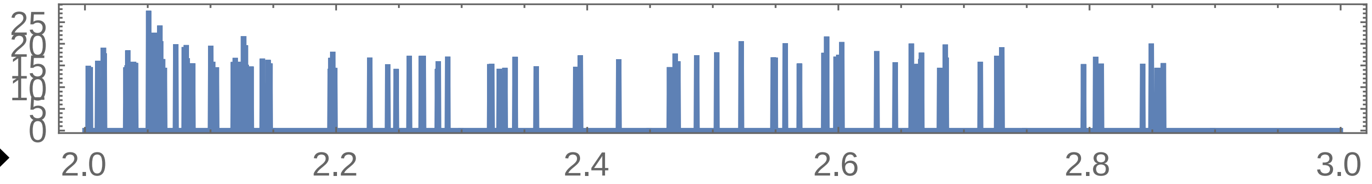
Robustness and Inheritance of 1/f fl.

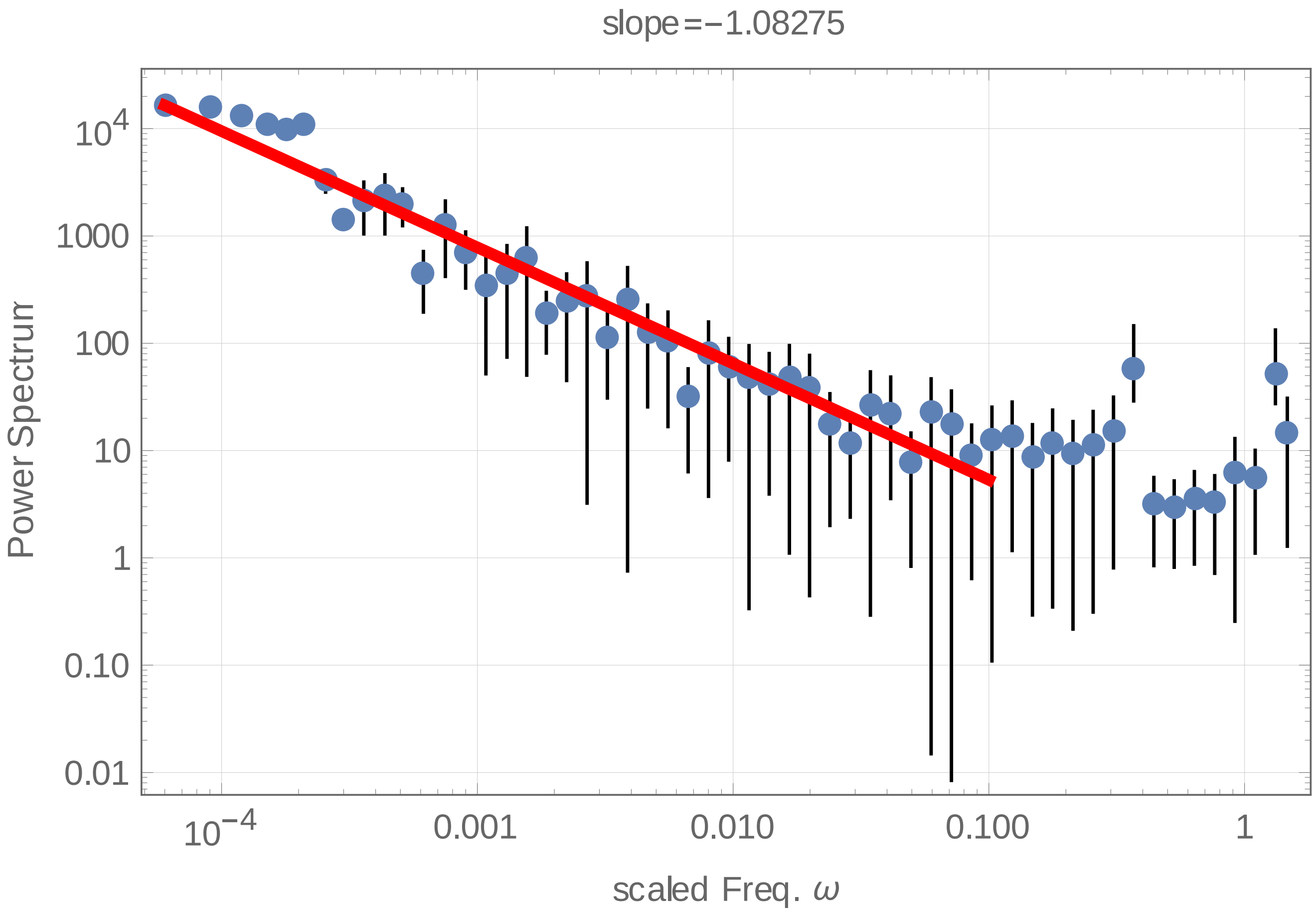
a) Current fl. ➡ root mean square sections… Vacuum tube f^{-1}



b) Voltage fl. in biological membrane ➡ neuron cell firing Nervous system f^{-1}

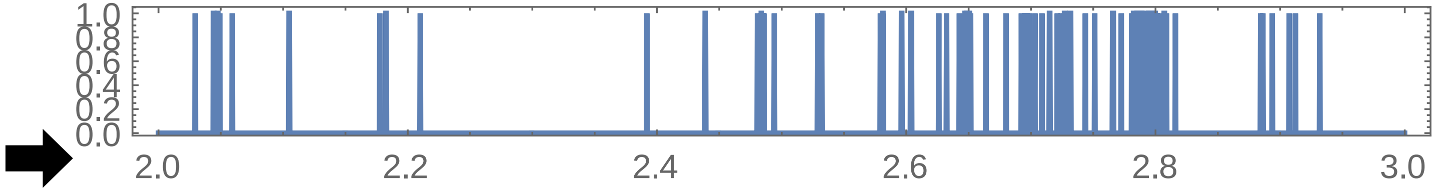
◆ Thresholding: select data larger than twice the average:

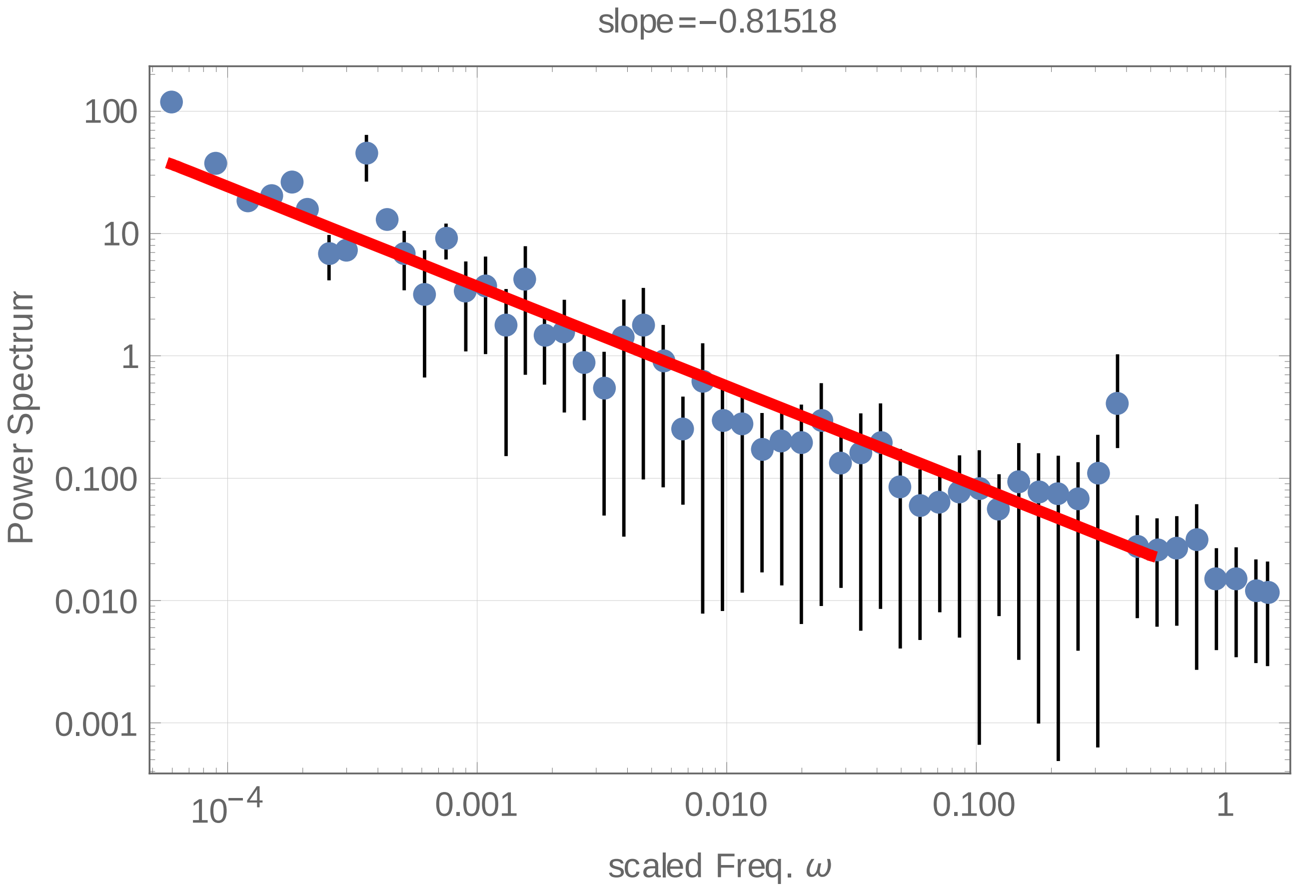
➡ 



◆ ON OFF data (1 or 0)…neuron firing

Select data larger than 3 times of average and set 1, smaller, set 0





Summary and Prospects

- We proposed the idea that the origin of 1/f fluctuations is **the beat of many waves with accumulating frequencies.**
  i.e. **1/f fluctuations are amplitude modulation (AM)**

- The frequency accumulation can naturally arise from **synchronization, resonance, and infrared divergence , etc.**

- **Demodulation** process is crucial for 1/f fluctuation, and yields diversity.

→ Various examples are shows tomorrow evening.

	system	resonant mode	demodulation	description
1	earthquakes	earth free oscillation[22]	fault rupture	USGS World 30-year data shallower than 20km, magnitude 4-5 show pink noise of slope -1.2 below 2.5 months.
2	icequakes	iceberg eigenfrequency	ice fault rupture	NOAA Icequakes (Bloop) deep-sea sound[23] show pink noise with slope -0.8.
3	solar flare	five minute oscillation[24]	magnetic reconnection	HESSI[25] solar flare luminosity curve for 16 years shows pink noise with slope -0.9
4	sunspots	same as above or macro spin model[13]	(intrinsic)	Sunspot number time sequence from the year 1820 to 2010 shows pink noise of slope -1.1 over the entire period[13].
5	NO_3^-	same as above	(intrinsic)	NO_3^- - Concentration during the years 1610–1904 in the DF01 antactica ice core[26] shows pink noise with slope -1.1.
6	variable stars	same as above	(intrinsic)	Some of the variable stars show pink noise. The light courve of Mira (Red giant) for about six years[27] shows pink noise with slope -1.2.
7	Suikinkutsu	2-meter pottery cavity underground	data squared	The water harp cave at HosenIn Kyoto shows pink noise of slope -0.8(left) and -0.6(right) for about four decades.
8	Big gong	eigenfrequency	data squared	The big gong in Kyoto shows pink noise with slope -1.6. The small gong shows NO pink noise.