Gravastar-like black hole solutions in q-theory

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Gravastars

A gravastar is a hypothetical astrophysical object alternative to Kerr-Newman black holes. Its interior is subject to a different phase of the gravitational vacuum as compared to the exterior region.

Types of gravastars:

- Mazur-Mottola gravastar: Schwarzschild black hole metric outside, de Sitter metric inside, both are separated by a thin shell of matter of positive energy density
- q-theory gravastar: "hairy" black hole with inside and outside regions resembling a Schwarzschild black hole, separated by a shell of negative energy density in horizon proximity with transition towards positive energy density further outwards

Basics of q-theory

The q-field is a scalar field describing the low energy dynamical vacuum. It depends on a more fundamental field directly present only at high energies. Its description is universal as compared to its possible origin from high energy degrees of freedom.

- example: q-field from a three form field $A_{\alpha\beta\gamma}$:
  $q^2 = -\frac{1}{4!} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta}$, $F_{\alpha\beta\gamma\delta} = \nabla_{[\alpha} A_{\beta\gamma\delta]}$
- action: $S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \epsilon(q) - \frac{1}{2} g^{\alpha\beta} \nabla_\alpha q \nabla_\beta q \right)$,
  symmetric Higgs potential $\epsilon(q) = \frac{\lambda}{4}(q^4 - \frac{1}{aG} q^2)$, $\lambda, a > 0$,
  $q = q(F_{\alpha\beta\gamma\delta}(x), g_{\mu\nu}(x))$
- require $|\nabla_\alpha q \nabla^\alpha q| \ll 1$ in units where $G = 1$

The q-field and the cosmological constant

- combined field equations:
  $R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta} = 8\pi G(T_q)_{\alpha\beta}$
  $(T_q)_{\alpha\beta} = -[g_{\alpha\beta}(\rho(q) + \frac{1}{2}\nabla_\alpha q \nabla^\alpha q) - \nabla_\alpha q \nabla_\beta q]$
- asymmetric Higgs potential $\rho(q) = \epsilon(q) - \mu q$, $\mu = \frac{d\epsilon(q)}{dq}\Big|_{q=q_{eq}}$
- a self-sustained quantum vacuum fulfills $0 = P = -\rho$ in thermodynamic equilibrium
- $\rho(q_{eq}) = \frac{d\rho}{dq}(q_{eq}) = 0$, $\frac{d^2\rho}{dq^2}(q_{eq}) > 0$
- vacuum stability: positive vacuum compressibility
  $\chi_{vac}^{-1} = (q^2 \frac{d^2\epsilon}{dq^2})\Big|_{q=q_{eq}} = \frac{\lambda}{6a^2G^2} > 0$
- I set $G = 1$ from now on

Theory and equations of motion

- general relativity with a minimally coupled, ordinary scalar field and asymmetric Higgs potential
- assumptions: stationarity, spherical symmetry, asymptotic flatness
- metric: $ds^2 = -f(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2(d\theta^2 + sin^2(\theta)d\phi^2)$,
  $f(r) = h(r)e^{2\delta(r)}$, $h(r) = 1 - \frac{2m(r)}{r}$
- field equations:
  1. $m' = 4\pi r^2(\rho + \frac{1}{2}h(q')^2)$
  2. $\delta' = 4\pi r(q')^2$
  3. $q'' = \frac{1}{h}\frac{d\rho}{dq} - \frac{h'}{h}q' - \frac{2}{r}q' - 4\pi r(q')^3$

Boundary conditions and numerical techniques

- choose boundary conditions at the black hole horizon $r = r_h$ with $lim_{r \to r_h} q''(r) < \infty$ (and $lim_{r \to \infty} q(r) = q_{eq}$)
- $h(r_h) = 0$, $q(r_h) =?$, $q'(r_h) = \frac{d\rho}{dq}(q(r_h)) \frac{r_h}{1 - 8\pi r_h^2 \rho(q(r_h))}$
- $q(r_h)$: shooting parameter, determined by asymptotic flatness
- numerical solver: non-adaptive, refined, fourth order Runge-Kutta method
- to avoid singularity at $r = r_h$: use the prescription $r \to r(1 + i\epsilon)$, $0 < \epsilon \ll 1$, $lim_{\epsilon \to 0}$ implied

Black hole no-hair theorems

The space of solutions is subject to conditions imposed by black hole no-hair theorems:

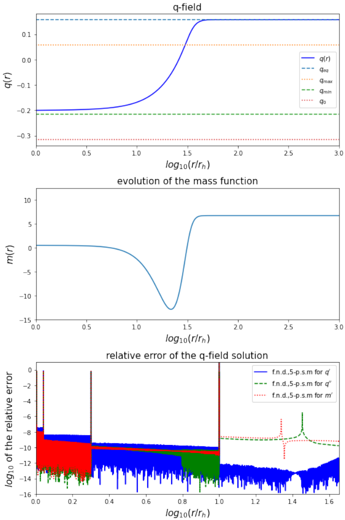
- If the dominant energy condition holds but the strong energy condition does not, in the presence of an event horizon in a static, spherically symmetric and asymptotically flat spacetime, no non-trivial regular scalar field solution exists outside the event horizon.
- Any black hole solution must necessarily have $V(r_h) < 0$ where r_h denotes the radial location of the event horizon and V is the potential energy density of the scalar field. The vicinity of the event horizon is enveloped by a region of negative scalar field energy density.

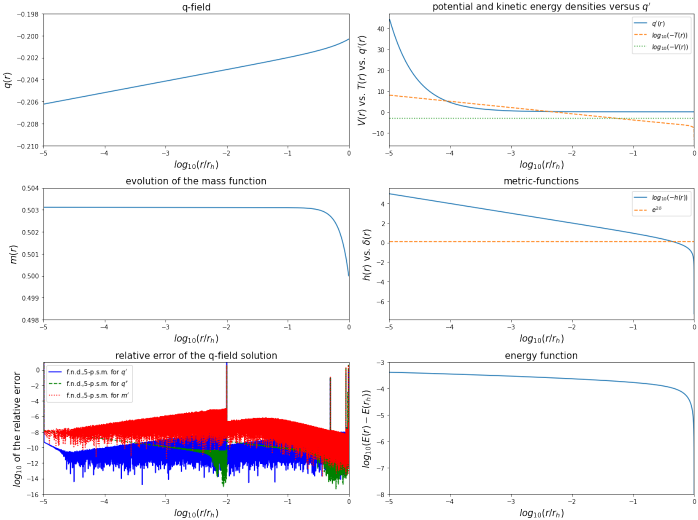
The q-field asymptotically relaxes to its equilibrium value but sweeps over field values corresponding to negative energy density as the horizon is approached, for sure in the horizon proximity.

Black hole solutions comform with this statement carry "hair" and may be called scalar hair black holes (SHBH's).
SHBH's have to fulfill an integral equation as a necessary condition for existence which reads $lim_{r\to\infty} nhf(r) = 0$ with

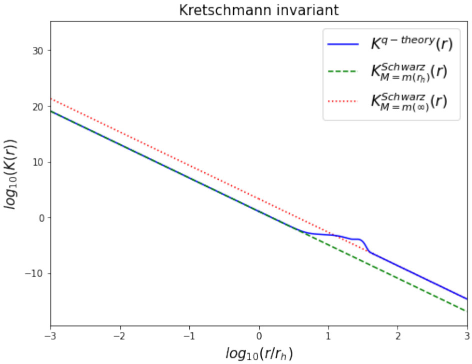
$$nhf(r) = \int_{r_h}^{r} s^2 e^{\delta(s)}((\tfrac{2r_h}{s}(1-\tfrac{m(s)}{s})-1)E_{kin}^{flat} + (\tfrac{2r_h}{s}-3)V(r))ds$$

$$E_{kin}^{flat}(r) = \tfrac{1}{2}(q'(r))^2,\; V(r) = \rho(q(r))$$


q-field
q(r)
log10(r/rh)
potential and kinetic energy densities versus q'
V(r) vs. T(r) vs. q'(r)
q'(r)
T(r)
50V(r)
evolution of the mass function
m(r)
metric functions
h(r) vs. f(r) vs. δ(r)
h(r)
f(r)
−δ(r)
relative error of the q-field solution
log10 of the relative error
f.n.d.,5-p.s.m for q'
f.n.d.,5-p.s.m for q''
f.n.d.,5-p.s.m for m'
no-hair function
nhf(r)


q-field
q(r)
log10(r/rh)
potential and kinetic energy densities versus q′
V(r) vs. T(r) vs. q′(r)
q′(r)
log10(−T(r))
log10(−V(r))
evolution of the mass function
m(r)
metric-functions
h(r) vs. δ(r)
log10(−h(r))
e^2δ
relative error of the q-field solution
log10 of the relative error
f.n.d.,5-p.s.m. for q′
f.n.d.,5-p.s.m. for q″
f.n.d.,5-p.s.m. for m′
energy function
log10(E(r) − E(rh))

Kretschmann invariant $K(r) = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, $K^{Schwarz}(r) = \frac{48G^2M^2}{r^6}$

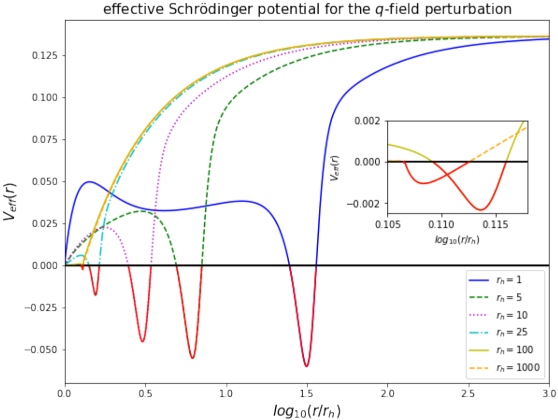


Classical stability for spherically symmetric perturbations

Do growing perturbations around the numerical solutions exist somewhere in the parameter space of solutions?
We consider only spherically symmetric perturbations.

$$\tilde{q}(r,t) = q(r) + \delta q(r,t)$$
$$\tilde{f}(r,t) = f(r)(1 - h_1(r,t))$$
$$\tilde{h}(r,t) = h(r)(1 - h_2(r,t))$$

h_1 and h_2 may be written in terms of δq. The linearized field equations for δq reduce to an effective Schrödinger equation. We have the relation $E_{min} < 0 \Leftrightarrow$ classically unstable.


effective Schrödinger potential for the q-field perturbation
$V_{eff}(r)$
0.125
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$log_{10}(r/r_h)$
0.002
0.000
−0.002
0.105
0.110
0.115
$r_h = 1$
$r_h = 5$
$r_h = 10$
$r_h = 25$
$r_h = 100$
$r_h = 1000$

Conclusions

- Both at the center $r \to 0$ and asymptotically for $r \to \infty$ the solutions asymptote towards Schwarzschild spacetime.
- In the limit $r_h \to 0$ the region of change of the q-field is moving away from the horizon. A Schwarzschild-anti de Sitter spacetime with (negative) cosmological constant $\Lambda = 8\pi G \rho(q_{shoot}(0))$ results locally to a good approximation.
- In the limit $r_h \to \infty$ the region of change of the q-field approaches the event horizon relative to its size and finally saturates. It then forms a thin shell. This shell carries negative energy density in the horizon proximity and positve energy density further outside.

- The positive energy contribution dominates such that $0 < m(r=0) < lim_{r \to \infty} m(r)$. The integration of the energy density over the shell thus yields a total positive energy resulting in the ADM mass perceived by the distant observer.
- The parameter space of solutions is two and not three dimensional.
- The black hole spacetime is classically (s-wave) stable for sufficiently large black holes.
- open questions: What about including rotation and electric charge? Is classical stability given in general?...

Thank you for your attention!