Fundação para a Ciência e a Tecnologia

LABORATÓRIO DE INSTRUMENTAÇÃO E FÍSICA EXPERIMENTAL DE PARTÍCULAS

Feynman Diagram Complexity in the High Energy Limit

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Outline

- Introduction
- The Reggeon
- The Pomeron (BFKL equation), the Odderon (BKP equation)… more than 3 Reggeons
- Iterative numerical solutions
- Outlook

Defining the context

- Perturbative QCD
- High energy scattering
- BFKL equation (Balitsky-Fadin-Kuraev-Lipatov)
- BKP equation (Bartels-Kwiecinski-Praszalowicz)
- Pomeron (Named after Pomeranchuk)
- Odderon (Lukaszuk & Nicolescu)
- Reggeon

Intro: Importance of high energy QCD

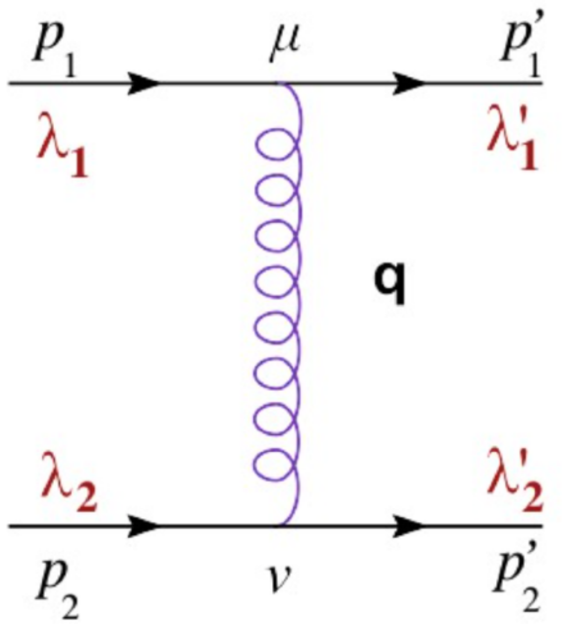
- High energy QCD studies only a part of the phase space, a certain limit, the limit of scattering at very high energies, however, there is a plethora or things we access from studying that limit:

  - Integrability
  - Gravity
  - AdS/CFT
  - BDS amplitudes
  - Factorization
  - Separation between transverse and longitudinal d.o.f.
  - Transition from hard to soft scale physics

- And this is only from the ‘pure’ theory point of view

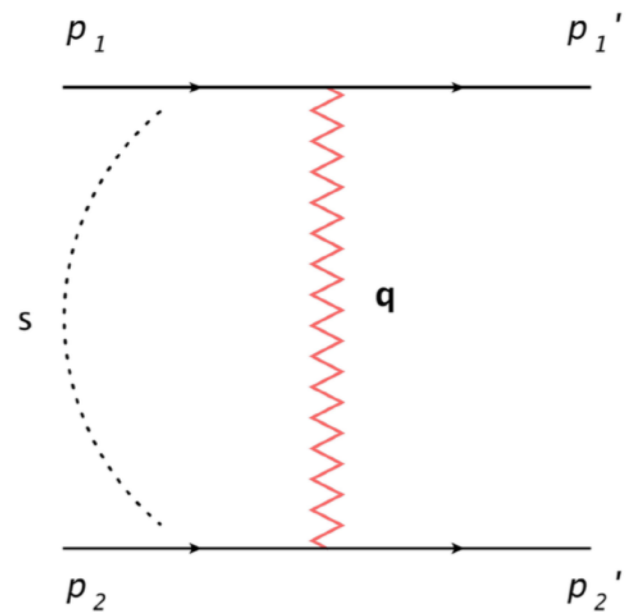
The Reggeon

A normal gluon propagator: $D_{\mu\nu}(s,q^2) = -i\frac{g_{\mu\nu}}{q^2}$



The reggeized gluon is a gluon with modified propagator:

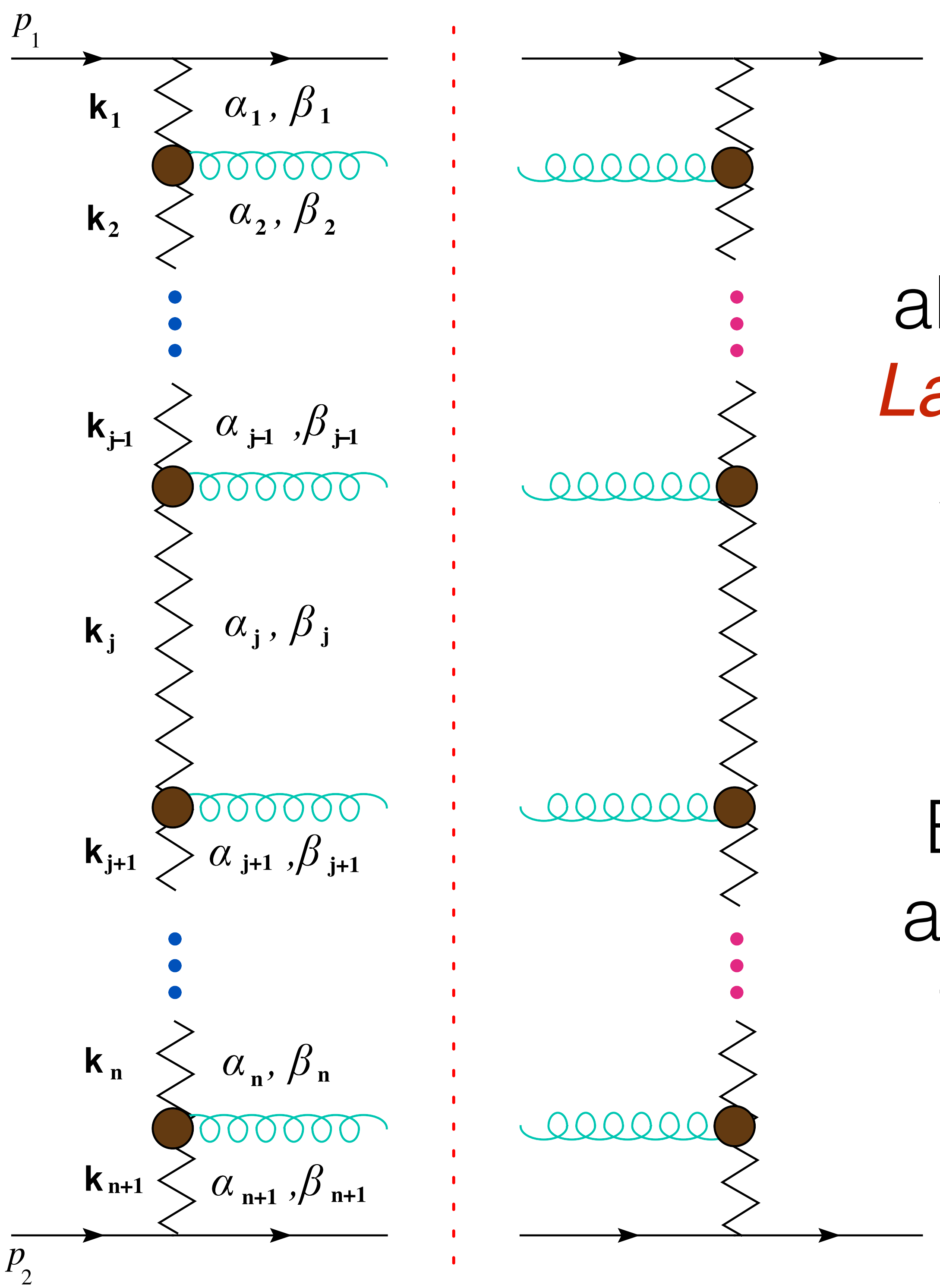
$$D_{\mu\nu}(s,q^2) = -i\frac{g_{\mu\nu}}{q^2} \left(\frac{s}{\mathbf{k}^2}\right)^{\omega(q^2)}$$



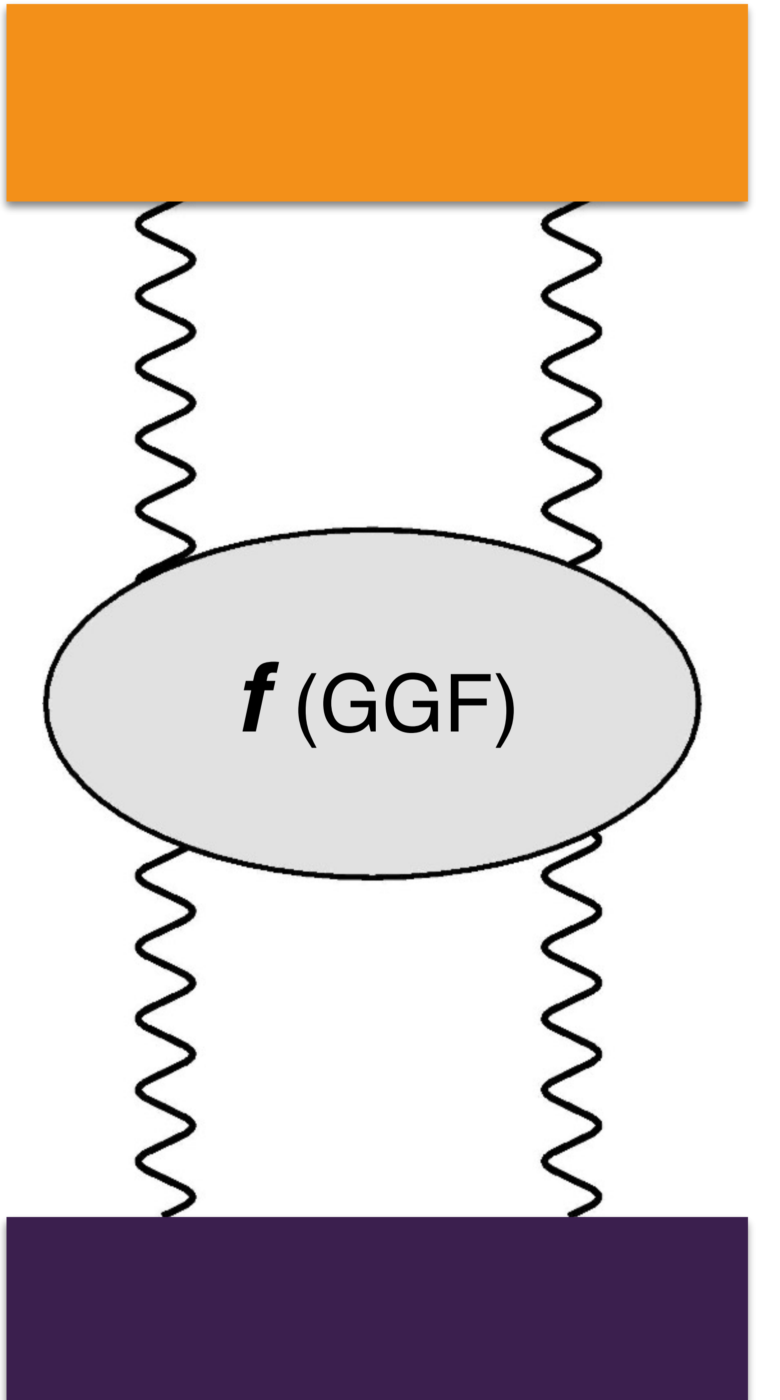
(where $\mathbf{k}^2$ is a hard scale in the process at hand)

From now on, vertical propagators represent Reggeons

Large logs from real emission corrections

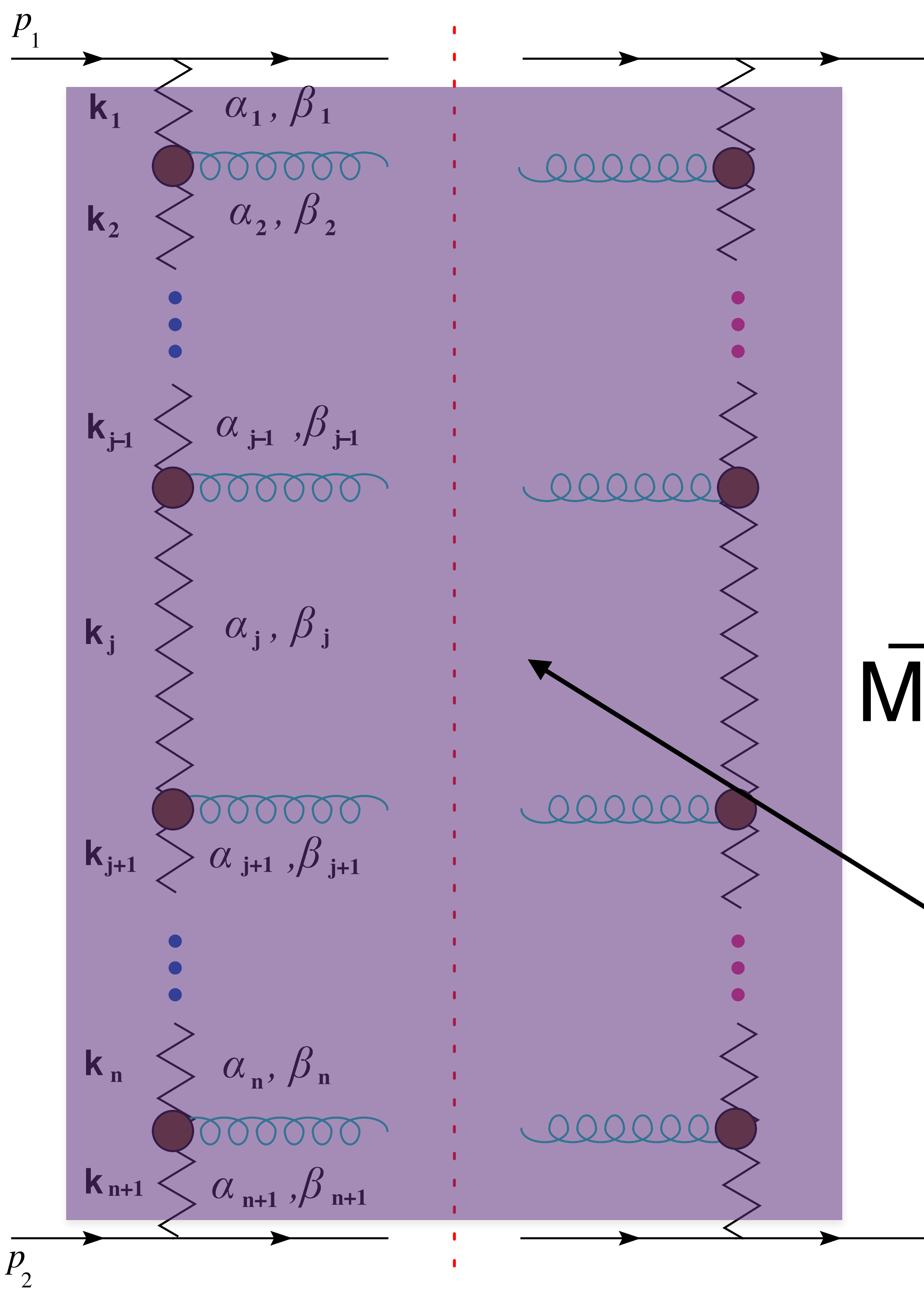


To resum to all n, use BFKL.
Ladder diagrams
An important notion:
KERNEL ITERATION
Each iteration adds one **rung** to the **ladder** diagram



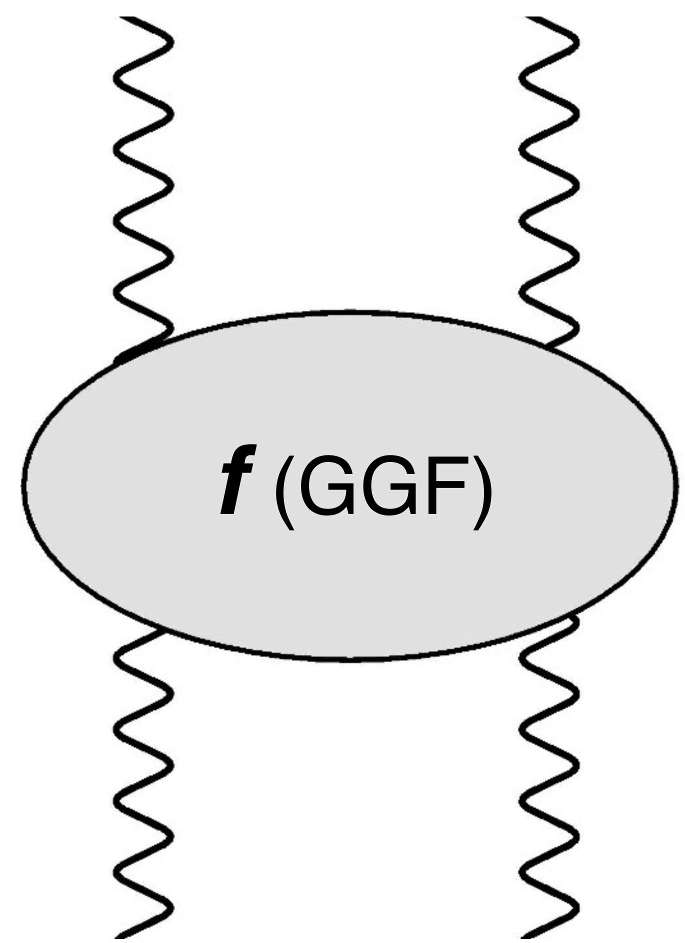
The Pomeron

From now on, ‘rungs’ instead of real emissions



BFKL

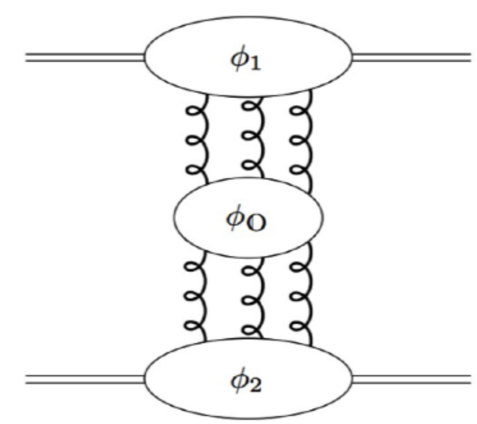
Monte Carlo

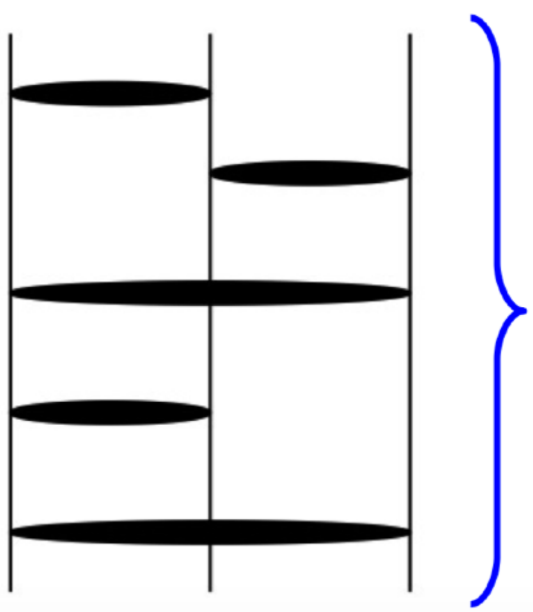


Many many
phase-space diagrams

Pomeron vs Odderon

- Pomeron is the state of two interacting reggeized gluons in the t-channel in the color singlet. It has the quantum numbers of the vacuum
- Odderon is the state of three interacting gluons exchanged in the t-channel in the color singlet but with C =-1 and P=-1
- Any pair of two gluons in the Odderon forms symmetric color octet subsystems

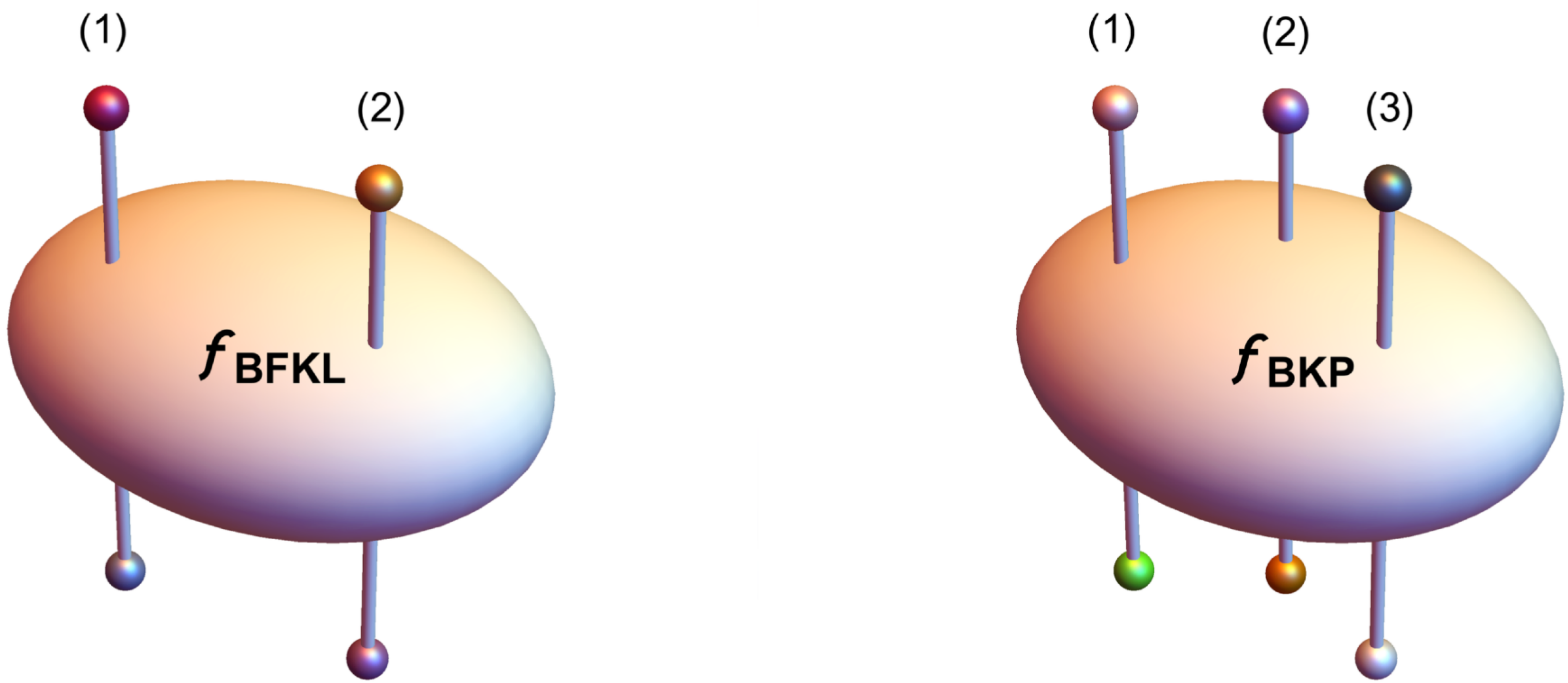




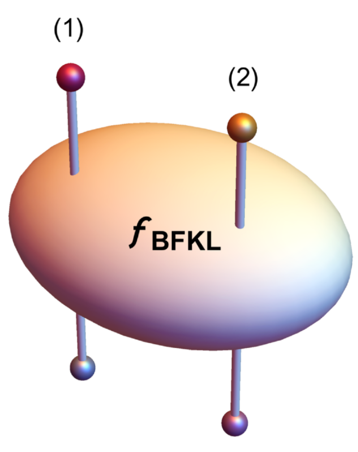
Ladder structure of the Odderon. BKP resums term of the form $\alpha_s(\alpha_s \log s)^n$

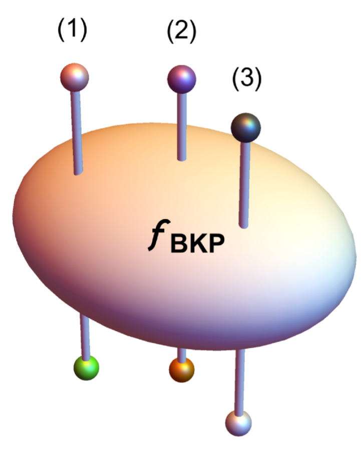
NLO corrections recently available
Bartels, Fadin, Lipatov, Vacca (2012)

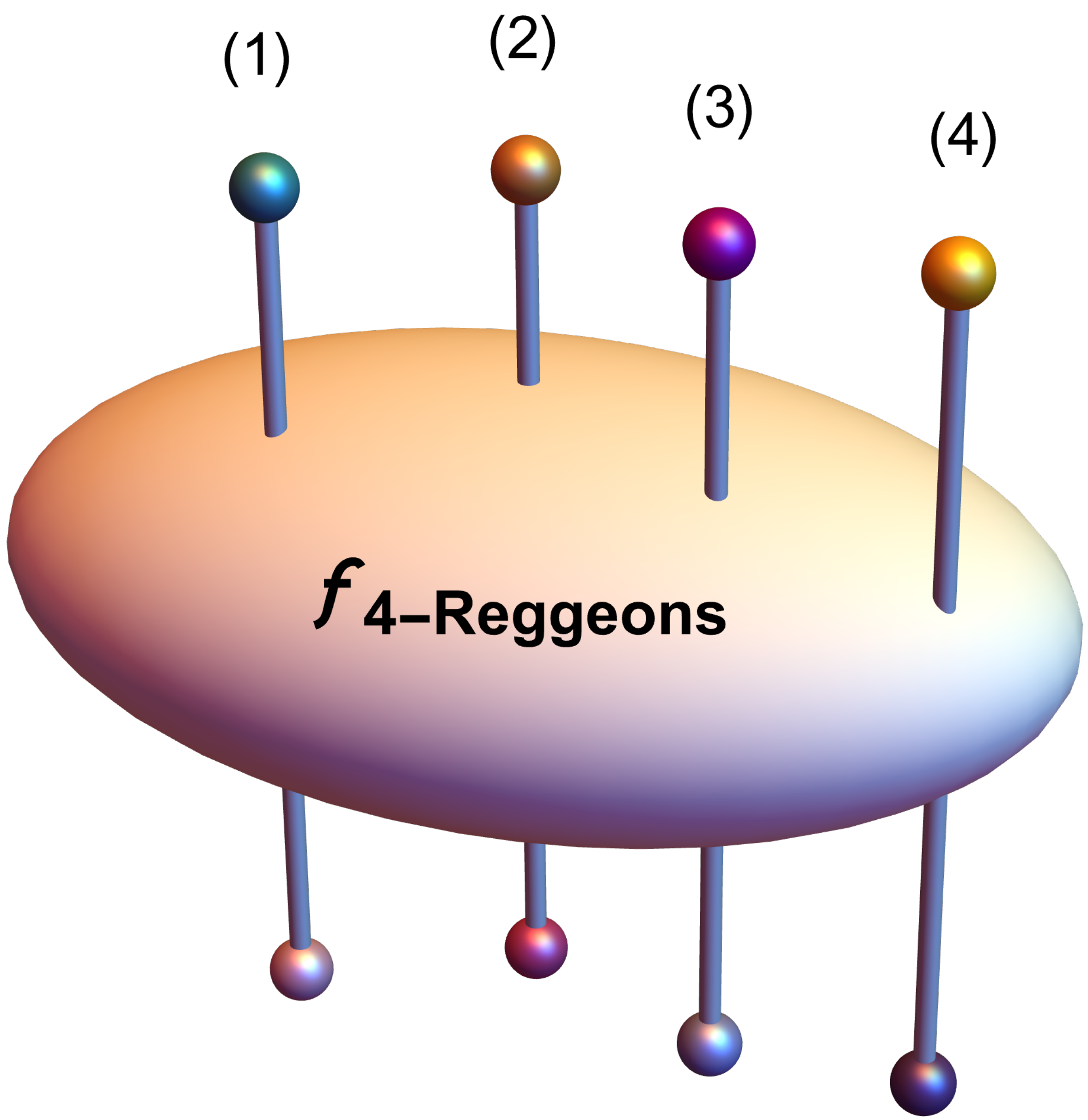
From two to three Reggeons



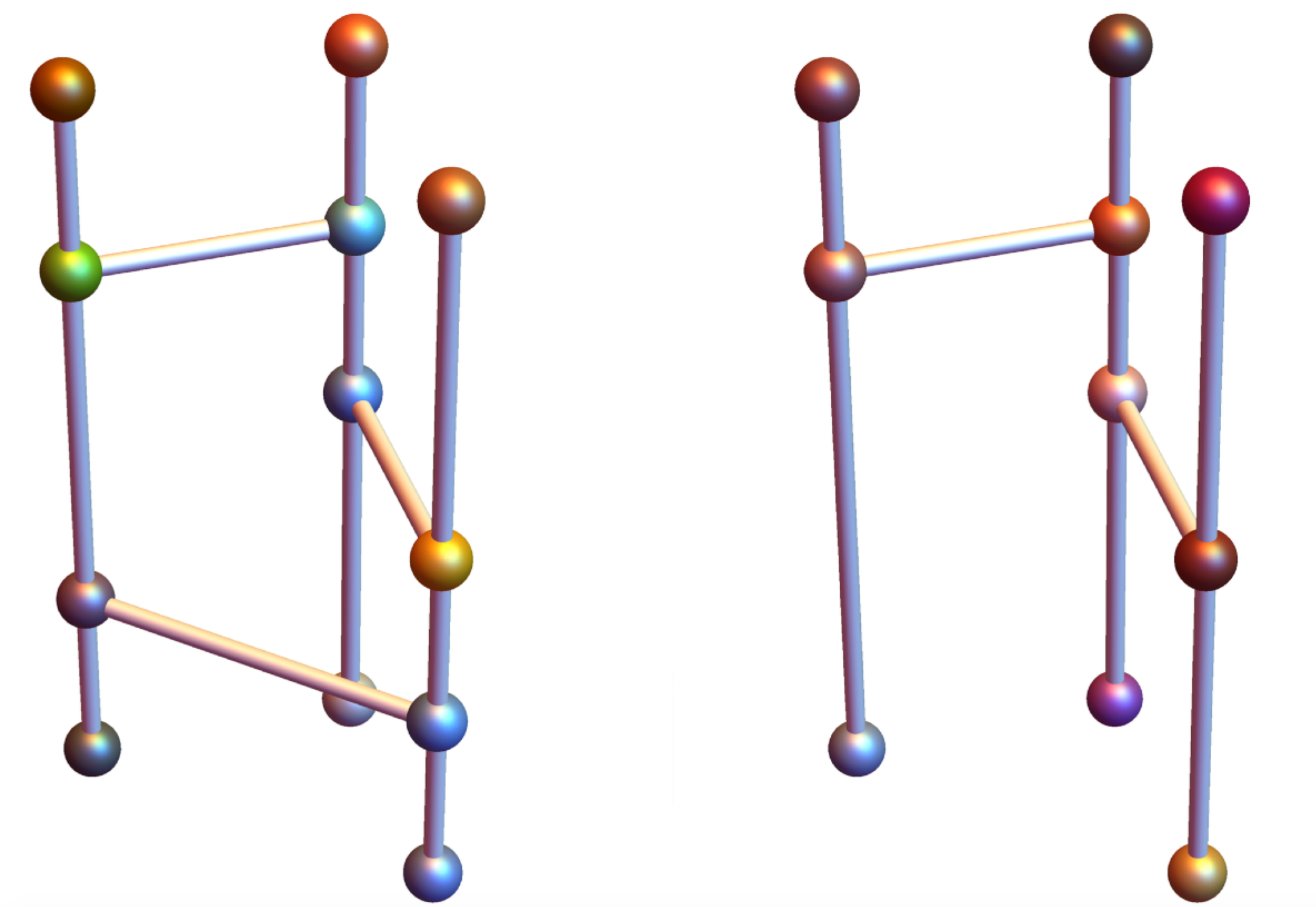
… and four Reggeons







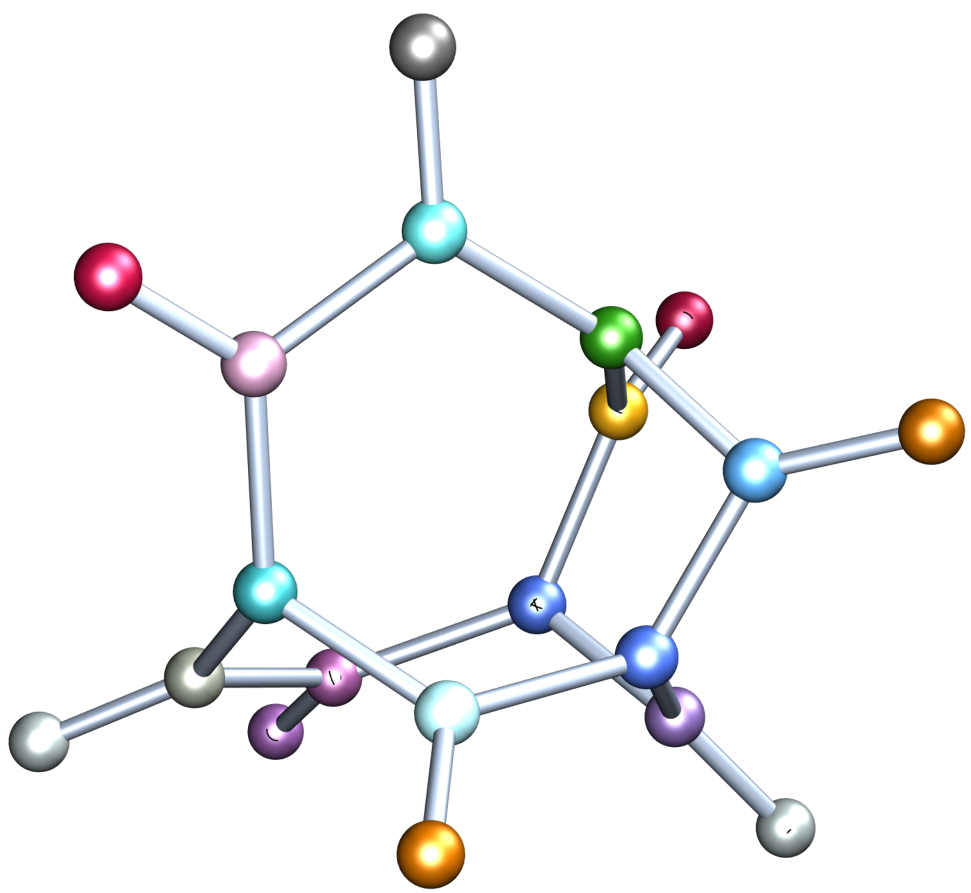
Closed vs Open

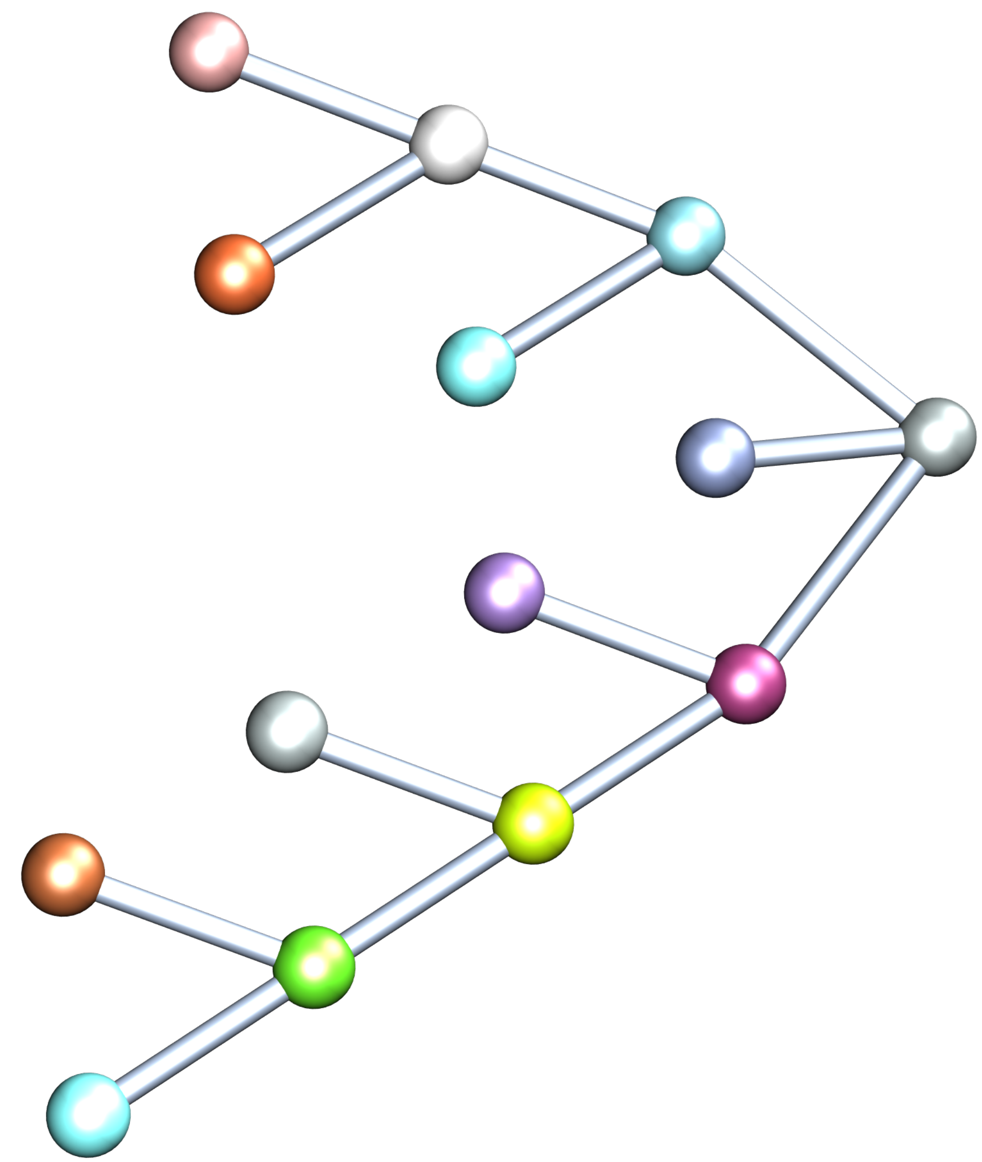


BKP was found to have a hidden integrability being equivalent to a periodic spin chain of a XXX Heisenberg ferromagnet. This was the first example of the existence of integrable systems in QCD

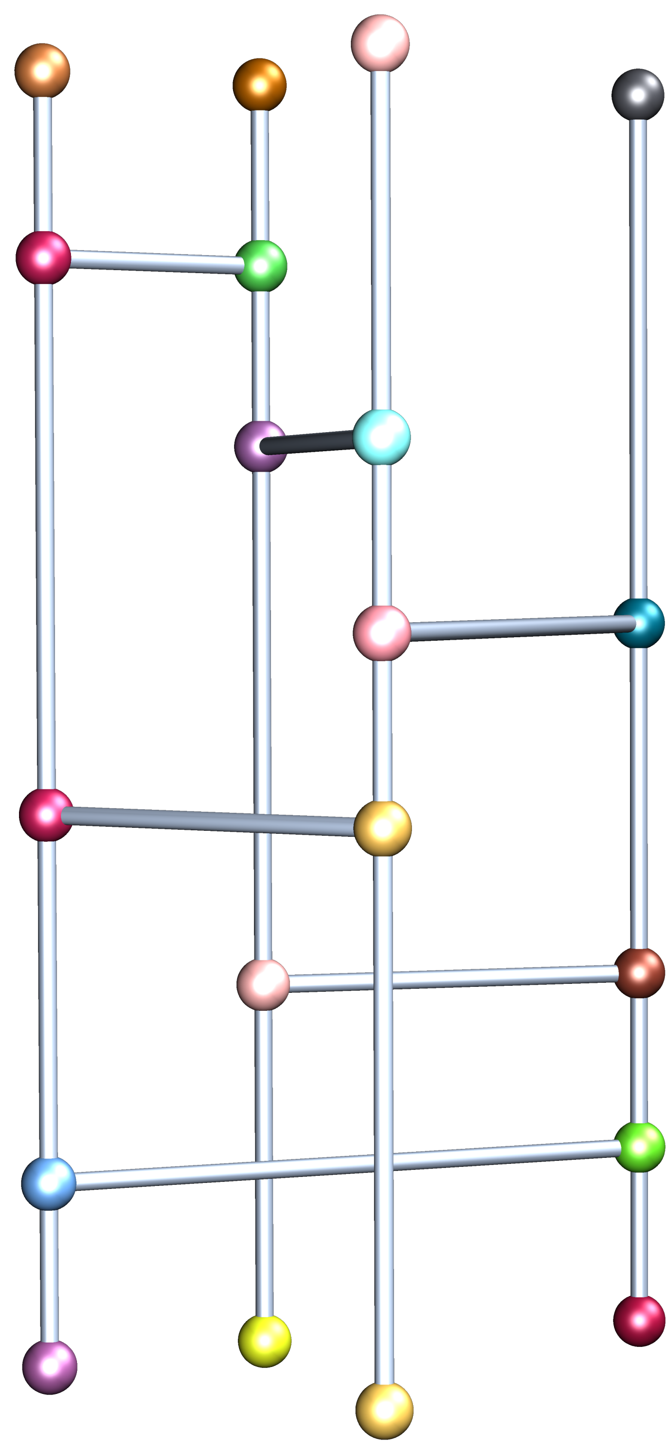
Lipatov (1986, 1990, 1993)

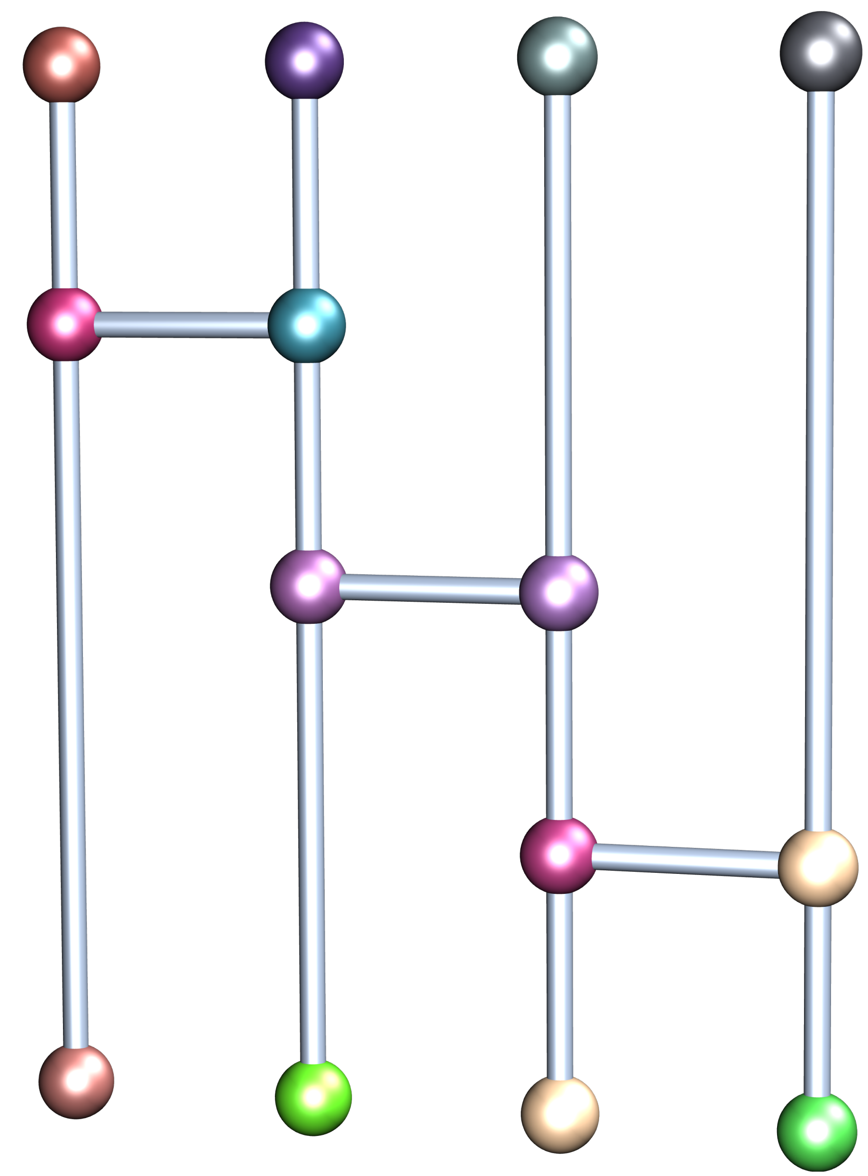
Closed vs Open



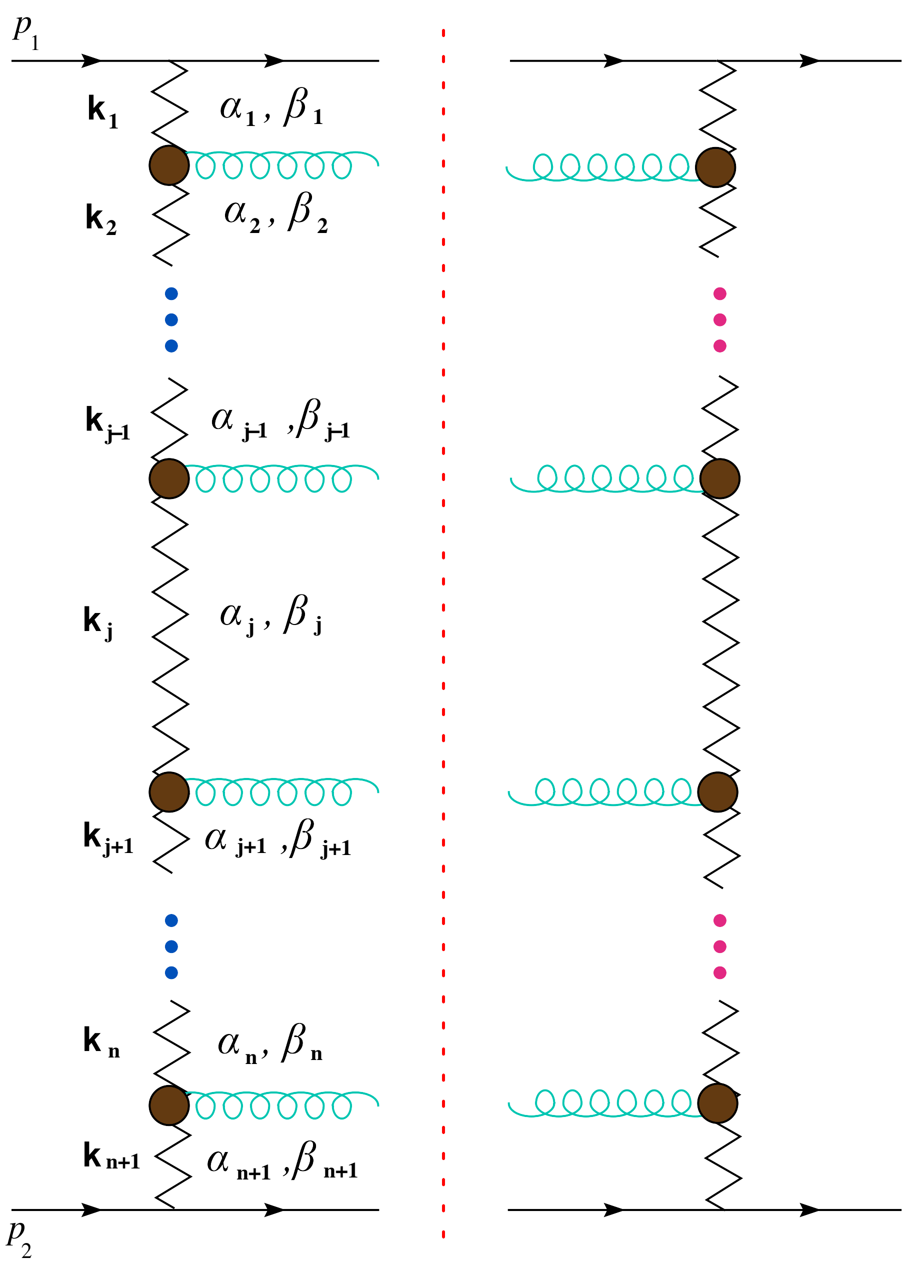


Closed vs Open

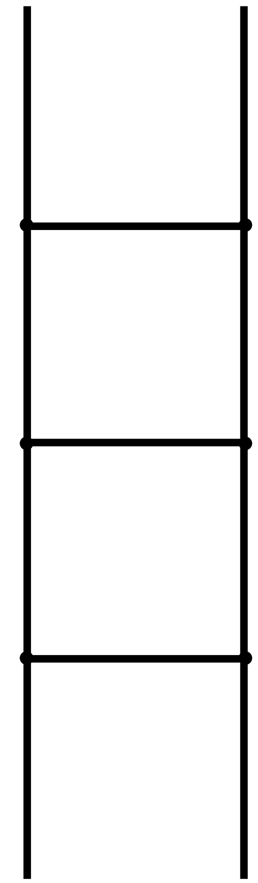




Let us iterate

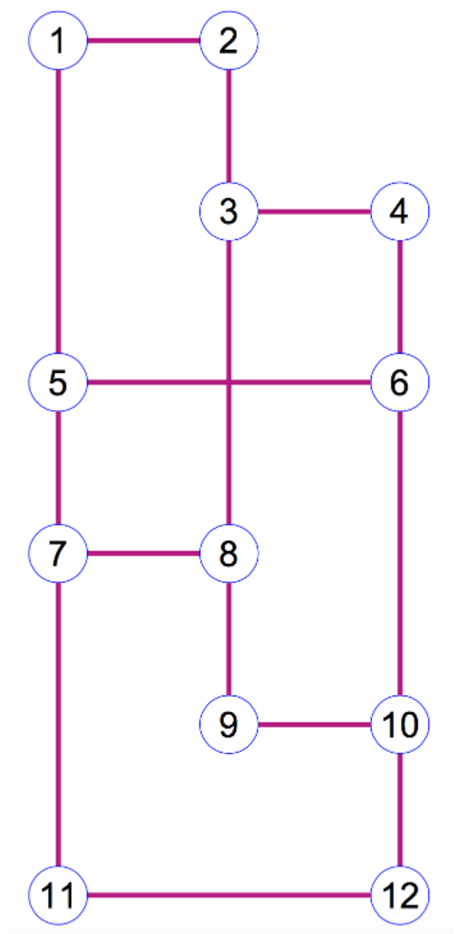


OK



BFKL

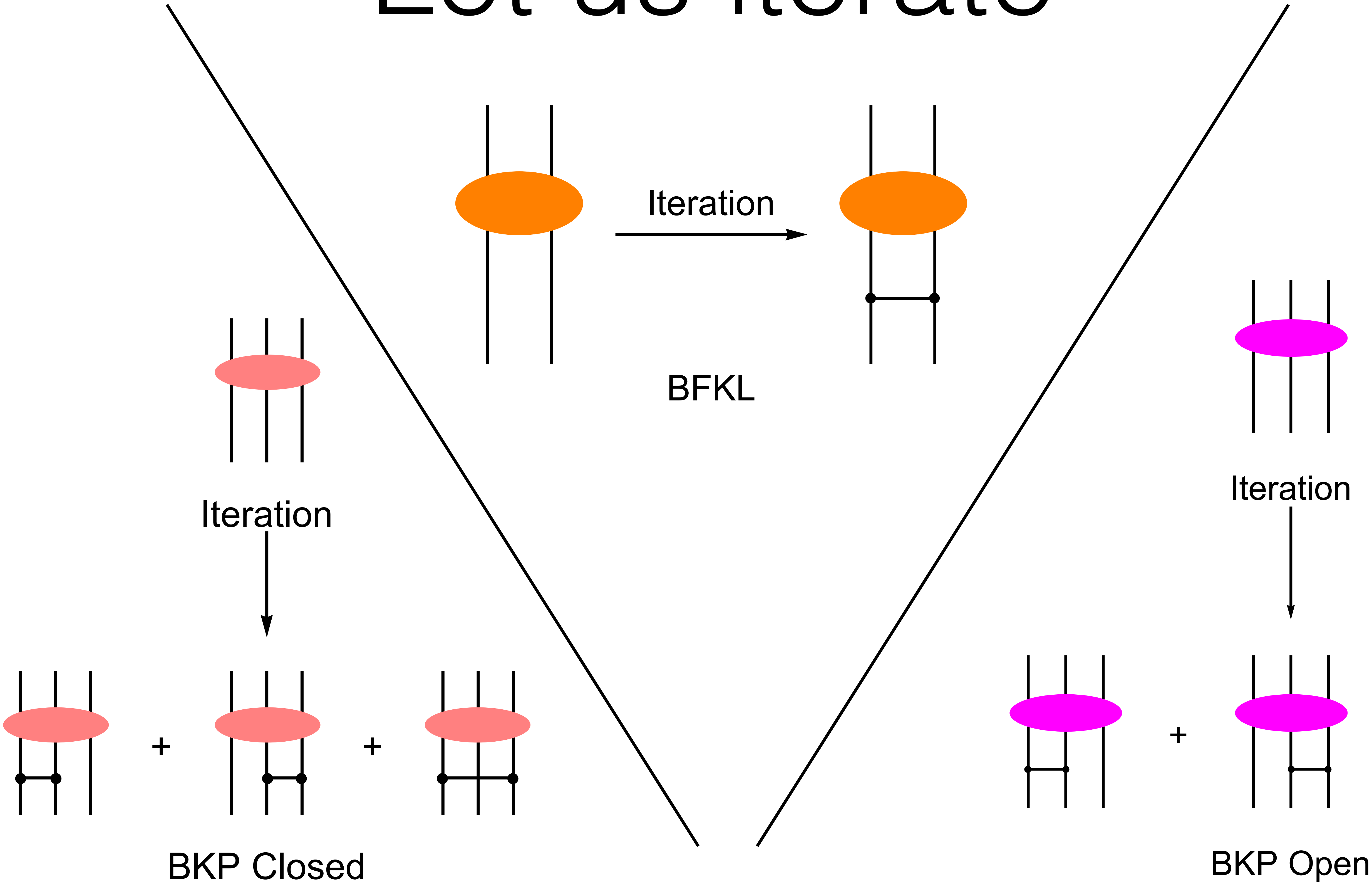
?



BKP

Vertical lines are Reggeons
horizontal ones are gluons

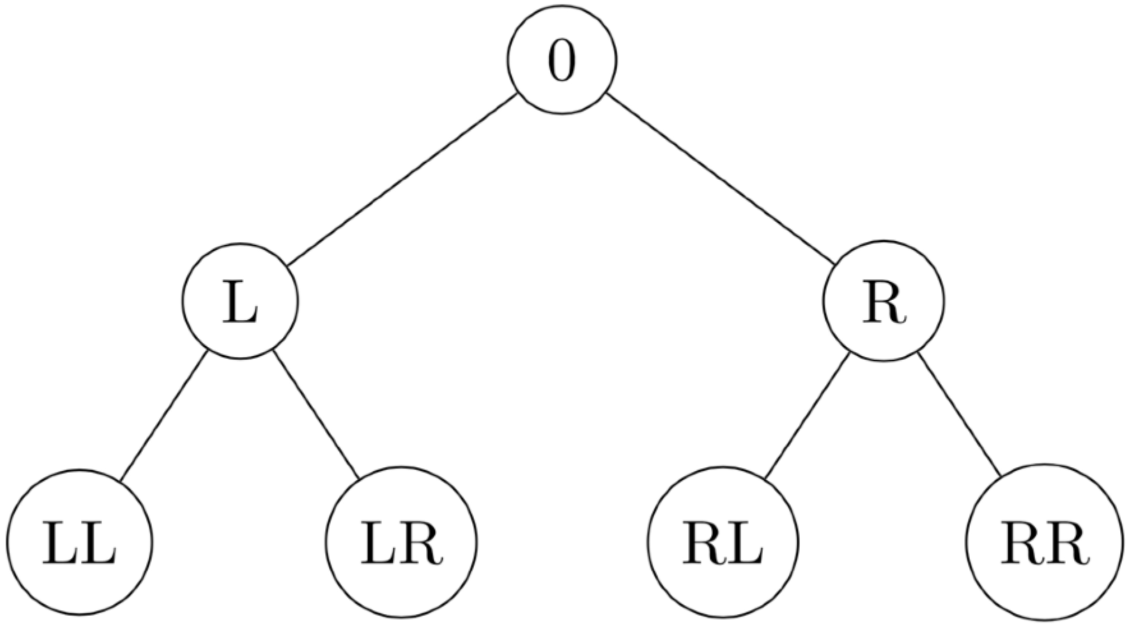
Let us iterate

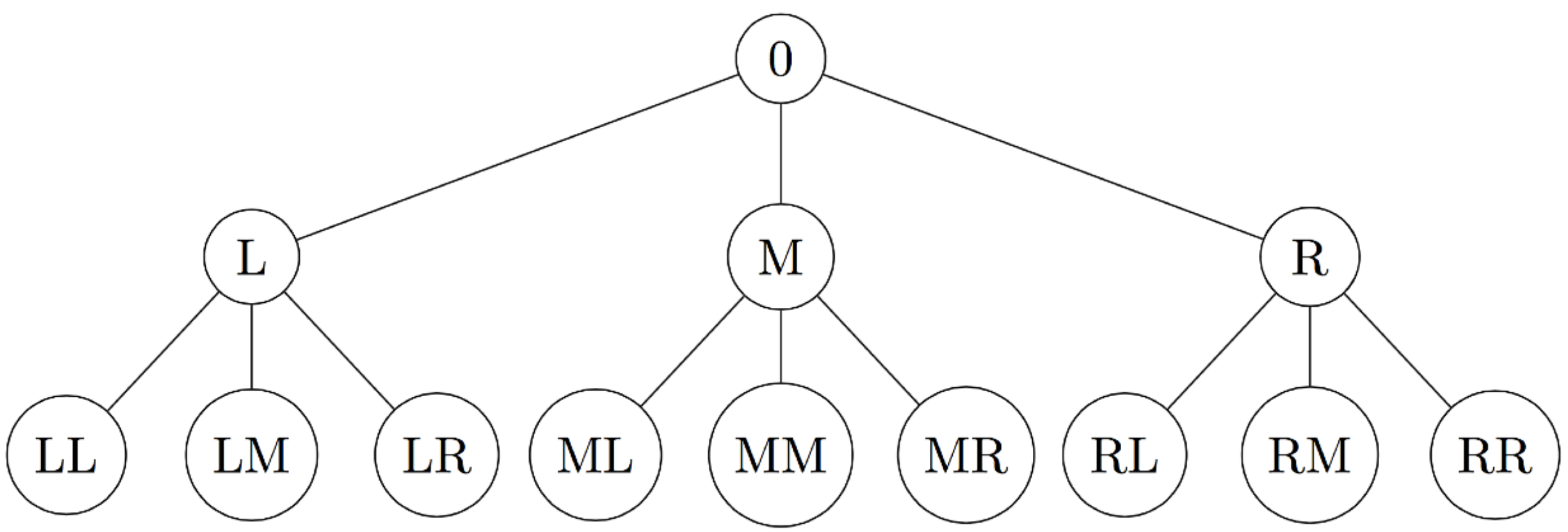


Binary/ternary tree structure

number of diagrams:
2^n (open) and 3^n (closed)

n rungs	Number of diagrams
2	9
3	27
4	81
5	243
6	729
7	2187
8	6561
9	19683
10	59049
11	177147
12	531441
13	1594323
14	4782969.

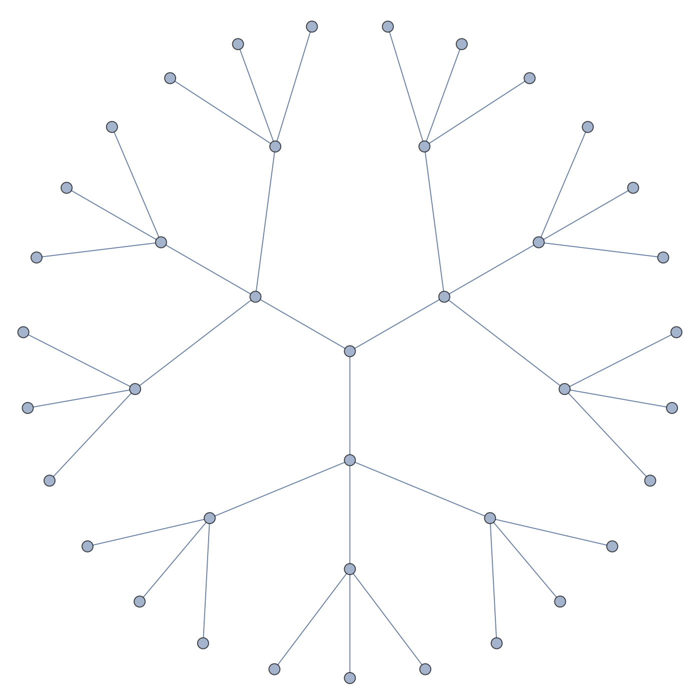


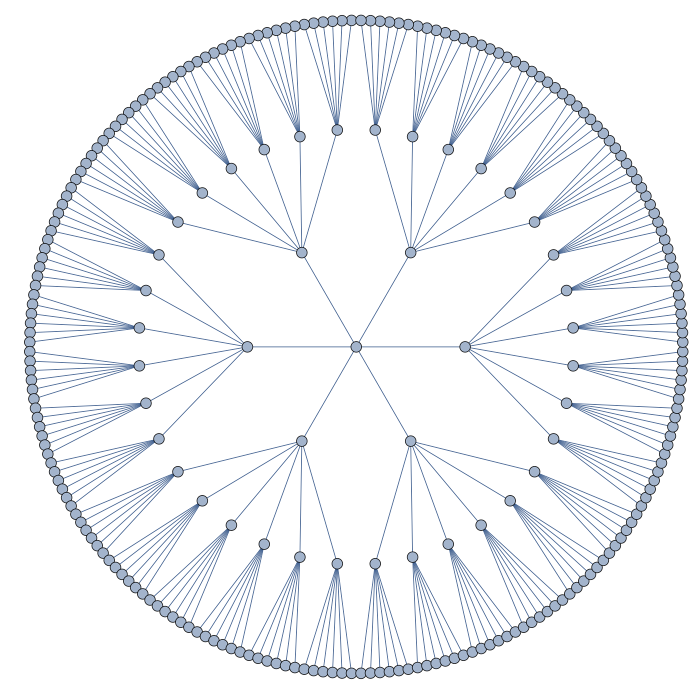


Ternary/senary tree structure

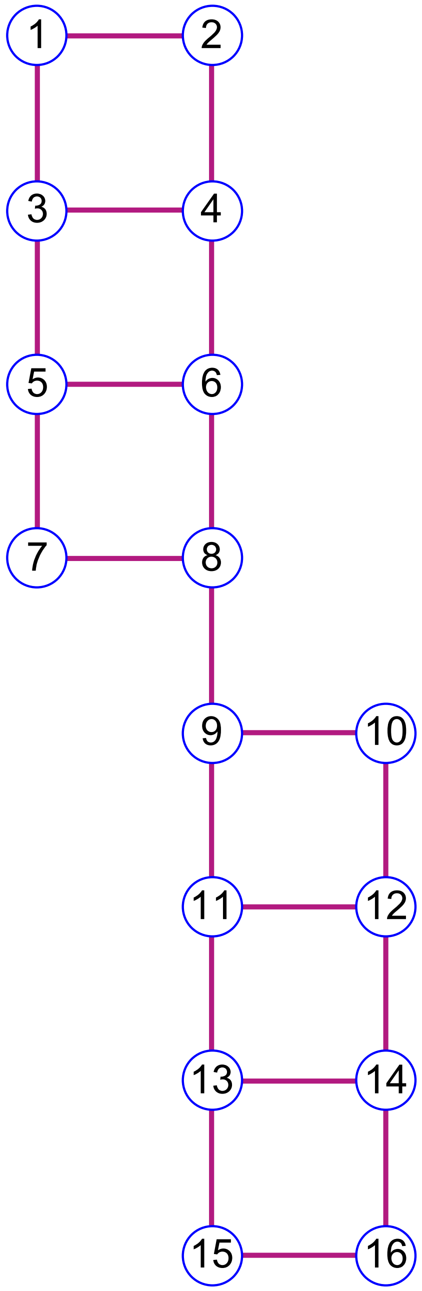
number of diagrams:
3^n (open) and 6^n (closed)

n-rungs	number of diagrams
2	36
3	216
4	1296
5	7776
6	46656
7	279936
8	1679616
9	10077696
10	60466176
11	362797056
12	2176782336
13	13060694016
14	78364164096





Topologies



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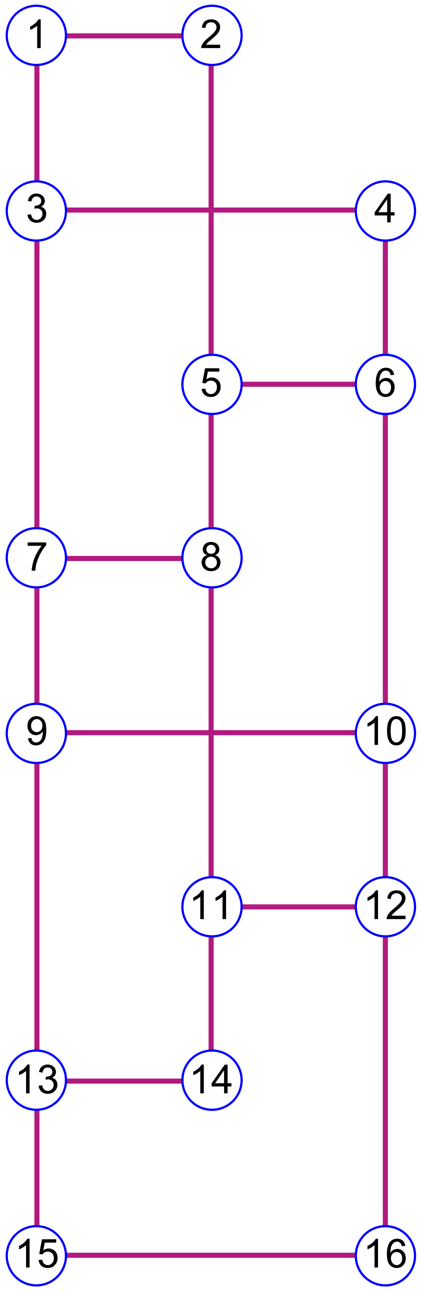

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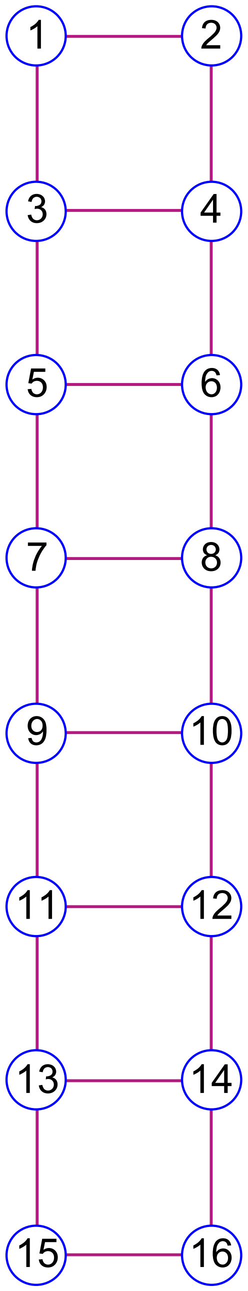

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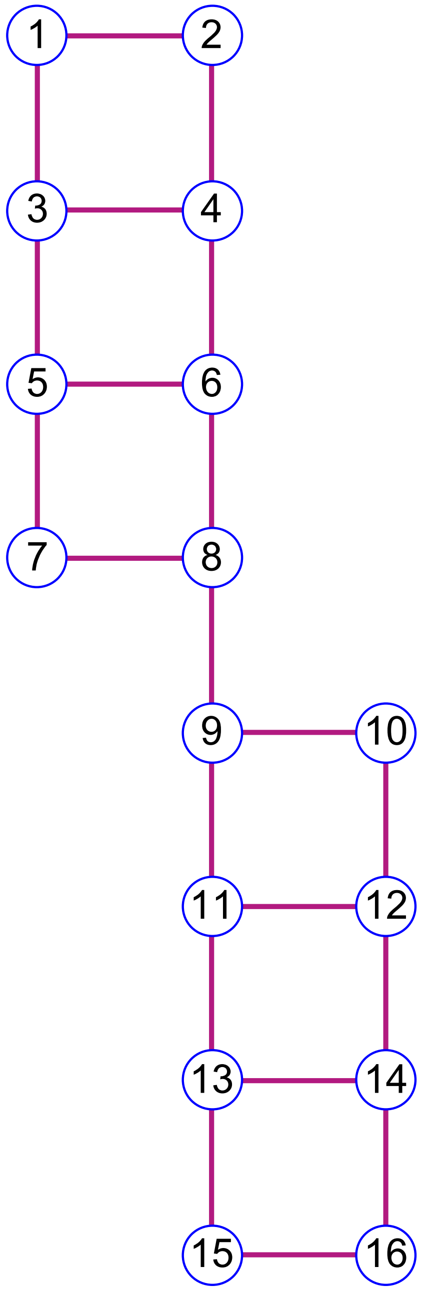
Topologies & complexity



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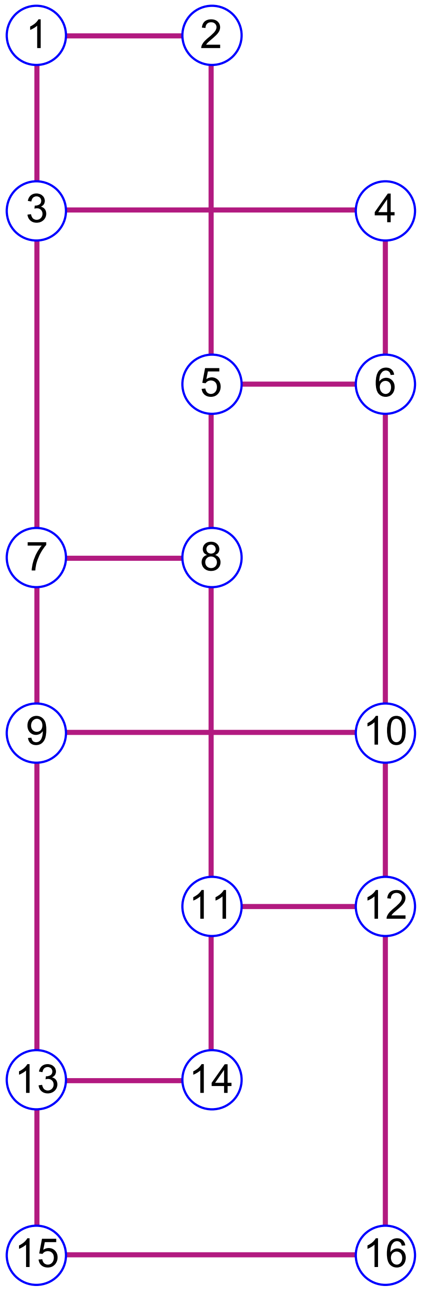

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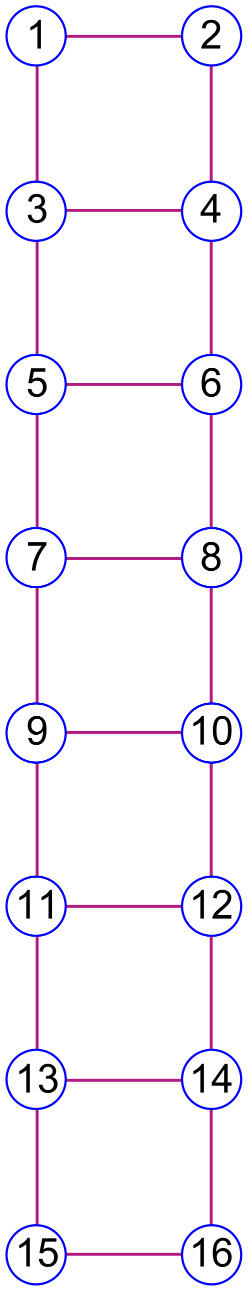

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Graph Complexity

The matrix-tree theorem (Kirchhoff, 1847)

A spanning tree T of an undirected graph G is a subgraph that is a tree which includes all of the vertices of G, with minimum possible number of edges.

The complexity of an undirected connected graph corresponds to the number of all possible spanning trees of the graph.

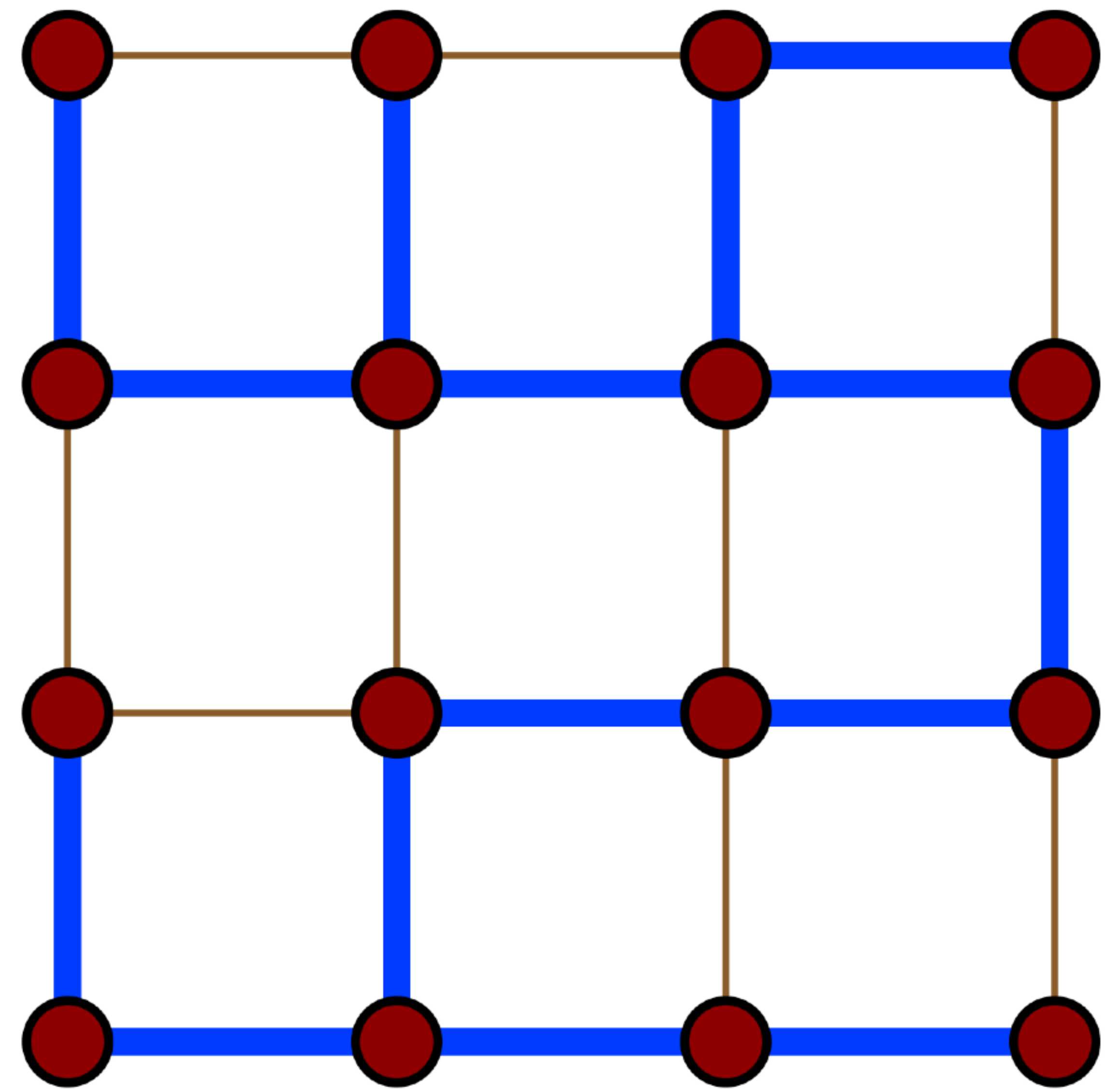
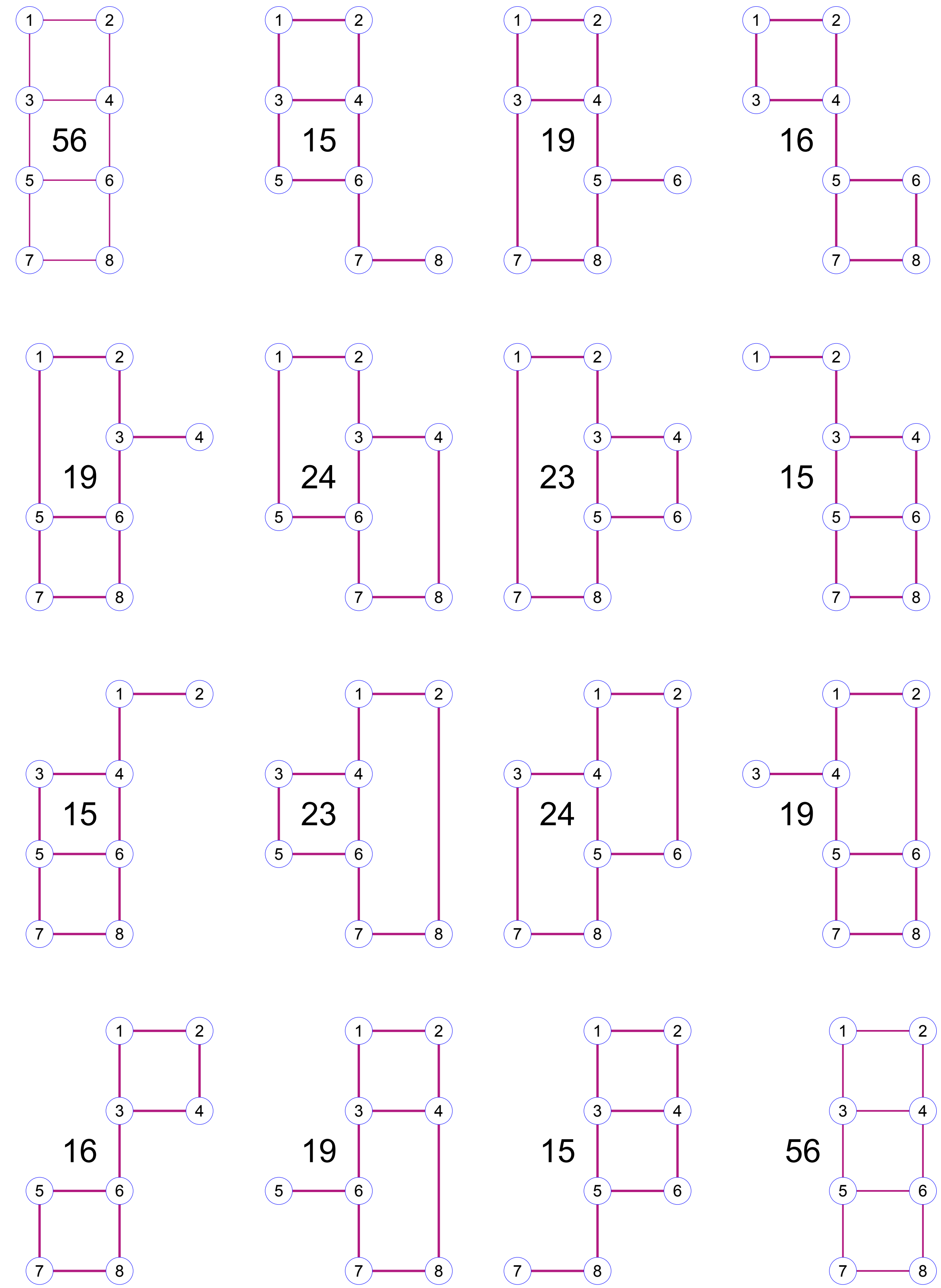
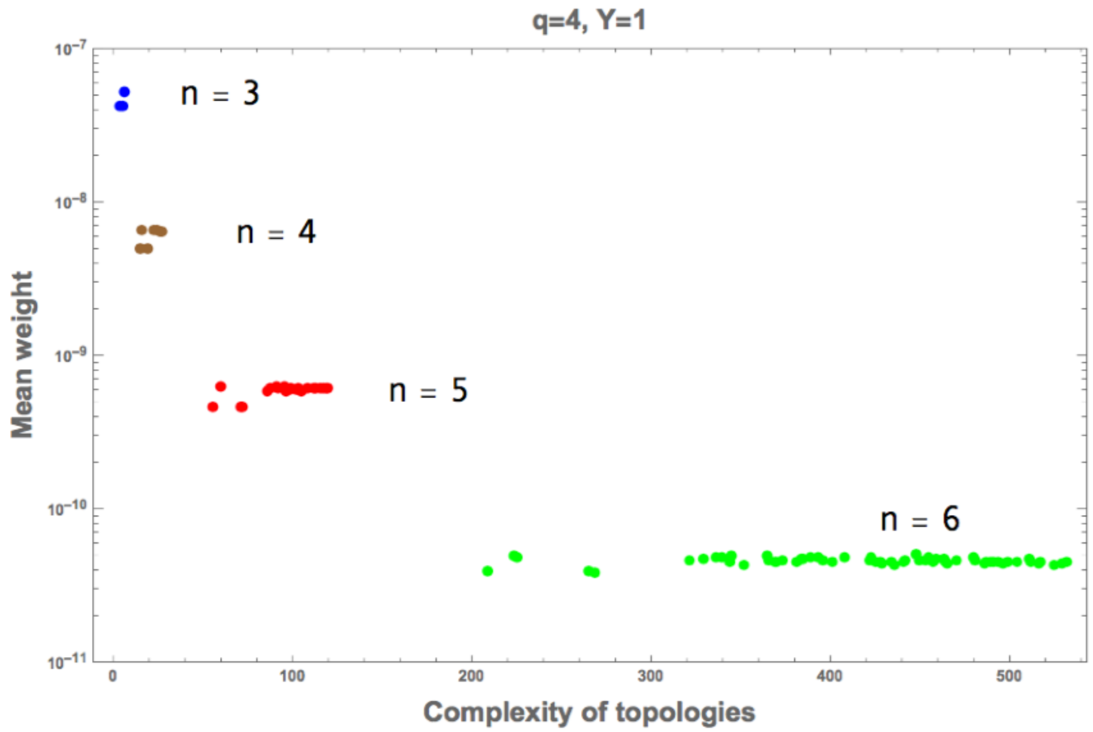


Figure from
https://en.wikipedia.org/wiki/Spanning_tree

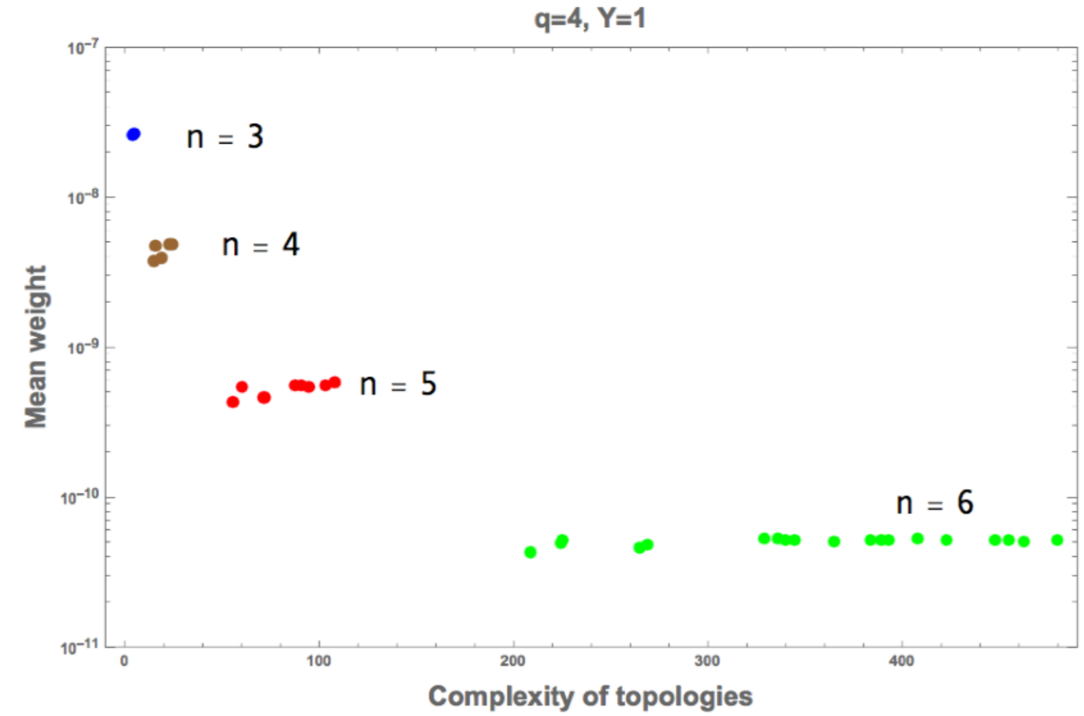
Open 4-rung diagrams & complexities



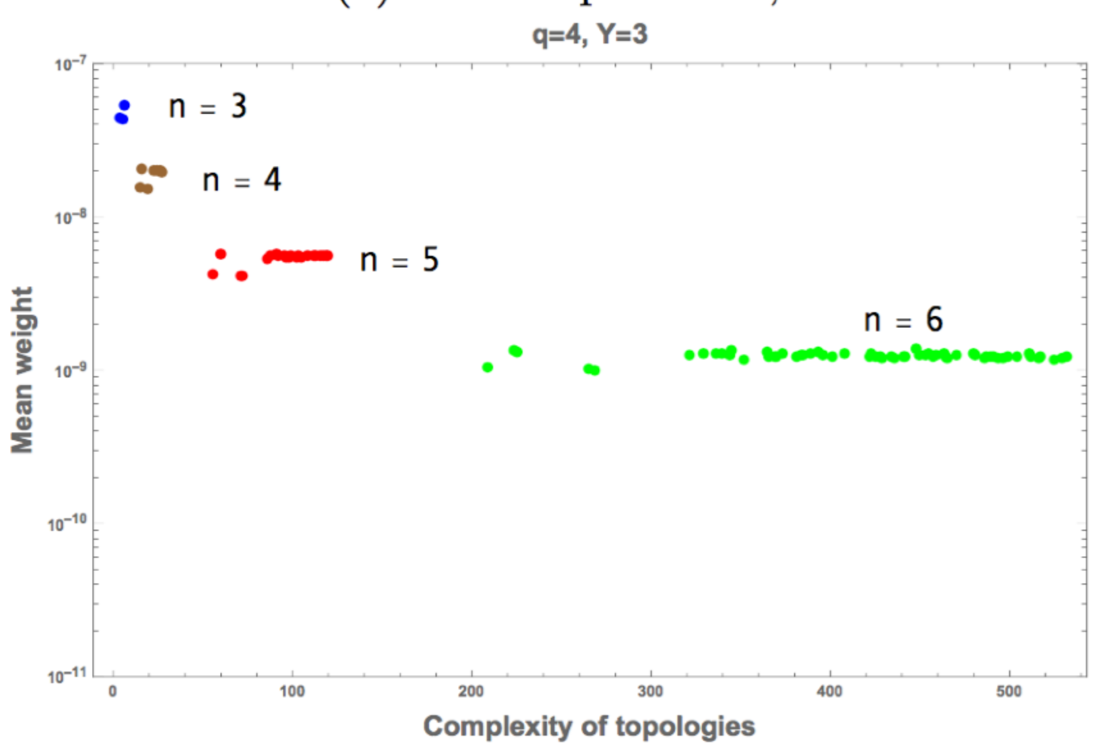
Average weight per complexity



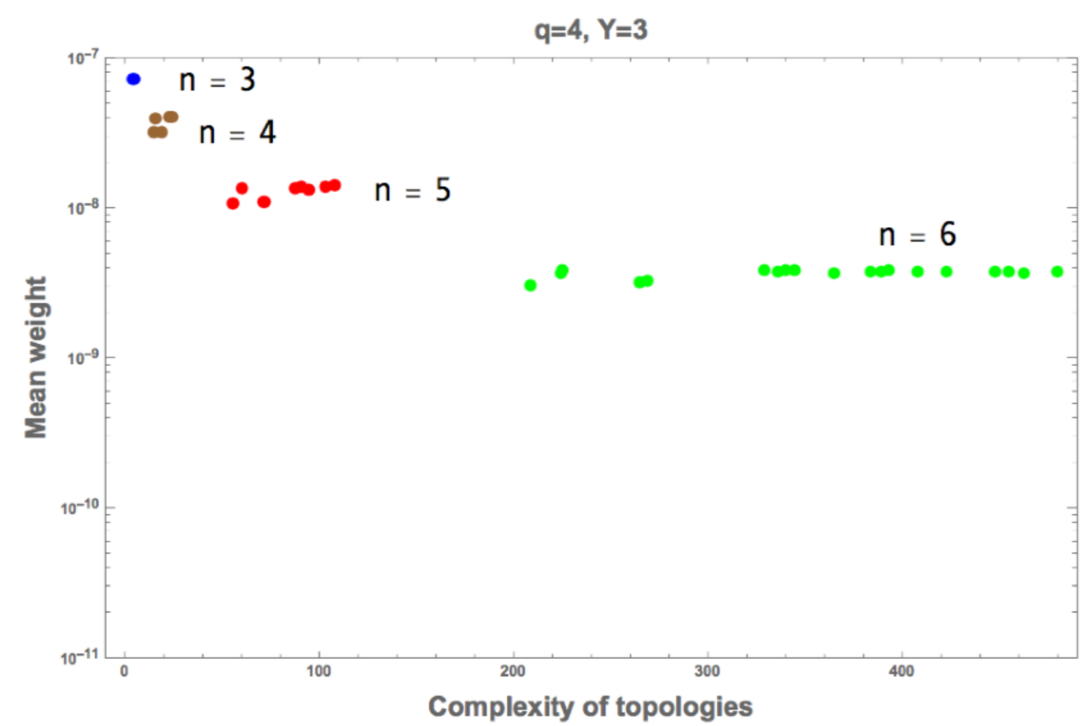
(a) Closed spin chain, $Y = 1$.



(a) Open spin chain, $Y = 1$.



(b) Closed spin chain, $Y = 3$.



(b) Open spin chain, $Y = 3$.

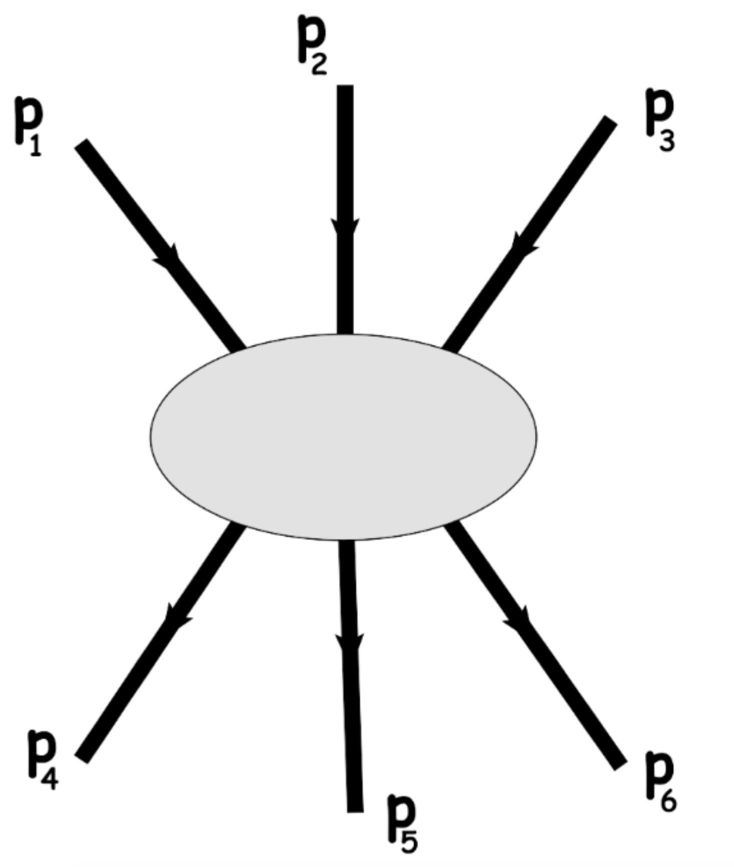
Conclusions and Outlook

- We have used a Monte Carlo numerical integration of iterated integrals in transverse momentum and rapidity space to solve the BKP equation with three Reggeized gluons in the t-channel.
- Numerical convergence of the solution is achieved after applying the BKP kernel on the initial condition for a finite number of times at a given value of the strong coupling and the center-of-mass energy (in terms of rapidity, Y).
- The formalism can be applied to the BKP equation with a higher number of exchanged Reggeons. It can also be used beyond the leading logarithmic approximation and for cases with a total t-channel color projection not being in the singlet but in the adjoint representation. This is very important for the calculation of scattering amplitudes in $N = 4$ supersymmetric theories in the Regge limit.
- Investigating the connection between the complexity and the numerical contribution of certain type of topologies will make the computation of the total interaction between 4-Reggeons faster and in some cases even feasible. (G.C., A. Sabio Vera and D. Vaccaro, work in progress)
- Our approach also has obvious applications in the study of phenomenological cross sections devoted to the study of the Odderon at hadron colliders.

Back up

Results: kinematical configuration

All vectors live in the transverse momentum space



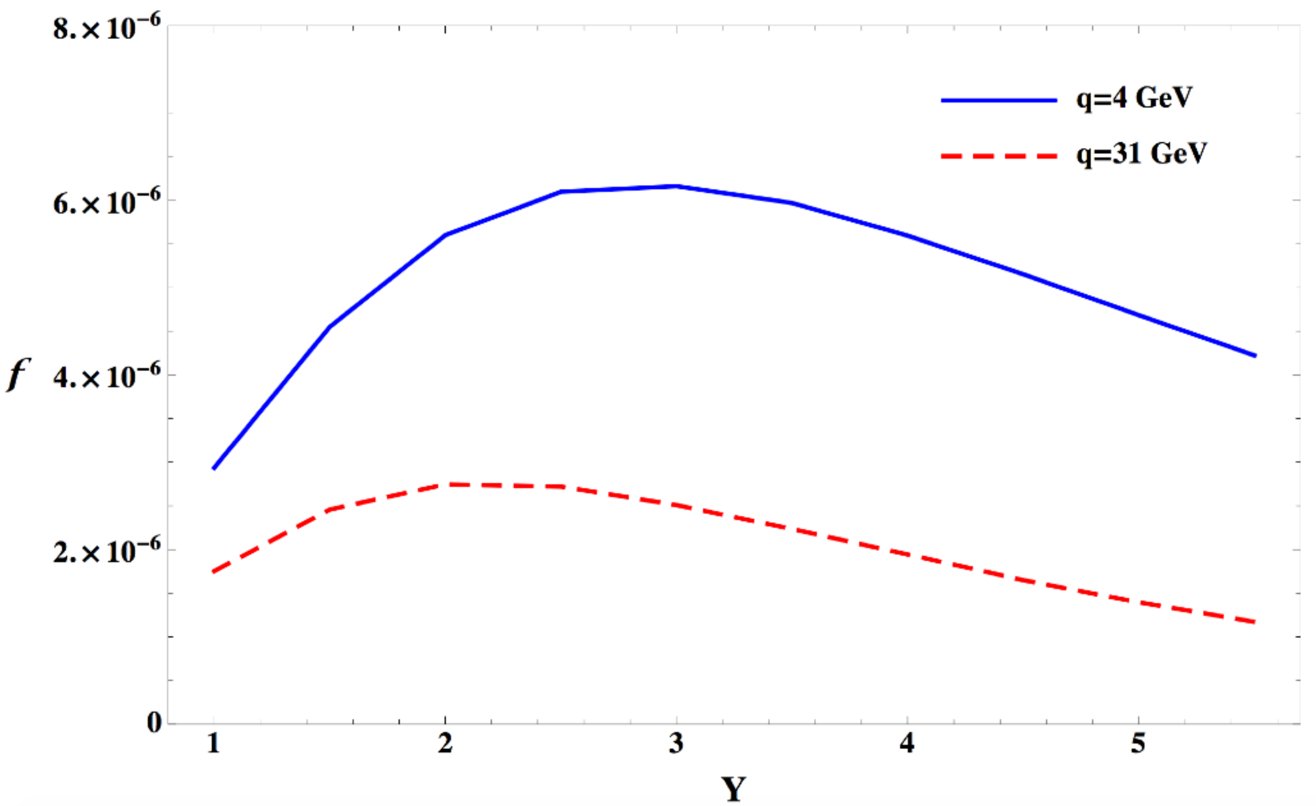
$$\mathbf{q} = (4, 0)$$
$$\mathbf{p}_1 = (10, 0)$$
$$\mathbf{p}_2 = (20, \pi)$$
$$\mathbf{p}_3 = (\mathbf{q} - \mathbf{p}_1) - \mathbf{p}_2 = (14, 0)$$
$$\mathbf{p}_4 = (20, 0)$$
$$\mathbf{p}_5 = (25, \pi)$$
$$\mathbf{p}_6 = (\mathbf{q} - \mathbf{p}_4) - \mathbf{p}_5 = (9, 0)$$

$$\mathbf{q} = (31, 0)$$
$$\mathbf{p}_1 = (10, 0)$$
$$\mathbf{p}_2 = (20, \pi)$$
$$\mathbf{p}_3 = (\mathbf{q} - \mathbf{p}_1) - \mathbf{p}_2 = (41, 0)$$
$$\mathbf{p}_4 = (20, 0)$$
$$\mathbf{p}_5 = (25, \pi)$$
$$\mathbf{p}_6 = (\mathbf{q} - \mathbf{p}_4) - \mathbf{p}_5 = (36, 0)$$

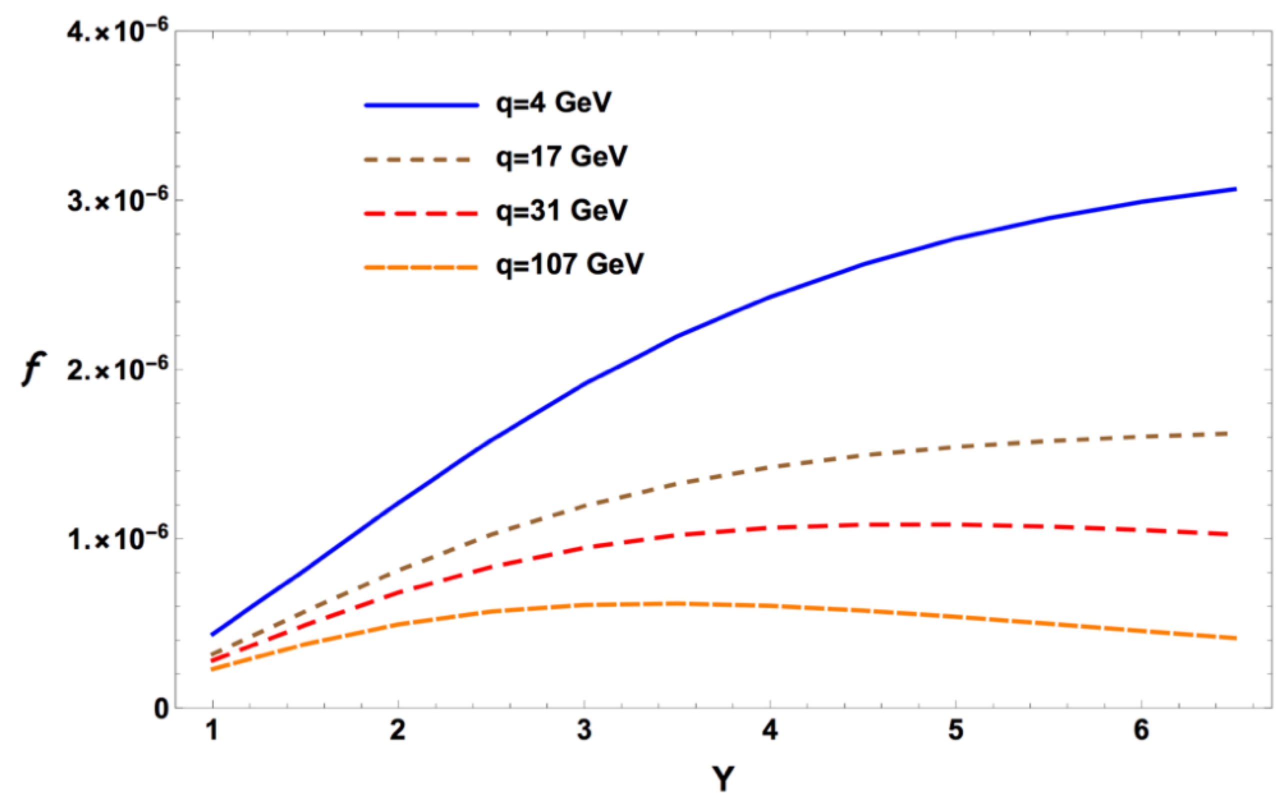
(r,θ): first component is in GeV, the second component in radians

Results: energy plots

Closed

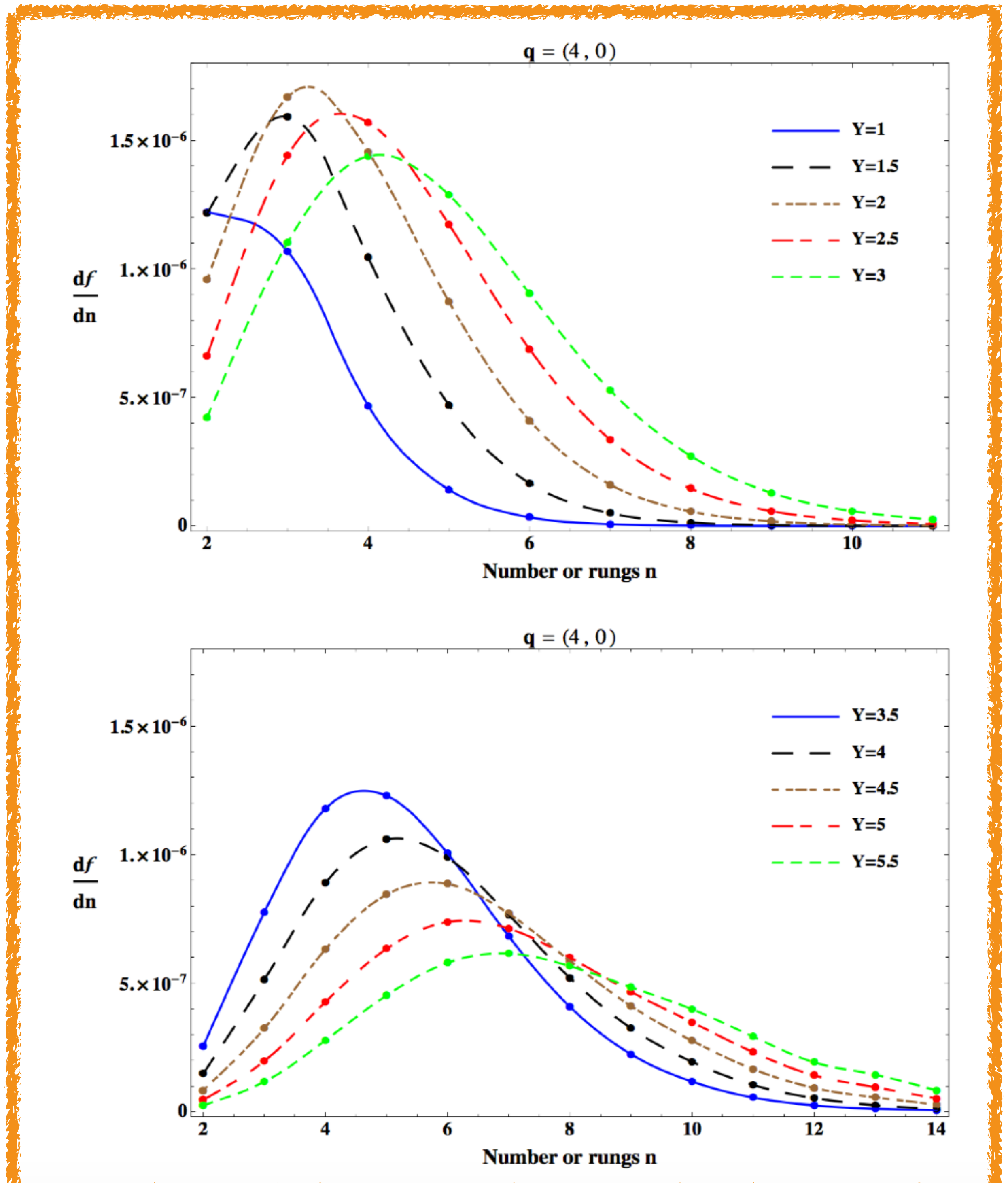


Open



Results: “multiplicity”

Closed



Open