

The Origin of $1/f$ Fluctuations in Music and Physics

(in between random and boredom)

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Special Thanks to

Larissa Bravina for all the setup

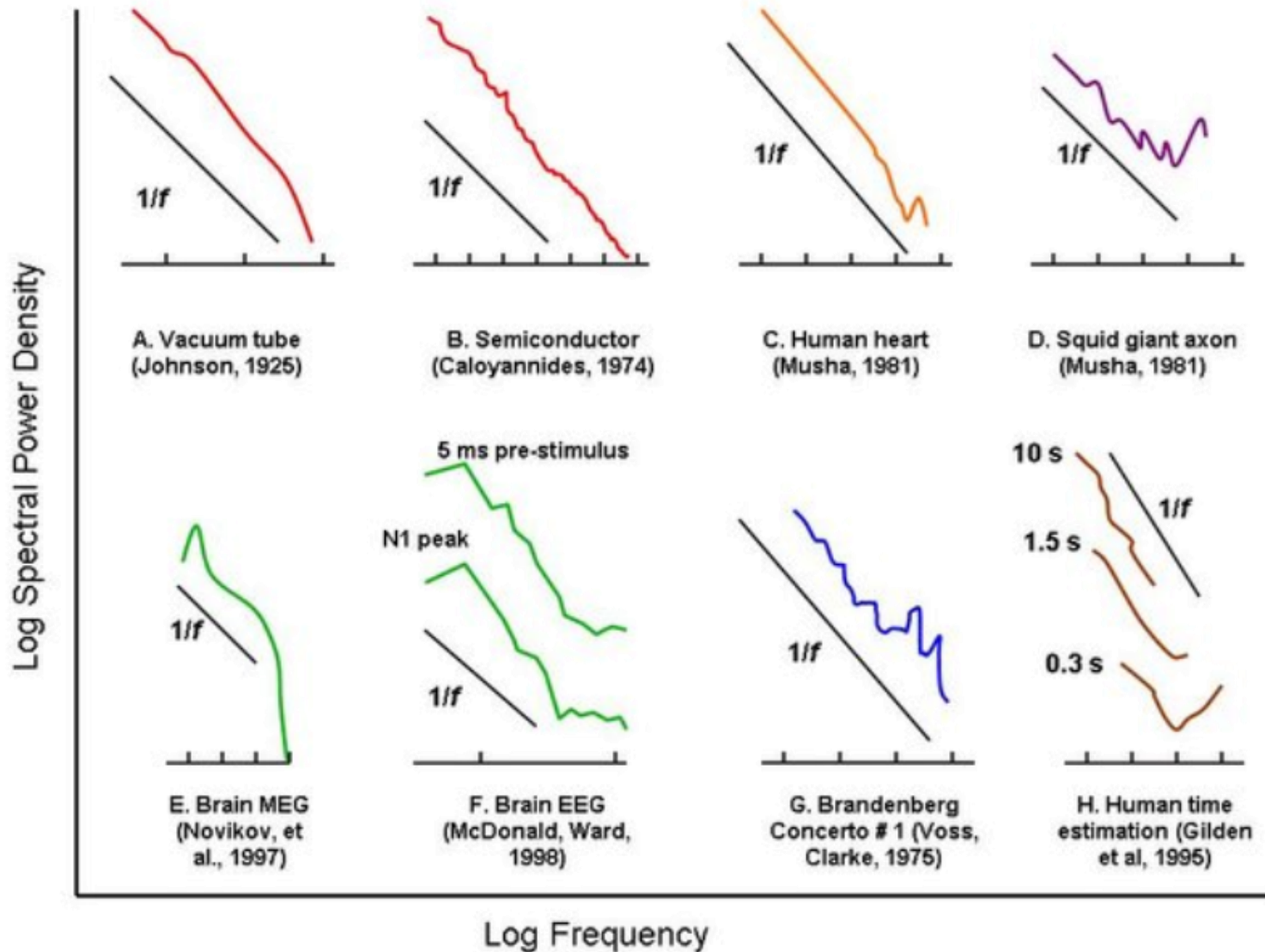
Motova Mary for supreme poster design

<https://www.nature.com/articles/s41598-023-34816-2>

<https://arxiv.org/abs/2307.03192> <https://arxiv.org/abs/2104.08872>

1. Introduction

- Many 1/f fluctuations $S(\omega) \propto \omega^{-\beta}$ with $\beta \approx 0.5$ to 1.5



- 1/f fluctuation (pink noise) exists **everywhere**

Semiconductors, thin metals, potential fluctuations in bio-membranes, current in electrolytes, crystal oscillators, high-stability frequency standard oscillators, oscillation frequency fluctuations, ultra-long-term temperature fluctuations, flow fluctuations on highways, the magnitude of symphonies, Variations in the rotation speed of the Earth, variations in the intensity of cosmic rays, heart rate, postural control, MEG and EEG (brain), ...,...

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MM: density fluctuations in early Universe(quantum origin)

- People often say:

- classical music exhibits 1/f fl,
- 1/f fl. is comfortable and good for your body.

Are these true?

● Three mysteries of 1/f fluctuations

1. A small system appears to yield long memory

small system

1/f noise below ultra-low frequency from the small system...

[[Liu2013](#)], in semiconductor films, 1/f noise from 2.5nm layers.

apparent long memory

small semiconductor yields 1/f up to 10^{-7} Hz [[Dukelov1974](#)],
from around 1Hz to $10^{-6.3}$ Hz [[Caloyannides1974](#)]

2. **Fluctuation-dissipation relation does not hold**

Even a conservative system can show 1/f fl.

3. **Data squared** was needed to yield 1/f fl.

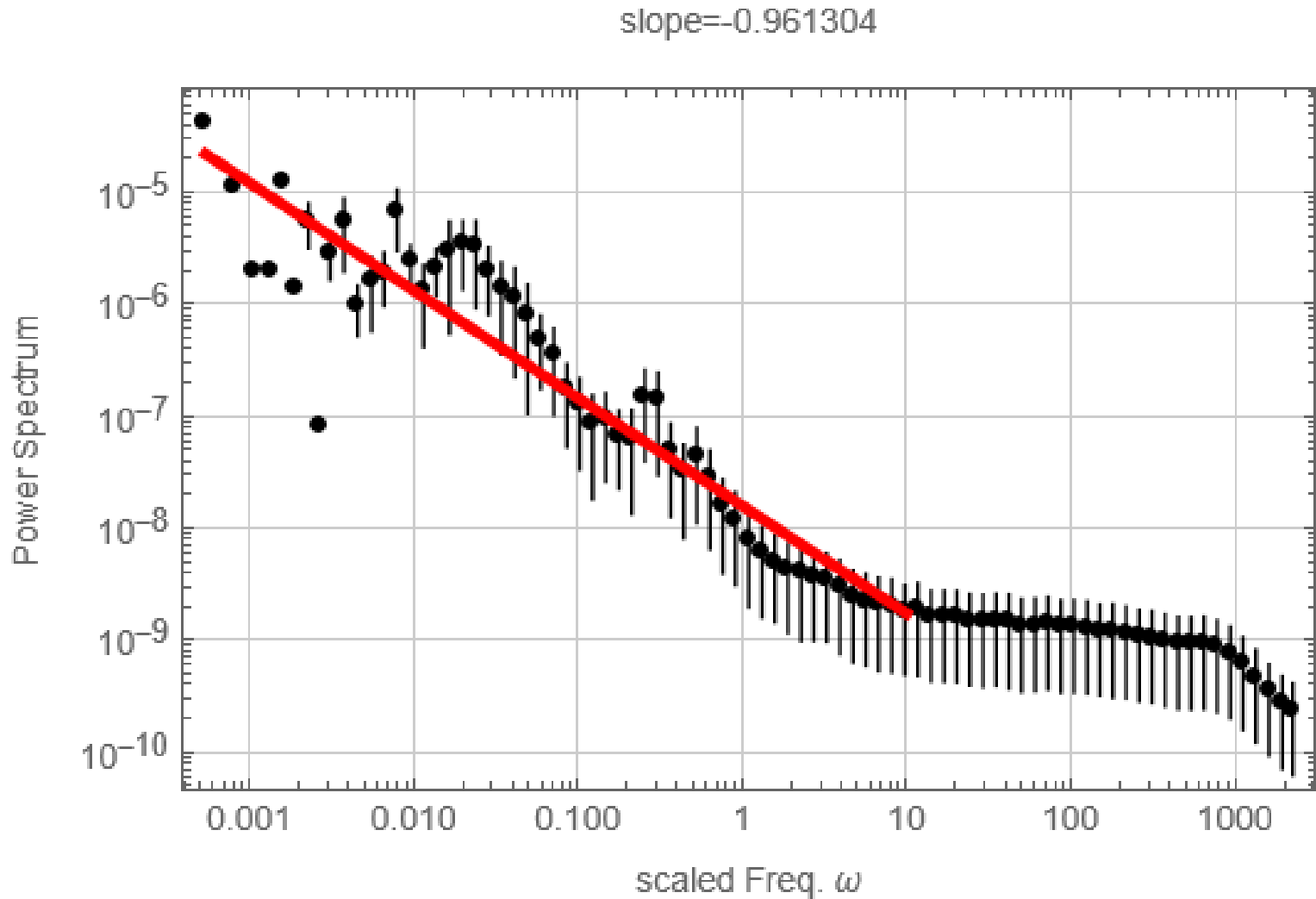
for HMF [Yamaguchi 2017]... M^2

case of music [[Voss1977](#)]... piecewise RMS was analyzed

● 1/f around OAC

a) Cretan Sea sound (whisper? Roar?) from OAC 13 July 4:53

[VID_20230713_105315.wav](#)



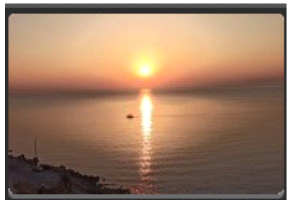
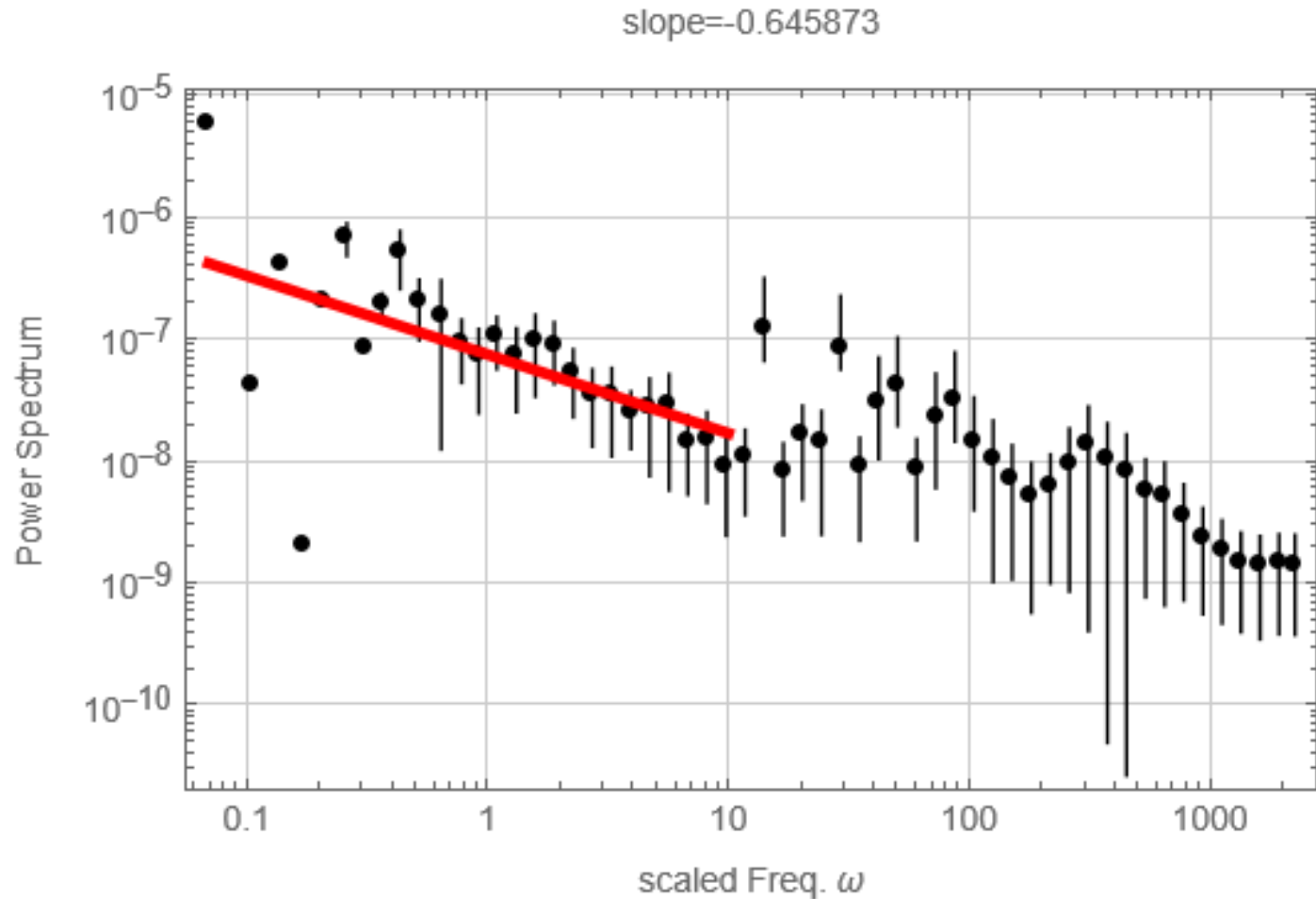
→ Almost 1/f fl. Power index -0.96

b) Aegean Sea sound **after sunrise** 14 July -6:37

[VID_20230714_123722.wav](#)



.... in contamination of noisy cicada



→ The power index reduces to -0.65

c) Chamber Music Concert, OAC, 12 July, 19:00-
by Kalliopi Petrou (soprano), Stefano Menegus (Piano)

[VID_20230713_020215.wav](#)

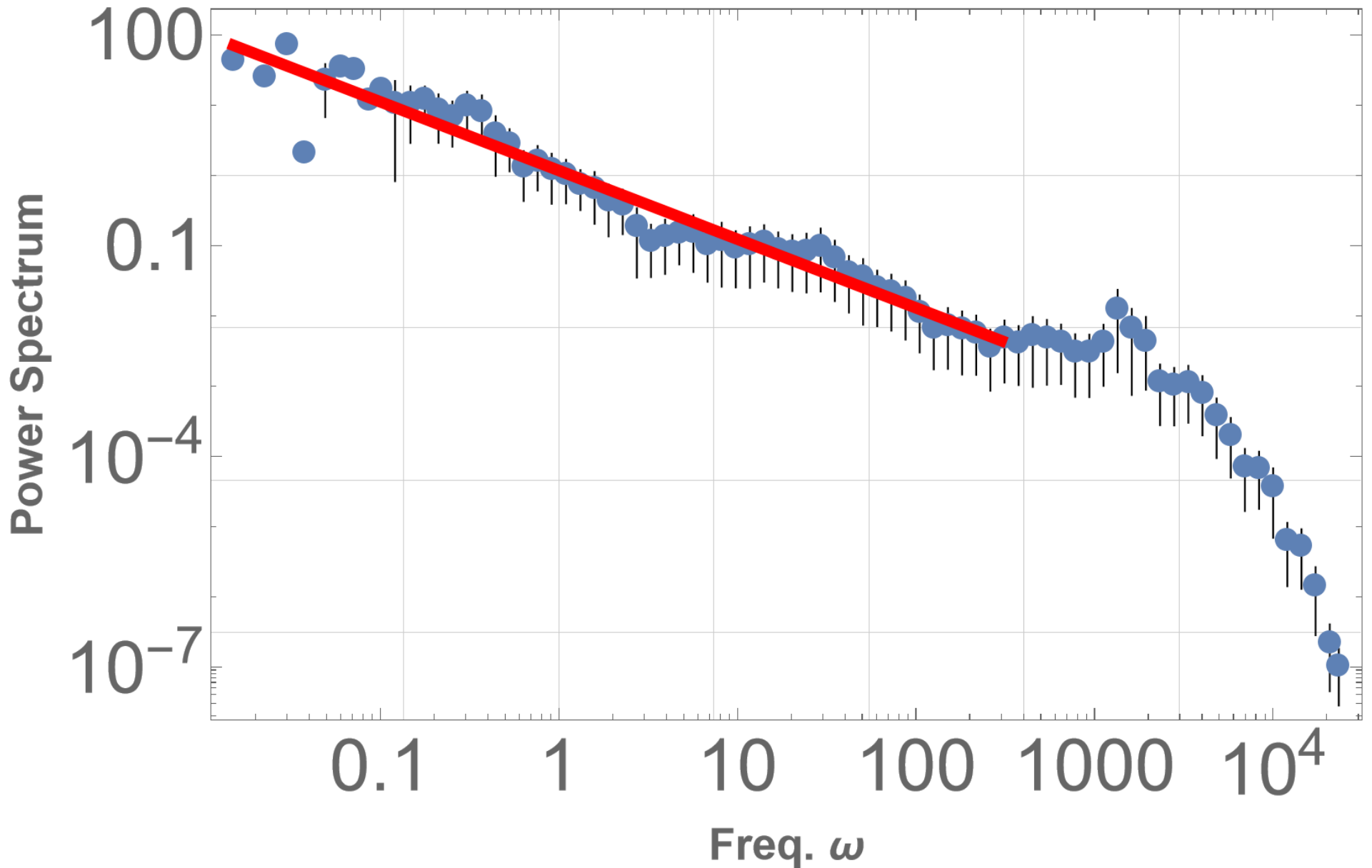


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<https://www.youtube.com/watch?v=7BWMJ7m0fC8>

PSD of signal squared shows 1/f fl.

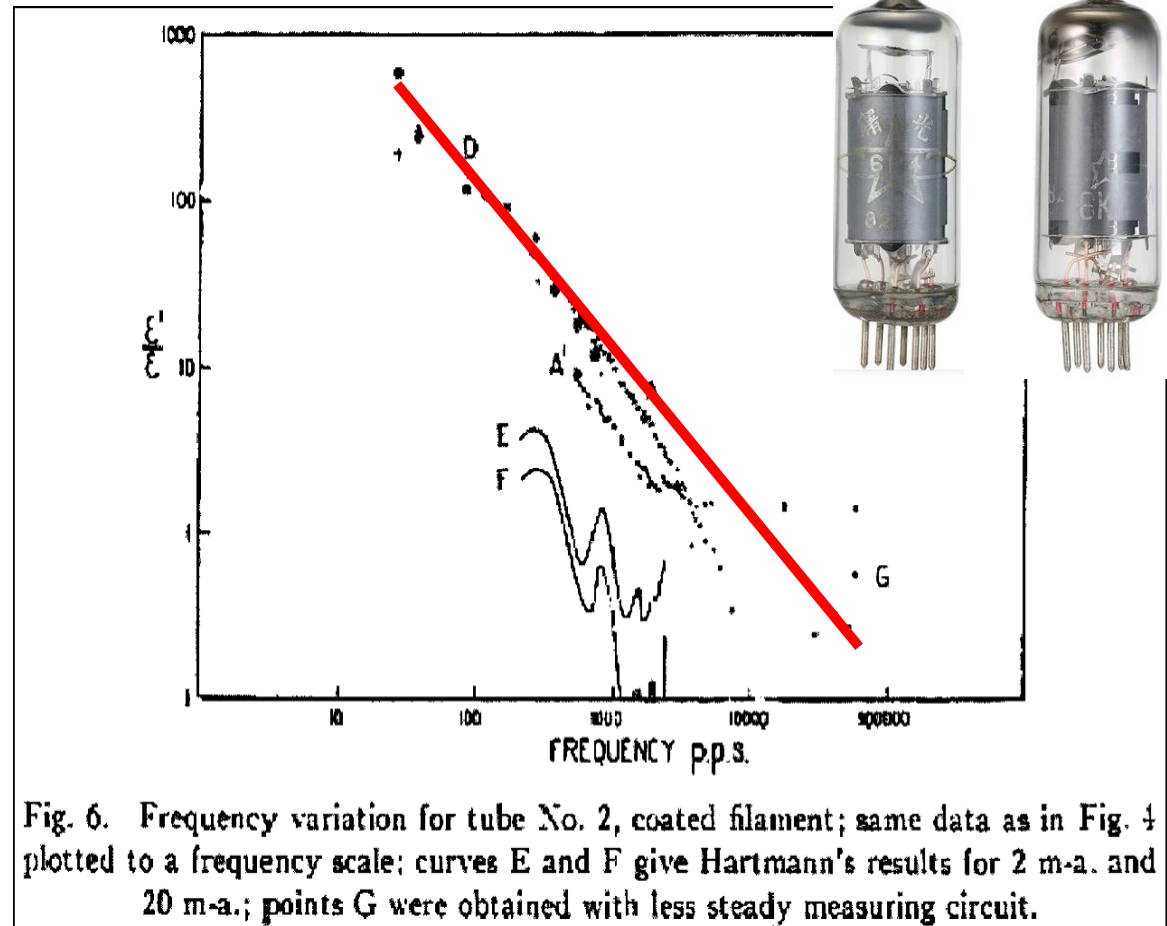
slope=-0.978073



● First observation of 1/f fluctuation 1925

The power spectrum (PSD) of the **voltage squared fluctuation** $\overline{V^2}$ (obs.) in the vacuum tube behaves 1/f on the low-frequency side. (1925). (f: frequency)

JB Johnson, Phys. Rev. 26 (1925) 71.



- There have been many theories for the origin of 1/f fluctuations. However, none of them were universal explanations.

-People often say that classical music exhibits 1/f fl, 1/f fl. is comfortable and good for your body. Let's check it in Music.

[Tchaikovsky-Serenade-for-Strings-Ozawa-Saito-Kinen-from-YouTube-part.wav](#)

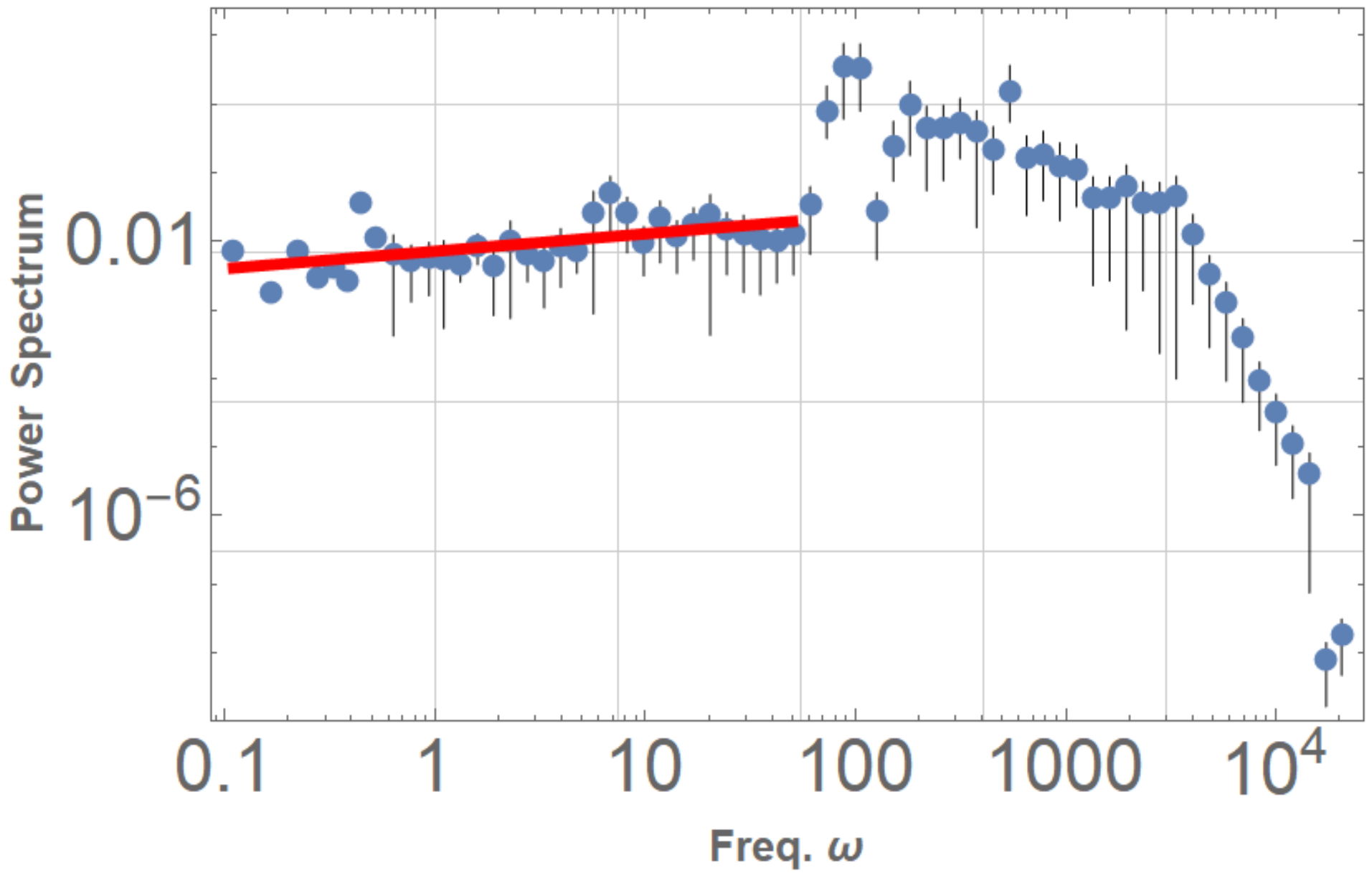


$$P(\omega) = \left| \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \right|^2$$

Or discrete version FFT is calculated.

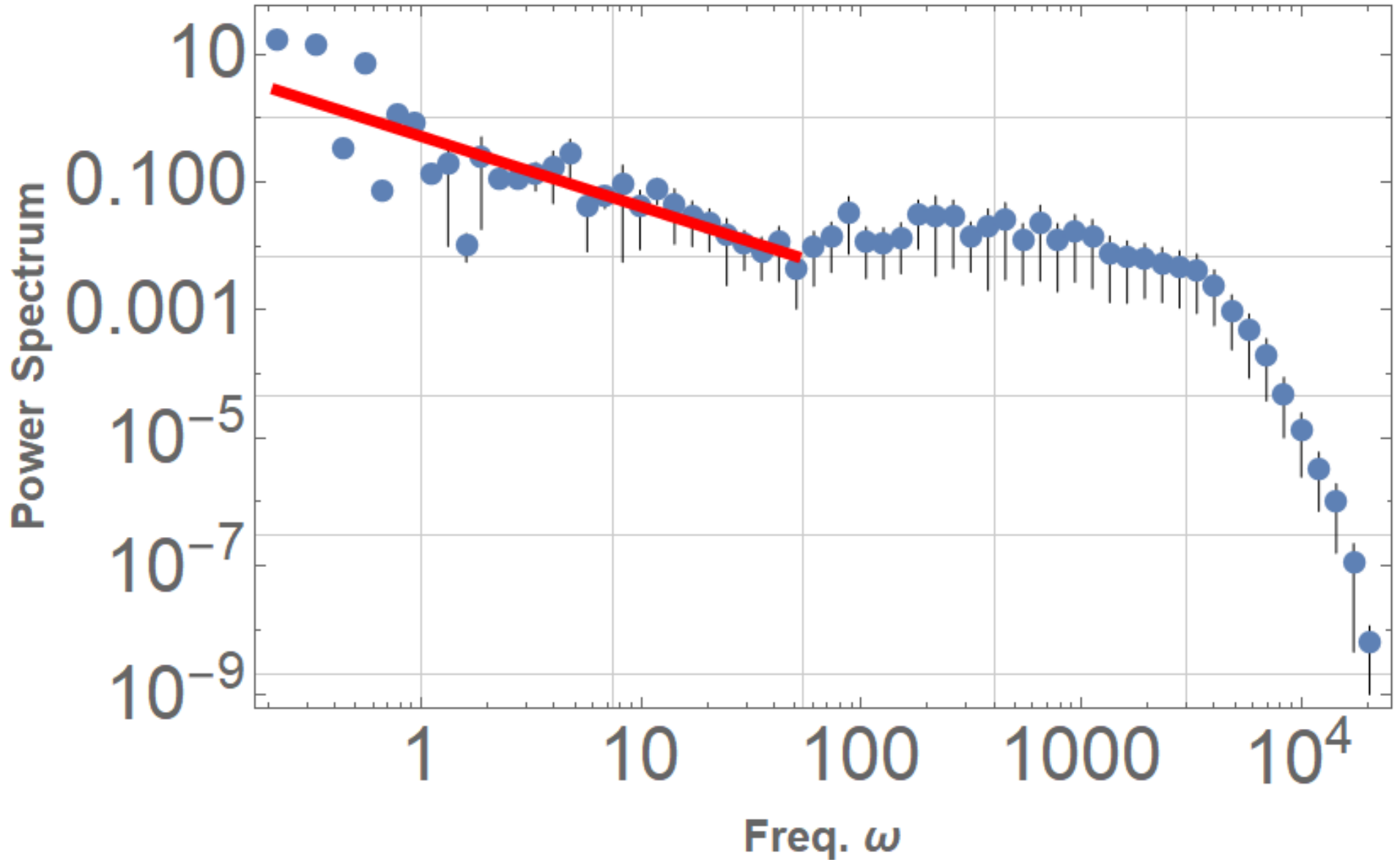
d) PSD of the music signal ϕ ← .wav file [SerenadeBare.wav](#)

slope=0.254319



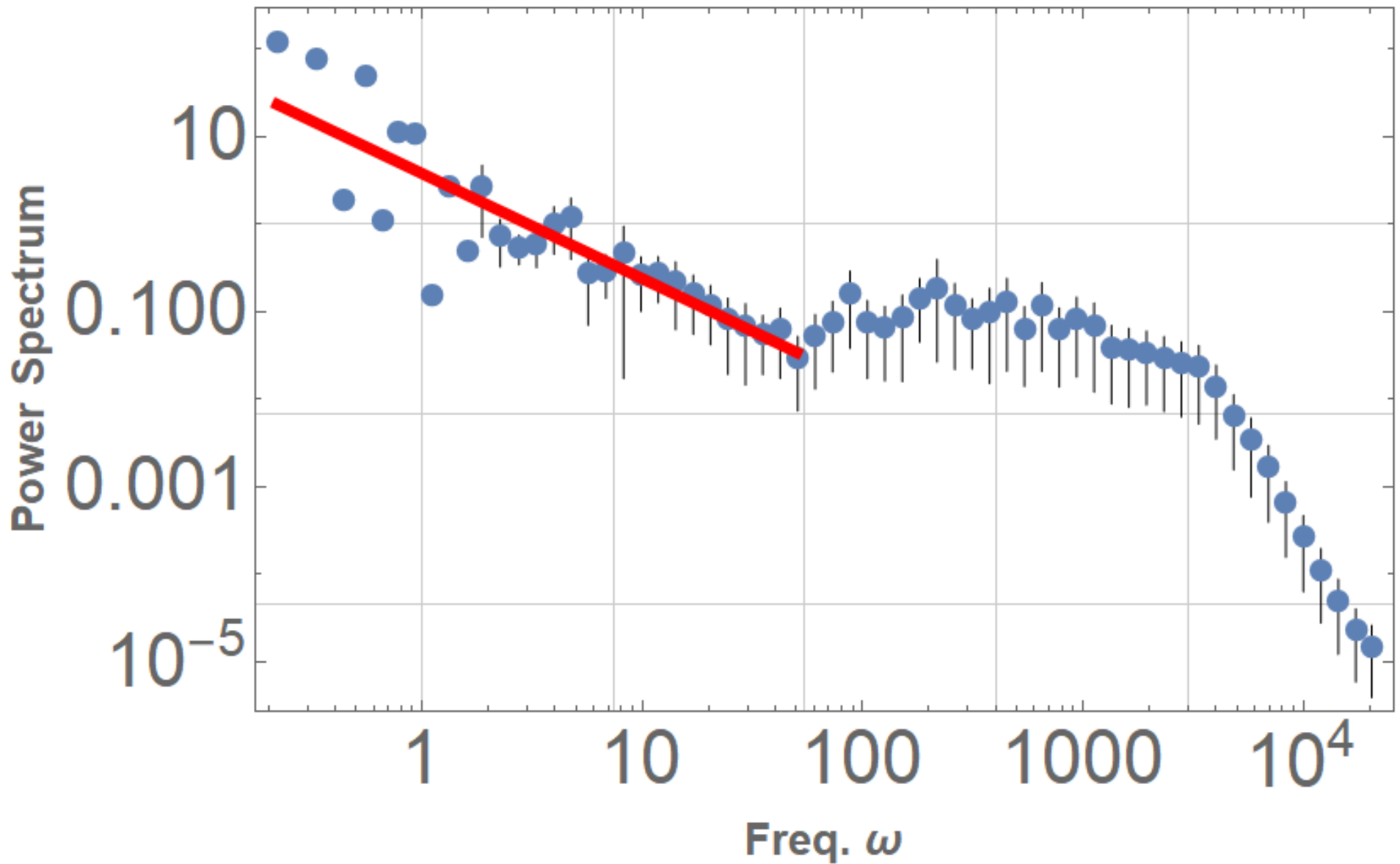
e) PSD of the music signal ϕ^2 [SerenadeSquared.wav](#)

slope = -1.09961



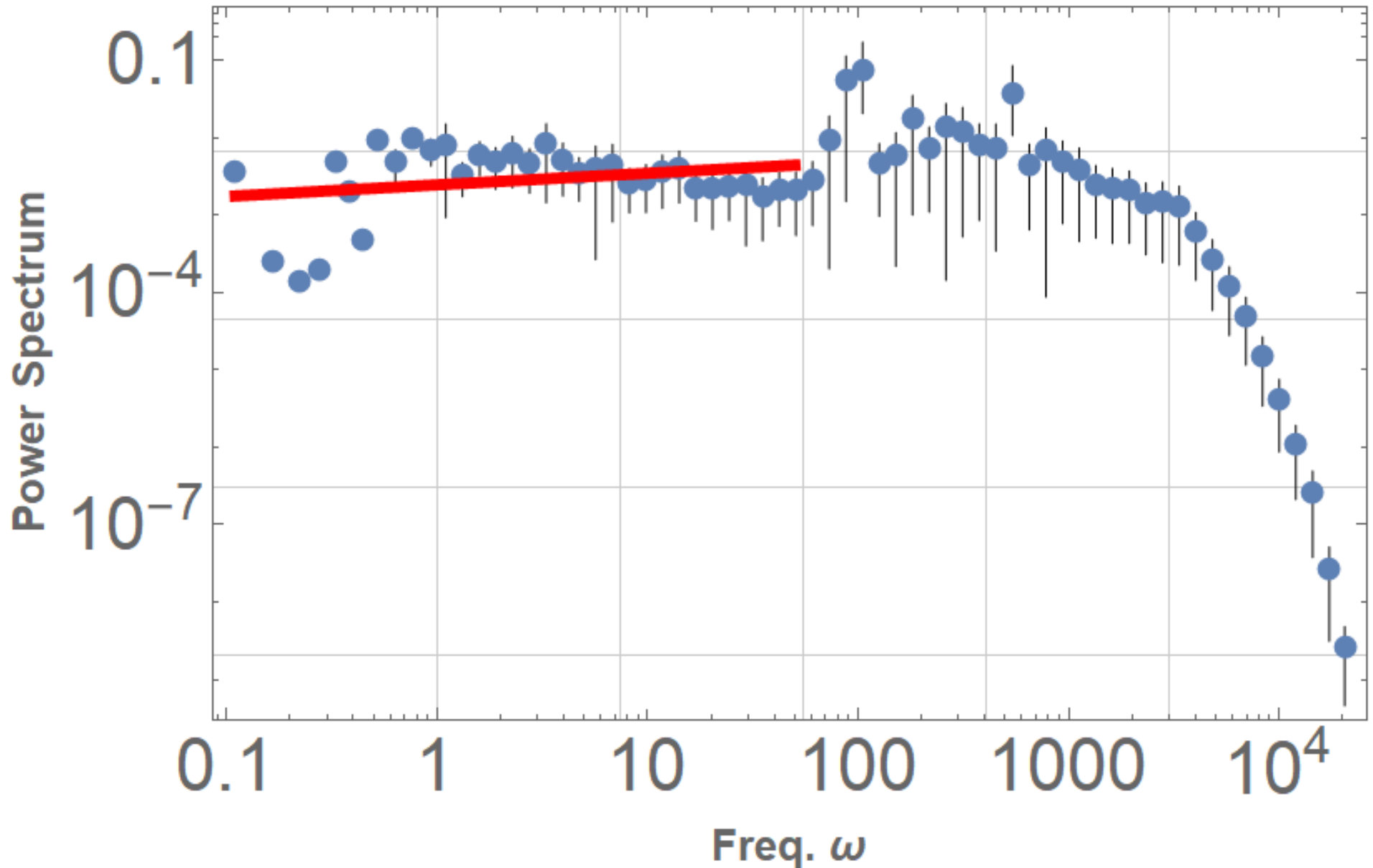
f) PSD of the music signal $|\phi|$ [SerenadeAbs.wav](#)

slope = -1.20071



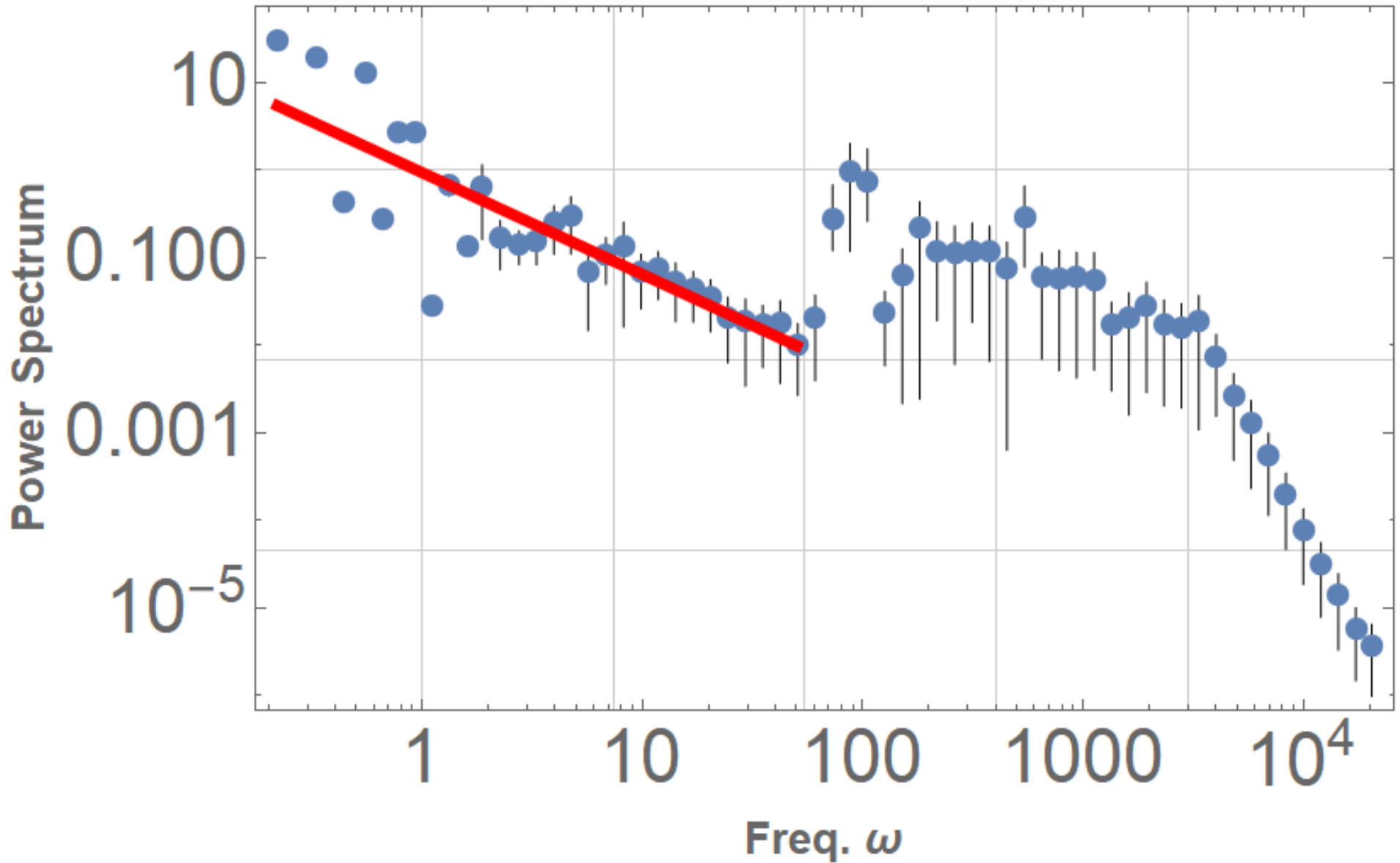
g) PSD of the music signal ϕ^3 [SerenadeCubic.wav](#)

slope=0.149523



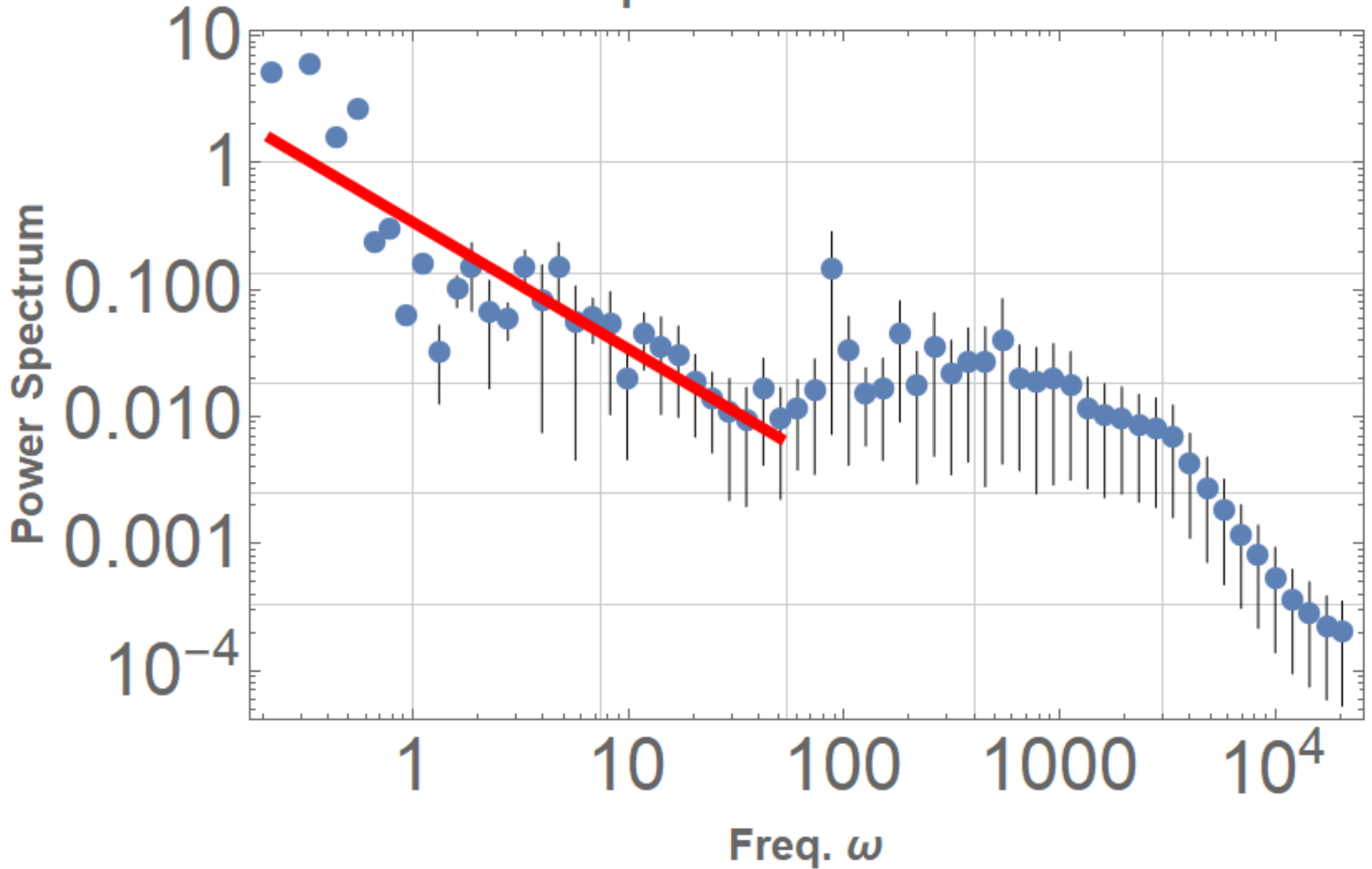
h) PSD of the music signal $\phi > 0$ [SerenadeRectified.wav](#)

slope = -1.16262



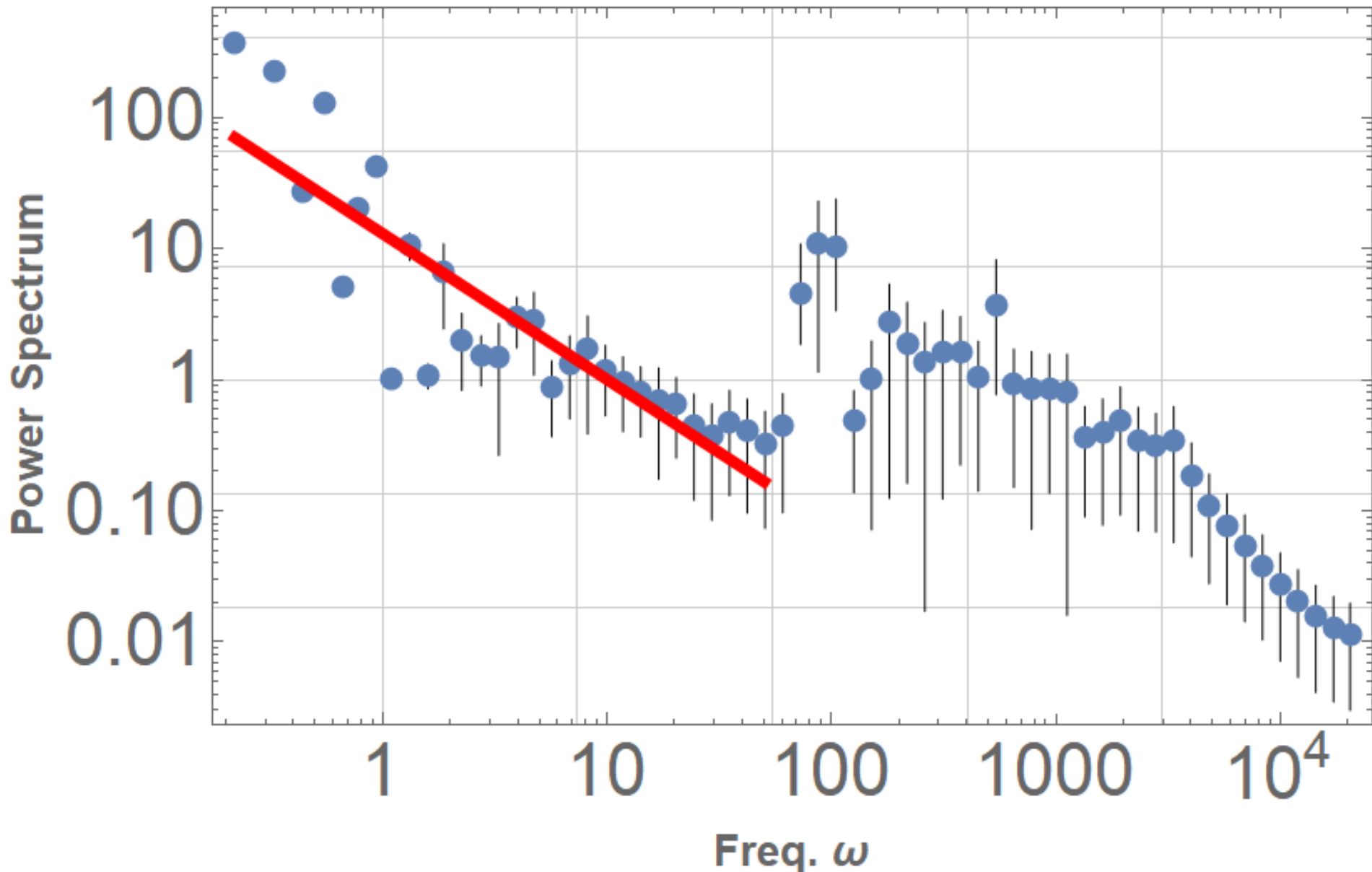
i) PSD of the music signal $\phi > \text{mean}|\phi|$ [SerenadeThresh.wav](#)

slope = -0.998499



j) PSD of the Timing [SerenadeTimingThresh.wav](#)

slope = -1.11319



Do you like 1/f music (or pink music)?

- Pink music [SerenadeSquared.wav](#)
- Pre-pink music [Tchaikovsky-Serenade-for-Strings-Ozawa-Saito-Kinen-from-YouTube-part.wav](#)

Curiously, the original signal does not show 1/f fl. at all.

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However, the **square** of the signal shows 1/f fl.

Rectification, thresholding, ...as well.

2 1/f fluctuation is the wave beat

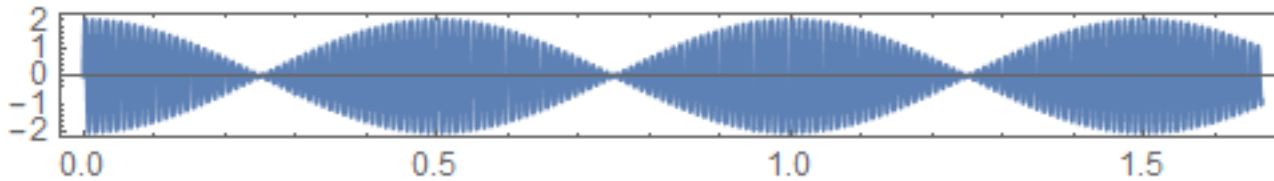
- What is happening?

ϕ No 1/f

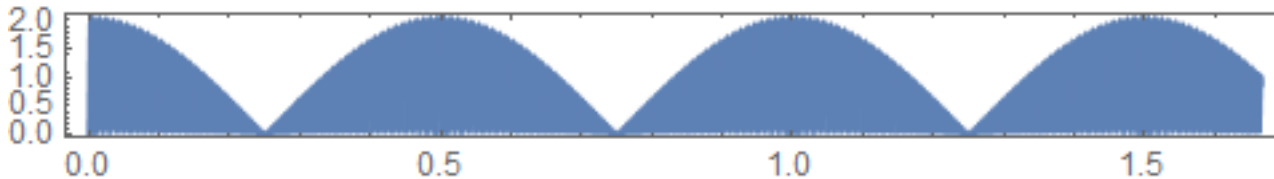
ϕ^2 1/f

ϕ^3 No 1/f ...

- Rapidly fluctuating + and - parts cancel and yield nothing.



But, square the signal yields 1/f fluctuation



- We speculate **1/f fluctuation is the wave beat.**

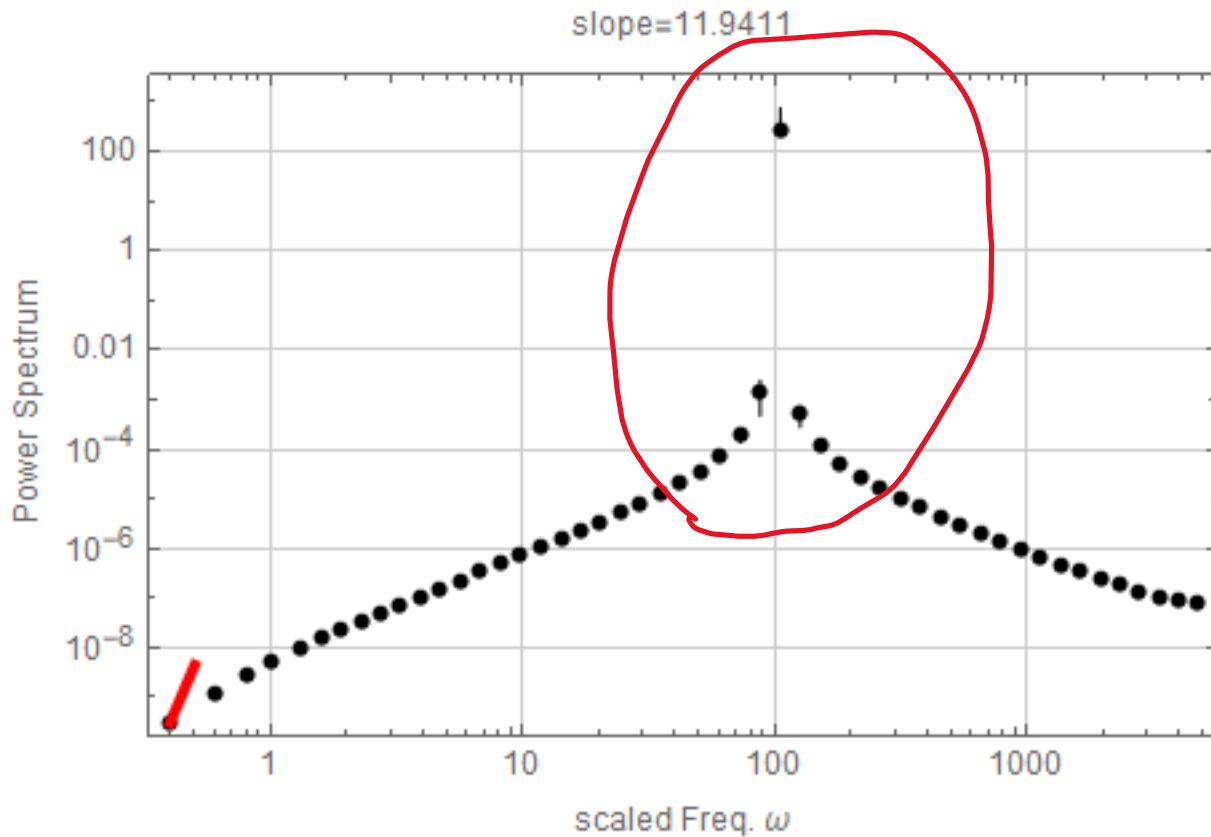
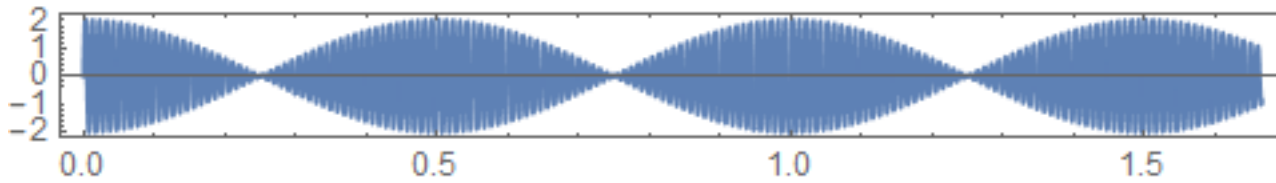
Or **Amplitude Modulation (AM)**

Then, **De-Modulation (DM)** is inevitable for 1/f fluctuation to appear

- ◆ Simple Fourier transform of a **sound wave** does not show beat

 $\sin(\omega t + \lambda t) + \sin(\omega t - \lambda t) = 2 \cos(\lambda t) \sin(\omega t)$ with $\omega \gg \lambda > 0$

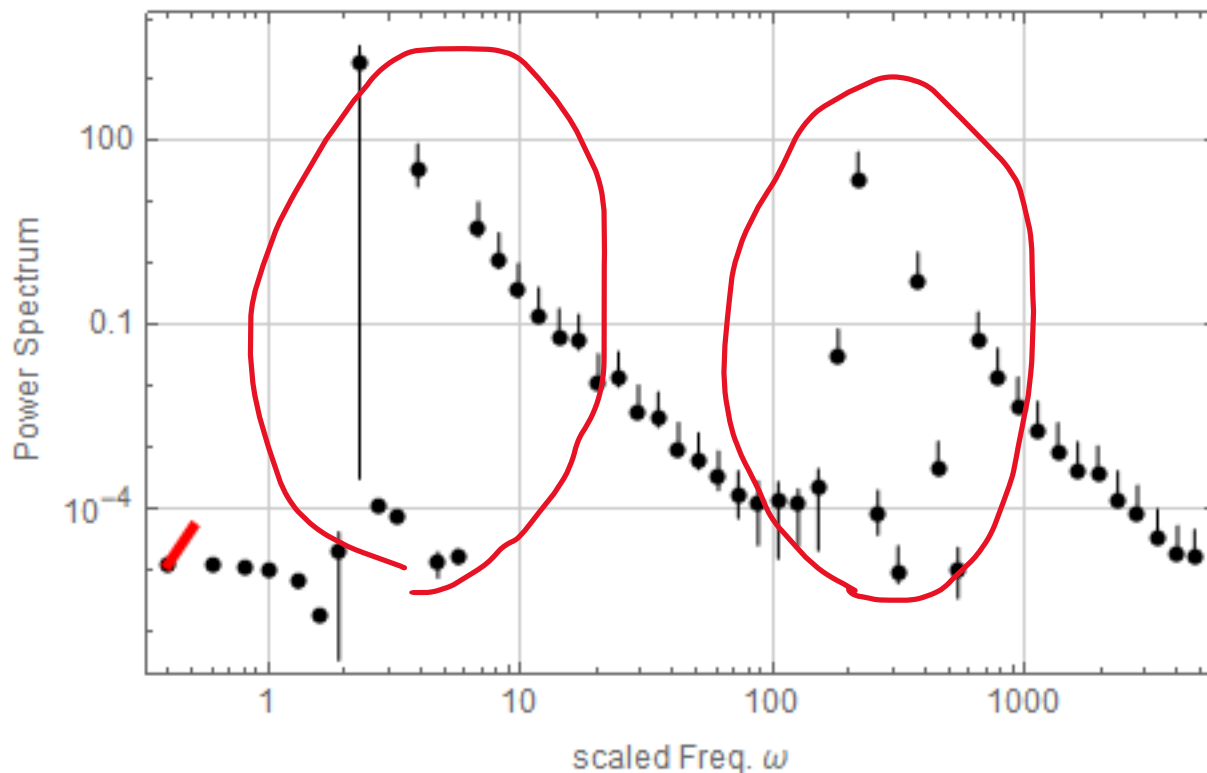
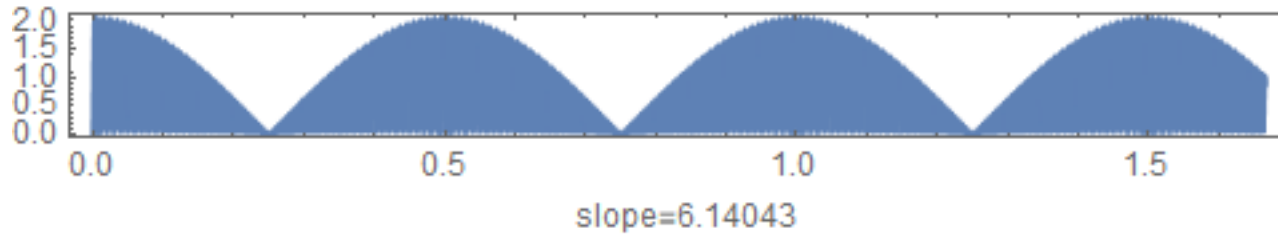
 \rightarrow Only $\omega \pm \lambda$ appears in PSD



◆ Fourier transform of **sound wave squared**

$$(\sin(\omega t + \lambda t) + \sin(\omega t - \lambda t))^2 > 0$$

→ $\pm 2\lambda$ comes out (beat)



- If $1/f$ fluctuation comes from wave beat, or AM, the previous **three mysteries would be dissolved**:
 1. A small system appears to yield long memory
 - If beat, **a denser accumulation of frequencies can yield an infinitely low-frequency** signal from a small system.
 2. Fluctuation-dissipation relation indicates that $1/f$ fl. must accompany dissipation.
 - If beat, it is **dissipation free**.
 3. Data squared was needed to yield $1/f$ fl.
 - These are **fully consistent with AM**.

4. Diversity sources of 1/f fluctuations

Many AM methods

a) Synchronization



b) Resonance



c) infrared divergence



→ promotes frequency accumulations

Many DM methods

a) **Operational** DM in Data Processing:

Square, absolute value, rectification of Data

b) **Intrinsic** DM associated with the system:

Nerve firing, fault rupture, magnetic reconnection,...

→ **Any type of demodulation works**

5. Synchronization in Music & Universe



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Synchronized swimming cute cats... Bing Image Creator

a) Kuramoto model describes the frequency accumulation.

$$\frac{d\theta_i[t]}{dt} = \omega - K \sum_{k=1}^N \text{Sin}(\theta_i[t] - \theta_k[t]) + \xi(t)$$

Where $\xi(t)$ is a random field.

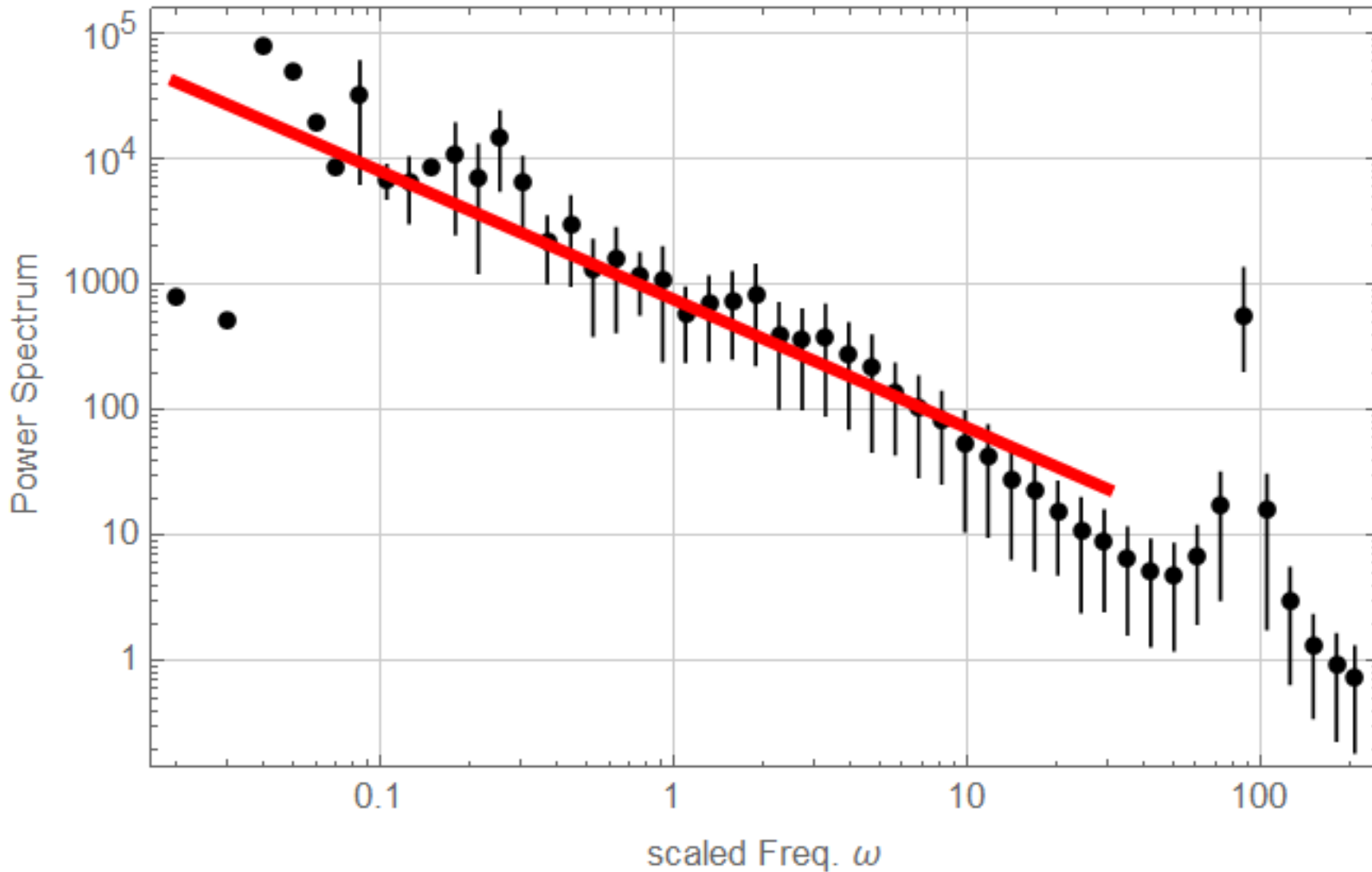
$$\varphi_i[t] = \int_0^t \theta_i[t] dt$$

And

$$\phi[t] = \sum_{i=1}^N (\text{Sin}[\varphi_i[t]])$$

PSD of $\phi[t]^2$ (PSD of $\phi[t]$ is flat and random)

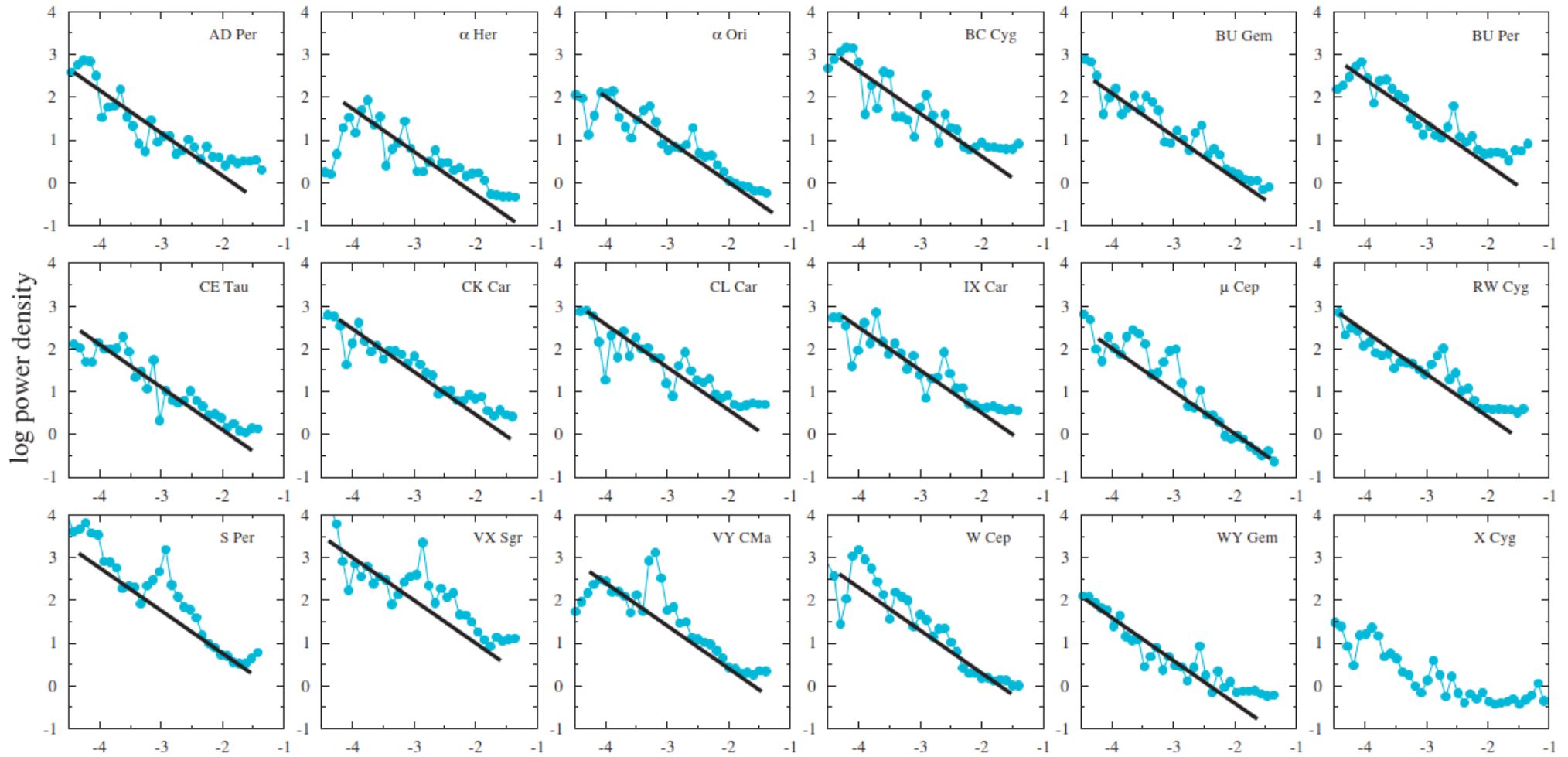
slope=-1.02138



Kuramoto sound: [Kuramoto.wav](#)

b) Variable stars

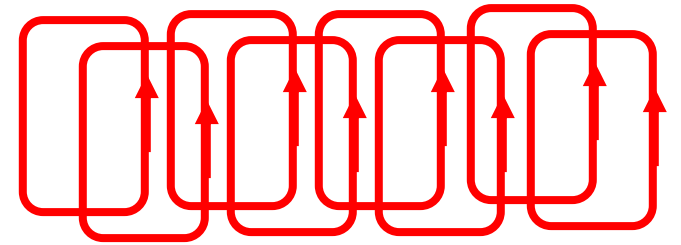
L. L. Kiss, Gy. M. Szabó, and T. R. Bedding, *Mon. Not. R. Astron. Soc.* 372, 1721–1734 (2006)



Variable stars: many convective zones synchronize with each other to form luminosity variation as a whole.

We model this variable star as a Coupling Lorentz Model

(x : stream function, y, z : lateral and longitudinal temperature difference)



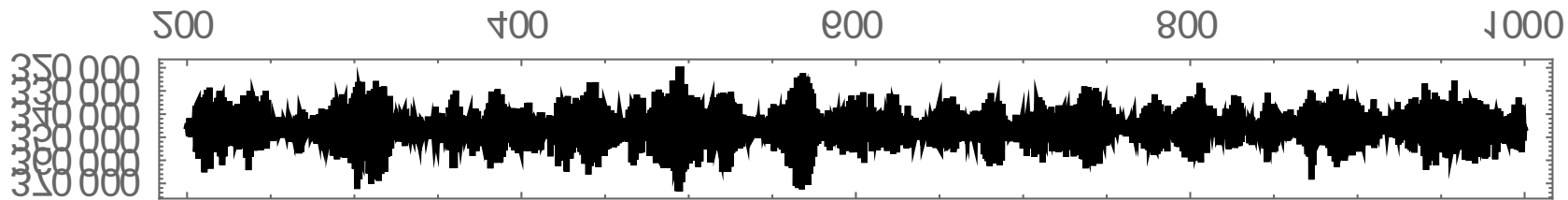
$$\dot{x}_i(t) = -3 (x_i(t) - y_i(t)),$$

$$\dot{y}_i(t) = x_i(t) (-z_i(t)) + 26.5 x_i(t) +$$

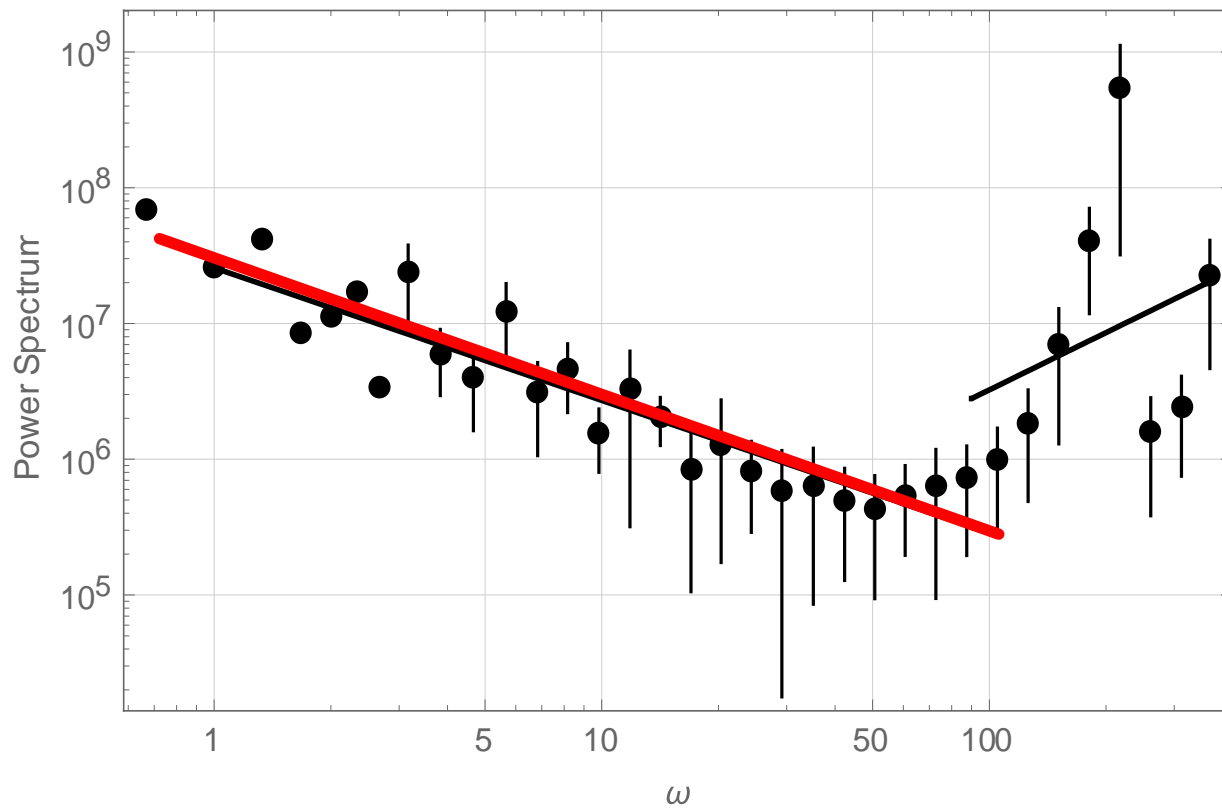
$$\frac{1}{2} \varepsilon (-y_{i-1}(t) + 2 y_i(t) - y_{i+1}(t)) - y_i(t)$$

$$\dot{z}_i(t) = x_i(t) y_i(t) - z_i(t)$$

$\varepsilon = -1, N = 1000$ yield:



slope=-0.976533 and 1.40092 before & after $\omega=90$



The power index is about -0.98

6. Resonance in music & physics



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Resonating cats... Bing Image Creator

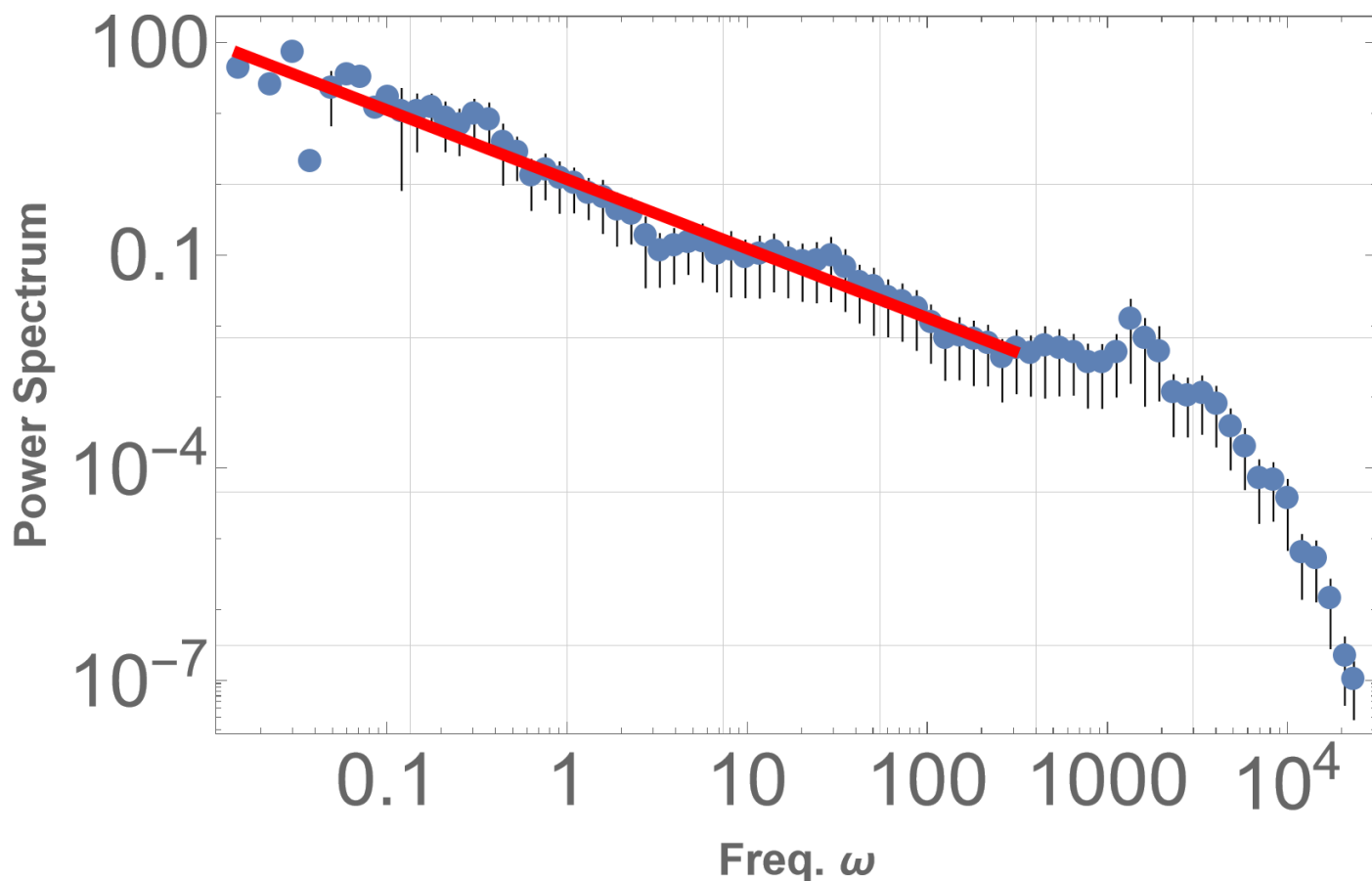
a) **Resonance: piano and a solo singer**

Chamber Music Concert, OAC, Petrou & Menegus

[VID_20230713_020215.wav](#)

PSD of signal squared shows 1/f fl.

slope=-0.978073



b) Water harp cave at Hosen-In Kyoto [00522.wav](#)



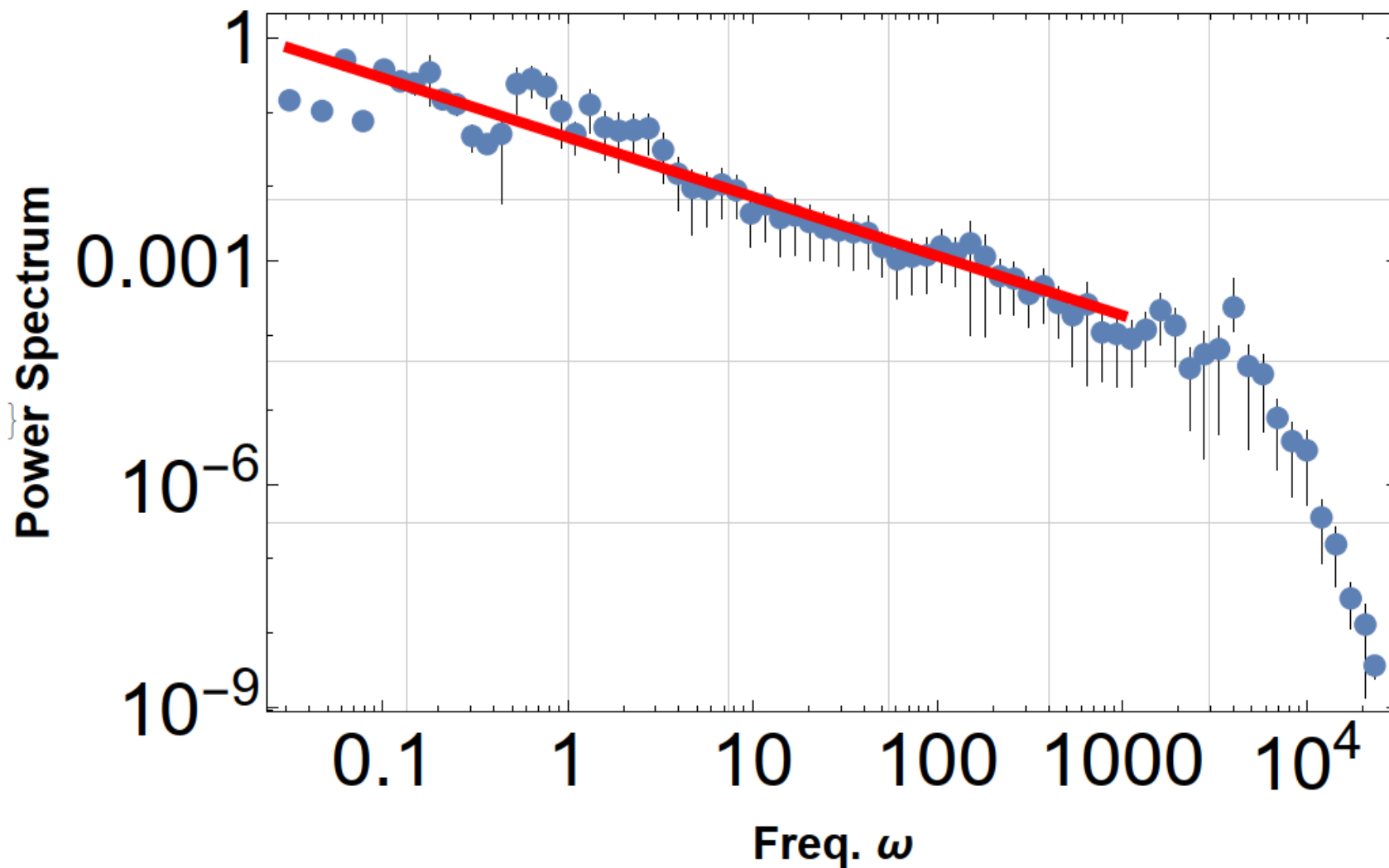
https://



PSD of the sound signal squared

2

slope=-0.796625

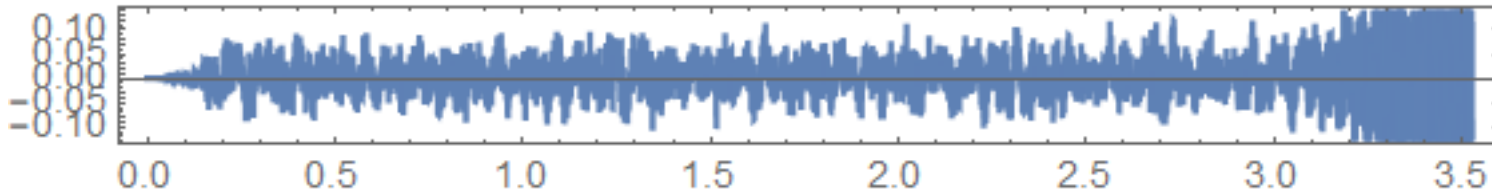


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c) **Iceberg?** [bloopNOAAtest.wav](#)

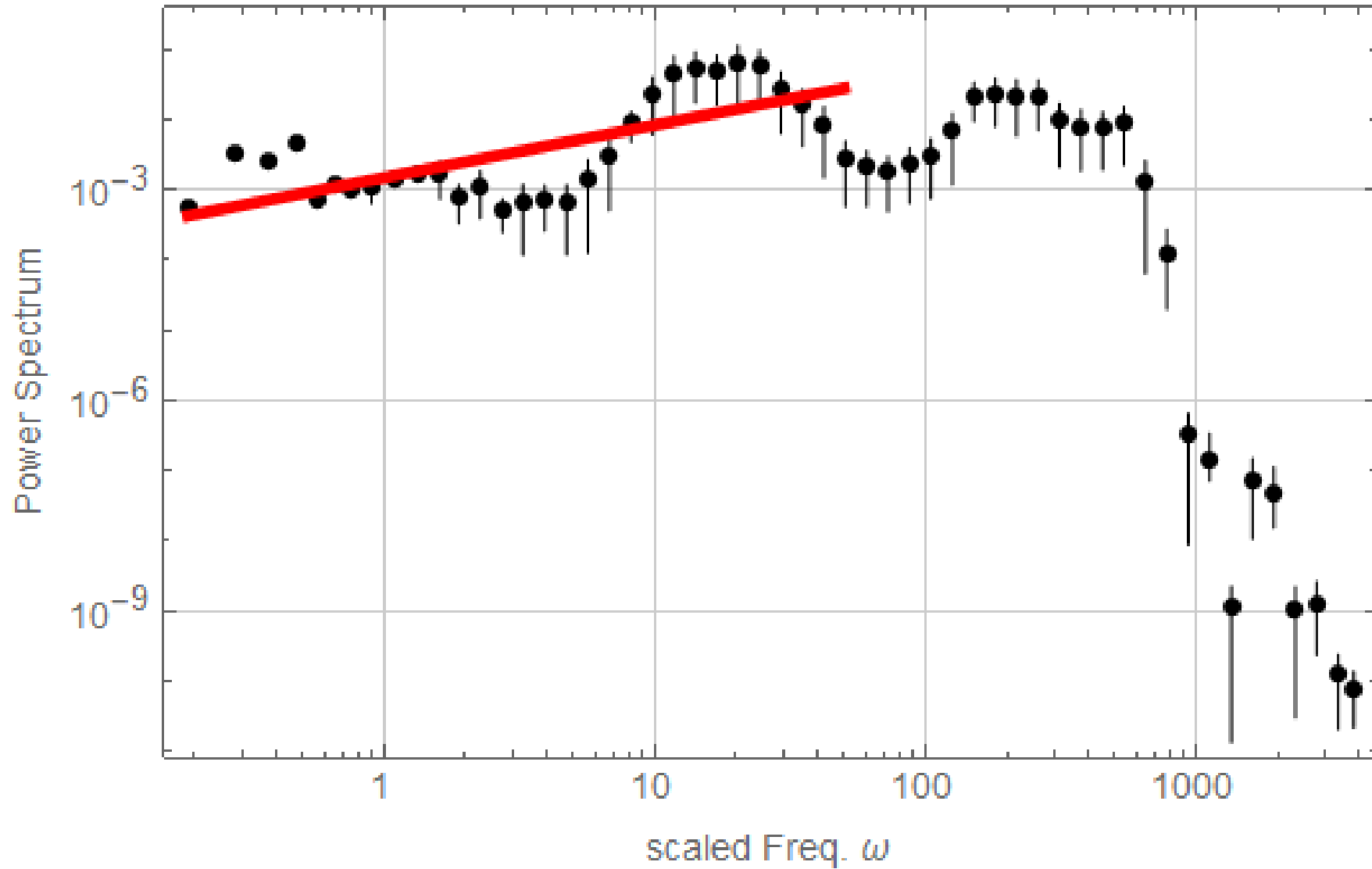
unexpected sound ...Is may be an iceberg icequake?

https://en.wikipedia.org/wiki/List_of_unexplained_sounds



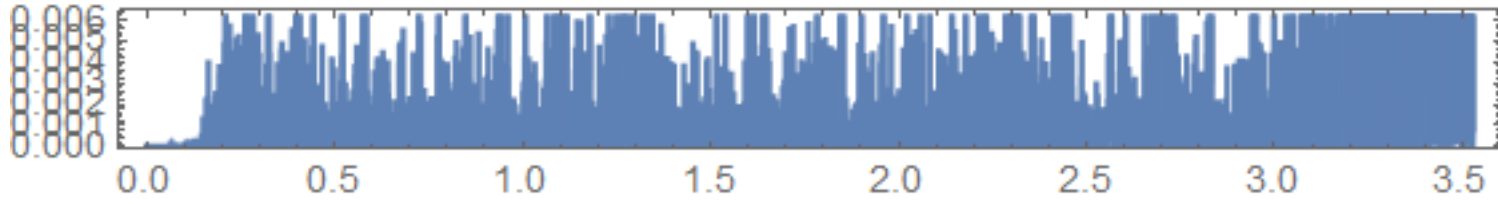
PSD of the **original** sound unexpected

slope=0.749779

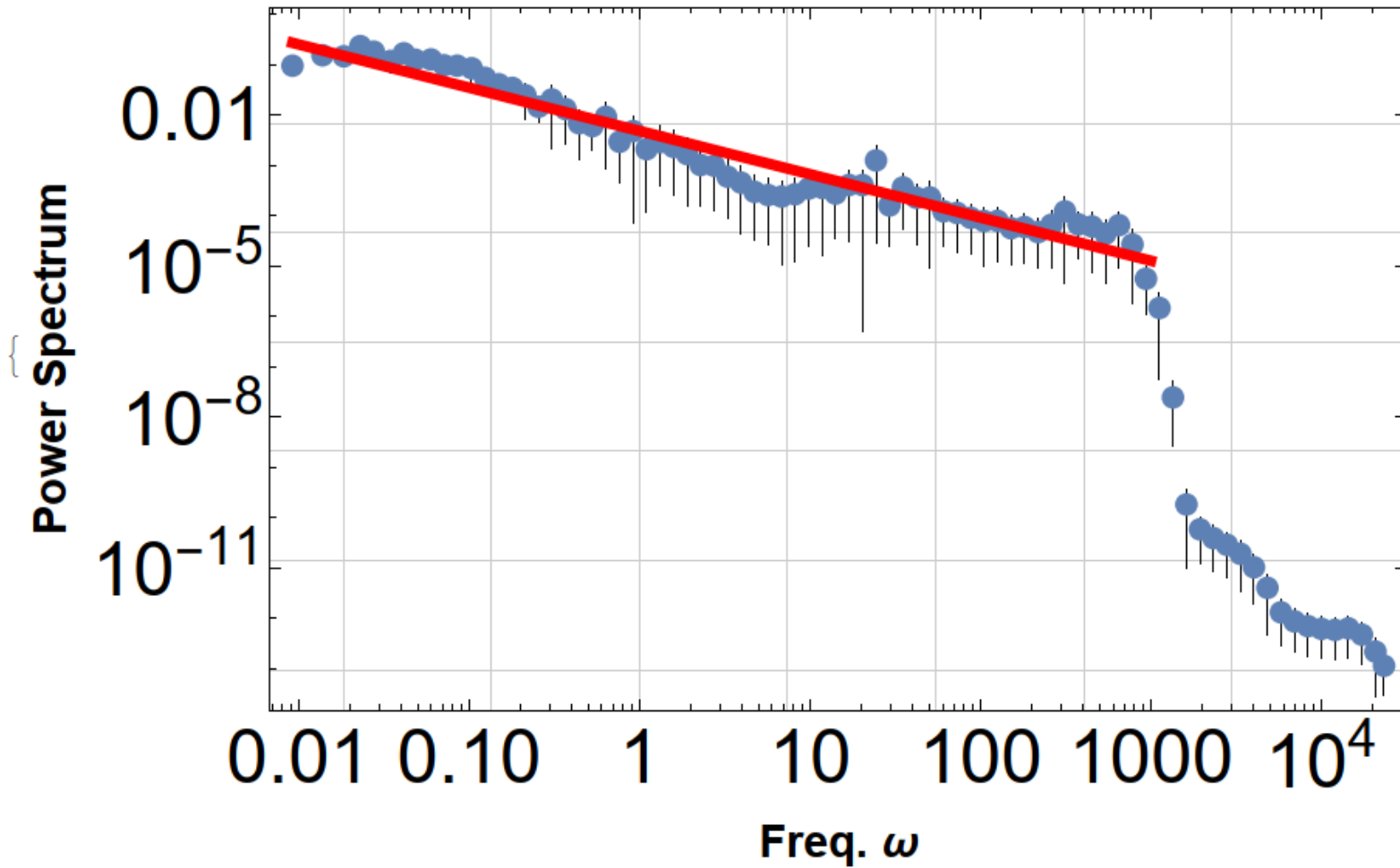


● No 1/f and flat

PDF of the **square** of the data: $\rightarrow 1/f$ fl.



slope=-0.860564

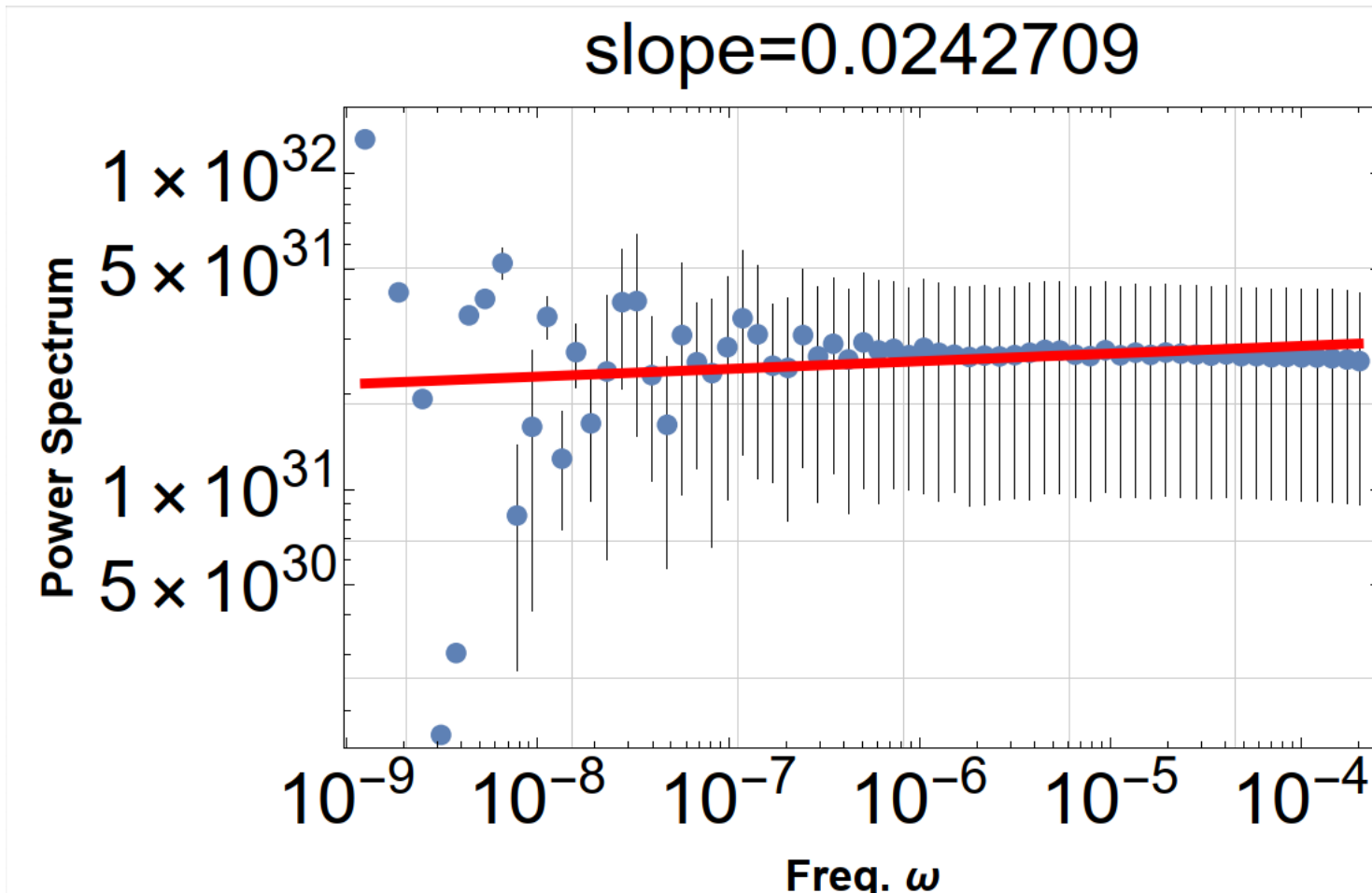


d) Seismic activities with possible verifications from EFO



<https://japaclip.com/earthquake/>

- Are earthquakes resonance? Using the USGS 50-year seismic data for the whole world's earthquakes of magnitude 3.5 and above,

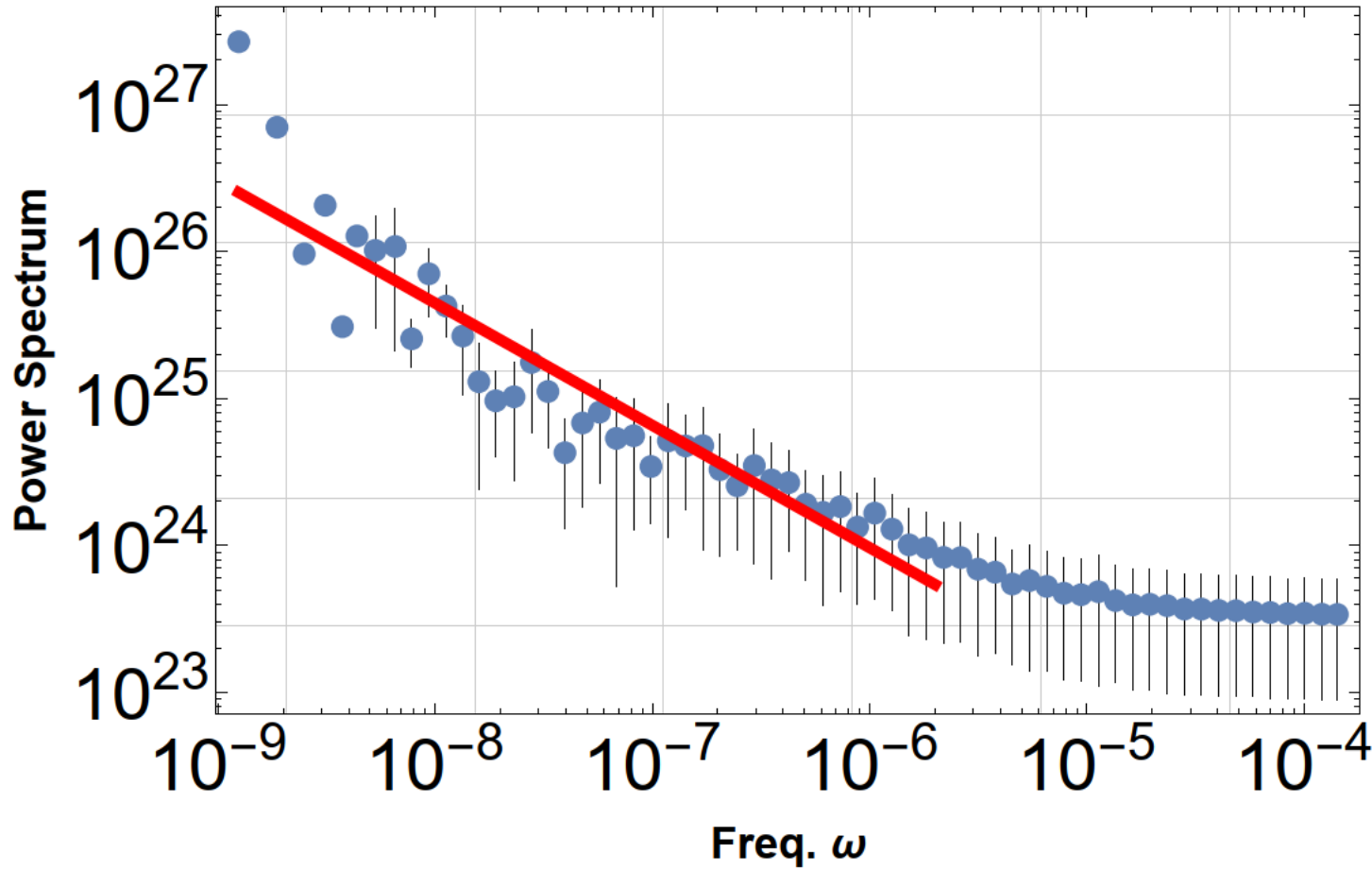


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PSD is flat and random.

- However, if we restrict magnitude 3.5 to 5,

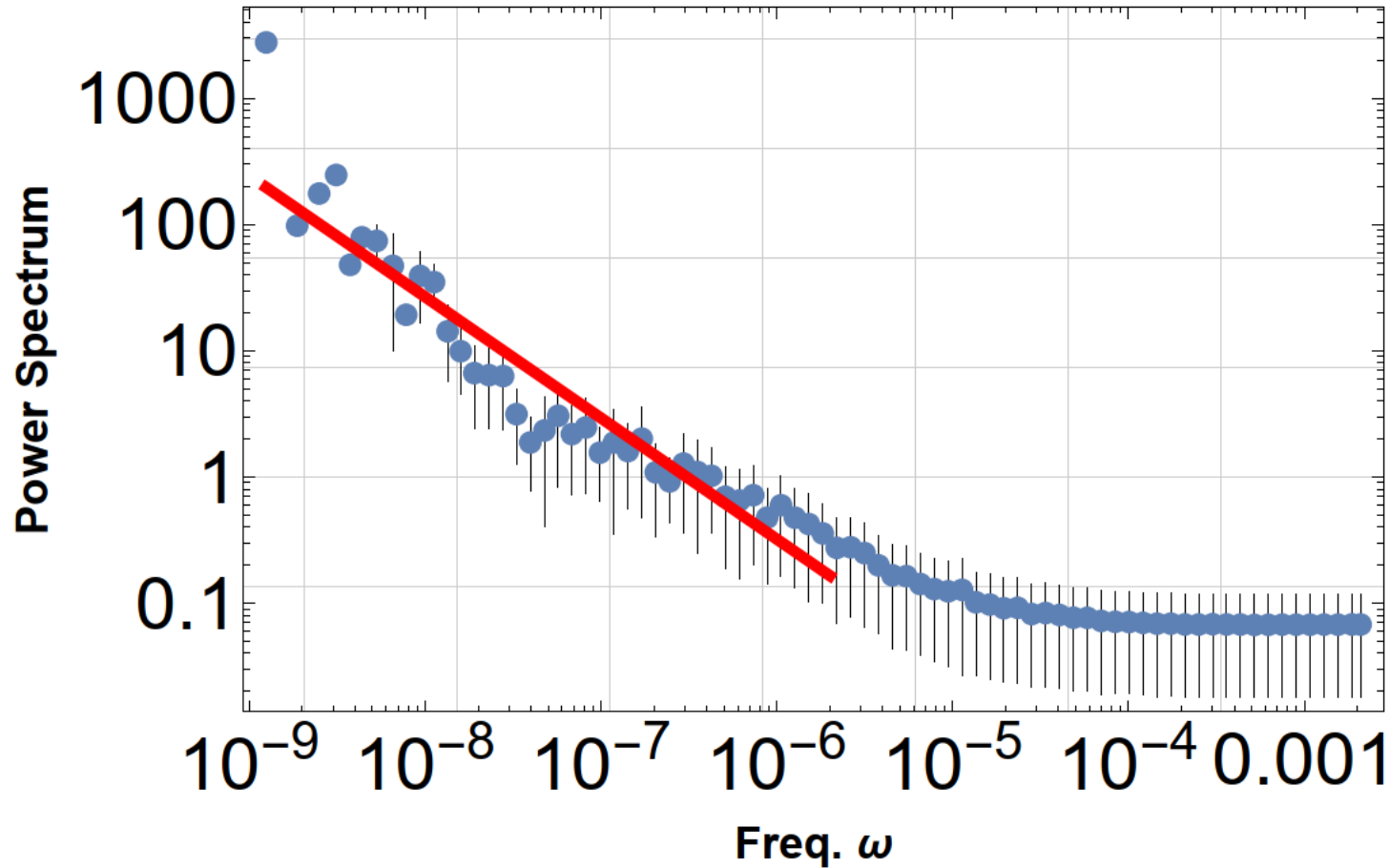
slope=-0.831697



PSD starts to show 1/f fluctuations.

- Further, for the earthquake occurring timing sequence, disregarding the whole energy information,

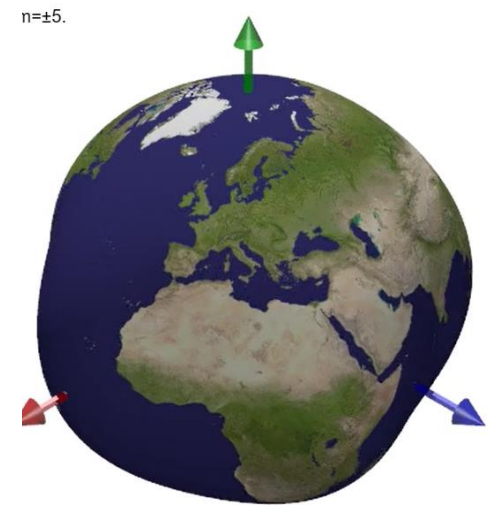
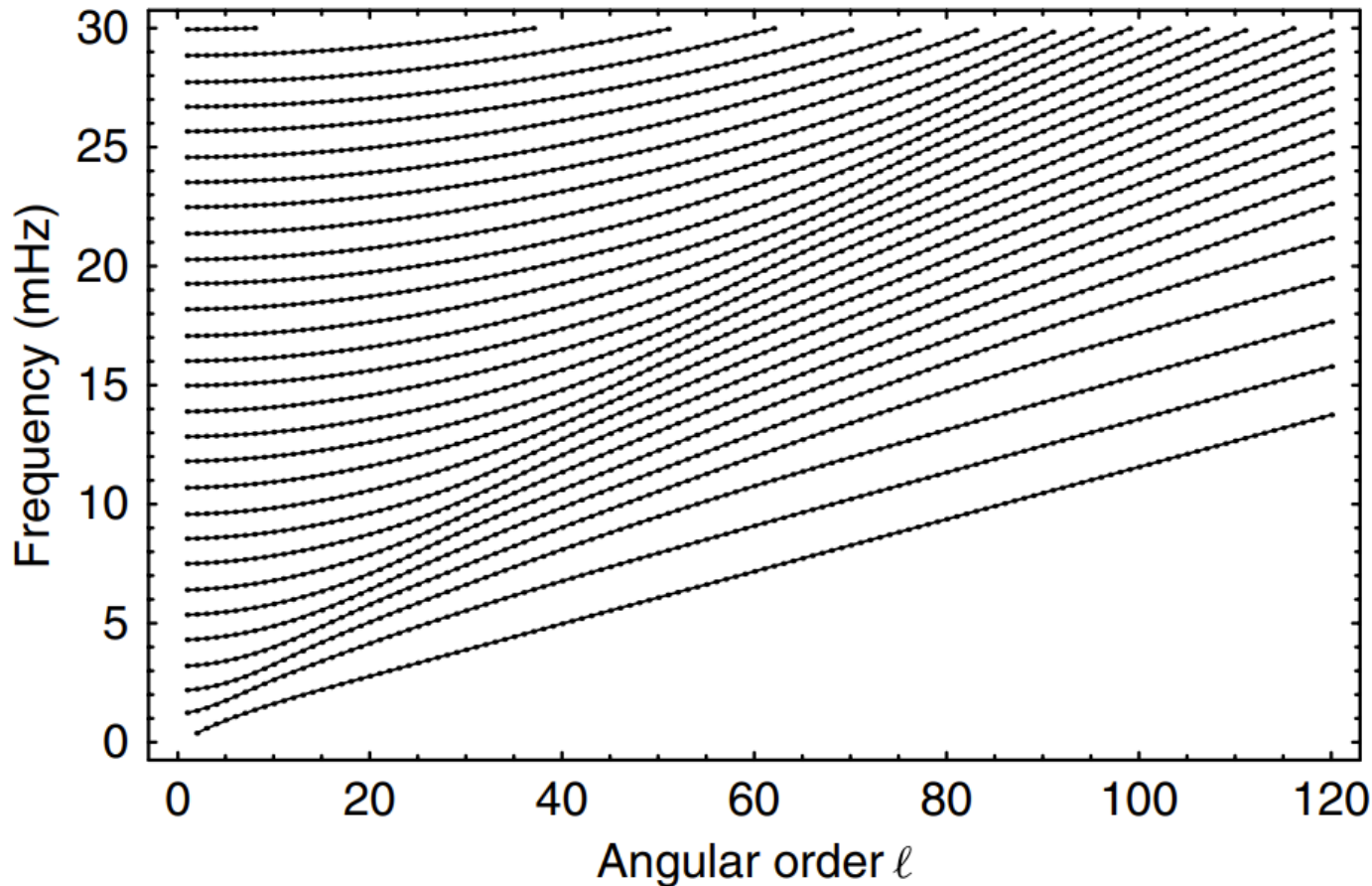
slope=-0.959939



PSD shows 1/f fluctuations more clearly.

Therefore 1/f of earthquakes is a **low-energy phenomenon**.

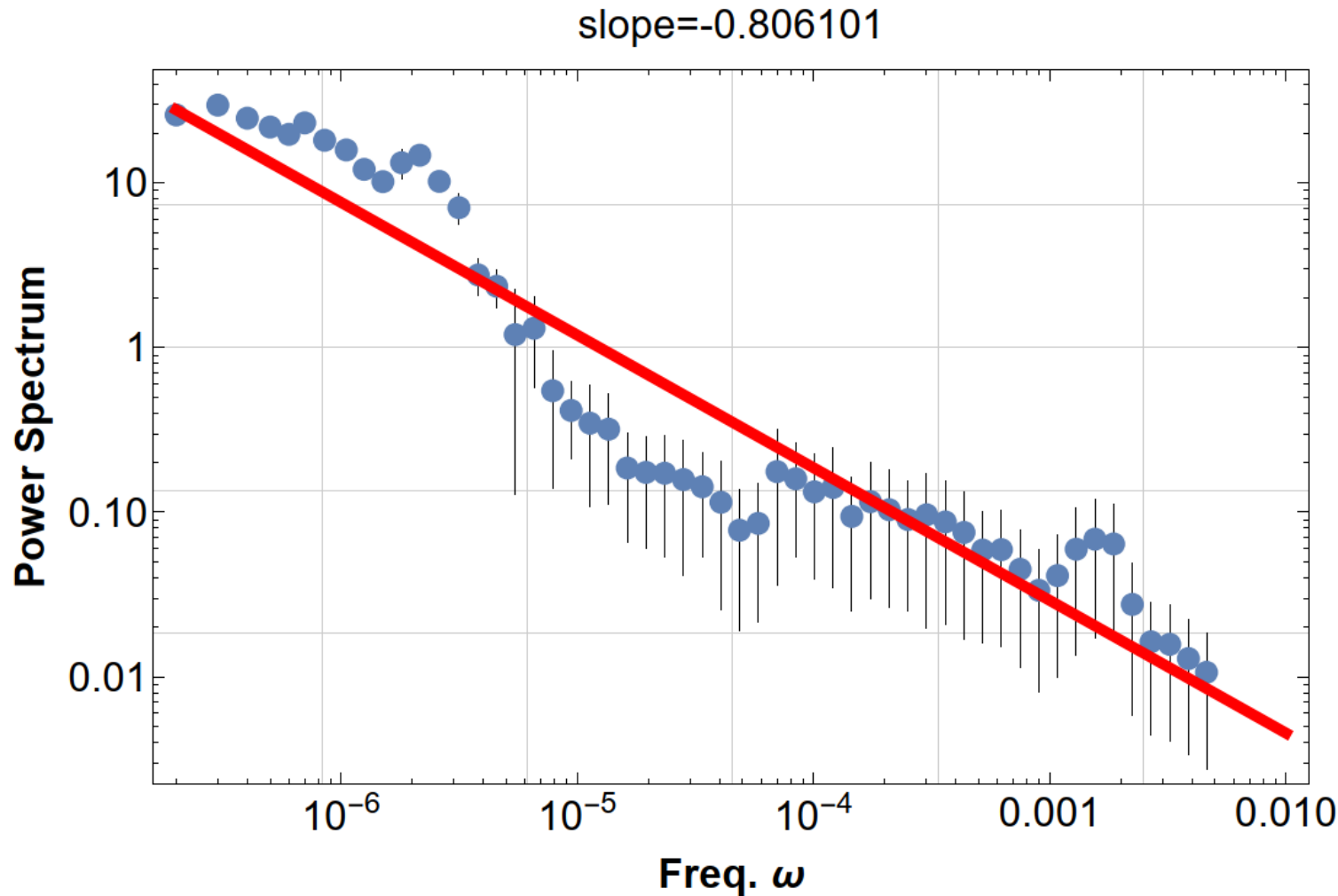
- If the origin of this $1/f$ fluctuation is "the beat of many waves with accumulating frequencies due to resonance," then what on Earth is resonating? → Earth's Free Oscillation (EFO)



<https://saviot.cnrs.fr/terre/index.en.html>

[00002 31..66 \(elsevier.com\)](#) Woodhouse and Deuss *1.02 Theory and Observations – Earth's Free Oscillations* 2007.

- Assuming EFO is resonating, the PSD created by it (including Earth rotation and resonance) shows $1/f$ fluctuation.



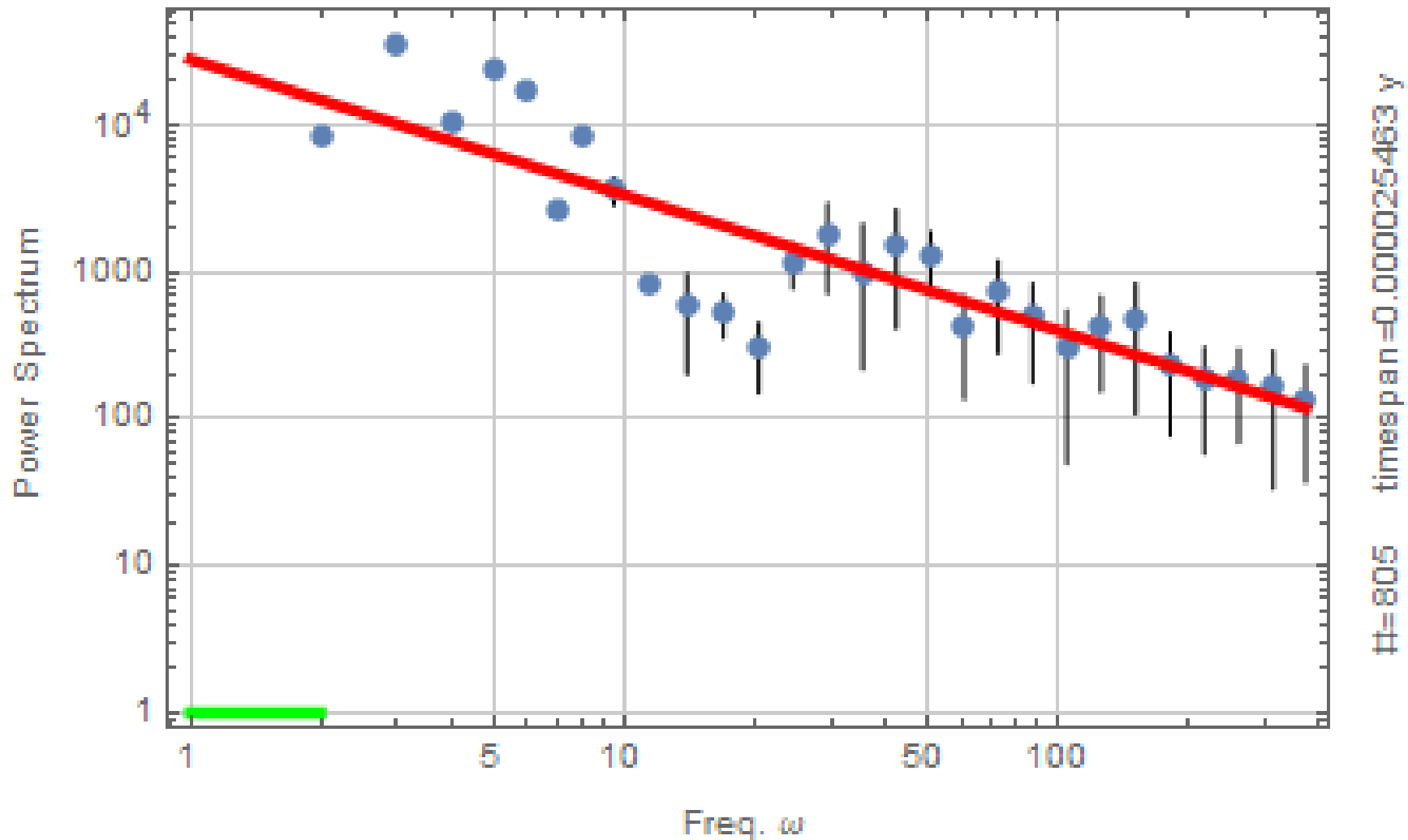
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The frequency range ($\sim 10^{-6}$) overlaps with that of the observed earthquake. **→ EFO may trigger seismic activities**

e) **Volcano eruption** ←? EFO

If EFO triggers an earthquake by breaking fault rupture, EFO also breaks the rocks that work as pressure valves.

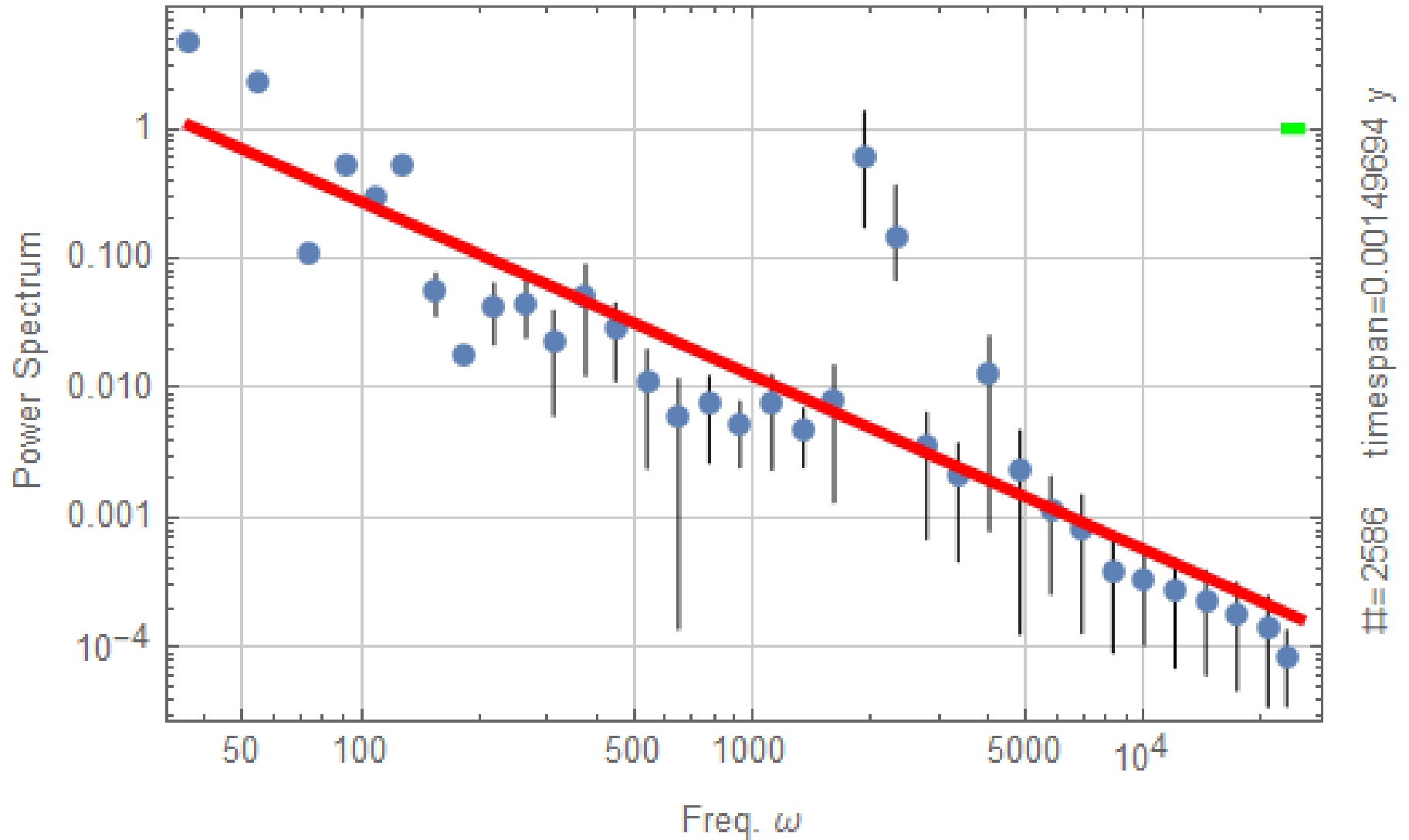
slope=0 and -0.923898 before & after $\omega=1$



f) **Fluctuation in the Earth's rotation axes** ← ? EFO

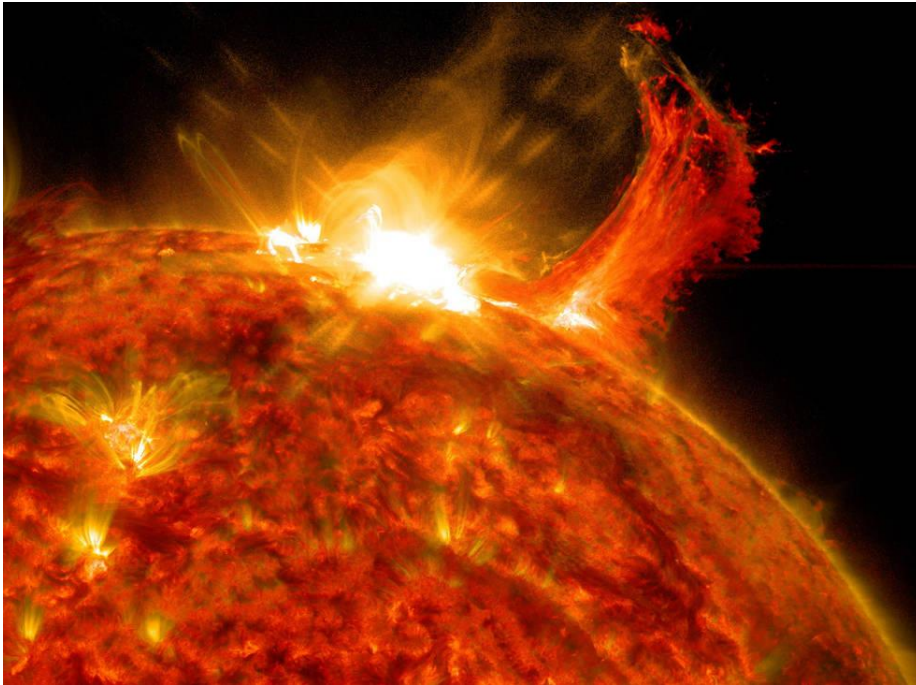
data from IERS, Earth orientation parameter PM-X fluctuation

slope = -1.34085 and 0 before & after $\omega = 25000$.



g) **Solar flare** ...physics is the same as earthquakes

NASA image of a solar flare on Oct. 2, 2014.



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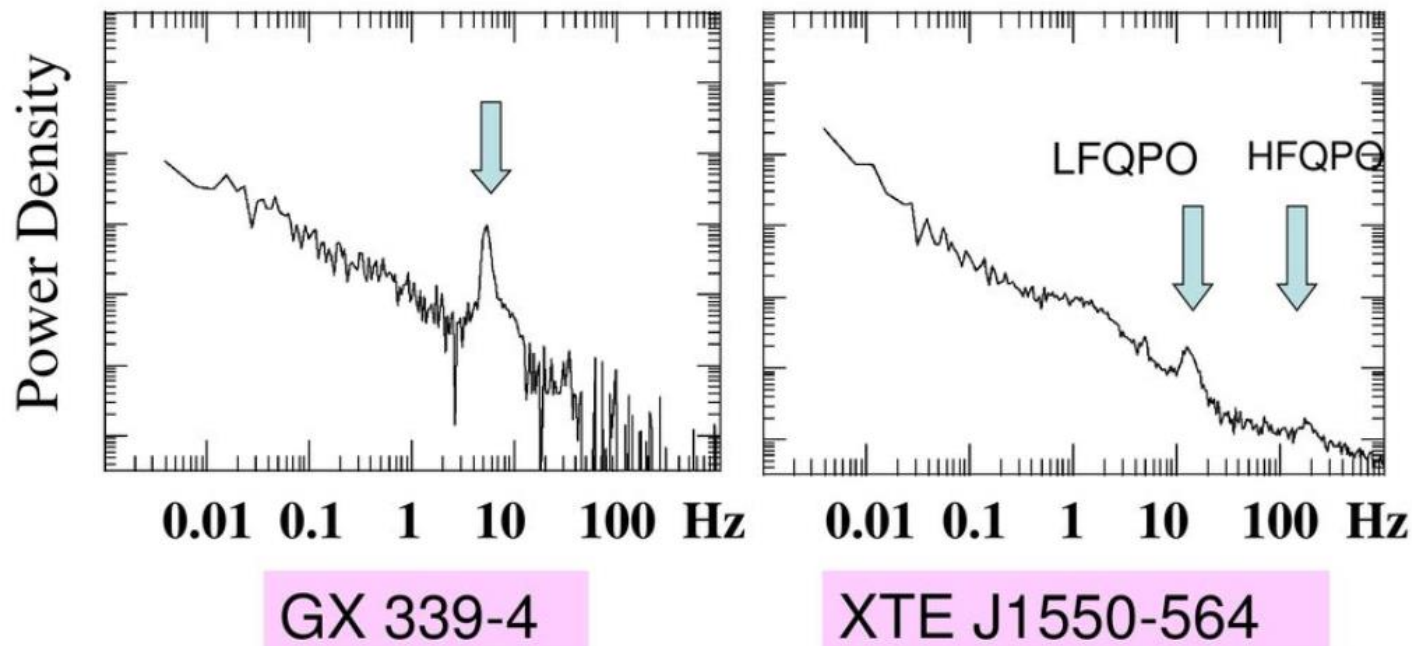
Physics is almost the same as seismic activities:

Seismic $1/f$	Earth's Free Oscillation	fault rupture
Solar flare	5 min. Solar oscillation	magnetic reconnection

h) BH QPO 1/f fl. ???

... from the resonance in the disk eigenfrequencies?

Quasi-Periodic Oscillations (QPOs) in Black Hole Candidates



GX 339-4

XTE J1550-564

McClintock and Remillard 2004

...

7. IR divergence



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Infrared diverging cats ... Bing Image Creator

a) Electron wave packet

- Electric currents in semiconductors often exhibit $1/f$ fluctuations.
- In semiconductors, $\text{mfp} \approx 10\text{nm} \approx$ several tens of lattice size.

Within this size, electrons form a **wave packet**:

$$\psi(x, t) = e^{i(k_0x - \omega_0t)} \int \phi(k_0 + k') \exp [ik'(x - v_g t)] dk'$$

- then, 10^{10} packets exist in a sample of 1mm^3 semiconductor.
- these packets are scattered by impurities with the **photon emission**.

- The packet behaves as

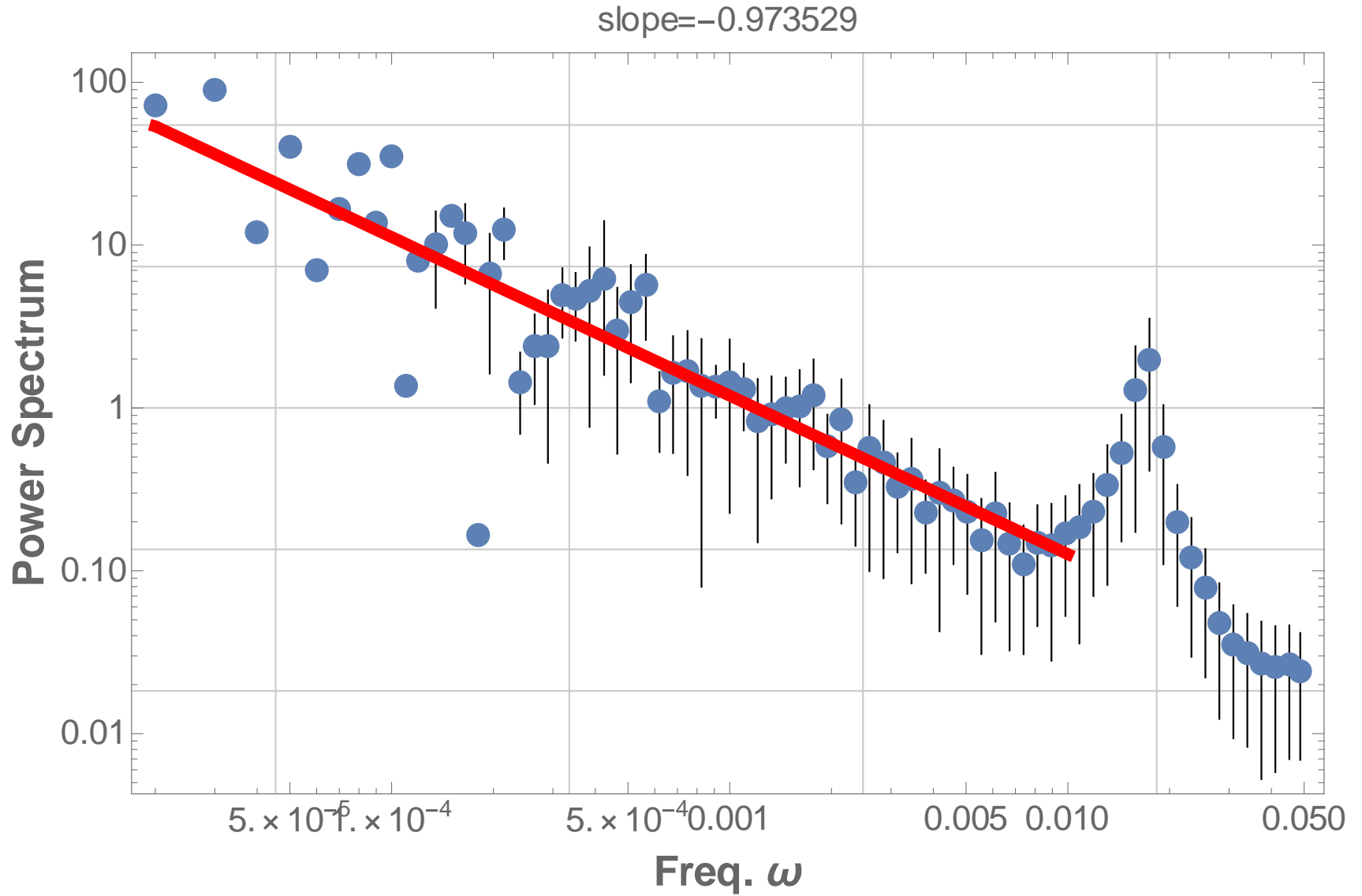
$$\psi''(t) = -\psi(t) + \xi(t)\psi(t) - \kappa\psi'(t) - \lambda\psi(t)^3$$

where $\xi(t)$ reflects the back reaction of photon emission

$$dN = \frac{d\mathcal{E}}{k^0} = e^2 \left| \frac{\varepsilon \cdot p^i}{k \cdot p^i} - \frac{\varepsilon \cdot p^f}{k \cdot p^f} \right|^2 \frac{d^3k}{2(2\pi)^3 k^0} \quad \text{with probability } \propto \frac{1}{\omega} \dots \text{IR div.}$$

with $\kappa = 0.001, \lambda = 0, \xi$: back reaction of γ em.

PSD of $\psi(t)^2$



→
almost
1/f

SEC2.wav

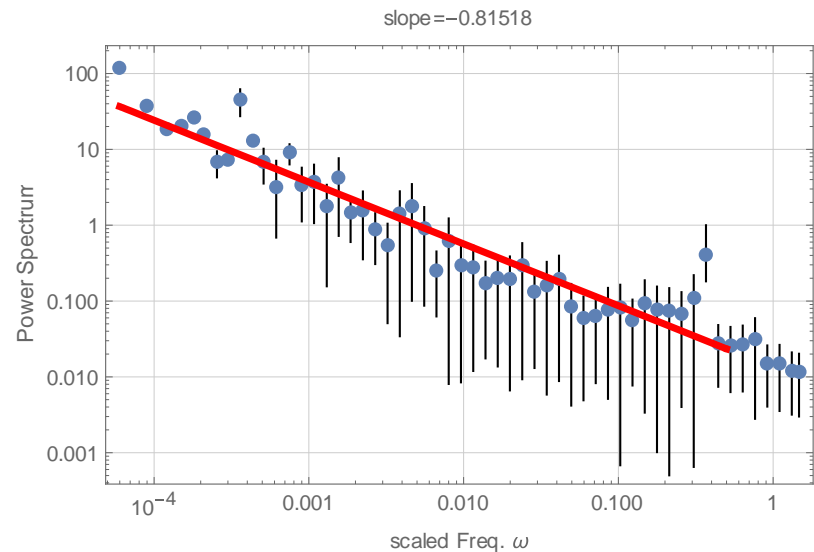
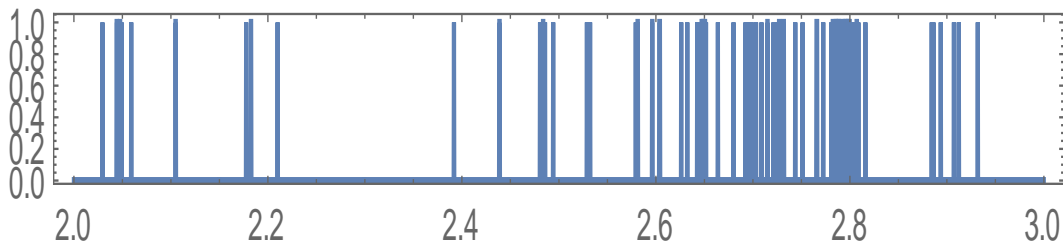
./SEC1.wav

b) **From an electric current to nerve firing: a proposal**

An electric current fl. f^{-1}

→ voltage fl. in biological membrane

→ **neuron cell firing (thresholding)** → nerve system f^{-1}



→ the living body, consciousness

→ heartbeat f^{-1}

→ MEG and EEG (brain) f^{-1}

→ Posture control (sway) f^{-1}

→ consciousness f^{-1}

8. Conclusions and Prospects

- Origin of $1/f$ fluctuations is **the beat of waves with accumulating frequencies (Amplitude Modulation, AM)**
- The frequency accumulation spontaneously arises from **synchronization, resonance, and infrared divergence.**
- Any **demodulation (DM) process** is inevitable for the appearance of $1/f$ fluctuations: this provides us with a variety of $1/f$ fl. ⁵²
- Comfortable music signal does not show $1/f$ fluctuations.
The squared signal does show $1/f$ fluctuations.
We can “hear” the wave beat even in the original signal of music.
i.e., We are listening to the beat of music.

● Plenty of further problems left:

Q. There are almost **no piano ensembles**

Q. The orchestra has **a few flutes** (cf. many violins).

Q. Enya has recorded a song **100 times and layered them.**

Q. The **Vienna Musikverein Hall** is a rectangular parallelepiped.

Q. Why do you apply "**vibrato**"?

Q. What is the difference between the sound of a **violin solo** and an **orchestra unison**?

Q. Relation between **Jazz Swing** and $1/f$ fluctuations?

Thank you all for your attention.

Thank Motova Mary, for the supreme poster design

Thank Larissa Bravina for her setup for this seminar..A. Nakamichi

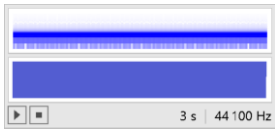
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Backup slides =====

1. UBR from Unison

A. Tonal violin unison

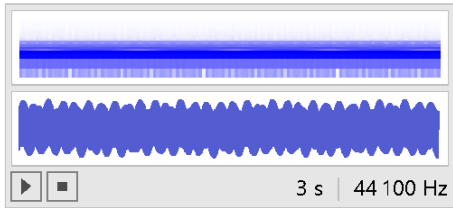
◆ One “violin” $v_0 = \sin(2\pi\omega t)$



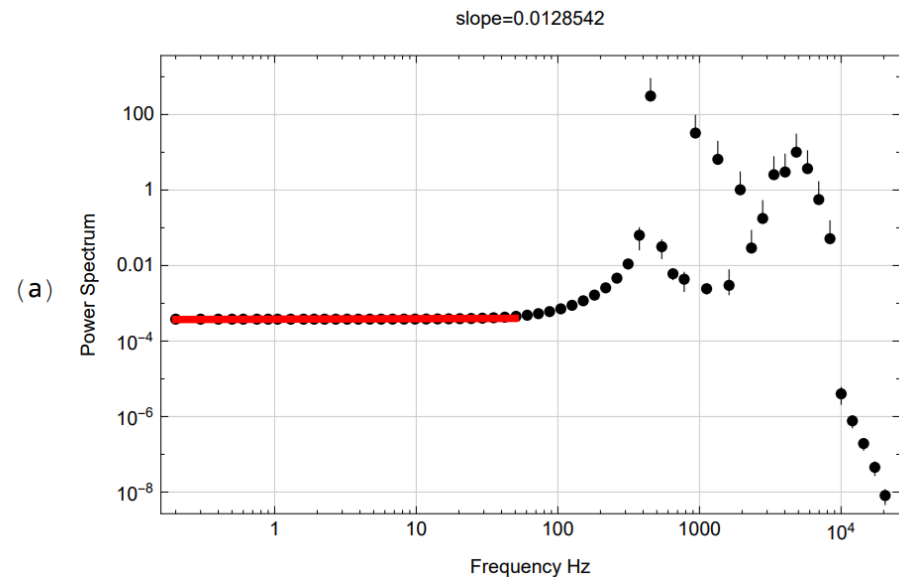
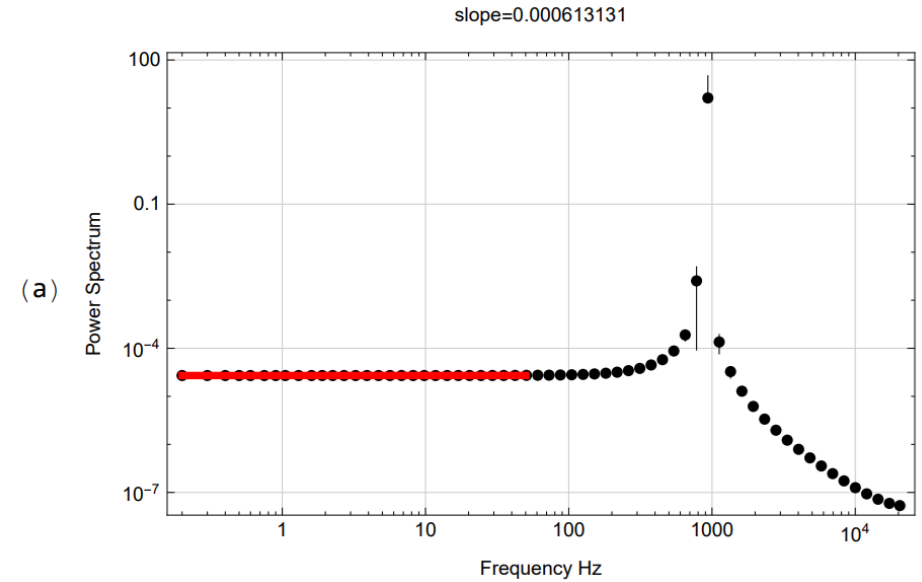
p3.wav

◆ Tone by adding overtones $v_0(\omega, t) =$

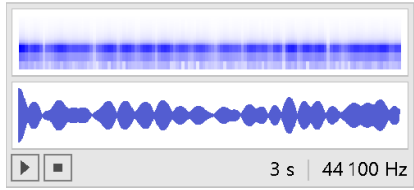
$$\sum_{\{m=1\}}^{\{M\}} n^\beta \sin(2 \pi m \omega t + \eta)$$



ph43.wav



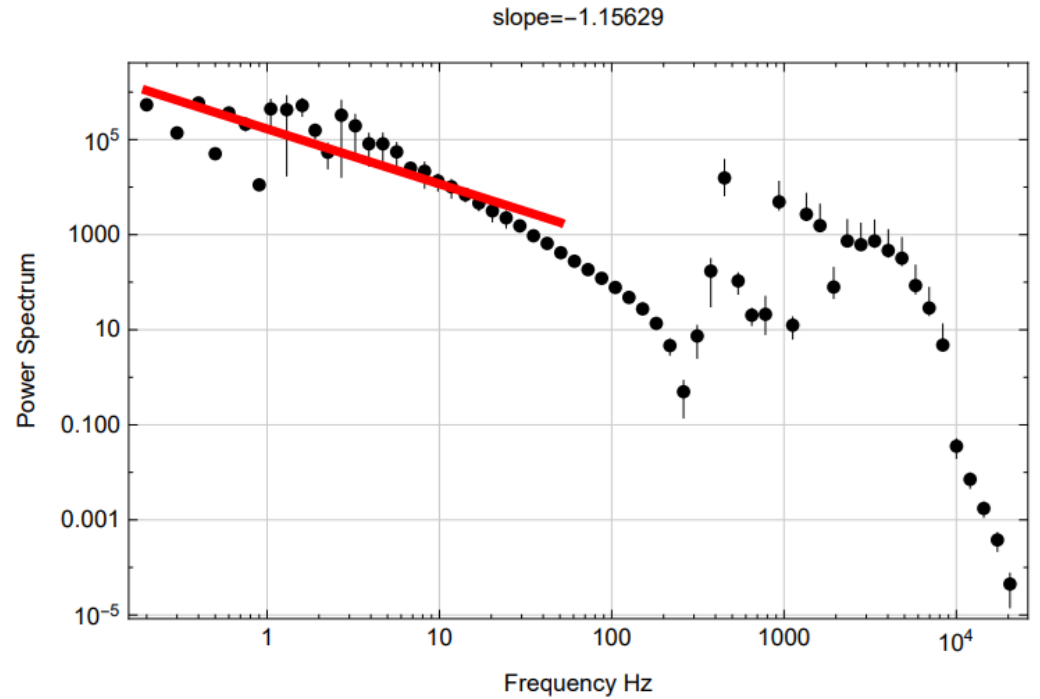
◆ Double quintet (10 layers)



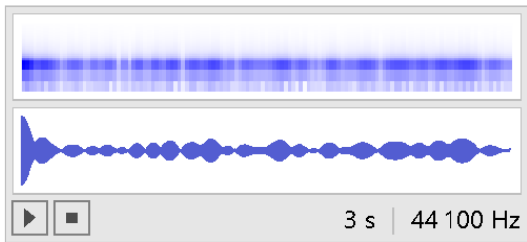
orc3-10-

10.wav

(c)



◆ 30 “violins” (30 layers)



orc3-10-30.wav

[Development 2] Sound wave \Rightarrow **electron wave**

◆ All the same arguments as above \rightarrow Electron waves (Vacuum tubes, semiconductors, human hearts, Undersecretary's guess, giant axons of squids, MEG and EEG in the brain.)

\Rightarrow Does most of the $1/f$ fluctuations related to electrons originate in beats?

\Rightarrow Rebuttal to Handel (1980), Kampen, Kish ...!

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◆ Cooperative phenomenon + closer approach is exponential \Rightarrow UBR...Relationship with infrared divergence

\Rightarrow Infrared divergence occurs in both quantum electrodynamics and inflation.

2. beyond music

- The above analysis is reminiscent of $1/f$ fluctuations and pink noise, which are characterized by low-frequency powers with

exponents from -1.5 to -1, unlike white noise (index 0) and brown noise (index 2).

- This pink noise appears everywhere in nature and has various origins.

→ Does UBR appear universally?

- ◆ The roar of all the waves that are generated in the energy and heat flow— isn't it? 58

◆ Infrared divergence in both quantum electrodynamics and inflation

This breaks the perturbation theory, but when expressed in terms of Keldysh- type path integrals, the divergence can be isolated and the statistical fluctuations can be brought into isolation. This leads to (classical) Langevin evolution and creates a coherent state → SSB, etc.

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$$\begin{aligned}
 Z[J] &= \int D\xi P[\xi] \delta[\ddot{x}(t) + \gamma\dot{x}(t) + V'(x(t)) - \xi(t)] \\
 &\quad \times \exp\left[-i \int dt J(t)x(t)\right] \quad x'(t): \text{Lagrange multiplier} \\
 &= \int Dx' D\xi P[\xi] \exp\left[-i \int dt x' \left(\ddot{x} + \gamma\dot{x} + V'(x) - \xi + Jx\right)\right] \\
 &= \int Dx' \exp\left[i \int dt \left(\underbrace{\dot{x}'\dot{x} + \gamma x'\dot{x} + x'V'(x)}_{S_{eff}} - i\sigma^2 x'^2 + Jx\right)\right] \\
 &\quad \text{Complex action}
 \end{aligned}$$

◆ Unison of violin solo and orchestra violin part

The sounds of (apart from the chords) are clearly different.

- a) Solo [Partita for Solo Violin No. 2-Giga - from YouTube part.wav](#)
- b) Unison [Tchaikovsky Serenade for Strings _Ozawa Saito.wav](#)

◆ Unison: sobbing, comfortable tension, divine rumbling

. . . (Even though it's physics, I have emotions, sorry)

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◆ Unison: From the growl of sound

sound characterization

power spectrum

