Time-energy uncertainty relations, non-inertial quantum clocks and the effective appearance of non-unitarity

Ismael L. de Paiva

Eliahu Cohen

Bar-Ilan University

אוניברסיטת בר אילן

BINA

Bar-Ilan Institute of Nanotechnology & Advanced Materials

From Foundations To Quantum Technology

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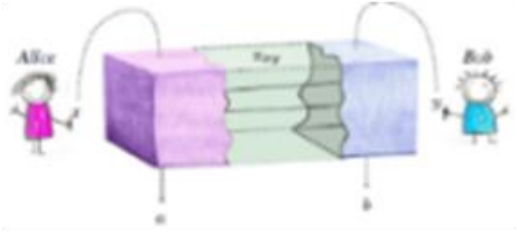
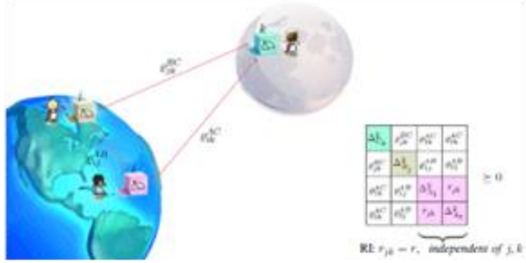
From Quantum Foundations To Technology

Entanglement&Nonlocality

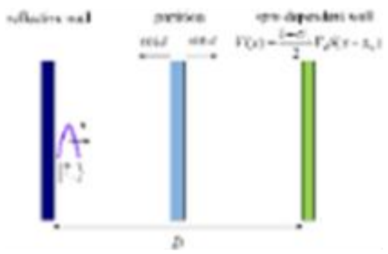




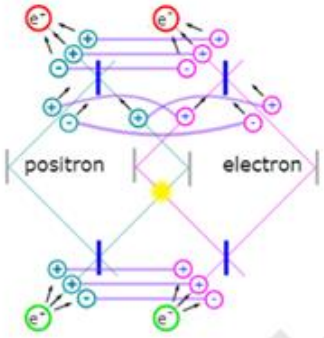
(*Sci. Adv.* 5, eaav8370 (2019)

NJP 21, 073032 (2019), etc.)

Weak values and Weak measurements

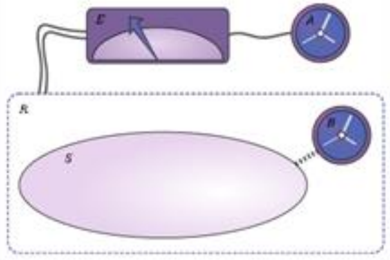


(*Nat. Commun.* 12, 4770 (2021)

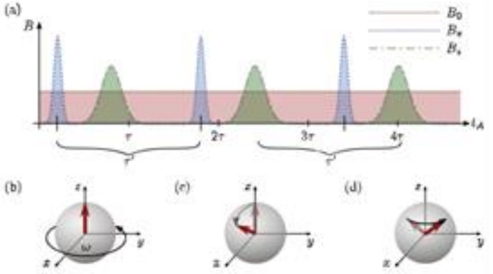


(accepted to PNAS)

Quantum time



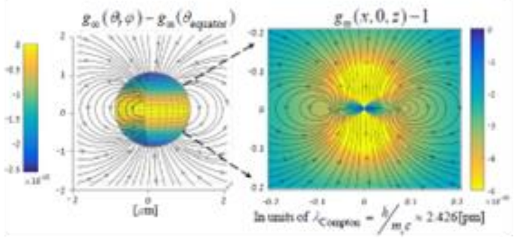
(Quantum 6, 683 (2022))



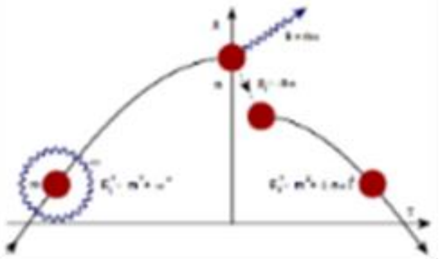
(PRA 105, 042207 (2022))

(Comm. Phys. 5, 298 (2022))

Information-based Quantum Gravity



(Ann. Phys. 534, 2100348 (2022))



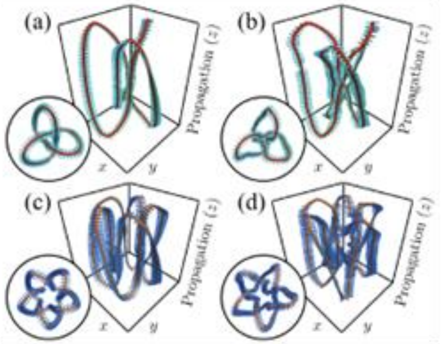
(*PRD* 104,021015 (2021))

(*NJP* 212, 083038 (2019))

See also: *PNAS* 115, 11730 (2018) – Quantum Emergent Phenomena
PNAS 114, 6480 (2017) – Quantum Interference via nonlocal dynamics

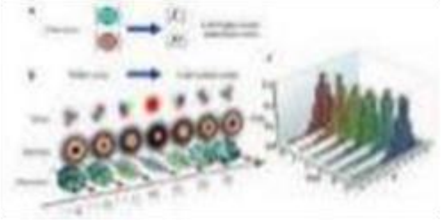
From Quantum Foundations To Technology

Quantum Computation and Communication

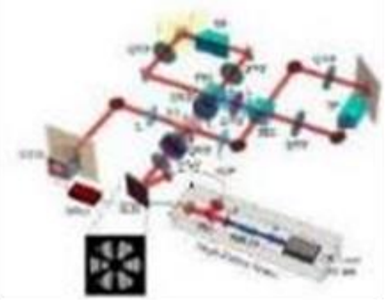


(*Nat. Commun.* 11, 5119 (2020))

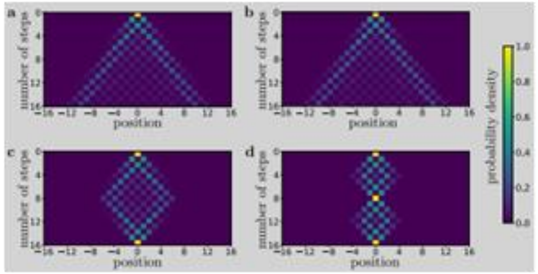
Photonic Quantum Walks for Quantum Simulations



(*Nat. Rev. Phys.* 1, 437-449 (2019))

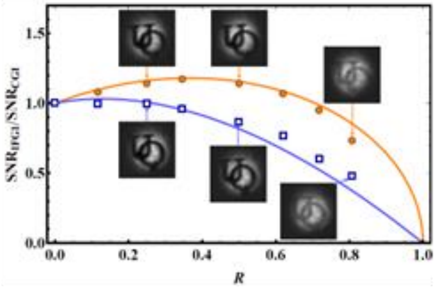


(*Optica* 6, 174-180 (2019))



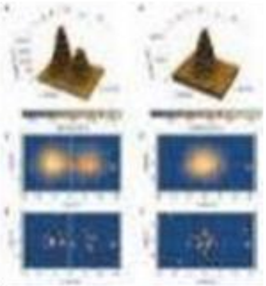
- (PRA 105, 032413 (2022))

Quantum Imaging



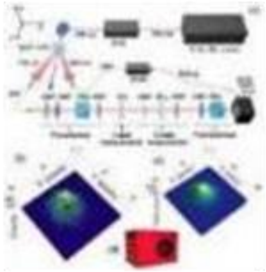
(*Opt. Express* 27, 2212 (2019))

Quantum Measurements & Quantum Metrology



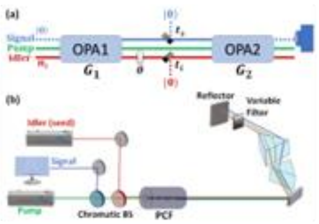
(*Nat. Phys.* 13, 1191 (2017)

Light: Sci. Appl. 10, 106 (2021)).

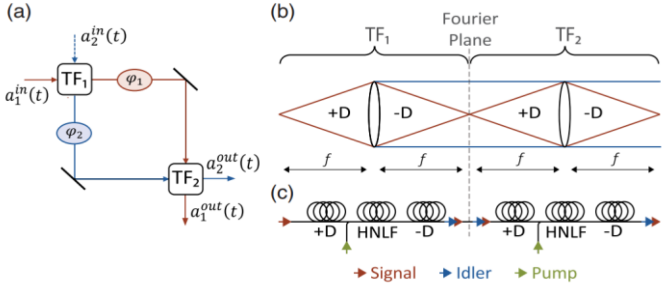


(*PRL* 117, 170402 (2016))

(*Nat. Phys.* 16, 1206-1210 (2020))



(*PRL* 127, 173603 (2021))



(PRL 130, 253601 (2023))

Outline

- Introduction to the "problem of time" and time-energy uncertainty relations
- How we used the Page-Wootters scheme for addressing the above topics
- What happens to time in accelerating/gravitating quantum frames of reference?
- Dynamical nonlocality in time

Based on:

- I. L. P., A. C. L., and E. C., Flow of time during energy measurements and the resulting time-energy uncertainty relations, Quantum 6, 683 (2022).

- I. L. P., A. T., B.P., E.C., Y.A., Noninertial quantum clock frames lead to non-Hermitian dynamics, arXiv:2204.04177, Communications Physics 5, 298 (2023).

- Y.A., E. C., F.C. et al., Finally making sense of the double-slit experiment, PNAS 114, 6480 (2017).

- I. L. P., M. N., and E. C., Dynamical nonlocality in quantum time via modular operators, Phys. Rev. A 105, 042207 (2022).

- M.S., I. L. P., and E. C., Non-relativistic spatiotemporal quantum reference frames, arXiv:2307.01874.

Background

- Problem of time
- Time-energy uncertainty relations

Problem of time

Apparent conflict between the concept of time in quantum theory and in diffeomorphism-invariant theories like general relativity

<u>Quantum Mechanics</u>

- Time is a background parameter (external to, and unaffected by, the system)
- Time is universal and absolute

<u>General relativity</u>

- Time is one of four dimensions affected by mass and energy
- Time is dynamical and relative

Wheeler-DeWitt Equation in quantum gravity: (Hamiltonian constraint)

$$H|\Psi\rangle = 0$$

Seemingly implying timelessness (no evolution, no Schrödinger eq. etc.)

Time Operator in Quantum Mechanics

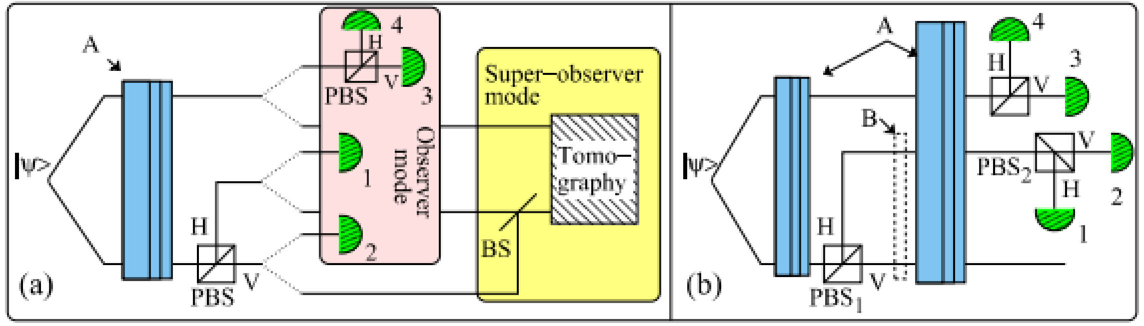
- Time is typically understood as a parameter rather than a dynamical variable
- On the one hand, a time operator could be instructive
- One the other hand, it is believed to create instability since a time operator T obeying $[H, T] = i\hbar$ leads to Hamiltonian H whose spectrum is unbounded from below
- We will show next, following **Page and Wootters**, how to overcome this problem and regain a notion of time flow even in a timeless framework
- Then we will make things just a bit more complicated…

Main Idea ("Evolution without evolution")

- Dividing the whole system into a "clock" and "the rest" and using
  $H|\Psi\rangle = 0$ to entangle them
- This was already demonstrated in the lab:

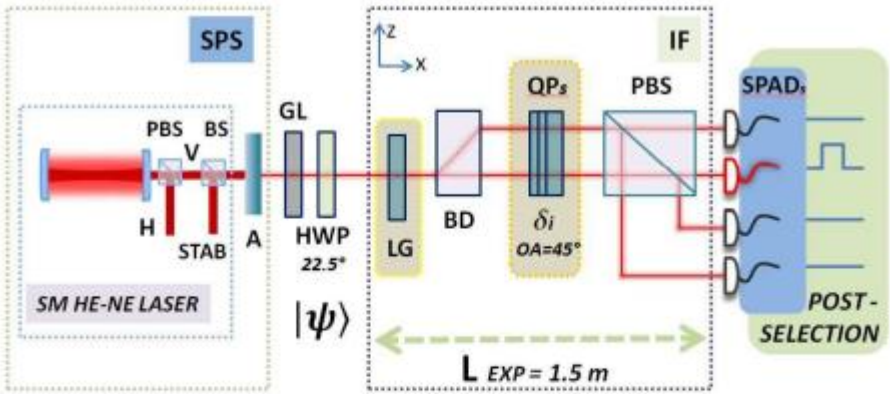
Time from quantum entanglement: An experimental illustration

Ekaterina Moreva, Giorgio Brida, Marco Gramegna, Vittorio Giovannetti, Lorenzo Maccone, and Marco Genovese
Phys. Rev. A **89**, 052122 – Published 20 May 2014



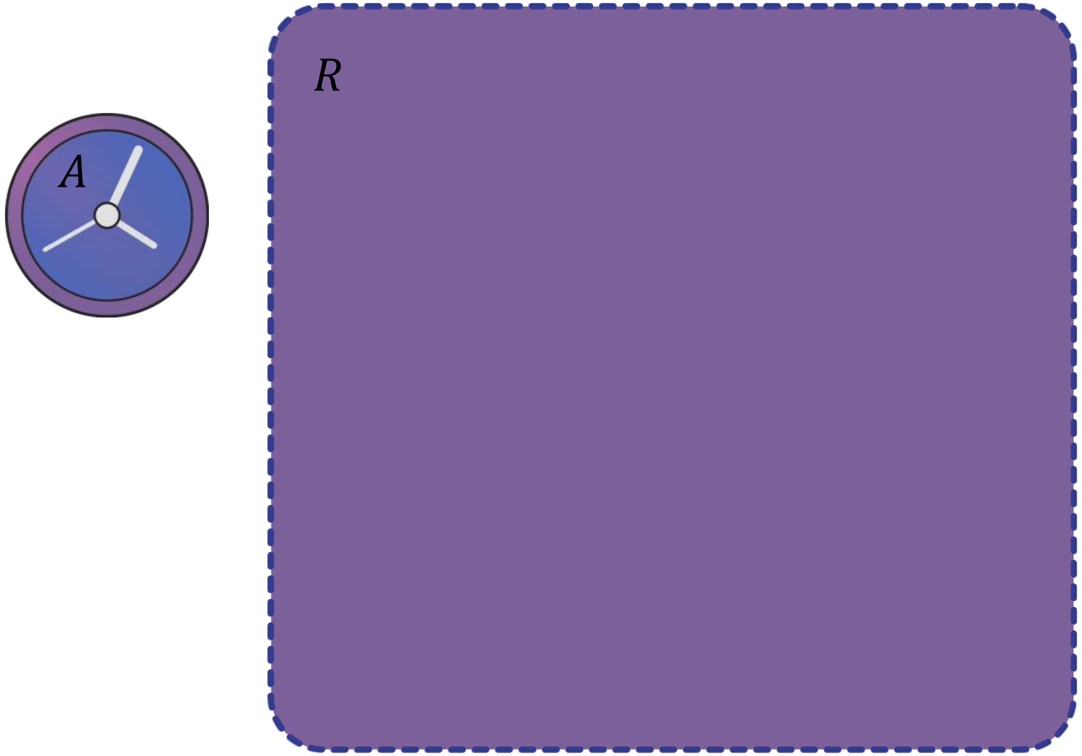
Quantum time: Experimental multitime correlations

Ekaterina Moreva, Marco Gramegna, Giorgio Brida, Lorenzo Maccone, and Marco Genovese
Phys. Rev. D **96**, 102005 – Published 16 November 2017



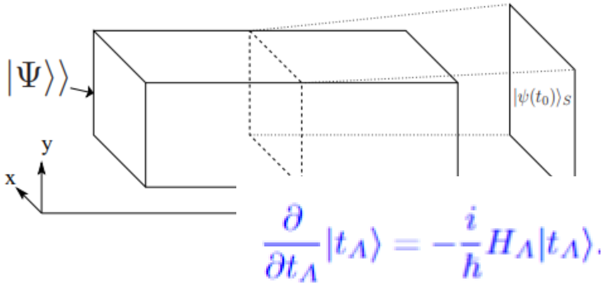
Developed by: Page-Wootters, Giovannetti-Lloyd-Maccone, Marletto-Vedral,....

Page and Wootters timeless framework



Wheeler-DeWitt equation

$$H_T|\Psi\rangle\rangle = 0$$



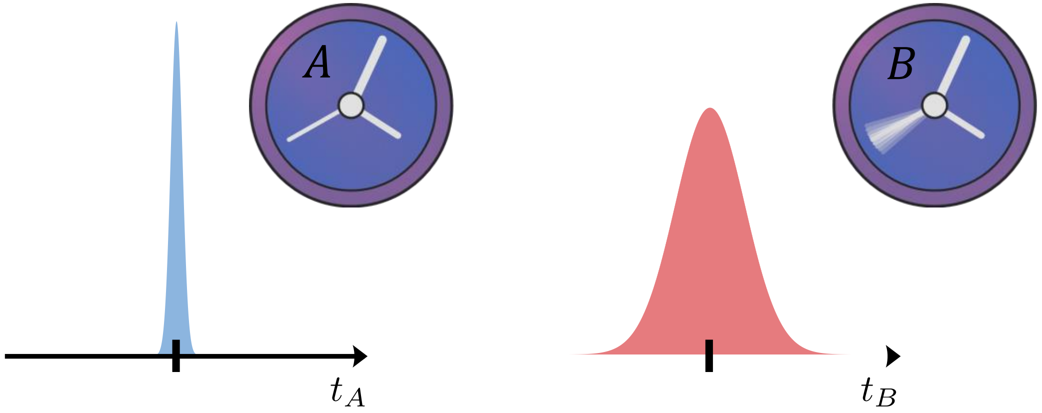
Page and Wooters

$$[H_A + H_R + H_{int}(T_A)]|\Psi\rangle\rangle = 0$$

$$\langle t_A|$$

$$i\hbar\frac{\partial}{\partial t_A}|\psi(t_A)\rangle = [H_R + H_{int}(t_A)]|\psi(t_A)\rangle$$

$$H^A_{eff} = H_R + H_{int}(t_A)$$



D. N. P. and W. K. W., Phys. Rev. D 27, 2885 (1983)

E. C.-R., F. G., A. B., and Č. B., Nat. Commun. 11, 2672 (2020)

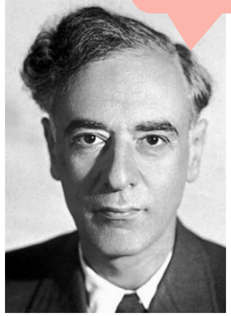
Formal details

- The clock should have an observable T_A associated with its time
- It obeys $|t_o + t_A\rangle = e^{-iH_A t_A/\hbar}|t_0\rangle$
- The wavefunction obeys $|\psi\rangle\rangle = \int dt_A |t_A\rangle \otimes |\psi(t_A)\rangle$
- T_A needn't be a self-adjoint operator, canonically conjugated to H_A
- It can be constructed as a POVM
- However, it is common to assume an *ideal* clock:
  - $[T_A, H_A] = i\hbar I$
  - $T_A|t_A\rangle = t_A|t_A\rangle$
  - $H_A = -i\partial/\partial t_A$

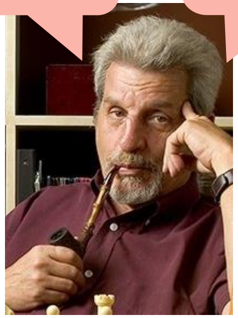
Time-energy uncertainty relations

Is there a minimum duration for energy measurements with a certain precision?

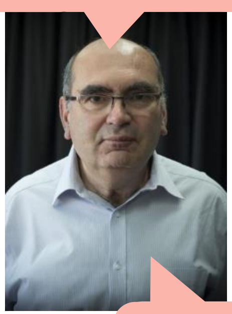
Yes, there is!



Not really...



... but only if the Hamiltonian is known!



... and they can be performed arbitrarily fast on the internal clock.

L. L. and R. P., Z. Phys. 69, 56 (1931)

Y. A. and D. B., Phys. Rev. 122, 1649 (1961)

Y. A., S. M., and S. P., Phys. Rev. A 66, 052107 (2002)

Y. A. and B. R., Phys. Rev. Lett. 84,1368 (2000)

S. M. and S. P., Phys. Rev. A 71, 042106 (2005)

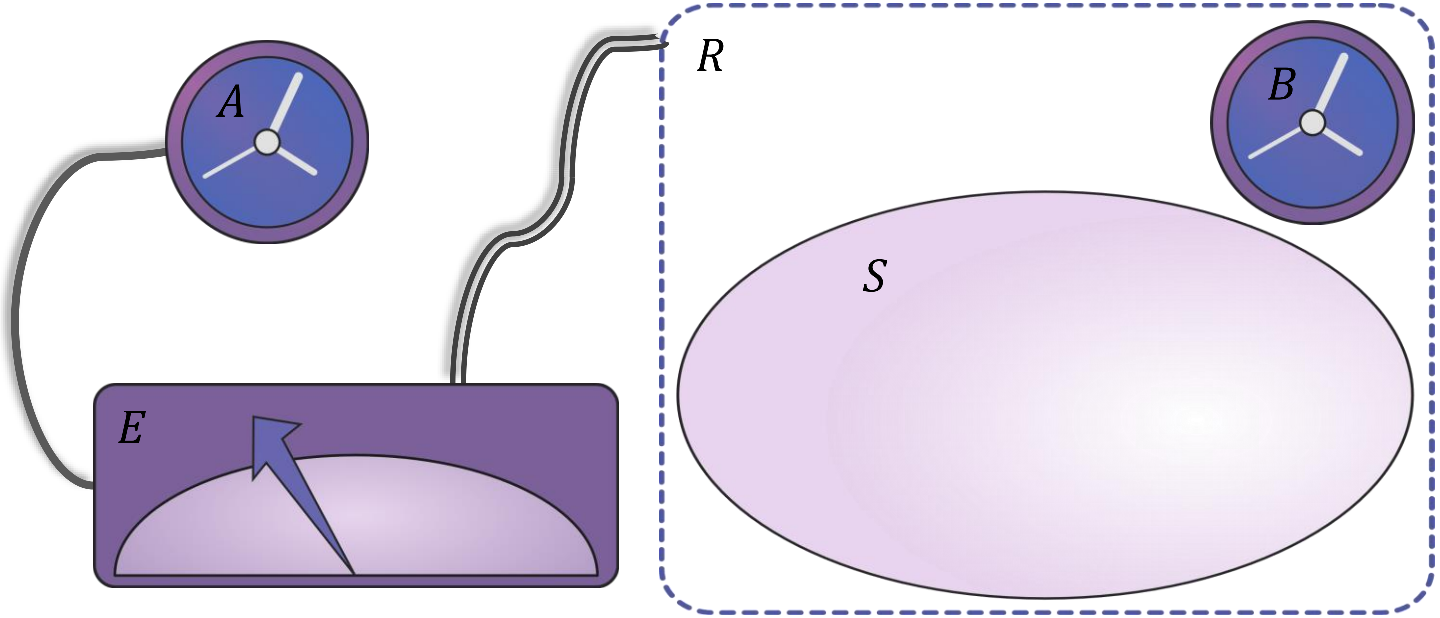
Time-energy uncertainty relations

Is there a minimum duration for energy measurements with a certain precision?

System carrying out measurement	Time optimized	Hamiltonian known?	Uncertainty relation?
External	External	Yes	No
External	External	No	Yes
External	Internal	Yes	?
External	Internal	No	?
Internal	External	Yes	?
Internal	External	No	?
Internal	Internal	Yes	No
Internal	Internal	No	?

Energy measurements

Energy measurement



Carried out by external system

$$H_R = H_B + H_S + H_{int}(T_B)$$

$$H_{VN} = g(T_A)H_R P_E$$

$$H_T = H_A + H_R + H_{VN}$$

$$H_{eff}^A = H_R + g(t_A)H_R P_E$$

$$\frac{d}{dt_A}T_B = I + g(t_A)P_E$$

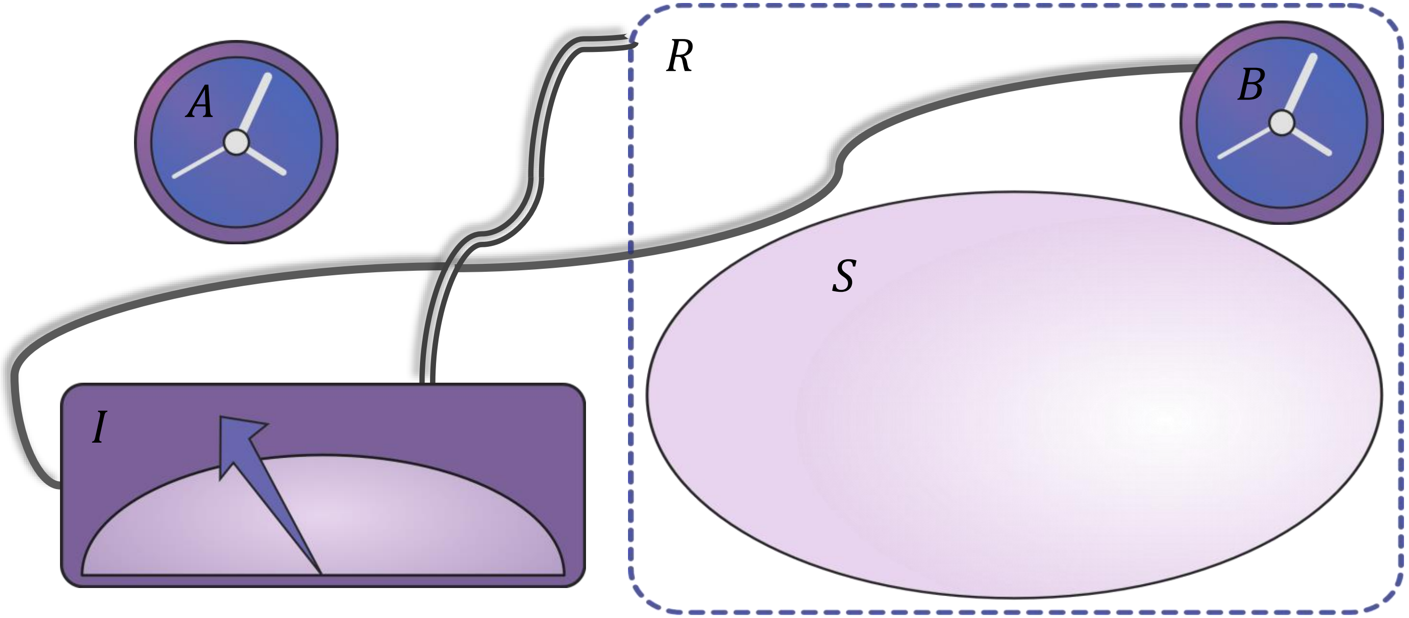
$$T_B(\tau) - T_B(0) = \tau + KP_E$$

$$H_{eff}^B = [I + g(T_A)P_E]^{-1}H_A + H_S + H_{int}(t_B)$$

$$\frac{d}{dt_B}T_A = [I + g(T_A)P_E]^{-1}$$

$$\langle T_B(\tau) - T_B(0)\rangle \geq K\langle P_E\rangle$$

Energy measurement



Carried out by internal system

$$H_R = H_B + H_S + H_{int}(T_B)$$

$$H_{VN} = \frac{1}{2}[g(T_B)H_R + H_R g(T_B)]P_I$$

$$H_T = H_A + H_R + H_{VN}$$

$$H_{eff}^A = H_R + \frac{1}{2}[g(T_B)H_R + H_R g(T_B)]P_I$$

$$\frac{d}{dt_A}T_B = I + g(T_B)P_I$$

$$H_{eff}^B = [I + g(t_B)P_I]^{-1}\left[H_A - \frac{i\hbar}{2}g'(t_B)P_I\right] + H_S + H_{int}(t_B)$$

$$\frac{d}{dt_B}T_A = [I + g(t_B)P_I]^{-1}$$

Carried out by external system

$$H_{eff}^A = H_R + g(t_A)H_R P_E$$

$$\frac{d}{dt_A}T_B = I + g(t_A)P_E$$

$$\langle T_B(\tau) - T_B(0)\rangle \geq K\langle P_E\rangle$$

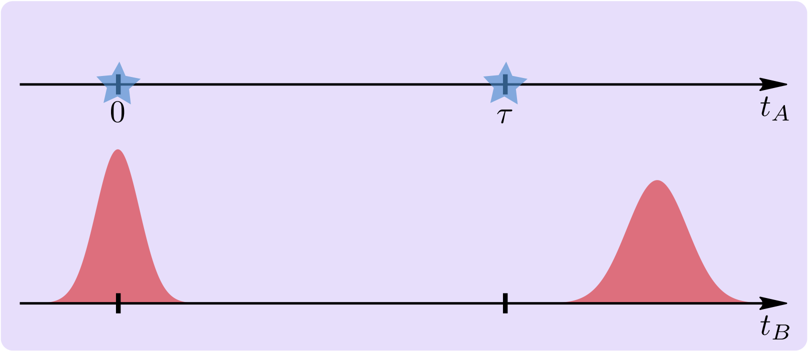
$$H_{eff}^B = [I + g(T_A)P_E]^{-1}H_A + H_S + H_{int}(t_B)$$

$$\frac{d}{dt_B}T_A = [I + g(T_A)P_E]^{-1}$$

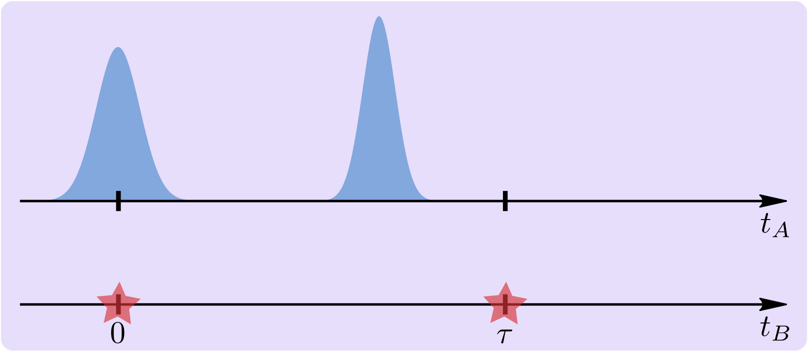
Non-Hermitian!

Results

External Measurement



Internal Measurement



System carrying out measurement	Time optimized	Hamiltonian known?	Uncertainty relation?
External	External	Yes	No
External	Internal	Yes	Yes
Internal	External	Yes	No
Internal	Internal	Yes	No

Non-unitarity

Non-unitarity from acceleration

- Hamiltonian of a free particle: H

- How is it modified if the particle starts accelerating?

$$H \mapsto H + V(x) \qquad\qquad V(x) = -m\int_{x_0}^{x} a(x')\,dx' = mf(x)$$

$$\boxed{H \mapsto H + mf(X)}$$

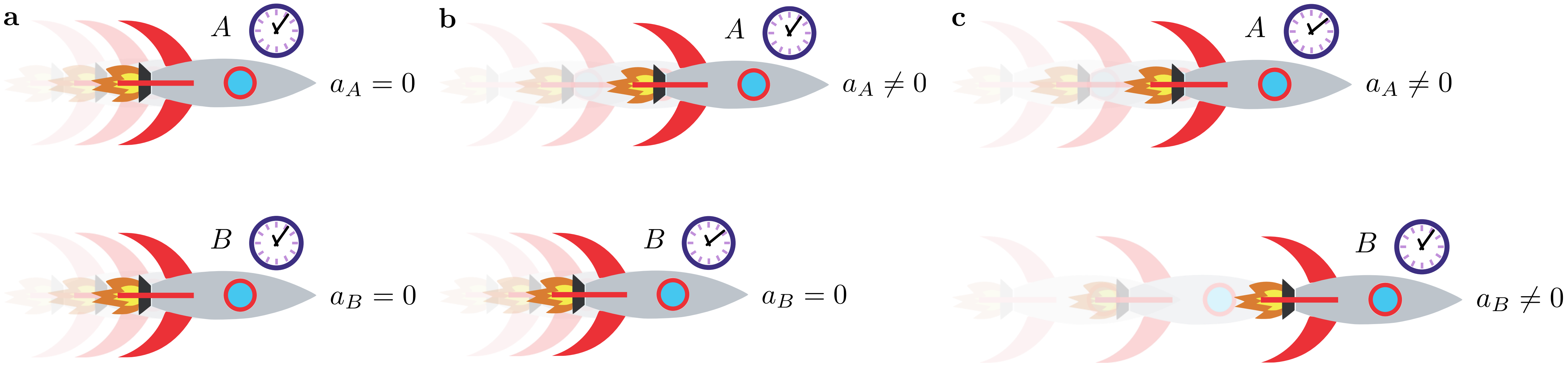
- Relativistic correction: $m \mapsto m + H/c^2$

- Then: $H \mapsto H + \frac{1}{2}[Hf(X) + f(X)H]$

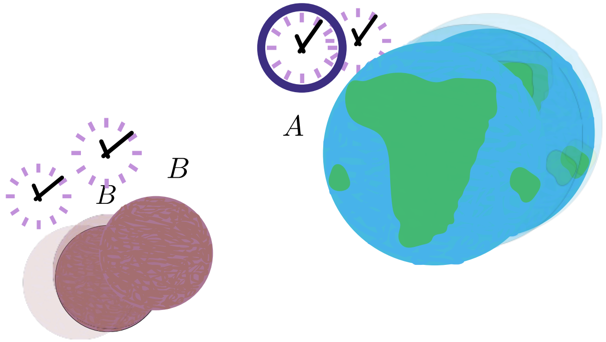
If $H = H_M + H_A$: $$H_{eff}^A = H_M + \frac{1}{2}[1 + f(X_M)]^{-1}\left\{ H_A(X_M), H_M - \frac{1}{2}[f(X_M), H_M] \right\}$$

Non-Hermitian!

Non-unitarity from acceleration



Gravitationally interacting clocks



$$V(x_N - x_M) = -G\frac{m_M m_N}{|x_N - x_M|}$$

$$V(X_N) = -GH_A H_B |X_N|^{-1}/2c^4$$

$$V(X_N) = -\frac{G}{2c^2}\{X_N^{-1}, (H_B + H_N)\}(H_A + H_M) = f(X_N, H_B + H_N)(H_A + H_M)$$

$$H = [I + f(X_N, H_B)]H_A + H_B + H_M + H_N \qquad f(X_N, H_B) = -GH_B |X_N|^{-1}/2c^4$$

$$H^A_{eff} = [I + f(X_N, H_B)]^{-1}(H_B + H_M + H_N)$$ Non-Hermitian!

Is this a feature of the framework or a hint regarding quantum gravity?

Summary (till now)

- In the studied model:
  - Time is emergent (as was claimed e.g. by Rovelli)
  - Time is quantized (as in many recent approaches)
  - Time is relational (as in the works of Gambini&Pullin)
- Leading to:
  - New time-energy uncertainty relations
  - Effective non-unitary dynamics from the perspective of the internal clock
  - Dynamically nonlocal phenomena-Next slides

Modular variables

How to describe interference phenomena in the Heisenberg picture?

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\psi(x)\rangle + e^{i\varphi}|\psi(x+\ell)\rangle\right]$$

$$\langle\psi(x)|\psi(x+\ell)\rangle = 0$$

$$\langle X^n\rangle \neq f(\varphi)$$

$$\langle P^n\rangle \neq g(\varphi)$$

$\langle e^{iP\ell/\hbar}\rangle$ depends on φ!

$$X_{mod} \equiv X \bmod 2\pi\hbar I/p_0$$

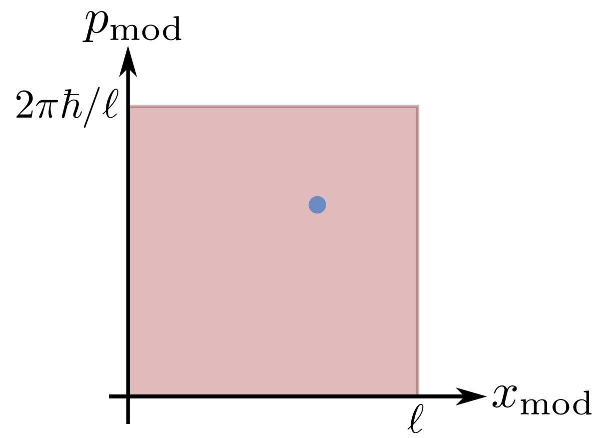
$$[X_{mod}, P_{mod}] = 0$$

$$P_{mod} \equiv P \bmod 2\pi\hbar I/\ell$$

$$H = \frac{1}{2m}P^2 + V(X)$$

$$p_0 = 2\pi\hbar/\ell$$

$$\frac{d}{dt}e^{iP\ell/\hbar} = -\frac{i}{\hbar}\left[V(X+\ell I) - V(X)\right]e^{iP\ell/\hbar}$$



Y.A., E. C., F.C. et al., Finally making sense of the double-slit experiment, PNAS 114, 6480 (2017)

Modular variables

How to describe interference phenomena in the Heisenberg picture?

Complete uncertainty relation

$$\Phi_{mod} = \Phi \bmod 2\pi I$$

$\langle e^{in\Phi}\rangle$ vanishes $\forall n \in \mathbb{Z}$ $\Leftrightarrow$ Φ_{mod} is completely uncertain $P_{\Phi_{mod}}(\varphi)$ is uniform

Modular energy

$e^{iH\tau/\hbar}$

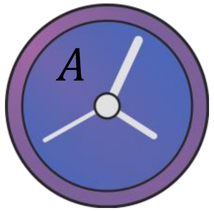
$$\left\langle e^{iH\tau/\hbar}\right\rangle = \langle\psi(t)|e^{iH\tau/\hbar}|\psi(t)\rangle = \langle\psi(t+\tau)|\psi(t)\rangle$$

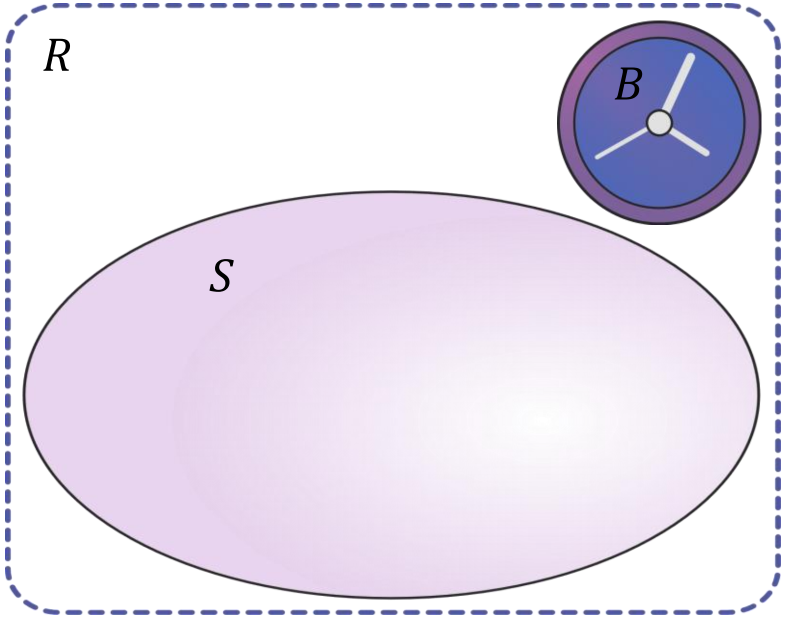
Depends on a future state!

Y.A., E. C., F.C. et al., Finally making sense of the double-slit experiment, PNAS 114, 6480 (2017)

Modular energy

$$e^{iH\tau/\hbar}$$





$$H_T = H_A + H_B + H_S + H_{int}(T_A)$$

$$H_{eff}^{A} = H_B + H_S + H_{int}(t_A)$$

$$H_{eff}^{B} = H_A + H_S + H_{int}(T_A)$$

$$\frac{d}{dt_B} e^{iH_A\tau/\hbar} = -\frac{i}{\hbar}\left[H_{int}(T_A + \tau I) - H_{int}(T_A)\right] e^{iH_A\tau/\hbar}$$

Classical variable

$$\frac{d}{dt_B} e^{2\pi i E_A/E_0} = -i\frac{2\pi}{E_0}\frac{dH_{int}}{dt_A} e^{2\pi i E_A/E_0}$$

$$\frac{d}{dt_B} T_A = I$$

Same flow of time!

$$\frac{d}{dt_B} e^{2\pi i T_A/\tau} = i\frac{2\pi}{\tau} e^{2\pi i T_A/\tau}$$

Thank you

For more details:

I. L. P., M. N., and E. C., Dynamical *nonlocality in quantum time via modular operators,* Phys. Rev. A 105, 042207 (2022)

I. L. P., A. C. L., and E. C., *Flow of time during energy measurements and the resulting time-energy uncertainty relations,* Quantum 6, 683 (2022).

I. L. P., A. T., B.P., E.C., Y.A., "Noninertial quantum clock frames lead to non-Hermitian dynamics", Communications Physics 5, 298 (2022).

M.S., I. L. P., and E. C., *Non-relativistic spatiotemporal quantum reference frames,* arXiv:2307.01874.

Contact: eliahu.cohen@biu.ac.il
Website: https://www.eng.biu.ac.il/cohenel4/