# Projections of Discovery Potentials for future $0\nu\beta\beta$ Decay Experiments


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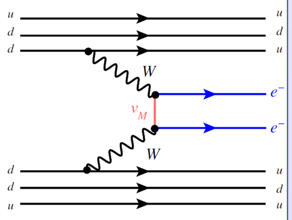
## Introduction

§ Neutrinoless double beta decay ($0\nu\beta\beta$) [Furry, 1939]

$$^{N}_{Z}A_{\beta\beta} \rightarrow {}^{N-2}_{Z+2}A_{\beta\beta} + 2\bar{e}$$

- Forbidden in Standard Model
- $\Delta L = 2$

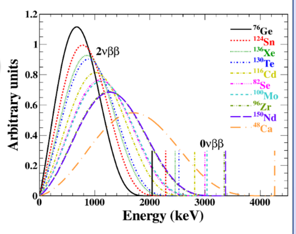


§ Observation of $0\nu\beta\beta$ implies new physics:

- Neutrinos are Majorana particles ($\nu = \bar{\nu}$)
- Lepton number violations
- Effective Majorana Neutrino Mass $\langle m_{\beta\beta}\rangle \neq 0$

§ Energetically possible for 35 nuclei

- A few are experimentally relevant




§ Present work: Required Sensitivity: Exposure vs Background

## Formalism

▲ Half-life in Mass Mechanism : $\left[\frac{1}{T^{0\nu}_{1/2}}\right] = G^{0\nu} g_A^4 \left|M^{0\nu}\right|^2 \left|\frac{\langle m_{\beta\beta}\rangle}{m_e}\right|^2$

▲ Effective Mass: $\langle m_{\beta\beta}\rangle = \left|U_{e1}^2\right| m_1 + \left|U_{e2}^2\right| m_2 e^{i\alpha} + \left|U_{e3}^2\right| m_3 e^{i\beta}$

▲ Experimentally measurable Half-life:

$$T^{0\nu}_{1/2} = \ln 2 \, . N(A_{\beta\beta}) . t_{DAQ} . \left[\frac{\varepsilon_{RoI}}{N^{0\nu}_{obs}}\right] = \ln 2 . \left[\frac{N_A}{M(A_{\beta\beta})}\right] . \Sigma . \left[\frac{\varepsilon_{RoI}}{N^{0\nu}_{obs}}\right]$$

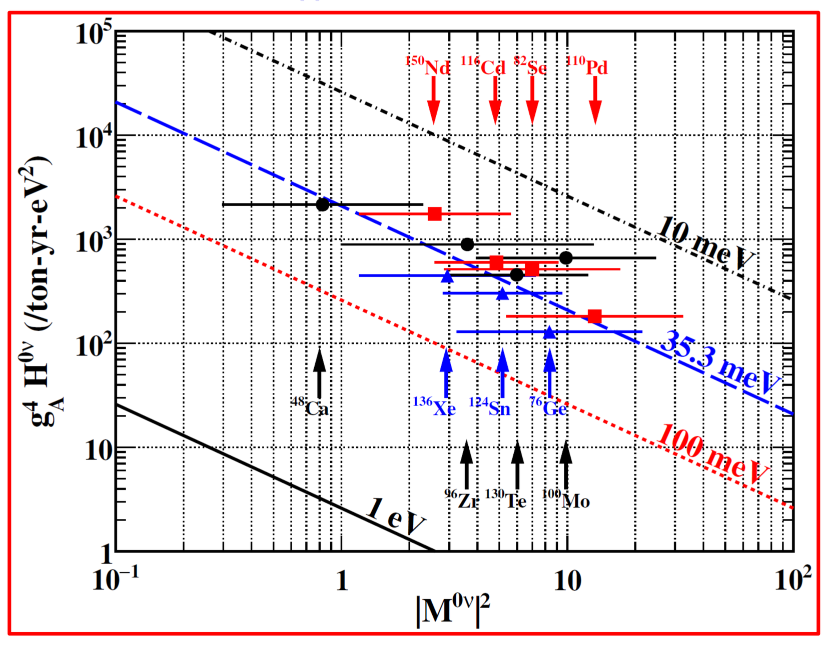
▲ Combined Half-life:

$$\left|M^{0\nu}\right|^2 \left[g_A^4 . H^{0\nu}\right] = \frac{1}{\langle m_{\beta\beta}\rangle^2} \left[\frac{1}{\Sigma} . \frac{N^{0\nu}_{obs}}{\varepsilon_{RoI}}\right]; \; H^{0\nu} \equiv \ln 2 \left(\frac{N_A}{M(A_{\beta\beta}) . m_e^2}\right) . G^{0\nu}$$

## A Model for NME's Uncertainty

❖ In Theory - Inverse correlation between $G^{0\nu}$ and $|M^{0\nu}|^2$

❖ Decay rates (1 event/ton-yr with full efficiency) are similar at given $\langle m_{\beta\beta}\rangle$ and constant $g_A$




R. G. H. Robertson, Mod. Phys. Lett. A 28, 1350021 (2013).

❖ No favored $0\nu\beta\beta$ isotope.

$$\sum (\text{ton-year}) . \left(\frac{\varepsilon_{RoI}}{N^{0\nu}_{obs}}\right) \alpha \left(\frac{1}{\langle m_{\beta\beta}\rangle}\right)^2$$

❖ Realistic interpretation lies within a factor of [0.5, 2.0].

## Statistics & Theme

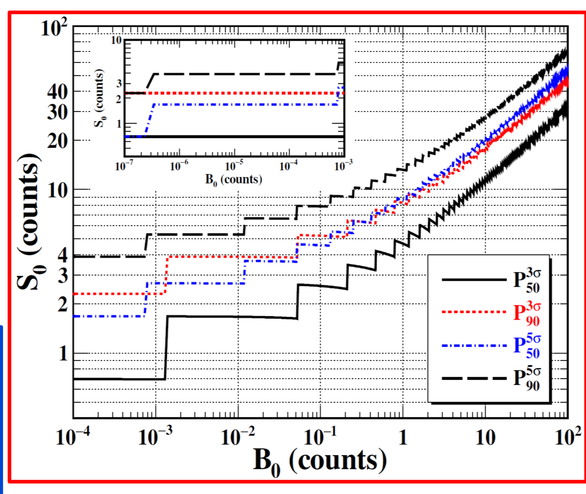
❑ Discrete/Complete Poisson
❑ Continuous Approximation
➡ (i) Low Background (ii) Rare Processes

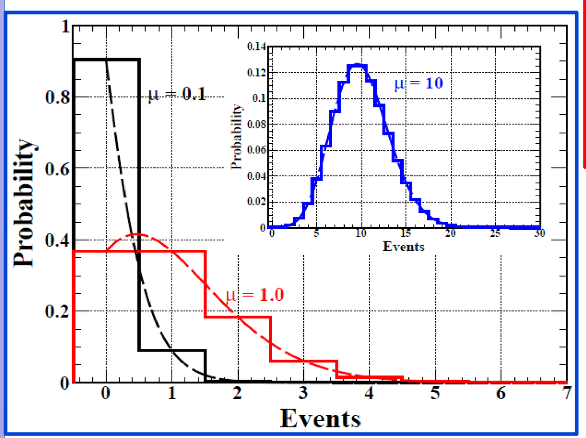
STEP-1

$$\sum_{i=0}^{N^{3\sigma}_{obs}-1} P(i; B_0) \geq (1 - 0.00135)$$

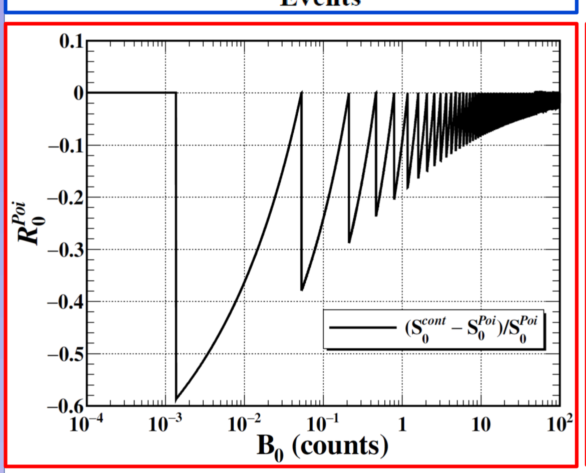
STEP-2

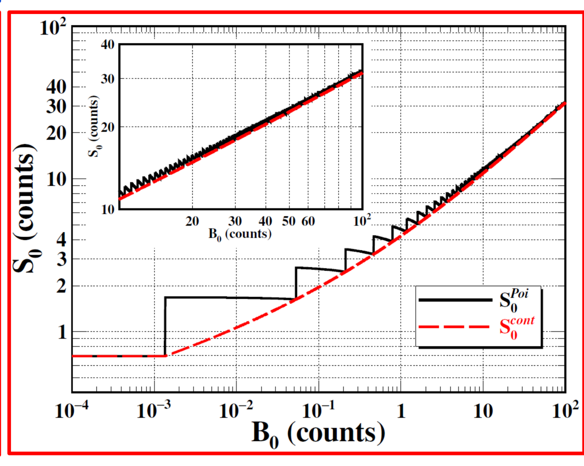
$$\sum_{i=N^{3\sigma}_{obs}}^{\infty} P(i; [B_0 + S_0]) \geq 0.5$$




$$\mathrm{CPoi}(\leq C; \mu) = \left[\frac{\Gamma(C+1; \mu)}{\Gamma(C+1)}\right]$$




❑ $S_0$ derived with complete Poisson ➡ Always ≥ Continuous Approximation

❑ Continuous approximation ➡ Always Underestimate $S_0$

❑ Deviation ➡ As much as 60% @ Low Background ($B_0$~$10^{-3}$)

❑ Both Consistent ➡ Within 3% @ Large $B_0$ ≥ 100

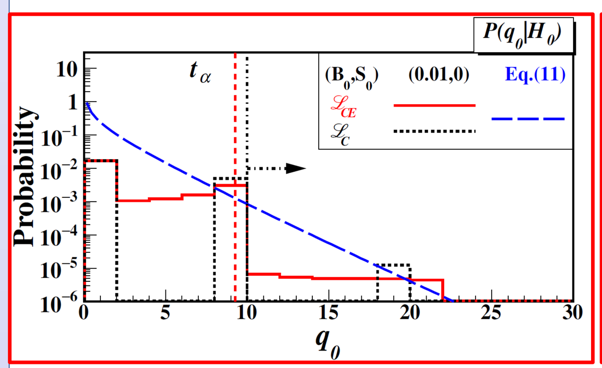
## Likelihood Analysis

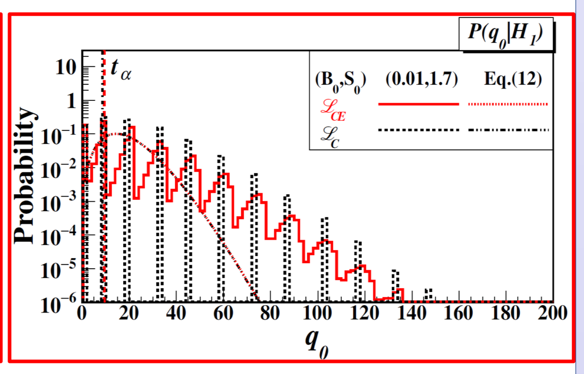
$$\mathscr{L}_C \equiv \mathscr{L}(S|N,B) = \frac{e^{-(B+S)}(B+S)^N}{N!}$$

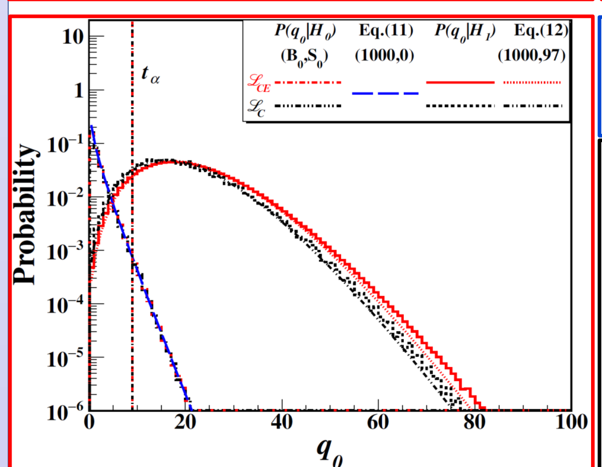
$$q_0 \equiv t(S{=}0) = -2 \ln\left[\frac{\mathscr{L}(S{=}0)}{\mathscr{L}(\hat{S})}\right]$$

$$\mathscr{L}_{CE} \equiv \mathscr{L}(S|\mathbb{E},B) = \frac{e^{-(B+S)}(B+S)^N}{N!} \times \prod_{i=1}^{N}\left[\frac{B \cdot f_B + S \cdot f_S(E_i)}{(B+S)}\right]$$

$$\alpha \equiv \int_{t_\alpha}^{\infty} P(q_0|H_0)\, dq_0 \qquad \alpha \geq \sum_{q_0 \geq t_\alpha} P(q_0|H_0) \qquad \beta \equiv \int_{0}^{t_\alpha} P(q_0|H_1)\, dq_0 \qquad \beta = \sum_{q_0 \leq t_\alpha} P(q_0|H_1)$$




### *Approximate Distribution of $q_0$ for Large-Samples*

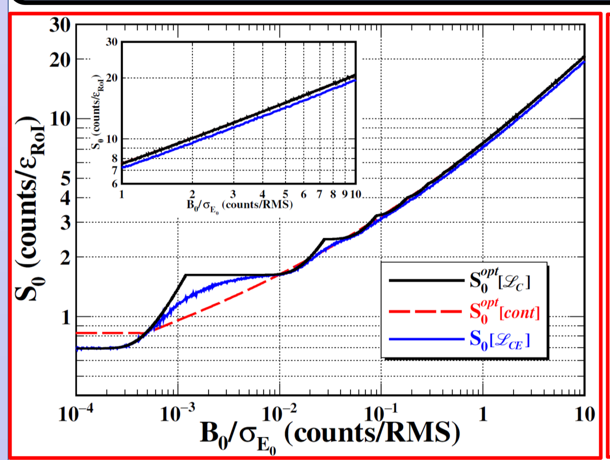
$$P(q_0|H_0) \approx \frac{1}{2}\delta(q_0) + \frac{1}{2}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{q_0}}e^{-q_0/2}$$

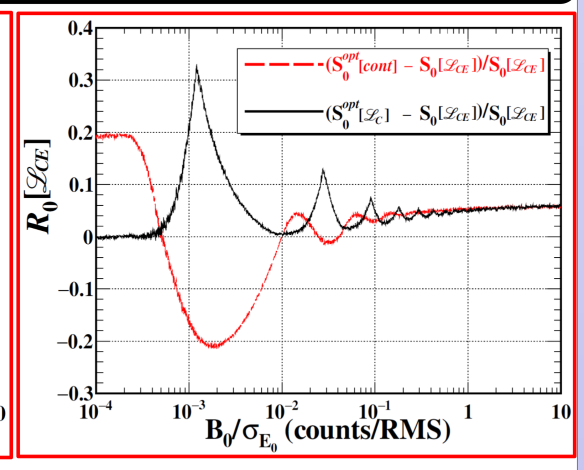
$$P(q_0|H_1) \approx (1 - \Phi(\sqrt{\Lambda}))\delta(q_0) + \frac{1}{2}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{q_0}}e^{-(\sqrt{q_0}-\sqrt{\Lambda})^2/2}$$

$$\mathscr{L}(S|\{n_i\},B) \approx \prod_{i=1}^{n} \mathrm{Poi}(n_i|F(E_i|S,B)) \text{ , wher}$$

$$F(E_i|S,B) = [\, B \cdot f_B + S \cdot f_S(E_i) \,] \cdot w(E_i)$$

## Counting vs Extended Likelihood




➢ $S_0[\mathscr{L}_{CE}] \leq S_0^{opt} S_0[\mathscr{L}_C]$ ➡ *Less Events Required to Establish Positive Signals*

➢ At $[(B_0/\sigma_{E0}) \leq 0.01]$ ➡ Criteria of $P^{50}_{3\sigma}$ Satisfied for all $B_0$ in $\mathscr{L}_{CE}$ NOT in $\mathscr{L}_C$

➢ At $[(B_0/\sigma_{E0}) > 1]$ ➡ Counting-only Analysis Overestimate $S_0[\mathscr{L}_{CE}]$ by 6%

➢ At $[(B_0/\sigma_{E0})$~$10^{-3}]$ ➡ $S_0^{opt}$[cont] Underestimate Strength of $S_0[\mathscr{L}_{CE}]$ by 20%

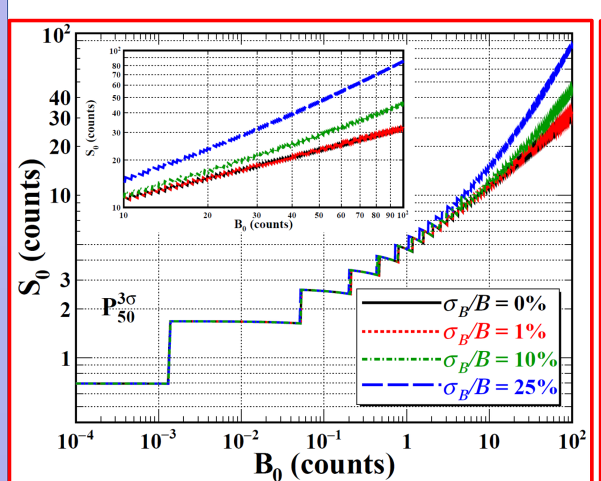
➢ $S_0^{opt}[\mathscr{L}_C]$ ➡ Overestimated by ~ 30% & > $S_0[\mathscr{L}_{CE}]$ for all $(B_0/\sigma_{E0}) > 5 \times 10^{-4}$

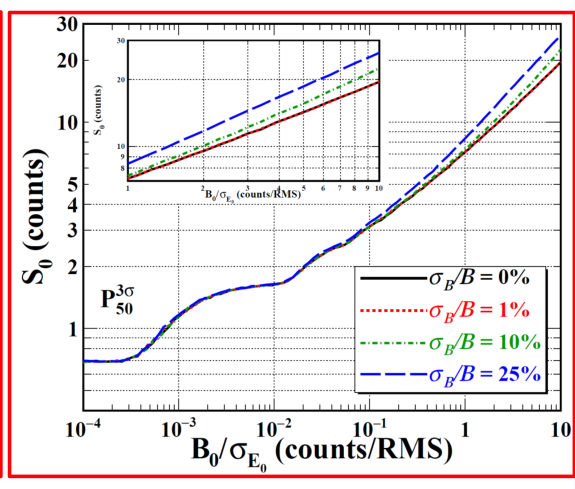
## Background Uncertainties

$$\mathscr{L}_{CEB} \equiv \mathscr{L}(S,B|\mathbb{E}) = \frac{e^{-(B+S)}(B+S)^N}{N!}\frac{e^{-\tau B}(\tau B)^{n_0}}{n_0!} \times \prod_{i=1}^{N}\left[\frac{B \cdot f_B + S \cdot f_S(E_i)}{(B+S)}\right],$$

$$q_0 \equiv t(S=0) = -2 \ln\left[\frac{\mathscr{L}_{CEB}(S=0, \hat{\hat{B}})}{\mathscr{L}_{CEB}(\hat{S}, \hat{B})}\right]$$

$$\mathscr{L}(S|\{n_i\},B) \approx \left[\prod_{i=1}^{n} \mathrm{Poi}(n_i|F(E_i|S,B))\right] \cdot \mathrm{Poi}(n_0|\tau B)$$



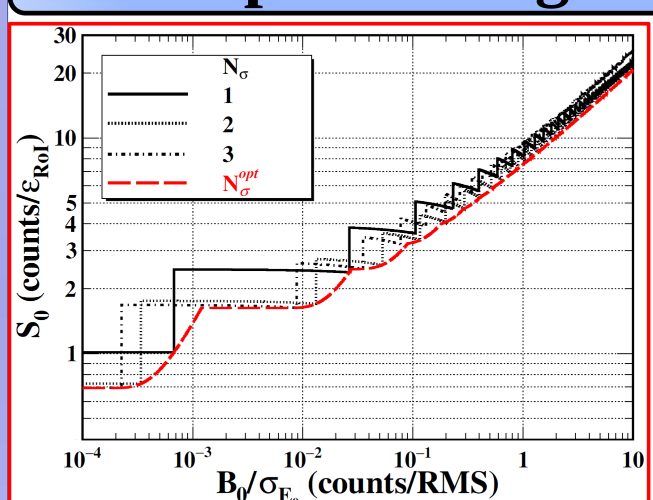




❖ Realistic Experiments ➡ Background $B_0$ can have Uncertainty $\sigma_B$

❖ At low-statistics ($B_0$<1) ➡ Negligible Effects of $\sigma_B$ [Larger in $\mathscr{L}_C$ than $\mathscr{L}_{CE}$]

❖ Statistical Fluctuations ➡ Dominate Over Uncertainty in $B_0$

## Optimal Region of Interest

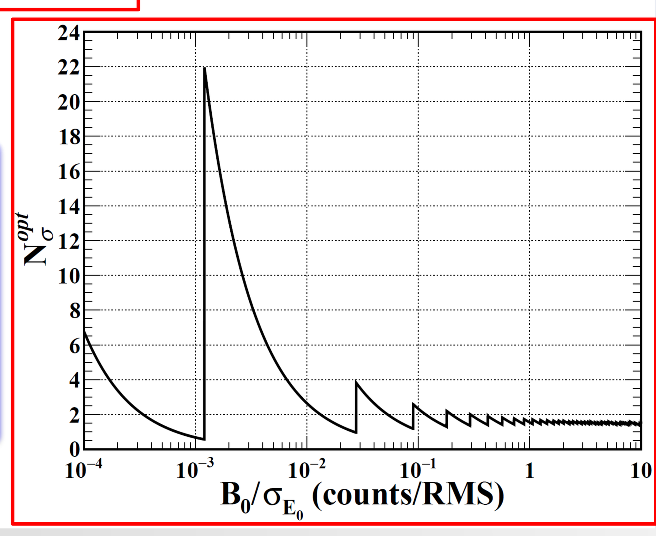



Choice of $N^{opt}_\sigma$ = 1.4

Optimal [@ minimal $S_0[\mathscr{L}_C]$]

➡ At large $(B_0/\sigma_{E0}) \geq 1$

At smaller $(B_0/\sigma_{E0})$ Vary Broadly ➡ Fluctuations @ Low counts & Discreteness




## Sensitivity Projection

**Standard-Model-allowed irreducible background**

$$^{N}_{Z}A_{\beta\beta} \rightarrow {}^{N-2}_{Z+2}A_{\beta\beta} + 2\bar{e} + 2\bar{\nu} \text{ [Goeppert-Mayer, 1935]}$$

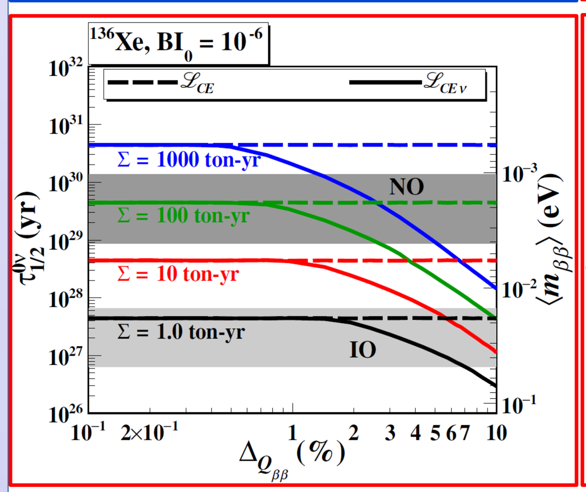
Worse resolution ($\Delta$) ⬌ Larger RoI ⬌ Larger $2\nu\beta\beta$ Background

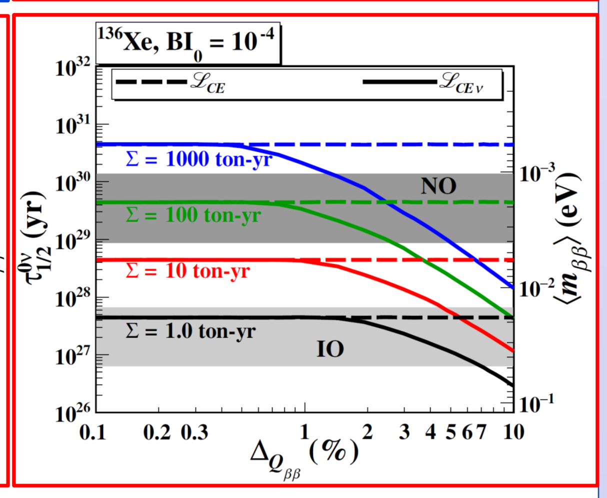
$$\mathscr{L}_{CEB\nu} \equiv \mathscr{L}(S,B|\mathbb{E}) = \frac{e^{-(B+\nu+S)}(B+\nu+S)^N}{N!}\frac{e^{-\tau B}(\tau B)^{n_0}}{n_0!} \times \prod_{i=1}^{N}\left[\frac{B \cdot f_B + \nu \cdot f_{2\nu}(E_i) + S \cdot f_S(E_i)}{(B+\nu+S)}\right]$$

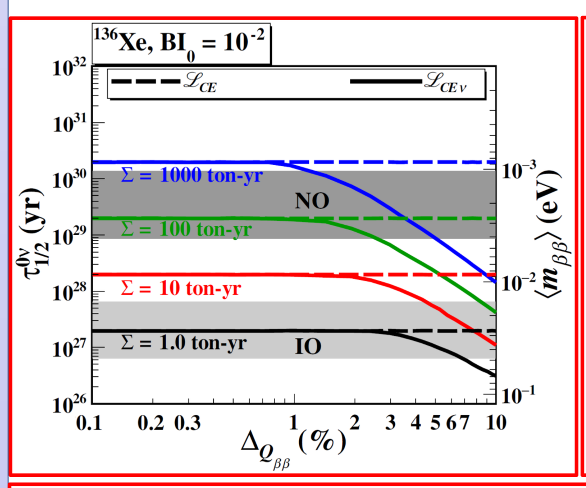
*$^{136}$Xe, $Q_{\beta\beta}$ = 2.458 MeV,*

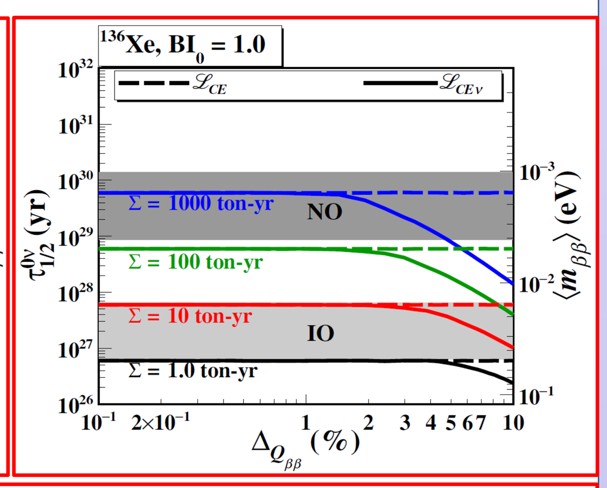
*$\tau^{2\nu}_{1/2}$=2.2 × $10^{21}$ yr, IA = 100%,*

*$\varepsilon_{expt.}$ = 100%, RoI = $Q_{\beta\beta} \pm 4\sigma_{E0}$*



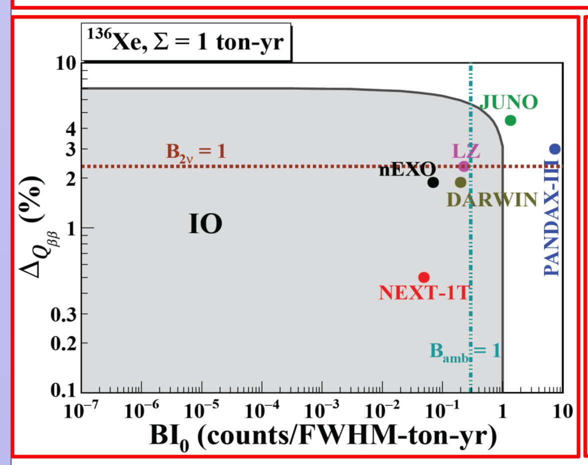



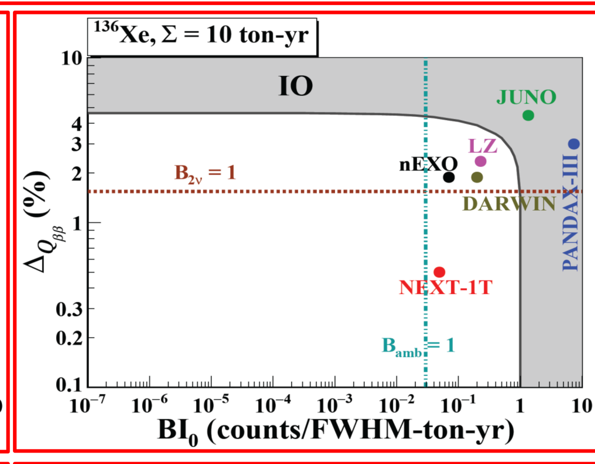

❑ Divergence: Solid & Dotted lines ➡ $2\nu\beta\beta$ Start to Dominate the Sensitivities

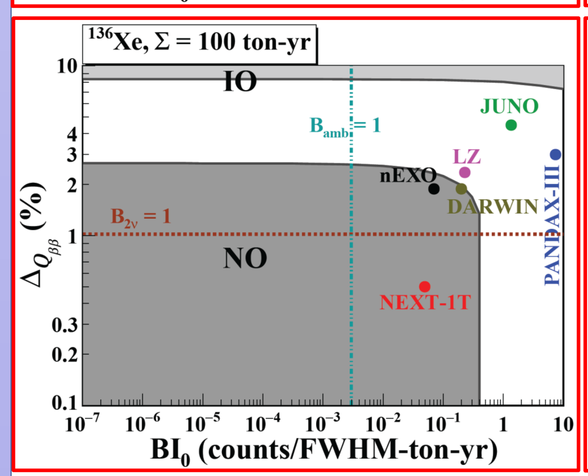
❑ At $BI_0 = 10^{-6}$ (counts/FWHM-ton-yr) ➡ ($\Delta_{Q_{\beta\beta}}$, $\Sigma$) ≈ (<1%, >1.5 ton-yr) & (<0.4%, >310 ton-yr) to cover IO & NO

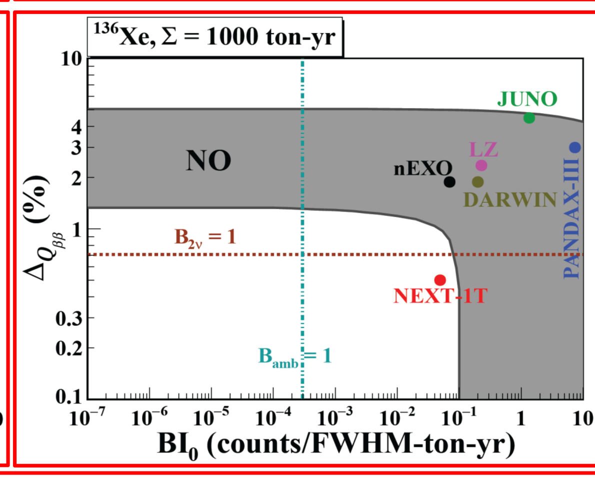
❑ At $BI_0$ = 1 (Best Achieved) ➡ *Overlap of Solid & Dotted lines*

$2\nu\beta\beta$ is insignificant




☑ $\Sigma$ of 10 ton-yr (with $\Delta Q_{\beta\beta}$<1.4%) & 100 ton-yr (with $\Delta Q_{\beta\beta}$ ~ 8%) Required to Cover IO

☑ Probing Entire NO ➡ *Not Possible even with 1000 ton-yr @ Best Achieved Resolution = 0.12% of $^{76}$Ge*

☑ Coming Generation of Projects ➡ Could Cover IO at $\Sigma$ >10 ton-yr

☑ Covering NO entirely ➡ *Require $\Sigma$ ~ 1000 ton-yr at $\Delta Q_{\beta\beta}$ ≤ 1% Together with $BI_0$ at ≤ $10^{-1}$ counts/FWHM-ton-yr*

☑ Required $\Sigma$ in Realistic Experiments: $\Sigma' = \Sigma / [IA . \varepsilon_{expt}]$

## Summary & Prospects

☑ Two Expected Features ➡ Required Signal Strength

☑ *In counting-only experiments:*

☑ Strength can be derived correctly with complete Poisson analysis

☑ Continuous Approximation would underestimate the values

☑ Incorporating continuous variables as additional constraints:

☑ Reduced Signal Strength relative to Counting-only analysis

## Acknowledgment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