QCD in the Early Universe

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The Weakly Interacting QCD: DGLAP and BFKL

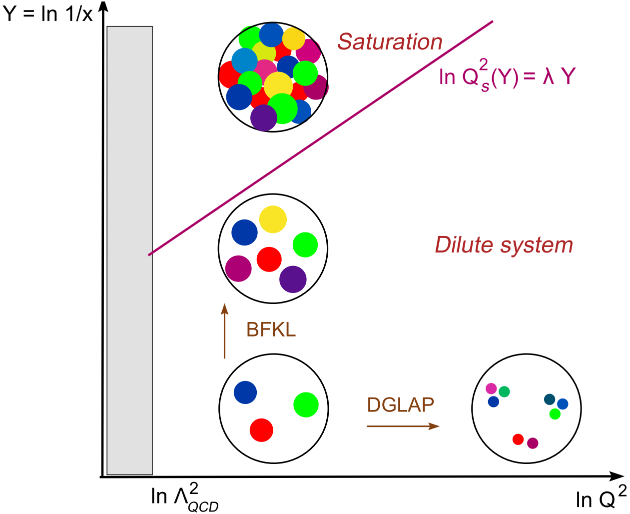


Figure 1: Parton density and size as a function $Y = \ln(1/x)$ and $\ln Q^2$. From [Gelis et al:2010].

- In the Bjorken limit of QCD $Q^2 \to \infty$ (impulse approximation) hadron viewed, in the infinite momentum frame (IMF), represents a large number of gluons and sea quark pairs with very small phase-space density.
- At large, fixed $Q^2 \gg \Lambda^2_{QCD}$ gluon distribution $xG(x, Q^2)$ in a proton rises very fast with decreasing x .

QCD at High Parton Densities and Saturation

[Gribov jr.,Levin,Ryskin:1983] , [McLerran,Venugopalan:1993]

- Partons "overlap" when $\sigma_{gg} \sim \alpha_S/Q^2$ times $xG_A(x, Q^2)$ – the probability to find at fixed Q a parton carrying a fraction x of the parent parton momentum – becomes comparable to the geometrical cross section πR_A^2 of the object A occupied by the gluons.

$$Q_s^2(x) = \frac{\alpha_S(Q_s)}{2(N_c^2-1)} \frac{xG_A(x, Q_s^2)}{\pi R_A^2} \sim \frac{1}{x^\lambda} \Rightarrow \ \ln Q_s^2(x) = \lambda Y \tag{1}$$

- Emergent "close packing" scale $Q_s(x) \gg \Lambda_{QCD}$ – fixed point of the PDF evolution in x.
- Repulsive gg interactions $\Rightarrow$ occupation number f_g (# of gluons with a given x times the area each gluon fills up divided by the transverse size of the object) saturates at $f_g \sim 1/\alpha_S$.
- The same scaling as for the Higgs condensate, superconductivity or the inflaton field.
- Saturated gluonic matter is weakly coupled. $\Rightarrow$ weakly interacting means semi-classical.

CGC–Black Hole correspondence

- Dvali, Venugopalan:2021: Correspondence between Color Glass Condensates as highly occupied gluon states and Black Holes as highly occupied condensates of weakly interacting gravitons.

- Both BH and CGC attain a maximal entropy S_{max} permitted by unitarity when the occupation number f and the coupling α of the respective constituents (gravitons, gluons) satisfy $f = 1/\alpha(Q_s)$, where Q_s represents the point of optimal balance between the kinetic energies of the individual constituents and their potential energies.

- In experiments with ultra-relativistic nuclei (RHIC, LHC) the CGC, aka Glasma, manifests itself as saturated state of weakly interacting off-shell gluons and quarks.

Evolution of the Early Universe: Effective Degrees of Freedom

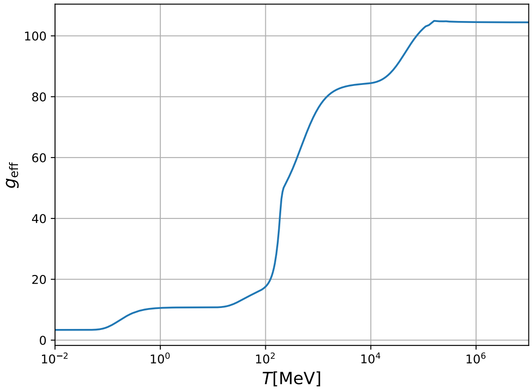
- Normalize to massless Bose gas with one d.o.f.

$$g_{\text{eff}}(T) \equiv \frac{\epsilon(T)}{\epsilon_0(T)}, \quad \epsilon_0(T) = \frac{\pi^2}{30} T^4 \tag{2}$$

$$h_{\text{eff}}(T) \equiv \frac{s(T)}{s_0(T)}, \quad s_0(T) = \frac{2\pi^2}{45} T^3 \tag{3}$$

- For non-interacting gas:

$$g^{id}_{\text{eff}}(T) = h^{id}_{\text{eff}}(T) = \frac{7}{8} 4N_{\text{F}} + 3N_{\text{V}} + 2N_{\text{V0}} + N_{\text{S}} \tag{4}$$



$g_{\text{eff}}(T)$ in the SM taking into account interactions between particles, obtained with both perturbative and lattice methods. [Hindmarsh et al.:2020].

Finite Temperature Glasma in Early Universe

- Hot medium with average momentum transfer Q among its constituents (partons) has temperature $T \cong Q/2\pi$.

- The QCD saturation scale $R_S \approx 1/Q_S \ll 1/\Lambda_{QCD} \approx \langle r_h \rangle$ is now given by the thermal de Broglie wavelength of massless gluons $\lambda_S = \pi^{2/3}/T_S$, where T_S is the saturation temperature.

- Consider a period of cosmological evolution when $T \gtrsim T_S \gg \Lambda_{\mathrm{QCD}}/(2\pi)$. For $T_S \gtrsim 1$ GeV the running coupling $\alpha_S(Q_S = 2\pi T_S) \lesssim 0.2$.

- At $T \approx T_{EW}$ for YM bosons $g_{\mathrm{eff}}^{\mathrm{QCD}}/g_{\mathrm{eff}}^{\mathrm{EW}} \simeq 8/3 \Rightarrow$
  Glasma might have been prevalent form of matter also during EW era.

- $T \gg T_{EW}$: the gluon exchange between quarks (antiquarks) becomes surpassed by the exchange of EW massless gauge bosons $W^{\pm}, W^0, B^0, \ldots$
  ... GUT YM bosons can also form the classical condensate.

Glass properties of the Glasma

- In condensed matter physics glass is a non-equilibrium, disordered state of matter acting like solids on short time scales but liquids on long time scales [Mauro:2014,Sethna:2021].

- Two scales of Glasma: $$\tau_{\mathrm{wee}} = \frac{1}{k^-} = \frac{2k^+}{k_\perp^2} = \frac{2xP^+}{k_\perp^2} \ll \frac{2P^+}{k_\perp^2} \approx \tau_{\mathrm{valence}} \,. \quad (5)$$

  $\Rightarrow$ Valence modes are static over the time scales of wee modes [Berges:2020].

- Glasses are formed when liquids are cooled too fast to form the crystalline equilibrium state. Fast cooling leads to an enormous # of possible configurations $N_{\mathrm{gl}}(T)$ into which the glasses can freeze $\Rightarrow$ large entropy $S = \ln N_{\mathrm{gl}}(T)$, such that $S(T=0) > 0$, [Sethna:1988].

- What's the entropy of the Color Glass Condensate?

Cosmological Parametrization of the EoS

- In cosmology the EoS is parametrized as $p = w\epsilon$

$$w(T) = \frac{sT}{\epsilon} - 1 = \frac{4h_{\rm eff}(T)}{3g_{\rm eff}(T)} - 1 \tag{6}$$

- Causality:

$$c_s \leq 1 \quad \Rightarrow \quad \frac{h_{\rm eff}(T)}{g_{\rm eff}(T)} \leq \frac{3}{2}\,, \tag{7}$$

- Upper bound $w = c_s = 1$ corresponds to absolutely stiff fluid, e.g. classical massive vector particles interacting with point charges [Zeldovich:1961]*, free scalar massless field etc.
- Holographic cosmology of the very early universe with the stiff fluid saturating the holographic covariant entropy bound [Banks,Fischler:2001].
- Can the stiff fluid represent the saturated QCD matter?

*) N.B. For $\epsilon = an^{\nu-1}$, where n is the density of charges $p = (\nu - 1)\epsilon$ [Zeldovich:1961].

Yang-Mills Field EoS in Cosmology

- Isotropic vector field $\mathcal{A}^a_\mu \equiv \mathbf{A}$ can not be Abelian $\Rightarrow$ one needs at least triplet of vector fields to ensure the isotropy.
- Early studies: Coupled Einstein-Yang-Mills equations with gauge group SU(2) in the FLRW universes [Galtsov, Volkov:1991].
- Non-linear nature of YM field configurations makes field-supported *radiation-dominated* universes (with zero temperature) possible. It may reconcile the Hot Big Bang geometry with the non-thermal matter which arises quite naturally within the context of GUT.
- Basic features of the EYM homogeneous and isotropic cosmological solutions can be attributed to the conformal nature of the YM field $\Theta(\epsilon_\mathbf{A}) \equiv (\epsilon_\mathbf{A} - 3p_\mathbf{A})/T^4 = 0$.

Q: How can we change the trace anomaly $\Theta(\epsilon_\mathbf{A})$ of the YM field?

A: By breaking its conformal invariance (needs effective action).

Ideal QCD Plasma with Power-like Corrections

- In pure gauge theory up to $T \approx (2 \div 5) T_c$, the dominant power-like correction to pQCD behavior is $\mathcal{O}(T^{-2})$ rather than $\mathcal{O}(T^{-4})$.

- Quadratic thermal terms in the deconfined phase can also be obtained from gauge/string duality [Zuo:2014vga].

$$\epsilon(T) = \sigma T^4 - CT^2 + \mathcal{B}\,, \qquad p(T) = \frac{\sigma}{3} T^4 - DT^2 - \mathcal{B}\,. \tag{8}$$

$$\sigma = \frac{\pi^2}{30} g_{\rm eff}^{\rm QCD}\;, \quad g_{\rm eff}^{\rm QCD} = 2 \times 8 + \tfrac{7}{8}(3 \times N_F \times 2 \times 2) \tag{9}$$

$$\sigma(N_F = 0,\ 2,\ 3,\ 4,\ 5) \approx 5,\ 12,\ 16,\ 18,\ 23\ ; \quad \mathcal{B}^{1/4} \approx 220 \text{ MeV}\ .$$

1. $C = -D > 0$: Gluonic q-particle EoS with $\mathcal{B}(T) = -CT^2 + \mathcal{B}$ [Schneider,Weise:2001].
2. $C = D > 0$: LQCD motivated "fuzzy" bag model EoS [Pisarski:2006, Megias:2007]. It has a sign change in entropy $s = \partial p / \partial T = (4/3)\sigma T^3 - 2DT$ at $T_c = \sqrt{3D/(2\sigma)}$!

Stiffer EoS from Power-like Corrections

- The trace anomaly (for $T \geq 1$ GeV $\mathcal{B} \approx 0$)

$$\Theta(T) \equiv \frac{\epsilon - 3p}{T^4} = \frac{4\mathcal{B}}{T^4} + \frac{3D - C}{T^2} \approx \frac{3D - C}{T^2} \equiv \frac{A}{T^2} \tag{10}$$

- The barotropic form of the EoS

$$p(\epsilon) = \frac{1}{3}\left(\epsilon - \Theta(T)T^4\right) = \frac{\epsilon}{3} - \frac{A}{3}T^2(\epsilon)\,, \quad T^2(\epsilon) \equiv \frac{C + \sqrt{C^2 + 4\sigma\epsilon}}{2\sigma} > 0\,. \tag{11}$$

- Speed of sound: $c_s^2(\epsilon) = \dfrac{dp(\epsilon)}{d\epsilon} = \dfrac{1}{3}\left(1 - \dfrac{2A\sigma}{\sqrt{C^2 + 4\sigma\epsilon}}\right) > \dfrac{1}{3} \Leftrightarrow A = 3D - C < 0$ (12)

- Recall: $h_{\mathrm{eff}} = s/s_0$ # eff.d.o.f. in entropy , $g_{\mathrm{eff}} = \epsilon/\epsilon_0$ # eff.d.o.f. in energy

$$w(T) = \frac{p(T)}{\epsilon(T)} = \frac{4h_{\mathrm{eff}}}{3g_{\mathrm{eff}}} - 1 = \frac{1}{3} - \frac{A}{3}\frac{T^2(\epsilon)}{\epsilon} \tag{13}$$

Stiff EoS Scenario for the Early Universe

- The fast cooling of the universe has stiffened its EoS $w = 1/3 \to w = 1$.
- Saturation energy density: $\epsilon_S(T_S) = \sigma T_S^4 - CT_S^2$
- Stiff EoS: $p = \epsilon$: $w(T) = 1 \;\; \Rightarrow A = 3D - C < 0\,, \quad \epsilon_S = \dfrac{3C^2 - 12CD + 9D^2}{4\sigma}$ (14)
- N.B. For $A = 3D - C < 0$ the trace anomaly $\Theta(T) < 0$. This is always true with $D < 0$.
- Particular case : gluonic q-particle EoS with $C = -D > 0$ [Schneider,Weise:2001]:

$$\epsilon_S = \frac{6C^2}{\sigma} = \sigma T_S^4 - CT_S^2 \;\Rightarrow\; C = \frac{\sigma}{3} T_S^2 \;\Rightarrow\; \epsilon_S = \left(1 - \frac{1}{3}\right)\sigma T_S^4\,,\; s = 2\times\frac{4}{3}\sigma T_S^3 \quad (15)$$

- $g_{\rm eff}$ decreases by 1/3, $h_{\rm eff}$ doubles – additional d.o.f. go to the condensate.
- $\epsilon - 3p = \langle G^2\rangle_{T=0} - \langle G^2\rangle_T$, where $\langle G^2\rangle_T$ is the gluon condensate.
- Gluonic q-particle model has $m_g(T) \Rightarrow$ [Zeldovich:1961] rediscovered!

Take-Home Message

- We propose that CGC – a state of QCD-saturated matter preceding the formation of the sQGP in heavy ion collisions may also exist at non-zero temperatures.
- The CGC has very likely played an important role in the evolution of our Universe over a broad range of temperatures $T_{GUT} \gtrsim T \gtrsim T_{QGP}$.
- Power-like corrections $\mathcal{O}(T^{-2})$ to conformal invariance of classical $SU(3)_c$ field result in the stiffening of its EoS and give rise to increase of its entropy.
- The gluonic quasi-particle model with $m_g(T)$ seems to be a promising candidate for the description of the properties of the saturated QCD matter up to $T \approx 5T_c$.
- TBD: Cherenkov radiation from QCD vacuum
  Strong field polarizes quantum fluctuations imbuing the vacuum with an effective anisotropic refractive index allowing the possibility of Cherenkov radiation from the quantum vacuum.
  (For QED case, see [Macleod,Noble,Jaroszynski:2019]).

Thank you for your attention!