Moment neutrino evolution equations:

application to fast-flavor instability in neutron star mergers

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Context: fast-flavor oscillations

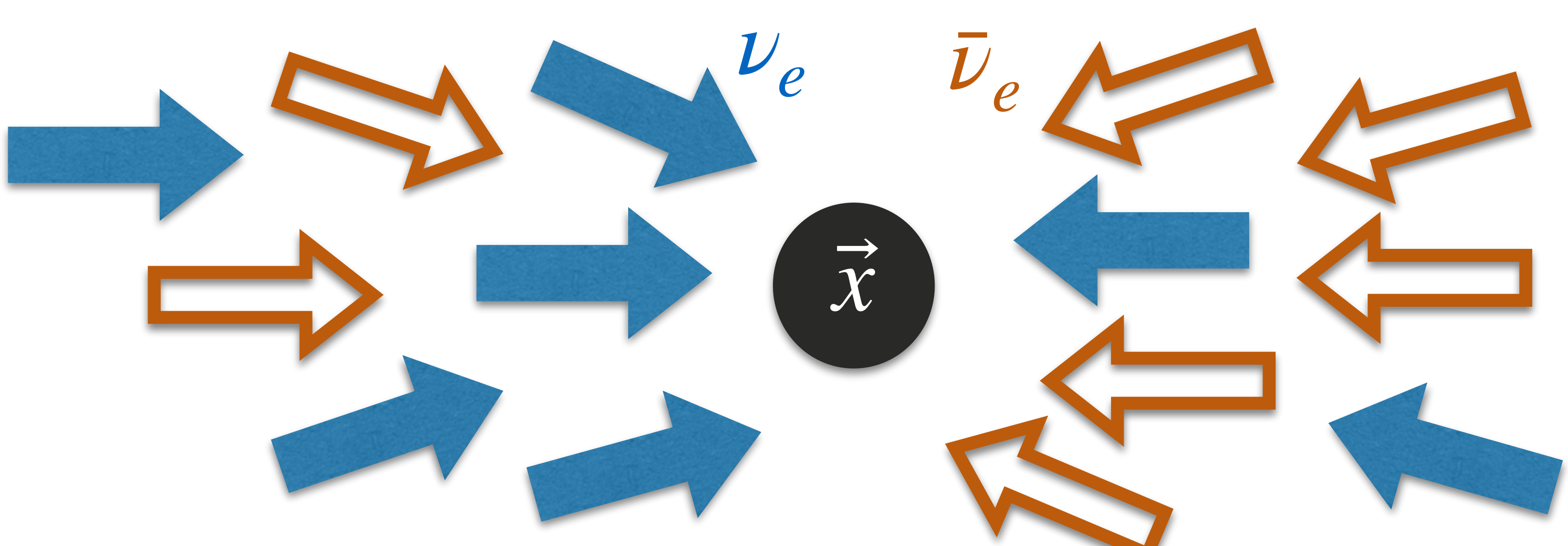
- Dense astrophysical environments (core-collapse supernovae, neutron star mergers): rich "zoology" of flavor oscillation regimes.

In particular: "**fast-flavor oscillations**": *R. F. Sawyer*, [0503013]

collective oscillation regime, uncovered when considering the *full angular distribution* in anisotropic environments.

- Condition: **electron lepton number crossing**

$$f_{\nu_e}(\vec{x}, p, \theta_1, t) - f_{\bar{\nu}_e}(\vec{x}, p, \theta_1, t) > 0 \text{ and } f_{\nu_e}(\vec{x}, p, \theta_2, t) - f_{\bar{\nu}_e}(\vec{x}, p, \theta_2, t) < 0$$



Context: fast-flavor oscillations

- First studies with **multi-angle linear stability analysis** (e.g. *Dasgupta et al.* [1609.00528], *Izaguirre et al.* [1610.01612], *Padilla-Gay & Shalgar* [2108.00012]), followed by numerical simulations to estimate the amount of flavor conversion.

- Fast-flavor oscillations "ubiquitous in compact binary merger remnants": *Wu & Tamborra* [1701.06580]

Recent reviews: *Tamborra & Shalgar* [2011.01948], *Capozzi & Saviano* [2202.02494], *Richers & Sen* [2207.03561].

- Most hydrodynamic simulations with neutrino transport use **moments** (density, flux) with an associated **closure** (for example, the *maximum entropy closure*).

$\Longrightarrow$ Develop a linear stability analysis of fast-flavor oscillations using directly <u>moments</u> of the neutrino distribution.

Introducing the QKEs

- In order to describe the evolution of a statistical ensemble of neutrinos: combination of kinetic theory and quantum mechanics.

Boltzmann equation   *Flavor mixing*

- Generalization of distribution functions: (1-body reduced) **“density matrix”**

$$\begin{pmatrix} f_{\nu_e} & \\ & f_{\nu_x} \end{pmatrix} \longrightarrow \begin{pmatrix} \varrho_{ee} & \varrho_{ex} \\ \varrho_{xe} & \varrho_{xx} \end{pmatrix}$$

- Evolution equation: the **Quantum Kinetic Equation**

$$\boxed{i\frac{\mathrm{d}\varrho(\vec{x},\vec{p},t)}{\mathrm{d}t} = [\mathcal{H}_{\mathrm{vac}} + \mathcal{H}_{\mathrm{mat}} + \mathcal{H}_{\mathrm{self}}, \varrho] + i\,\mathcal{I}(\varrho,\bar{\varrho})}$$

Mean-field   **Collisions**

"Moment" Quantum Kinetic Equations

- Angular moments of the density matrix:

$$\begin{array}{r}\textbf{Number density}\\ \textbf{Flux}\\ \textbf{Pressure tensor}\end{array} \begin{bmatrix} N \\ F^i \\ P^{ij} \end{bmatrix} = p^2 \int \mathrm{d}\Omega \begin{bmatrix} 1 \\ p^i/p \\ p^i p^j/p^2 \end{bmatrix} \varrho(t, \vec{x}, \vec{p})$$

- Focus on fast-flavor instabilities, governed by the Hamiltonian:

$$\mathcal{H}_{\text{self}} = \frac{\sqrt{2} G_F}{(2\pi)^3} \int \mathrm{d}^3\vec{q}\,(1 - \cos\theta)\,[\varrho(t, \vec{x}, \vec{q}) - \bar{\varrho}(t, \vec{x}, \vec{q})]$$

- QKEs for moments (simplifying assumption: mono-energetic p):

$$i\left(\frac{\partial N}{\partial t} + \frac{\partial F^j}{\partial x^j}\right) = \sqrt{2} G_F \left[N - \overline{N}, N\right] - \sqrt{2} G_F \left[(F - \overline{F})_j, F^j\right]$$

$$i\left(\frac{\partial F^i}{\partial t} + \frac{\partial P^{ij}}{\partial x^j}\right) = \sqrt{2} G_F \left[N - \overline{N}, F^i\right] - \sqrt{2} G_F \left[(F - \overline{F})_j, P^{ij}\right]$$

Closure $P^{ij}_{\alpha\beta}\left(N_{\alpha\beta}, F^k_{\alpha\beta}\right)$

Linear stability analysis

- Possible (although computationally expensive) to numerically solve these QKEs with a moment code.

- To quickly and systematically study the existence and timescales of FFI: **linear stability analysis**.

Previous study restricted to a particular “zero mode”: *Dasgupta et al.* [1807.03322]

$$N = \begin{pmatrix} N_{ee} & A_{ex} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} \\ A_{xe} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} & N_{xx} \end{pmatrix}$$

$$F^j = \begin{pmatrix} F^j_{ee} & B^j_{ex} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} \\ B^j_{xe} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} & F^j_{xx} \end{pmatrix}$$

Linear stability analysis

- Linearly expand the QKEs to get the system of equations:

$$\boxed{S_{\vec{k}} \cdot Q + \Omega\,\mathbb{I} \cdot Q = 0}$$

"Stability matrix"

$$Q = \begin{pmatrix} A_{ex} \\ B^{\mathrm{x}}_{ex} \\ B^{\mathrm{y}}_{ex} \\ B^{\mathrm{z}}_{ex} \\ \bar{A}_{ex} \\ \bar{B}^{\mathrm{x}}_{ex} \\ \bar{B}^{\mathrm{y}}_{ex} \\ \bar{B}^{\mathrm{z}}_{ex} \end{pmatrix}$$

- Non-zero solution only if:

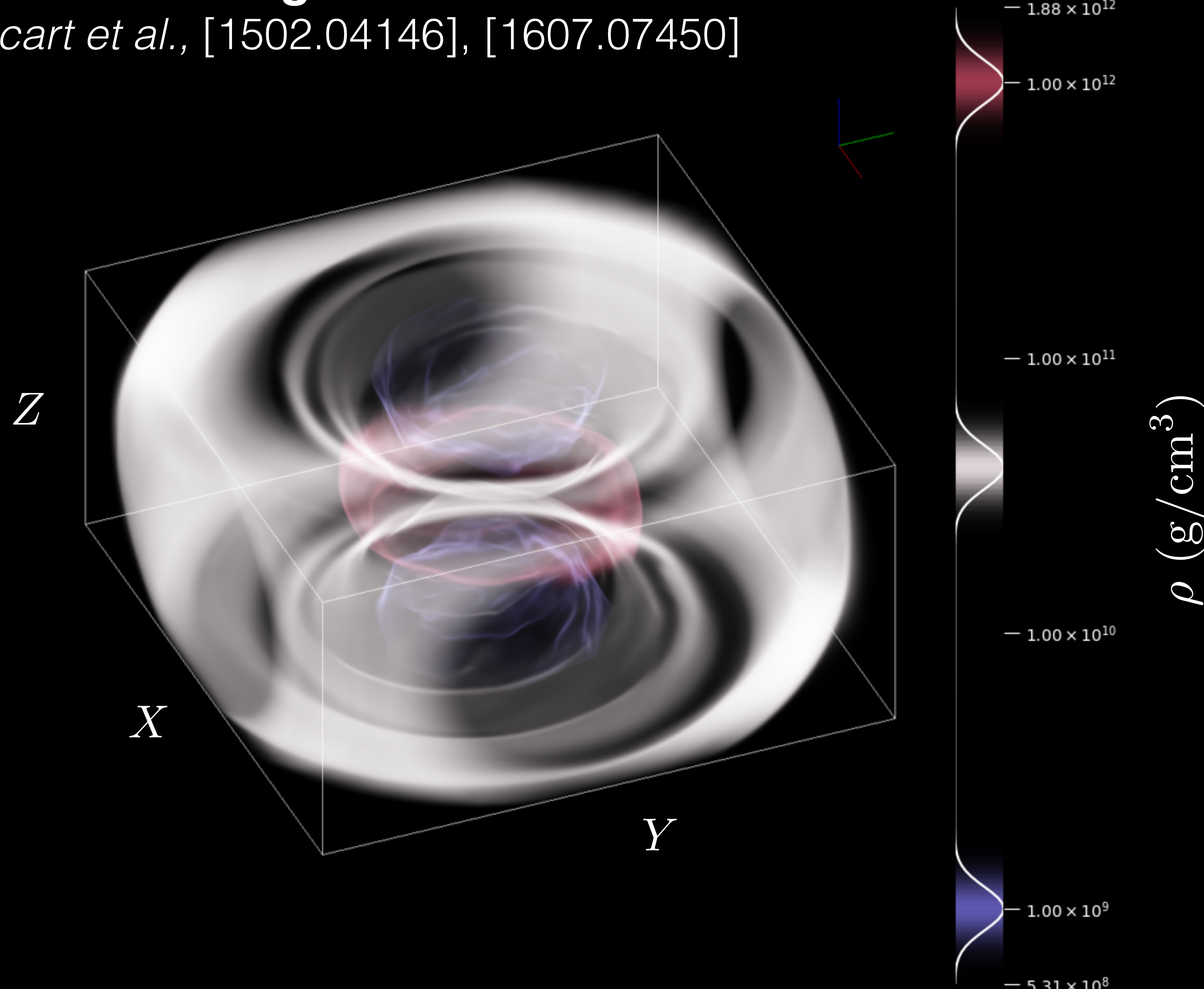
$$\det\left(S_{\vec{k}} + \Omega\,\mathbb{I}\right) = 0 \implies \Omega(\vec{k})$$

- Fastest growing mode:

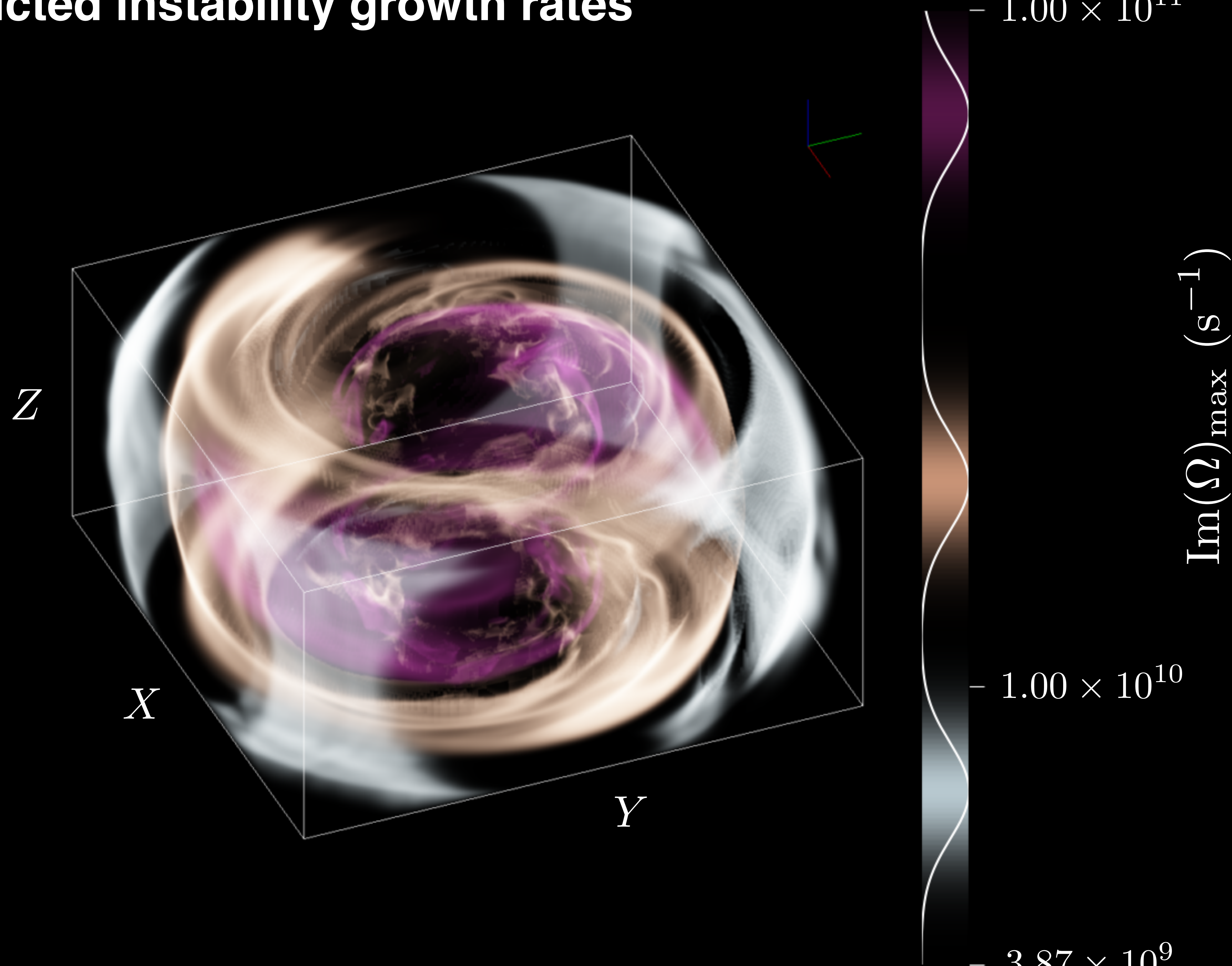
$$\max_{\vec{k}}\left\{\mathrm{Im}[\Omega(\vec{k})]\right\} \equiv \mathrm{Im}(\Omega)_{\max}$$ **Instability growth rate**

Neutron star merger simulation

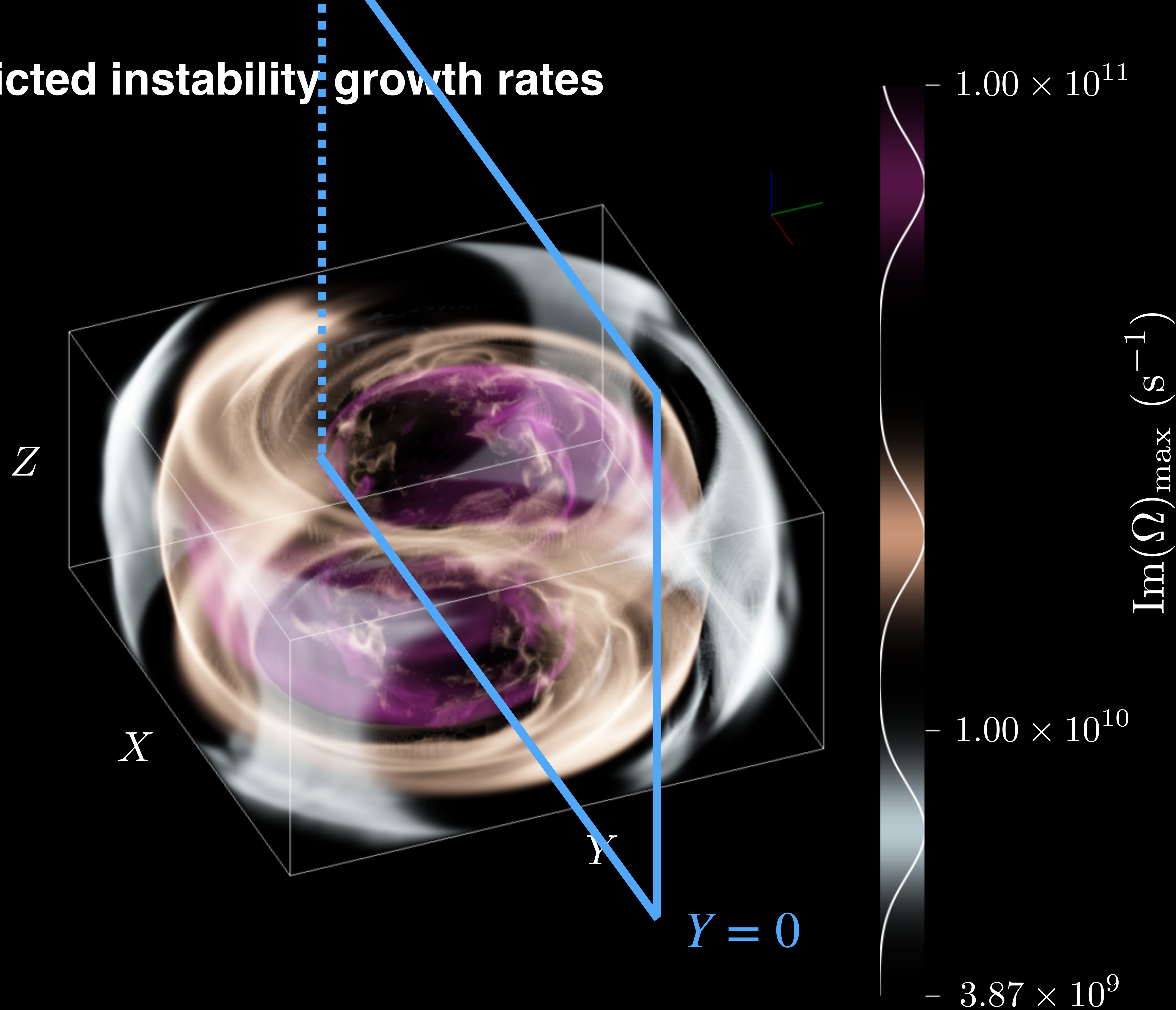
F. Foucart et al., [1502.04146], [1607.07450]



Predicted instability growth rates

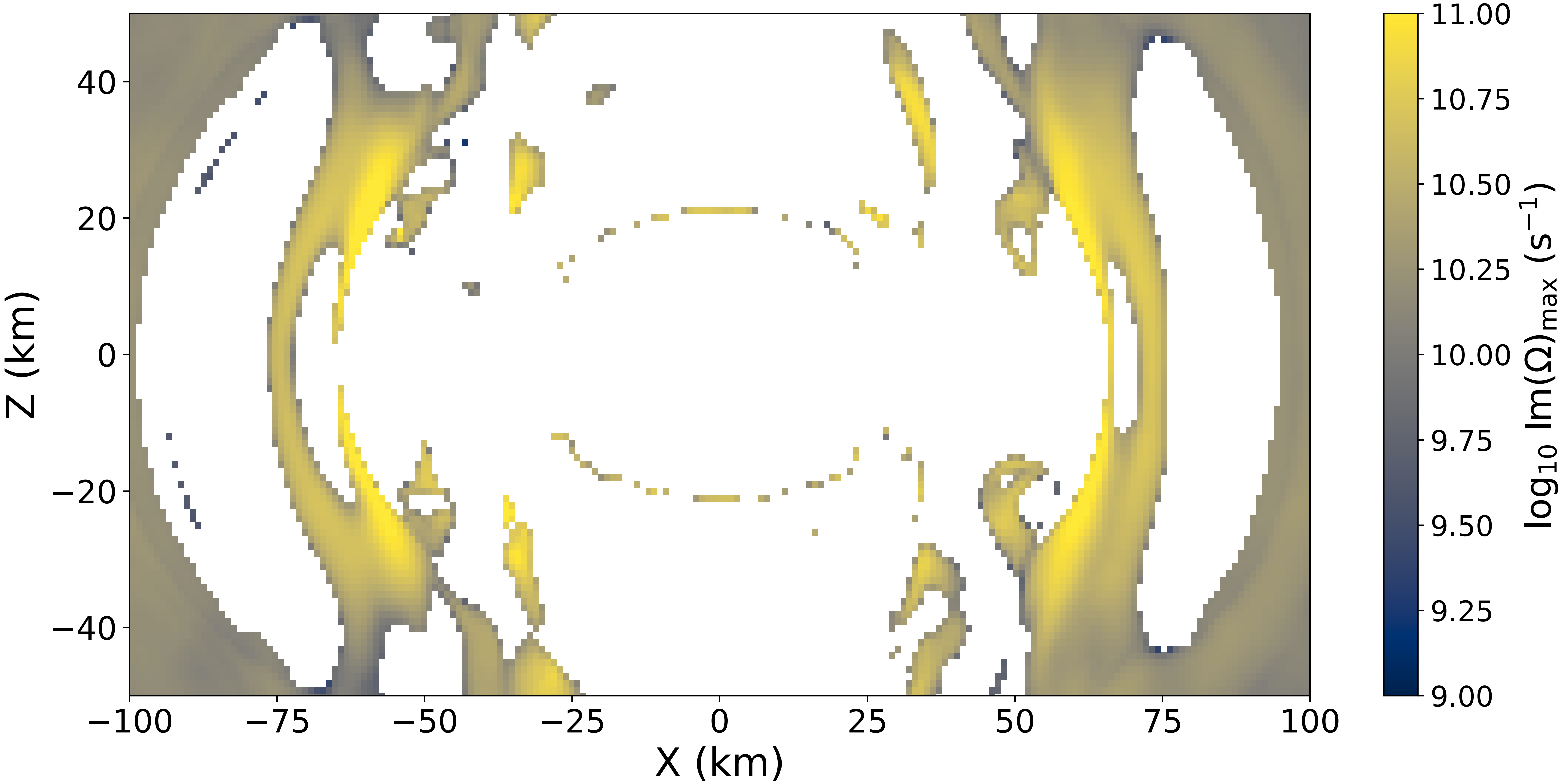


Predicted instability growth rates



Linear stability analysis — Results

Slice across the disk



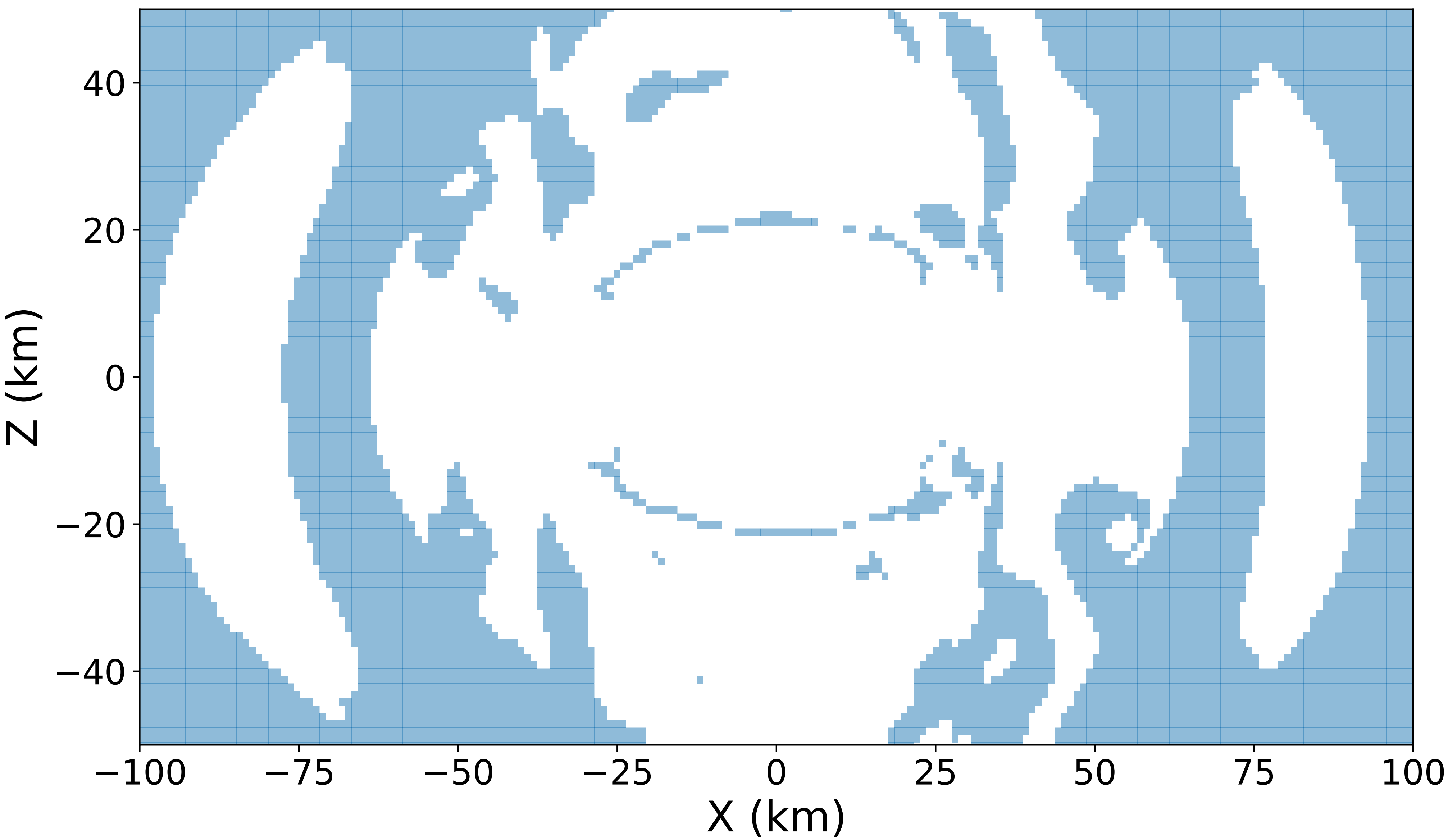
$\Longrightarrow$ Presence of fast-flavor instabilities across the post-merger remnant

$\Longrightarrow$ Typical timescale $0.01 - 0.1\,\mathrm{ns}$

Linear stability analysis — Results

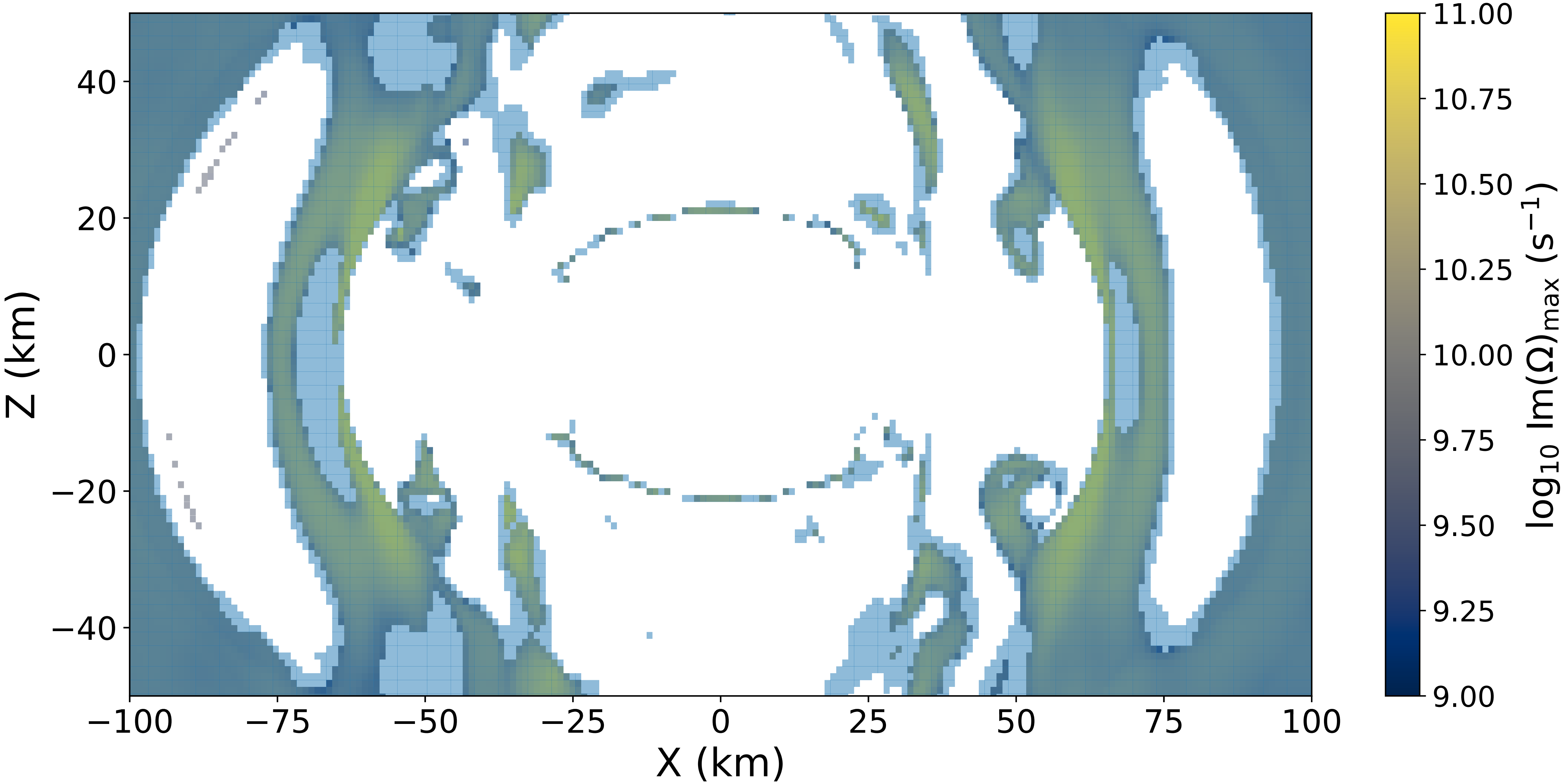
Regions with an electron lepton number crossing

Angular distributions obtained via the maximum entropy closure, cf. *S. Richers* [2206.08444]



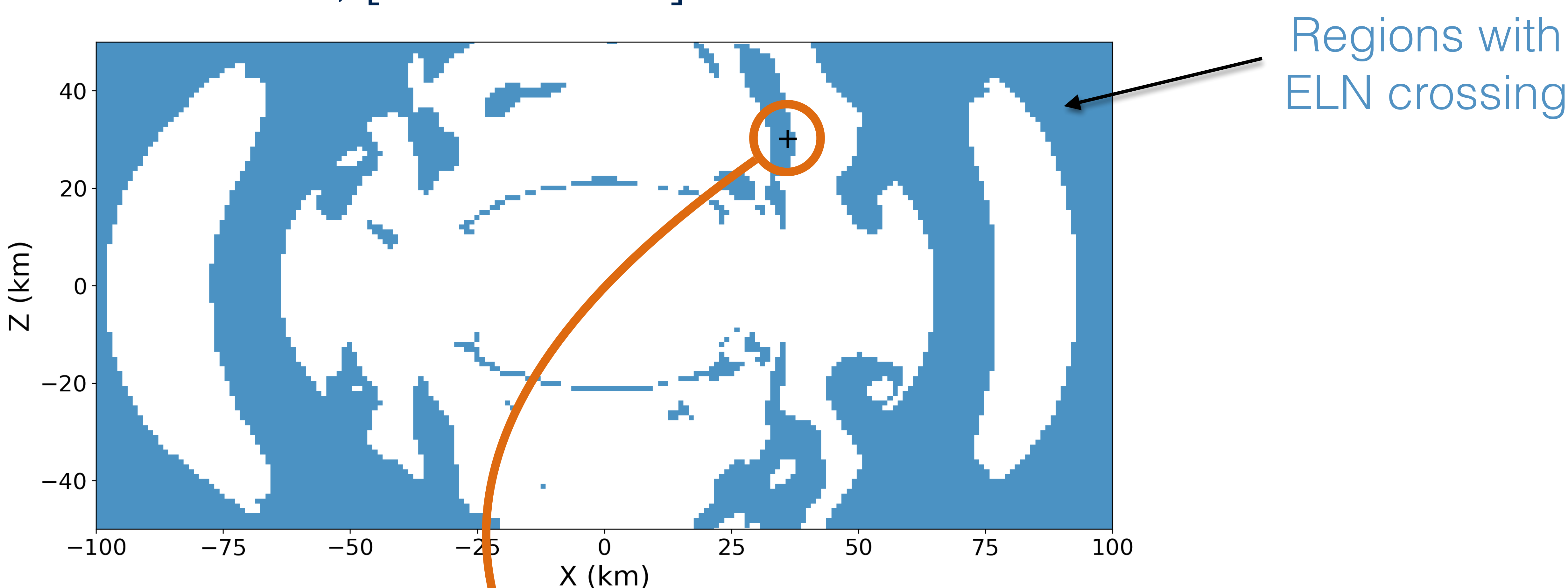
Linear stability analysis — Results

Regions with an electron lepton number crossing



Moment calculation

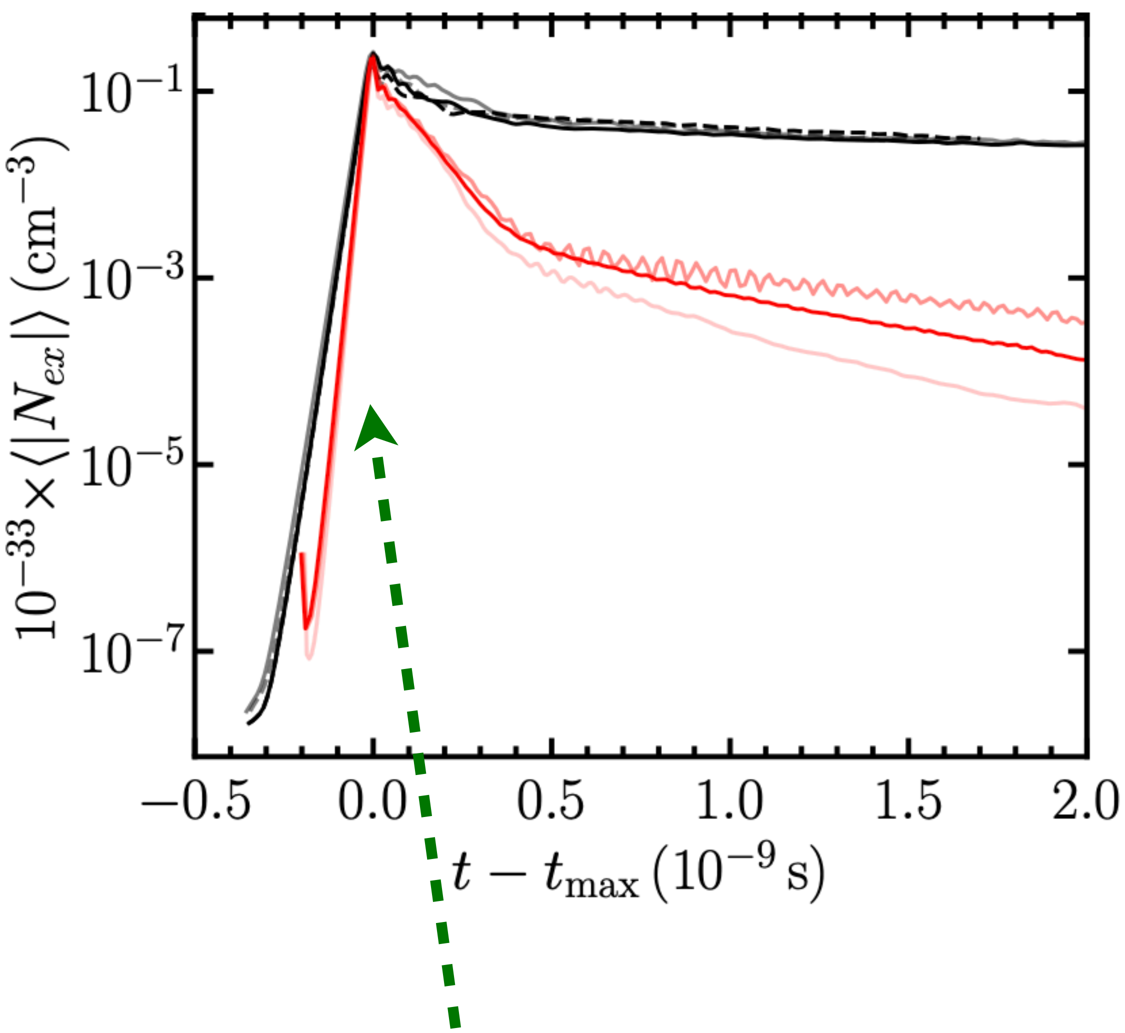
E. Grohs et al., [2207.02214]

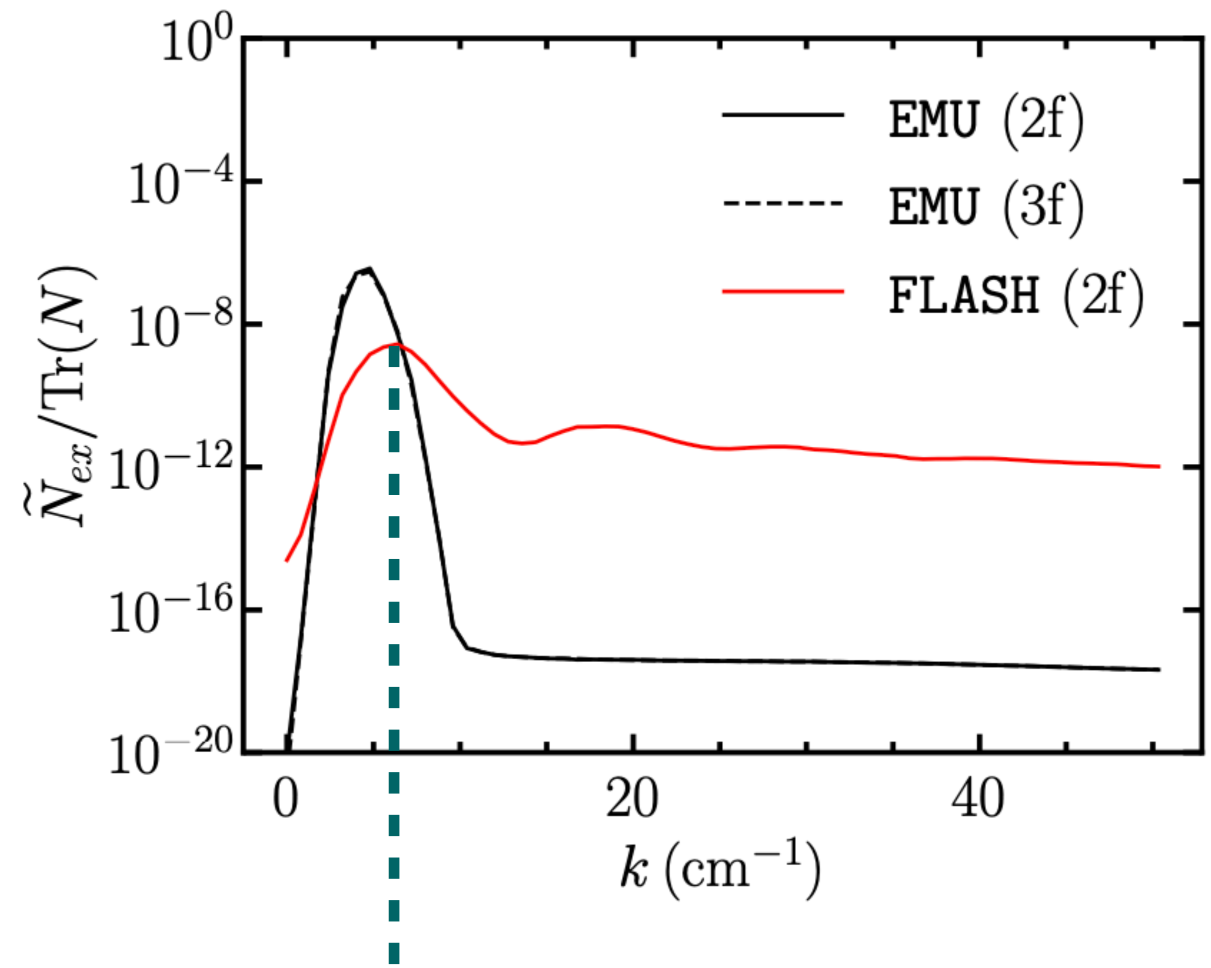


$N_{ee}\,(\mathrm{cm}^{-3})$	1.422×10^{33}
$\overline{N}_{ee}\,(\mathrm{cm}^{-3})$	1.915×10^{33}
$N_{xx} = \overline{N}_{xx}\,(\mathrm{cm}^{-3})$	1.965×10^{33}
$\vec{F}_{ee}/N_{ee}$	$(0.0974, 0.0421, -0.1343)$
$\vec{\overline{F}}_{ee}/\overline{N}_{ee}$	$(0.0723, 0.0313, -0.3446)$
$\vec{F}_{xx}/N_{xx} = \vec{\overline{F}}_{xx}/\overline{N}_{xx}$	$(-0.0216, 0.0743, -0.5354)$

Moment calculation

- Good agreement between a moment (**FLASH**) and a particle-in-cell multi-angle calculations (**EMU**).





$$\mathrm{Im}(\Omega)_{\max} \simeq 8.1 \times 10^{10}\,\mathrm{s}^{-1}$$
$$\mathrm{Im}(\Omega)^{\mathsf{LSA}}_{\max} \simeq 7.4 \times 10^{10}\,\mathrm{s}^{-1}$$

$$k_{\max} \simeq 6.4\,\mathrm{cm}^{-1}$$
$$k^{\mathsf{LSA}}_{\max} \simeq 5.7\,\mathrm{cm}^{-1}$$

Summary & Prospects

<u>Moments can describe fast-flavor instabilities.</u>

- Linear stability analysis provides a powerful, “cheap” framework to provide guidance for calculations.
- We confirm the “ubiquity” of FFI in neutron star mergers.
- Further exploration of oscillation regimes and instabilities.

But no prescription yet regarding the amount of flavor transformation!

- Need for exploratory studies in simulations, with a crude treatment of flavor oscillations
    - Assess the potential changes due to (some) flavor transformation.
    - At least, estimate associated uncertainties.

(cf. for instance *J. Ehring et al.* [2305.11207, 2301.11938]…)