

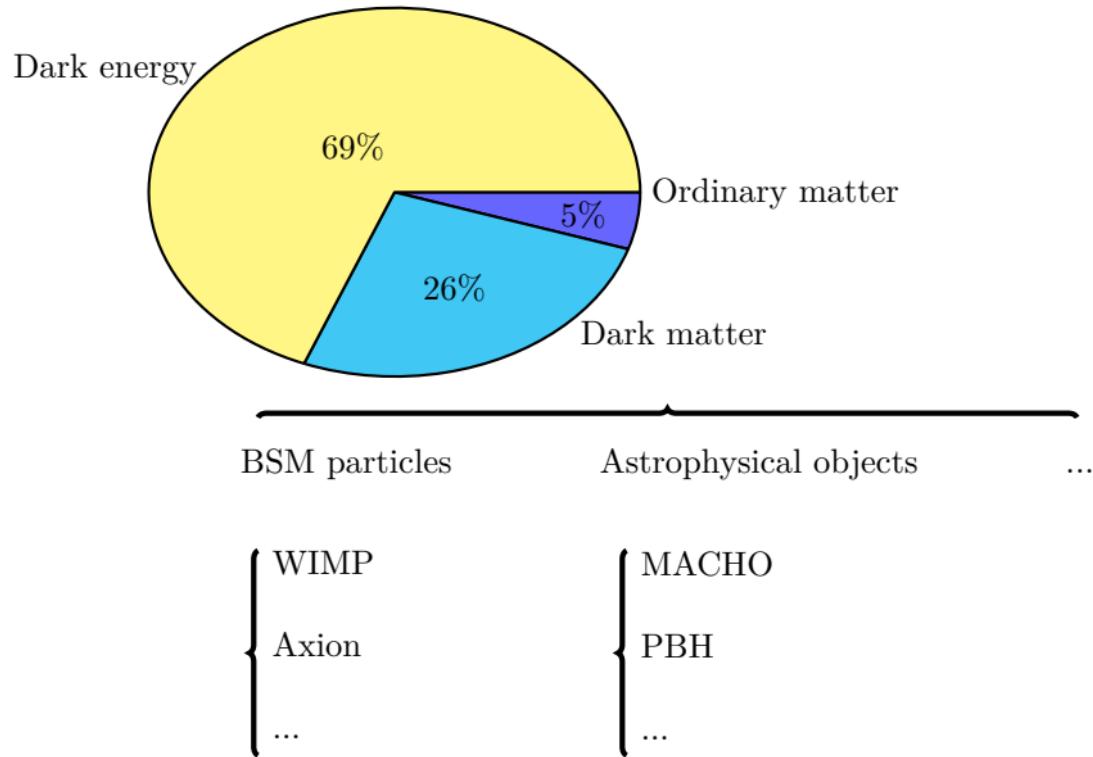
# *Gravitational Wave Probes on Self-Interacting Dark Matter Surrounding an Intermediate Mass Black Hole*

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K. Kadota, J. H. Kim, P. Ko, XYY [[2306.10828](#)]



- $\Lambda$ CDM is an extremely successful model for the large scale structure of the Universe, corresponding to distances greater than  $\mathcal{O}(\text{Mpc})$  today.
- On small scales, there are several discrepancies between CDM predictions and observations.
  - Core-cusp problem
  - Diversity problem
  - Missing satellites problem
  - Too-big-to-fail problem
- Self-interacting dark matter is proposed as a promising alternative to collisionless CDM.
  - Solving problems of CDM model.
  - Many dark matter models can give strong self-interaction.



Black hole + Cold dark matter

- Spike halo:  
the adiabatic growth of a black hole creates a high density dark matter region.

$$\rho_{\text{halo}}(r) = \begin{cases} \rho_{\text{spike}}(r), & r_{\min} \leq r < r_{\text{sp}} \\ \rho_{\text{NFW}}(r), & r_{\text{sp}} \leq r \end{cases}$$

$$\rho_{\text{spike}}(r) = \rho_{\text{sp}} \left( \frac{r_{\text{sp}}}{r} \right)^{\gamma_{\text{sp}}}, \gamma_{\text{sp}} = 7/3$$

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\textcolor{red}{m}^2}{2} \phi^2 - \frac{\lambda}{4} \phi^4 \right]$$

$\lambda > 0$ , repulsive interaction



$$\rho_{\text{soliton}}(r) = \rho_{\sin} \frac{\sin(r/r_c)}{r/r_c} + \rho_{\cos} \frac{\cos(r/r_c)}{r/r_c}$$

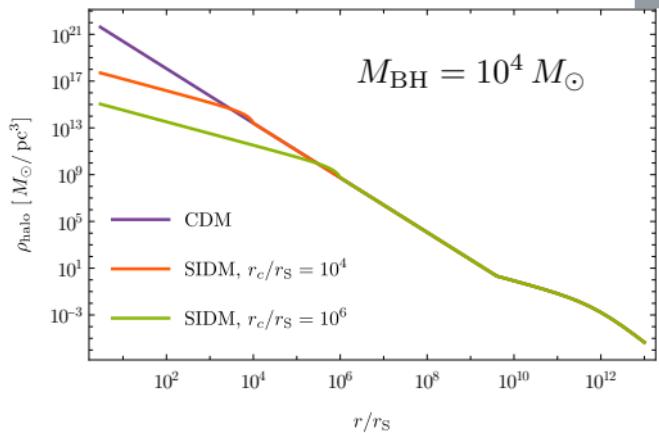
$$\textcolor{red}{r}_c \equiv \sqrt{\frac{3\lambda}{16\pi G \textcolor{red}{m}^4}}$$

Black hole + Self-interacting dark matter

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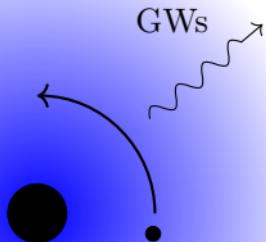
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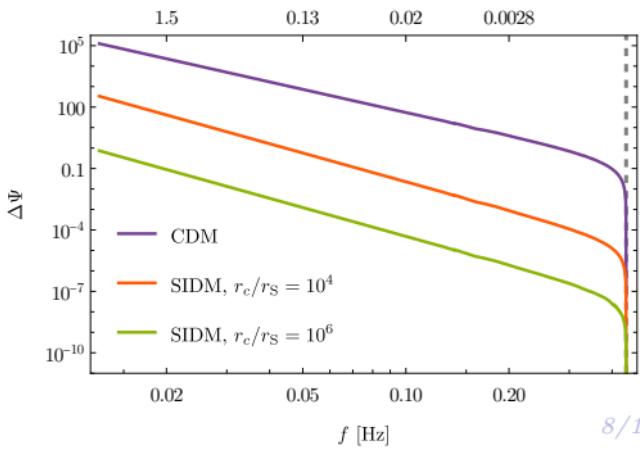
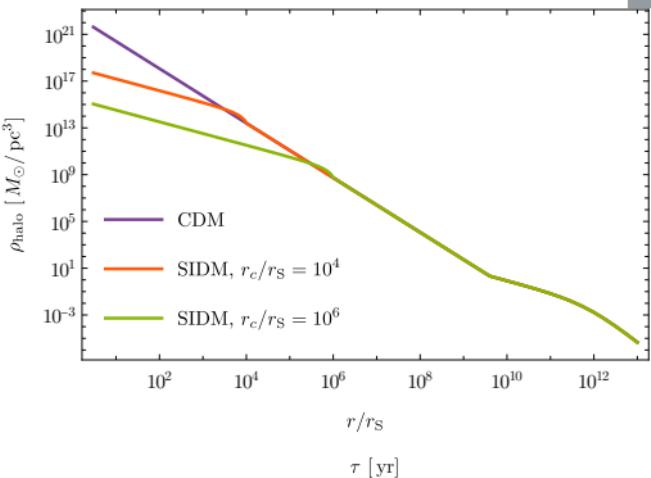


- Dynamical friction
  - Accretion
- ⇒ Dephasing of GWs:  $\Delta\Psi = \Psi(\text{vacuum}) - \Psi(\text{with DM halo})$



$$m_1 = 10^4 M_{\odot}, m_2 = 1 M_{\odot}$$

LISA



Fisher information matrix:

$$\Gamma_{ij} = \left( \frac{\partial \mathbf{d}(f)}{\partial \theta_i}, \frac{\partial \mathbf{d}(f)}{\partial \theta_j} \right)_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}$$

$$\boldsymbol{\theta} = \{\textcolor{red}{r_c}; m_1, m_2, D_L, \iota, \chi, \vartheta, \varphi, \phi_{\text{ISCO}}, t_{\text{ISCO}}\}$$

$$\mathbf{d}(f) = \left[ \frac{\tilde{h}_1(f)}{\sqrt{S_1(f)}}, \frac{\tilde{h}_2(f)}{\sqrt{S_2(f)}}, \dots, \frac{\tilde{h}_N(f)}{\sqrt{S_N(f)}} \right]^T$$

$$\sigma_{\theta_i} = \sqrt{\Sigma_{ii}} \quad , \quad \boldsymbol{\Sigma} = \boldsymbol{\Gamma}^{-1}$$

The point of the Fisher matrix formalism is to **predict how well the experiment will be able to constrain the model parameters before doing the experiment.**

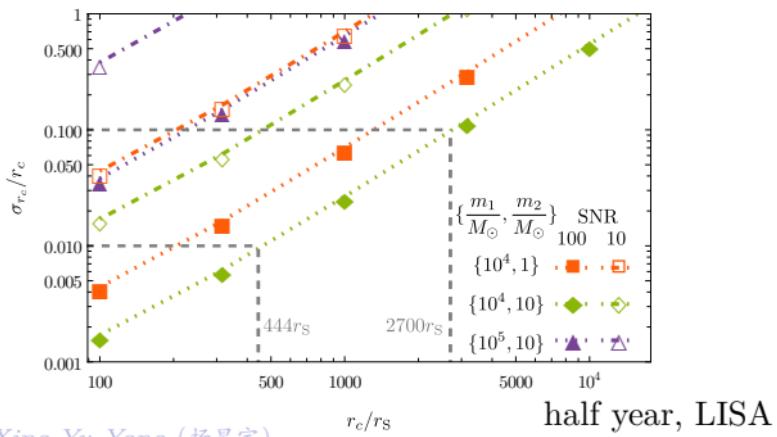
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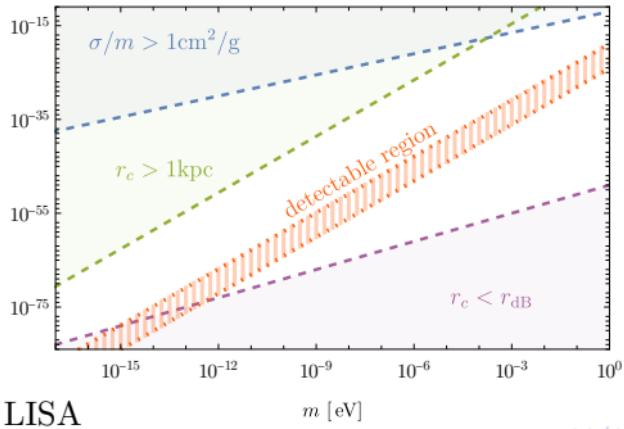
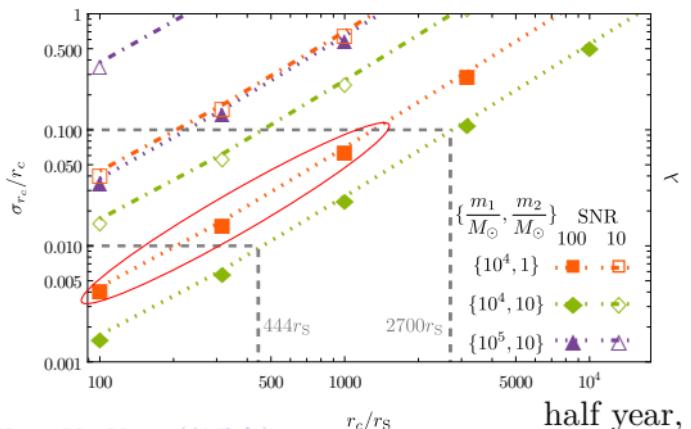
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$$r_c \equiv \sqrt{\frac{3\lambda}{16\pi Gm^4}}$$



## Summary

- Self-interacting dark matter is proposed as a promising alternative to collisionless cold dark matter.
- The accretion of self-interacting dark matter around black holes could lead to the formation of surrounding halo with soliton core.
- The surrounding halo of black holes could lead to the dephasing of gravitational waves, which contains valuable information of the self-interacting dark matter.
- The gravitational waves from intermediate mass ratio inspiral with surrounding halo can be probes on the self-interacting dark matter.