Dark Matter subhalos from semi-analytical perspectives

(as applied to indirect DM searches)

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(incl. important contribs from G. Facchinetti, T. Lacroix, M. Stref)
(+ D. Maurin, J. Pérez-Romero, M.A. Sanchez-Conde)

[Hyperlinks to arXiv refs. 1610.02233, 2007.10392, 2201.09788, 2203. 16440, 2203.16491]

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DM subhalos: connecting fundamental unknowns

Origin of cosmological perturbations / Inflation

→ Primordial power spectrum (PS)

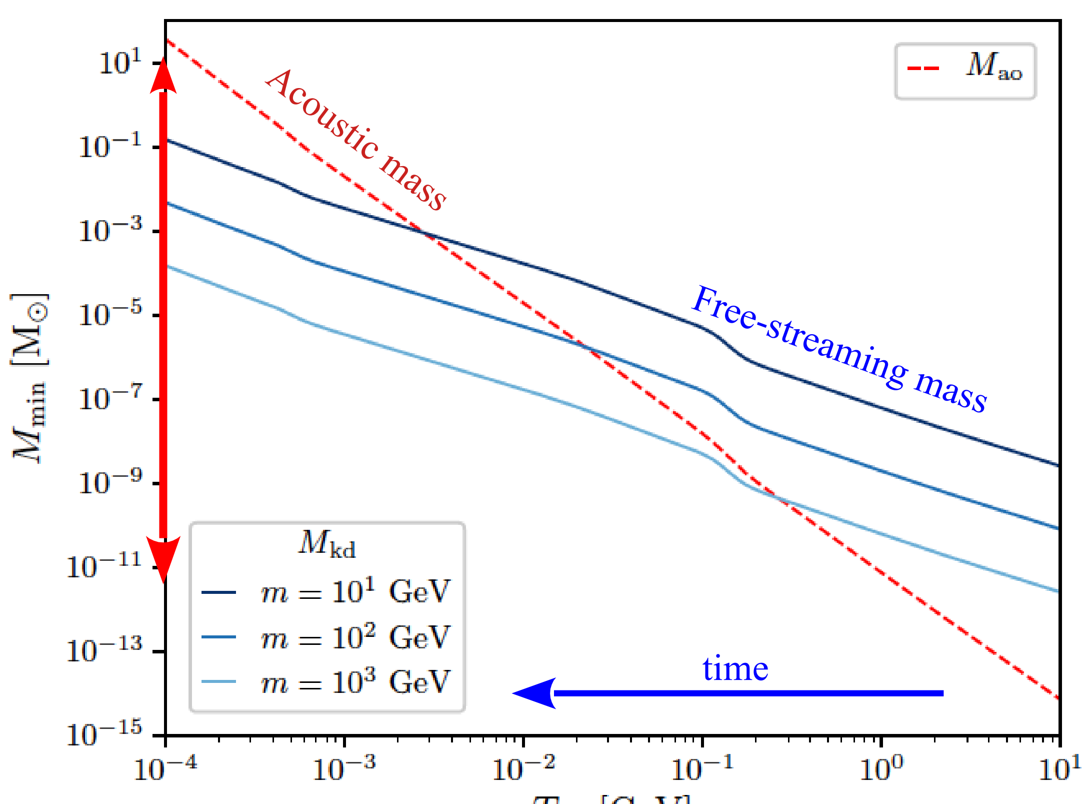
Nature and origin of dark matter

DM: - grows primordial perturbations (matter PS)
- imprints its own features (interactions, etc.)
- might even generate additional perturbations

→ Smallest dark structures carry invaluable information down to much smaller scales than CMB+LSS can probe

Setting the minimal halo mass (thermal DM)

Mind the range!



Facchinetti+ (in prep)

Kinetic decoupling
(~ end of collisions with plasma)

→ onset of DM free-streaming
→ sets minimal DM halo mass

Roughly $\propto \lambda_{fs}^3 \propto (1/m_\chi)^3$

Structure formation in ΛCDM
=
1-parameter model (mass)
+ density profile
(non-linear collapse)

See also:
Hoffman+'01, Green+'04,
Bertschinger'06,
Bringmann+'07,
Gondolo+'08, etc.

$$\lambda_{fs} = a_{eq} \int_{t_{kd}}^{t_{eq}} dt \frac{v(t)}{a(t)} \approx v_{kd}(a_{kd}/a_{eq})/H_{eq}$$

Routes to modeling DM subhalos

Cosmological simulations

(+) Great for non-linear evolution (halo shapes, impact of baryons)
(+) Great for galaxy/cluster population studies + systematics in LSS cosmology
(+) Test/validate analytical models

(-) Resolution limited (subhalos>$10^5 M_{sun}$)
(-) Cosmology limited
(-) Cannot be extrapolated to known target objects (e.g. MW, M31, Coma, etc.)

(Semi)-analytical models

(-) Simulation inputs (e.g. profiles, pdf for concentration)
(-) A few simplifying assumption (assess pros/cons)

(+) Includes properties related to DM candidates
(+) No resolution limit
(+) No cosmology limit
(+) Fast (~min-hr)
(+) Can account for details of real/constrained hosts

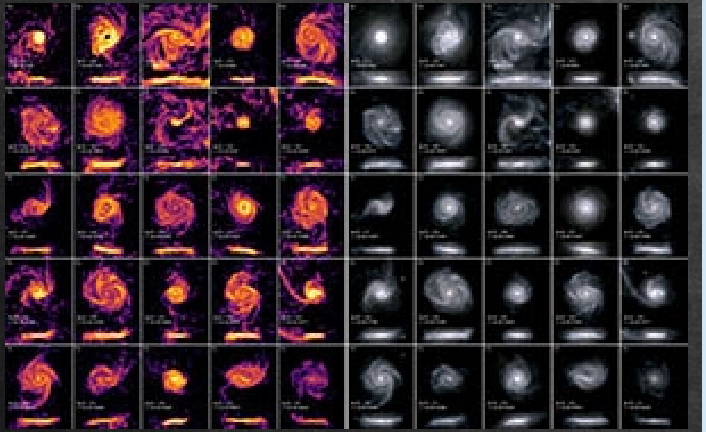
Semi-analytical (sub)halo models for cosmology

→ Power spectrum and halo mass functions
[e.g. ETHOS – Bringmann, Cyr-Racine, Vogelsberger+]

→ Halo models and populations of galaxies
[e.g. Galacticus – Benson+]

→ Subhalo mass functions (w/o spatial distribution)
[e.g. van den Bosch, Giocoli+, see also SASHIMI – Ando+]

DM subhalos in known galaxies/clusters



TNG50 – Pillepich+

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Subhalo populations in specific galaxies/clusters

* Local mass function carries information on primordial PS and DM nature
* Observational features expected:
→ gravitational/kinematic (~DM candidate dependent)
→ impact on other types (DM candidate dependent)

CAUTION: specific hosts are constrained (content, kinematics, etc.)
→ Extrapolations from simulations hardly trustable

NEEDS:
=> Spatial distribution (of properties) required beyond mass function
=> Down to cutoff mass
=> Characterize subhalo (impact on) searches in specific galaxies or clusters

Stimulating ideas for gravitational searches:
Lensing: e.g. Vegetti+
Pulsar timing: e.g. Ramani+'20
Halometry: e.g. Van Tilburg+'18

Analytical population model for a constrained host halo

1. Facts

→ Real galaxies constrained by observations:
- baryonic content
- overall DM profile

→ Non-linear predictions for profiles:
- NFW or Einasto (scale invariance)

$$\rho_{\rm tot}(R) = \left\langle \rho_{\rm smooth}(R) + \sum_i^{N_{\rm tot}} \rho_i(|\vec{R} - \vec{r}_i|) \right\rangle$$

Observation+theory constraints

Analytical population model for a constrained host halo

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2. Reasoning

→ If subhalos were hard spheres
- would trace overall DM profile
- would retain initial properties (no spatial dependence)

→ Can be considered as hard spheres at host halo collapse and at accretion (initial conditions)

→ Changes induced by tidal evolution/stripping

$$\rho_{\rm tot}(R) = \left\langle \rho_{\rm smooth}(R) + \sum_i^{N_{\rm tot}} \rho_i(|\vec{R}-\vec{r}_i|) \right\rangle$$

Observation+theory constraints

Initial conditions: homogeneous mass and concentration pdfs

$$\frac{{\rm d}^n N^0}{{\rm d}\omega^n} = N_0 \underbrace{\frac{{\rm d}\mathcal{P}_V^0(\vec{x})}{{\rm d}V}}_{\text{spatial distrib.}} \times \underbrace{\frac{{\rm d}\mathcal{P}_m^0(m)}{{\rm d}m}}_{\text{mass distrib.}} \times \underbrace{\frac{{\rm d}\mathcal{P}_c^0(c,m)}{{\rm d}c}}_{\text{concentration distrib.}}$$

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3. Model

→ Assume hard spheres initially

→ Determine tidal evolution from gravitational interactions with host + baryons

→ Final phase space non-trivial + intricate (non separable anymore)

$$\rho_{\rm tot}(R) = \left\langle \rho_{\rm smooth}(R) + \sum_i^{N_{\rm tot}} \rho_i(|\vec{R} - \vec{r}_i|) \right\rangle$$

Observation+theory constraints

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Tidal stripping

Intricate final phase space

$$\frac{d^n \bar{N}}{d\omega^n} = \frac{\bar{N}_{\rm tot}}{\bar{K}_w} \frac{d\bar{\mathcal{P}}_V(\vec{x})}{dV} \times \frac{d\bar{\mathcal{P}}_m(m,\vec{x})}{dm} \times \frac{d\bar{\mathcal{P}}_c(c,m,\vec{x})}{dc}$$

(Analytical) properties of dark matter halos

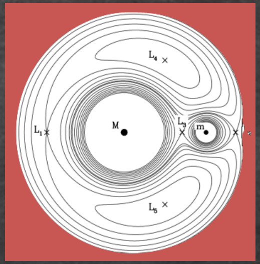
* DM halos have similar profiles (e.g. NFW, Einasto) – scale invariance in non-linear shaping
→ 1-parameter class of model (scale invariance – see e.g. NFW '95-'96, etc.)
=> Cosmological mass → halo parameters (concentration given profile shape)

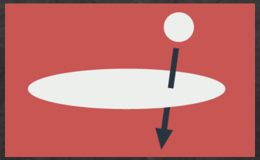
* Scatter in concentration for a given cosmological halo mass
→ Log-normal distribution of concentration (e.g. Bullock+ '01) with scale-invariant dispersion

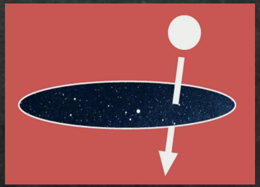
Caution: valid for halos in flat background (not subhalos *after accretion*)

* Mass function from peak statistics theory (Press&Schechter '74, Bond+'86-91, etc.)

Sources of tidal stripping



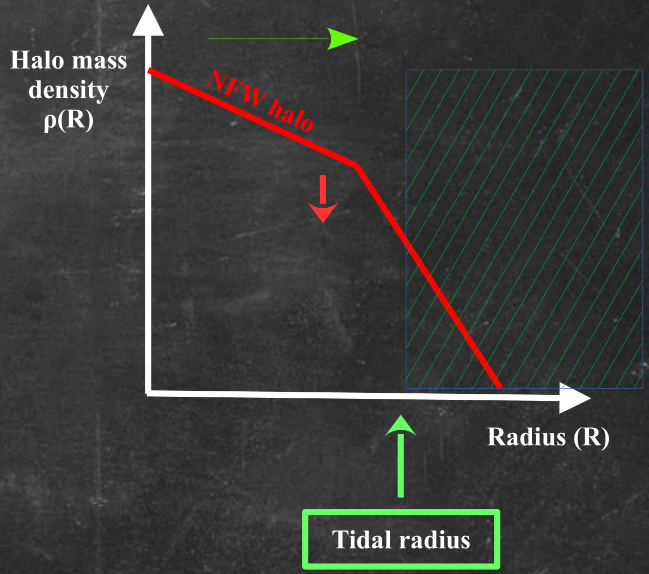




1. Tidal field of the host

2. Baryonic disk shocking

3. Direct encounters with stars



+ Disruption criterion: $r_t / r_s < \varepsilon_t$

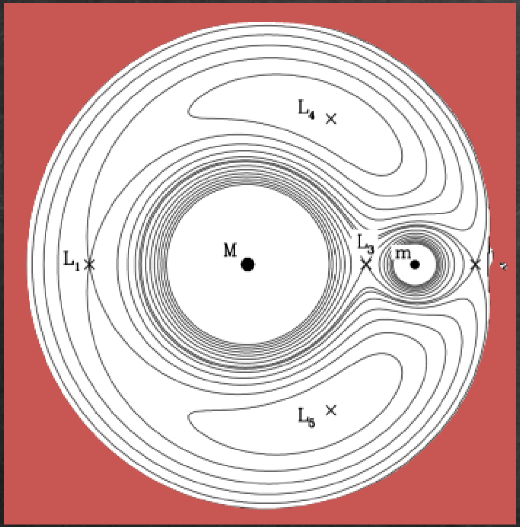
Guesses for ε_t?

→ Hayashi+'04: $\varepsilon_t \sim 1$
→ van Den Bosch+'15: $\varepsilon_t < 1$
→ Physical principles: $\varepsilon_t \ll 1$

Fragile subhalos: **ε_t=1**
Resilient subhalos: **ε_t=0.01**

(see also Errani+, Drakos+, Stücker+'21-23)

Tidal field of the host



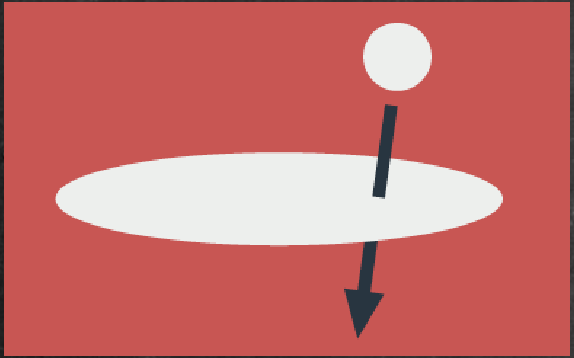
Binney & Tremaine ‘08
[Also, Tormen+, Springel+’08]

$$\ddot{x} = \frac{G\,m}{x^2} - \frac{G\,M}{(R-x)^2} - \omega^2 \{(\mu/m)R - x\} = 0$$

$$r_t = \left\{ \frac{m(r_t)}{3\,M(R)\left(1 - \frac{1}{3}\frac{\mathrm{d}\ln M(R)}{\mathrm{d}\ln R}\right)} \right\}^{1/3} R$$

Disk shocking

$$\delta E = E_{\rm after} - E_{\rm before}$$



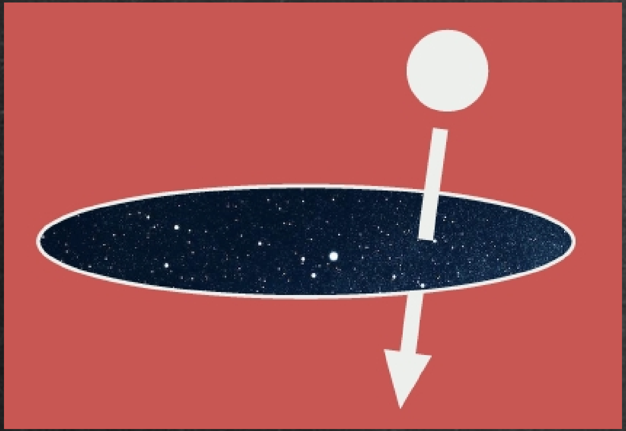
$$\delta E > |\Phi(r)| \,?$$

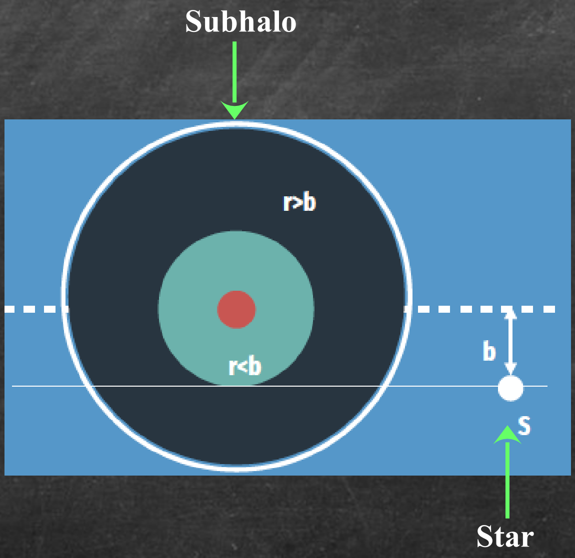
$$\left\langle \frac{\delta E}{m_\chi} \right\rangle = \frac{2}{3}\frac{g_{\rm d}^2}{V_z^2} A(\eta) r^2$$

More efficient for big subhalos
(impulsive shocks)

Impulse approximation
+ adiabatic invariance for central regions
(crossing timescale >> internal orbital period)
[Weinberg'91, Gnedin, Ostriker'98, etc.]

Encounters with individual stars





Approximate analytical results for 2 extended objects by Gerhard & Fall'83.
Extrapolations used several times in context of DM (incl. PBHs), e.g.: Carr+'93, Green+'05, etc.

Simulations: Angus+'07, Goerdt+'07, Schneider+'10, Delos'19

Fully analytical result (improved wrt G&F)

$$\delta E = \frac{1}{2}(\delta \mathbf{v})^2 + \mathbf{v} \cdot \delta \mathbf{v}$$

$$(\delta \mathbf{v})^2(\mathbf{r}) = \left(\frac{2G_N m_\star}{v_r b}\right)^2 \left[I^2 + \frac{b^2(1-2I) - 2I\mathbf{r}\cdot\mathbf{b}}{(\mathbf{r}+\mathbf{b})^2 - (\mathbf{r}\cdot\hat{\mathbf{e}}_{v_r})^2} \right]$$

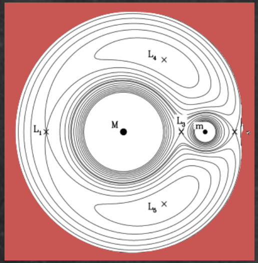
(see Facchinetti+'22)

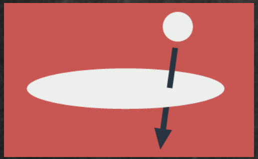
More efficient for small subhalos (stellar masses + impulsive shocks)

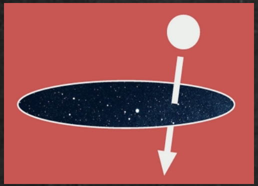
Caution: tricky part is statistics

(see also Delos, Stücker+'21-23)

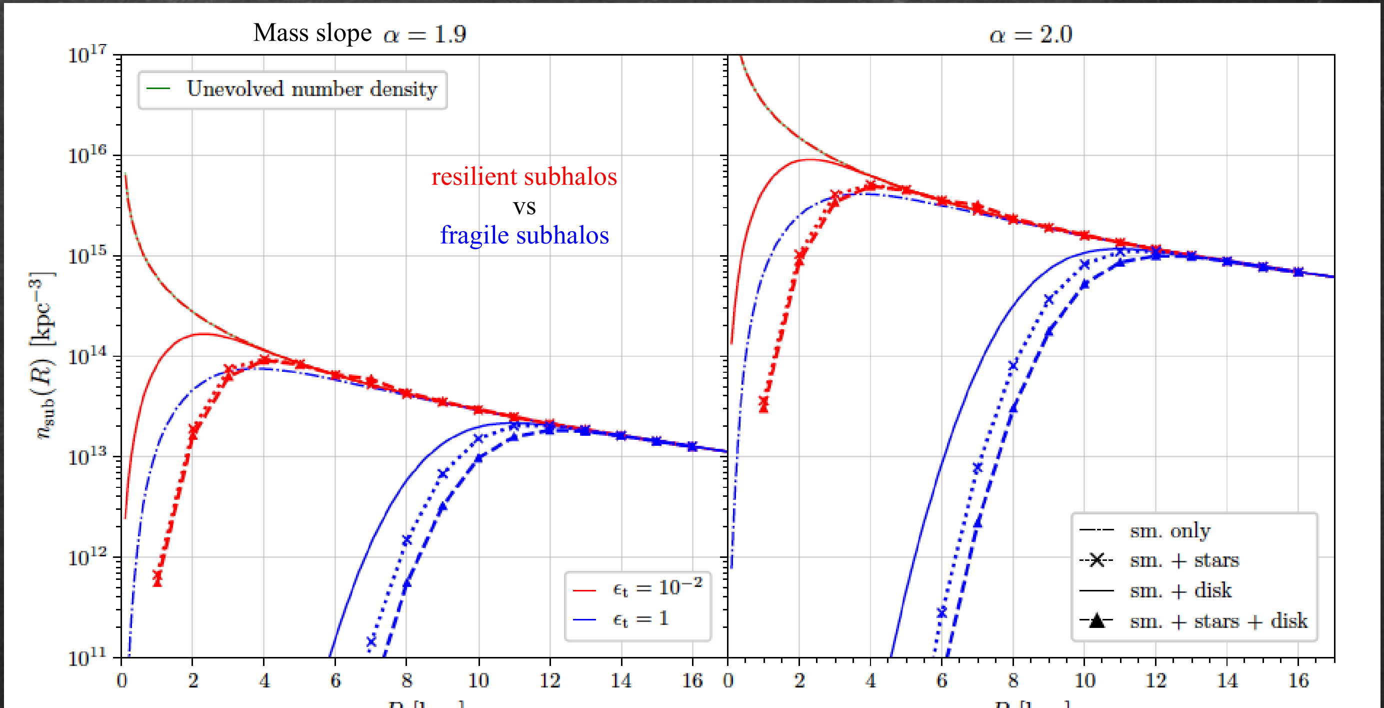
Tidal stripping: all effects







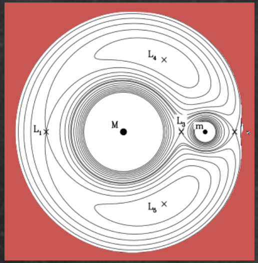
Spatial distribution of subhalos with baryonic-dependent tidal stripping (MW)

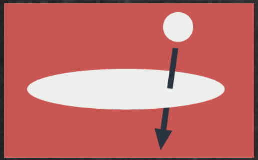


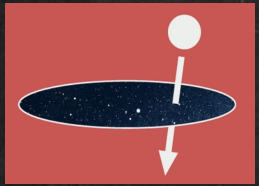
Facchinetti+'22

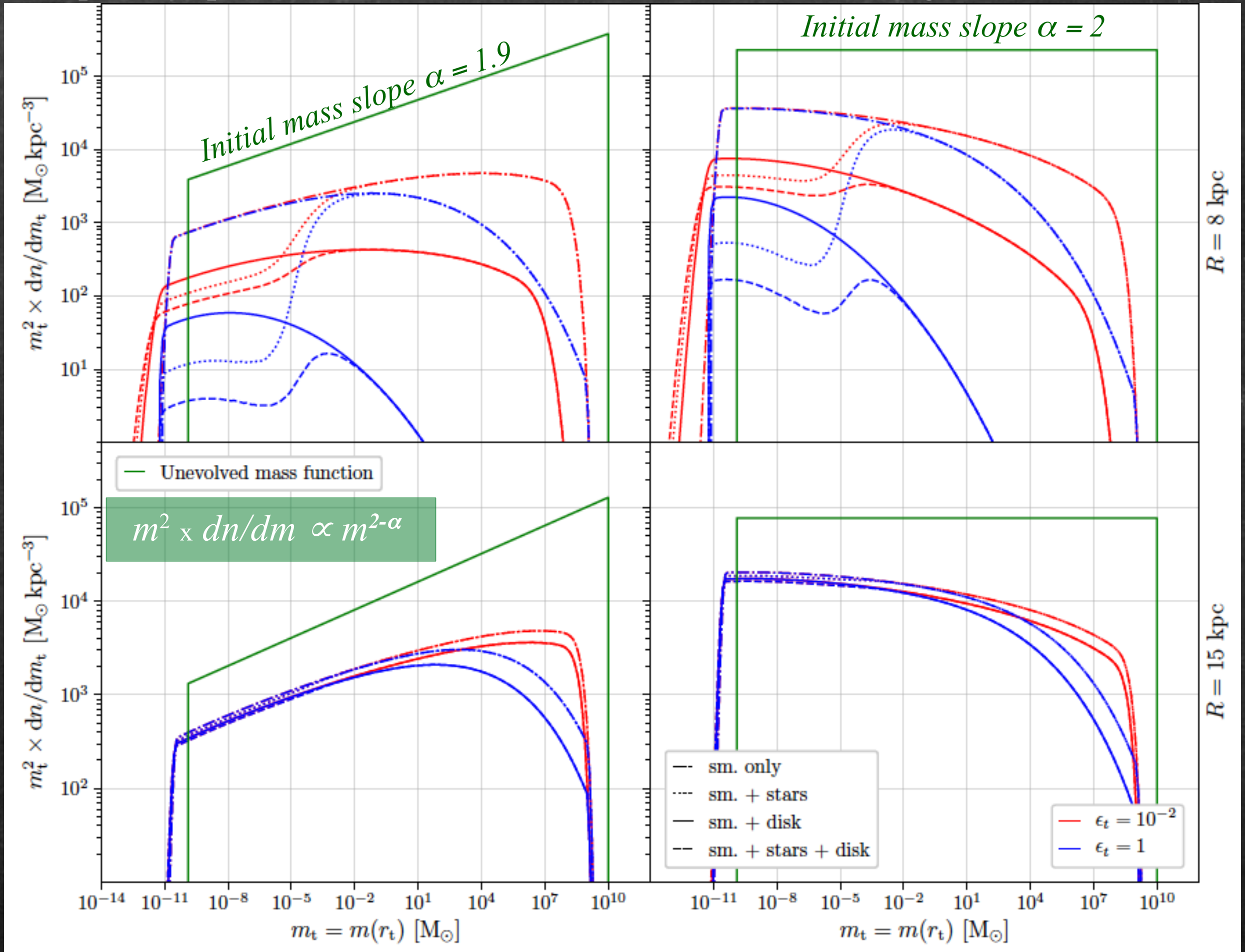
Tidal stripping: all effects

Spatial dependence of mass function (tidal selection of high concentrations)







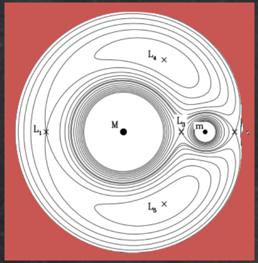


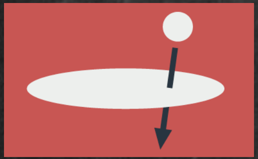
Inner mass function

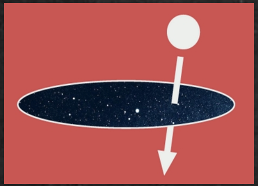
Outer mass function

Facchinetti+'22

Tidal stripping: all effects







$$\rho_{\rm tot}(R) = \left\langle \rho_{\rm smooth}(R) + \sum_{i}^{N_{\rm tot}} \rho_i(|\vec{R} - \vec{r}_i|) \right\rangle \xrightarrow{\text{smooth limit}} \rho_{\rm smooth}(R) + \rho_{\rm sub}(R)$$

$$\boxed{\frac{\mathrm{d}^n \bar{N}}{\mathrm{d}\omega^n} = \frac{\bar{N}_{\rm tot}}{\bar{K}_w} \frac{\mathrm{d}\bar{\mathcal{P}}_V(\vec{x})}{\mathrm{d}V} \times \frac{\mathrm{d}\bar{\mathcal{P}}_m(m, \vec{x})}{\mathrm{d}m} \times \frac{\mathrm{d}\bar{\mathcal{P}}_c(c, m, \vec{x})}{\mathrm{d}c}}$$

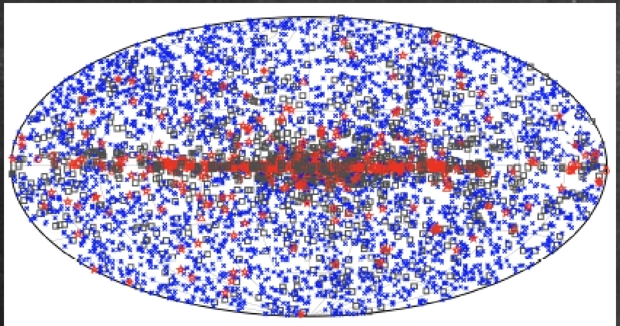
Take home:

Tidal selection of concentrated objects in hosts (resilient to tides)
=> spatial dependence of mass and concentration pdfs
[observed in simulations]

=> ∃ Subhalos much lighter than intitial cutoff mass
(tidal mass function shifted to the left)

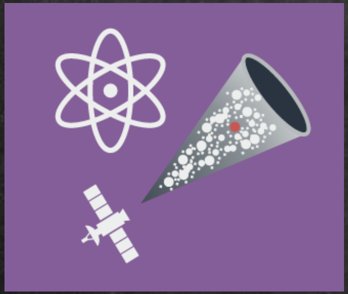
[Limitation: disruption criterion could be improved]
[See e.g. Delos, Stücker+'22-23]

Applications to gamma-ray searches



Fermi-LAT ‘19

Subhalo searches in the Milky-Way

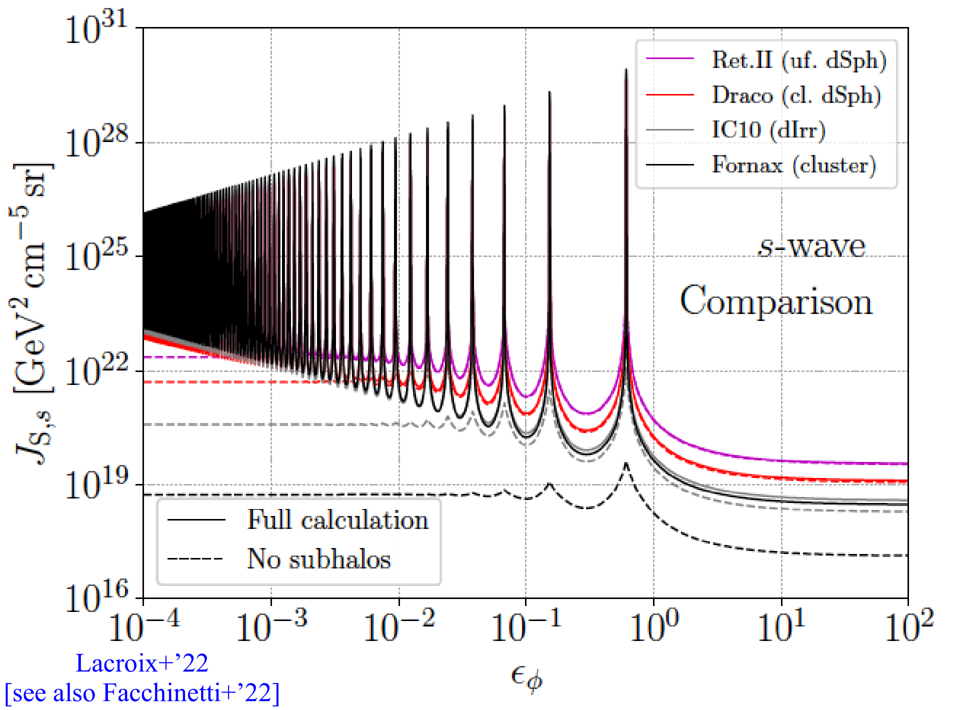


Detailed spatial + mass + concentration pdfs matter!

1525 unassociated sources in 4FGL

→ Subhalos ? [spectral analysis]
[e.g. Belikov+’12, Bertoni+’15, Mirabal+’16, Schoonenberg+’16, Hooper+’17, Coronado-Blazquez+’19, etc.]
→ A few subhalo candidates

Sommerfeld effect in external targets

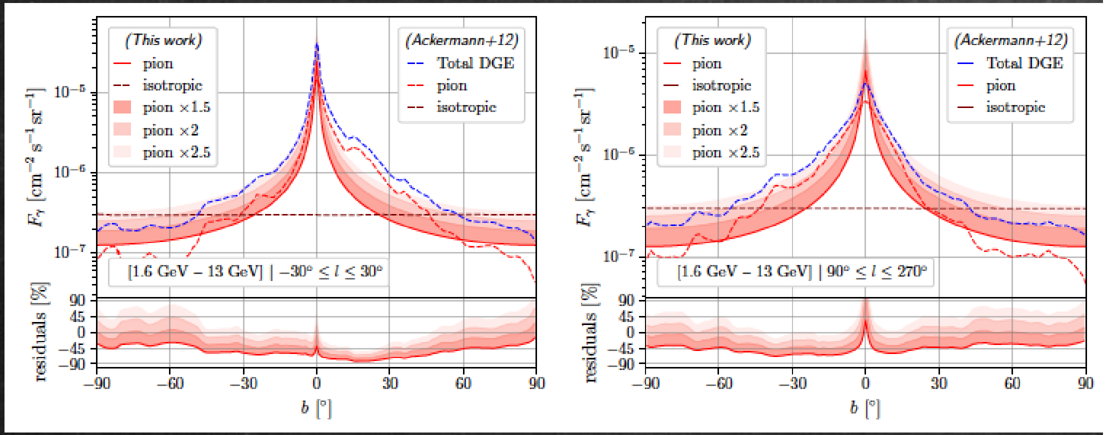


$$\epsilon_v \equiv \frac{v}{\alpha_{\rm D}\, c} \quad \text{and} \quad \epsilon_\phi \equiv \frac{m_\phi}{\alpha_{\rm D}\, m_\chi}$$

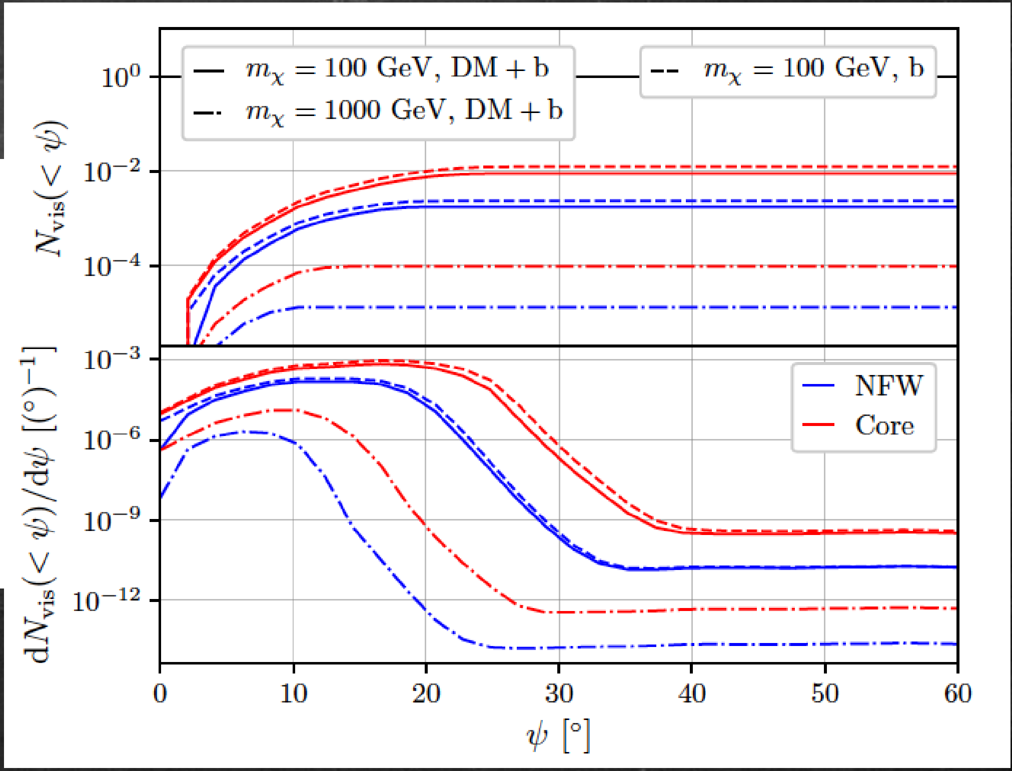
Enhancement at small velocity
→ Subhalo contrib. dominates

Gamma-ray searches of MW subhalos

Foreground model



Bg/fg model vs data from Fermi-LAT+'12

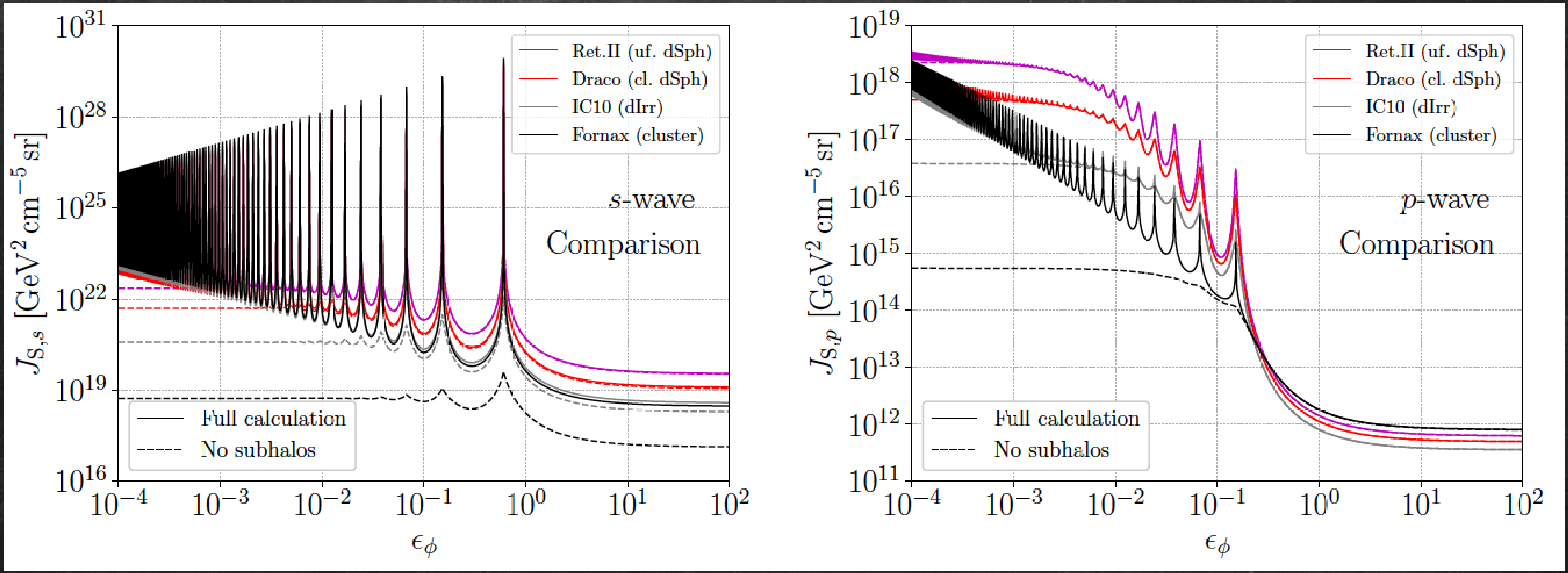


Facchinetti+'20
Expected number of detected subhalos after 10 yrs

Results:

* Best angular region: ~10°-20° from GC
* P(1 subhalo) before smooth halo ~ 0
* P(1 subhalo / 20 yrs) ~ 0.95 after smooth halo detected (10 yrs)
* Need to go for extended source searches (~1°)

Sommerfeld enhancement



Lacroix+'22
[see also Facchinetti+'22]

$$\epsilon_v \equiv \frac{v}{\alpha_D\, c} \quad \text{and} \quad \epsilon_\phi \equiv \frac{m_\phi}{\alpha_D\, m_\chi}$$

Results:
* Huge boost factors (> 3 OM)
* Exacerbate/revert hierarchy btw targets

Take home

* DM subhalos connect DM properties and primordial PS (=> DM candidate + inflation model)

* Different DM candidates => different properties on subgalactic scales

* Subhalo (impacts on DM) searches require spatial+mass+concentration distributions over full mass range
=> challenging with simulations, not extrapolable to dynamically constrained objects (e.g. MW)

* (Semi)-analytical population models to the rescue (with simplifying assumptions)
→ Rely on physical principles + self-consistency (e.g. smooth/subhalo separation)
→ no resolution limit
→ no cosmology limit
→ can account for detailed properties of constrained target hosts
→ complementary to simulations

* Predictive: spatial dependence of concentration+mass function (selection effect), flattening of spatial distribution

* Fast + flexible + can be used to optimize search strategies for any DM candidate [+ ongoing improvements]

* Effective: e.g. gamma-ray predictions, lensing searches, etc.

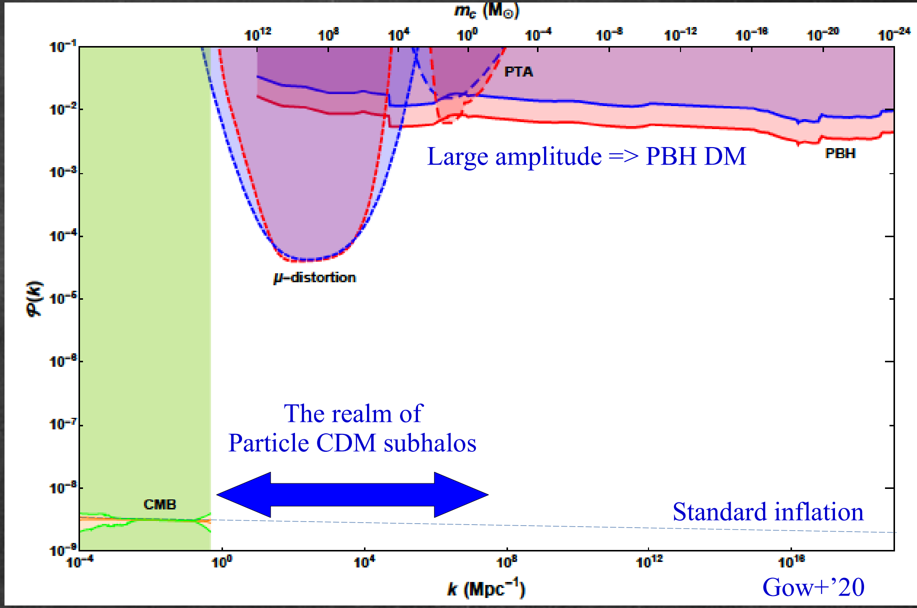
Backup

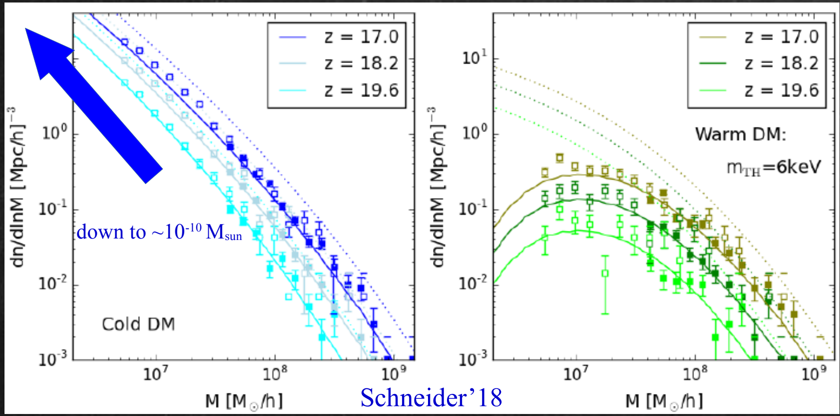
DM subhalos: connecting fundamental unknowns

Origin of cosmological perturbations

→ Primordial power spectrum (PS)

(on scales much lower than CMB+LSS can touch)





Nature and origin of dark matter

DM: - grows primordial perturbations (matter PS)
- imprints its own features (interactions, etc.)
- might even generate additional perturbations

→ *Smallest dark structures carry invaluable information*

Concentration

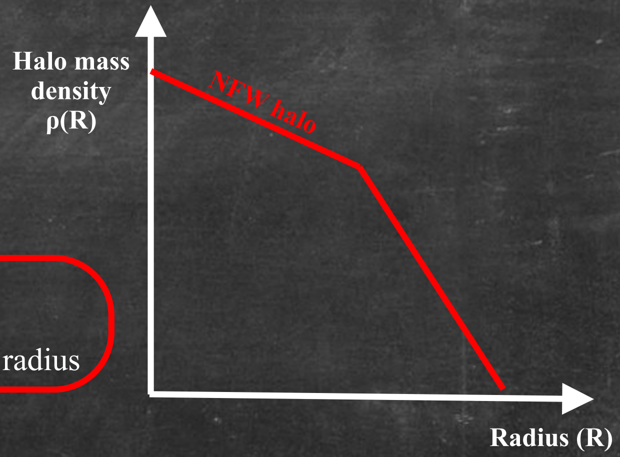
$$\rho_{\rm nfw}(r) \quad = \quad \rho_s \frac{(r/r_s)^{-1}}{(1+r/r_s)^2}$$

2 free parameters

$$m_{200} = \frac{4\pi}{3}\,(200\,\rho_c)\,r_{200}^3$$

1 constraint (mass + volume)

CAUTION:
r_{200} not physical!
Physical radius = tidal radius



$$c_{200} \quad = \quad \frac{r_{200}}{r_{-2}}$$

2nd constraint (concentration)

Physical meaning:
(central density / background density)$^{1/3}$

=> decreases with redshift!
(increases with time from collapse)

Concentration

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2 free parameters

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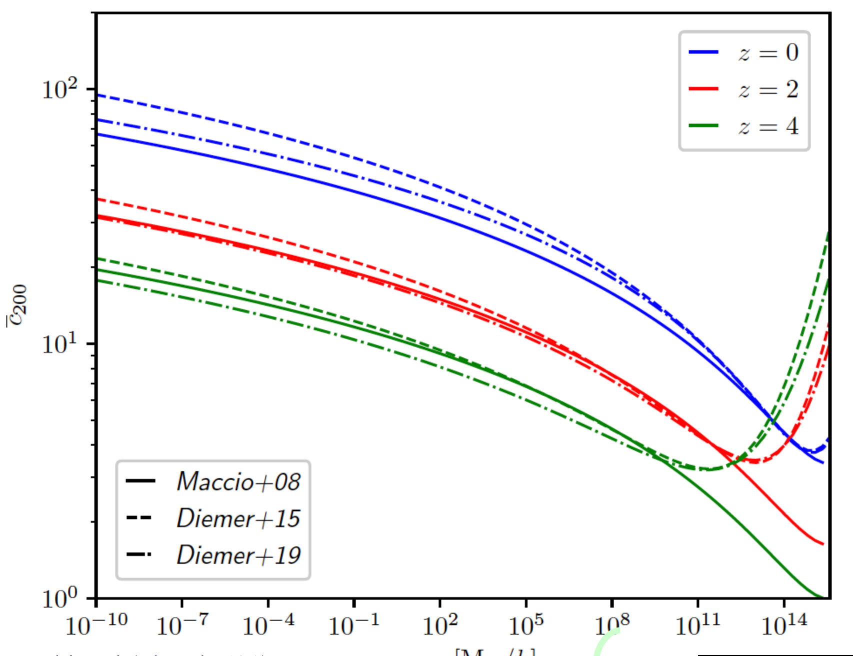
2nd constraint (concentration)



Facchinetti (PhD th. '21)

Bullock+'01 model
+ refinements in Maccio+'08, Prada+'11, Sanchez-Conde+'12, Okoli+'16, Diemer+'19

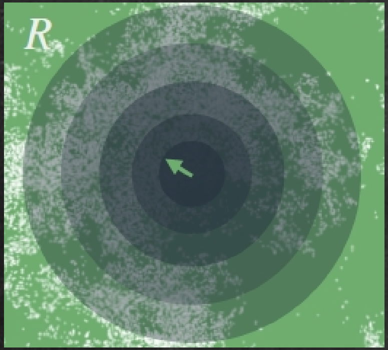
$$\bar{c}_{200}(m_{200}, z) = K_{200}\left[\frac{\rho_c(z_c)}{\rho_c(z)}\right]^{1/3}$$

$$m_{200}(z) = G_z\, m_*(z_c)$$

$$\sigma(m_*(z_c)) = \sigma_c^0\, D_+(z)$$

Initial mass function: excursion set

Halo mass function:
Press+’74, Bardeen+’86,
Bond+’91, Lacey+’93,
Cole+’93, Sheth+’99, etc.



Facchinetti (PhD th.)

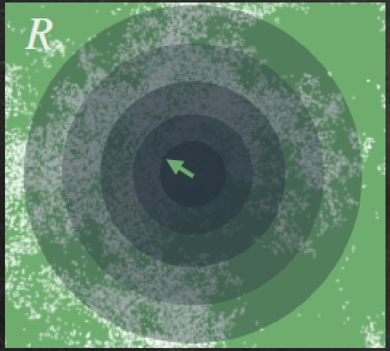
$$\delta_R(\mathbf{x}) = \int \mathrm{d}\mathbf{y}\frac{\delta\rho}{\bar{\rho}}W_R(|\mathbf{x}-\mathbf{y}|)$$ (smoothed density contrast)

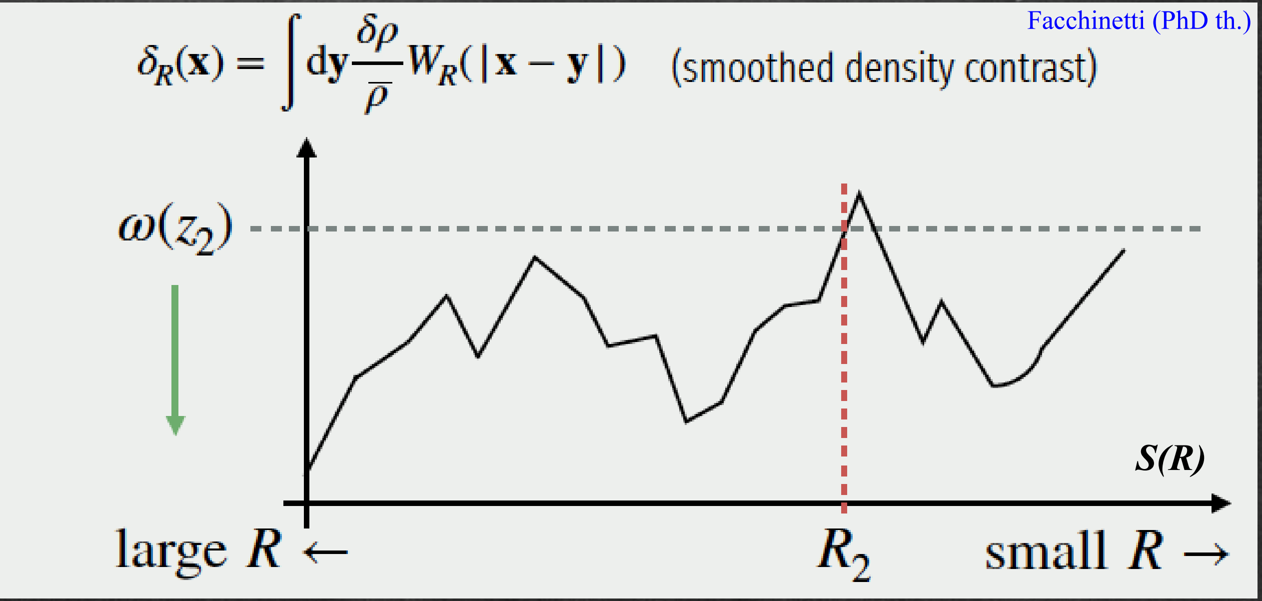
$$P_\mathrm{m}(k,z) = \frac{8\pi^2 k}{25}\left\{\frac{\mathrm{d}_1(z)}{\Omega_{\mathrm{m},0}H_0^2}T(k)\right\}^2 \mathcal{A}_S\left(\frac{k}{k_0}\right)^{n_s-1}$$

$$S(R) \equiv \sigma^2 = \frac{1}{2\pi^2}\int_0^{1/R}\mathrm{d}k\,k^2\,P_\mathrm{m}(k,z=0)$$

Initial mass function: excursion set

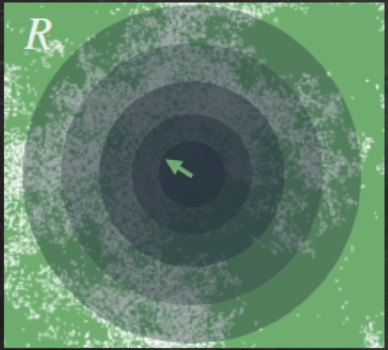
Halo mass function:
Press+’74, Bardeen+’86,
Bond+’91, Lacey+’93,
Cole+’93, Sheth+’99, etc.





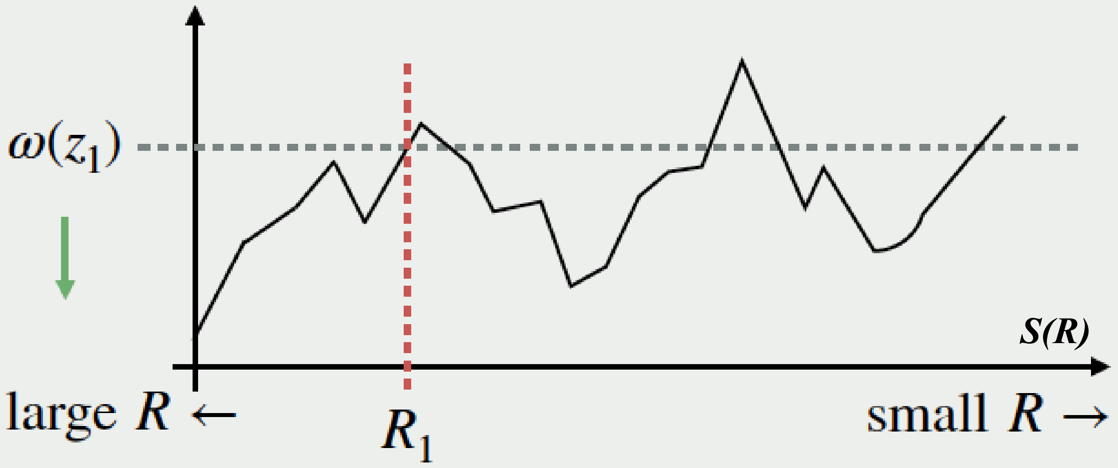
Initial mass function: excursion set

Halo mass function:
Press+'74, Bardeen+'86,
Bond+'91, Lacey+'93,
Cole+'93, Sheth+'99, etc.



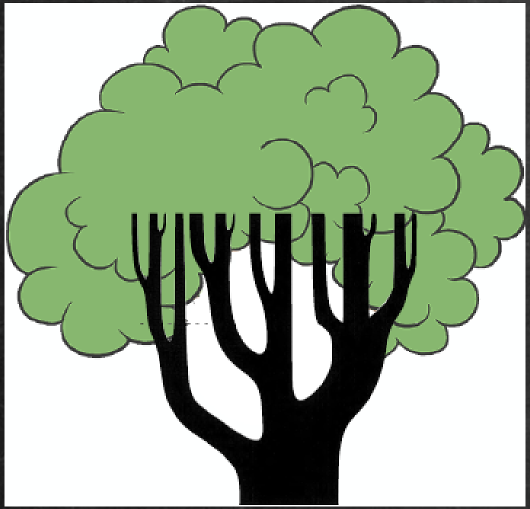
Facchinetti (PhD th.)

$$\delta_R(\mathbf{x}) = \int \mathrm{d}\mathbf{y} \frac{\delta\rho}{\bar{\rho}} W_R(|\mathbf{x}-\mathbf{y}|)$$ (smoothed density contrast)

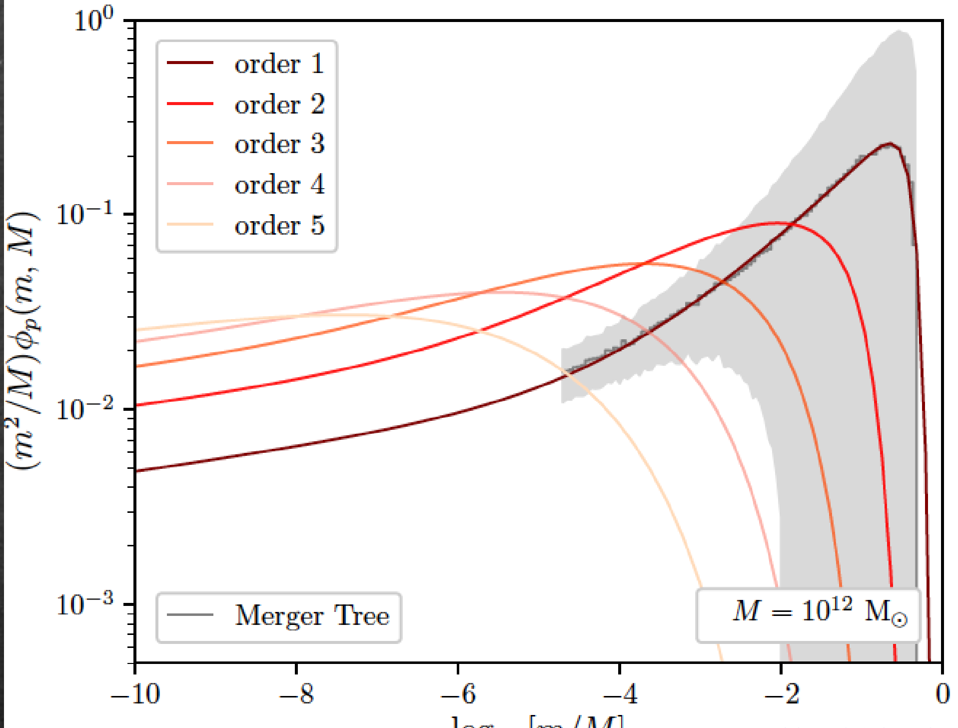


$$f(\omega_2, S(R_2) \,|\, \omega_1, S(R_1)) = \frac{\Delta\omega}{\sqrt{2\pi}\Delta S^{3/2}} \exp\left(-\frac{(\Delta\omega)^2}{2\Delta S}\right)$$

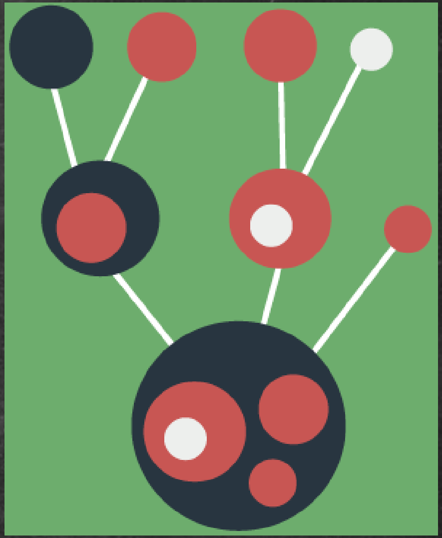
Initial mass function: excursion set



Adapted from Lacey & Cole



Facchinetti (PhD th.)



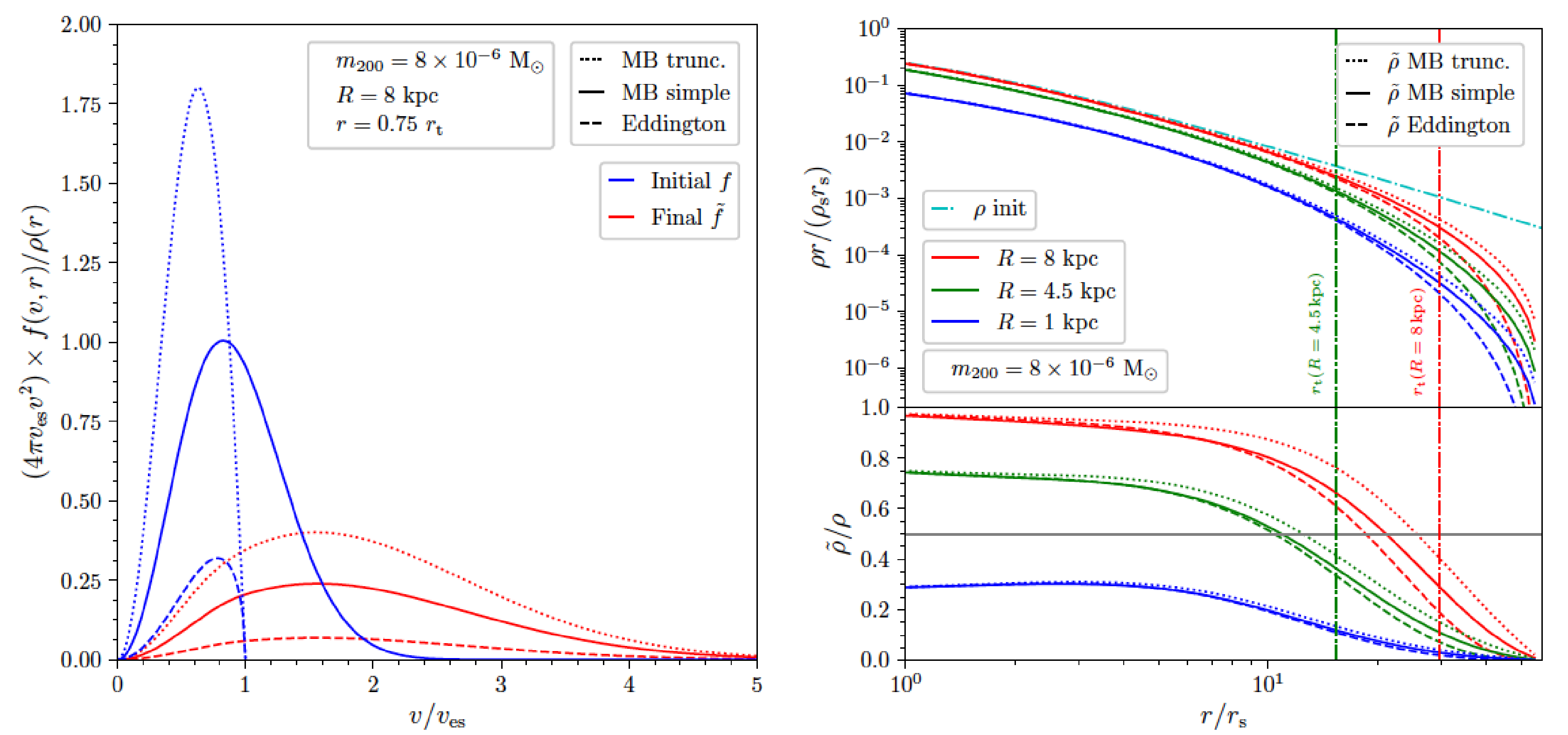
$$\frac{\mathrm{d}N(m,M)}{\mathrm{d}m} = \frac{1}{m}\left[\sum_{i=1,2}\gamma_i\left(\frac{m}{M}\right)^{-\alpha_i}\right]\exp\left\{-\beta\left(\frac{m}{M}\right)^{\zeta}\right\}$$

See also:
Cole'01-'08, van den Bosch+, Giocoli+, Despali+, Hiroshima+, etc.

Absolute number + subhalo mass fuction for :
- any cosmology
- any host mass / cutoff mass
- any subhalo layer

=> slope ~1.95 (ΛCDM)

Tidal evolution of inner profiles



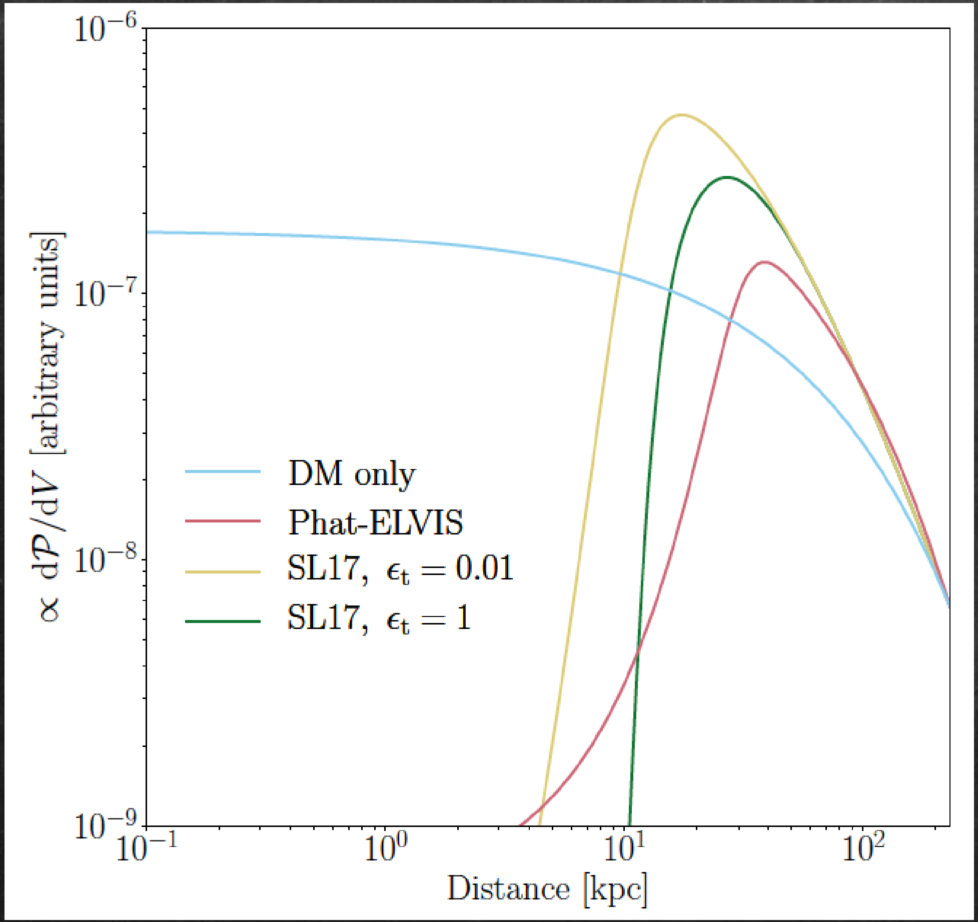
Inner shape is preserved, but with decreased inner density

[also e.g. Penarrubia'08, Delos'19, Errani+'21]

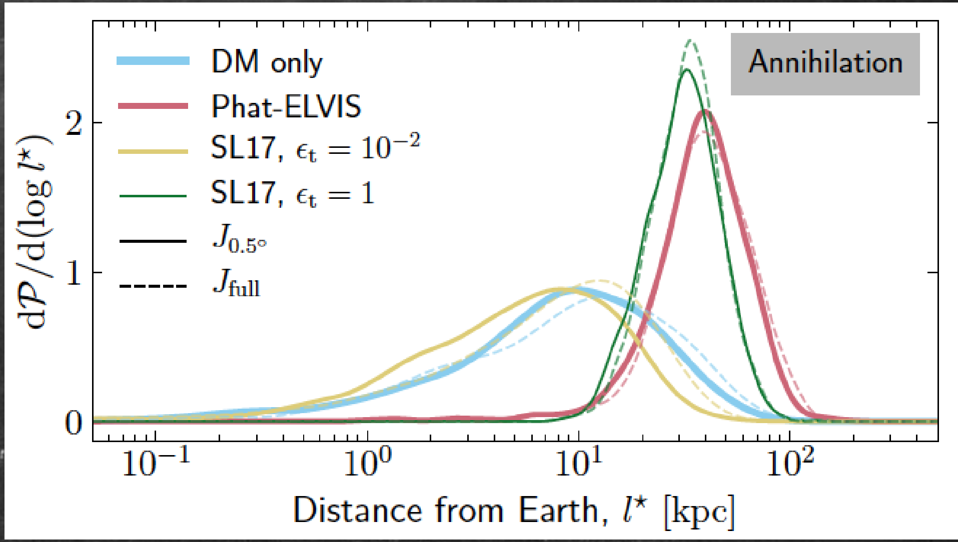
→ Currently not included in model

Comparisons with simulations

Spatial distribution of subhalos $M > 1.e6\ M_{sun}$

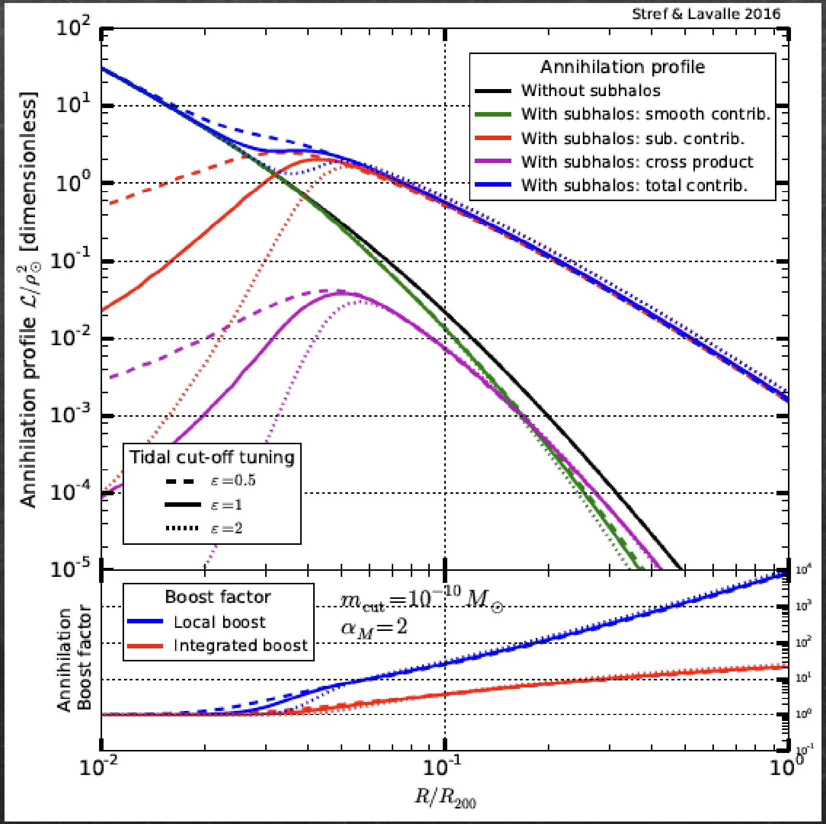


PDF distance to most "luminous" subhalo

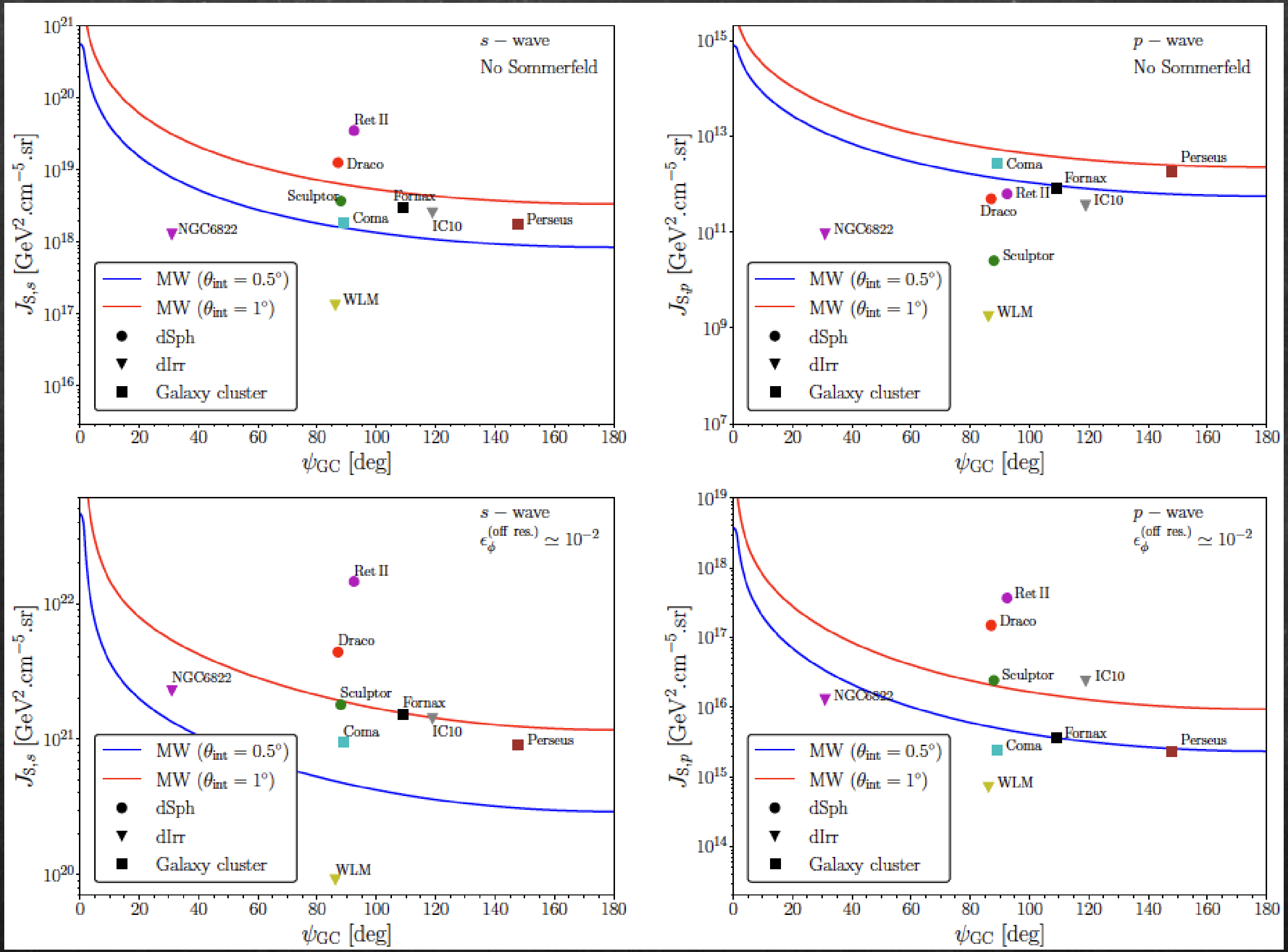


Hütten+'19 with Clumpy code [Charbonnier+'11]
[comparison with Phat Elvis, Kelley+'19]

Annihilation profile in MW with subhalos

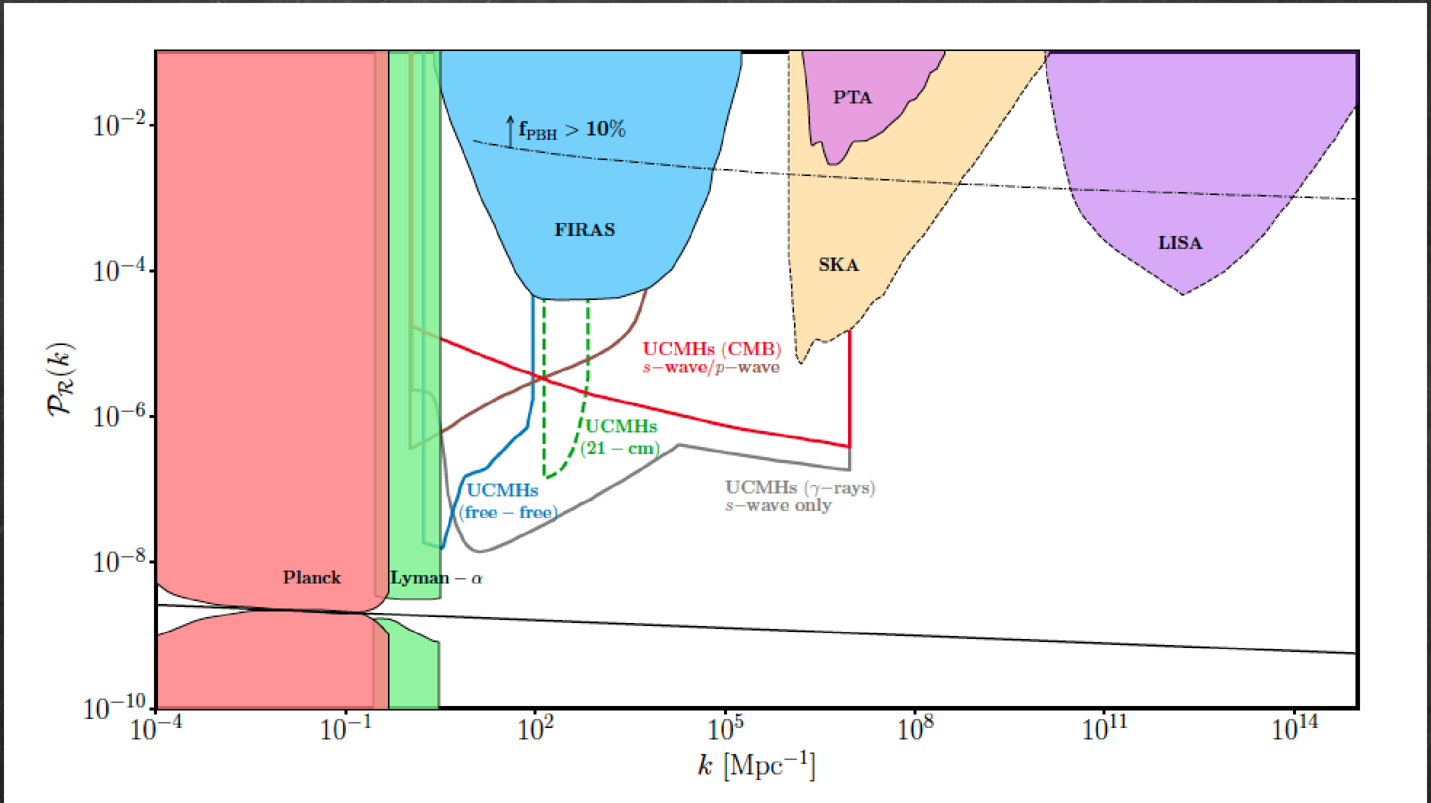


Impact on targets hierarchy



Lacroix+'22
[see also Facchinetti+'22]

Ultracompact minihalos



Abellan & Facchinetti ‘23