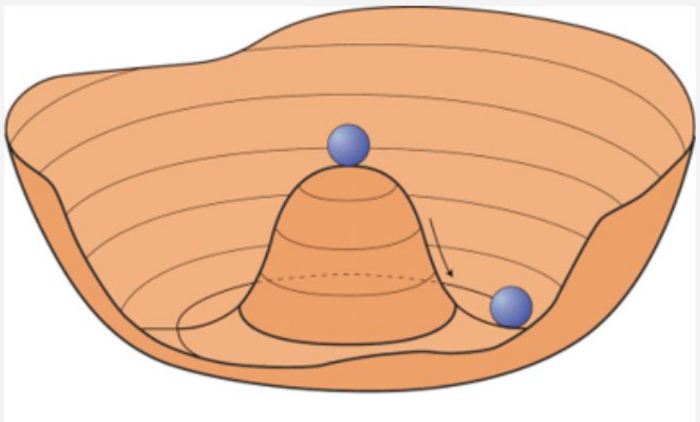
Axion Clumps meeting Neutron Stars

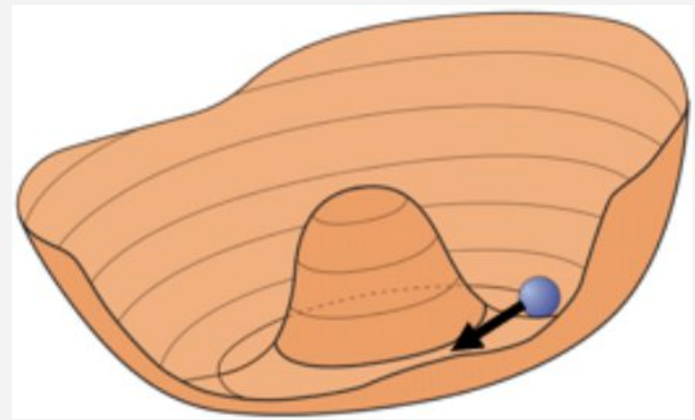
1710. 08910: Visinelli, SB, Redondo, Freese, Wilczek
2107.07399: Millar, SB, Lawson, Marsh
2212.08079: Witte, SB, Lawson, Marsh, Millar, Salinas

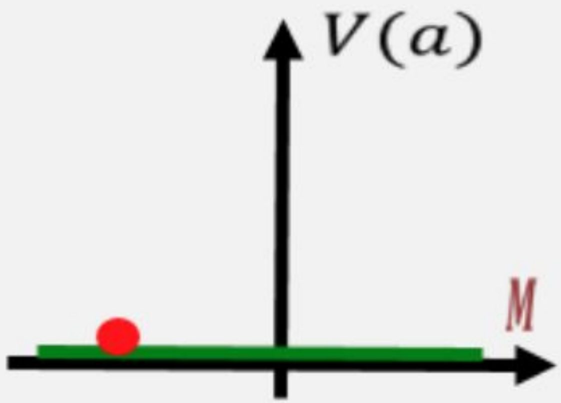
Sebastian Baum

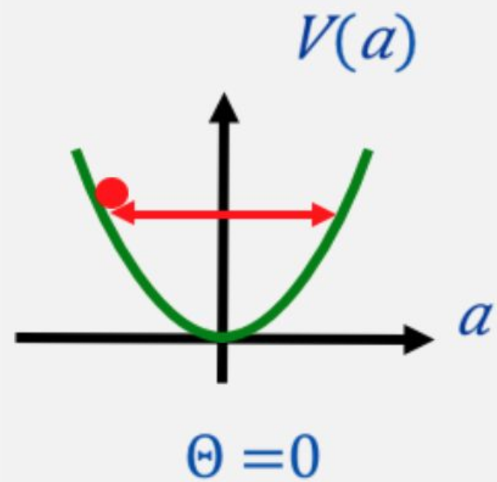
Axion Dark Matter Reminder: Re-alignment

- The axion field is "born" with random θ at the PQ phase transition
- At the QCD phase transition, the axion potential develops



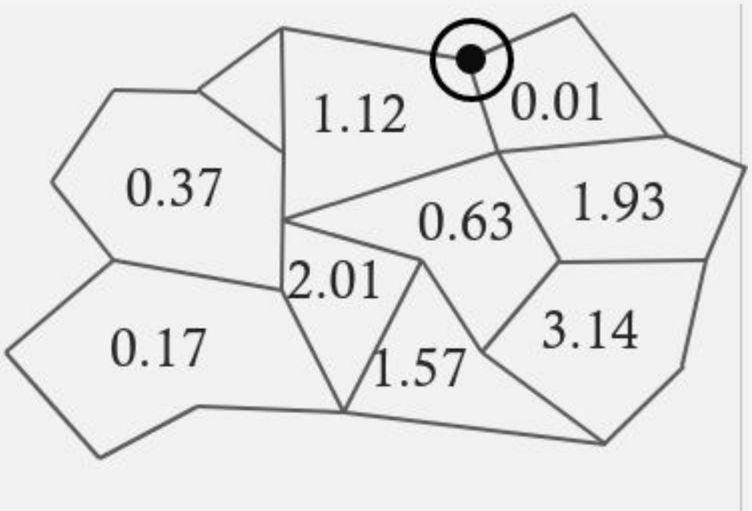






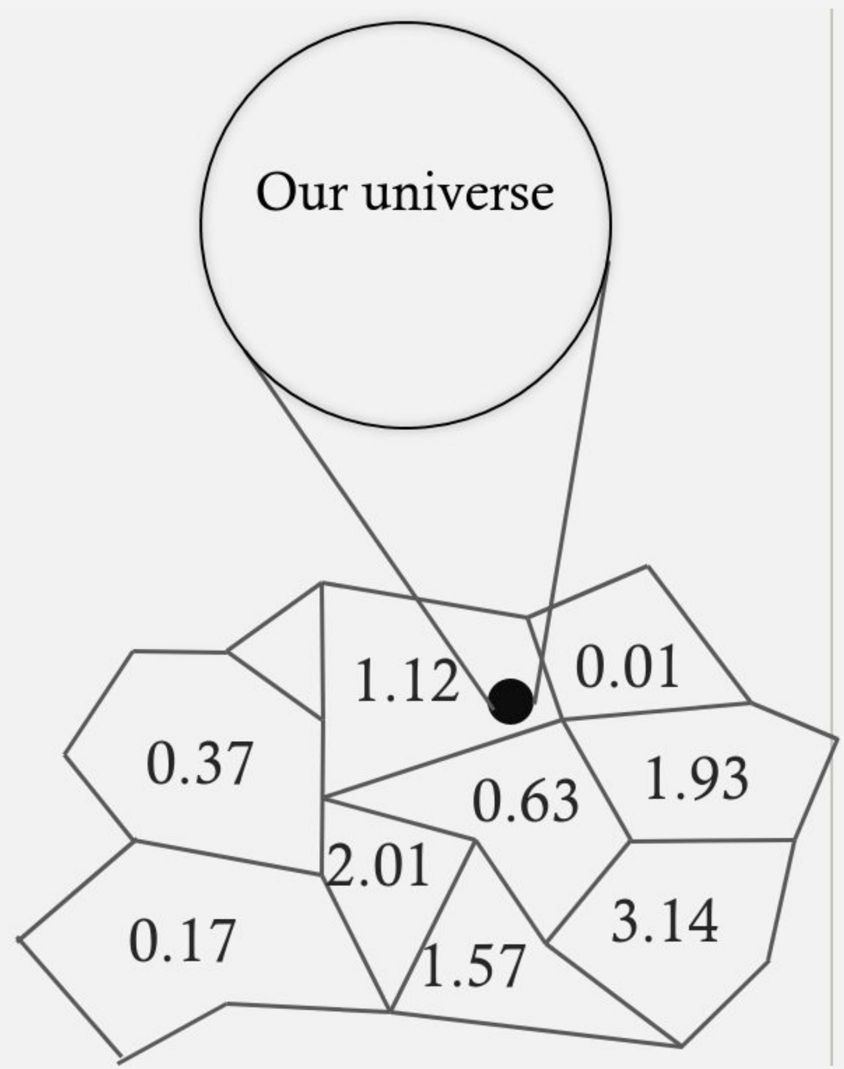
Axion DM: scenario 1

- PQ symmetry broken after the end of inflation
- Many patches with different misalignment angles in our Universe
- Topological defects (strings, domain walls, ...) exist in the early Universe

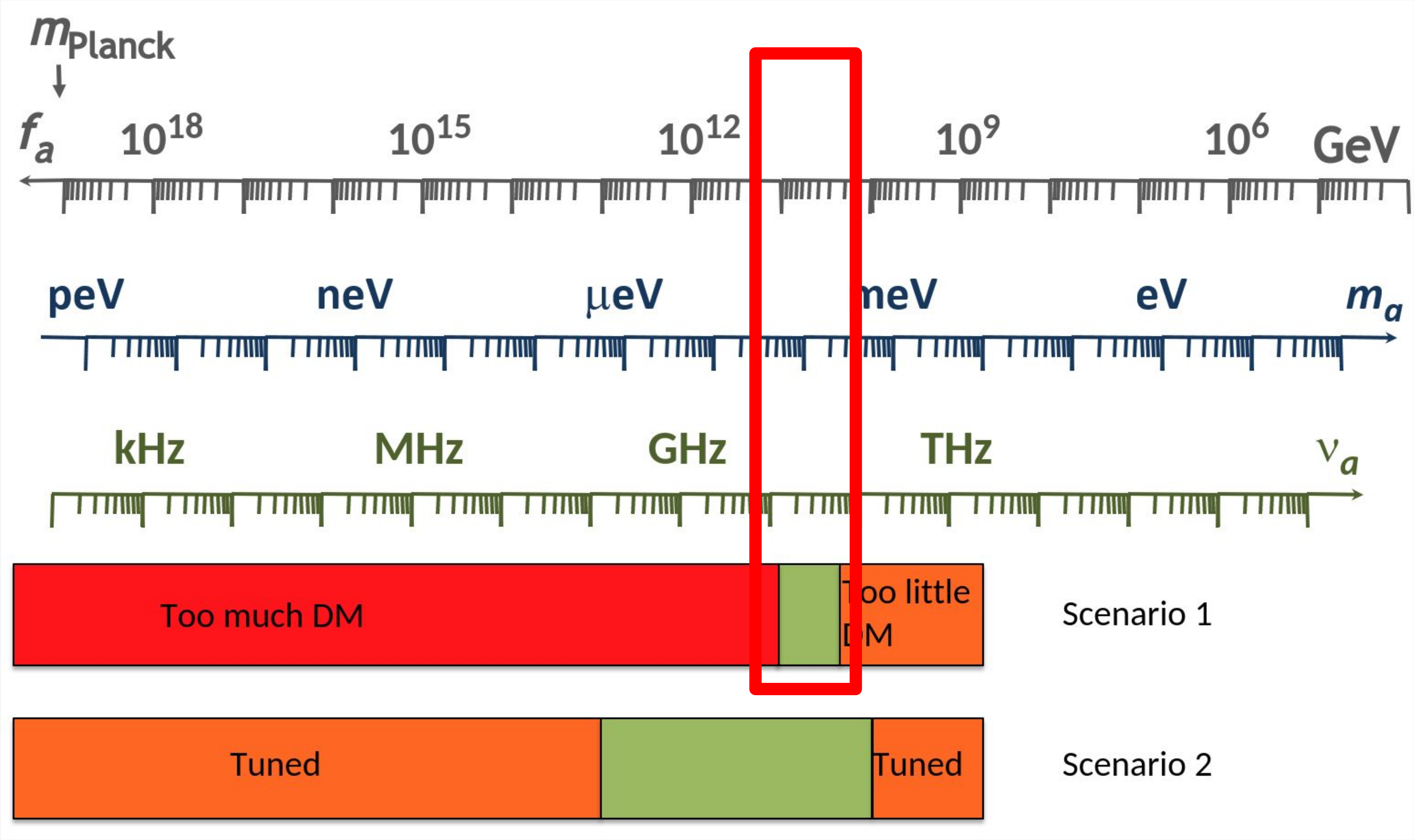


Axion DM: scenario 2

- PQ symmetry broken (sufficiently early) before (the end of) inflation
- One misalignment angle in the whole observable Universe
- No topological defects

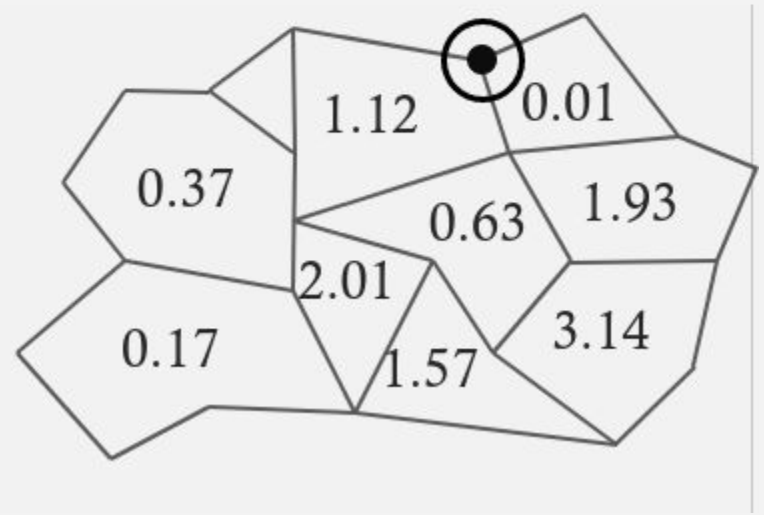


Axion Dark Matter



How much axion DM could be in compact objects?

In scenario 1 (PQ broken after inflation) the axion field in our Universe is "born" with O(1) inhomogeneities



- Topological defects decay when the axion mass turns on
- "Axitons" decay quickly after the QCD phase transition finishes
- Axion overdensities become gravitationally bound and form "minicluster" around matter-radiation equality

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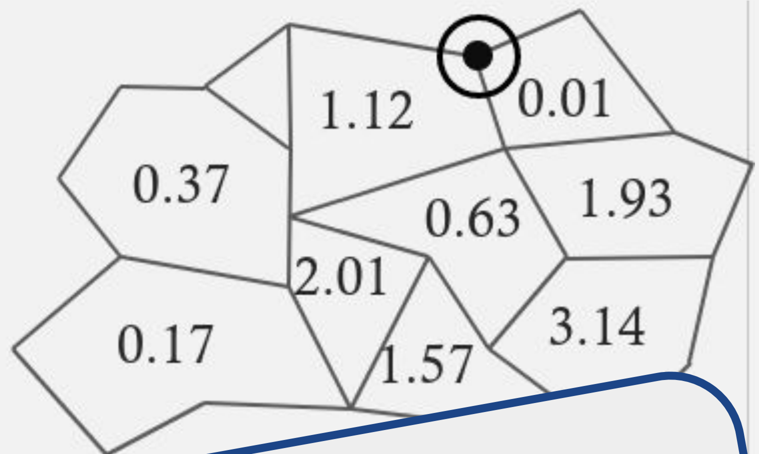


Most of axion DM could be in compact objects!

Levkov, Panin, Tkachev 1804.05857, Vaquero, Redondo, Stadler 1809.09241, Buschmann, Foster, Safdi 1906.00967, Eggemeier & Niemeyer 1906.01348, Eggemeier, Redondo, Dolag, Niemeyer, Vaquero 1911.09417, Eggemeier, O'Hare, Pierobon, Redondo, Wong 2212.00560, …

Axion Miniclusters, Stars, Axitons, ...

Bosonic Dark Matter can collapse to high densities (occupation numbers) and attain (hydrostatic) equilibrium

Relevant contributions:

- Kinetic Pressure (repulsive)
- Gravity (attractive)
- Self-interactions (both)

Summary I

In axion DM scenario 1 (PQ breaking after the end of inflation),

axion miniclusters and axion stars are ubiquitous

- Possible up to ~75% of axions bound in miniclusters
- At the center of miniclusters, axion stars form (quickly and efficiently!)
- Important open questions:
  - Evolution in hierarchical structure formation
  - Tidal disruption in Galaxies
  - Dynamical evolution of axion stars

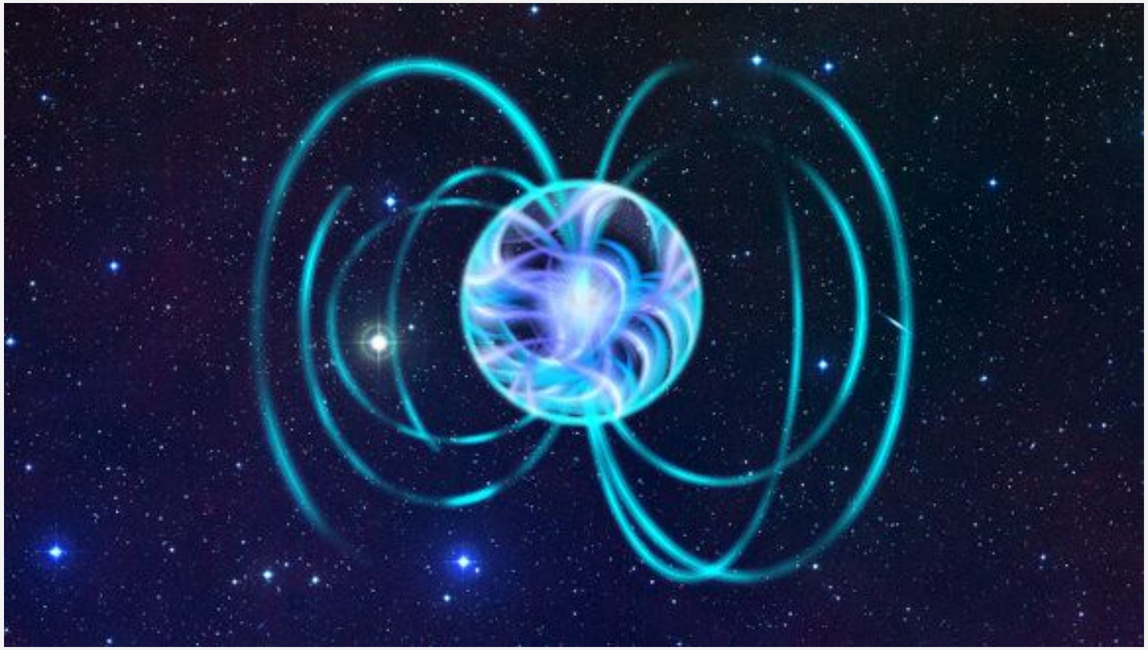
Dokuchaev, Eroshenko, Tkachev [1710.09586]
Kavanagh, Edwards, Visinelli, Weniger [2011.05377]

Fox, Weiner, Xiao [2302.00685]

- Typical mass, size, density:
  - Axion minicluster: $M \sim 10^{-12}\,M_\odot$, $R \sim 10^9$ km, $\rho \sim 10^3$ GeV/cm^3
  - Axion star: $M \sim 10^{-13}\,M_\odot$, $R \sim 10^4$ km, $\rho \sim 10^{17}$ GeV/cm^3

Enter the Neutron Star

- Radius: ~10 km
- Mass: ~ 1.4 $M_{\odot}$
- Magnetic field strength: ~ 10^{14} G = 10^{10} T
- Plasmas mass of photons $\lesssim$ 50 μeV



(Resonant) axion photon conversion

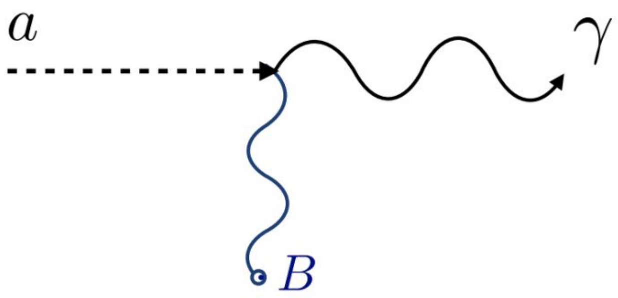
$$\mathcal{L} \sim g_{a\gamma\gamma} \, a \, E \cdot B$$

$$\downarrow$$

$$p_{a \to \gamma} \sim g_{a\gamma\gamma}^2 \, B^2 \, L^2$$

B: Magnetic Field

L: Length scale



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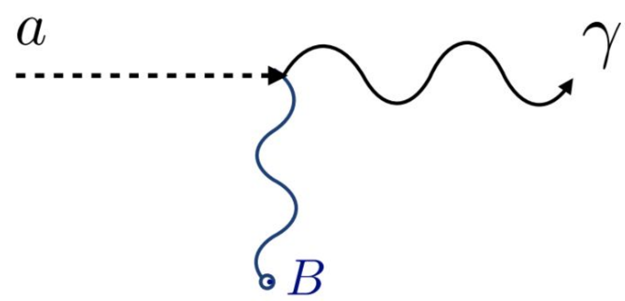

Two options for efficient axion-photon conversion of axion in astrophysical systems:

- Magnetic field with power at wavelength ~ axion mass
- Effective plasma mass of photons matching the axion mass

Enter the Neutron Star

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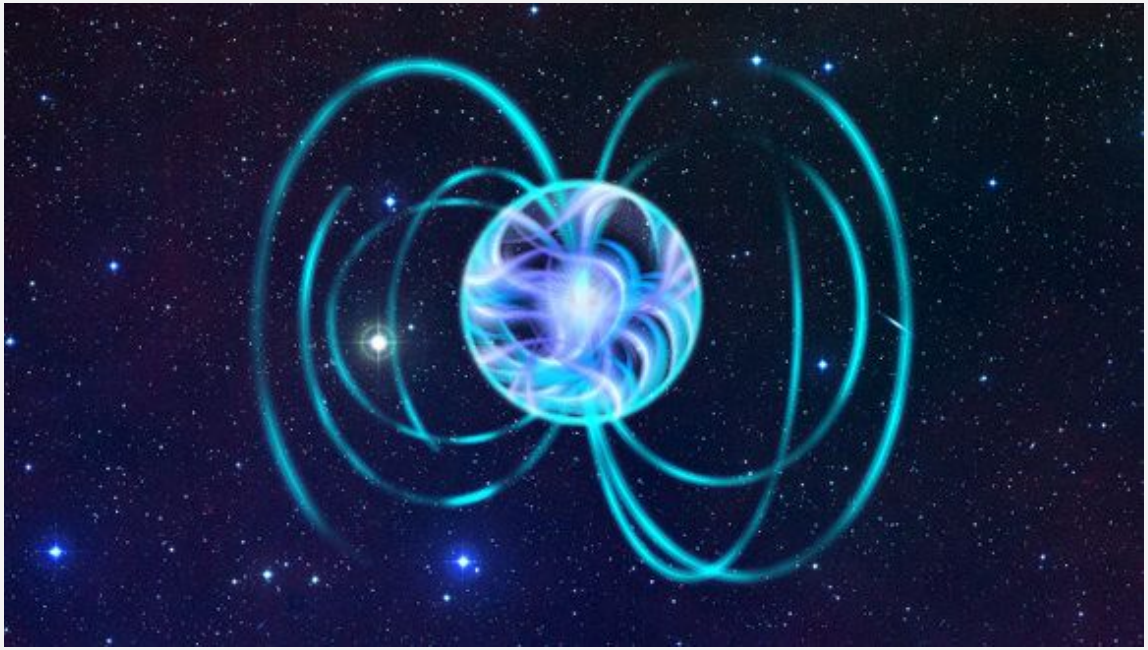
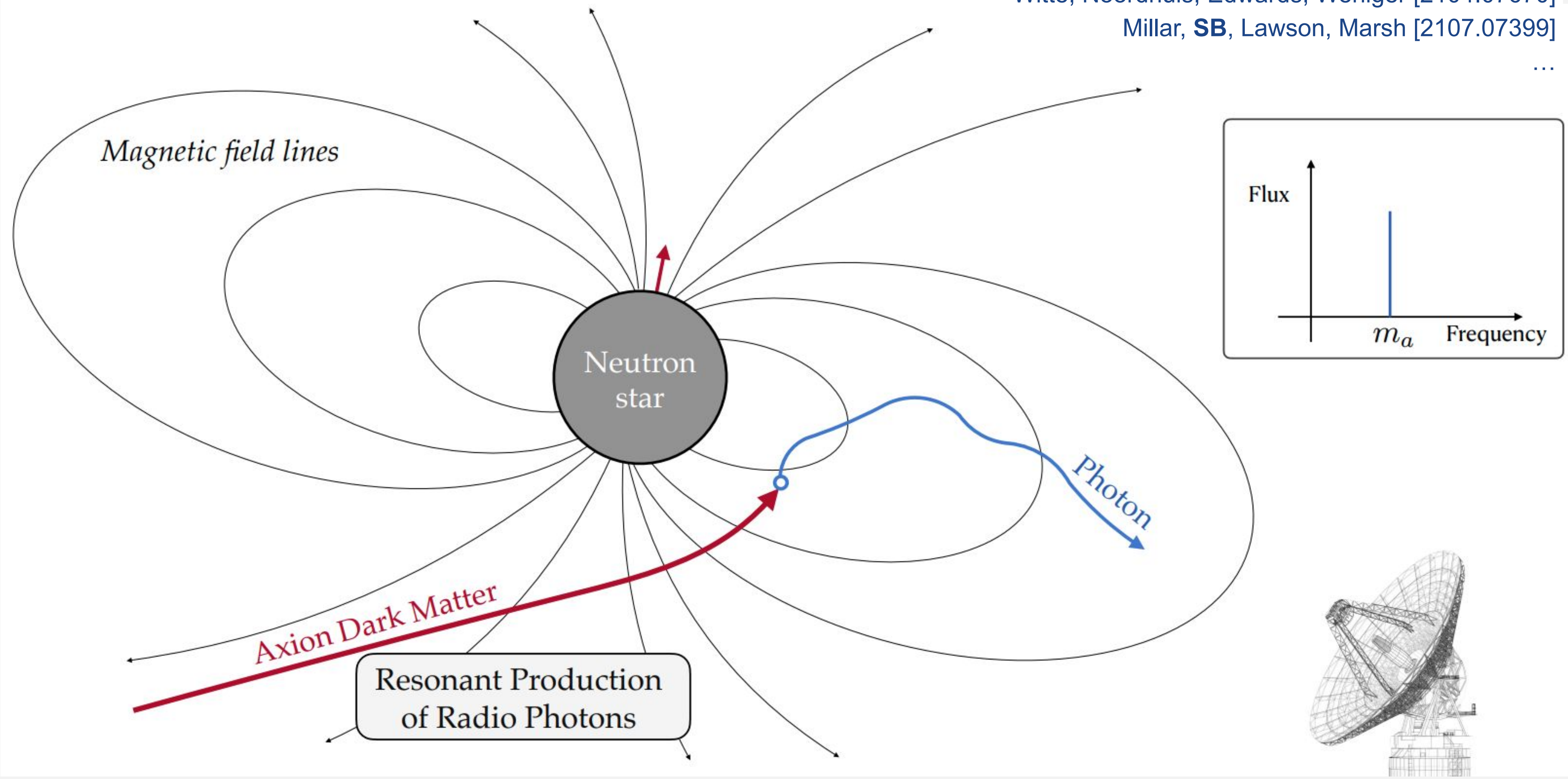


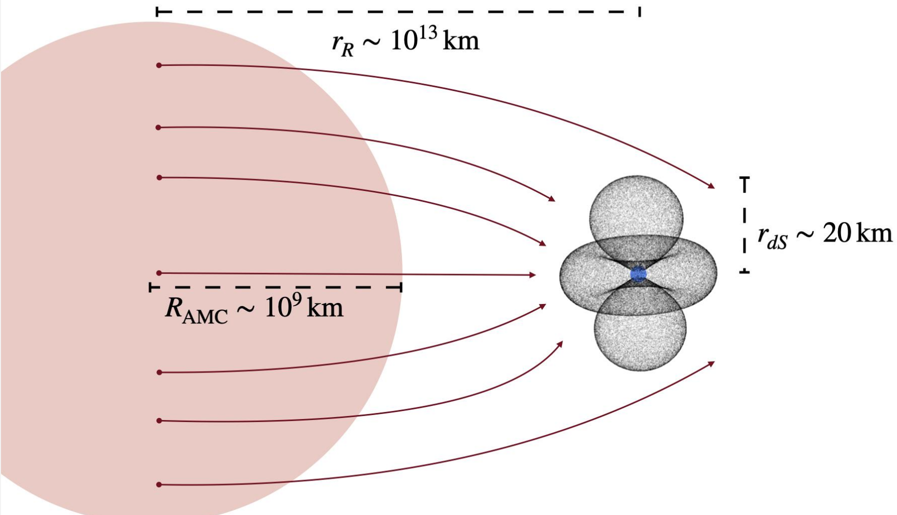
Axions meeting Neutron Stars

Pshirkov & Popov [0711.1264]
Hook, Kahn, Safdi, Sun [1804.03145]
…
Witte, Noordhuis, Edwards, Weniger [2104.07670]
Millar, **SB**, Lawson, Marsh [2107.07399]
…



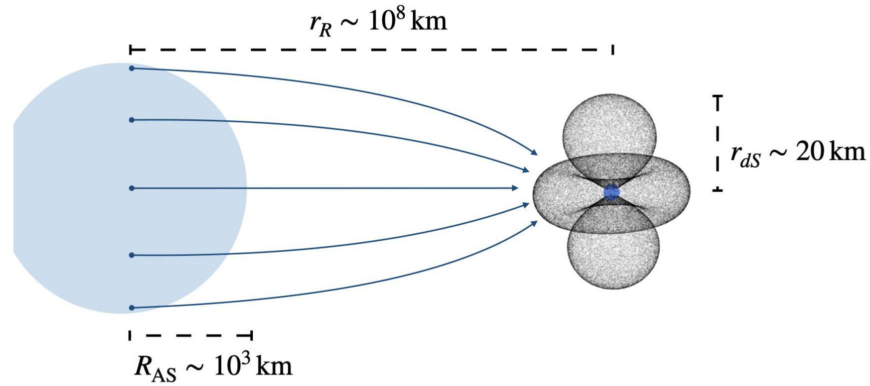
Axion Clumps & Neutron Stars

Axion Minicluster



Axion Star

Witte, **SB**, Lawson, Marsh, Millar, Salinas [2212.08079]



Axion Clumps & Neutron Stars

Three steps of the calculation:

- Gravitational infall of axions DM (see also Bai, Du, Hamada [2109.01222])
  - Treat axion clump as undisturbed until the Roche radius [$r_R = r_{AC} \times (2\, M_{NS}/M_{AC})^{1/3}$]
  - Treat axions as free particles in the NS potential insides of the Roche radius
- Axion-photon conversion
  - Use 3D resonant conversion calculation from Millar, **SB**, Lawson, Marsh [2107.07399]
- Tracing the "photons" out of the plasma
  - Ray-tracing calculation à la Witte, Noordhuis, Edwards, Weniger [2104.07670]

Evaluate the whole chain as a hybrid importance/weighted sampling Monte Carlo integral with trajectories drawn homogeneously over the resonant conversion surface/drawn from the axion velocity distribution at the Roche radius

Axion Clumps & Neutron Stars

Result of the calculation in Witte, SB, Lawson, Marsh, Millar, Salinas [2212.08079]:

For an axion clump – neutron star encounter, characterized by

- neutron star configuration ($M_{\rm NS}$, magnetic field, plasma),
- axion clump ($M_{\rm AC}$, $R_{\rm AC}$, density profile, velocity distribution),
- encounter geometry ($v_{\rm AC}$, $b_{\rm AC}$, θ_v),

we compute, as functions of direction and time:

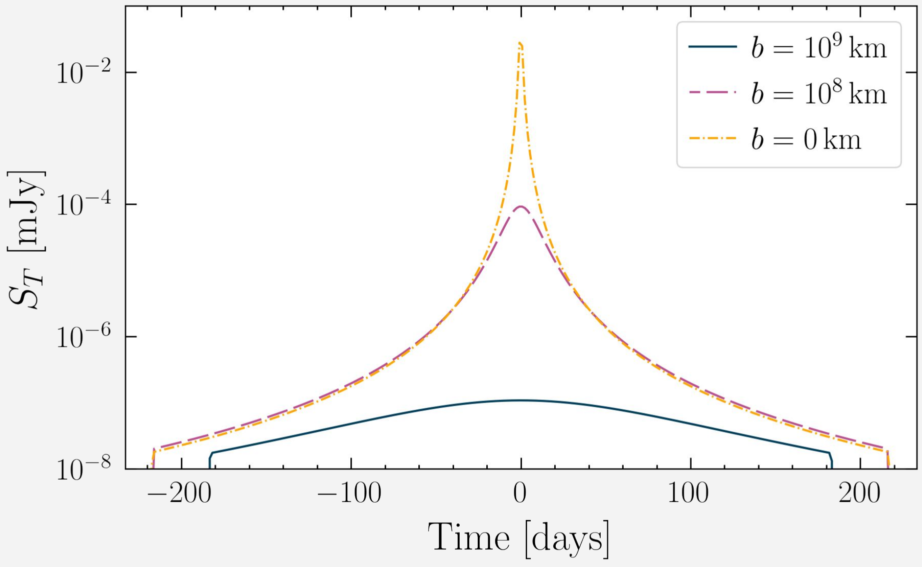
- Flux density at a telescope
- Characteristic linewidth

$$S_T(\theta,\phi) = \frac{dP}{d\Omega}(\theta,\phi)\,\frac{1}{\delta f_T \,\times\, d_T^2}$$

$$f_\sigma = \left[\frac{\sum_{j\in\mathrm{pixel}} (E_j - \bar{E})^2 \, R_{a\to\gamma,j}}{\bar{E}^2 \sum_{j\in\mathrm{pixel}} R_{a\to\gamma,j}}\right]^{1/2}$$

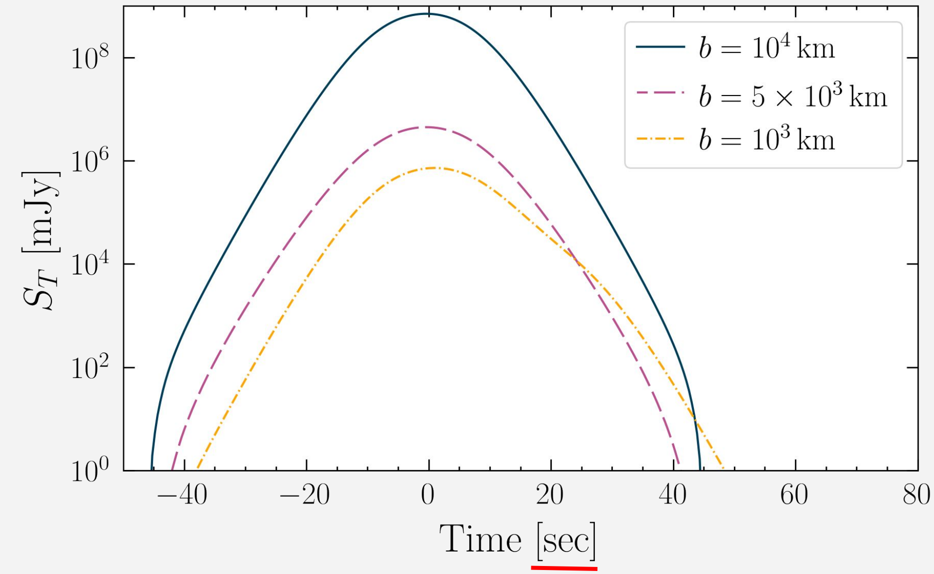
How long does the signal last?

Axion Minicluster



Axion Star

Witte, **SB**, Lawson, Marsh, Millar, Salinas [2212.08079]



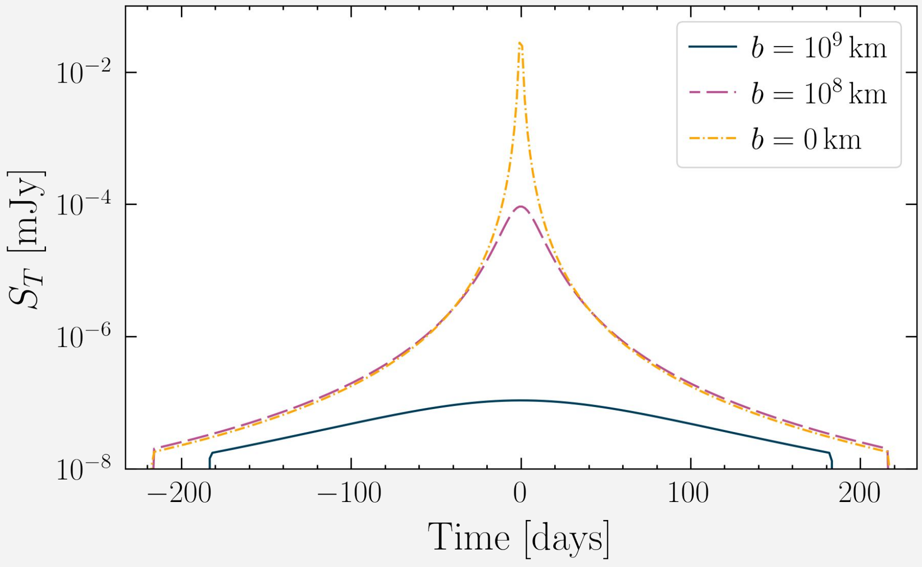
m_a	$26\,\mu\mathrm{eV}$
$g_{a\gamma\gamma}$	$10^{-14}\,\mathrm{GeV}^{-1}$
B_0	$1.6\times10^{14}\,\mathrm{G}$
P	$3.76\,\mathrm{s}$
θ_m	$0.2\,\mathrm{rad}$
θ_v	$0.0\,\mathrm{rad}$
δf_T	$10^{-5}\times m_a$
d_T	$1\,\mathrm{kpc}$

M_{AMC}	$10^{-12}\,M_\odot$
R_{AMC}	$1.86\times10^9\,\mathrm{km}$
M_{AS}	$10^{-13}\,M_\odot$
R_{AS}	$3905\,\mathrm{km}$
ρ_{AMC}	NFW
b_{AMC}	$10^8\,\hat{x}\,\mathrm{km}$
b_{AS}	$2\times10^3\,\hat{y}\,\mathrm{km}$
$\lvert v_{\mathrm{AMC/AS}}\rvert$	$100\,\mathrm{km/s}$

How long does the signal last?

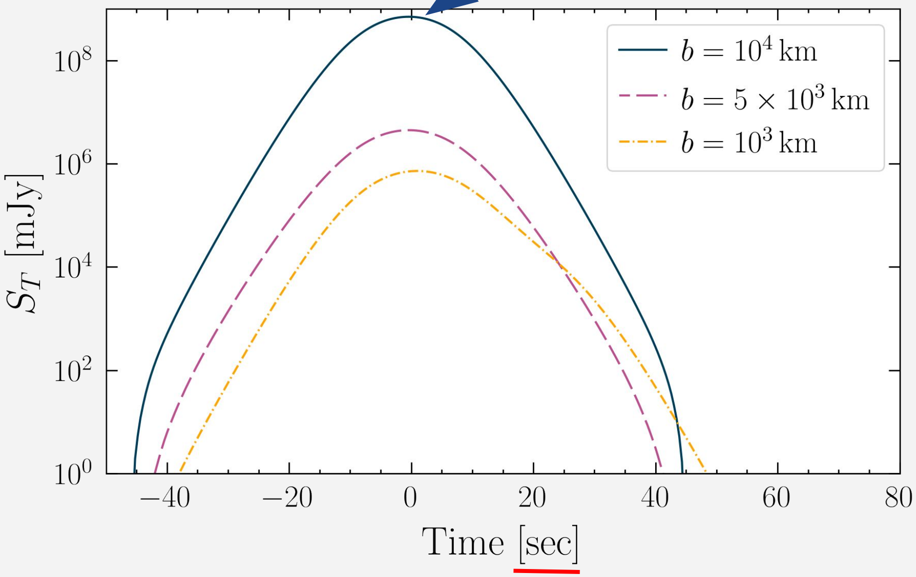
Still 10 mJy at 10 Mpc!

Axion Minicluster



Axion Star

Witte, **SB**, Lawson, Marsh, Millar, Salinas [2212.08079]



m_a	$26\,\mu$eV
$g_{a\gamma\gamma}$	10^{-14} GeV^{-1}
B_0	1.6×10^{14} G
P	3.76 s
θ_m	0.2 rad
θ_v	0.0 rad
δf_T	$10^{-5} \times m_a$
d_T	1 kpc

$M_{\rm AMC}$	$10^{-12}\,M_\odot$
$R_{\rm AMC}$	1.86×10^9 km
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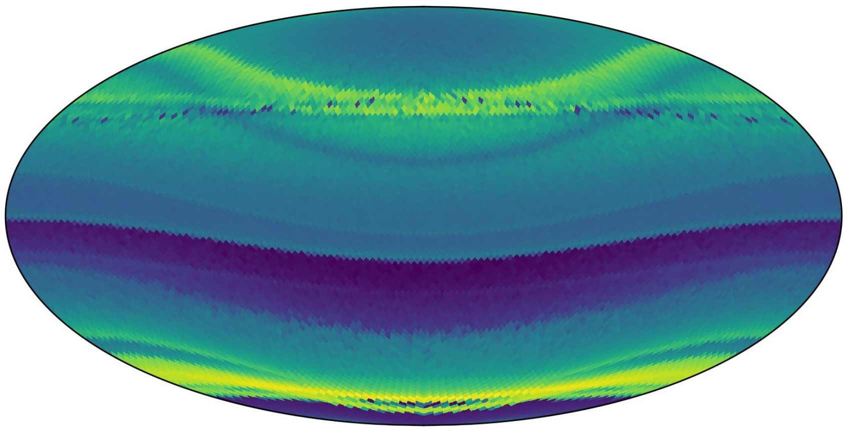
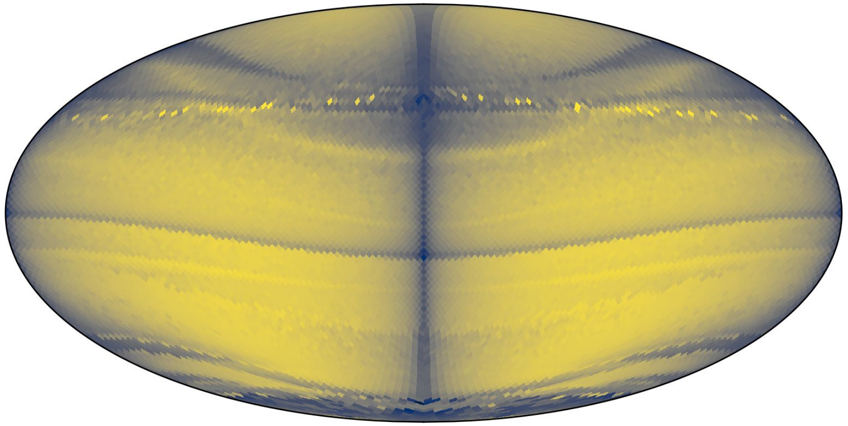
$\lvert v_{\rm AMC/AS}\rvert$	100 km/s

Minicluster - Neutron Star Collision





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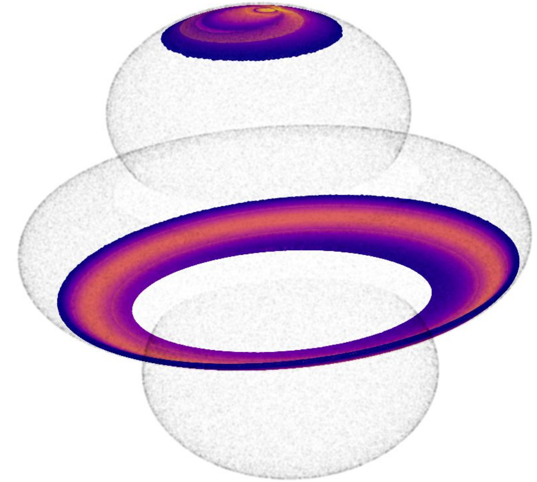
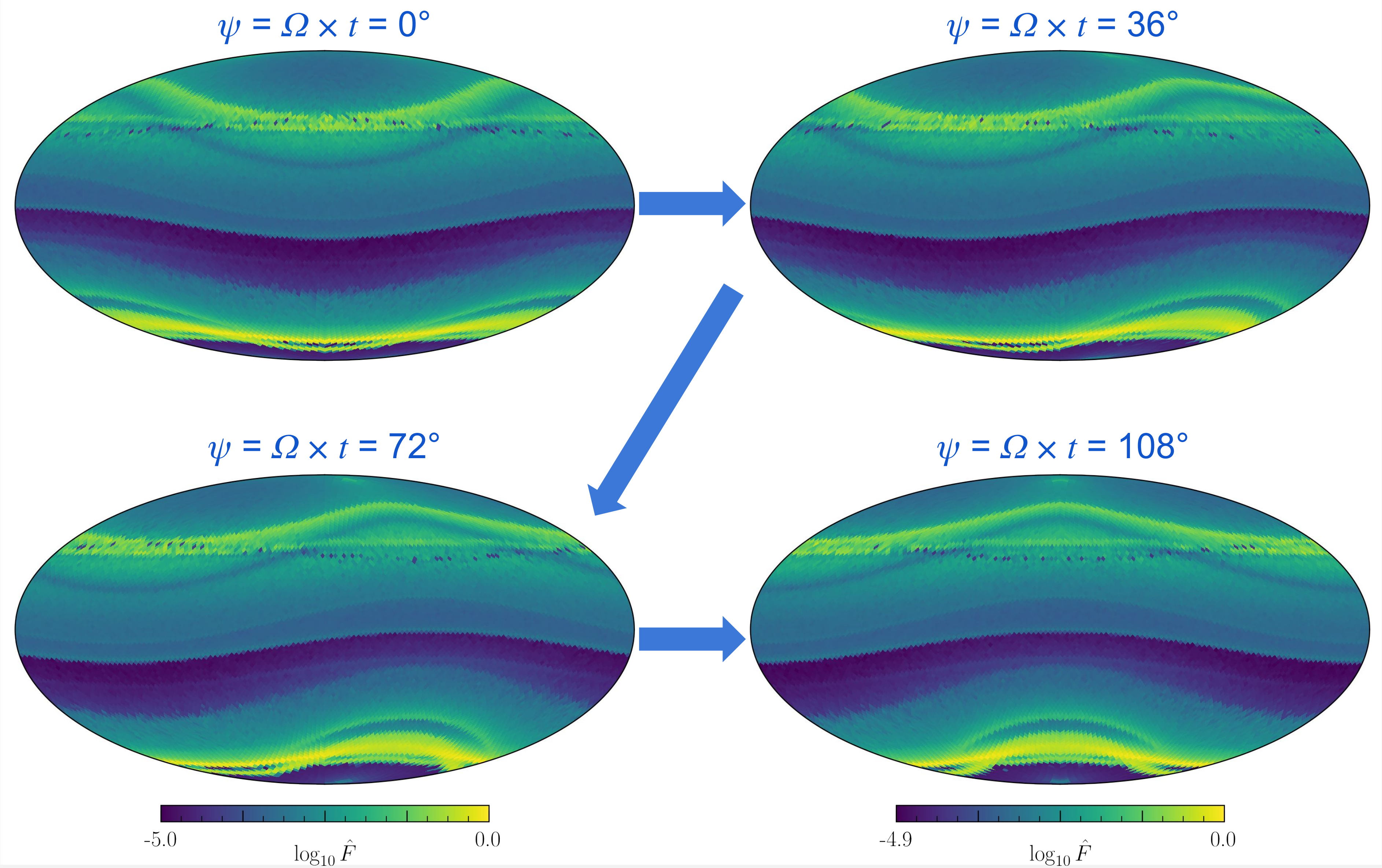
θ_m	$0.2\,\mathrm{rad}$		ρ_{AMC}	NFW
θ_v	$0.0\,\mathrm{rad}$		b_{AMC}	$10^{8}\,\hat{x}\,\mathrm{km}$
δf_T	$10^{-5}\times m_a$		b_{AS}	$2\times10^{3}\,\hat{y}\,\mathrm{km}$
d_T	$1\,\mathrm{kpc}$		$\lvert v_{\mathrm{AMC/AS}}\rvert$	$100\,\mathrm{km/s}$



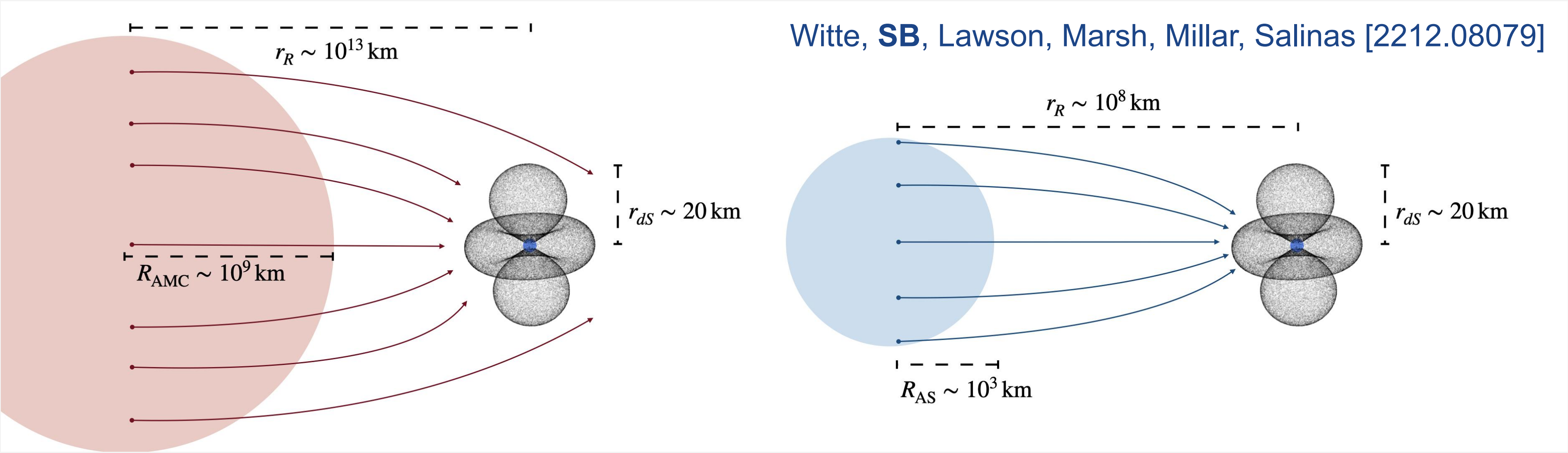
Minicluster - Neutron Star Collision: NS rotation



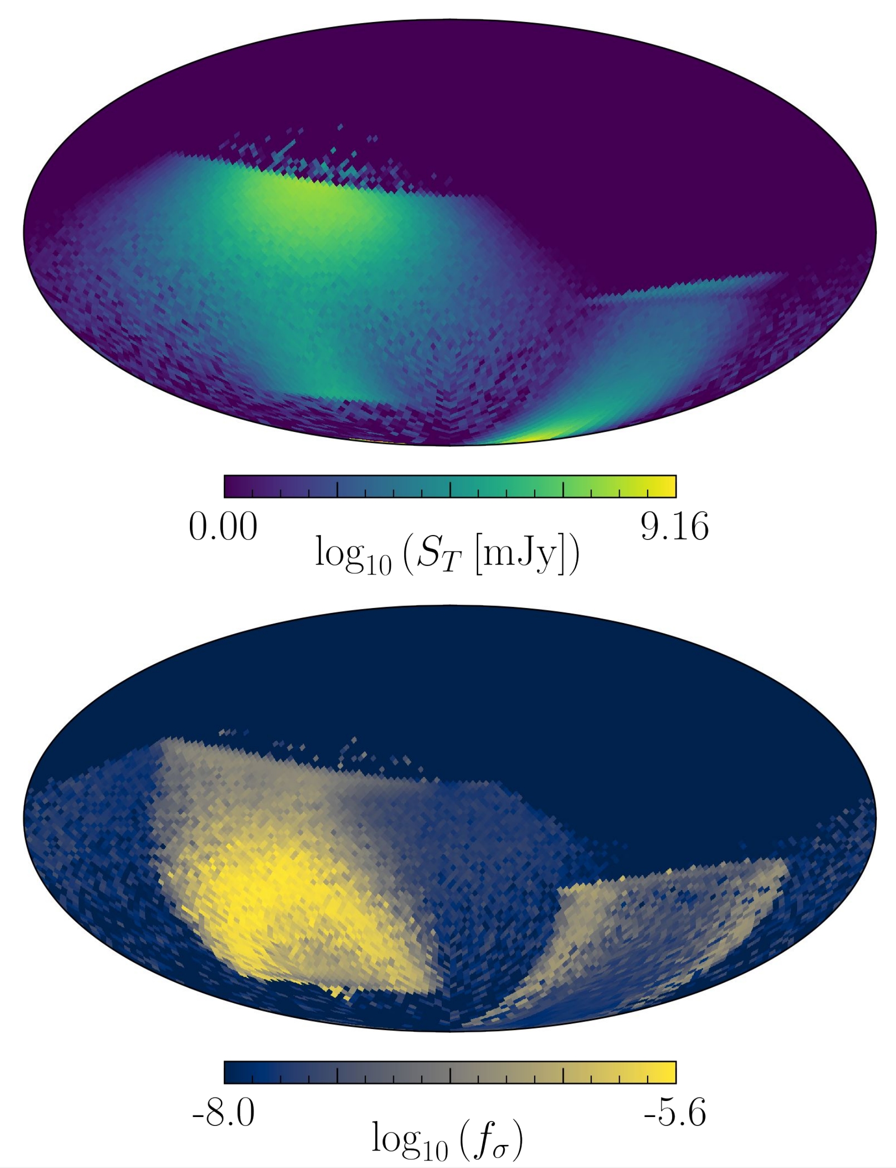
Axion Clumps & Neutron Stars

Axion Minicluster

Axion Star



Axion Star - Neutron Star Collision



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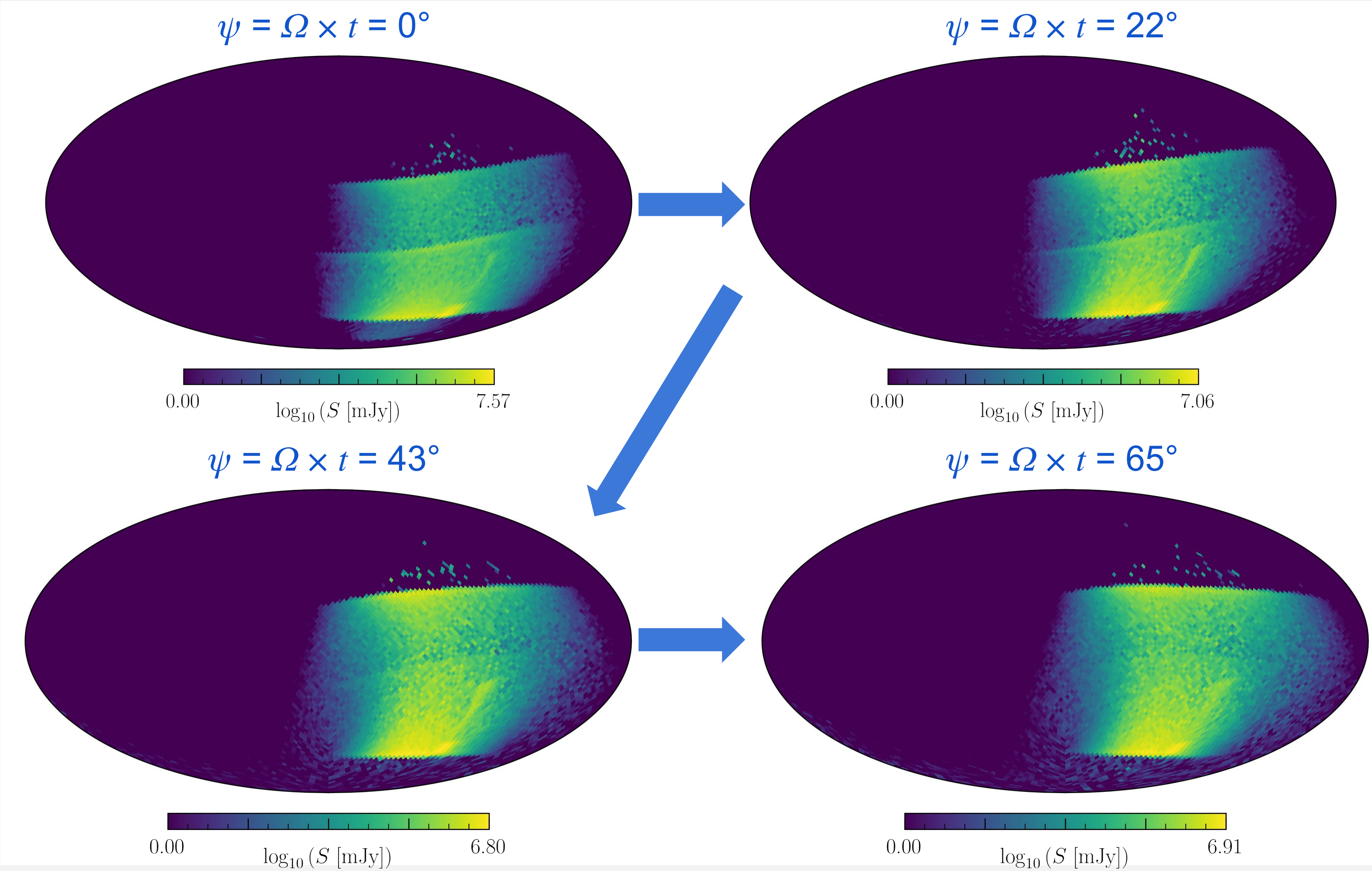
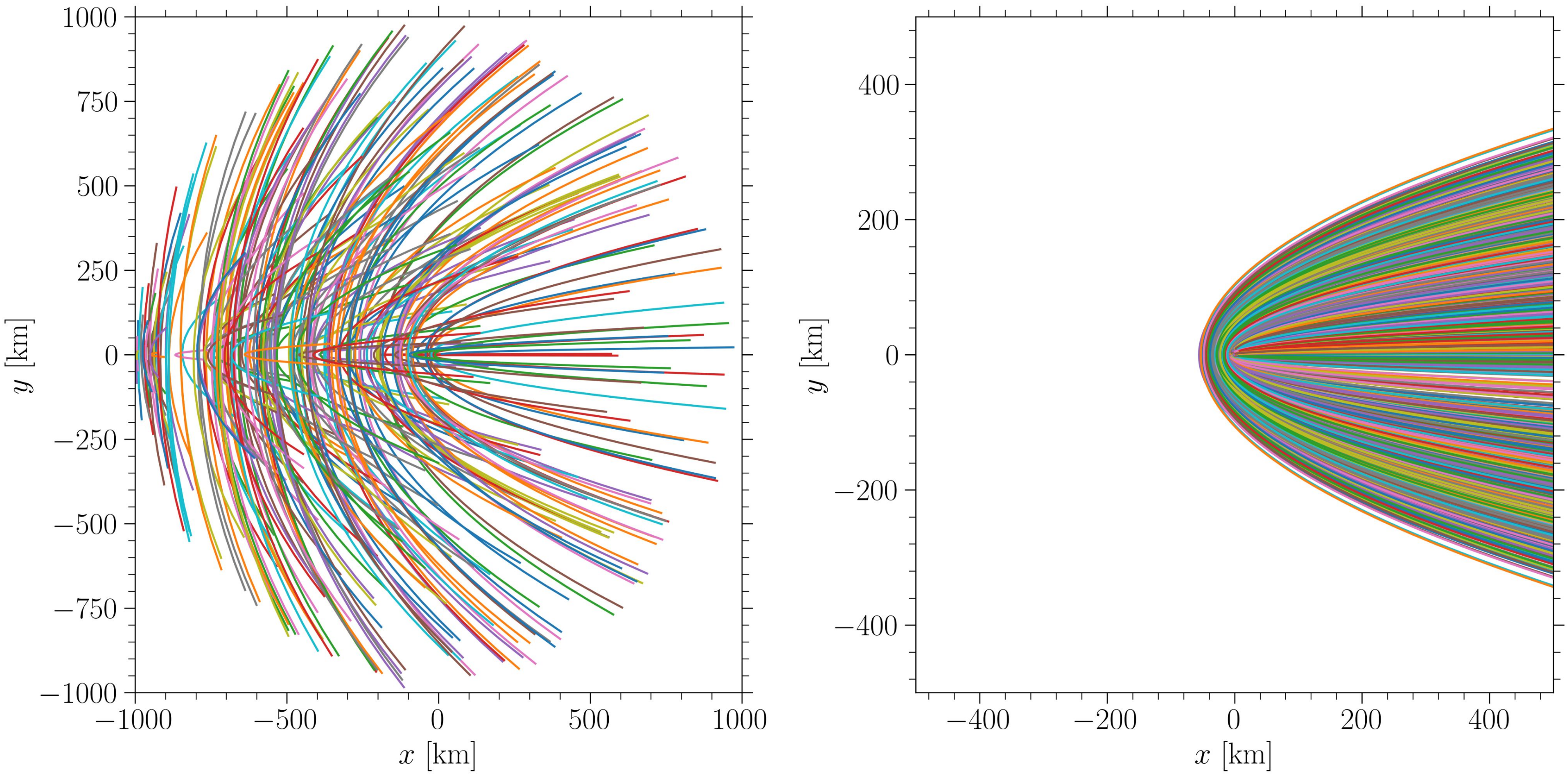
$\lvert v_{\mathrm{AMC/AS}} \rvert$	$100\ \mathrm{km/s}$

Axion Star - Neutron Star Collision: NS rotation



Summary II

- Axion clump - Neutron Star collisions would generate spectacular radio signals
  - month-long signal for a $10^{-12}\,M_\odot$ AMC – NS collision
  - minutes for a $10^{-13}\,M_\odot$ AS – NS collision
  - AS – NS collisions could be bright enough to be observable anywhere within the Local Group
- Might be the only way to search for axion clumps
- Where and how to look? Population study needed. Inputs:
  - Neutron star population model
  - Axion minicluster mass function (+ profiles, distributions in galaxies, …)
  - Axion star mass function (+ profiles, distributions in galaxies, …)


1000
750
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y [km]
−1000
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0
500
1000
x [km]
400
200
0
−200
−400
y [km]
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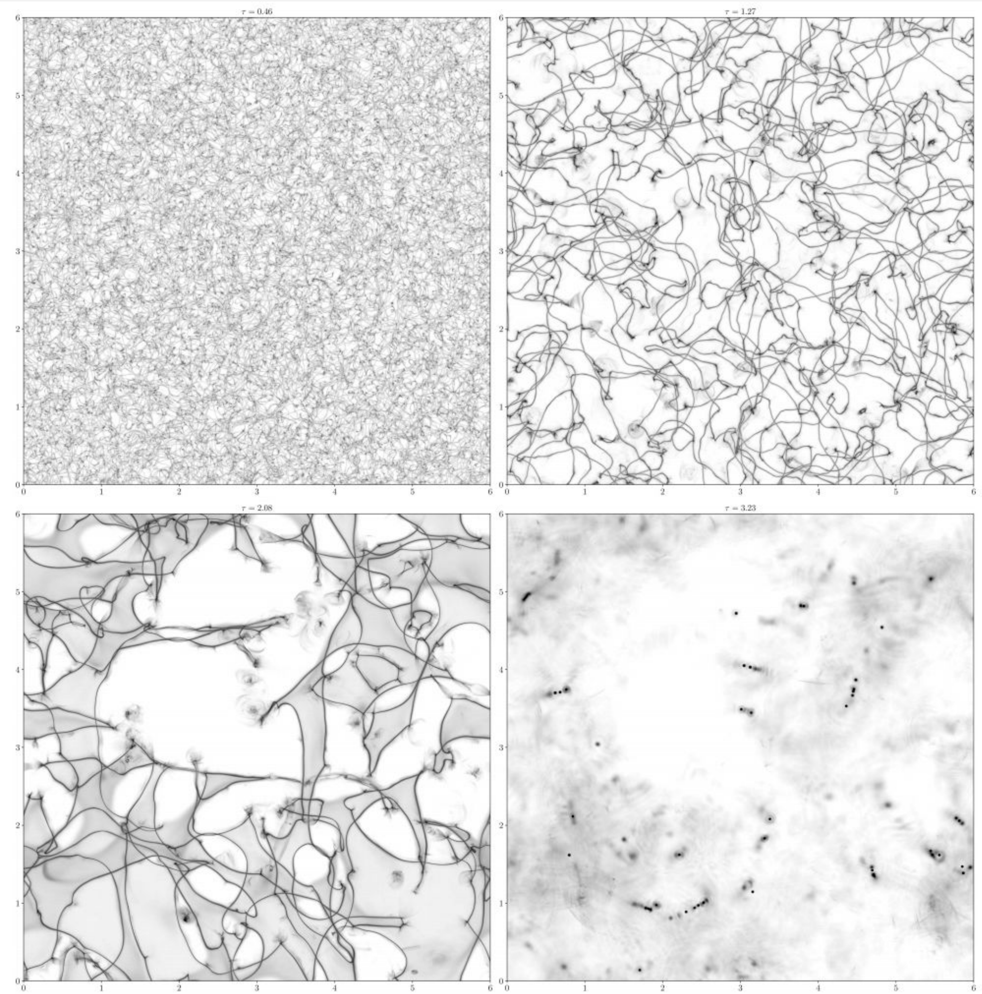
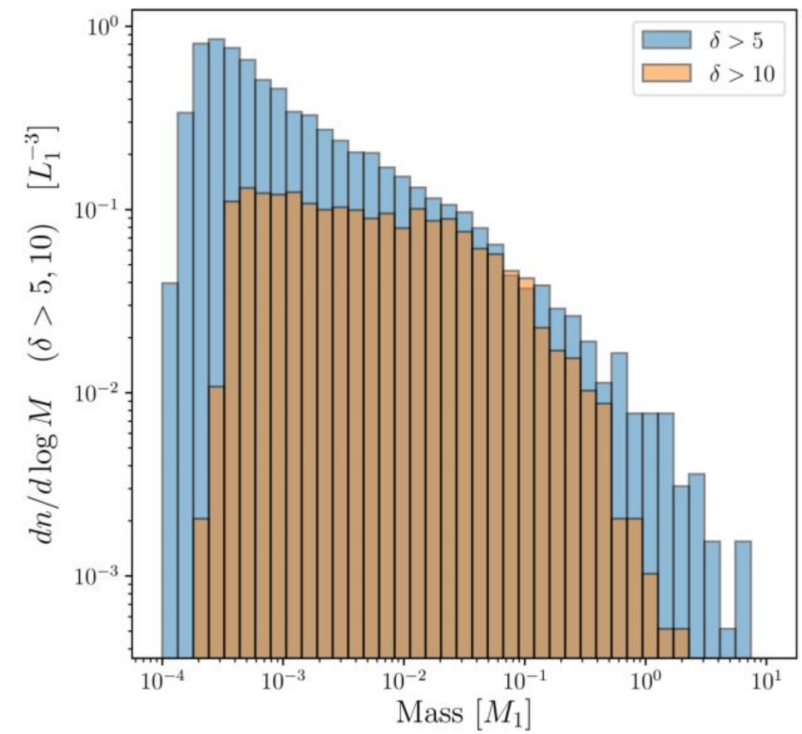
−200
0
200
400
x [km]

How much axion DM could be in compact objects?

Vaquero, Redondo, Stadler [1809.09241]



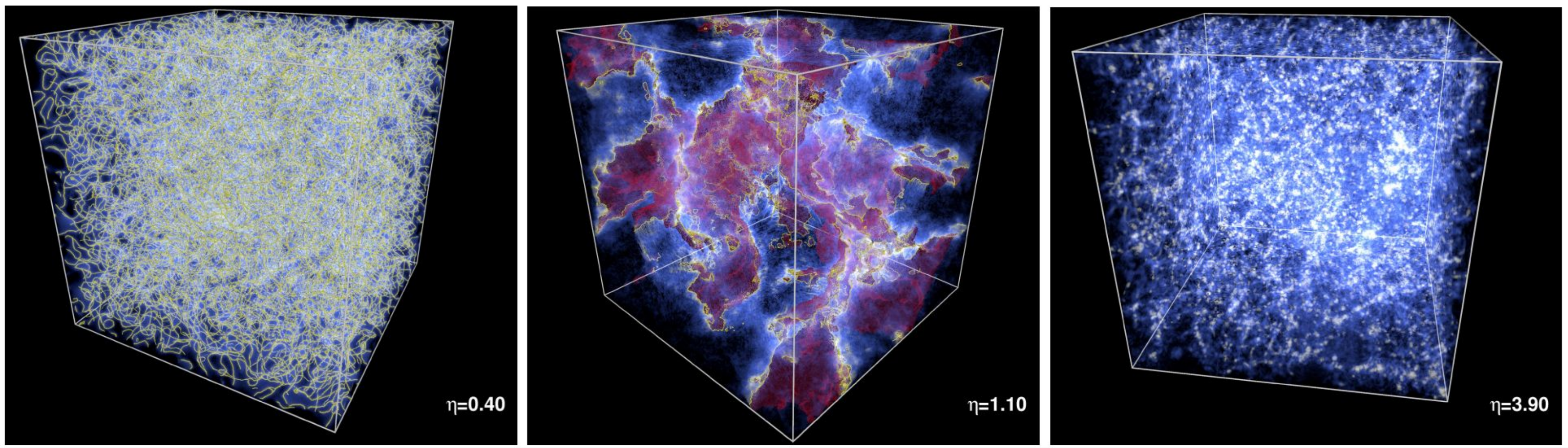
Minicluster “seed” mass function well after the end of the QCD phase transition

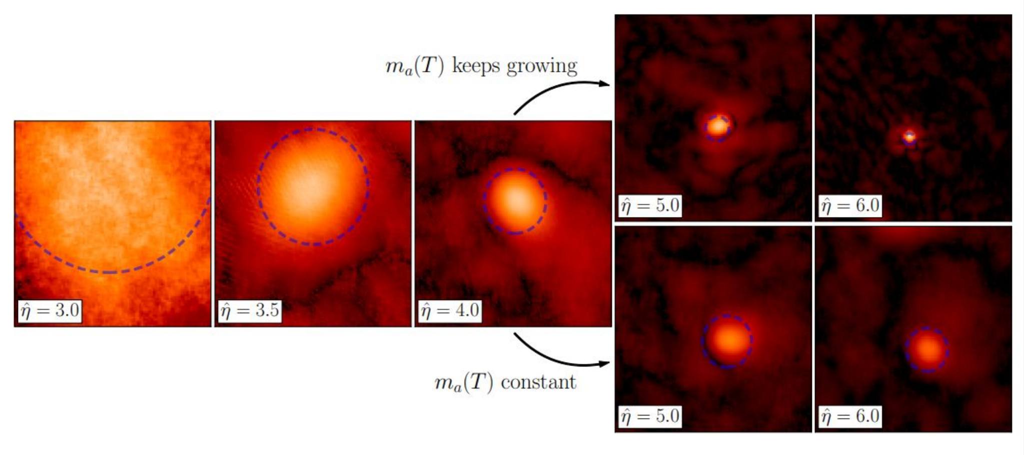


$$M_1 = 2.1 \times 10^{-12} M_\odot \, \frac{\Omega_{Ac} h^2}{0.12} \left(\frac{50\,\mu\text{eV}}{m_a} \right)^{0.49}$$

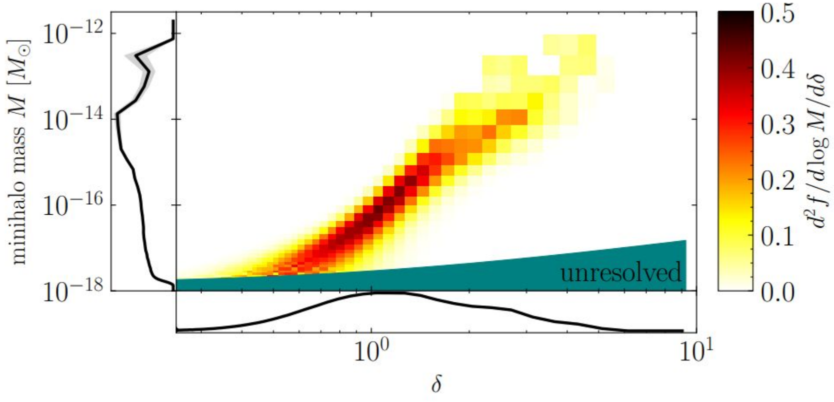
How much axion DM could be in compact objects?

Buschmann, Foster, Safdi [1906.00967]





Minicluster "seed" mass function at matter-radiation equality



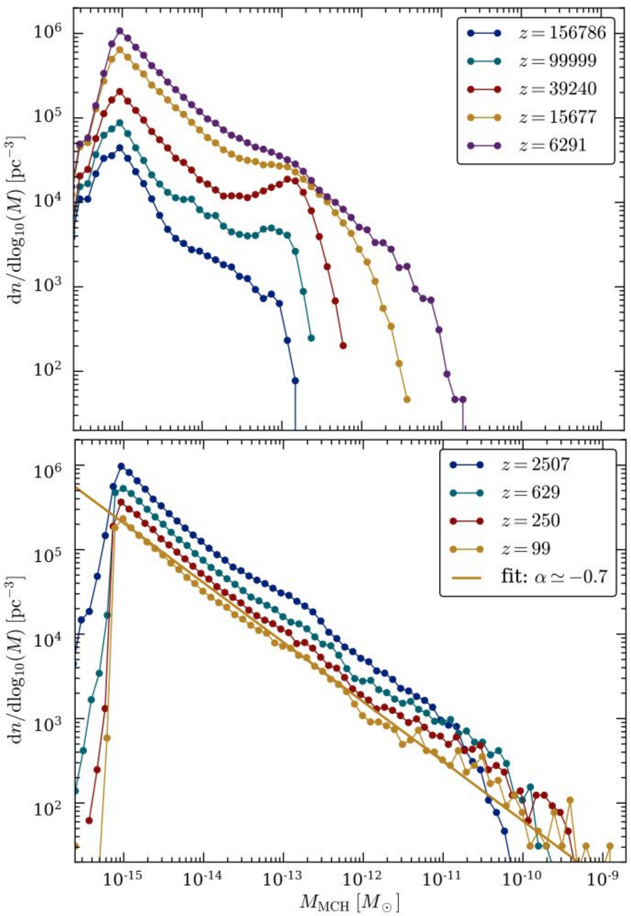
How much axion DM could be in compact objects?

Eggemeier, Redondo, Dolag, Niemeyer, Vaquero [1911.09417]

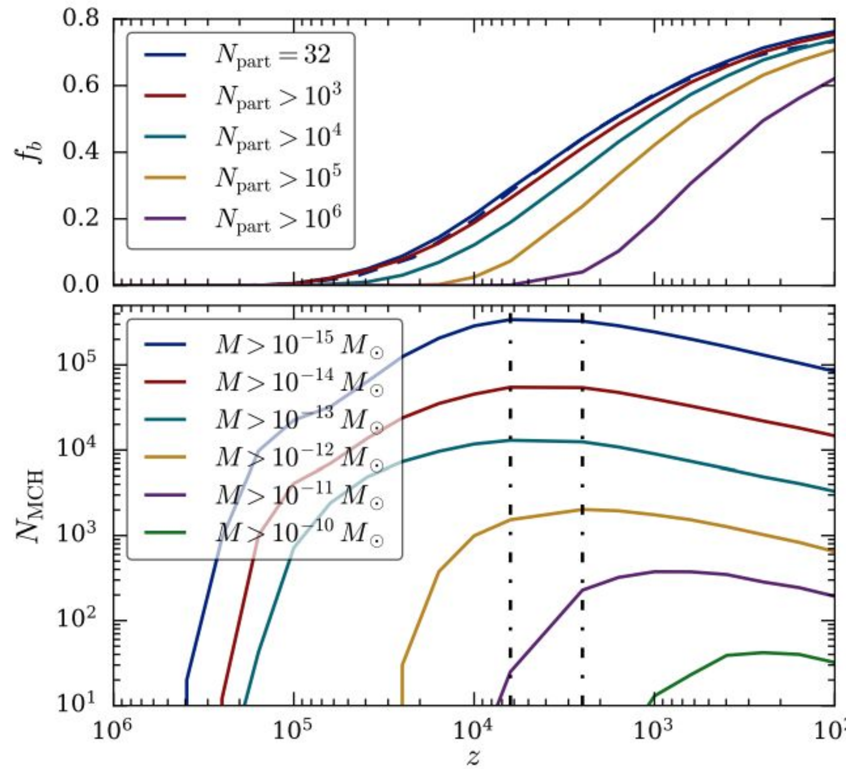
Axion minicluster mass function

& before matter-radiation equality

After

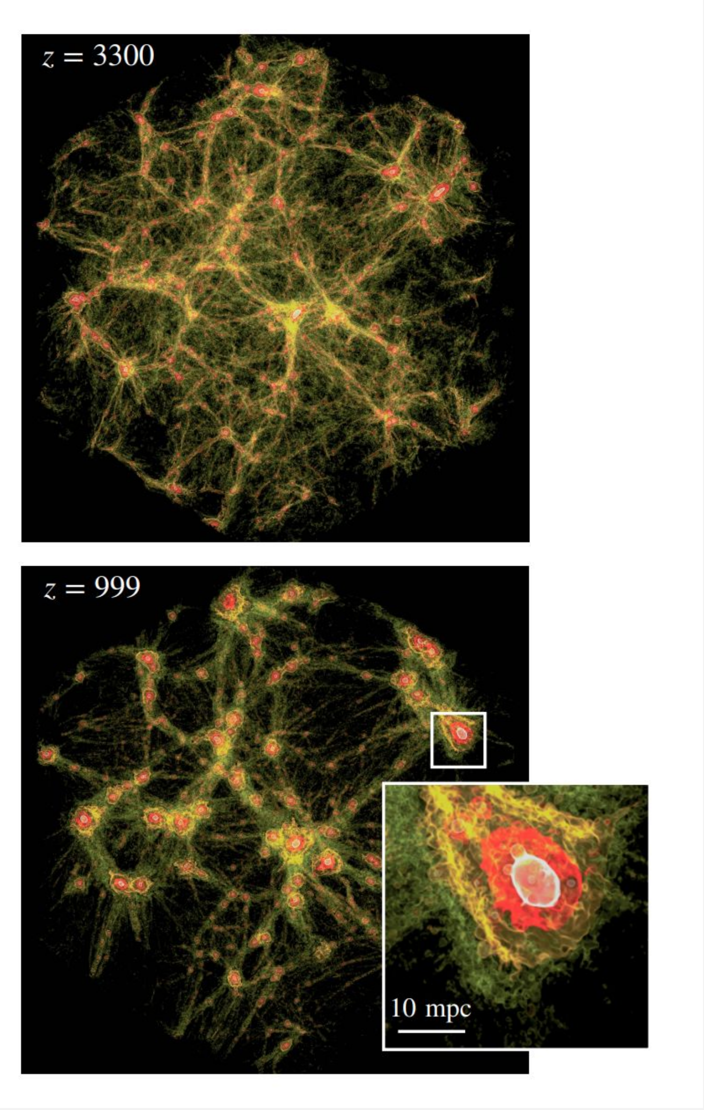


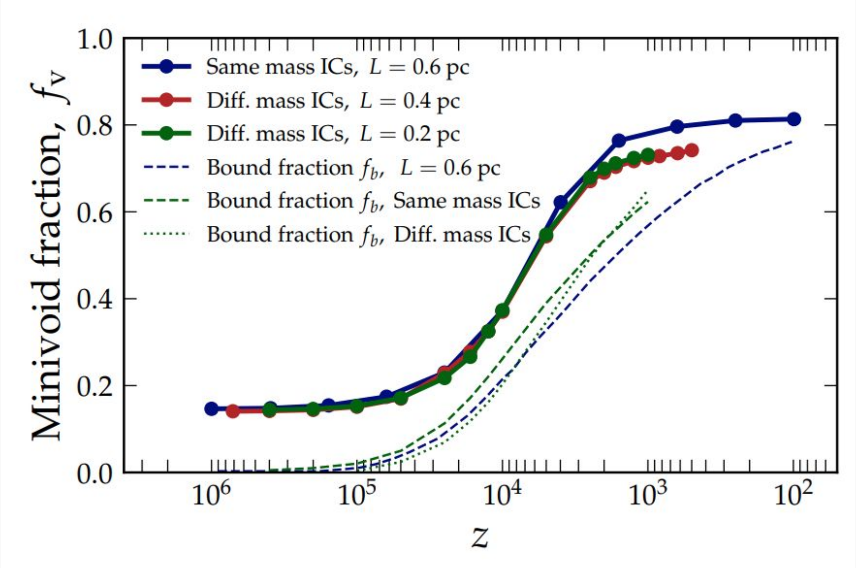
Bound fraction of axions



How much axion DM could be in compact objects?

Eggemeier, O'Hare, Pierobon, Redondo, Wong [2212.00560]





Axion Stars

- Dynamics of axion field ($a = f\,\theta$) controlled by action

$$S = \int d^4x \sqrt{-g} \left[\frac{f^2}{2} \left(\partial^\mu \theta\right) \left(\partial_\mu \theta\right) - V(\theta) \right]$$

- metric $g^{\mu\nu}$ determined by energy-momentum tensor of axion field
- Potential from chiral perturbation theory:

$$V(\theta) = \frac{\Lambda^4}{c_z} \left[1 - \sqrt{1 - 4c_z \sin^2(\theta/2)} \right] \quad \text{with} \quad c_z \approx \frac{m_u/m_d}{(1 + m_u/m_d)^2}$$

- First order self-interaction (quartic) is attractive!

Single harmonic approximation

- Approximation for $|\theta| \ll 1$

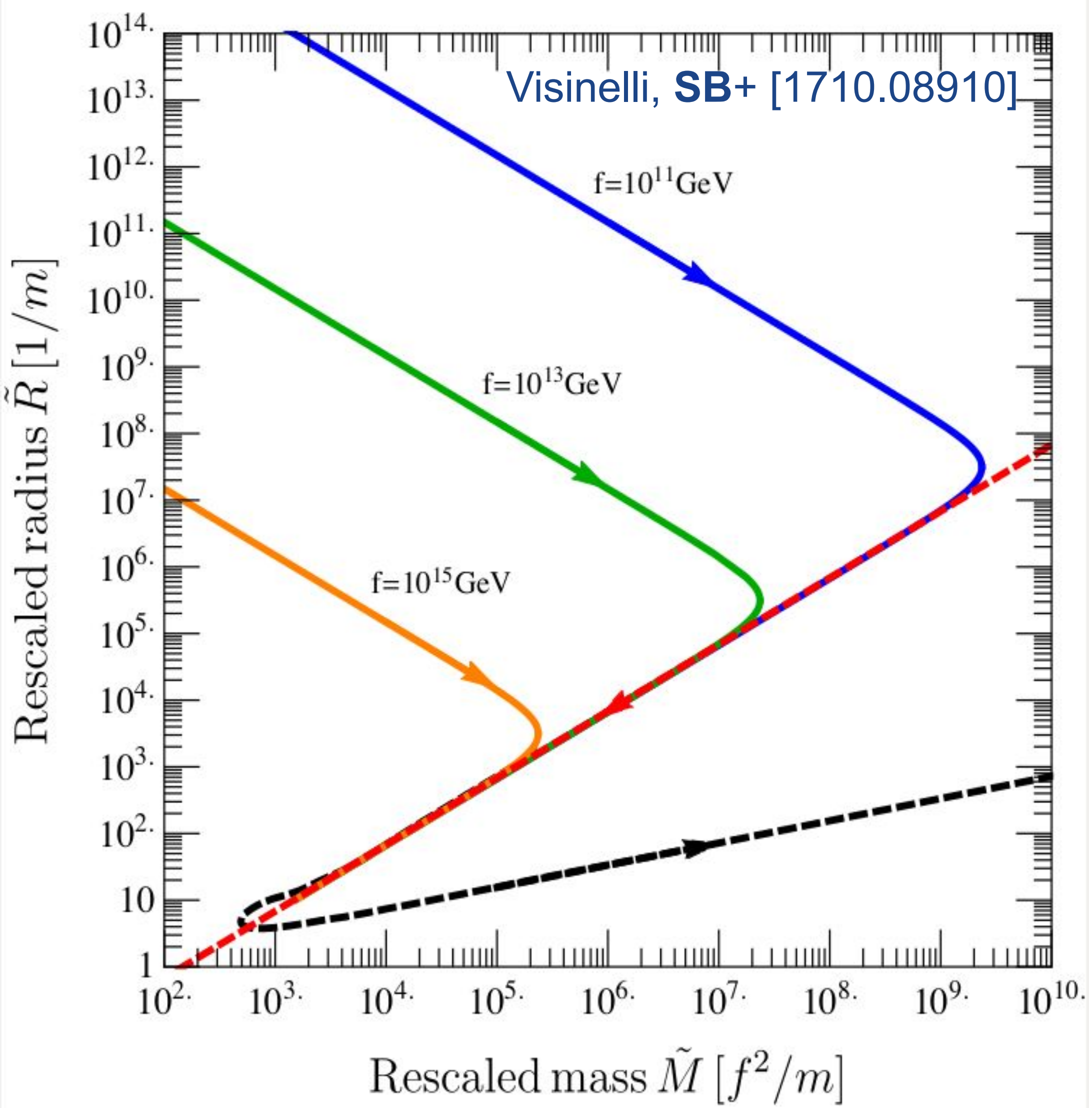
$$\theta = \Theta(r)\cos(\omega t)$$

- average over time-period

$2\pi\omega$

Units for mass and radius:

$$\frac{f^2}{m} = 3\times10^{-20} M_\odot \left(\frac{10^{-5}\,\mathrm{eV}}{m}\right)^3,$$
$$\frac{1}{m} = 3\times10^{-11} R_\odot \left(\frac{10^{-5}\,\mathrm{eV}}{m}\right),$$



Axion star mass-radius diagram

Solutions can be understood from dimensional analysis:

For chiral potential
$\lambda_\phi \approx -0.34$

$$U \propto \frac{f^2}{m}\left(-\frac{\beta\tilde{M}^2}{\tilde{R}} + \alpha_k \frac{\tilde{M}}{2\tilde{R}^2} + \frac{\alpha_4\lambda_\phi}{4!}\frac{\tilde{M}^2}{\tilde{R}^2}\right)$$

Gravity

Quantum/Gradient Pressure

quartic self-interaction

- Fitting to to numerical results yields α_k = 9.9 and α_4=1.7
- Parameter $\beta = (f/m_{\rm pl})^2$ measures strength of self-interaction vs gravity

Dilute Branch

Relevant contributions:

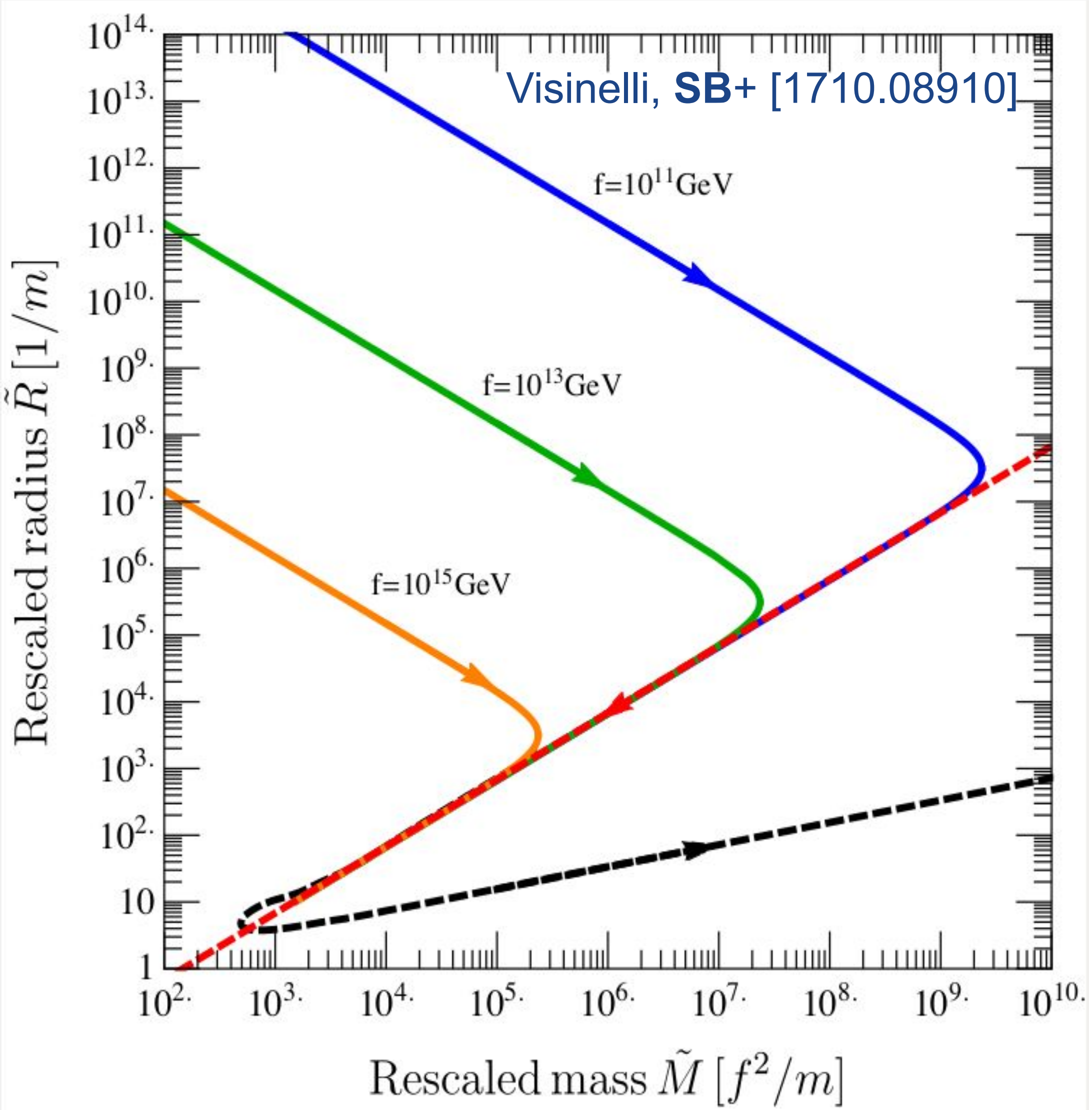
- quantum/gradient pressure
- gravity

From dimensional analysis:

$$\tilde{R}_+\Big|_{\lambda_\phi \to 0} = \frac{\alpha_k}{\beta \tilde{M}}$$

Turning point at

$$|\theta_0| \sim 10^{-6} \times (10^{-5}\,\mathrm{eV}/m)$$
$$M \sim 10^{-11}\, M_\odot \times (10^{-5}\,\mathrm{eV}/m)^2$$



Axion Stars: the “Dilute Branch”

Relevant contributions:

- Quantum/gradient pressure
- Gravity

From dimensional analysis:

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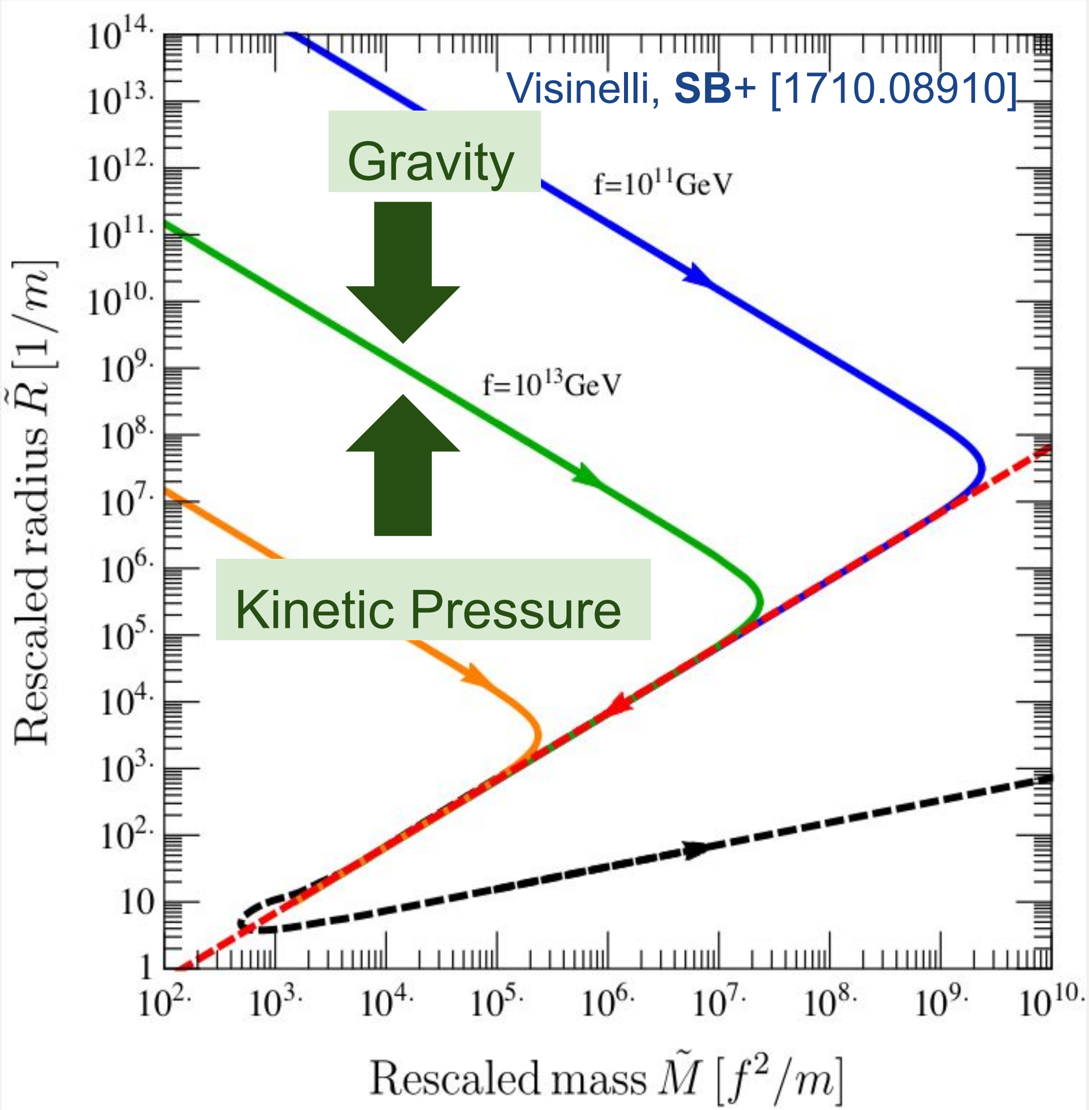


Critical Branch

Relevant contributions:

- Quantum/gradient pressure
- quartic self-interaction

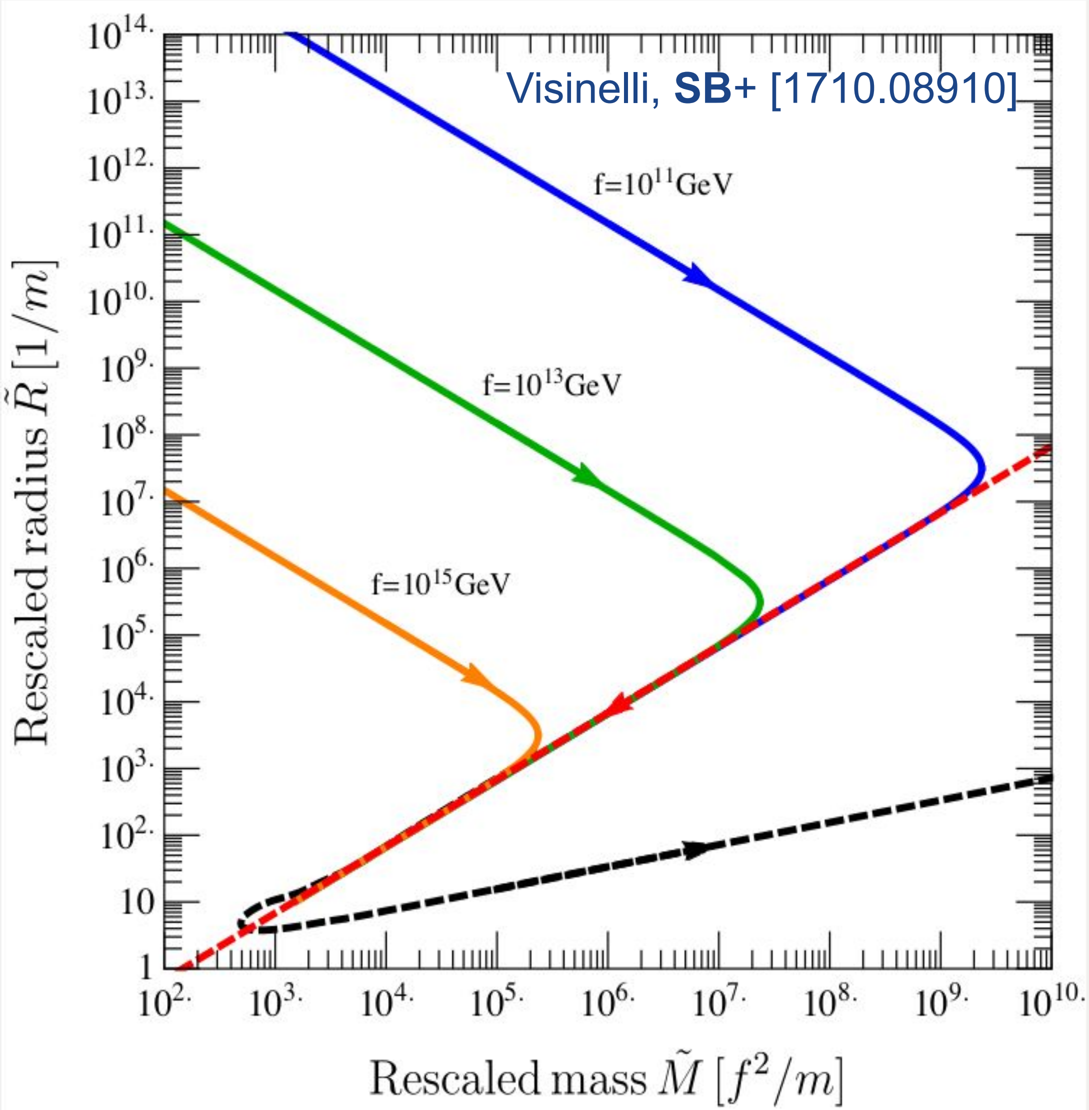
From dimensional analysis:

$$\tilde{R}_-\Big|_{G\to 0} = \frac{\alpha_4 |\lambda_\phi| \tilde{M}}{8\alpha_k}$$

Amplitude at center:

$$\Theta_0 \propto M^{-1}$$

$$\tilde{M}(\Theta_0 = 1) \approx \left(\frac{4\alpha_k}{\alpha_4|\lambda_4|}\right)^{3/2}$$



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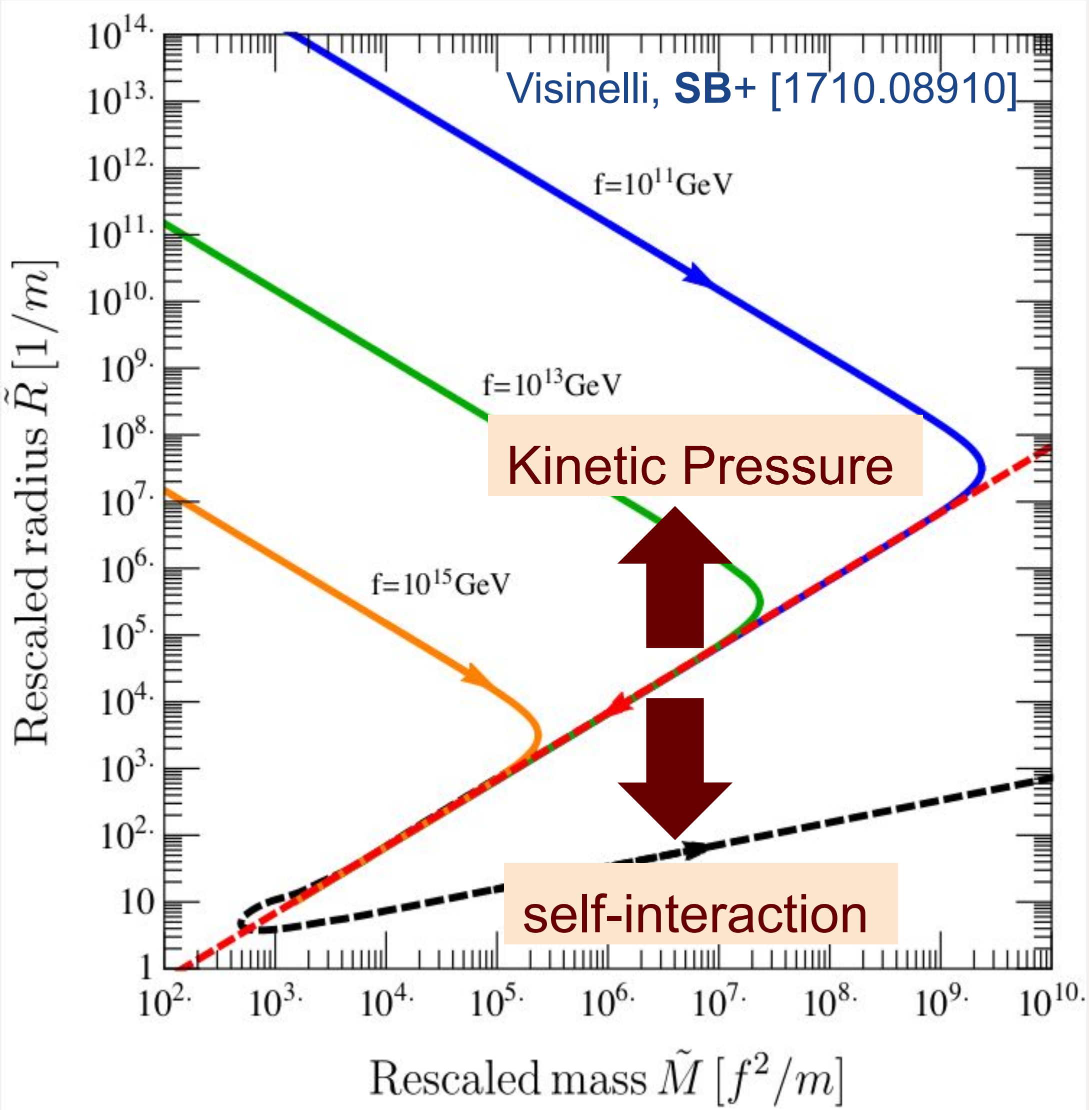


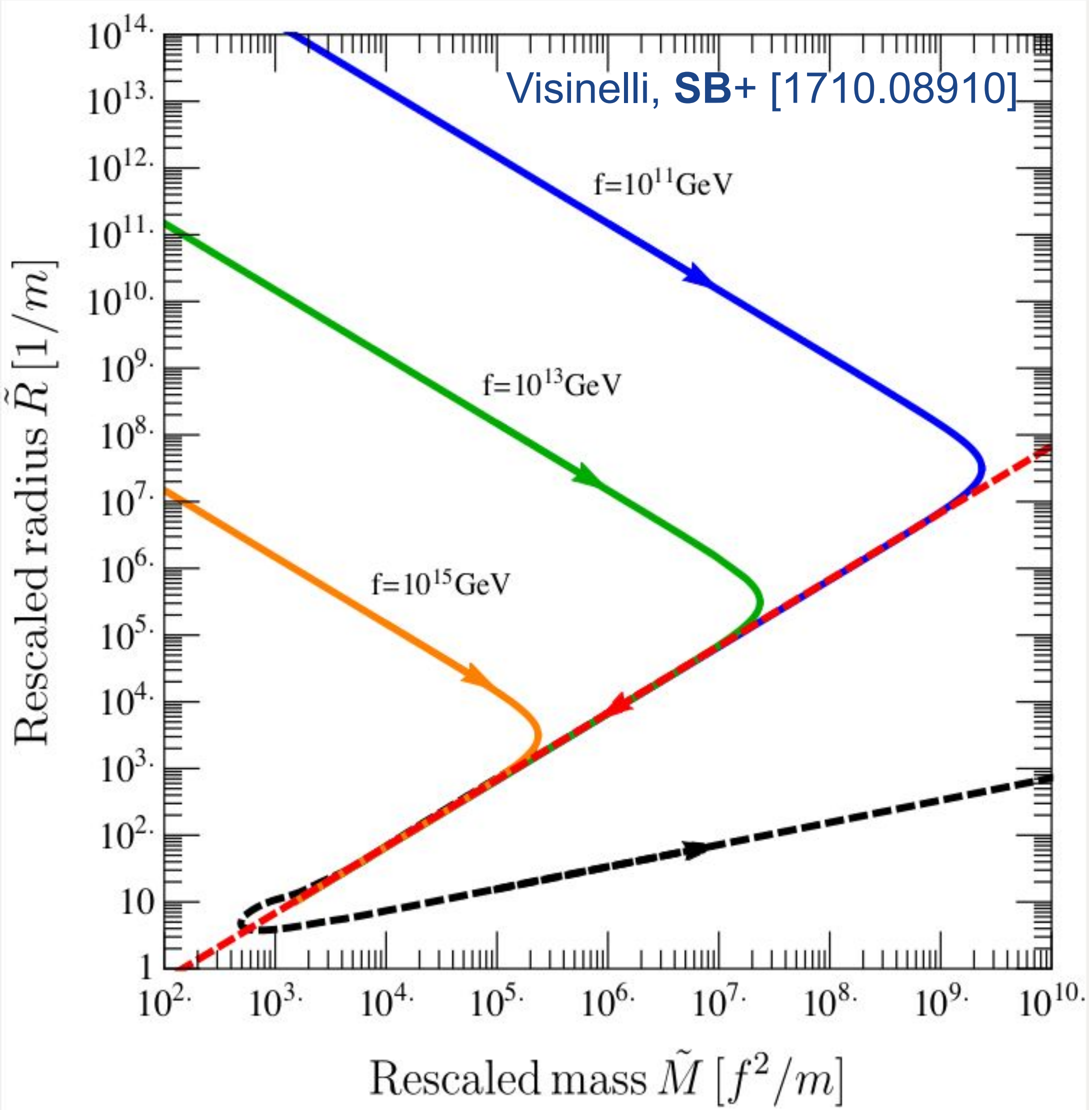
Dense Branch

Relevant contributions:

- Quantum/gradient pressure
- gradient energy
- all-order self-interaction

Saturated field: $\boldsymbol{\rho} \sim \Lambda^4$

$$\tilde{R} \approx 0.6 \tilde{M}^{1/3}$$



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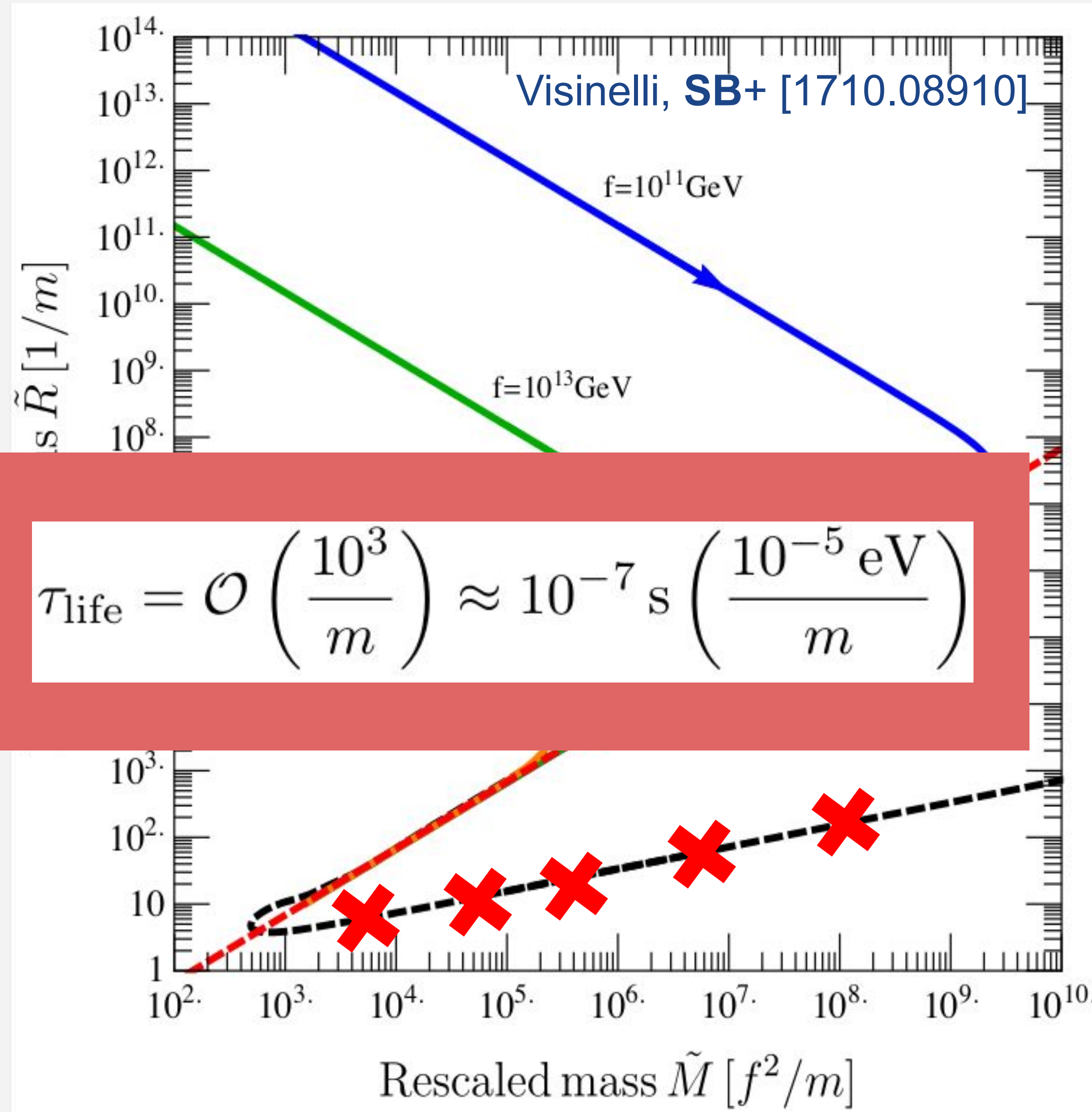


$$\tau_{\rm life} = \mathcal{O}\left(\frac{10^3}{m}\right) \approx 10^{-7}\,{\rm s}\left(\frac{10^{-5}\,{\rm eV}}{m}\right)$$

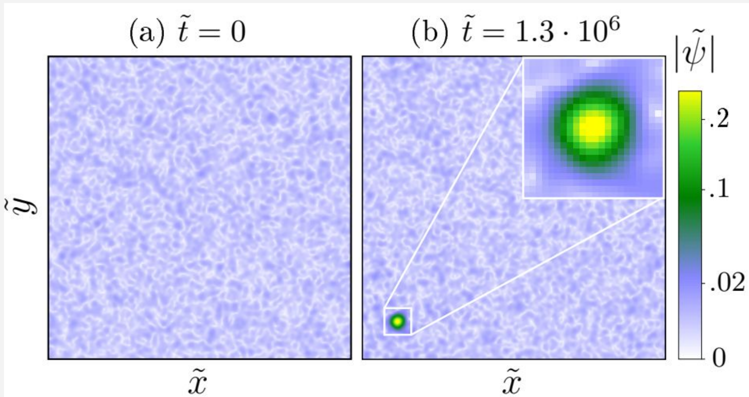
Axion Stars

- Only axion stars on the 'dilute branch' are stable, maximal mass $M \sim 10^{-11}\, M_\odot \times (10^{-5}\,\mathrm{eV}/m)^2$
- Dense axion stars either radiate relativistic axions until puffing out to the dilute branch, or might collapse to black holes

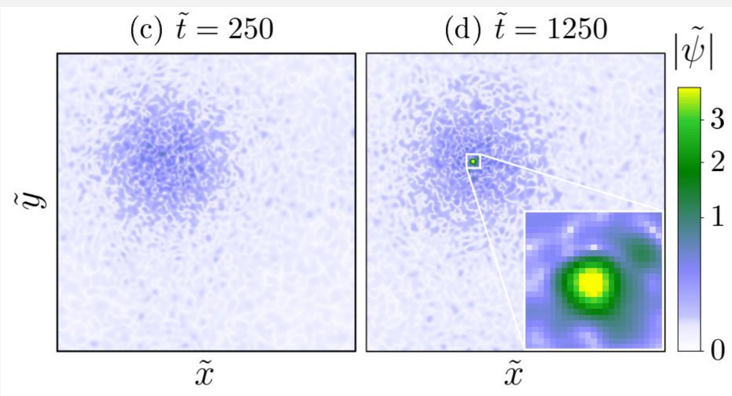
$$\tau_{\rm life} = \mathcal{O}\left(\frac{10^3}{m}\right) \approx 10^{-7}\,\mathrm{s}\left(\frac{10^{-5}\,\mathrm{eV}}{m}\right)$$

Formation of axion stars in miniclusters

- In random field



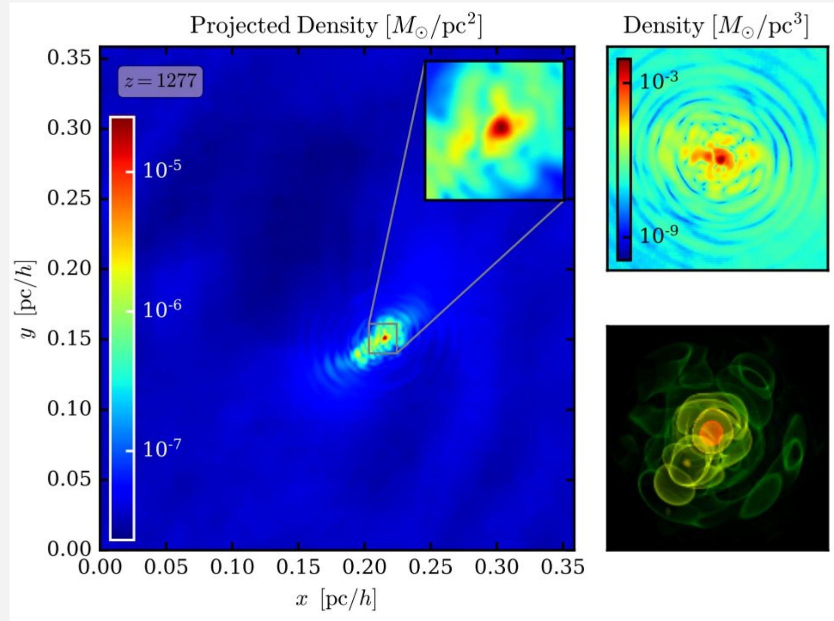
- In a minicluster

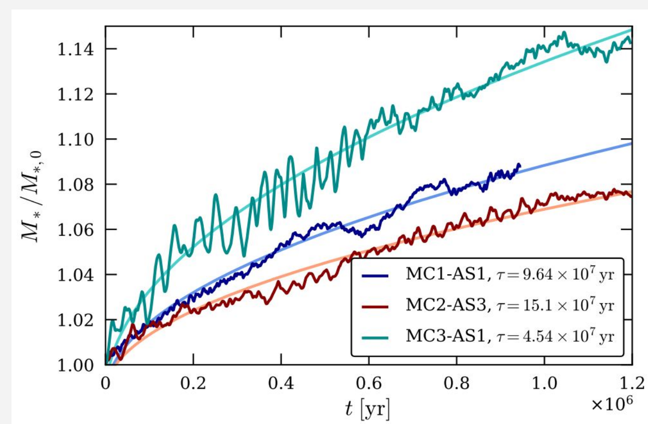


$$\tau_{gr} \sim \frac{10^9\,\text{yr}}{\Phi^3(1+\Phi)} \left(\frac{M_c}{10^{-13}\,M_\odot}\right)^2 \left(\frac{m}{26\,\mu\text{eV}}\right)^3$$

Levkov, Panin, Tkachev [1804.05857]

- In miniclusters





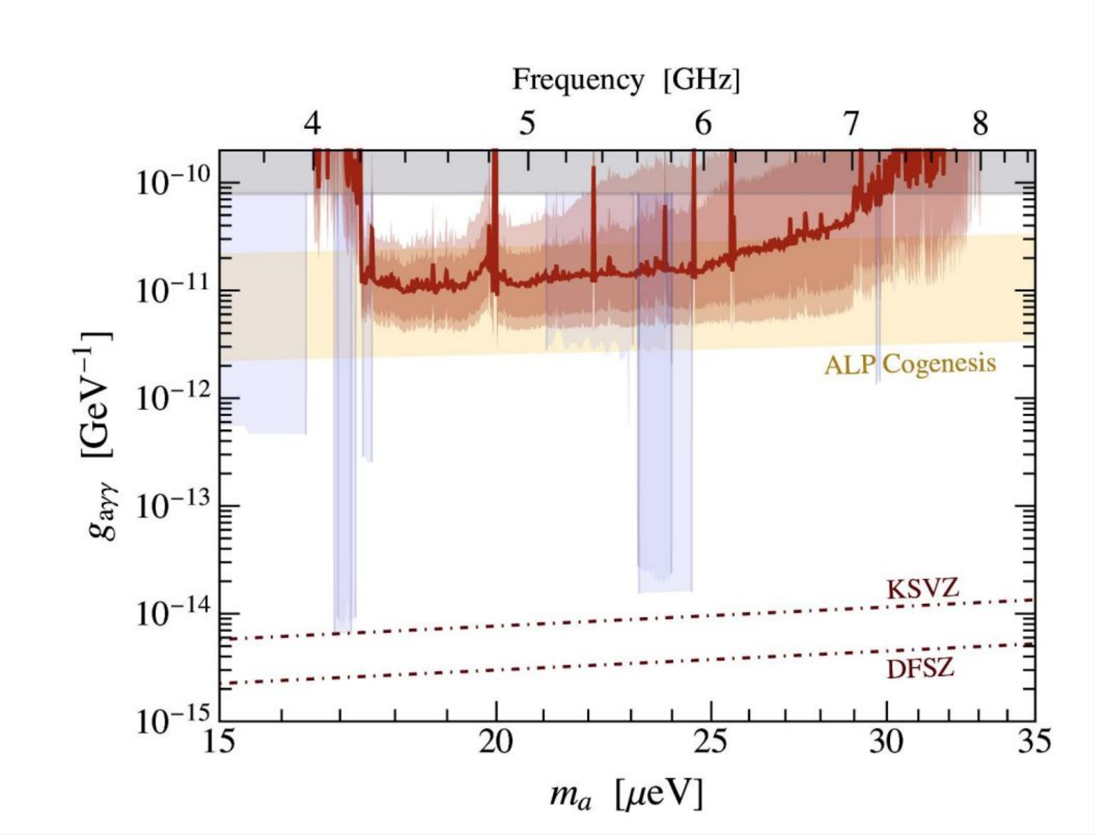
Eggemeier & Niemeyer [1906.01348]

Axions meeting Neutron Stars

Foster, Witte, Lawson, Linden, Gajjar, Weniger, Safdi [2202.08274]

Most up-to-date constraint:

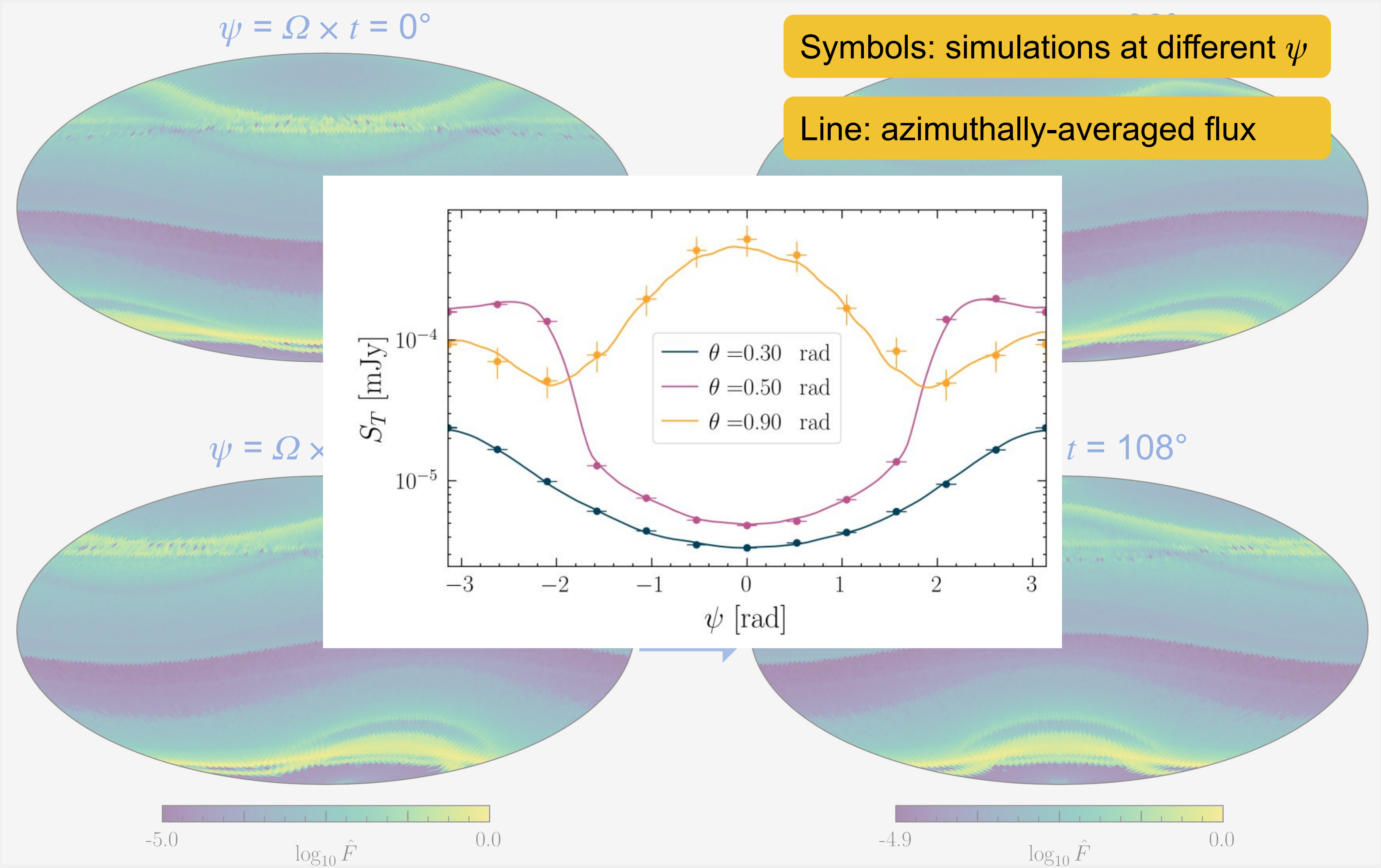
- Observations of the galactic center (GBT Breakthrough Listen Data)



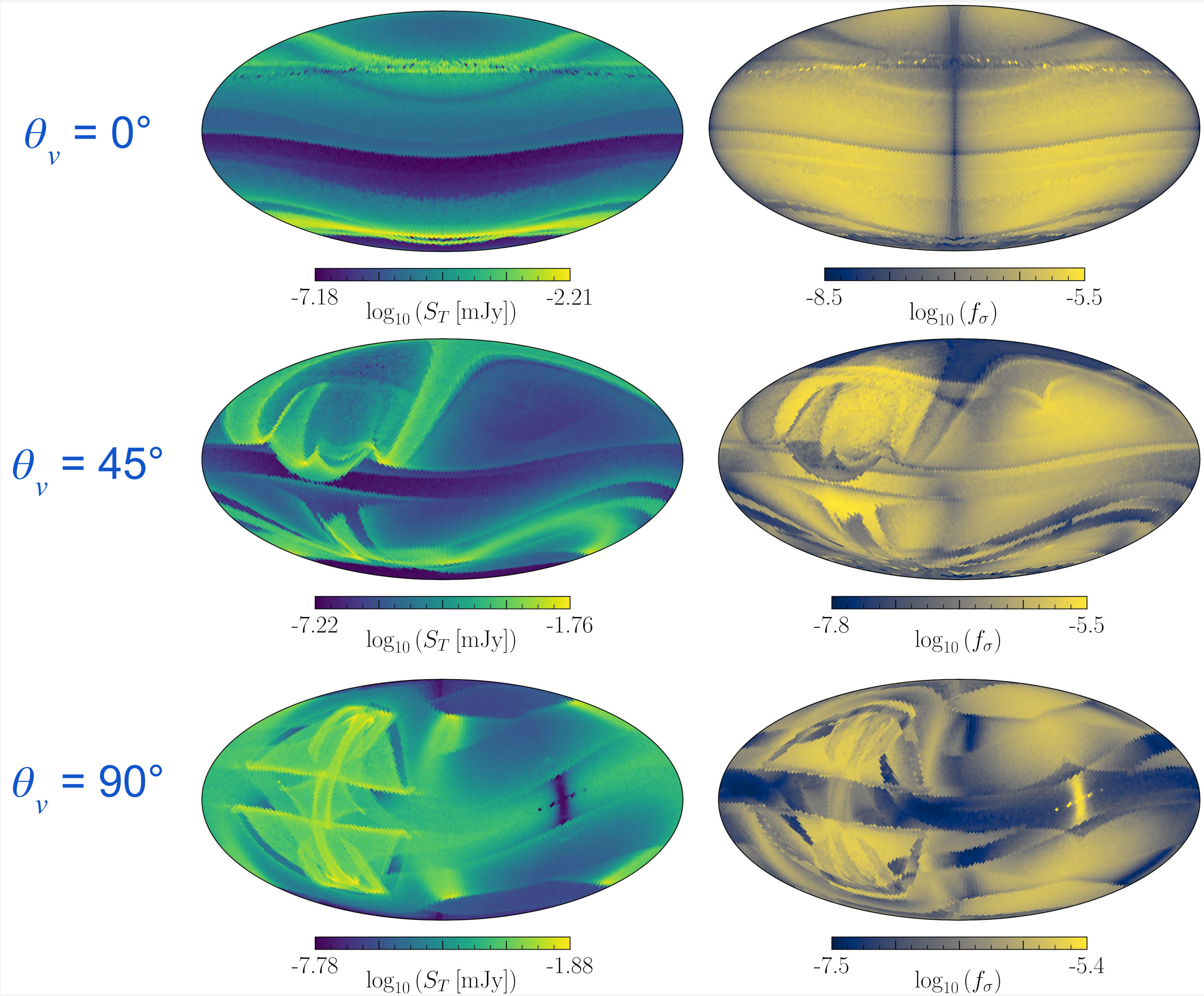
Minicluster - Neutron Star Collision: NS rotation

Symbols: simulations at different ψ

Line: azimuthally-averaged flux



AMC - NS Collision: Varying the Encounter-Geometry



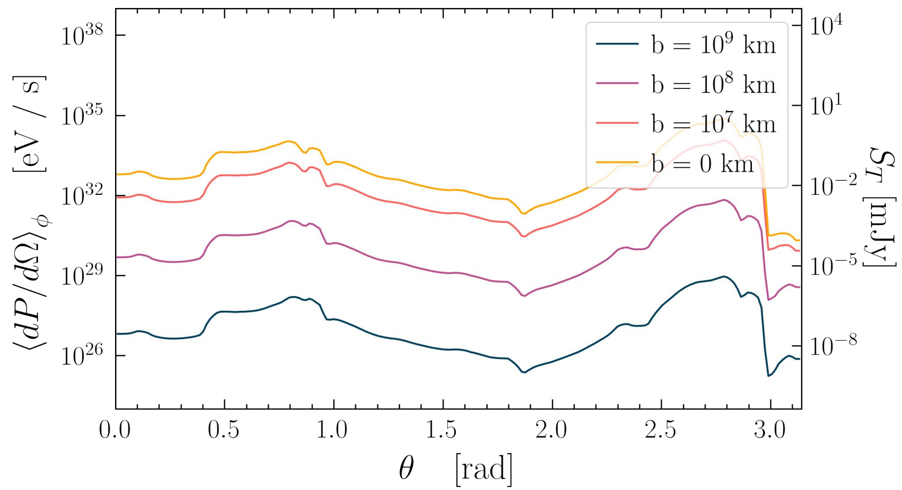
m_a	$26\,\mu$eV
$g_{a\gamma\gamma}$	$10^{-14}\,\mathrm{GeV}^{-1}$
B_0	$1.6\times10^{14}\,$G
P	$3.76\,$s
θ_m	$0.2\,$rad
θ_v	$0.0\,$rad
δf_T	$10^{-5}\times m_a$
d_T	$1\,$kpc

$M_{\rm AMC}$	$10^{-12}\,M_\odot$
$R_{\rm AMC}$	$1.86\times10^{9}\,$km
$M_{\rm AS}$	$10^{-13}\,M_\odot$
$R_{\rm AS}$	$3905\,$km
$\rho_{\rm AMC}$	NFW
$b_{\rm AMC}$	$10^{8}\,\hat{x}\,$km
$b_{\rm AS}$	$2\times10^{3}\,\hat{y}\,$km
$\lvert v_{\rm AMC/AS}\rvert$	$100\,$km/s

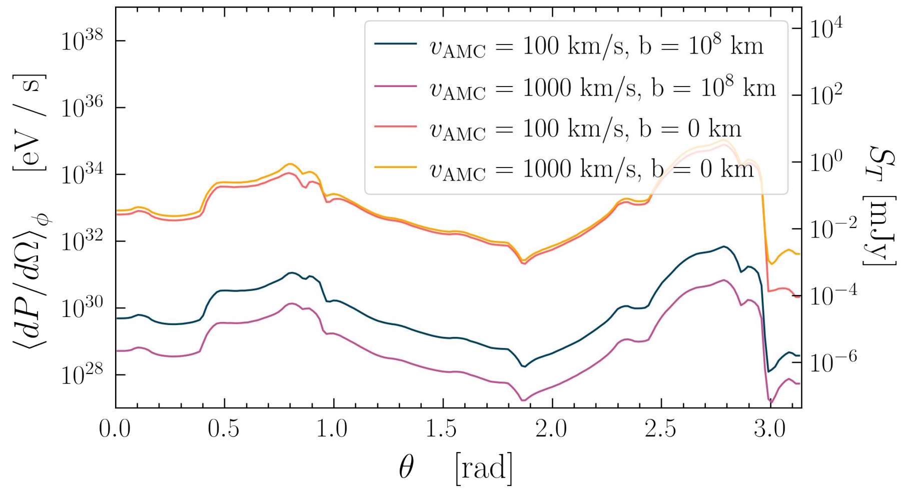
AMC - NS Collision: Varying the Encounter-Geometry

Azimuthally-averaged flux
$\langle dP/d\Omega \rangle_\phi \equiv \frac{\omega}{2\pi} \int d\phi \, \frac{dP}{d\Omega}(\phi, \theta)$

Varying the impact parameter



Varying the impact parameter & speed



m_a	26 μeV
$g_{a\gamma\gamma}$	10^{-14} GeV^{-1}
B_0	1.6×10^{14} G
P	3.76 s
θ_m	0.2 rad
θ_v	0.0 rad
δf_T	$10^{-5} \times m_a$
d_T	1 kpc

$M_{\rm AMC}$	$10^{-12}\, M_\odot$
$R_{\rm AMC}$	1.86×10^9 km
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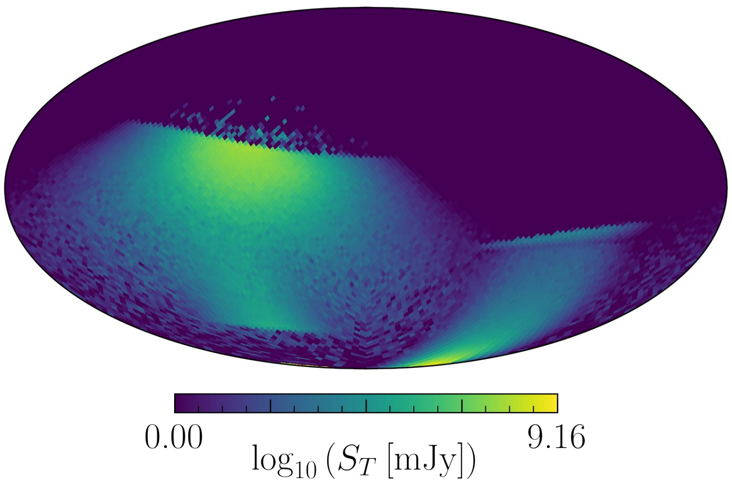
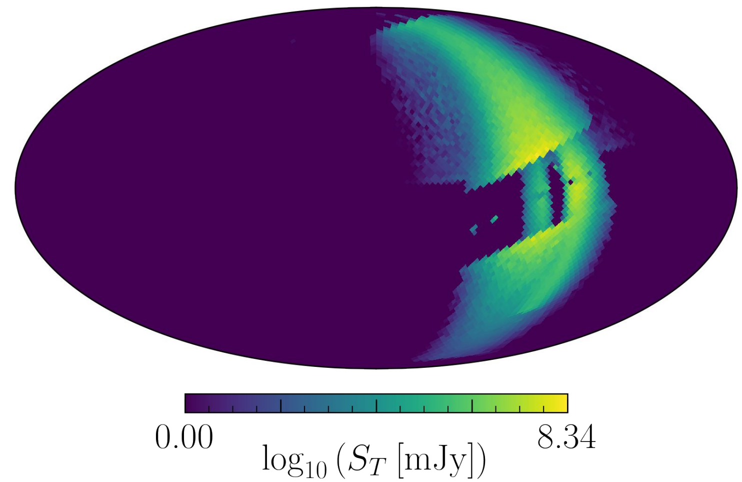
$R_{\rm AS}$	3905 km
$\rho_{\rm AMC}$	NFW
$b_{\rm AMC}$	$10^8\, \hat{x}$ km
$b_{\rm AS}$	$2 \times 10^3\, \hat{y}$ km
$\lvert v_{\rm AMC/AS} \rvert$	100 km/s

AS - NS Collision: Varying the Encounter-Geometry

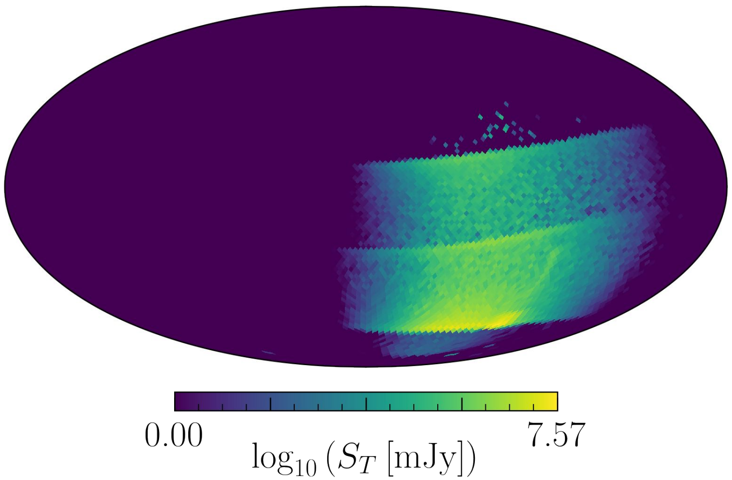
$\theta_v = 0°$



$\theta_v = 90°$



$\theta_v = 45°$



m_a	$26\,\mu\mathrm{eV}$
$g_{a\gamma\gamma}$	$10^{-14}\,\mathrm{GeV}^{-1}$
B_0	$1.6 \times 10^{14}\,\mathrm{G}$
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d_T	$1\,\mathrm{kpc}$

M_{AMC}	$10^{-12}\,M_\odot$
R_{AMC}	$1.86 \times 10^9\,\mathrm{km}$
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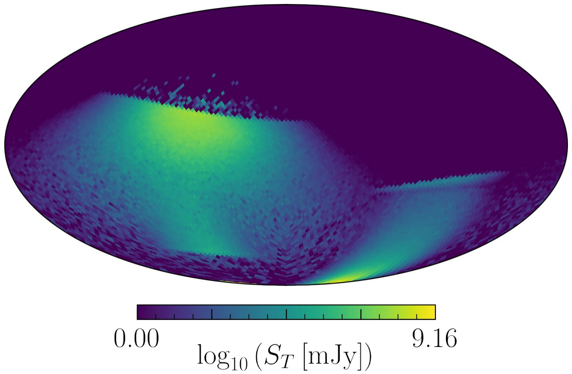
$\lvert v_{\mathrm{AMC/AS}} \rvert$	$100\,\mathrm{km/s}$

AS - NS Collision: Varying the Encounter-Geometry



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$g_{a\gamma\gamma}$	$10^{-14}\,\mathrm{GeV}^{-1}$
B_0	1.6×10^{14} G
P	3.76 s
θ_m	0.2 rad
θ_v	0.0 rad
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d_T	1 kpc

$M_{\rm AMC}$	$10^{-12}\,M_\odot$
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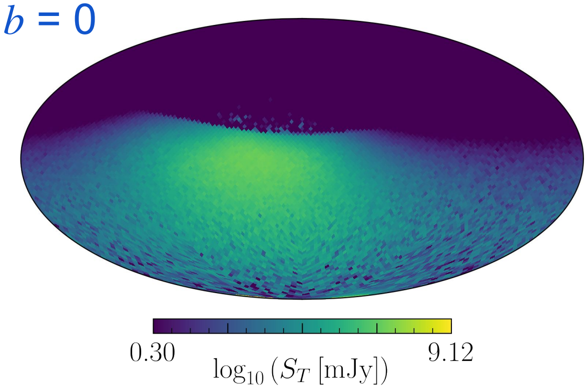
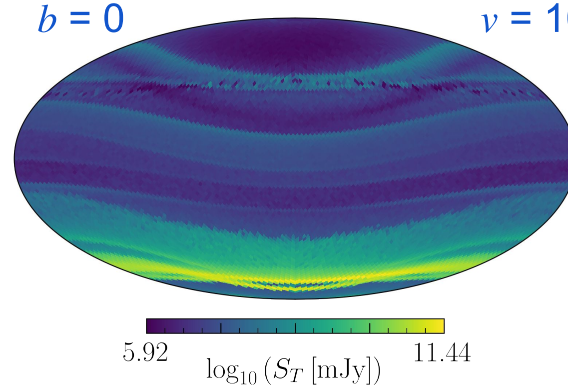
$\lvert v_{\rm AMC/AS}\rvert$	100 km/s

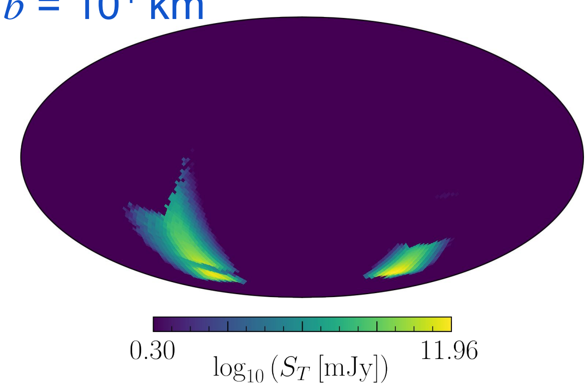
b = 0



b = 0 $\quad v = 10^3$ km/s



$b = 10^4$ km



$b = 2 \times 10^3$ km $\quad v = 10^3$ km/s