On leptogenesis in flipped SU(5)

Martin Zdráhal

August 29, 2023

Institute of Particle and Nuclear Physics @@ Faculty of Mathematics and Physics
Charles University, Prague

Work in cooperation with M. Malinský, V. Miřátský and R. Fonseca
Details will be published soon [FMMZ '23]

The minimal flipped SU(5) model

Flipped SU(5)

- favourite BSM framework based on the $SU(5) \times U(1)_X$ group
- fermionic matter field of SM embedded in $(\bar{5}, -3)$, $(10, -1)$, $(1, +5)$
- two ways of accommodation of the fields into SU(5) multiplets:

  – standard [Georgi, Glashow '74]

$$\bar{5}_F = \begin{pmatrix} d_R \\ l_L \end{pmatrix} \qquad 10_F = \left(\begin{array}{c|cc} \epsilon_{ijk}\, u_R^k & u & d \\ \hline -u & 0 & e_R \\ -d & -e_R & 0 \end{array}\right) \qquad 1_F = \begin{pmatrix} \nu_R \end{pmatrix}$$

- SM hypercharge: $Y = \boxed{T_{24}}$
- at unification scale: $\boxed{\hat{M}_u = \hat{M}_u^T}$ $\qquad$ $\boxed{\hat{M}_\ell = \hat{M}_d^T}$

Flipped SU(5)

- favourite BSM framework based on the $SU(5) \times U(1)_X$ group
- fermionic matter field of SM embedded in $(\overline{5}, -3)$, $(10, -1)$, $(1, +5)$
- two ways of accommodation of the fields into SU(5) multiplets:
  – standard [Georgi, Glashow '74] – flipped [Barr '82, Nanopoulos, Derendinger, Kim '84]

$$\overline{5}_F = \begin{pmatrix} u_R \\ l_L \end{pmatrix} \qquad 10_F = \left(\begin{array}{c|cc} \epsilon_{ijk}\, d_R^k & u & d \\ \hline -u & 0 & \nu_R \\ -d & -\nu_R & 0 \end{array}\right) \qquad 1_F = \begin{pmatrix} e_R \end{pmatrix}$$

- SM hypercharge: $Y = \boxed{\frac{1}{5}(X - T_{24})}$
- at unification scale: $\boxed{\hat{M}_d = \hat{M}_d^T}$ $\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$

Flipped SU(5)

- favourite BSM framework based on the $SU(5) \times U(1)_X$ group
- fermionic matter field of SM embedded in $(\overline{5}, -3)$, $(10, -1)$, $(1, +5)$
- two ways of accommodation of the fields into SU(5) multiplets:
  – standard [Georgi, Glashow '74] – flipped [Barr '82, Nanopoulos, Derendinger, Kim '84]

$$\overline{5}_F = \begin{pmatrix} u_R \\ l_L \end{pmatrix} \qquad 10_F = \left(\begin{array}{c|cc} \epsilon_{ijk}\, d_R^k & u & d \\ \hline -u & 0 & \nu_R \\ -d & -\nu_R & 0 \end{array}\right) \qquad 1_F = \begin{pmatrix} e_R \end{pmatrix}$$

- SM hypercharge: $Y = \boxed{\frac{1}{5}(X - T_{24})}$
- at unification scale: $\boxed{\hat{M}_d = \hat{M}_d^T}$ $\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$
- Gauge fields accommodated in $(24, 0) \oplus (1, 0) \Rightarrow$ 12+1 new bosons X
- Symmetry breaking pattern:
  – $SU(5) \times U(1)_X$ to the SM group: $(10, +1)_H$
  – SM electroweak breaking: $(5, -2)_H$

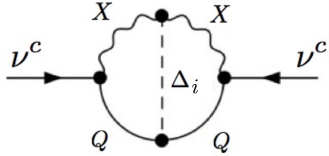
Right-handed neutrinos in the flipped SU(5) model

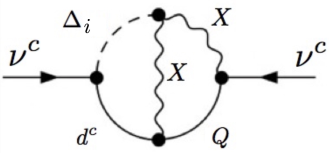
Right-handed neutrinos in the flipped SU(5) model

- at unification scale: $\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$ $\Rightarrow$ see-saw mechanism desirable
- addition of the scalar multiplet $(50, -2)$?

Right-handed neutrinos in the flipped SU(5) model

- at unification scale: $\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$ $\Rightarrow$ see-saw mechanism desirable
- addition of the scalar multiplet $(50, -2)$? Possible, but definitely not minimal.
- Witten's loop radiative generation [Witten '80, Arbelaez-Rodriguez et al. '13]





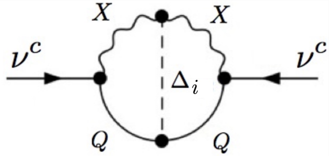
$$M_\nu^M = -\frac{3g_5^4}{(4\pi)^4}\,(-8Y_{10})\;V_G\;\boxed{\tilde{I}}$$

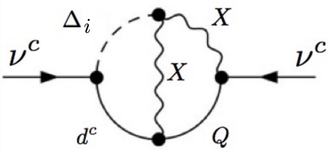
(Δ_i is color triplet scalar from 10_H and 5_H)

Right-handed neutrinos in the flipped SU(5) model

- at unification scale: $\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$ $\Rightarrow$ see-saw mechanism desirable
- addition of the scalar multiplet $(50, -2)$? Possible, but definitely not minimal.
- Witten's loop radiative generation [Witten '80, Arbelaez-Rodriguez et al. '13]

(Δ_i is color triplet scalar from 10_H and 5_H)



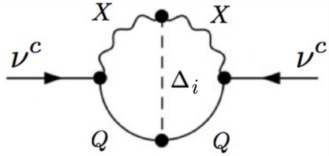


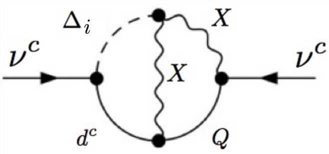
$$M_\nu^M = -\frac{3g_5^4}{(4\pi)^4}\,(-8Y_{10})\;V_G\;\tilde{I}$$

Evaluated to be $|\tilde{I}| \leq 3$.
[Harries, Malinský, MZ '18]

Right-handed neutrinos in the flipped SU(5) model

- at unification scale: $\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$ $\Rightarrow$ see-saw mechanism desirable
- addition of the scalar multiplet $(50, -2)$? Possible, but definitely not minimal.
- Witten's loop radiative generation [Witten '80, Arbelaez-Rodriguez et al. '13]





(Δ_i is color triplet scalar from 10_H and 5_H)

$$M_\nu^M = -\frac{3g_5^4}{(4\pi)^4}\boxed{(-8Y_{10})}\, V_G\, \boxed{\tilde{I}}$$

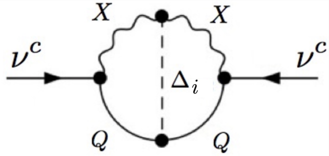
gives also mass of d-type quarks, $\hat{M}_d \sim Y_{10} v$

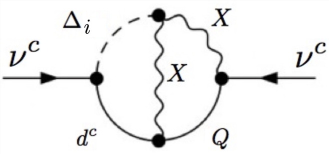
Evaluated to be $|\tilde{I}| \leq 3$. [Harries, Malinský, MZ '18]

$\Rightarrow$ Hierarchical structure of the neutrino masses and its scale given by the quark masses.

Right-handed neutrinos in the flipped SU(5) model

- at unification scale: $\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$ $\Rightarrow$ see-saw mechanism desirable
- addition of the scalar multiplet $(50, -2)$? Possible, but definitely not minimal.
- Witten's loop radiative generation [Witten '80, Arbelaez-Rodriguez et al. '13]





(Δ_i is color triplet scalar from 10_H and 5_H)

$$M_\nu^M = -\frac{3g_5^4}{(4\pi)^4}\,(-8\,Y_{10})\,V_G\,\tilde{I}$$

gives also mass of d-type quarks, $\hat{M}_d \sim Y_{10}\,v$

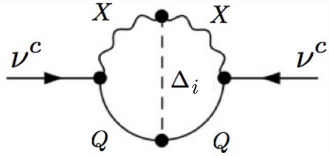
Evaluated to be $|\tilde{I}| \leq 3$.
[Harries, Malinský, MZ '18]

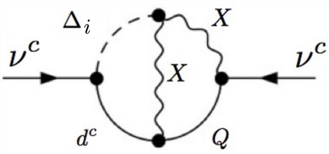
$\Rightarrow$ Hierarchical structure of the neutrino masses and its scale given by the quark masses.
Inconsistent with experimental data.

Right-handed neutrinos in the flipped SU(5) model

- at unification scale: $\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$ $\Rightarrow$ see-saw mechanism desirable
- addition of the scalar multiplet $(50, -2)$? Possible, but definitely not minimal.
- Witten's loop radiative generation [Witten '80, Arbelaez-Rodriguez et al. '13]

(Δ_i is color triplet scalar from 10_H and 5_H)





$$M_\nu^M = -\frac{3g_5^4}{(4\pi)^4} \boxed{(-8Y_{10})}\, V_G\, \boxed{\tilde{I}}$$

gives also mass of d-type quarks, $\hat{M}_d \sim Y_{10} v$

Evaluated to be $|\tilde{I}| \leq 3$.
[Harries, Malinský, MZ '18]

$\Rightarrow$ Hierarchical structure of the neutrino masses and its scale given by the quark masses.
Inconsistent with experimental data.

- We include additional $(5, -2)_H$ with Y'_{10}. Minimal potentially realistic model.

The minimal realistic flipped SU(5) model

The minimal realistic flipped SU(5) model

- gauge group: $SU(5) \times U(1)_X$
- content:
  - fermionic: $\overline{5}_F$, 10_F, 1_F
  - gauge fields: $(24, 0) \oplus (1, 0)$
  - scalar: $(10, +1)_H$, $(5, -2)_H$, $(5', -2)_H$
- M_ν^M given by Witten's loop $\rightarrow$ see-saw gives physical neutrino masses

$$\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$$

The minimal realistic flipped SU(5) model

- gauge group: $SU(5) \times U(1)_X$
- content:
  - fermionic: $\overline{5}_F$, 10_F, 1_F
  - gauge fields: $(24, 0) \oplus (1, 0)$
  - scalar: $(10, +1)_H$, $(5, -2)_H$, $(5', -2)_H$

$$\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$$

- M_ν^M given by Witten's loop $\rightarrow$ see-saw gives physical neutrino masses
- theoretical constraints:
  - boundeness from below $\rightarrow$ any mass spectrum
  - perturbativity $\rightarrow$ $m_0 \gtrsim 10^{-10}\,\mathrm{eV}$; no constraint for $m_0 \gtrsim 0.2\,\mathrm{eV}$;

  the smaller m_0 is the stricter constraints on U_ν we have

m_0 = the lightest of LH neutrinos (in NH m_1)

The minimal realistic flipped SU(5) model

- gauge group: $SU(5) \times U(1)_X$
- content:
  - fermionic: $\overline{5}_F$, 10_F, 1_F
  - gauge fields: $(24, 0) \oplus (1, 0)$
  - scalar: $(10, +1)_H$, $(5, -2)_H$, $(5', -2)_H$

$$\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$$

- M_ν^M given by Witten's loop $\rightarrow$ see-saw gives physical neutrino masses
- theoretical constraints:
  - boundeness from below $\rightarrow$ any mass spectrum
  - perturbativity $\rightarrow$ $m_0 \gtrsim 10^{-10}\,\mathrm{eV}$; no constraint for $m_0 \gtrsim 0.2\,\mathrm{eV}$;

  the smaller m_0 is the stricter constraints on U_ν we have

m_0 = the lightest of LH neutrinos (in NH m_1)

- scales:

SM scale: $M_H \sim 10^2\,\mathrm{GeV}$ | see-saw: $M_\nu^M \sim 10^9\,\mathrm{GeV}$ | unification: $M_5 \sim 10^{16}\,\mathrm{GeV}$

The minimal realistic flipped SU(5) model

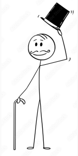
- gauge group: $SU(5) \times U(1)_X$
- content:
  - fermionic: $\overline{5}_F$, 10_F, 1_F
  - gauge fields: $(24, 0) \oplus (1, 0)$
  - scalar: $(10, +1)_H$, $(5, -2)_H$, $(5', -2)_H$

$$\boxed{\hat{M}_\nu^D = \hat{M}_u^T}$$

- M_ν^M given by Witten's loop $\to$ see-saw gives physical neutrino masses
- theoretical constraints:
  - boundeness from below $\to$ any mass spectrum
  - perturbativity $\to$ $m_0 \gtrsim 10^{-10}\,\mathrm{eV}$; no constraint for $m_0 \gtrsim 0.2\,\mathrm{eV}$;

  the smaller m_0 is the stricter constraints on U_ν we have

$\boxed{m_0 = \text{the lightest of LH neutrinos (in NH } m_1)}$



quantities with hat at unification scale

- scales:

SM scale $M_H \sim 10^2\,\mathrm{GeV}$

see-saw $M_\nu^M \sim 10^9\,\mathrm{GeV}$

unification $M_5 \sim 10^{16}\,\mathrm{GeV}$

Neutrino sector of the model

- the central equation (in the basis of diagonal up-type quark mass matrix)

$$\boxed{M_\nu^M = -\hat{D}_u U_\nu^T \hat{D}_\nu^{-1} U_\nu \hat{D}_u}$$

$$\hat{D}_u = \begin{pmatrix} \hat{m}_u & & \\ & \hat{m}_c & \\ & & \hat{m}_t \end{pmatrix}$$

- unitary (Takagi) decomposition

$$M_\nu^M = \tilde{U}^T D_\nu^M \tilde{U}$$

- from the properties of the determinant

$$\hat{D}_\nu = \begin{pmatrix} \hat{m}_1 & & \\ & \hat{m}_2 & \\ & & \hat{m}_3 \end{pmatrix}$$

$$\boxed{\hat{m}_1 \hat{m}_2 \hat{m}_3 M_1 M_2 M_3 = \hat{m}_u^2 \hat{m}_c^2 \hat{m}_t^2}$$

- experimental Δm_{12}^2 and Δm_{13}^2 $\Rightarrow$ the only independent variables for whole ν sector are m_0 and 6 parameters of U_ν (3 angles, 3 physical phases); (3 phases are unphysical)

Thermal leptogenesis in the flipped SU(5) model

Thermal leptogenesis in a ~~nut~~ bacteria-shell

The out-of-equilibrium decays of RH neutrinos

$$Y_\nu = \frac{1}{v} V_{PMNS} U_\nu \hat{D}_u \tilde{U}^\dagger$$

Leptonic asymmetry

$$\epsilon^i_{CP} = \frac{\Gamma(N_i \to \phi L) - \Gamma(N_i \to \phi^\dagger \overline{L})}{\Gamma(N_i \to \phi L) + \Gamma(N_i \to \phi^\dagger \overline{L})}$$

Washout effect $\mathcal{W}$

Flavor density matrix (Boltzmann eq.)

$$\frac{\mathrm{d}N^{B-L}_{\alpha\beta}}{\mathrm{d}z} = \sum_i \mathcal{D}^i \epsilon^i_{\alpha\beta} - \mathcal{W}_{\alpha\beta} - \mathcal{C}_{\alpha\beta}$$

Decoherence effects $\mathcal{C}$

Washout factor

$$k_i = \frac{|\tilde{U}_{i3}|^2 \hat{m}_t^2 + |\tilde{U}_{i2}|^2 \hat{m}_c^2 + |\tilde{U}_{i1}|^2 \hat{m}_u^2}{m_\star M_i}$$

$m_\star \approx 10^{-3}\,\mathrm{eV}$ equilibrium neutrino mass

$z = M_1/T$, T temperature of the Universe

Sphaleron interaction

$$\eta_B \approx 10^{-2} \times \mathrm{Tr}\, N_f^{B-L}(z \to \infty)$$

Baryon asymmetry

$$\eta_B = \frac{n_B - n_{\overline{B}}}{n_\gamma} = 6.1 \times 10^{-10}$$

Back-of-the-envelope calculations

- From $\hat{m}_1\hat{m}_2\hat{m}_3 M_1 M_2 M_3 = \hat{m}_u^2\hat{m}_c^2\hat{m}_t^2$, there is a maximal M_1 for a fixed $\hat{m}_1$.
  - For quasi-degenerate $\hat{m}_k$, behaving as $M_1 \lesssim \dfrac{\text{const.}}{\hat{m}_1}$.
  - For $\hat{m}_0 \lesssim 10^{-2}\,\text{eV}$, where $\hat{m}_{2,3}$ stay almost constant, $M_1 \lesssim \sqrt[3]{\dfrac{\text{const.}}{\hat{m}_1}}$.
- for fixed $\hat{m}_0$ and M_1,

$$M_1 \leq M_2 \leq \sqrt{\frac{\text{const.}}{M_1}}$$

  Triangle in $\log M_1$ vs. $\log M_2$ plot.

- The washout factor $k_i = \dfrac{|\tilde{U}_{i3}|^2\hat{m}_t^2 + |\tilde{U}_{i2}|^2\hat{m}_c^2 + |\tilde{U}_{i1}|^2\hat{m}_u^2}{m_\star M_i}$ typically large as $k_i = \dfrac{\hat{m}_t^2}{m_\star M_i}$.
  However, for specific forms of $\tilde{U}$, it can be suppressed as m_c^2/m_t^2, or even m_u^2/m_t^2.
  One can show that it occurs for

$$\frac{\hat{m}_c^2}{\hat{m}_3} \lesssim M_i \lesssim \frac{\hat{m}_c^2}{\hat{m}_1} \qquad \text{and} \qquad \frac{\hat{m}_u^2}{\hat{m}_3} \lesssim M_i \lesssim \frac{\hat{m}_u^2}{\hat{m}_1}\,\text{, respectively.}$$

Back-of-the-envelope calculations

- Davidson-Ibarra (DI) limit would lead to

$$|\epsilon_{CP}^{1}| \lesssim \frac{3}{8\pi} \frac{\Delta \hat{m}_{13}^2}{v^2(\hat{m}_0 + \hat{m}_3)} M_1$$

- the washout factor is always larger than

$$k_1 \gtrsim \frac{\hat{m}_u^2}{m_\star M_1}$$

- these two together would give

$$10^{-9} \lesssim \frac{2|\epsilon_{CP}|}{k_1} \lesssim \frac{3}{4\pi} \frac{m_\star M_1^2}{v^2 \hat{m}_u^2} \frac{\Delta \hat{m}_{31}^2}{\hat{m}_0 + \hat{m}_3} \qquad \Rightarrow \qquad \boxed{M_1 \gtrsim 10^7 \, \mathrm{GeV}}$$

- the saturation of both inequalities would dictate

$$\boxed{\hat{m}_0 \lesssim 10^{-4.6} \, \mathrm{eV}}$$

Back-of-the-envelope calculations

- Davidson-Ibarra (DI) limit would lead to

$$|\epsilon_{CP}^1| \lesssim \frac{3}{8\pi} \frac{\Delta \hat{m}_{13}^2}{v^2(\hat{m}_0 + \hat{m}_3)} M_1$$

- the washout factor is always larger than

$$k_1 \gtrsim \frac{\hat{m}_u^2}{m_\star M_1}$$

- these two together would give

$$10^{-9} \lesssim \frac{2|\epsilon_{CP}|}{k_1} \lesssim \frac{3}{4\pi} \frac{m_\star M_1^2}{v^2 \hat{m}_u^2} \frac{\Delta \hat{m}_{31}^2}{\hat{m}_0 + \hat{m}_3} \qquad \Rightarrow \qquad \boxed{M_1 \gtrsim 10^7 \,\mathrm{GeV}}$$

- the saturation of both inequalities would dictate

$$\boxed{\hat{m}_0 \lesssim 10^{-4.6} \,\mathrm{eV}}$$

- However, the DI limit can be violated, e.g., for $M_1 \approx M_2$ or N_2 generated asymmetry. Not a strict limit.

Back-of-the-envelope calculations

- Davidson-Ibarra (DI) limit would lead to

$$|\epsilon_{CP}^1| \lesssim \frac{3}{8\pi} \frac{\Delta \hat{m}_{13}^2}{v^2(\hat{m}_0 + \hat{m}_3)} M_1$$

- the washout factor is always larger than

$$k_1 \gtrsim \frac{\hat{m}_u^2}{m_\star M_1}$$

- these two together would give

$$10^{-9} \lesssim \frac{2|\epsilon_{CP}|}{k_1} \lesssim \frac{3}{4\pi} \frac{m_\star M_1^2}{v^2 \hat{m}_u^2} \frac{\Delta \hat{m}_{31}^2}{\hat{m}_0 + \hat{m}_3} \qquad \Rightarrow \qquad \boxed{M_1 \gtrsim 10^7\,\mathrm{GeV}}$$

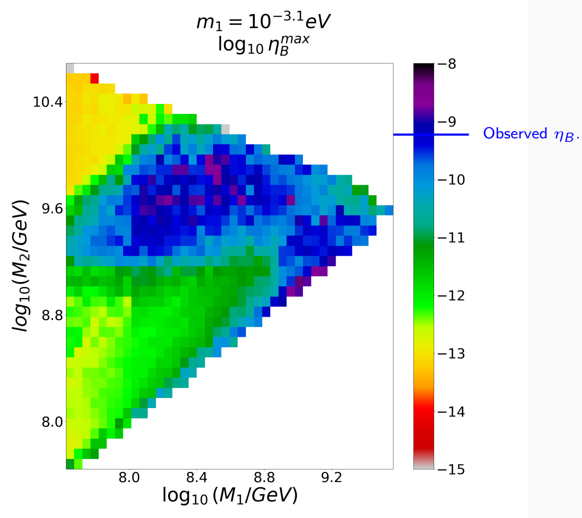
- the saturation of both inequalities would dictate

$$\boxed{\hat{m}_0 \lesssim 10^{-4.6}\,\mathrm{eV}}$$

- However, the DI limit can be violated, e.g., for $M_1 \approx M_2$ or N_2 generated asymmetry. Not a strict limit.
- In any case, for lower $\hat{m}_0$, it is easier to find large enough ϵ_{CP}, and the intersection of the permitted region with the region of small washout is larger.

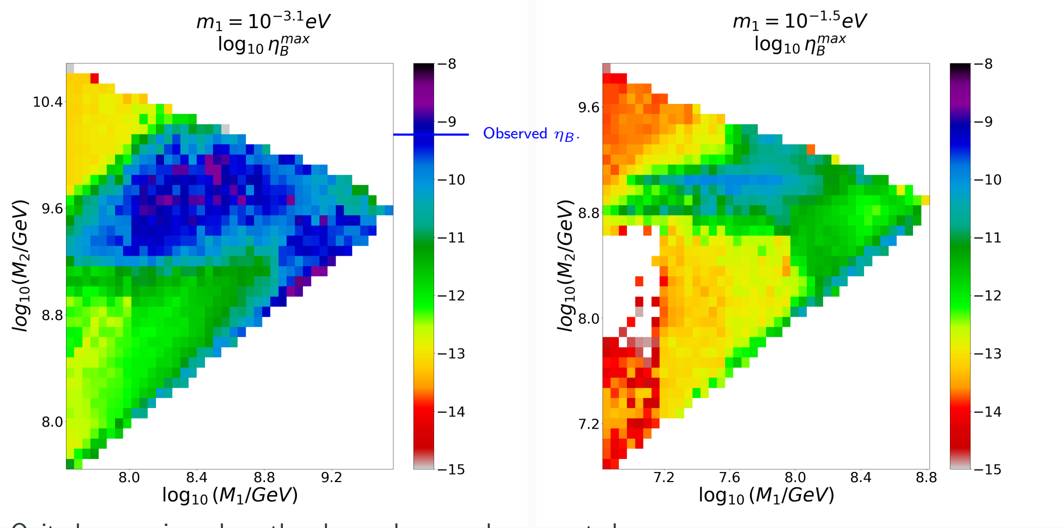
Numerical results

Numerical results for $\hat{m}_0 = 10^{-3}\,\mathrm{eV}$ and $\hat{m}_0 = 10^{-1.4}\,\mathrm{eV}$ (using ULYSSES [Granelli '21])



Quite large region where the observed η_B can be generated.

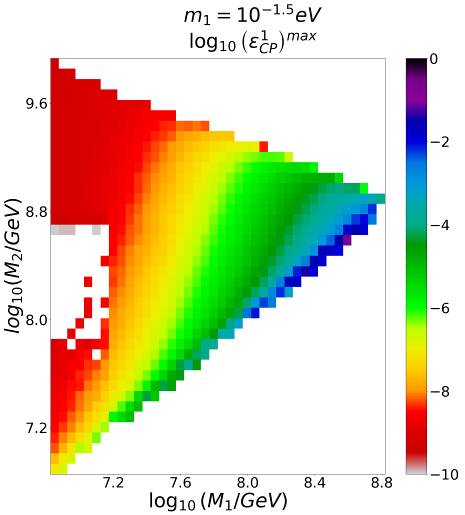
Numerical results for $\hat{m}_0 = 10^{-3}\,\mathrm{eV}$ and $\hat{m}_0 = 10^{-1.4}\,\mathrm{eV}$ (using ULYSSES [Granelli '21])

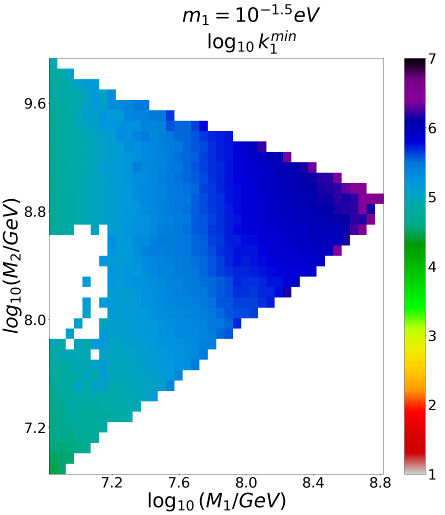


Quite large region where the observed η_B can be generated.

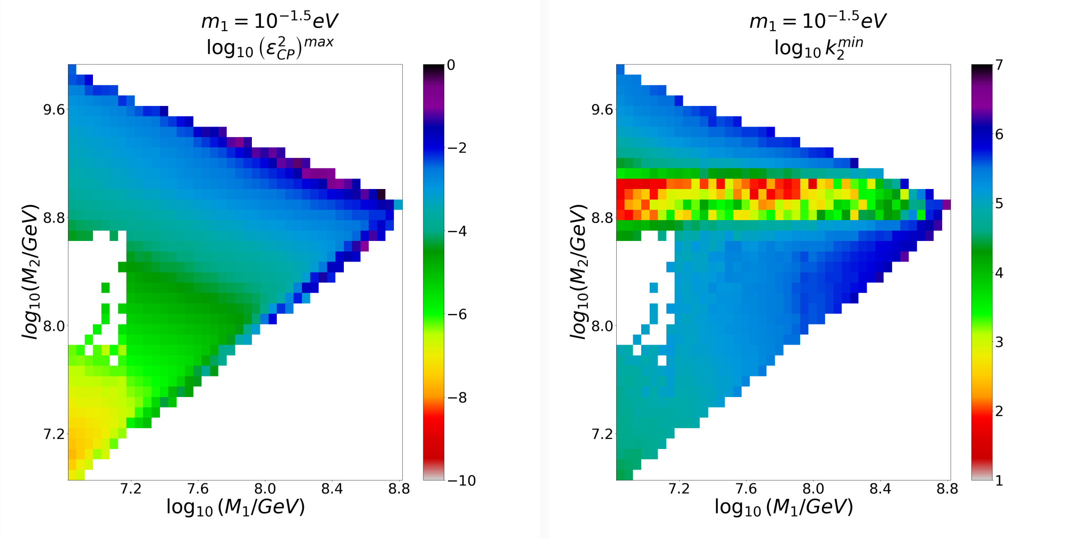
For larger $\hat{m}_0$, it gets smaller and smaller till $\hat{m}_0 = 10^{-1.4}\,\mathrm{eV}$.

Numerical results for $\hat{m}_0 = 10^{-1.4}\,\mathrm{eV}$. N_1 generated asymmetry.





Numerical results for $\hat{m}_0 = 10^{-1.4}\,\mathrm{eV}$. N_2 generated asymmetry.



Proton decay

- in flipped SU(5)

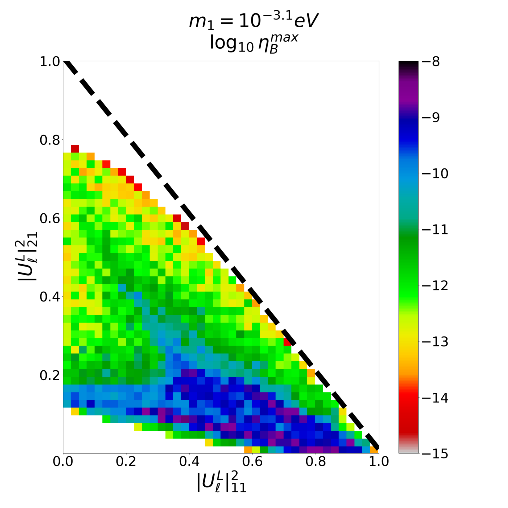
$$\boxed{\Gamma(p \to K^+\overline{\nu}) = 0}$$

- the ratios

$$\frac{\Gamma(p \to \pi^0 \ell_\alpha^+)}{\Gamma(p \to \pi^+ \overline{\nu})} = \frac{1}{2} \left|(V_{CKM})_{11}\right|^2 \left|\underbrace{(V_{PMNS} U_\nu)_{\alpha 1}}_{(U_\ell^L)_{\alpha 1}}\right|^2$$

driven by the matrix elements of U_ν

- similarly for $\Gamma(p \to K^0 \ell_\alpha^+)$ and $\Gamma(p \to \eta \ell_\alpha^+)$
- leptogenesis constrains U_ν, thereby constraining the decay rates $\mathrm{Br}(p \to \pi^0 e^+)$ and $\mathrm{Br}(p \to \pi^0 \mu^+)$ (constraints follow from the lower limit on M_1)
- for m_0 large, there is no such constraint, but such m_0 ruled out by the leptogenesis



Conclusions

- The flipped SU(5) with Witten loop and 2 scalar pentuplets is a viable and most compact model of perturbative baryon and lepton number violation (BLNV).
- It passes all theoretical and current experimental constraints (the perturbativity condition dictates $m_0 \gtrsim 10^{-11}\,\mathrm{eV}$).
- Thermal leptogenesis indicates an upper limit on $m_0 \lesssim 10^{-1.5}\,\mathrm{eV}$. The connected upper limit on the effective neutrino mass from β decay experiments

$$\boxed{m_\beta \lesssim 0.03\,\mathrm{eV}.}$$

  $\Rightarrow$ Model falsifiable in the near future (N.B.: KATRIN designed sensitivity m_β down to $0.3\,\mathrm{eV}$).
- Possible regimes of baryon asymmetry generation:
  - N_1-dominated
  - N_2-dominated with suppression of decoherence effects generating large leptonic asymmetry
  - N_2-dominated with small washout effect
- Branching ratios for proton decays on the neutral pion and charged leptons are partially constrained from the leptogenesis.

Conclusions

- The flipped SU(5) with Witten loop and 2 scalar pentuplets is a viable and most compact model of perturbative baryon and lepton number violation (BLNV).
- It passes all theoretical and current experimental constraints (the perturbativity condition dictates $m_0 \gtrsim 10^{-11}\,\mathrm{eV}$).
- Thermal leptogenesis indicates an upper limit on $m_0 \lesssim 10^{-1.5}\,\mathrm{eV}$. The connected upper limit on the effective neutrino mass from β decay experiments

$$\boxed{m_\beta \lesssim 0.03\,\mathrm{eV}.}$$

  $\Rightarrow$ Model falsifiable in the near future (N.B.: KATRIN designed sensitivity m_β down to $0.3\,\mathrm{eV}$).
- Possible regimes of baryon asymmetry generation:
  - N_1-dominated
  - N_2-dominated with suppression of decoherence effects generating large leptonic asymmetry
  - N_2-dominated with small washout effect
- Branching ratios for proton decays on the neutral pion and charged leptons are partially constrained from the leptogenesis.

Thank you for your attention.