Cosmic inflation

and the primordial universe

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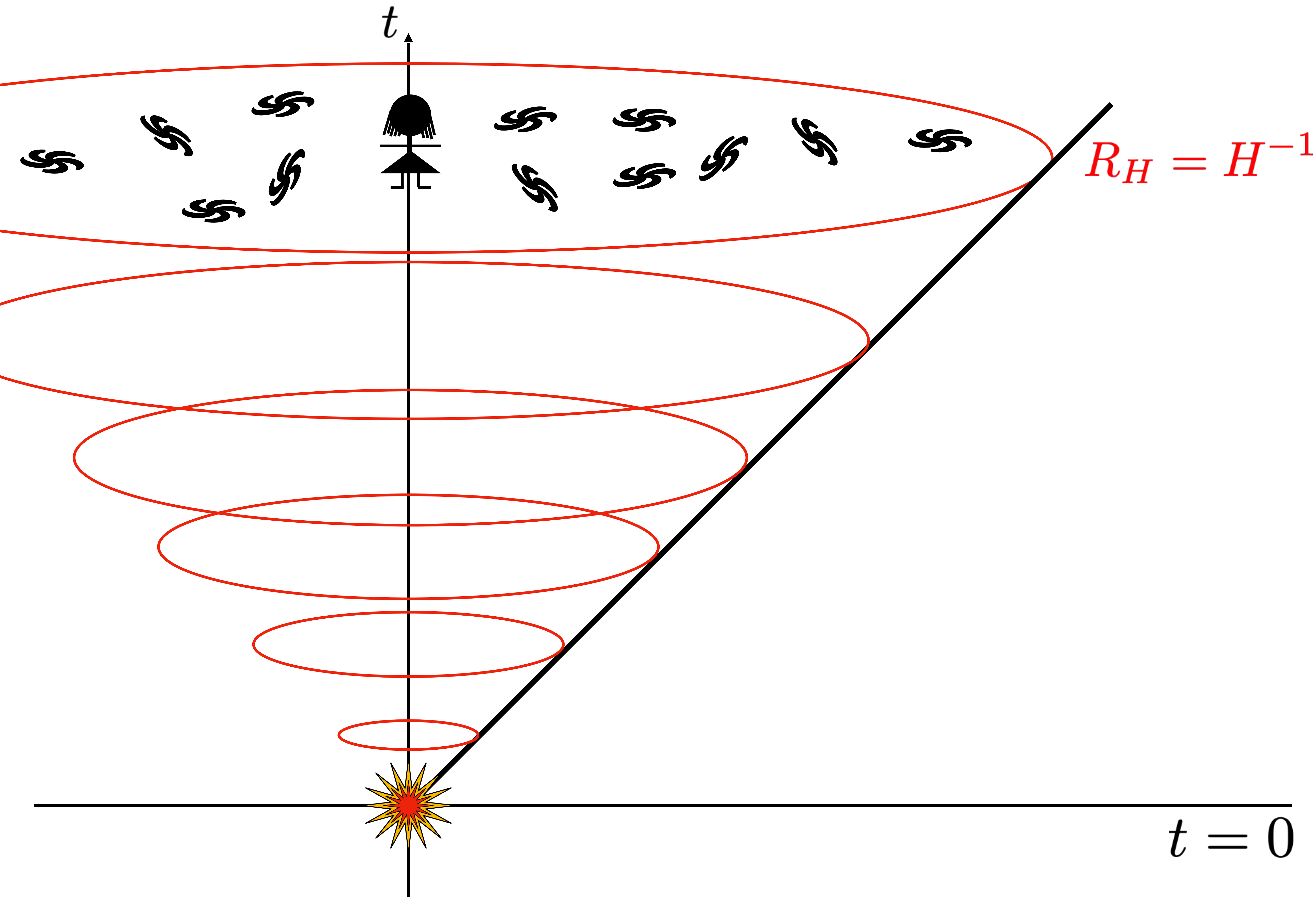
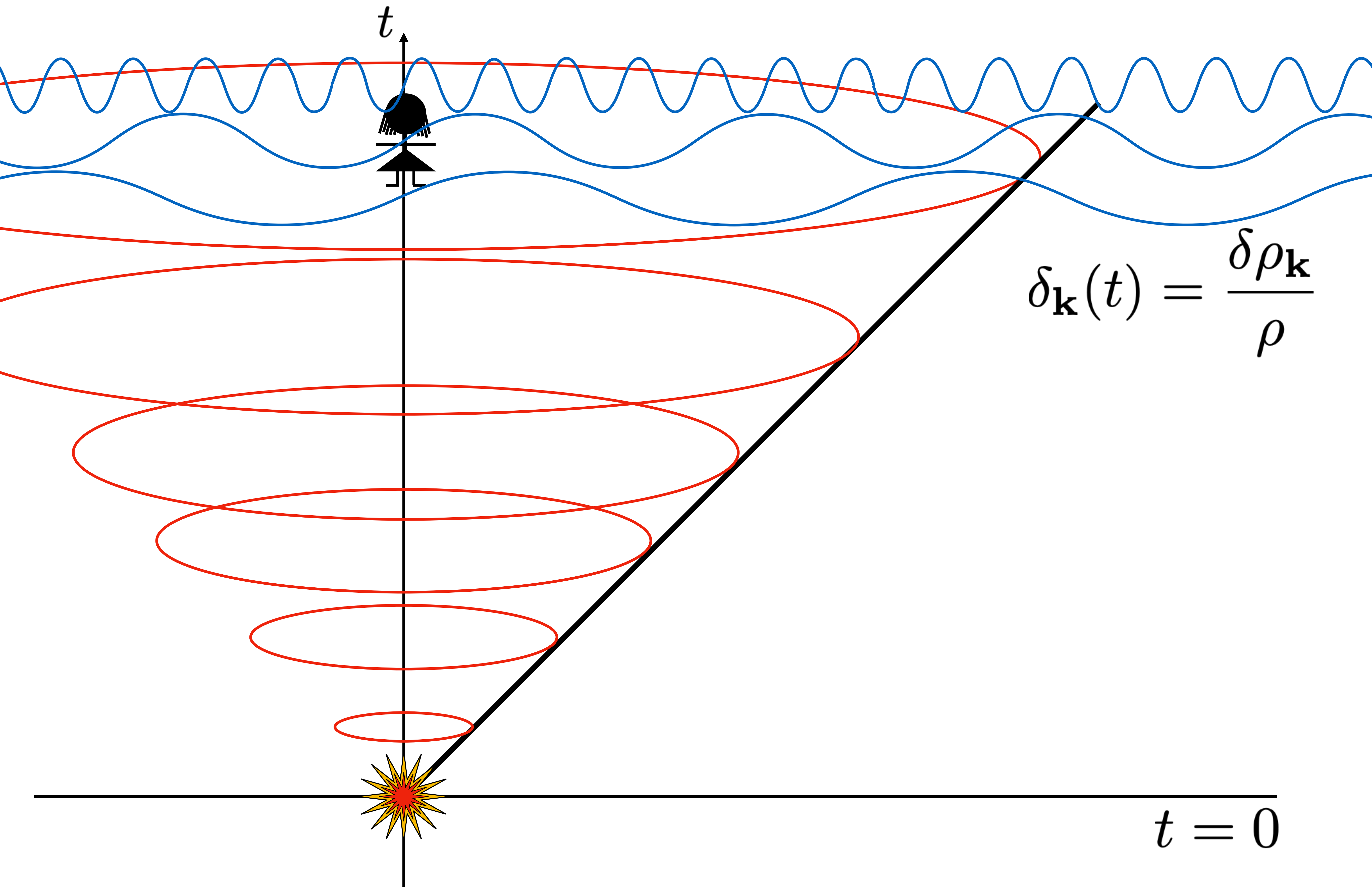
Primordial fluctuations



Primordial fluctuations



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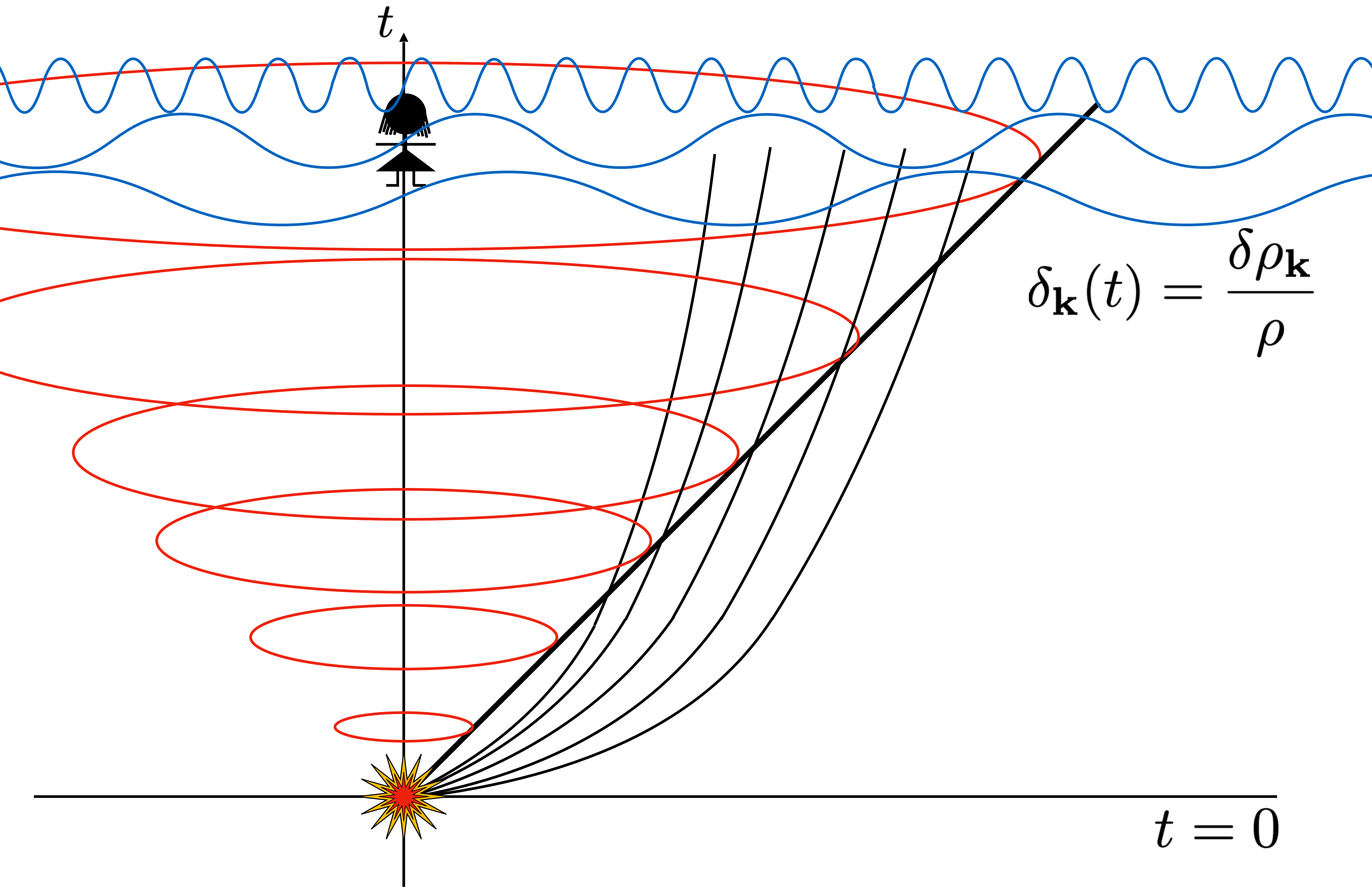

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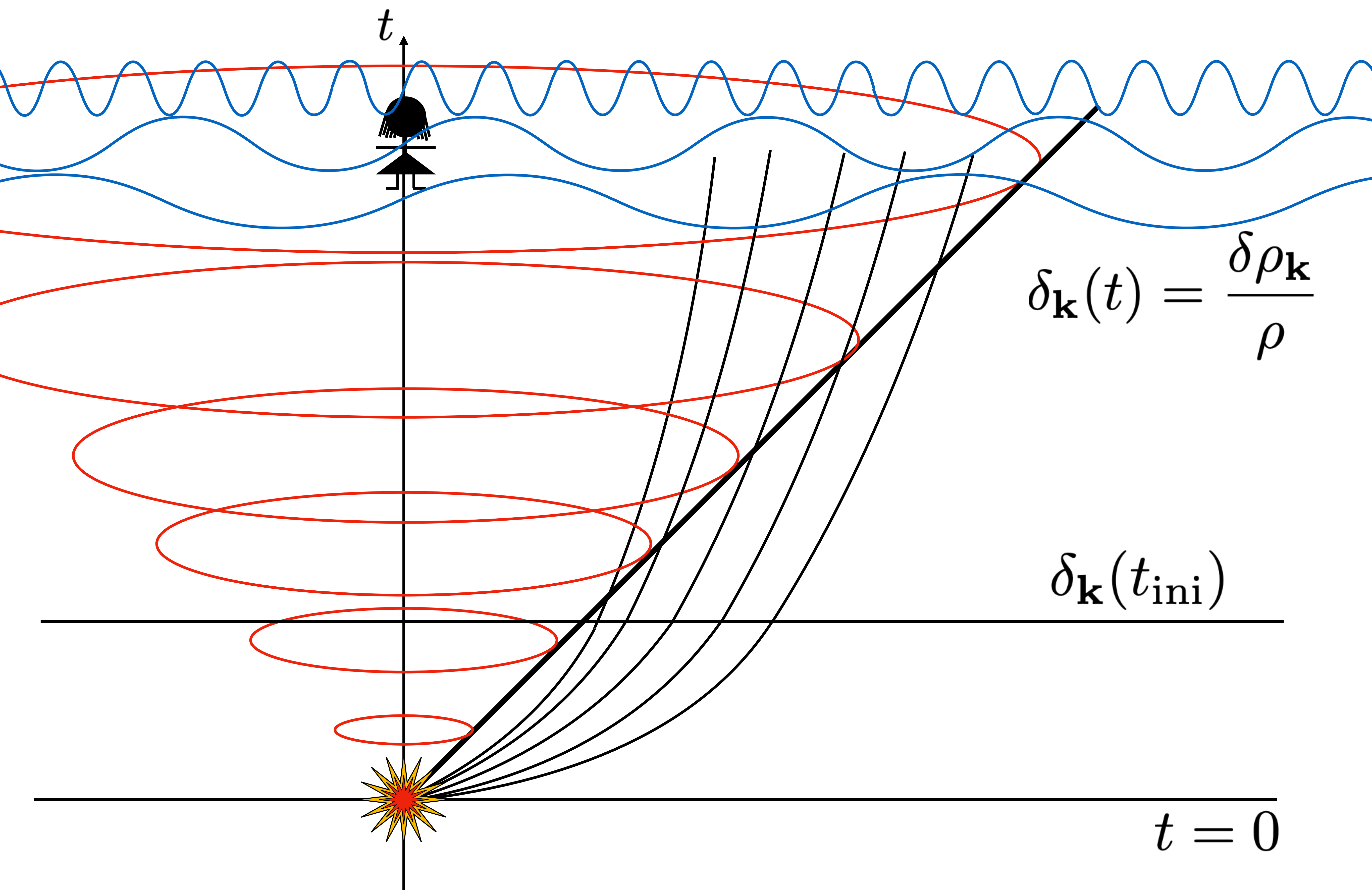
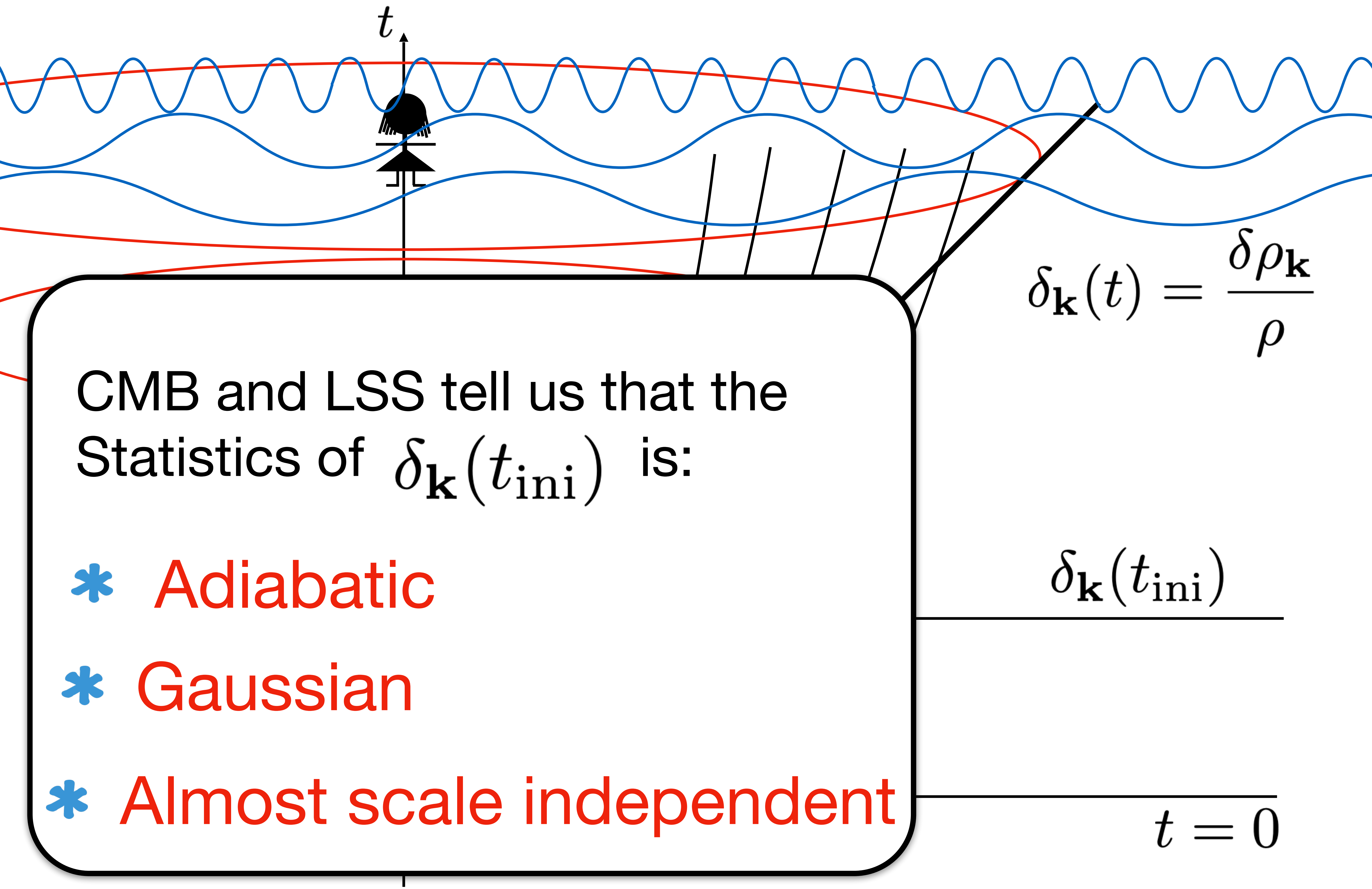


Primordial fluctuations



CMB and LSS tell us that the Statistics of $\delta_{\mathbf{k}}(t_{\rm ini})$ is:

- Adiabatic
- Gaussian
- Almost scale independent

Primordial fluctuations

Adiabaticity

Every inhomogeneity is determined by a single fluctuation

$$\delta^{\gamma}_{\mathbf{k}}(t_{\rm ini}) \propto \mathcal{R}_{\mathbf{k}}$$

$$\delta^{\nu}_{\mathbf{k}}(t_{\rm ini}) \propto \mathcal{R}_{\mathbf{k}}$$

$$\delta^{\rm Bar}_{\mathbf{k}}(t_{\rm ini}) \propto \mathcal{R}_{\mathbf{k}}$$

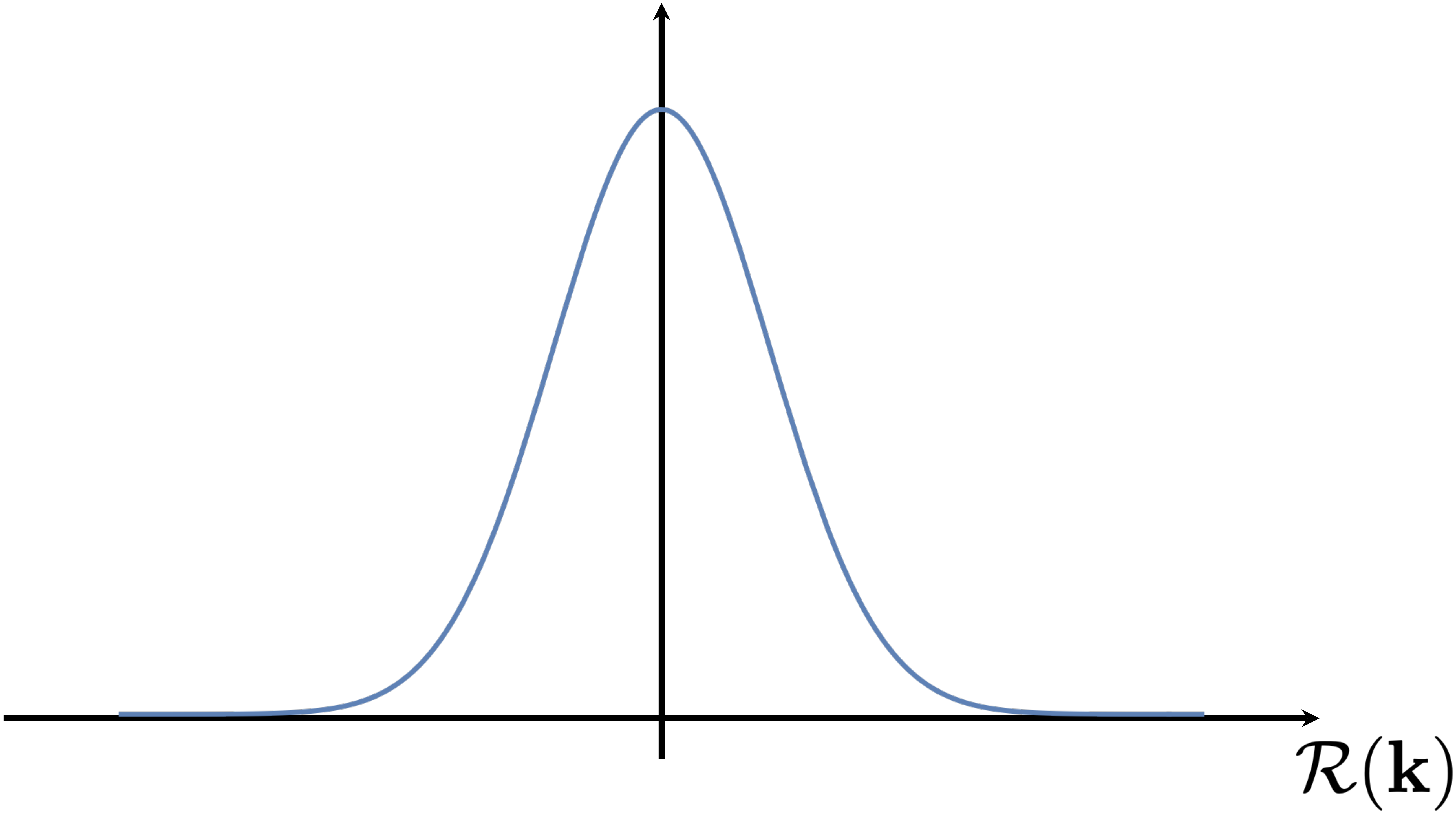
$$\delta^{\rm DM}_{\mathbf{k}}(t_{\rm ini}) \propto \mathcal{R}_{\mathbf{k}}$$

$$ds^2 = -dt^2 + a^2(t)e^{2\mathcal{R}(t,\mathbf{x})}d\mathbf{x}^2$$

Primordial fluctuations

* Gaussianity

$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2}\int_k \frac{|\mathcal{R}_\mathbf{k}|^2}{P(k)}}$$



Primordial fluctuations

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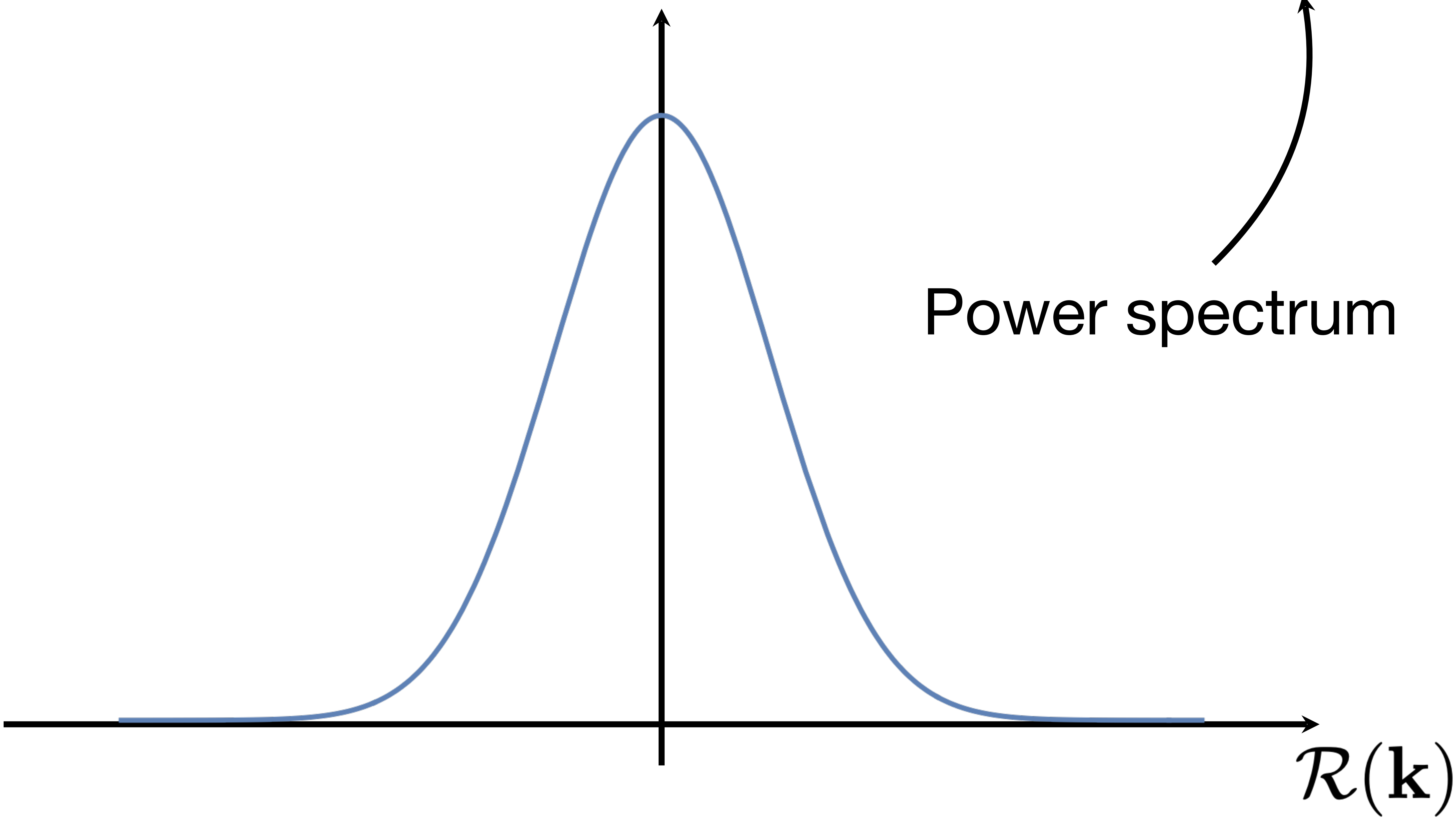
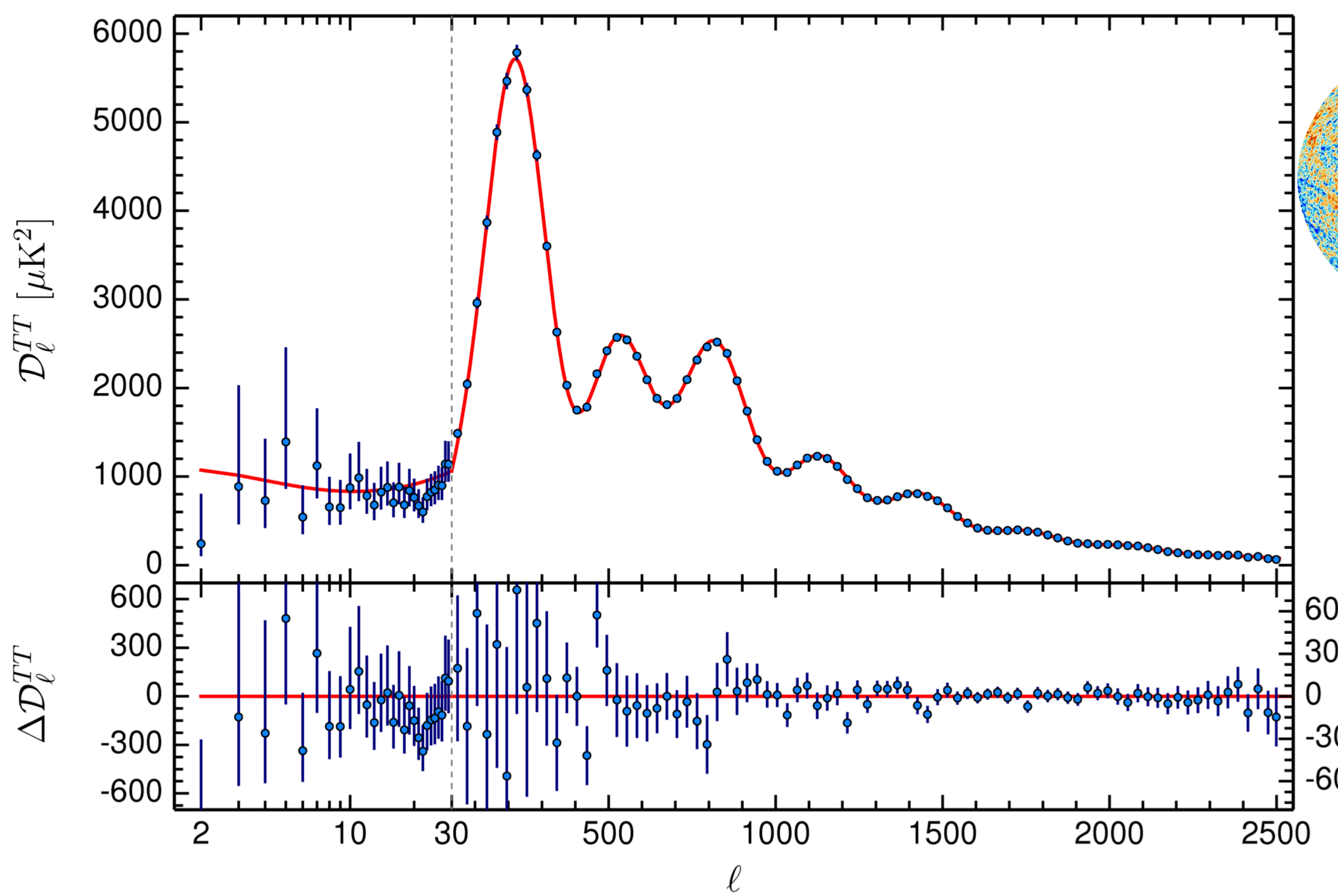
Power spectrum

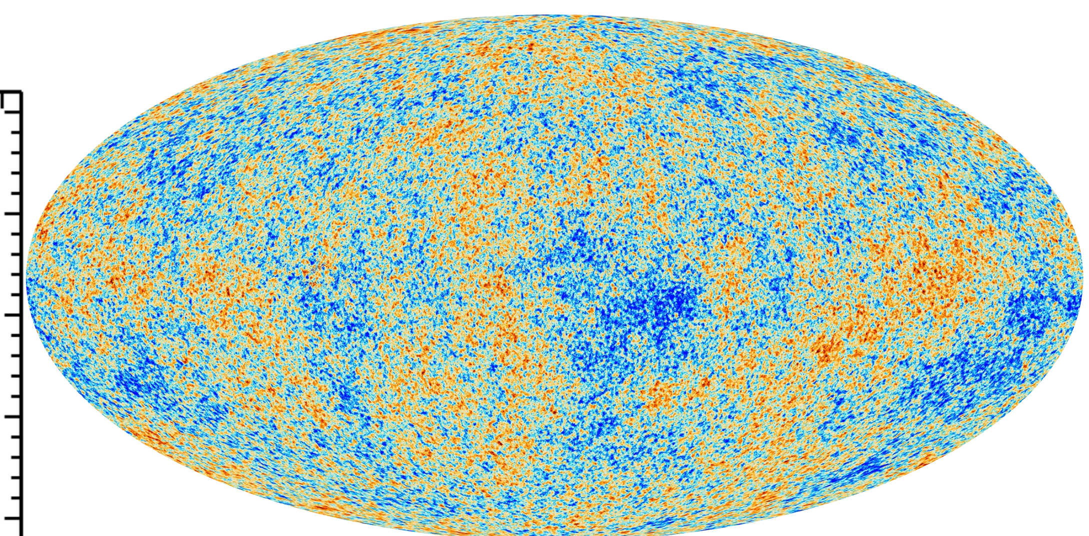


Primordial fluctuations

* Almost scale independent

$$P_{\mathcal{R}}(k) = \frac{2\pi^2}{k^3}\Delta_{\mathcal{R}}(k) \qquad \Delta_{\mathcal{R}}(k) = A\left(\frac{k}{k_*}\right)^{n_s-1}$$





$$n_s \simeq 0.96$$

Planck collaboration (2018)

Non-Gaussianity?

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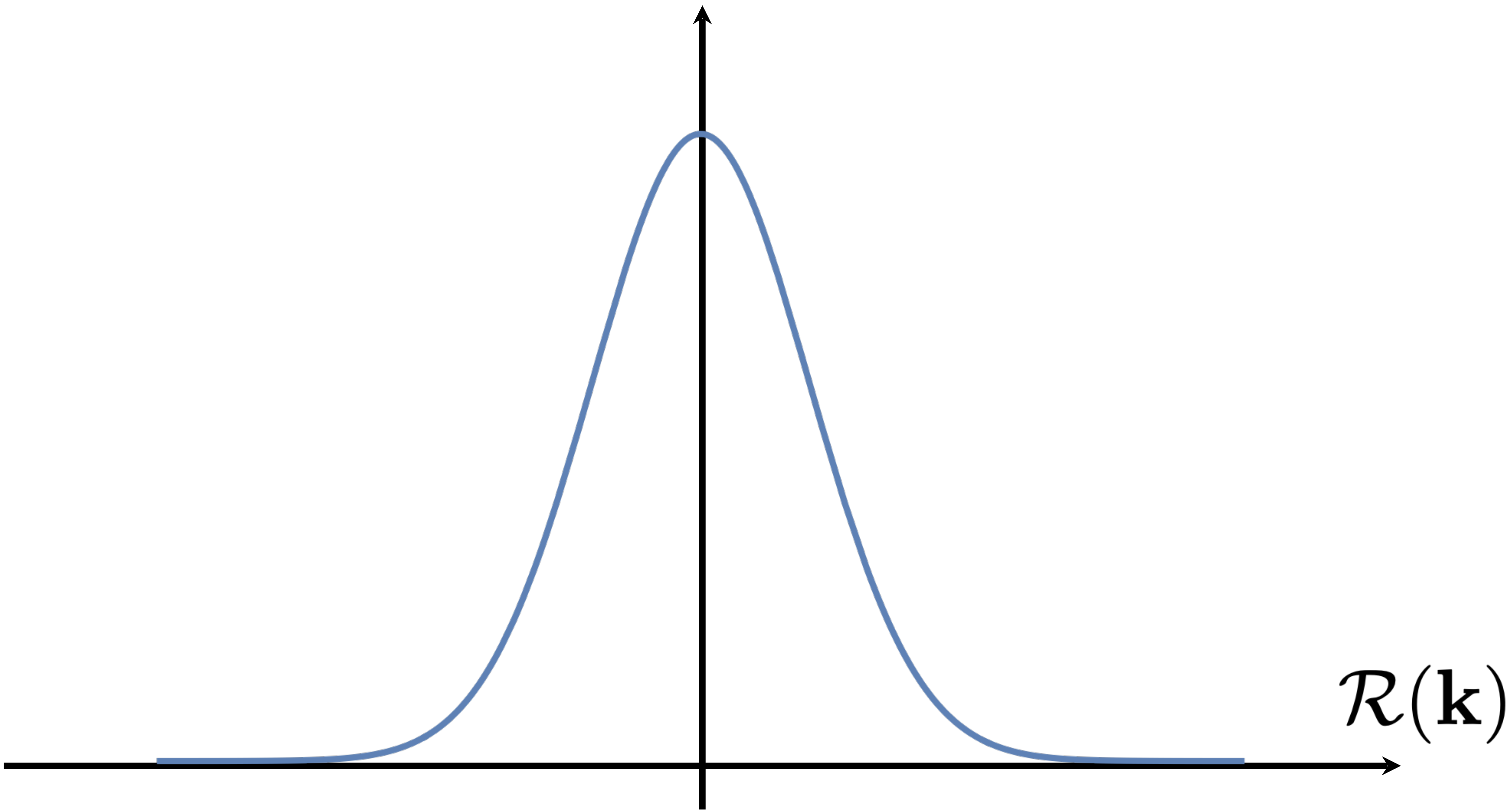
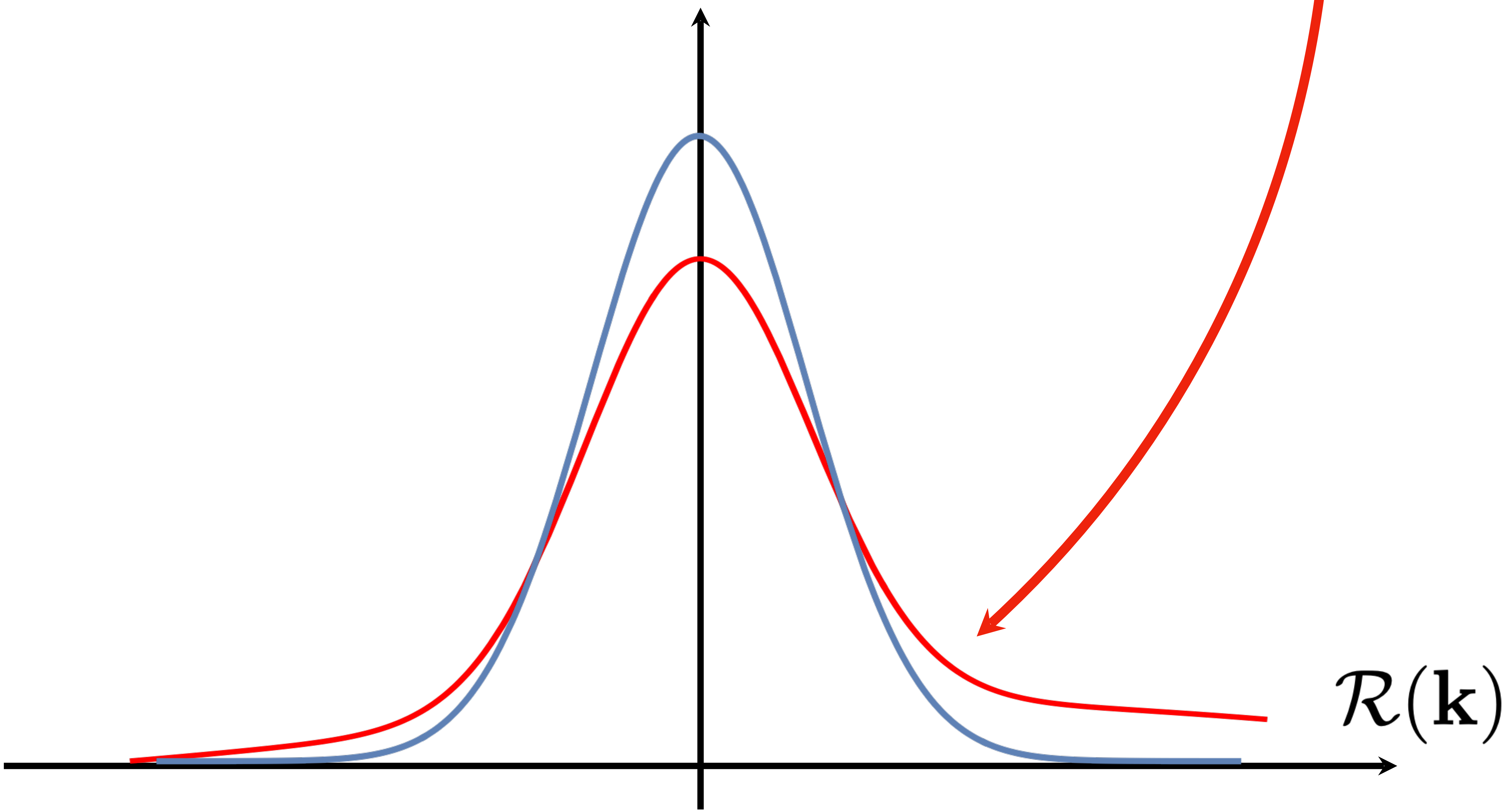


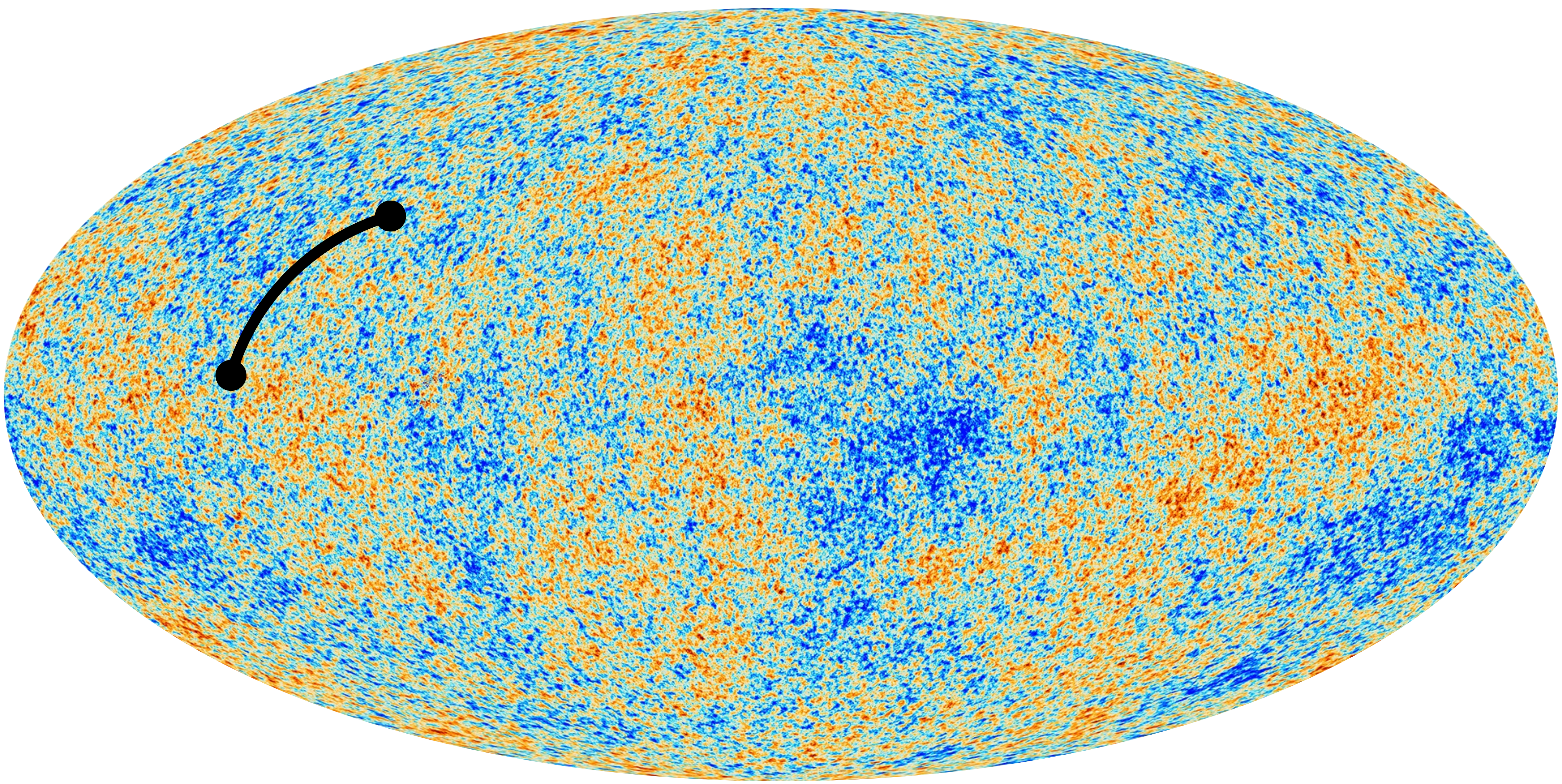
Non-Gaussianity?

$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2}\int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}} + \cdots$$



Non-Gaussianity?

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \rangle = \int \mathcal{D}\mathcal{R}\ \rho[\mathcal{R}]\ \times\ \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2}$$



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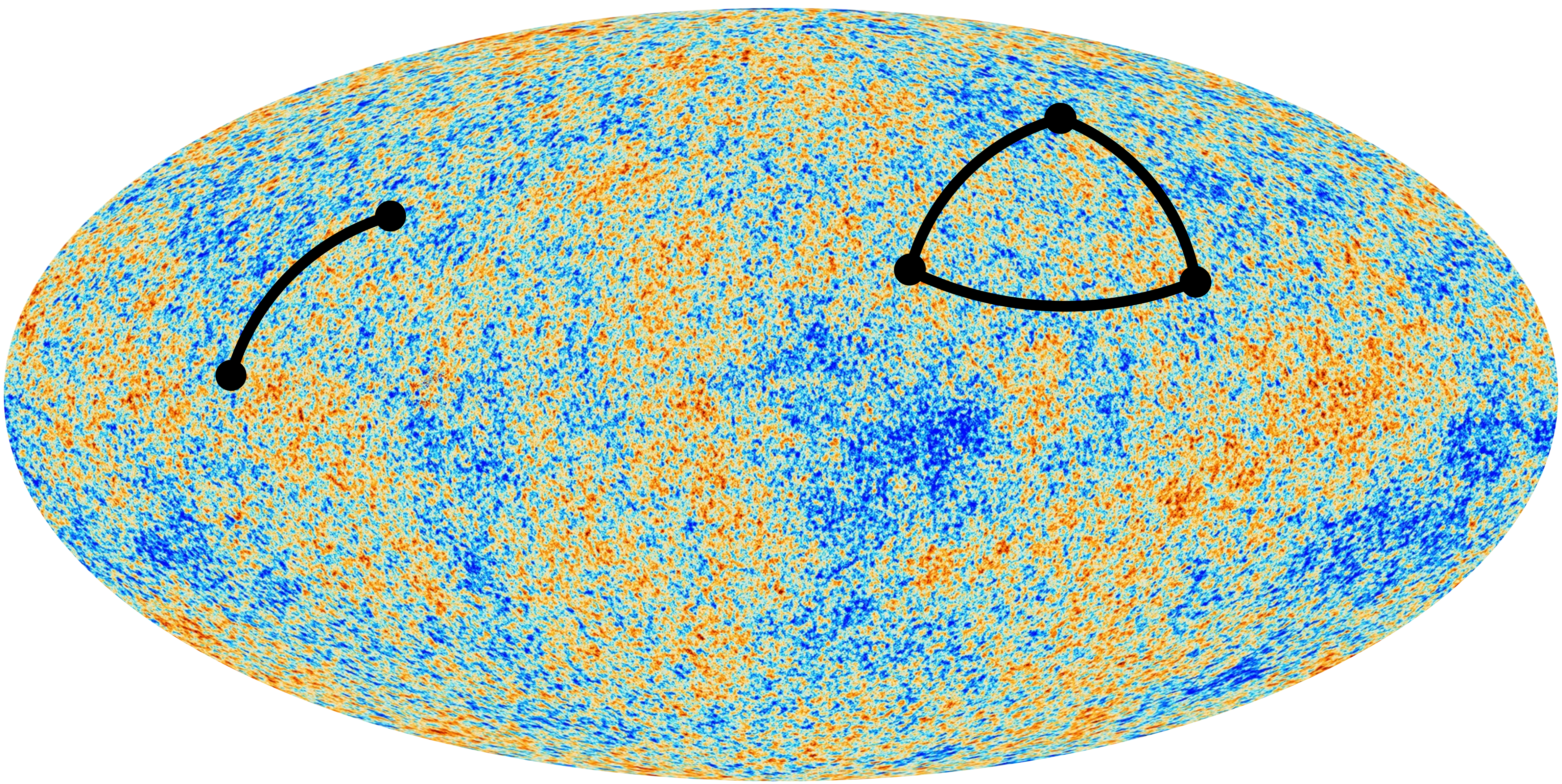
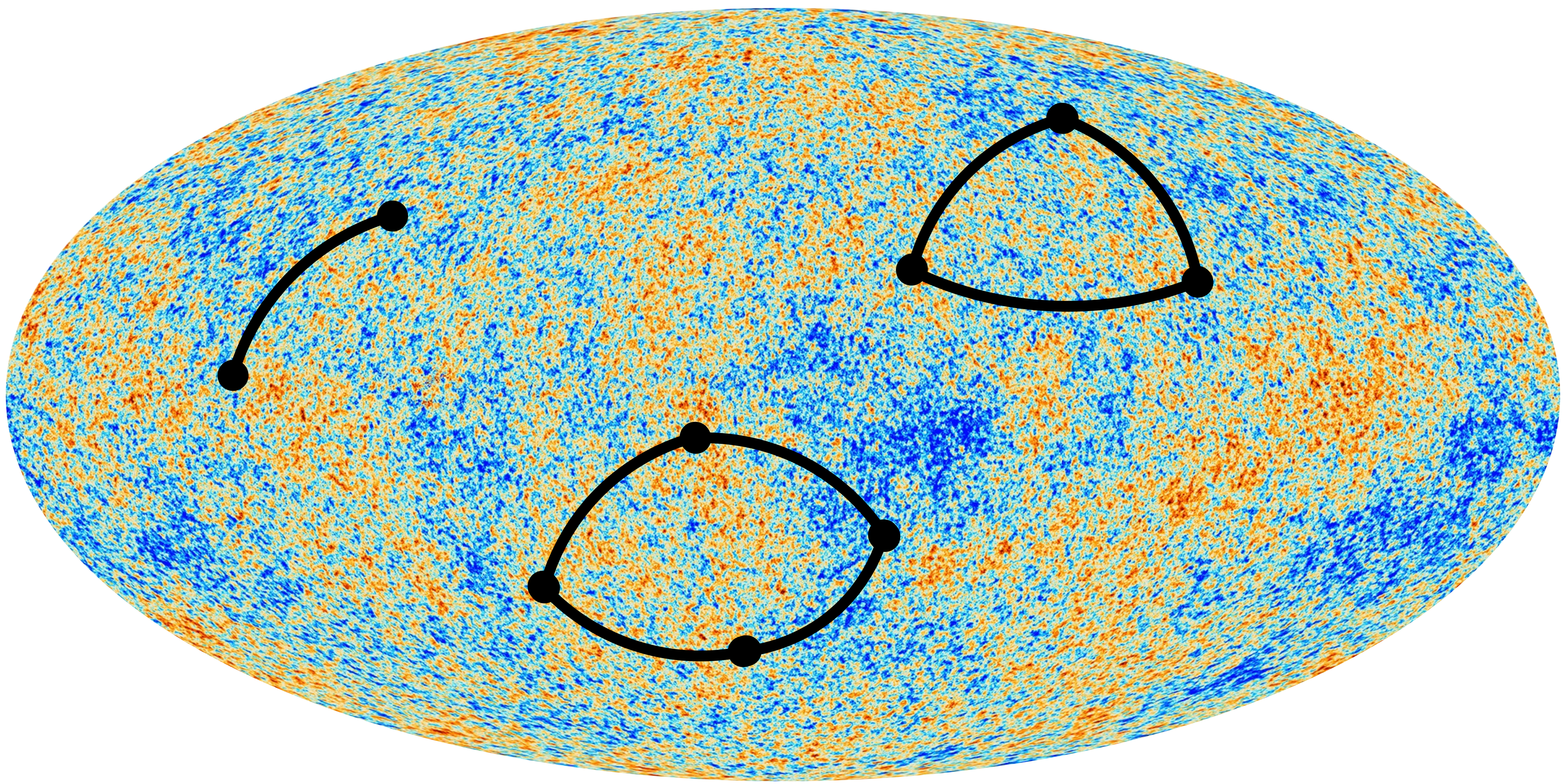


Non-Gaussianity?

$$\langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\mathcal{R}_{\mathbf{k}_4}\rangle = \int \mathcal{D}\mathcal{R}\ \rho[\mathcal{R}] \times \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\mathcal{R}_{\mathbf{k}_4}$$



Non-Gaussianity?

The bispectrum parametrizes the simplest deviation to NG

$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2}\int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}} + \cdots$$

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$$+\cdots = \int_{k_1}\int_{k_2}\int_{k_3} B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_2)\mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3} + \cdots$$

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$$\langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

Non-Gaussianity?

In the absence of a specific theory, a common parametrization for the bispectrum is the $f_{\rm NL}$ parameter

$$\mathcal{R}(\mathbf{x}) = \mathcal{R}_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL}^{\rm loc} \mathcal{R}_G^2(\mathbf{x})$$

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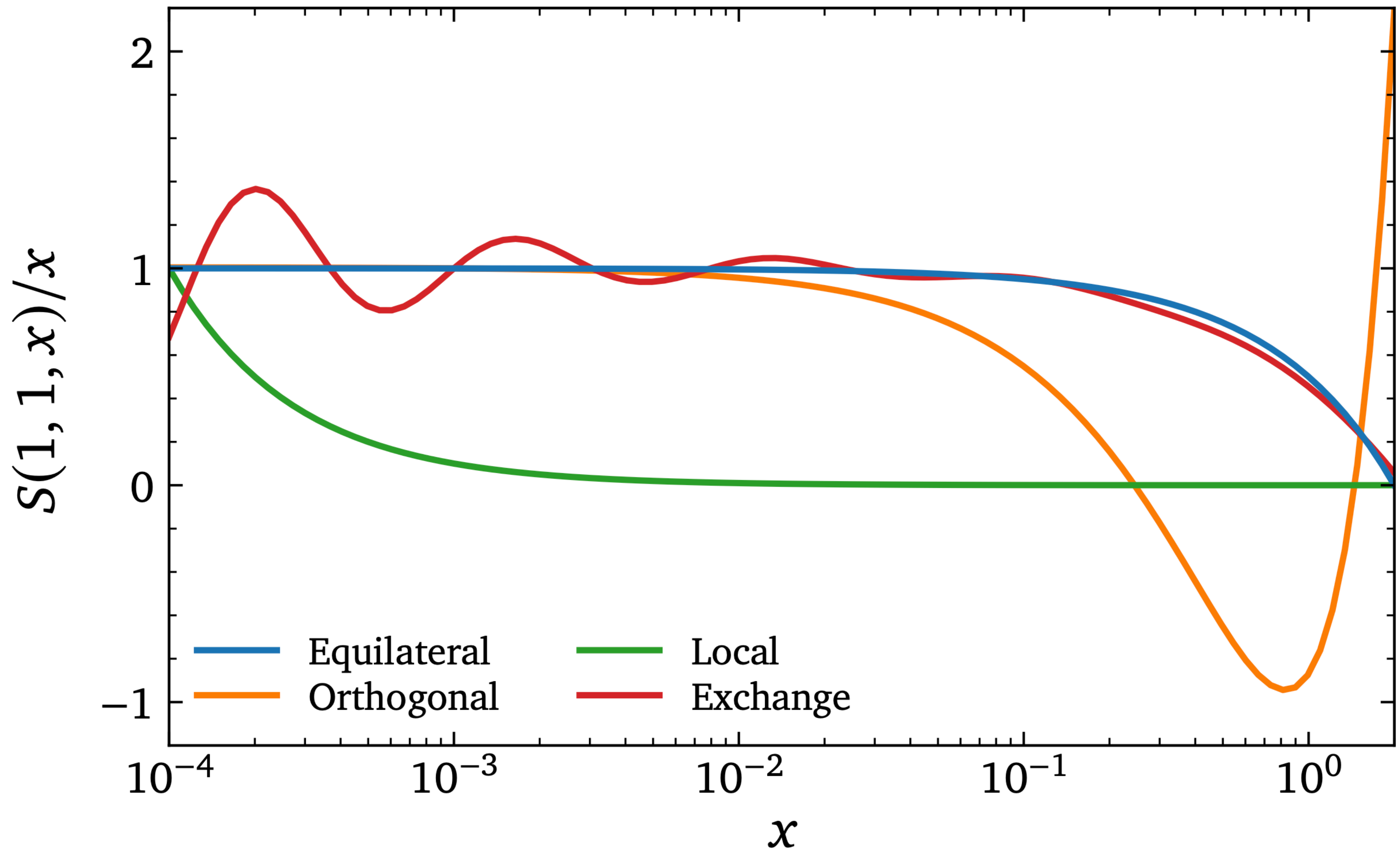
$$\boxed{B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{18}{5} A^2 \sum_{\rm type} f_{\rm NL}^{\rm type} S_{\rm type}(k_1, k_2, k_3)}$$

Non-Gaussianity?

In the … the
bispe…



Pimentel, Wallisch, Wu, Achúcarro, GAP et al. (2022)

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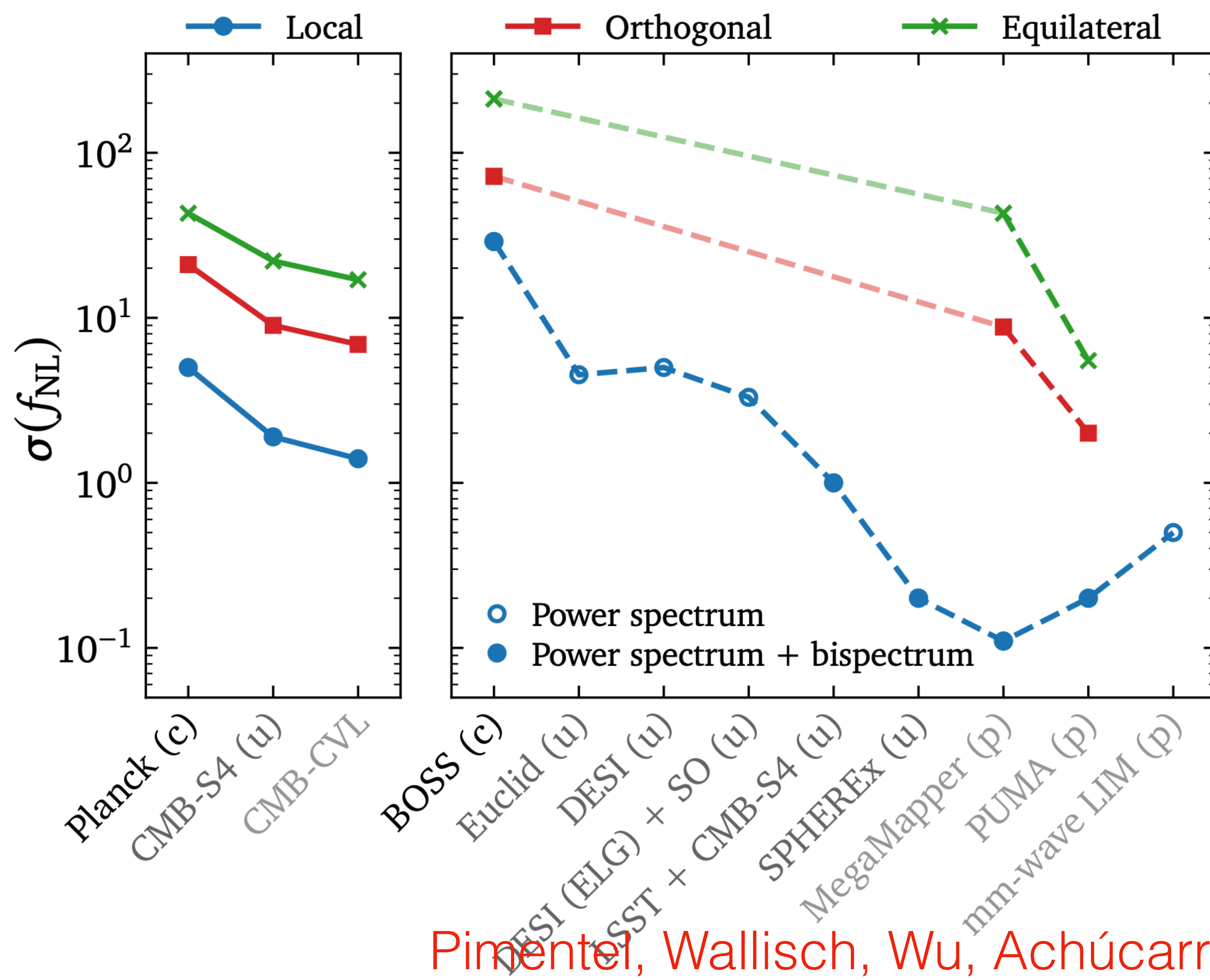

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$B($ $\mathbf{k}_1))$

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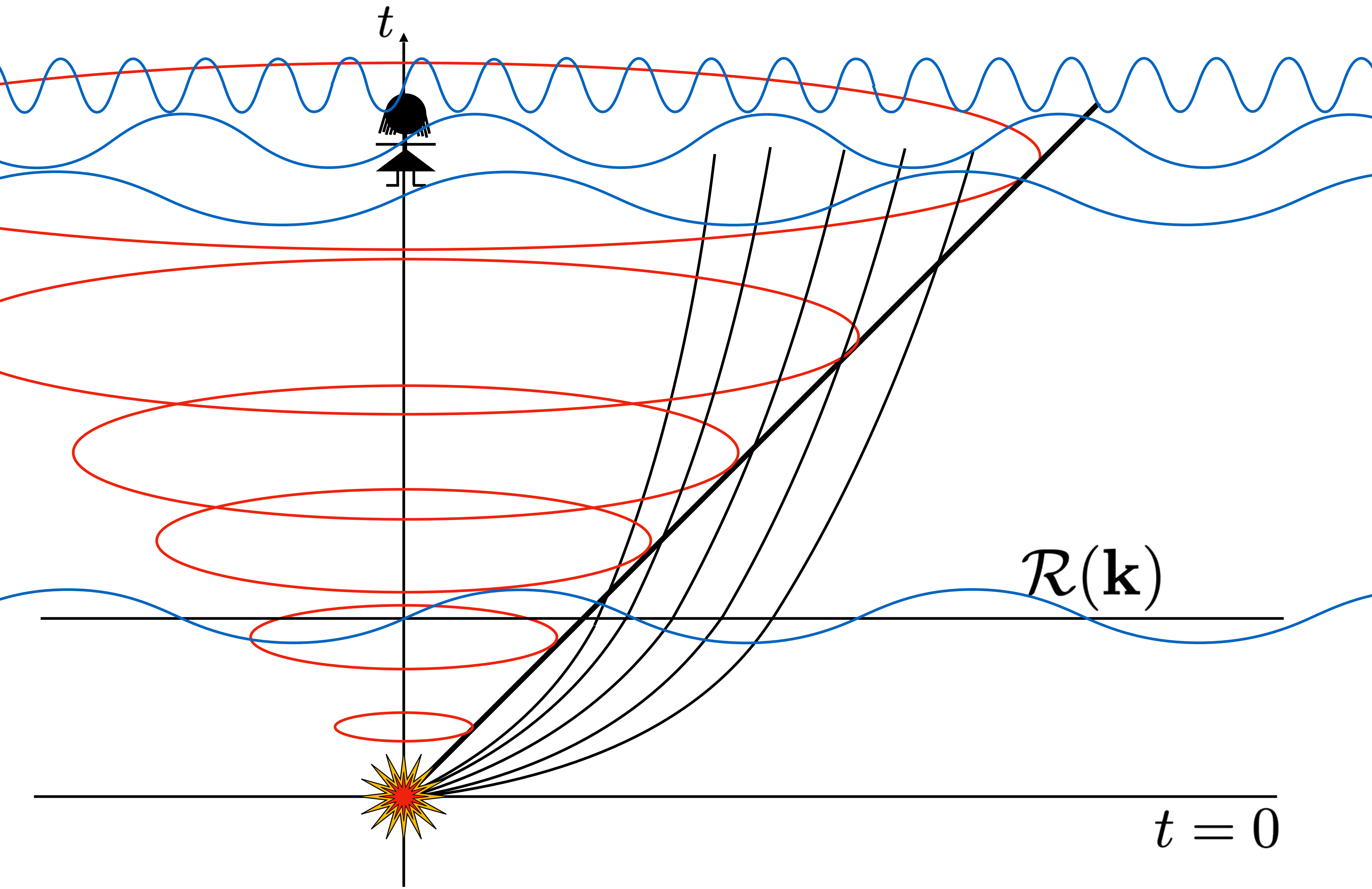
Inflation



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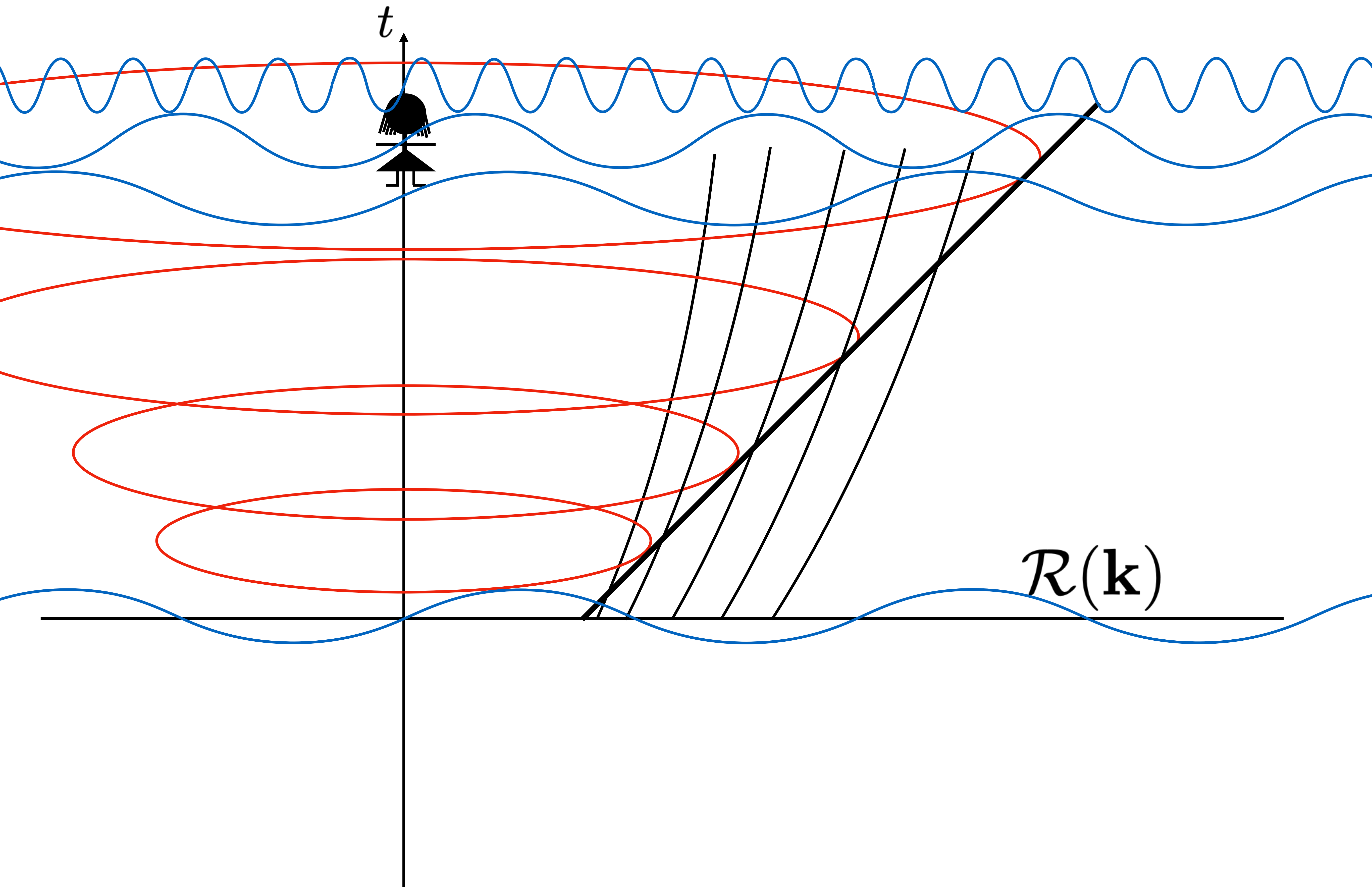
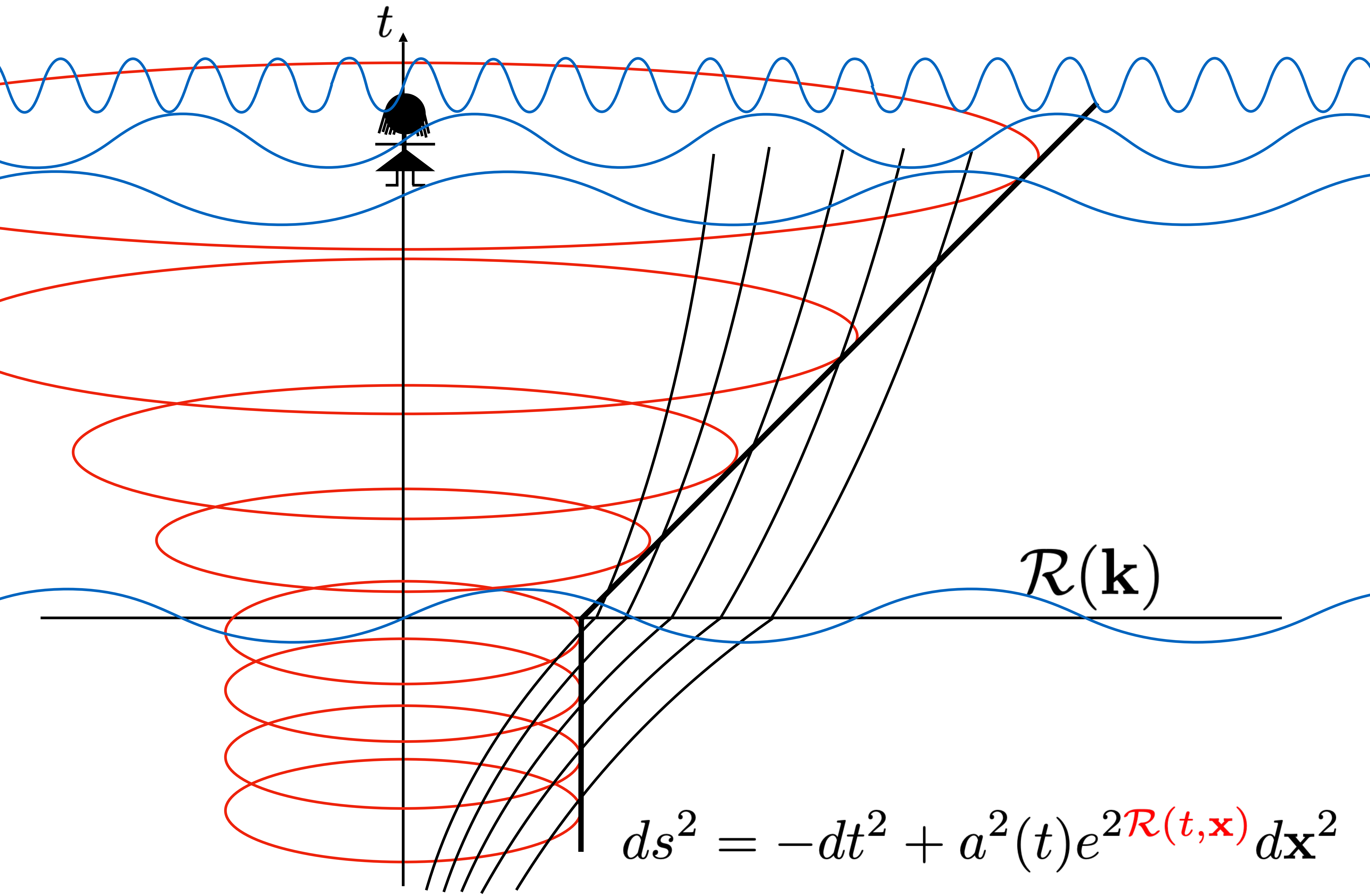
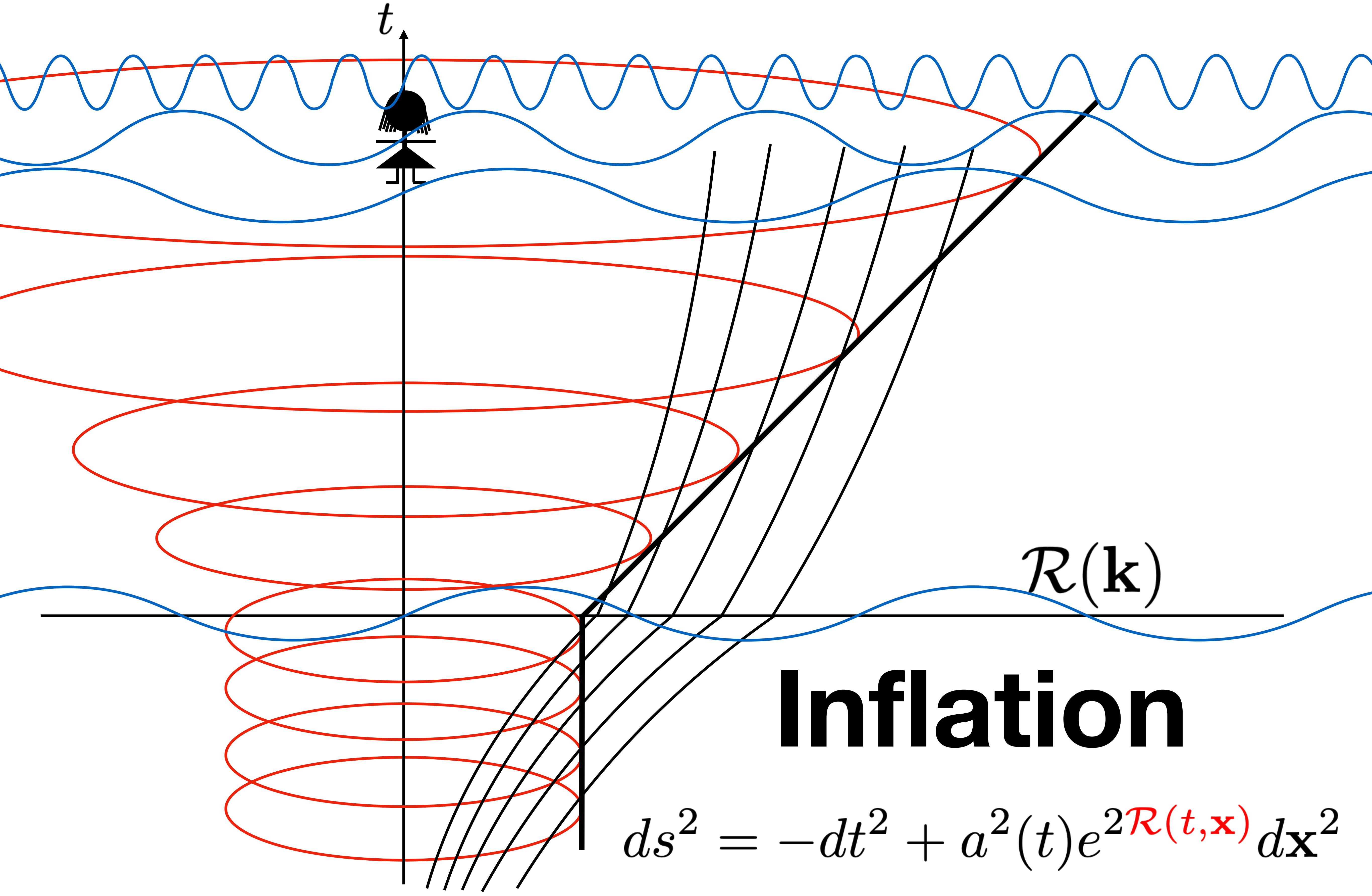




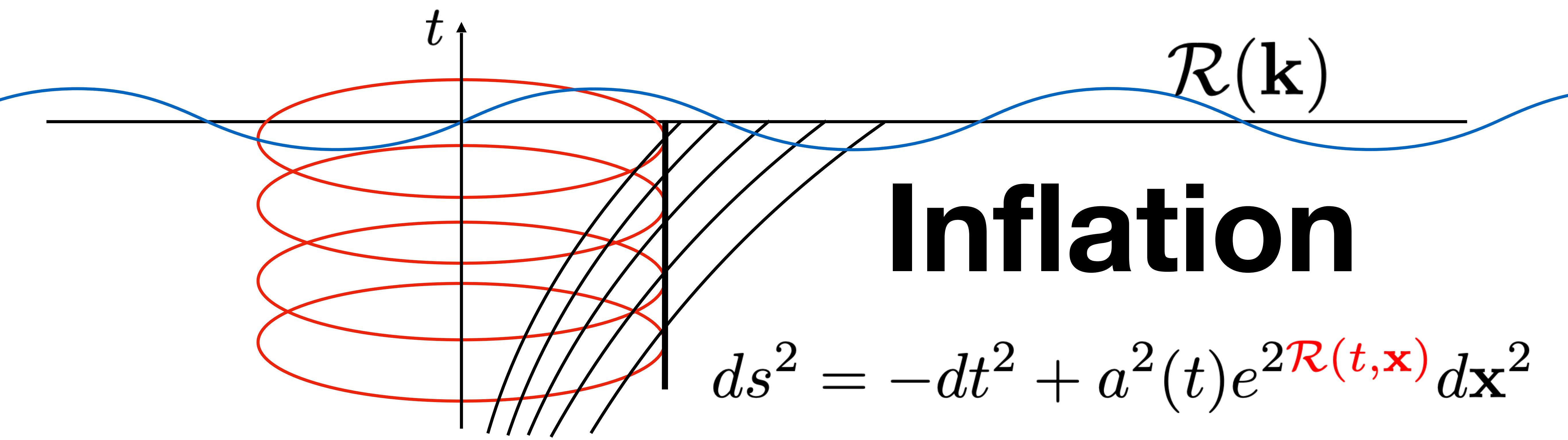
$$ds^2 = -dt^2 + a^2(t)e^{2\mathcal{R}(t,\mathbf{x})}d\mathbf{x}^2$$



Inflation

$$ds^2 = -dt^2 + a^2(t)e^{2\mathcal{R}(t,\mathbf{x})}d\mathbf{x}^2$$

We can now write a theory for $\mathcal{R}(\mathbf{k})$



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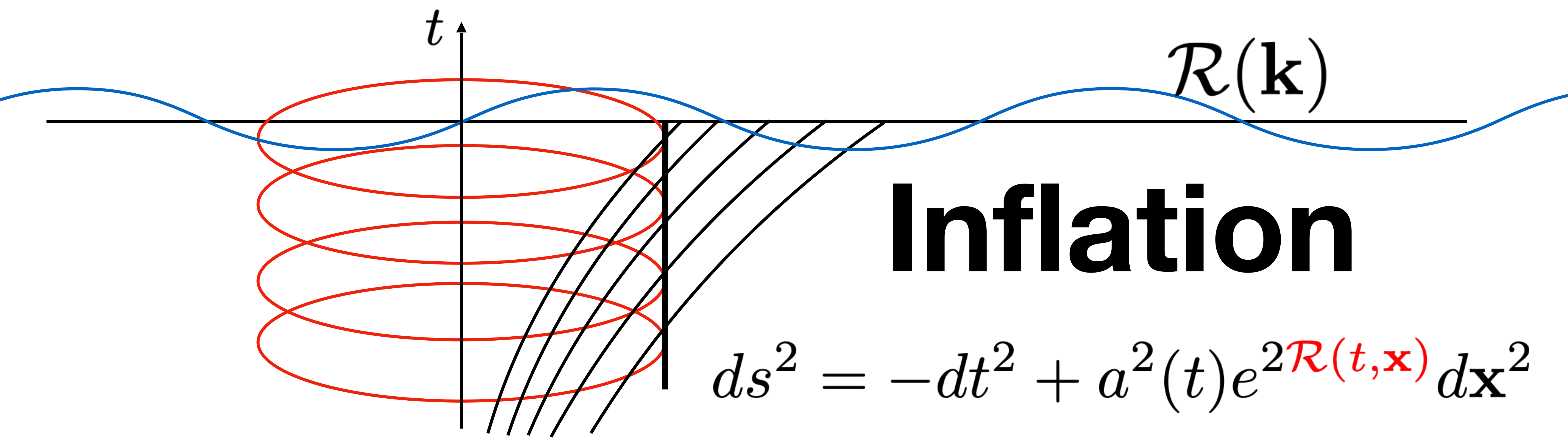
We can now write a theory for $\mathcal{R}(\mathbf{k})$

$$S = \int d^4x\, a^3 \epsilon \left[\dot{\mathcal{R}}^2 - \frac{1}{a^2} (\nabla \mathcal{R})^2 + \mathcal{O}(\mathcal{R}^3) + \mathcal{O}(\mathcal{R}^4) + \cdots \right]$$

There are three favorite ways to compute observables from this theory



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(1) Hamiltonian in-in formalism:

$$H(t) = H^{(2)}(t) + H^{(3)}(t) + \cdots$$

$$U(t) = \mathcal{T} \exp\left\{ -i \int_{-\infty}^{t} dt' \; H_I(t') \right\}$$

$$\langle \mathcal{R}_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t) \rangle = \langle 0 | U^\dagger(t) \mathcal{R}^I_{\mathbf{k}_1}(t) \cdots \mathcal{R}^I_{\mathbf{k}_N}(t) U(t) | 0 \rangle$$

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There are three favorite ways to compute observables from this theory

(2) Wavefunction of the Universe:

$$\Psi[\mathcal{R}(\mathbf{x})] = \int_{\mathcal{R}_{BD}}^{\mathcal{R}(\mathbf{x})} D\mathcal{R} e^{iS[\mathcal{R}]/\hbar} \qquad \rho[\mathcal{R}(\mathbf{x})] = \Big|\Psi[\mathcal{R}(\mathbf{x})]\Big|^2$$

$$\langle \mathcal{R}_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t) \rangle = \int D\mathcal{R}(\mathbf{x}) \rho[\mathcal{R}(\mathbf{x})] R_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t)$$

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There are three favorite ways to compute observables from this theory

(3) Schwinger-Keldysh in-in formalism:

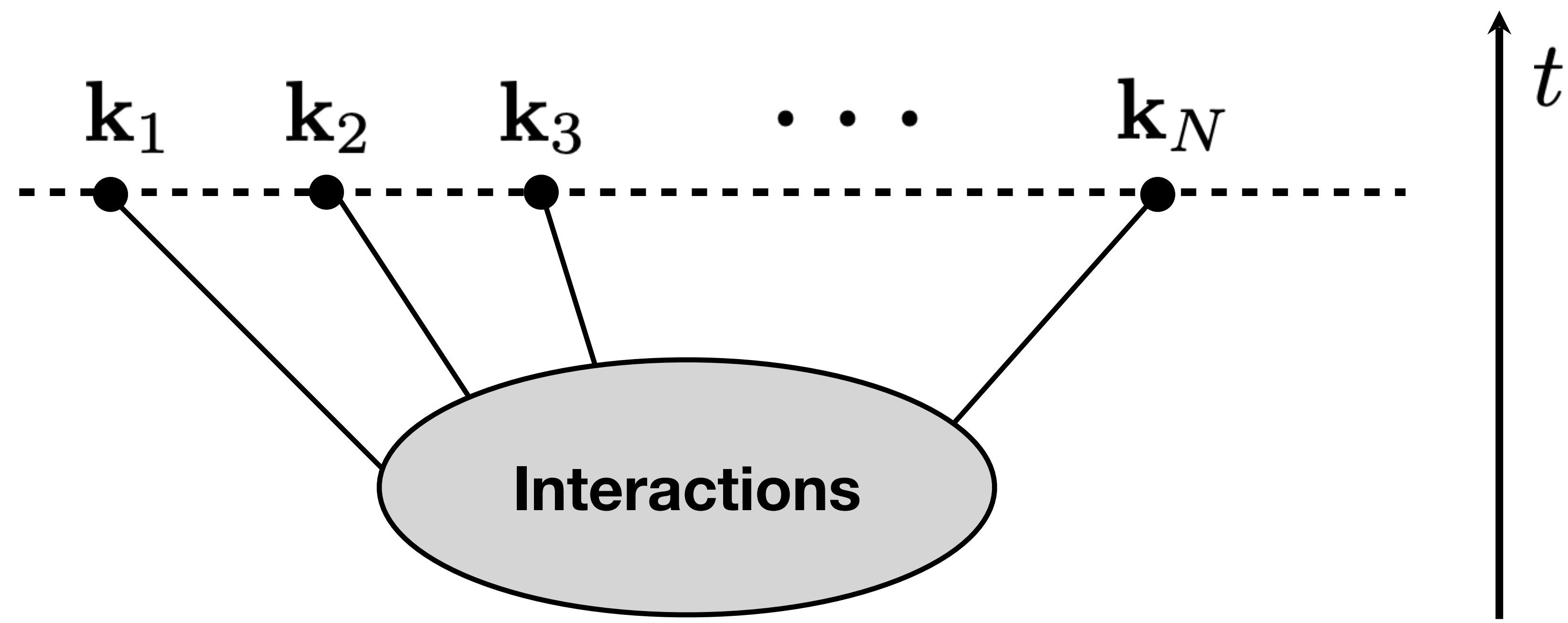
$$Z[J_+, J_-] = \int_{\mathcal{R}_+(t_{\rm end}) = \mathcal{R}_-(t_{\rm end})} D\mathcal{R}_+ D\mathcal{R}_- e^{iS[\mathcal{R}_+] - iS[\mathcal{R}_+] + i\int_x R_+ J_+ - i\int_x R_- J_-}$$

$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_N} \rangle = \frac{\delta}{\delta J_+(\mathbf{k}_1)} \cdots \frac{\delta}{\delta J_+(\mathbf{k}_N)} Z[J_+, J_-]$$

Inflation

These methods offer a diagramatic procedure to compute n-points

$$\langle \mathcal{R}_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t) \rangle =$$



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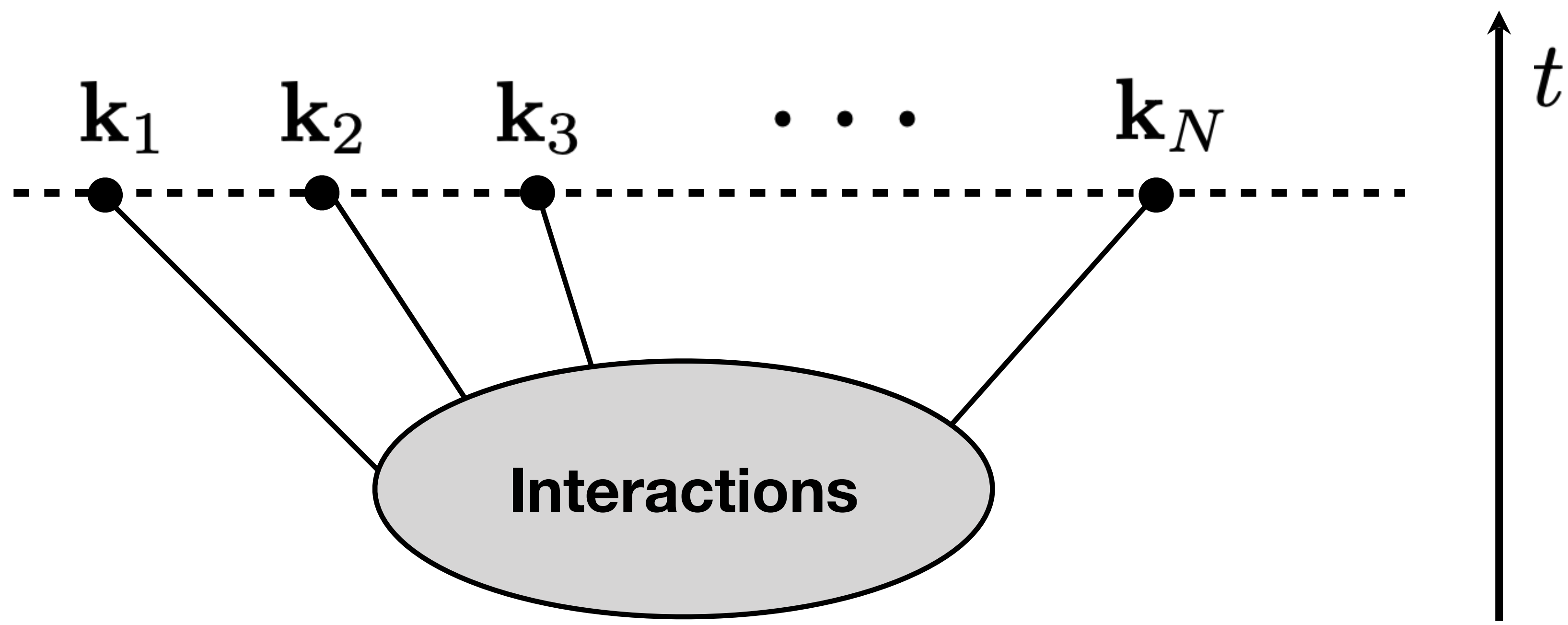
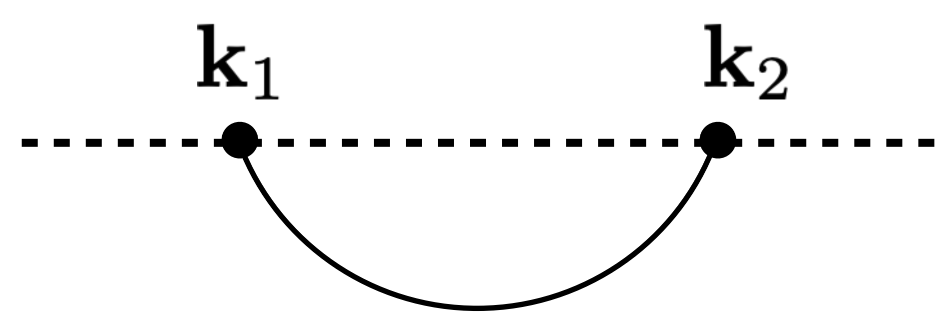


Power spectrum

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \rangle =$$



$$= (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}}(k)$$

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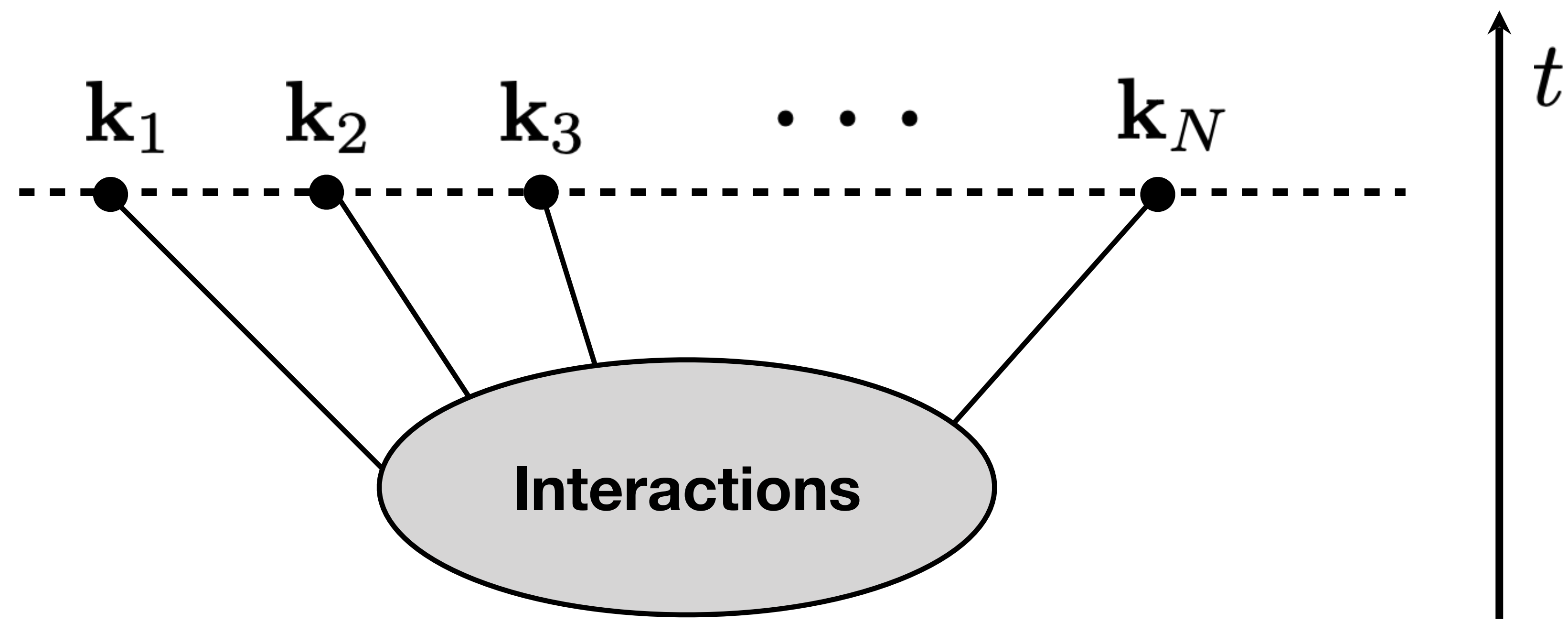
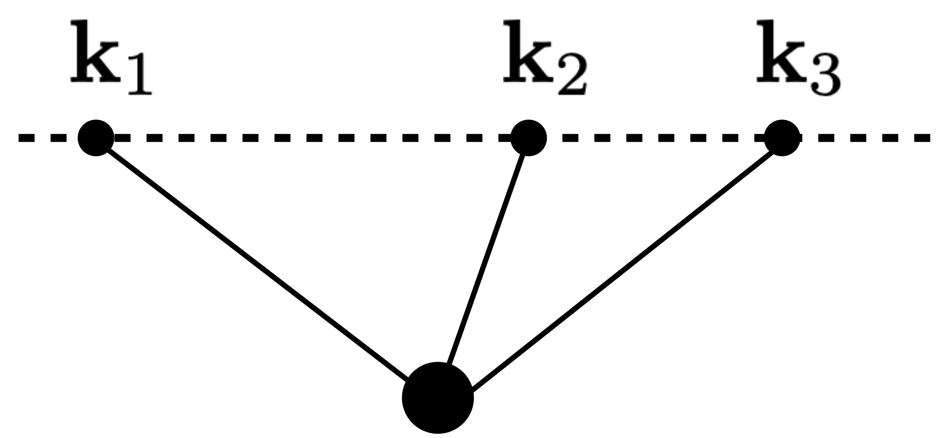


Bi-spectrum

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle =$$



$$= (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

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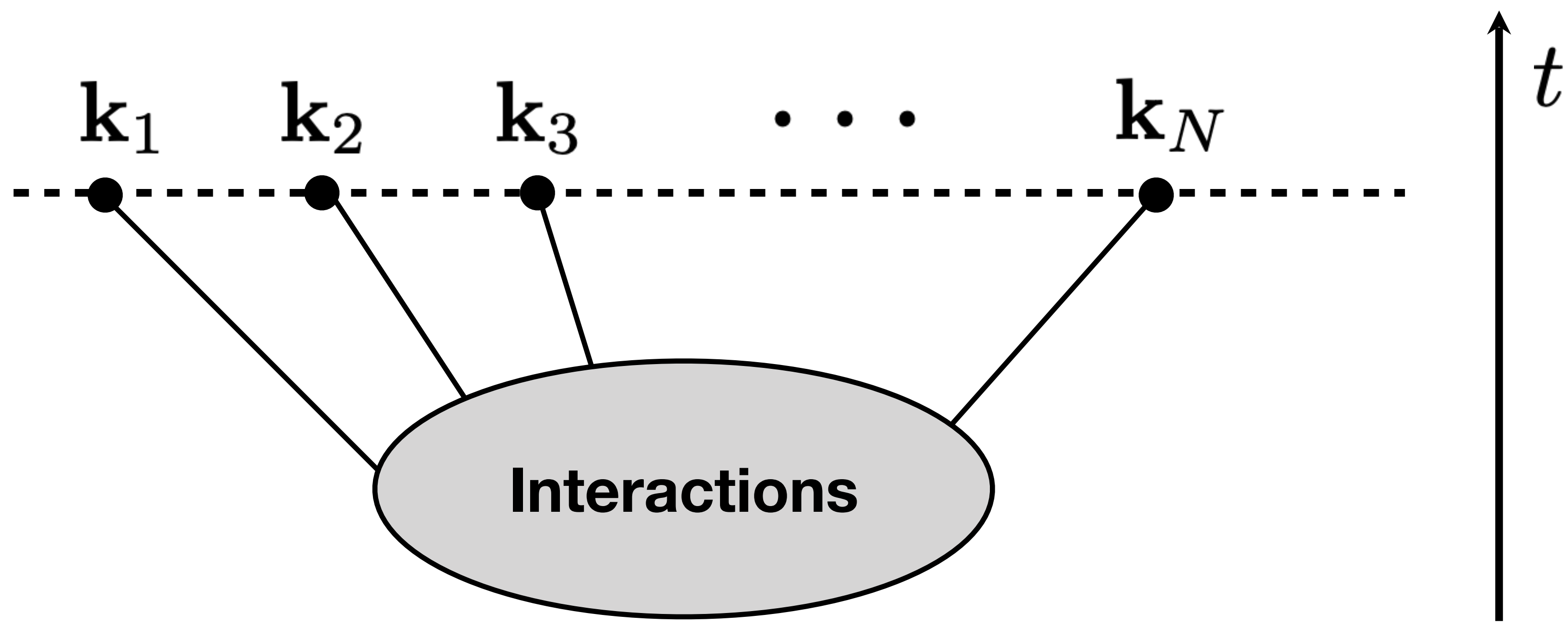
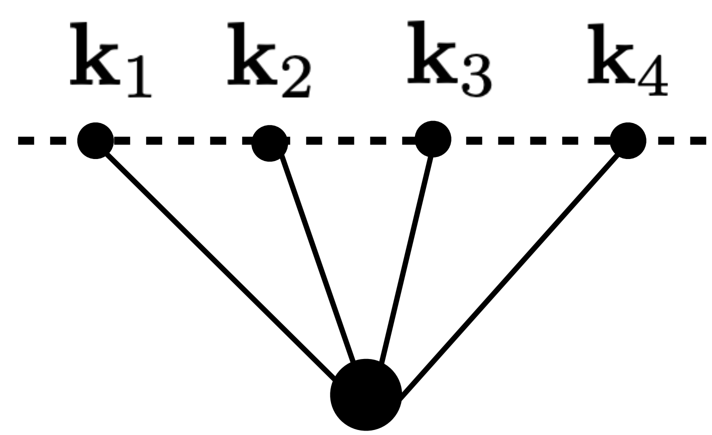


Tri-spectrum

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \mathcal{R}_{\mathbf{k}_4} \rangle =$$



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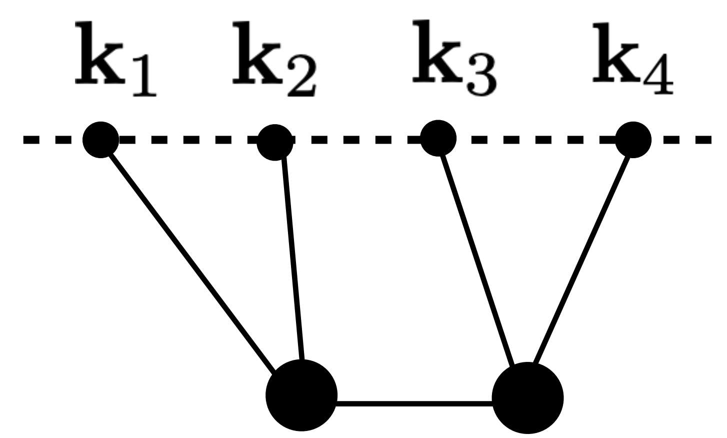

$$= (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) \tau(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_3)$$

Inflation

Single-field slow-roll inflation predicts small amounts of NG:

$$f_{\mathrm{NL}}^{\mathrm{type}} \simeq \mathcal{O}(\epsilon, \eta)$$

Maldacena (2002)

$$f_{\mathrm{NL}}^{\mathrm{loc}} = 0$$

Tanaka & Urakawa (2011)
Pajer, Schmidt & Zaldarriaga (2013)

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More general types of single-field inflation can enhance NG:

$$\mathcal{L} = \epsilon\left(c_s^2\dot{\mathcal{R}}^2 - \frac{1}{a^2}(\nabla\mathcal{R})^2\right) + \left(\frac{1}{c_s^2} - 1\right) \times \mathcal{O}(\mathcal{R}^3) + \cdots$$

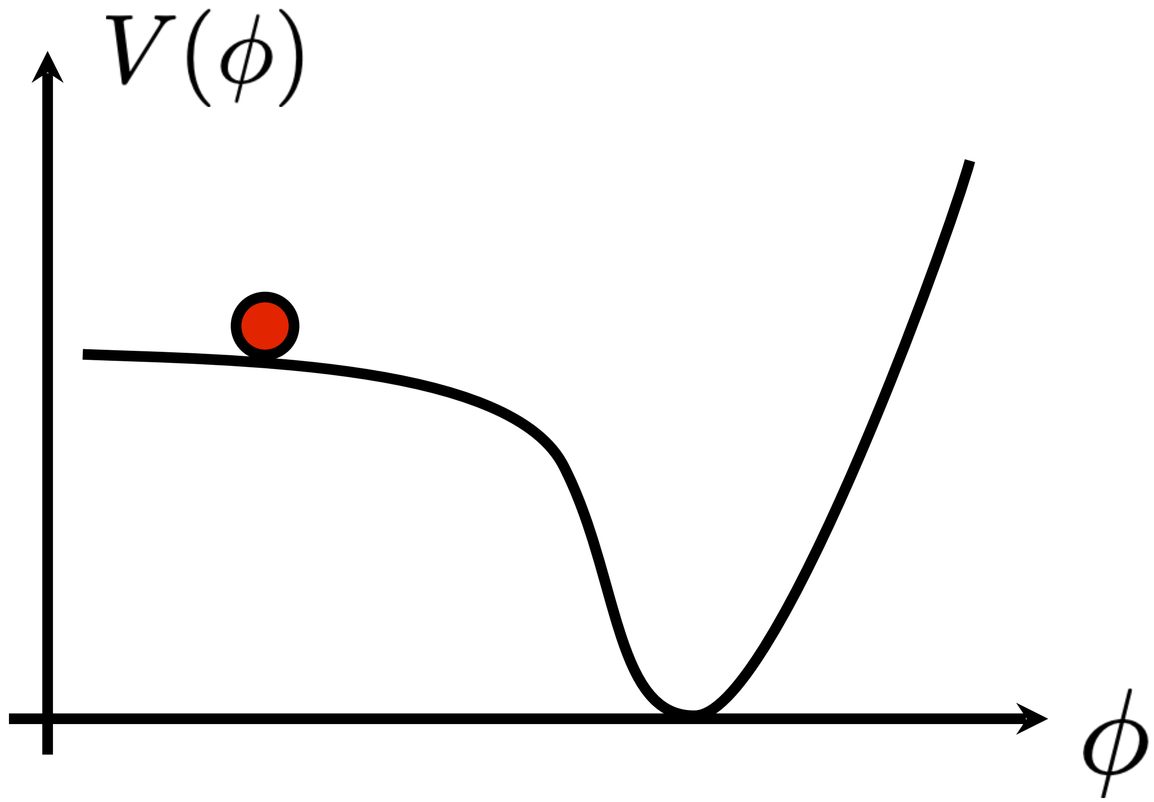
$$f_{\rm NL}^{\rm equil} \simeq \mathcal{O}\left(\frac{1}{c_s^2} - 1\right)$$

Chen, Huang, Kachru & Shiu (2007)

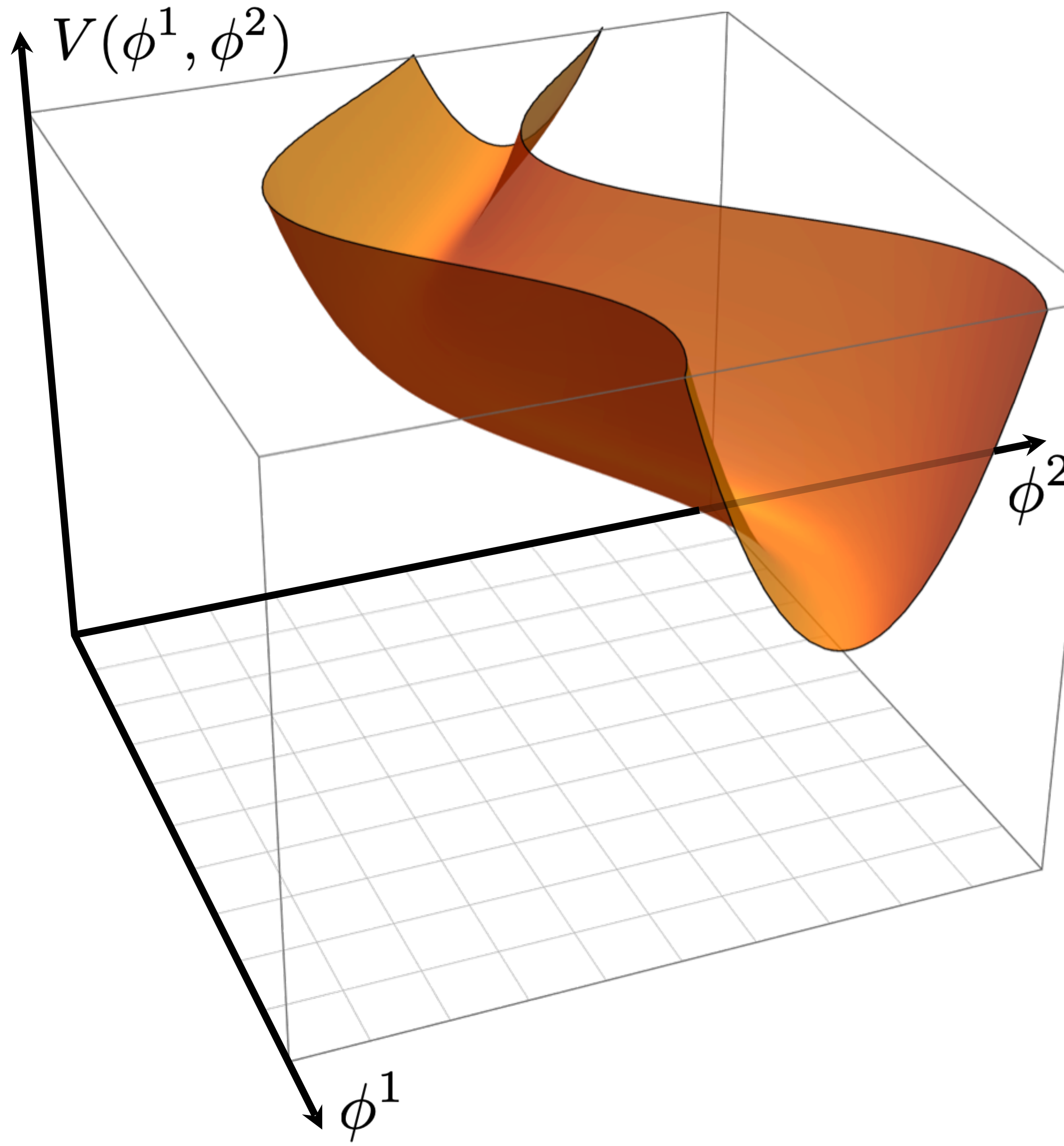
$$f_{\rm NL}^{\rm loc} = 0$$

Creminelli & Zaldarriaga (2004)

Multi-field inflation



Multi-field inflation



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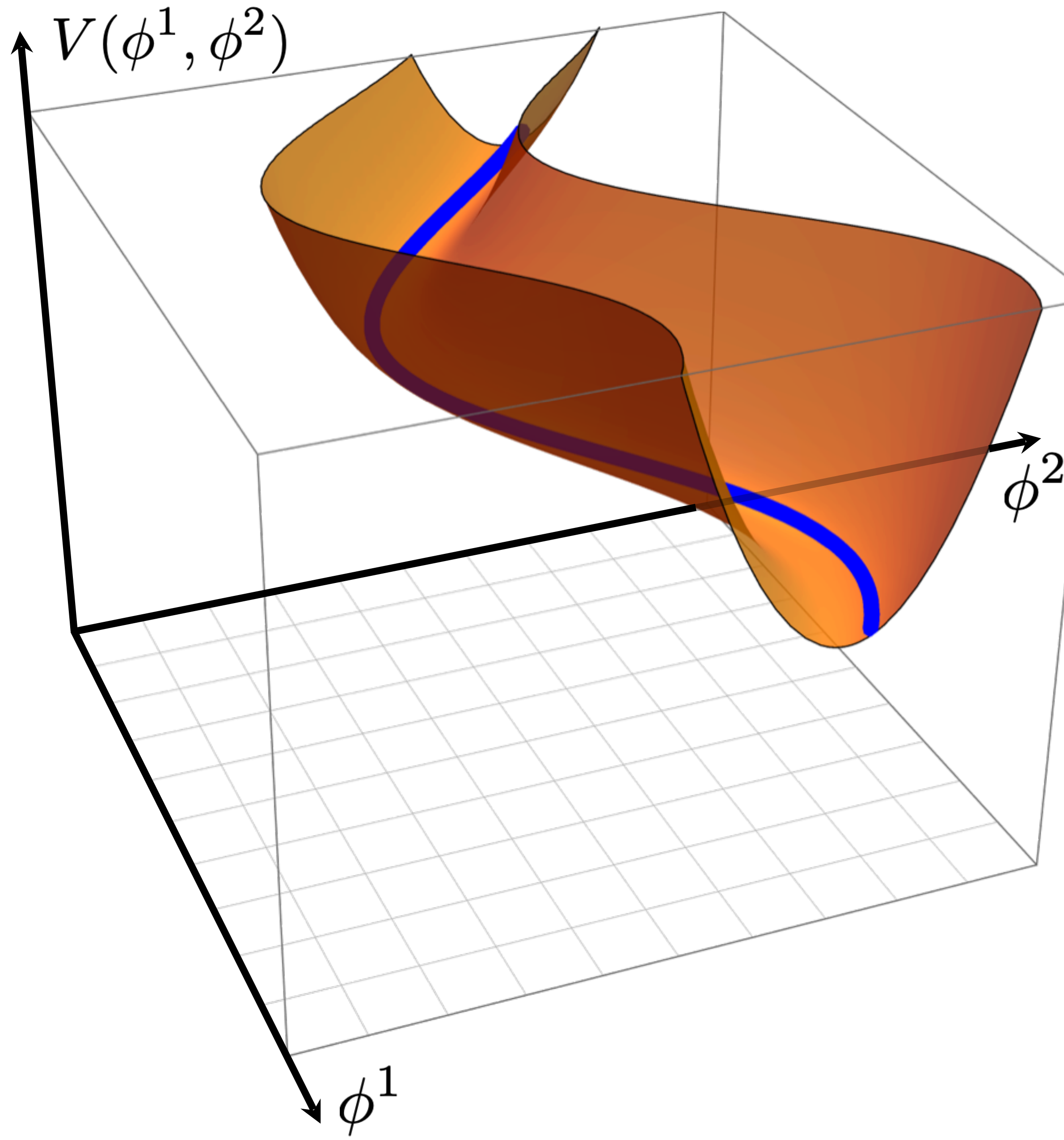

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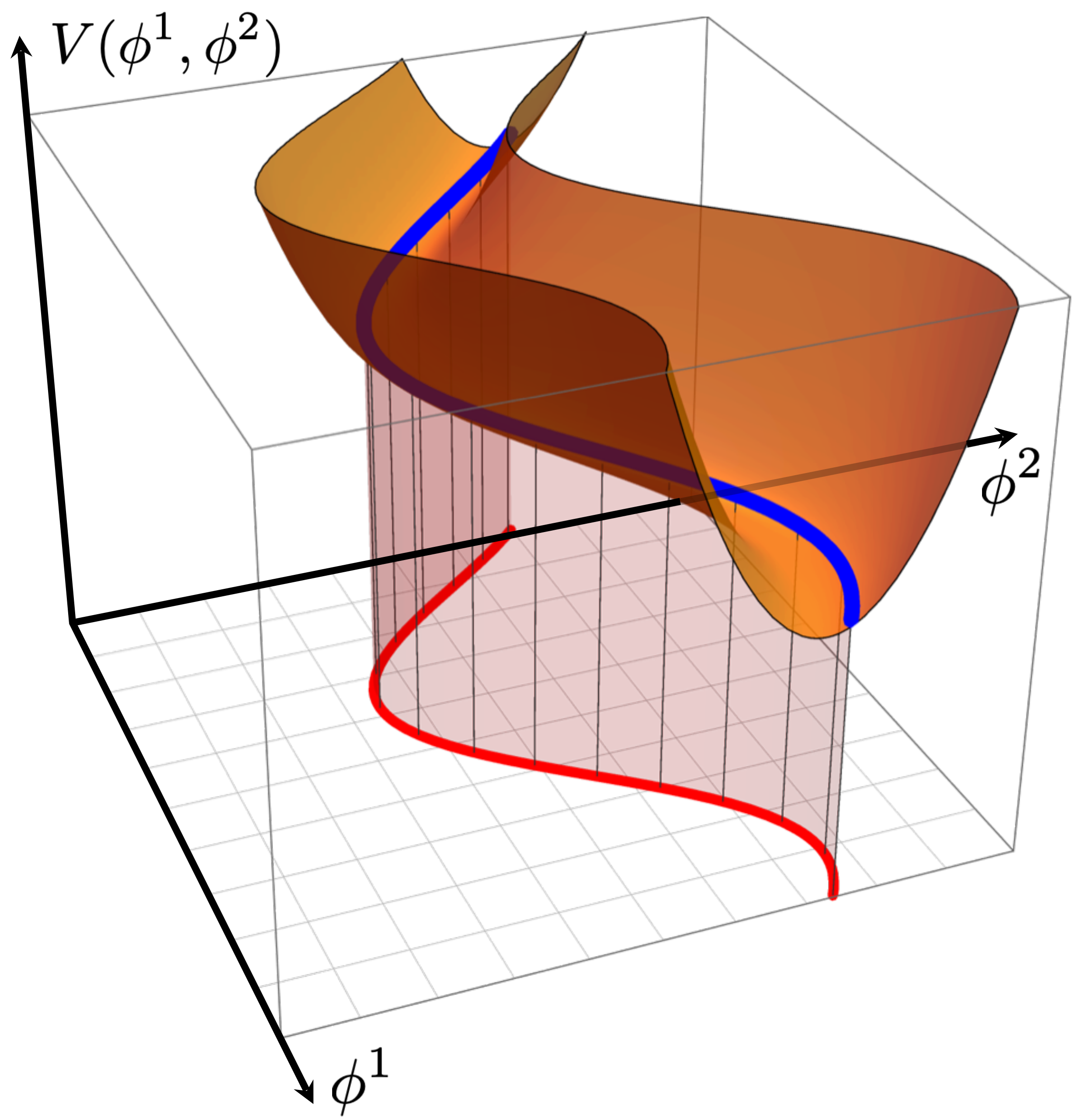
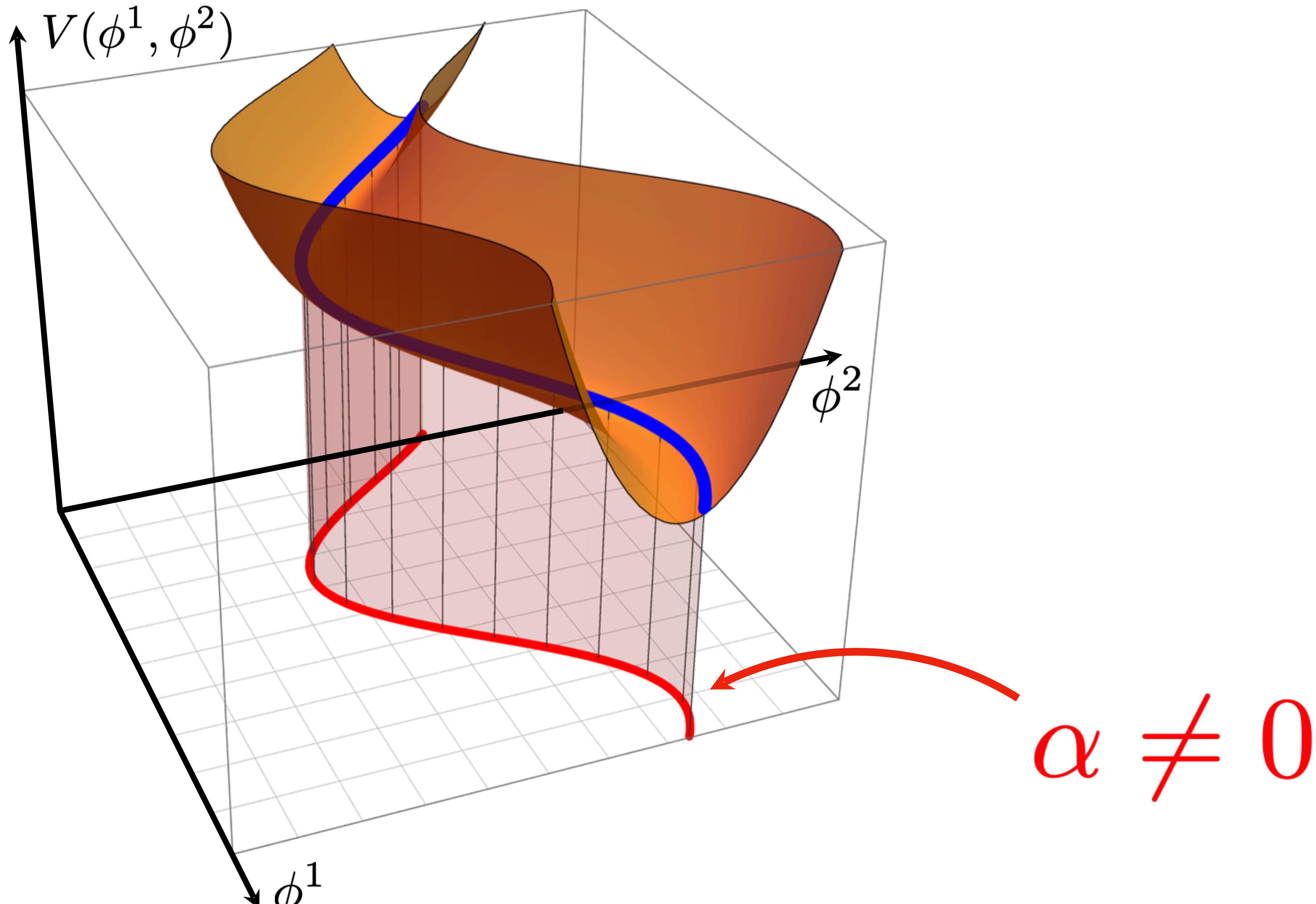


Multi-field inflation

$$\mathcal{L} = \epsilon\left(\dot{\mathcal{R}} - \alpha\,\psi\right)^2 - \frac{\epsilon}{a^2}(\nabla\mathcal{R})^2 + \frac{1}{2}\dot{\psi}^2 - \frac{1}{a^2}(\nabla\psi)^2$$

$$+\,\epsilon\left(\dot{\mathcal{R}} - \alpha\,\psi\right)^3 - V(\psi) + \cdots$$



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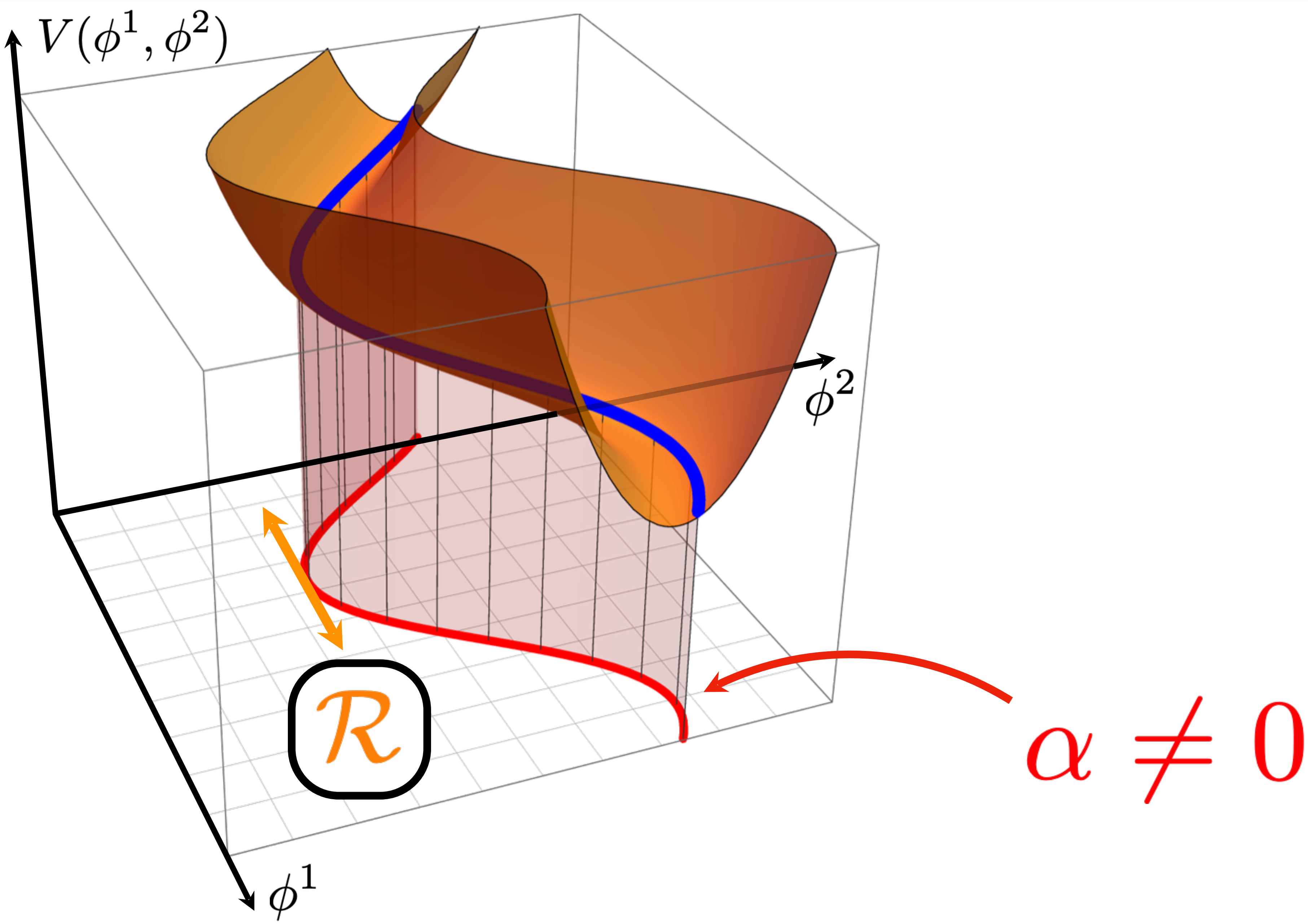
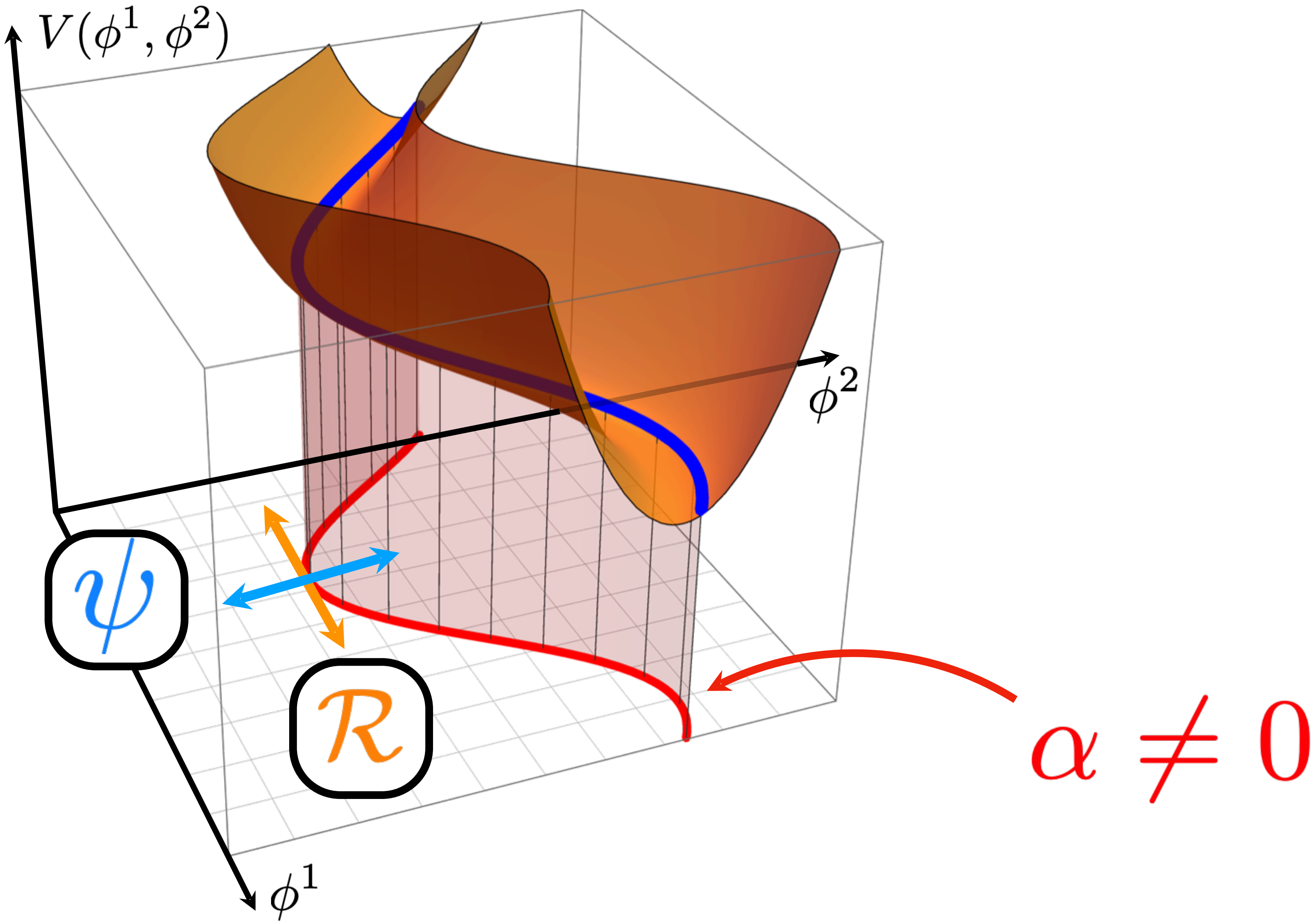


Multi-field inflation

$$\mathcal{L} = \epsilon\left(\dot{\mathcal{R}} - \alpha\,\psi\right)^2 - \frac{\epsilon}{a^2}(\nabla\mathcal{R})^2 + \frac{1}{2}\dot{\psi}^2 - \frac{1}{a^2}(\nabla\psi)^2 + \epsilon\left(\dot{\mathcal{R}} - \alpha\,\psi\right)^3 - V(\psi) + \cdots$$



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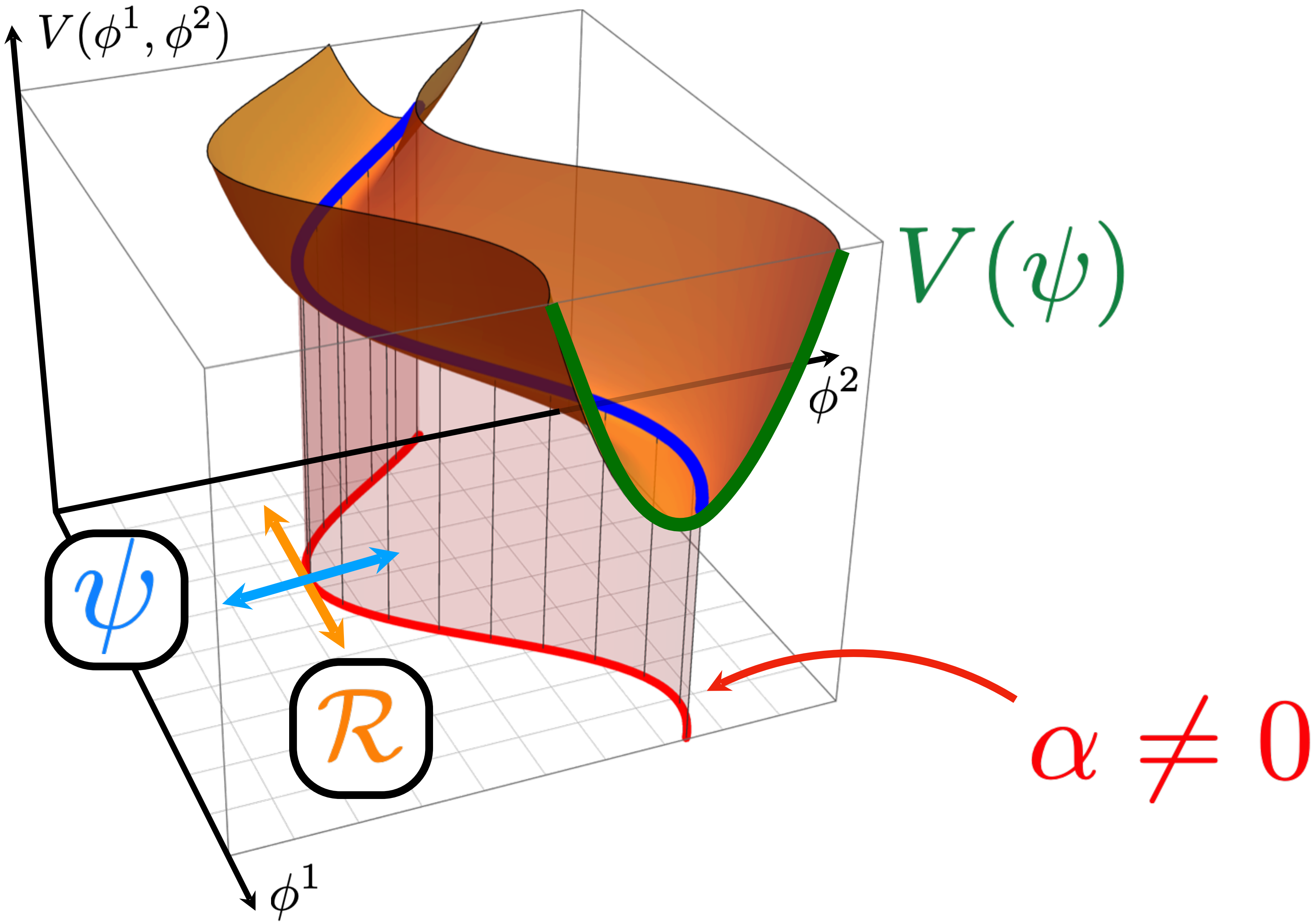

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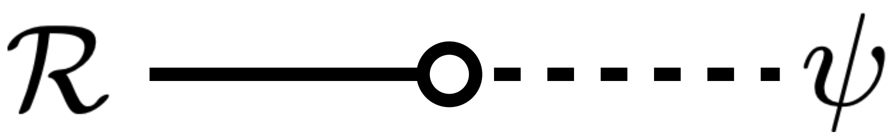

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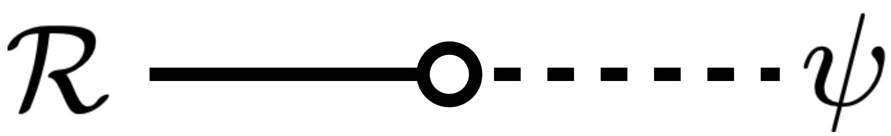

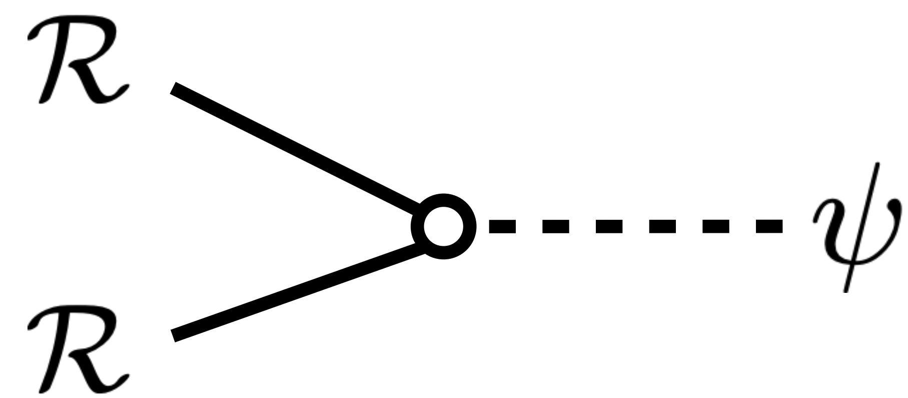

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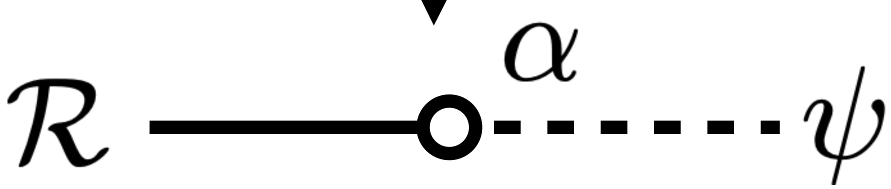
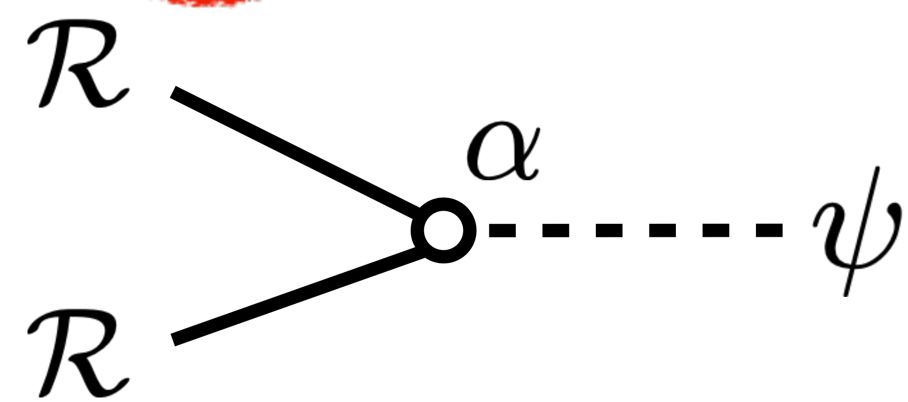
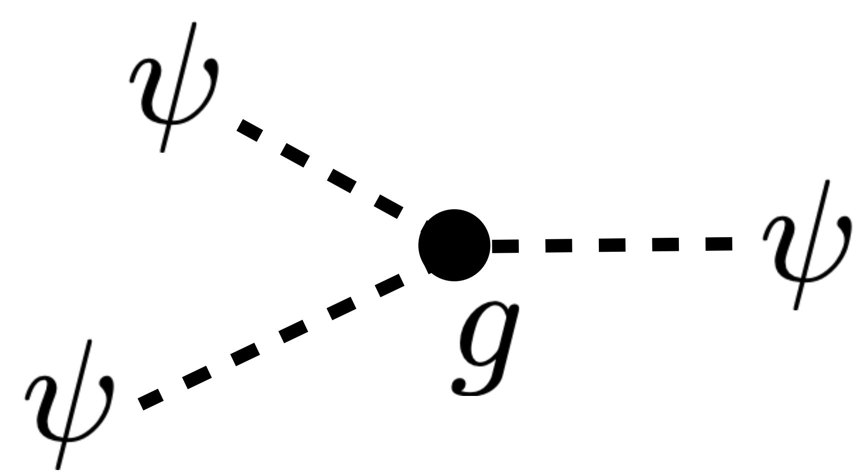




$$V(\psi) = \frac{1}{2}\mu^2\psi^2 + \frac{1}{3}g\,\psi^3 + \cdots$$

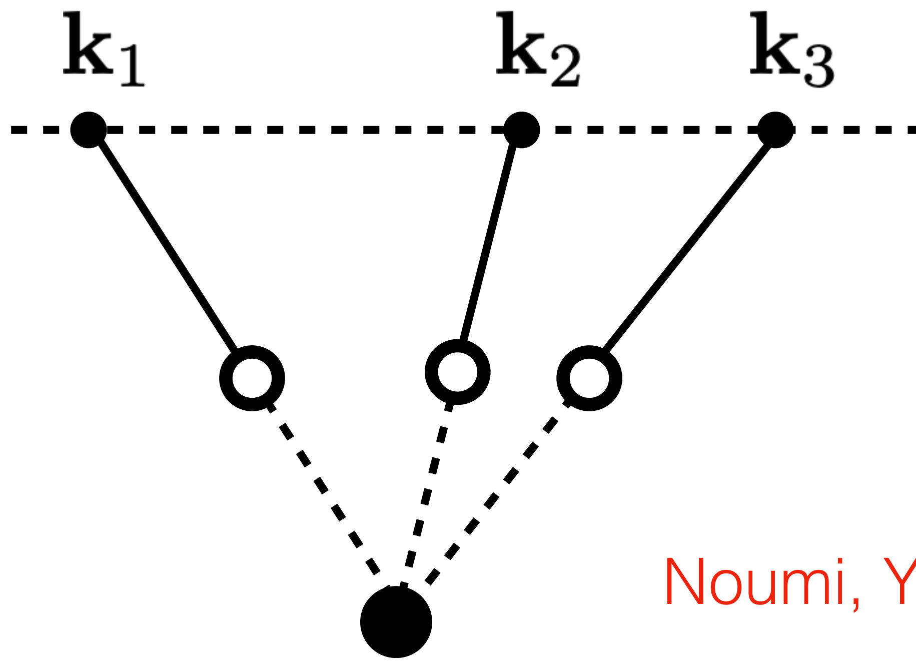


Multi-field inflation

Three-point statistics:

$\mu \neq 0$

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle =$$



(Quasi-single field)

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Noumi, Yamaguchi & Yokoyama (2013)
see also Assassi et al. (2013)

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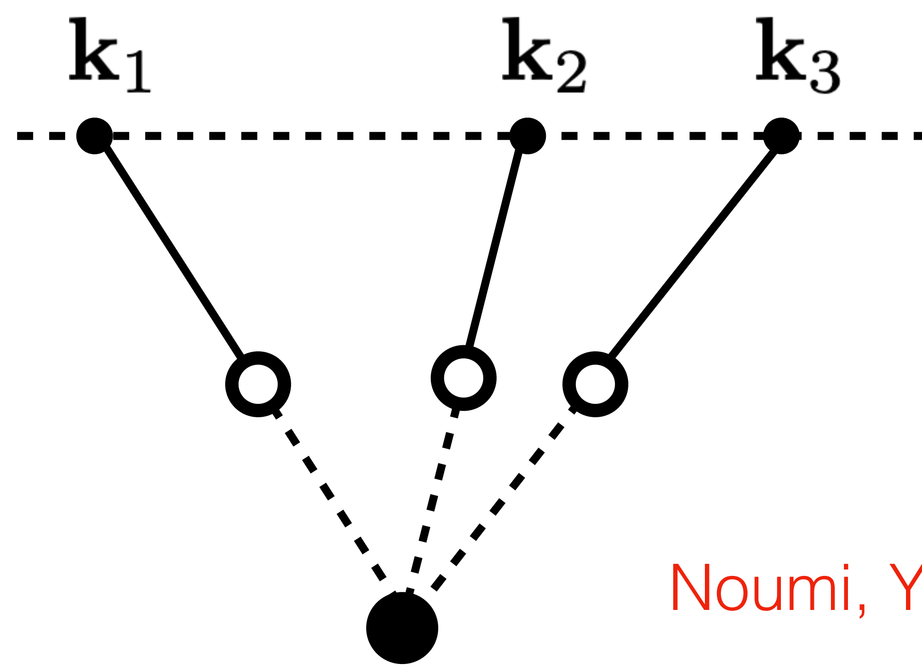

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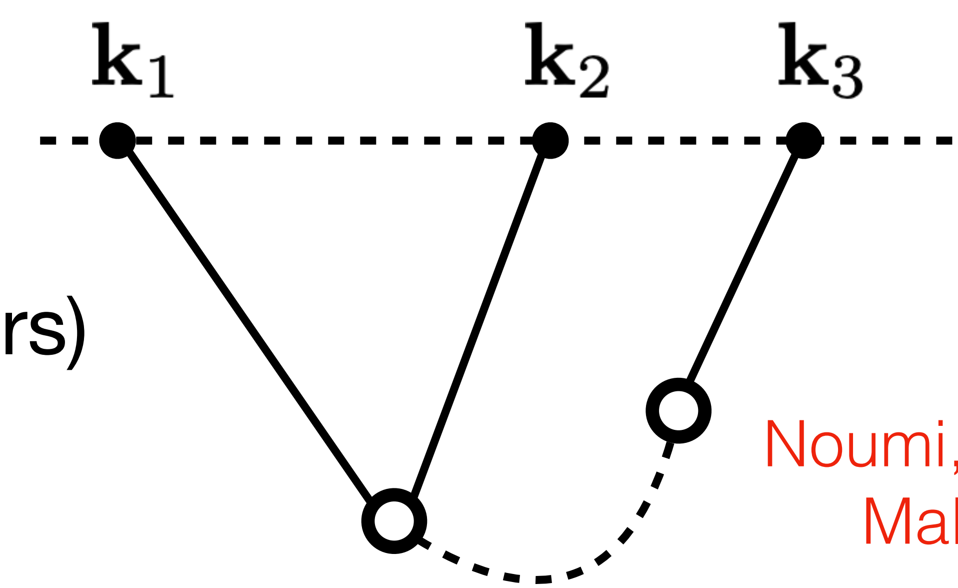


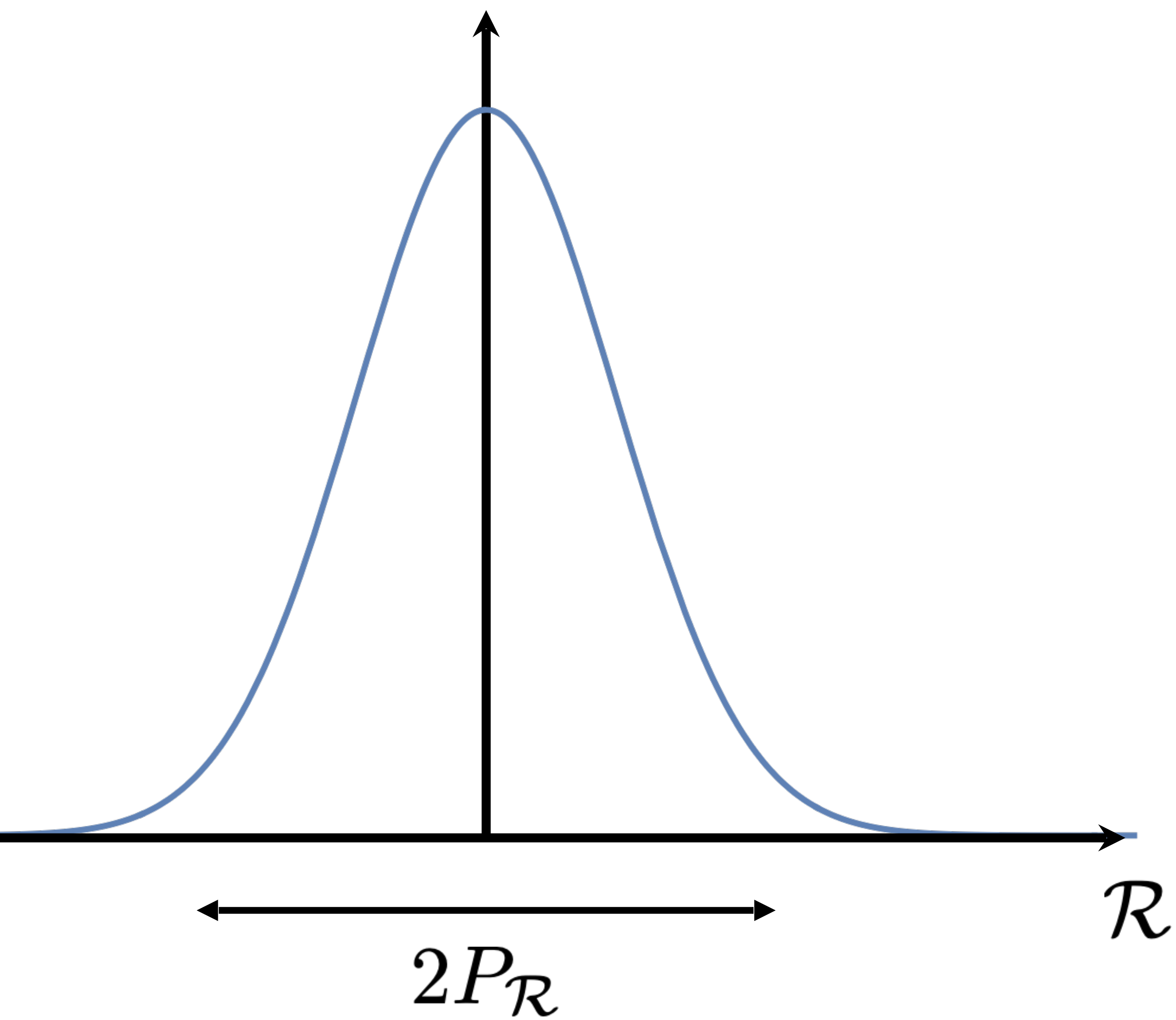
(Cosmological colliders)

Noumi, Yamaguchi & Yokoyama (2013)
Maldacena & Arkani-Hamed (2016)
Chen & Wang (2016)
Lee, Baumann & Pimentel (2016)

Beyond the bispectrum

The perturbative schemes seem to imply a hierarchical reconstruction of the probability distribution function

$$\rho[\mathcal{R}] \sim \exp\left\{-\frac{\mathcal{R}^2}{2P_{\mathcal{R}}}\left(1+f_{\rm NL}\mathcal{R}+g_{\rm NL}\mathcal{R}^2+\cdots\right)\right\}$$



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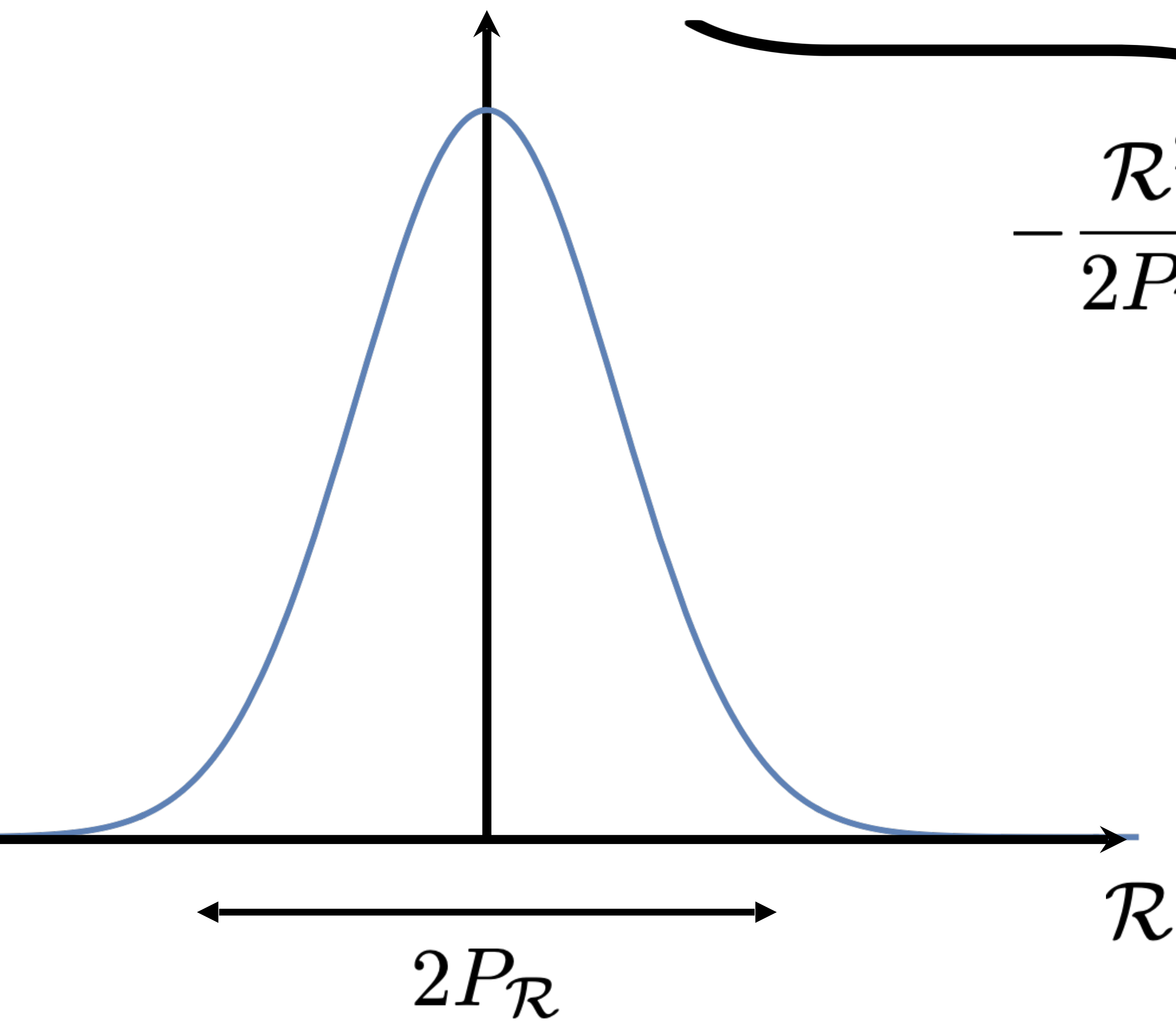

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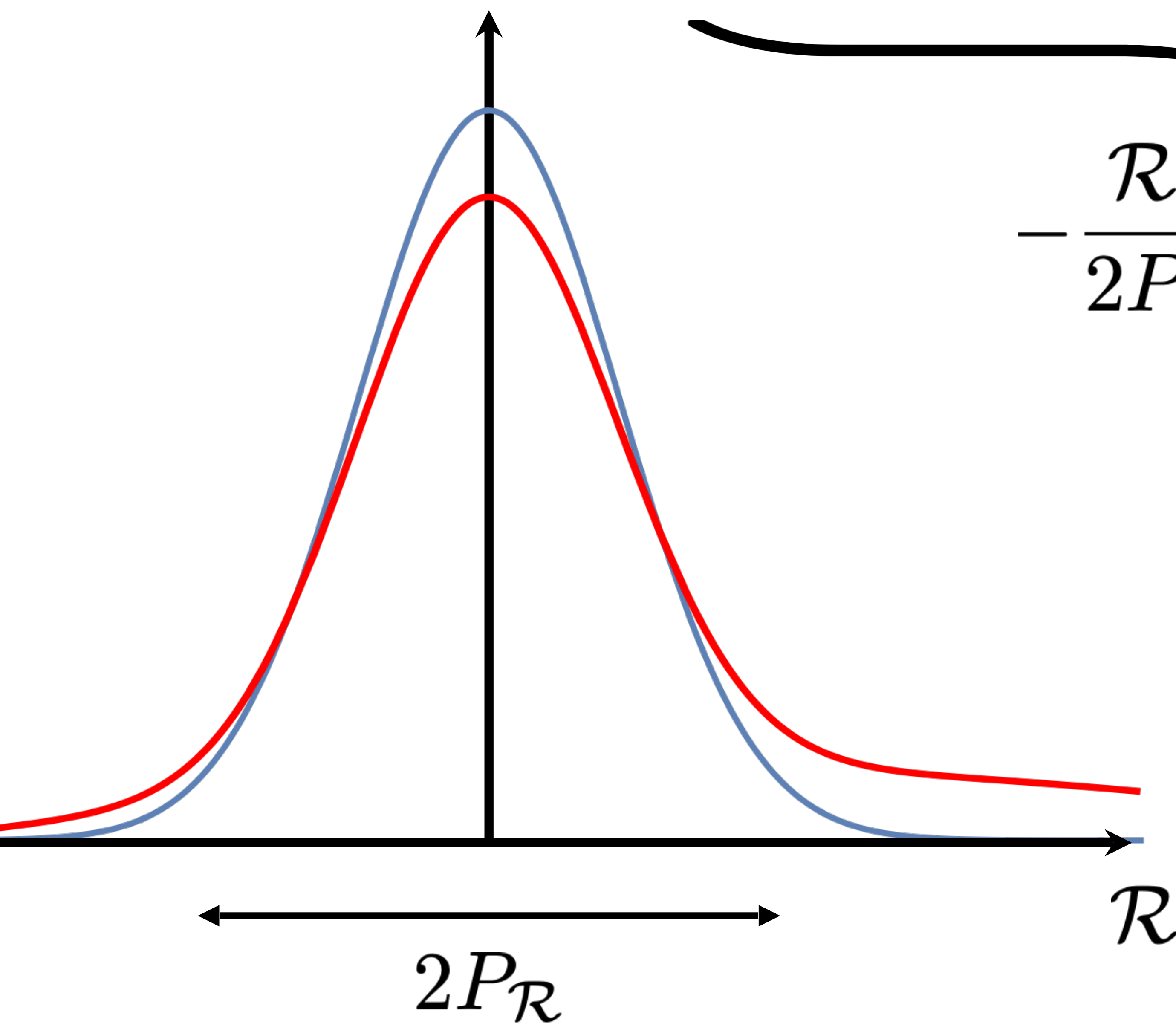

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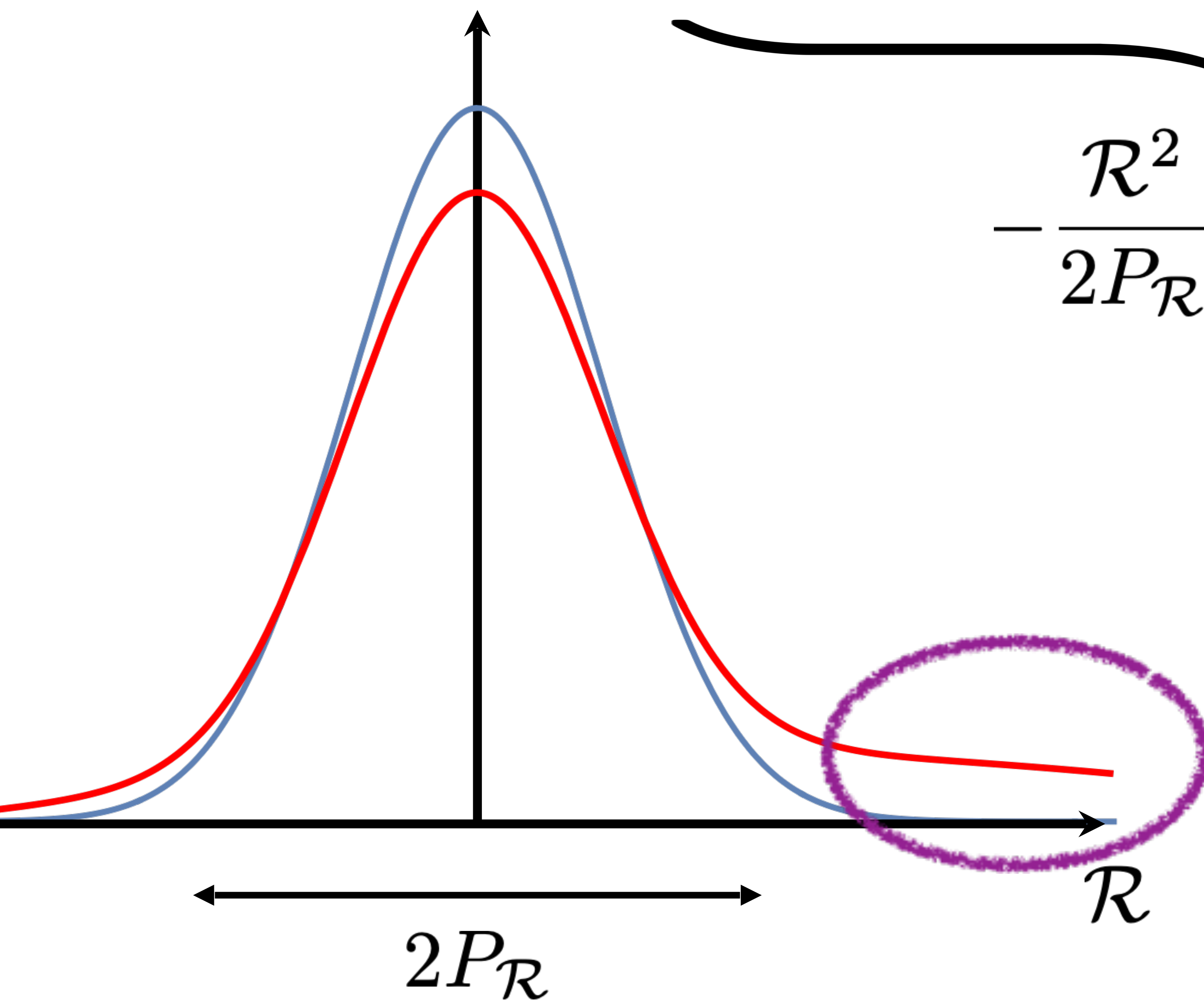

An opportunity to generate large (but rare) fluctuations leading to Primordial Black Holes

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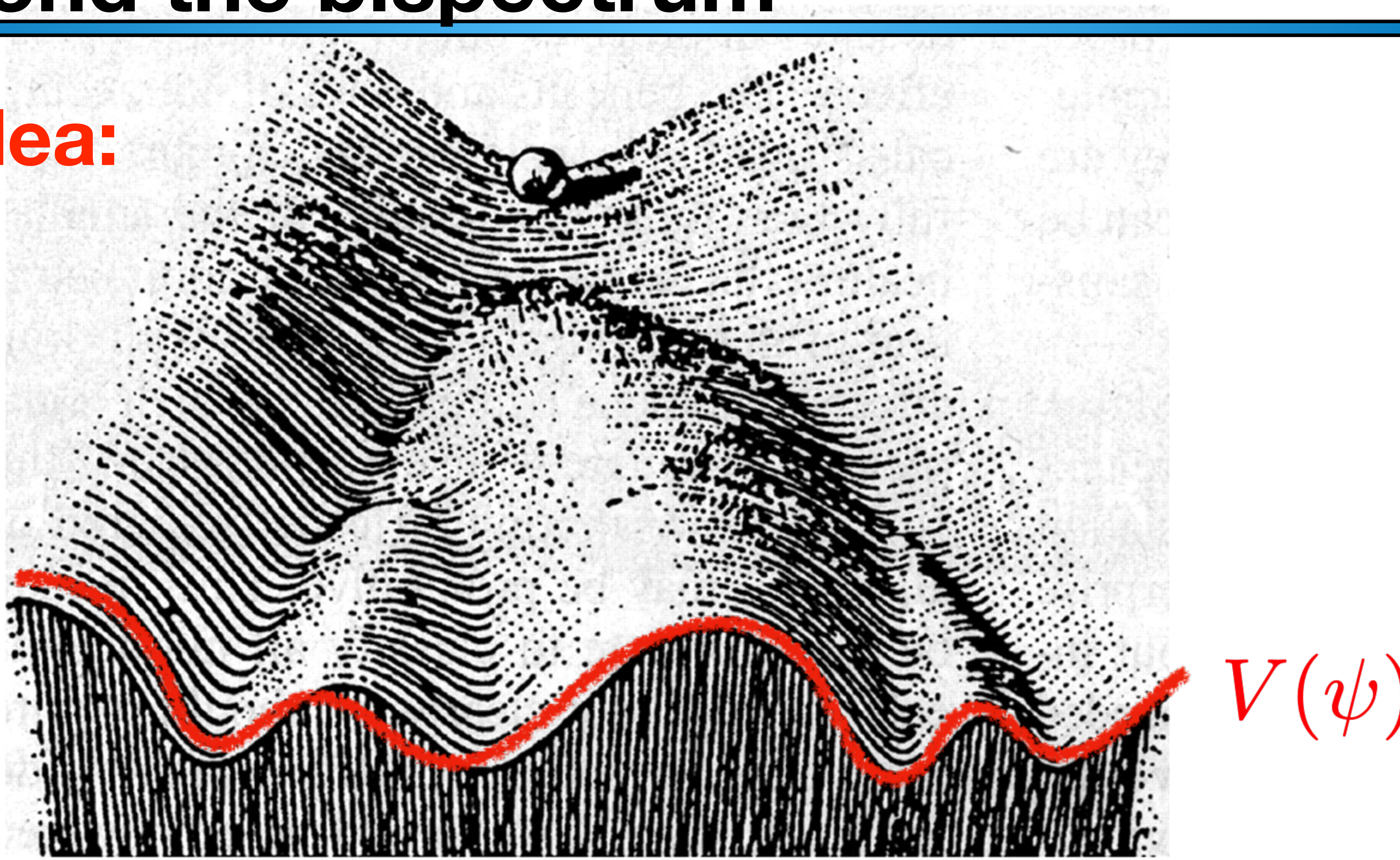
Main idea:



Consider a spectator field ψ during inflation with it's own potential $V(\psi)$

(This potential is not driving inflation)

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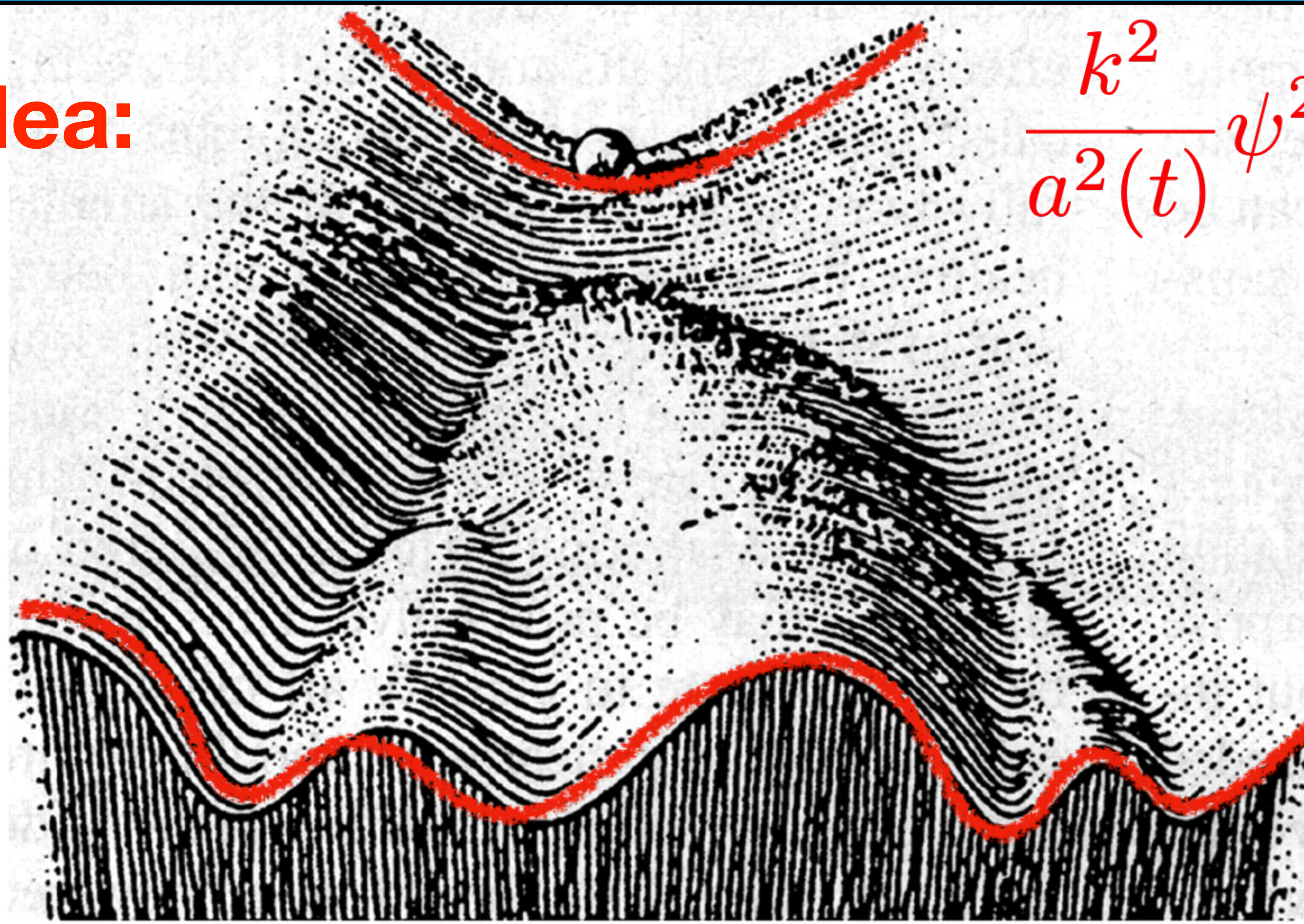

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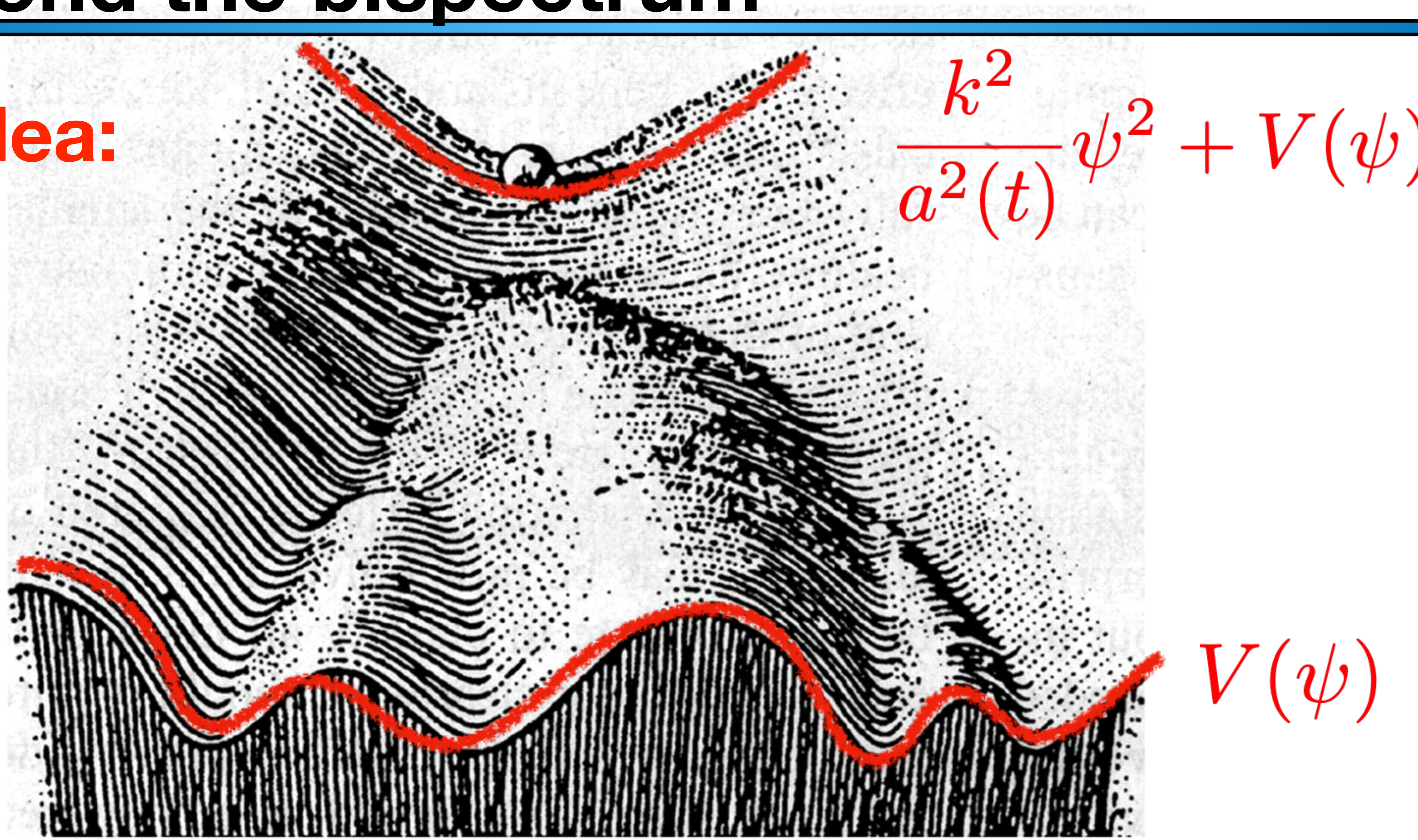

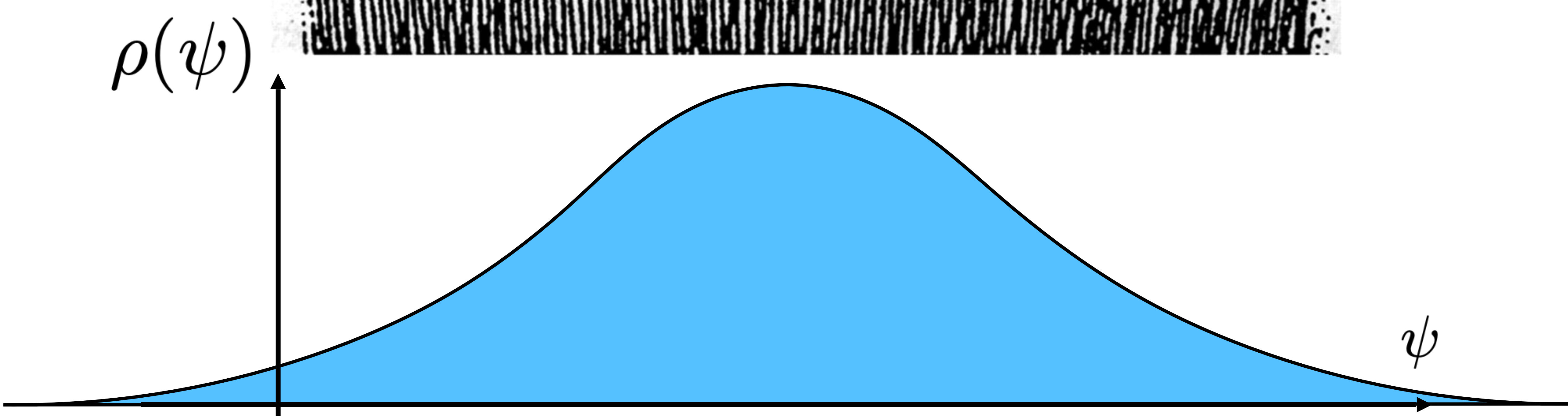

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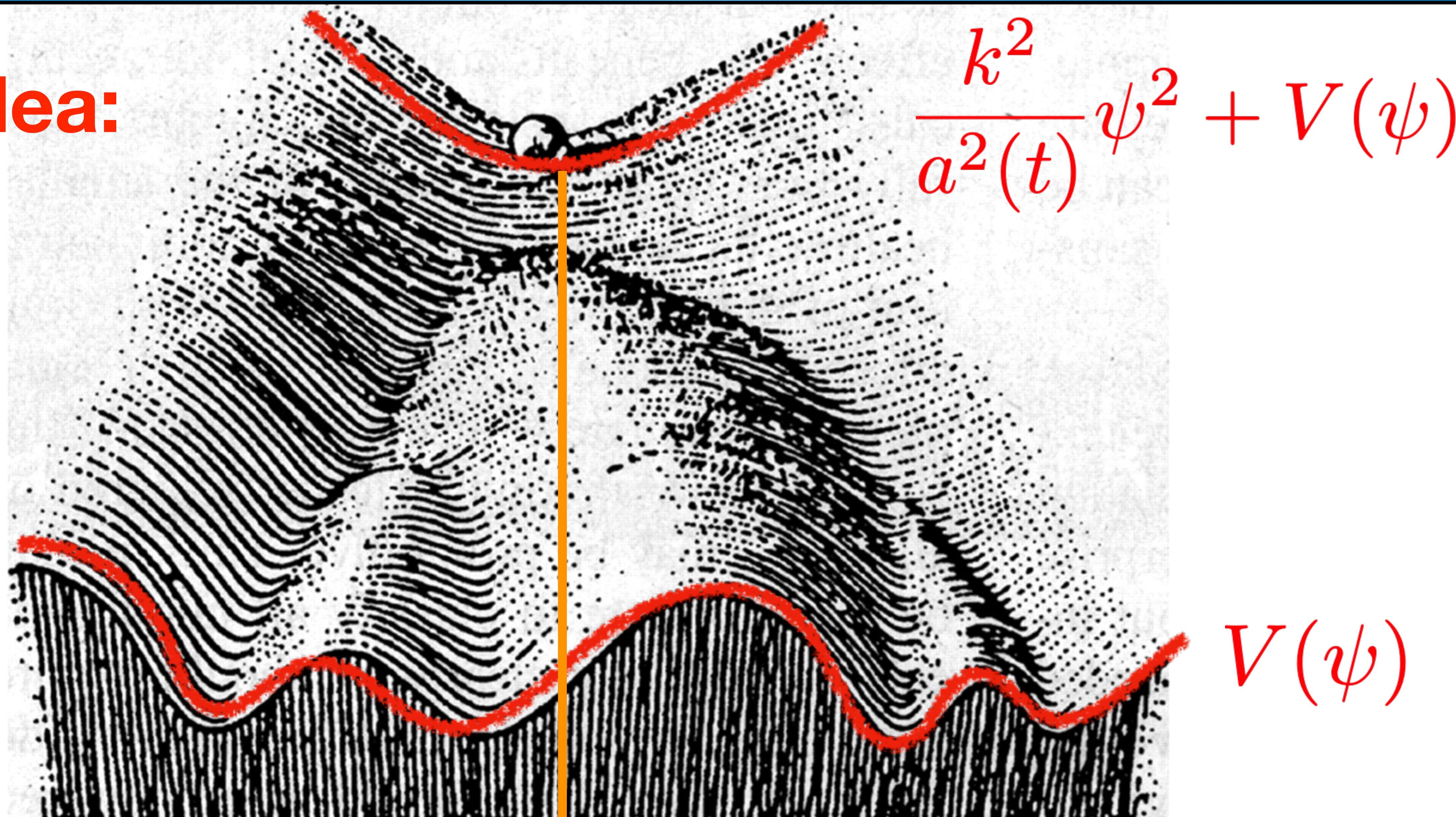

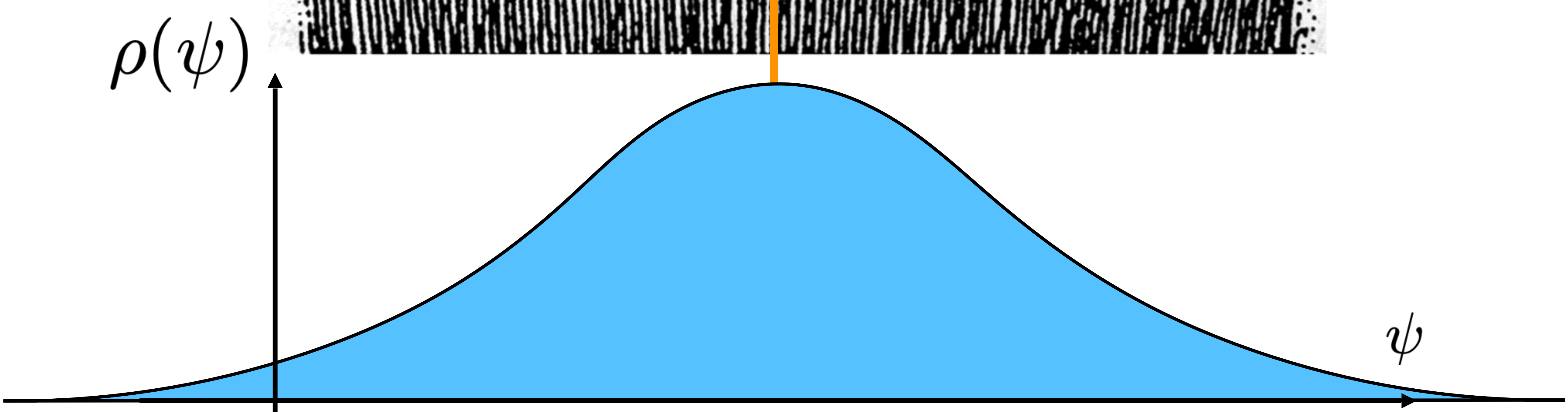

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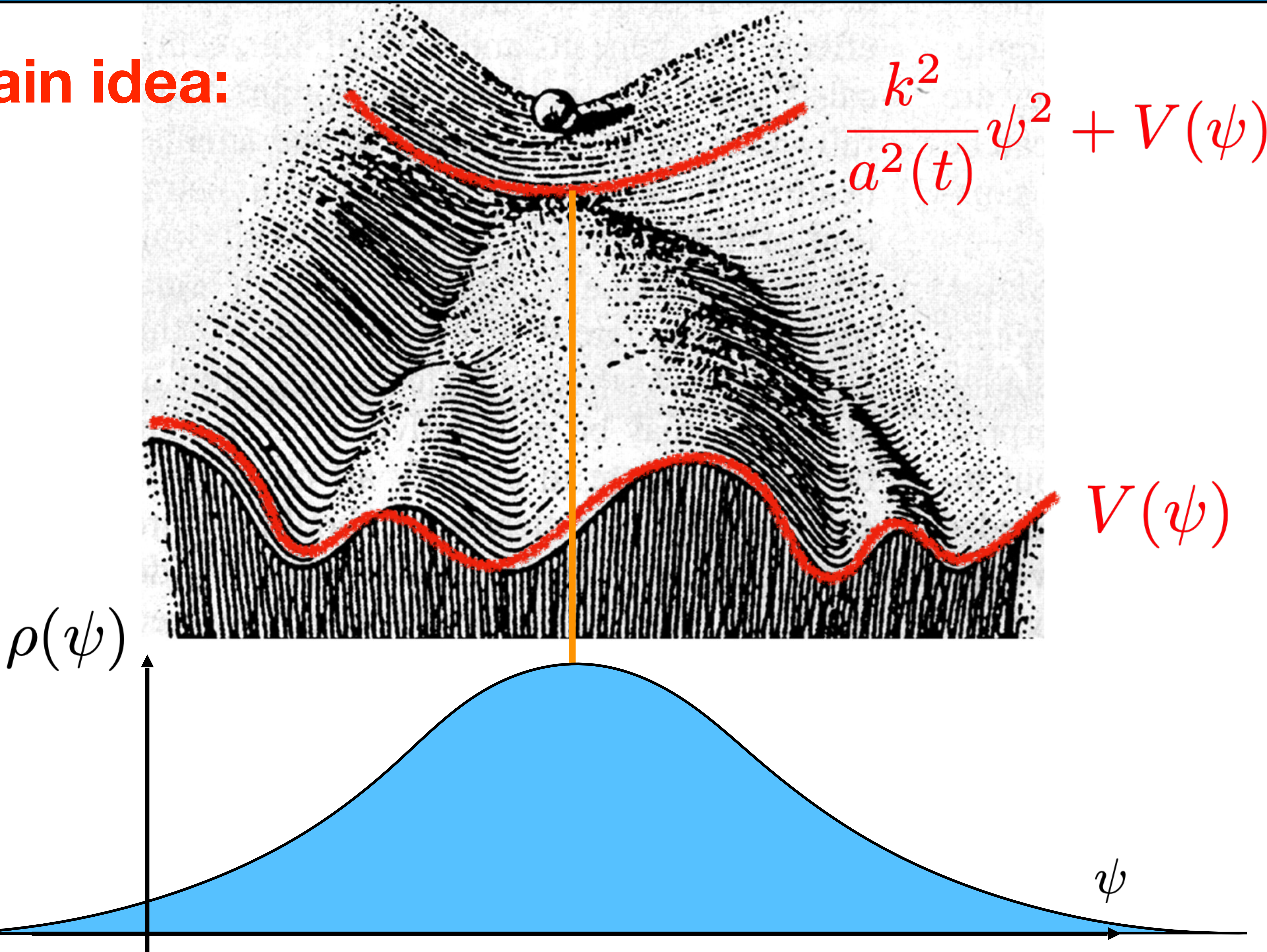

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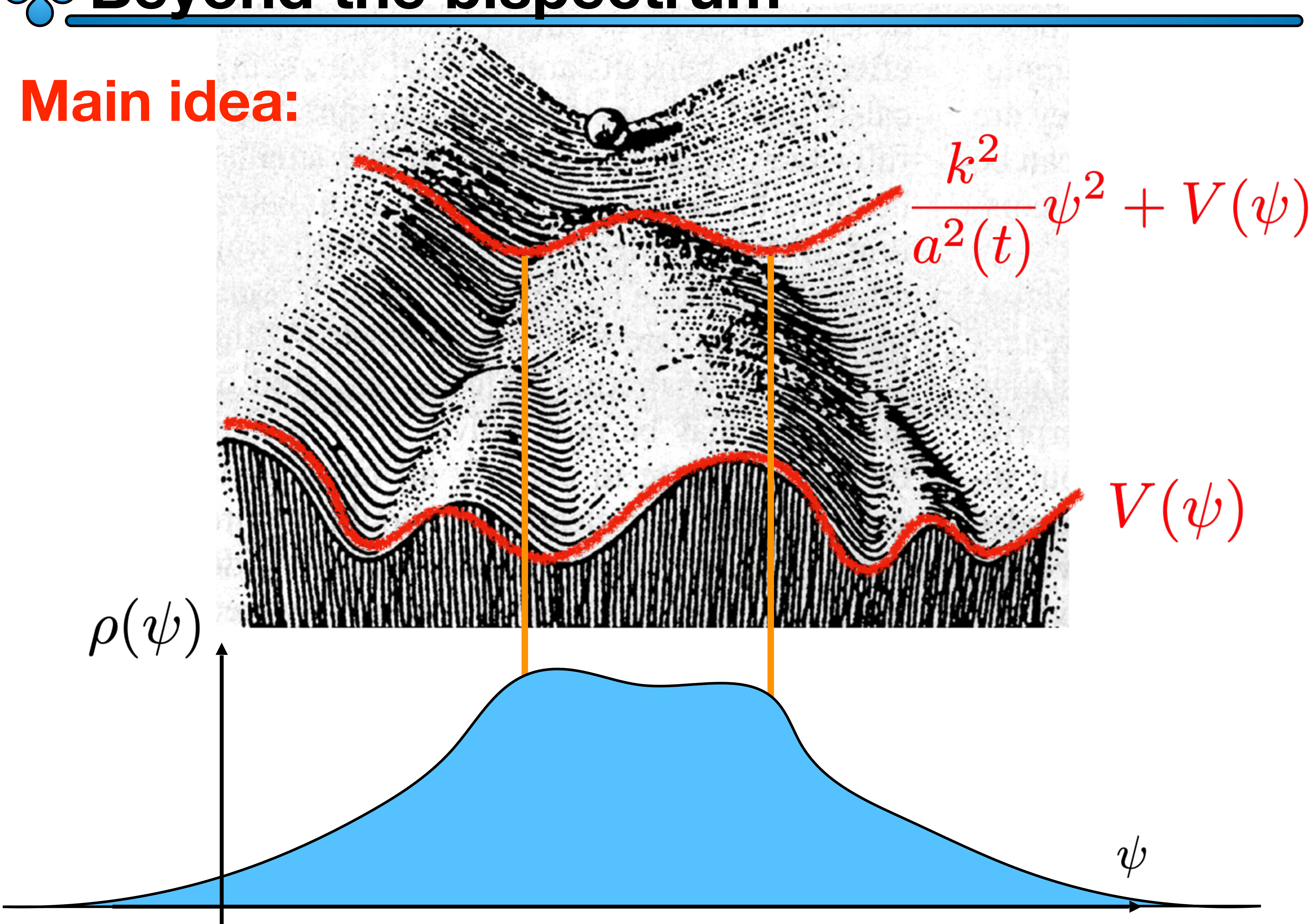

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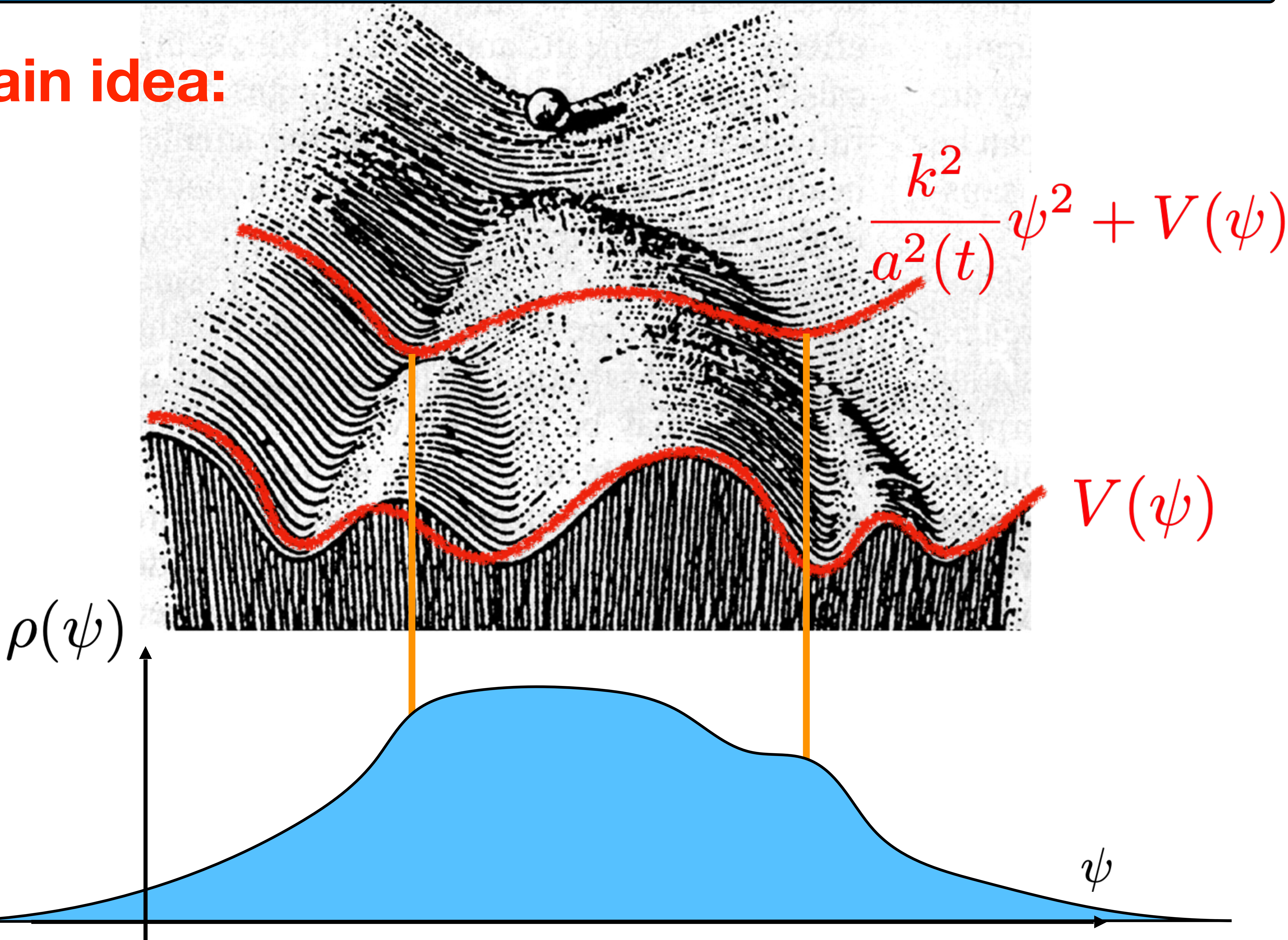

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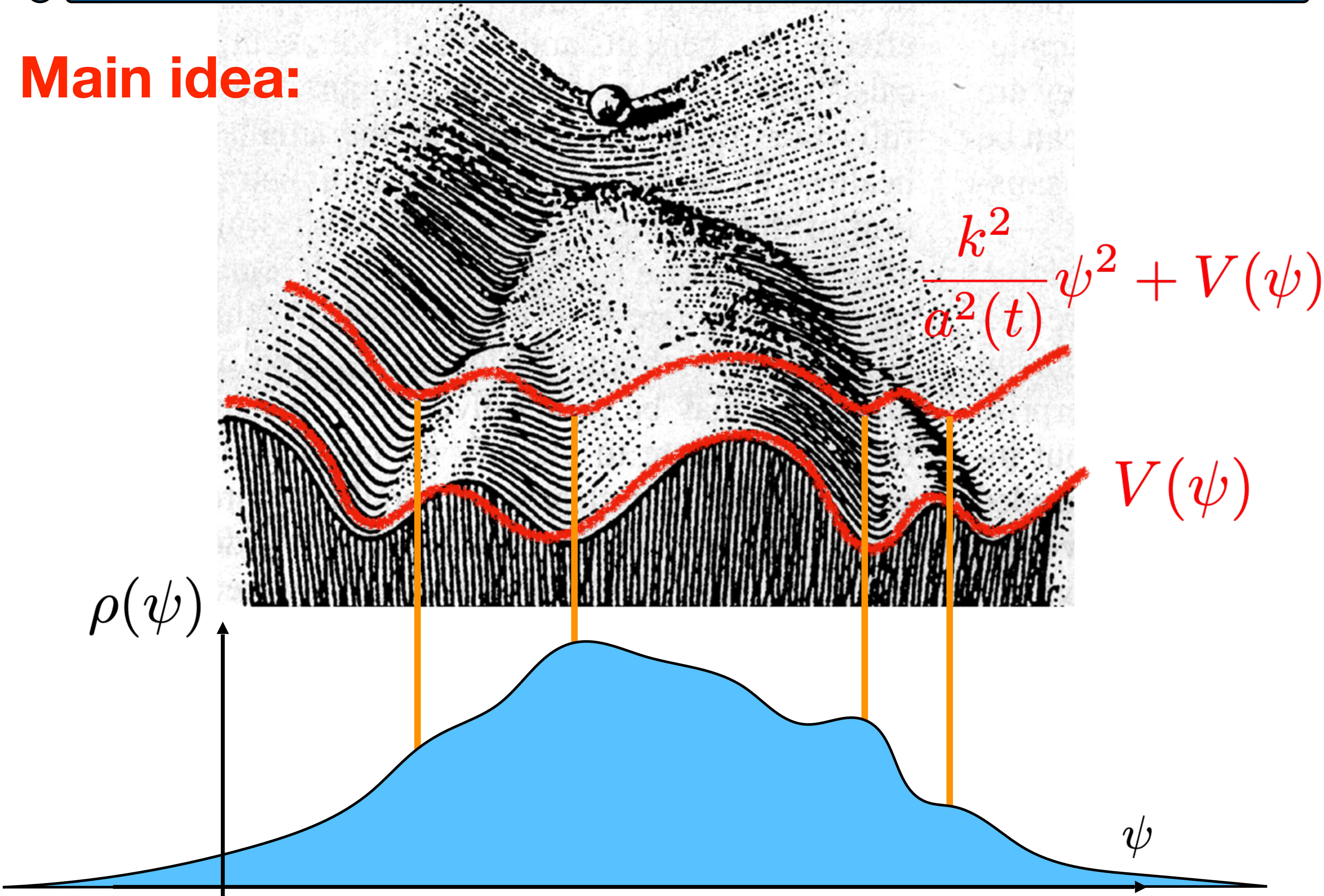

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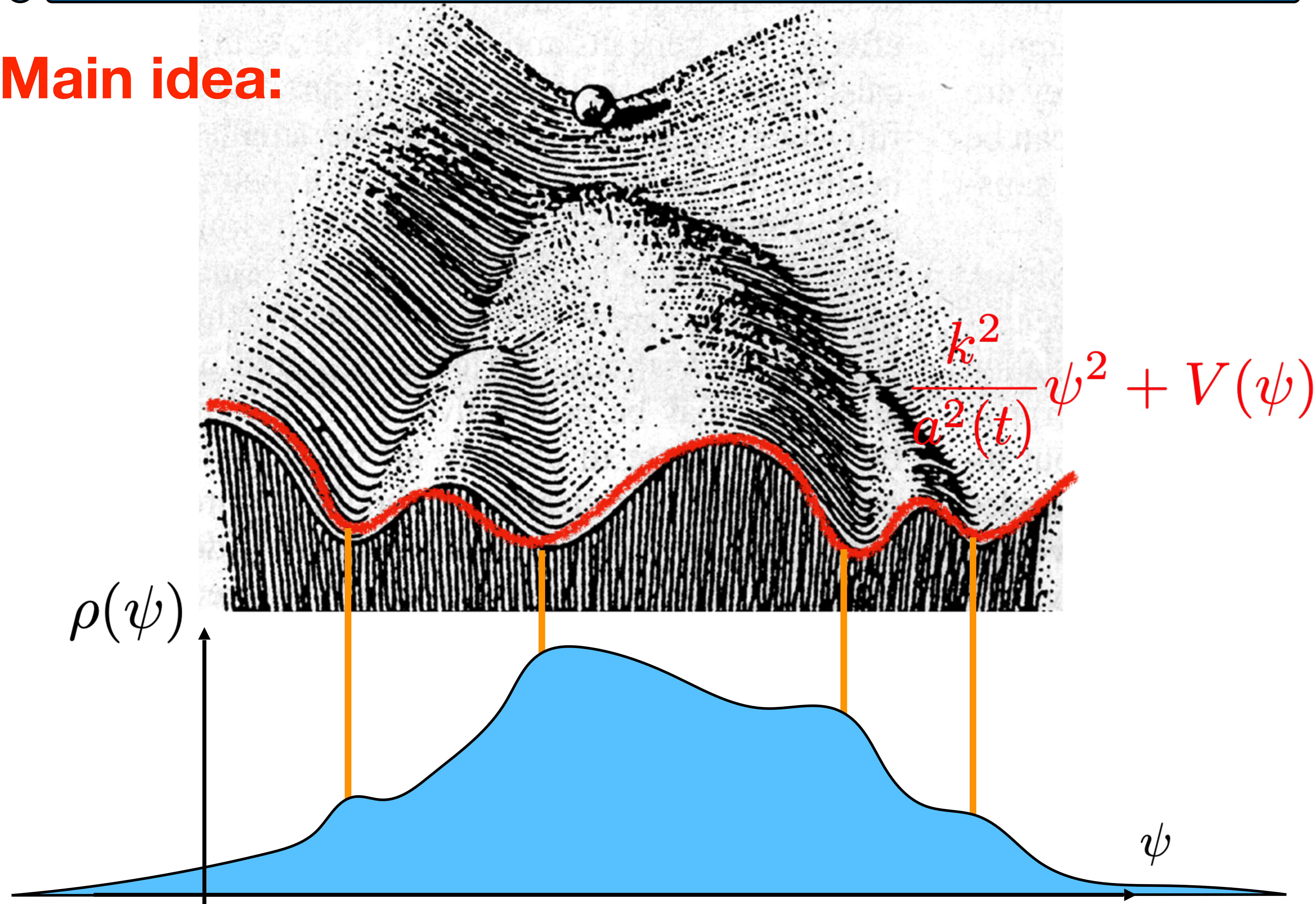

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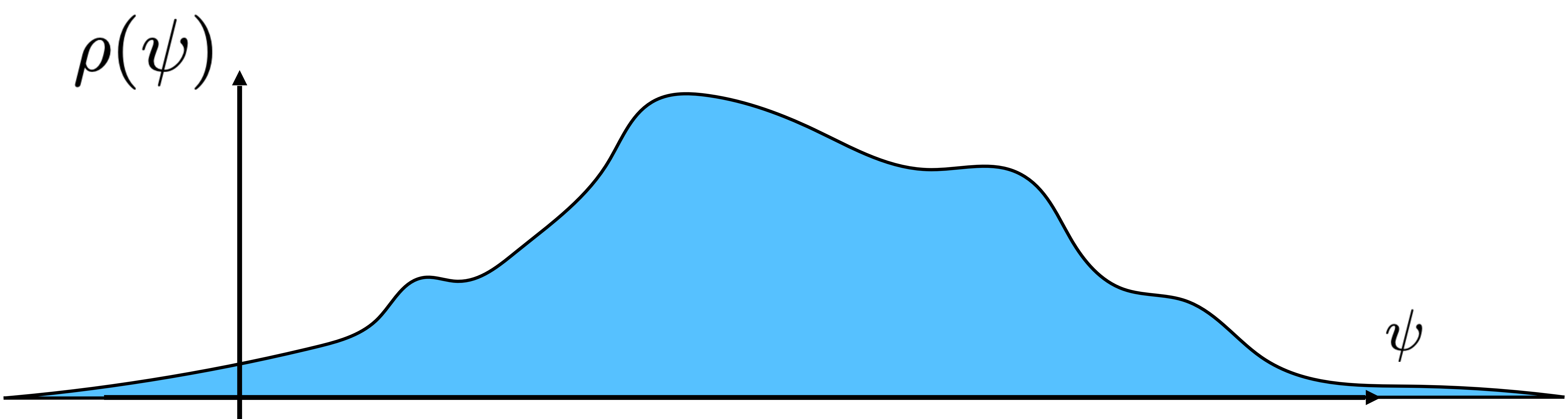

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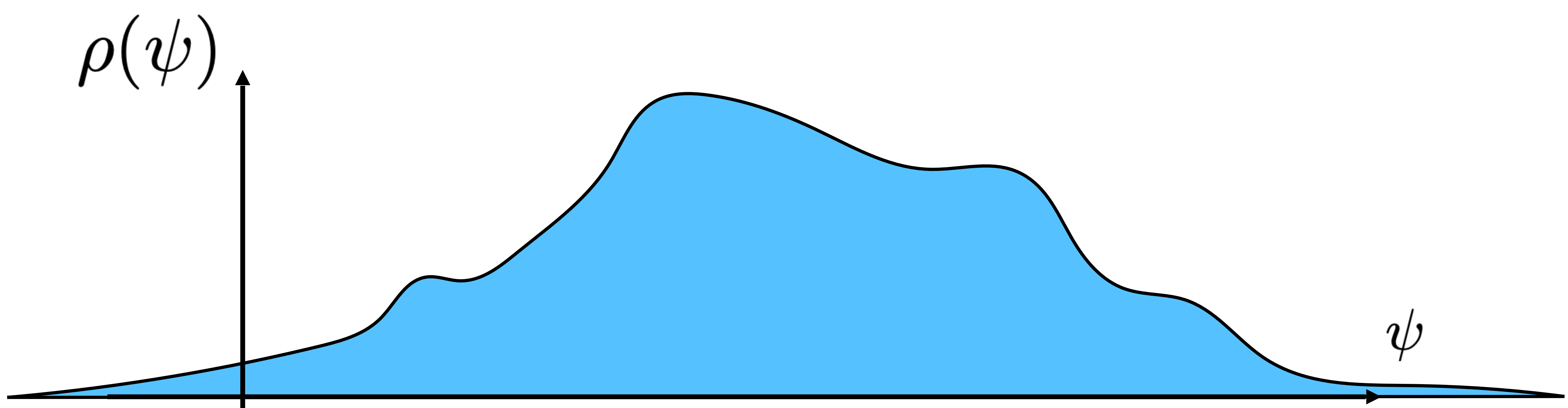

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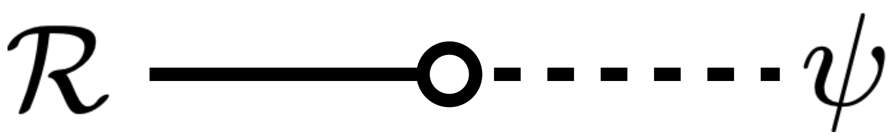

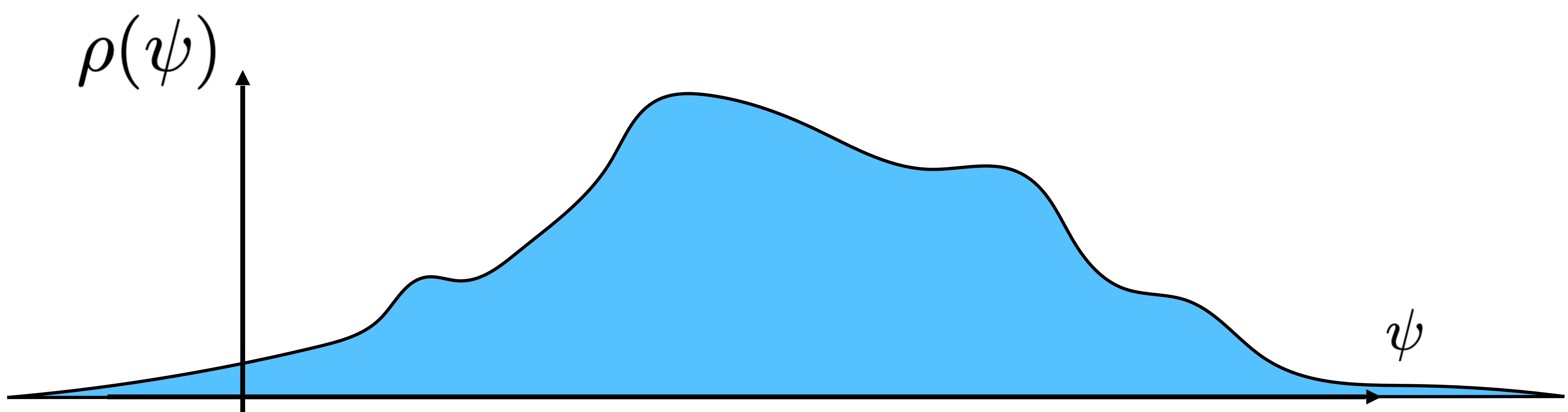

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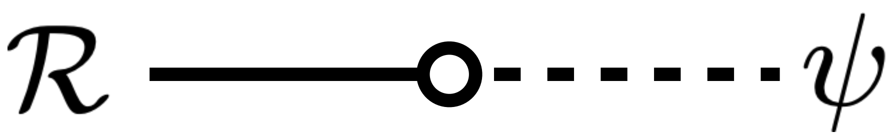
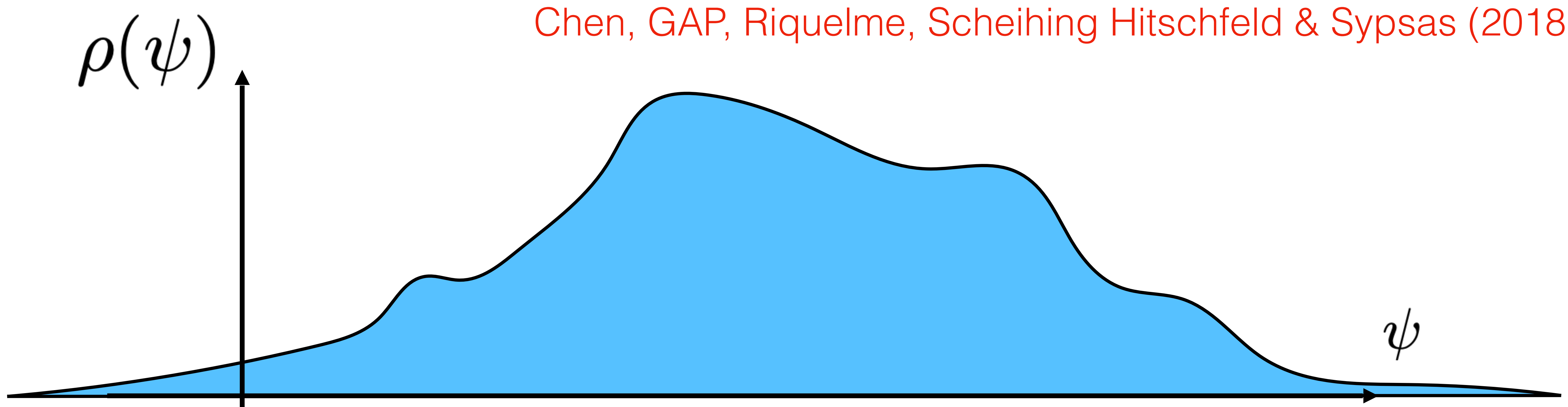


$$\Longrightarrow \rho(\mathcal{R}) \neq \exp\left[-\frac{\mathcal{R}^2}{2\sigma_{\mathcal{R}}^2}\right]$$

Flauger, Mirbabayi, Senatore & Silverstein (2016)

Chen, GAP, Riquelme, Scheihing Hitschfeld & Sypsas (2018)



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The previous idea can be examined non-perturbatively. On long wavelengths a spectator field satisfies the Fokker-Planck equation:

$$\frac{\partial \rho}{\partial t} = \frac{1}{3H}\frac{\partial}{\partial \psi}\Big(V'(\psi)\rho\Big) + \frac{H^3}{8\pi^2}\frac{\partial^2 \rho}{\partial \psi^2}$$

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Equilibrium solution:

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Starobinsky & Yokoyama (1994)

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Gorbenko & Senatore (2019)

Cohen, Green & Premkumar (2022)

Beyond the bispectrum

Another approach consists of directly reconstructing $\rho(\mathcal{R})$ out of n-point functions

$$V(\psi) = \sum_n \frac{\lambda_n}{n!} \psi^n$$

GAP & Sypsas (2023)

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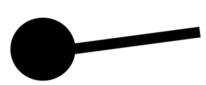

λ_1

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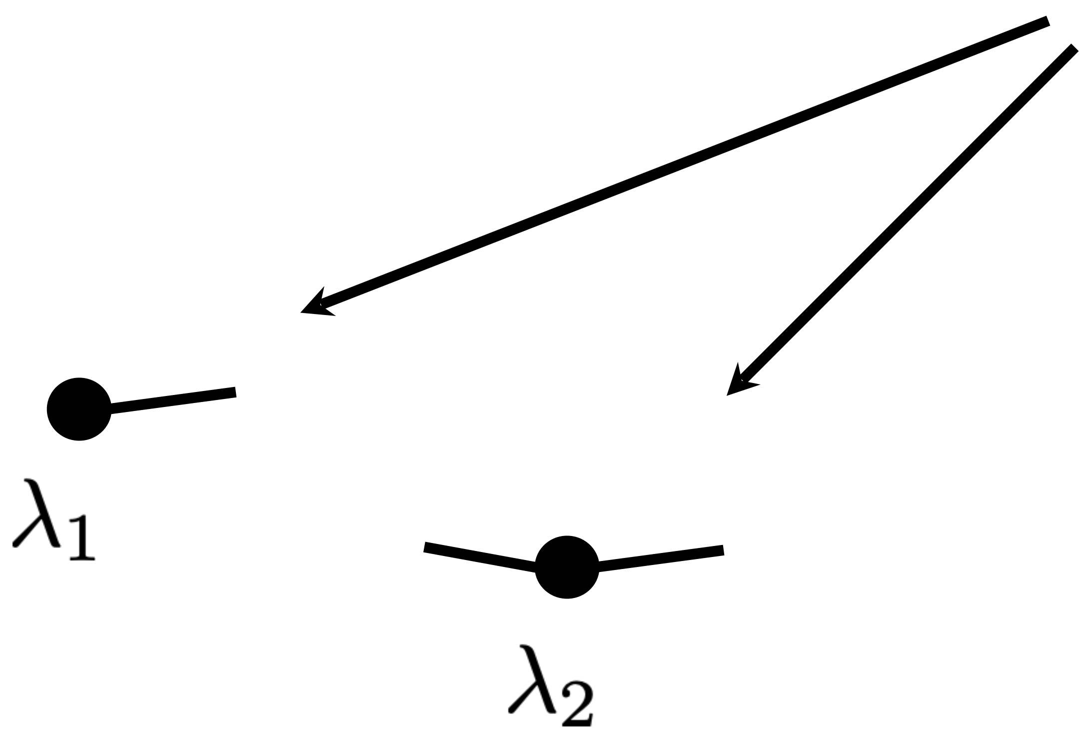

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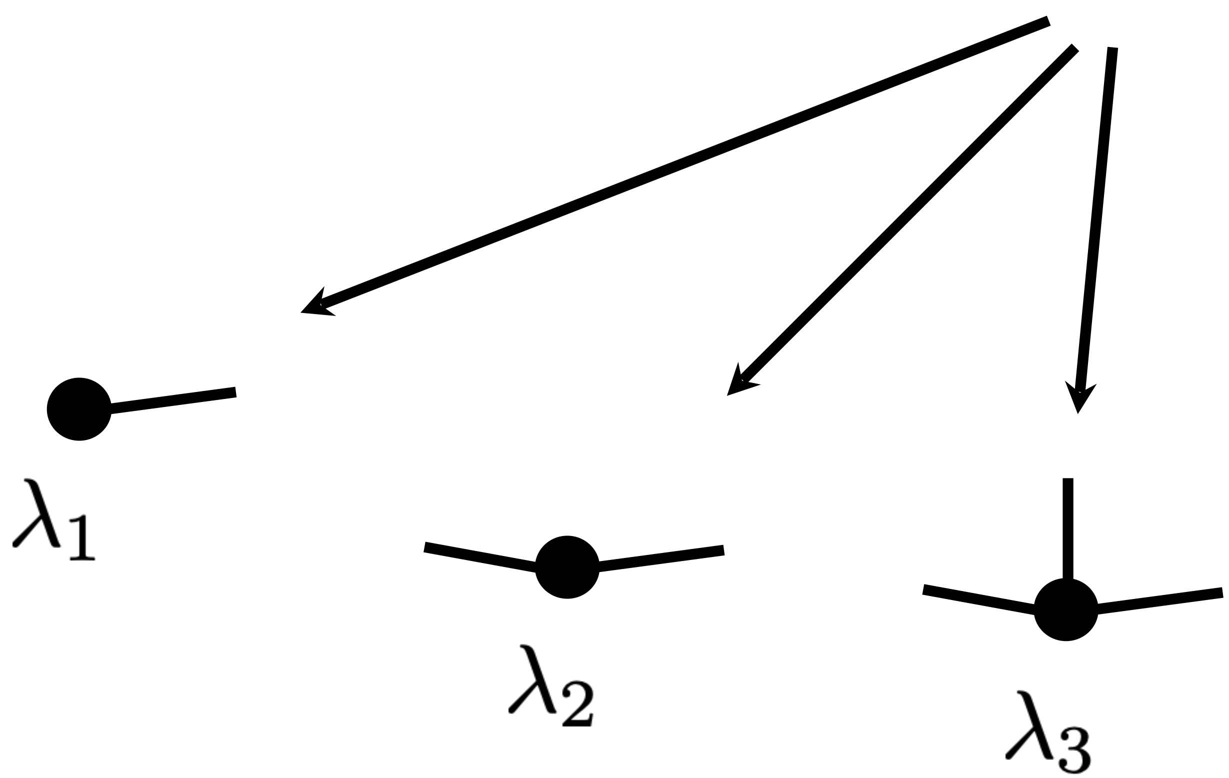

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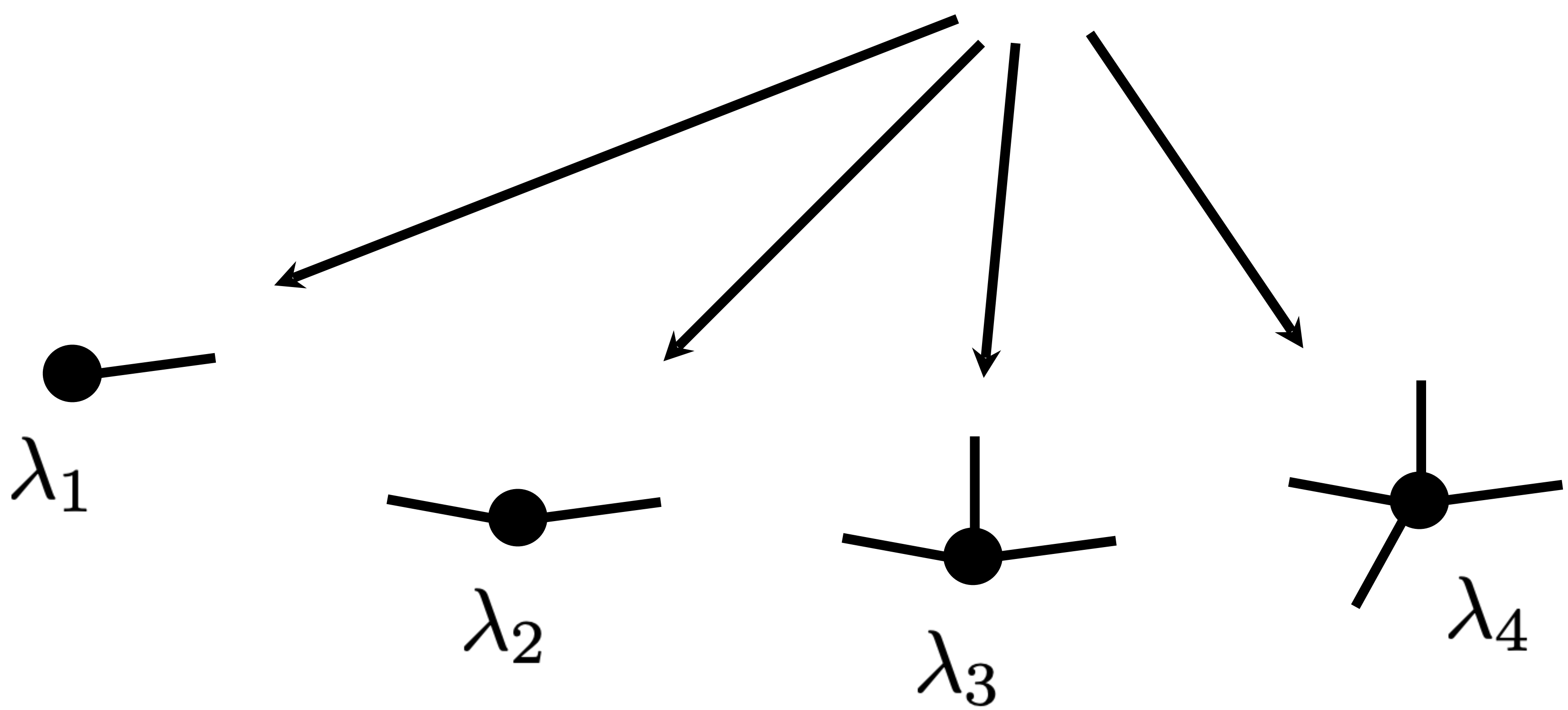

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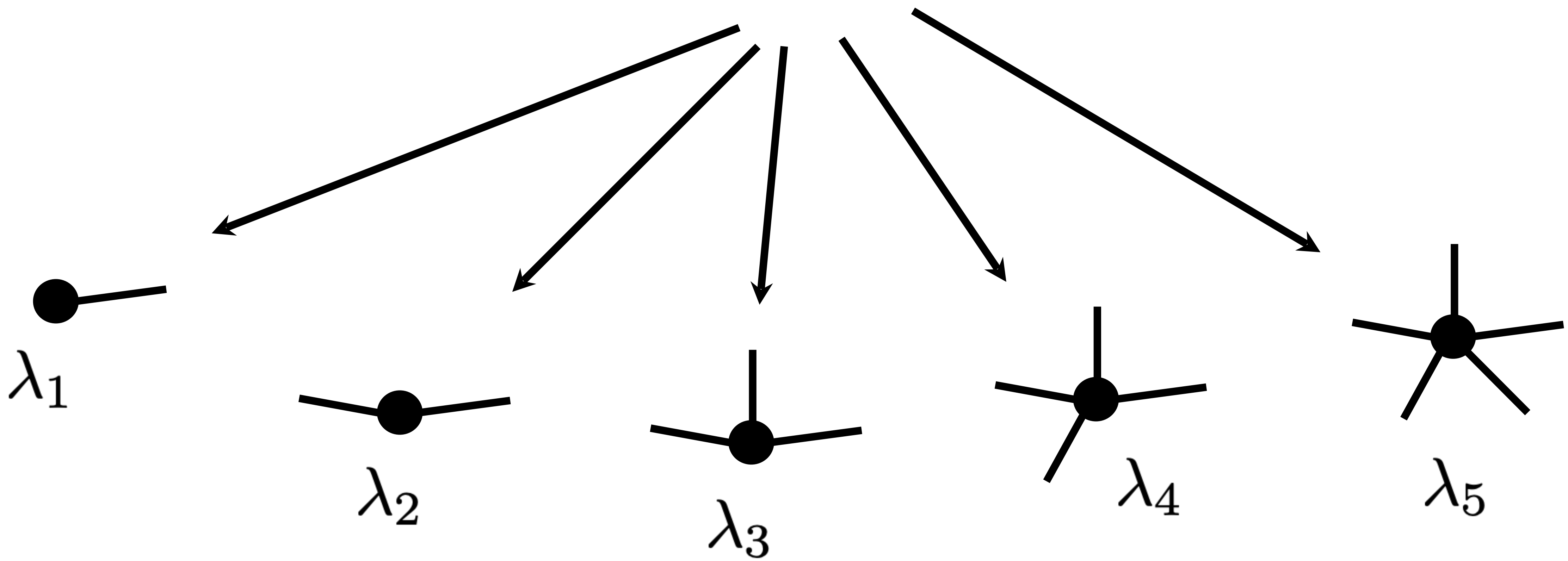

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One can compute every leading n-point function

$$\langle \psi(\mathbf{k}_1)\cdots\psi(\mathbf{k}_n)\rangle \sim \lambda_n + \mathcal{O}(\lambda^2)$$

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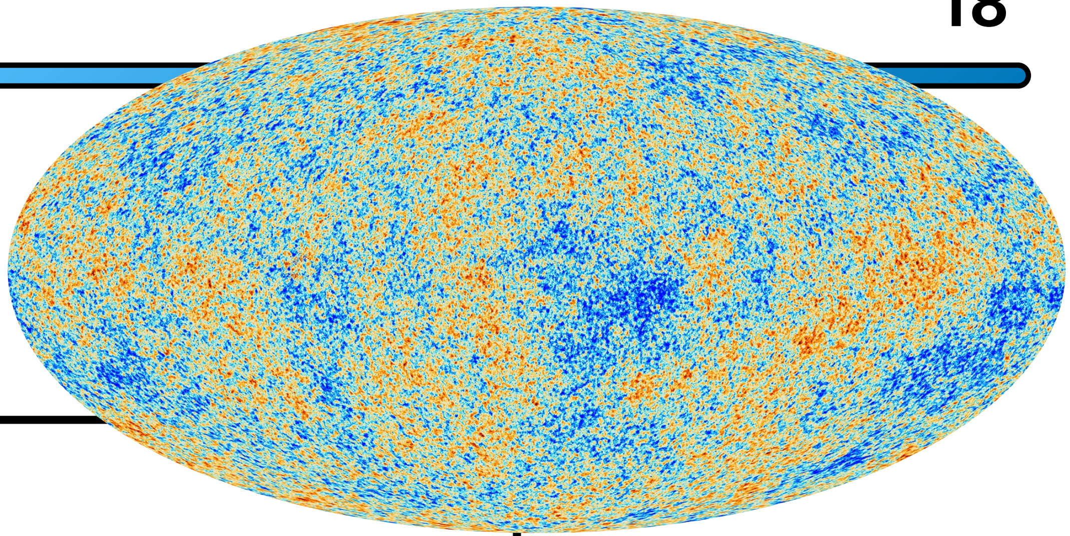
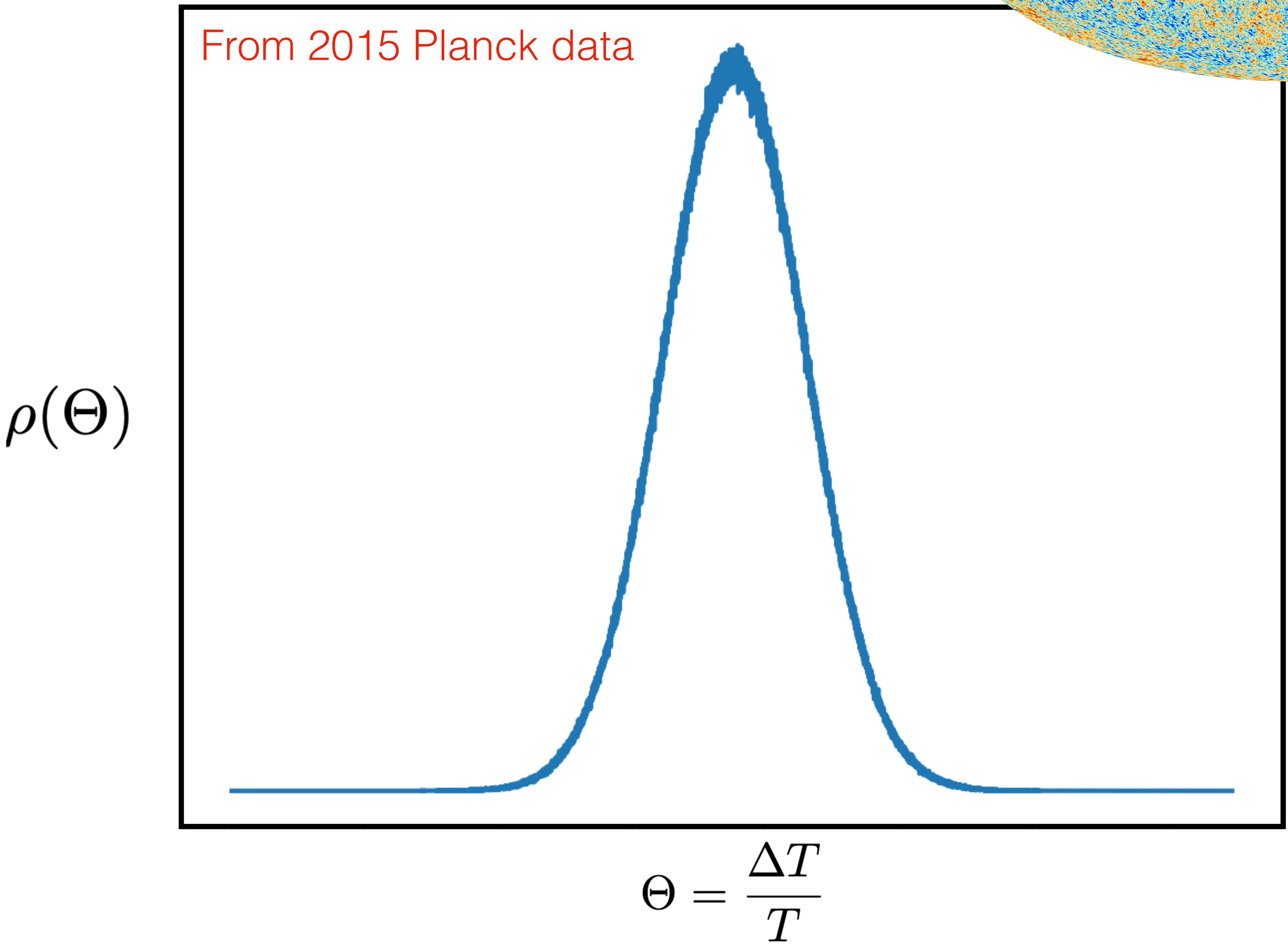
GAP & Sypsas (2023)

Beyond the bispectrum

For example, the reconstructed PDF from CMB data is





Beyond the bispectrum

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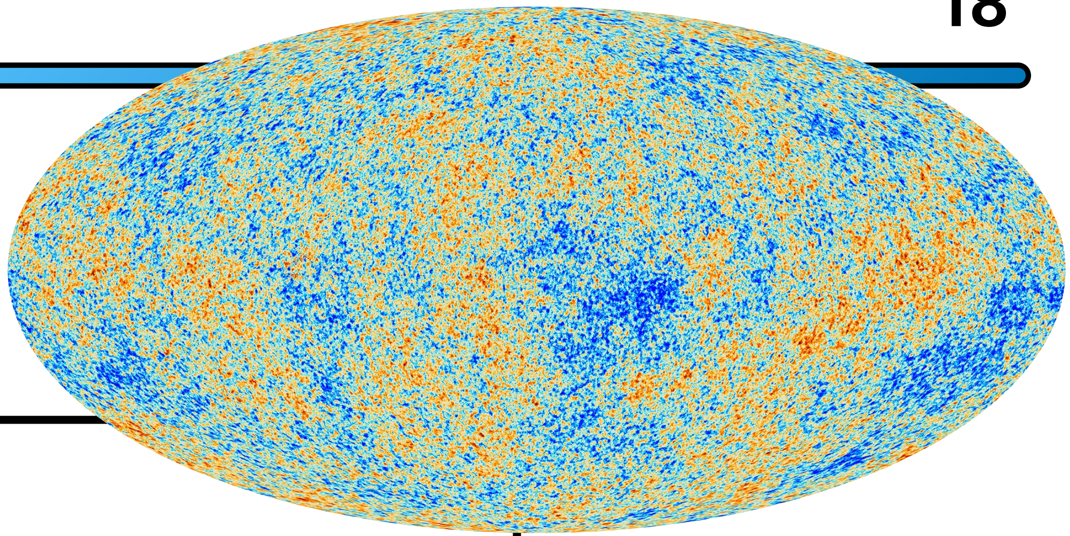

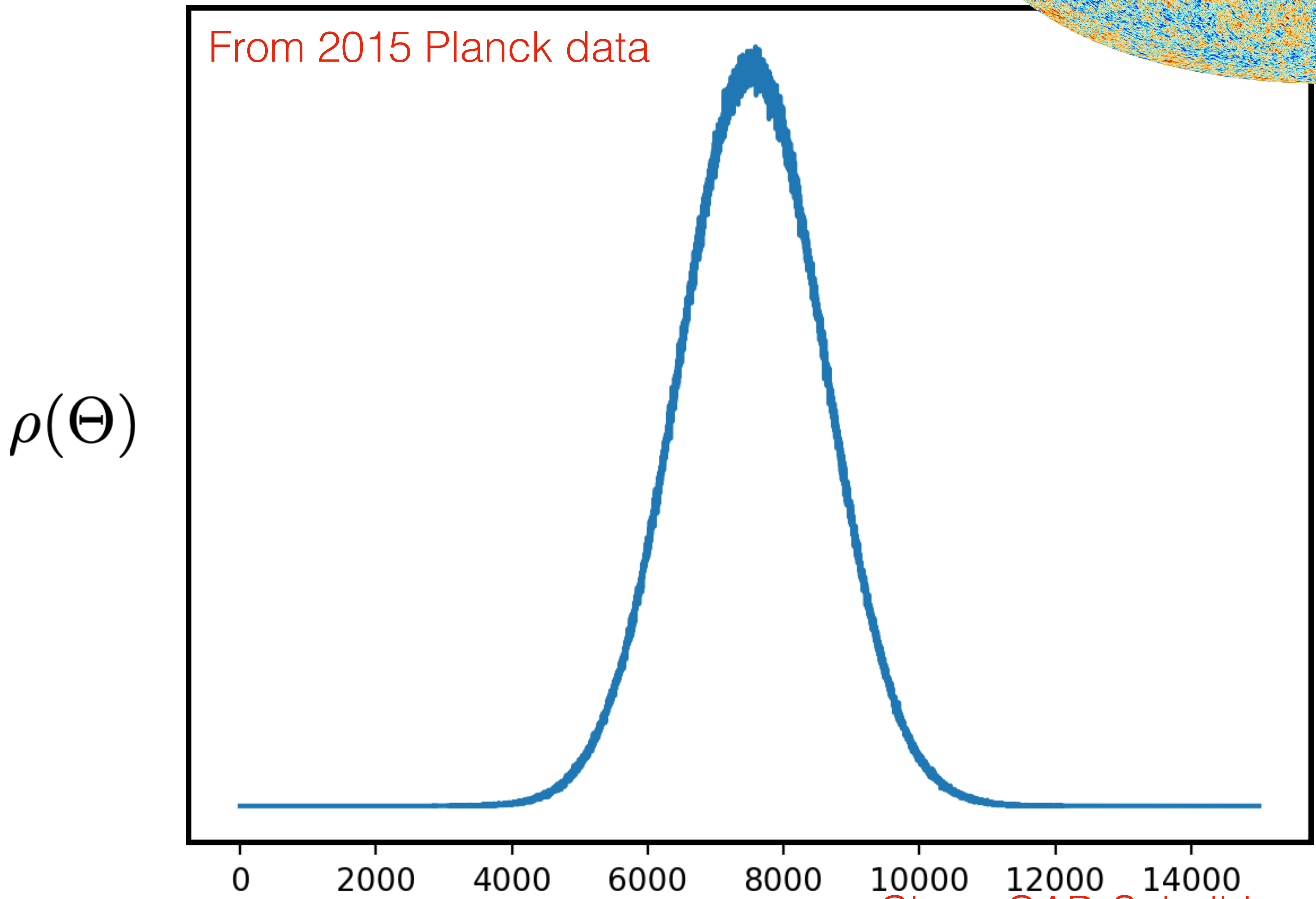

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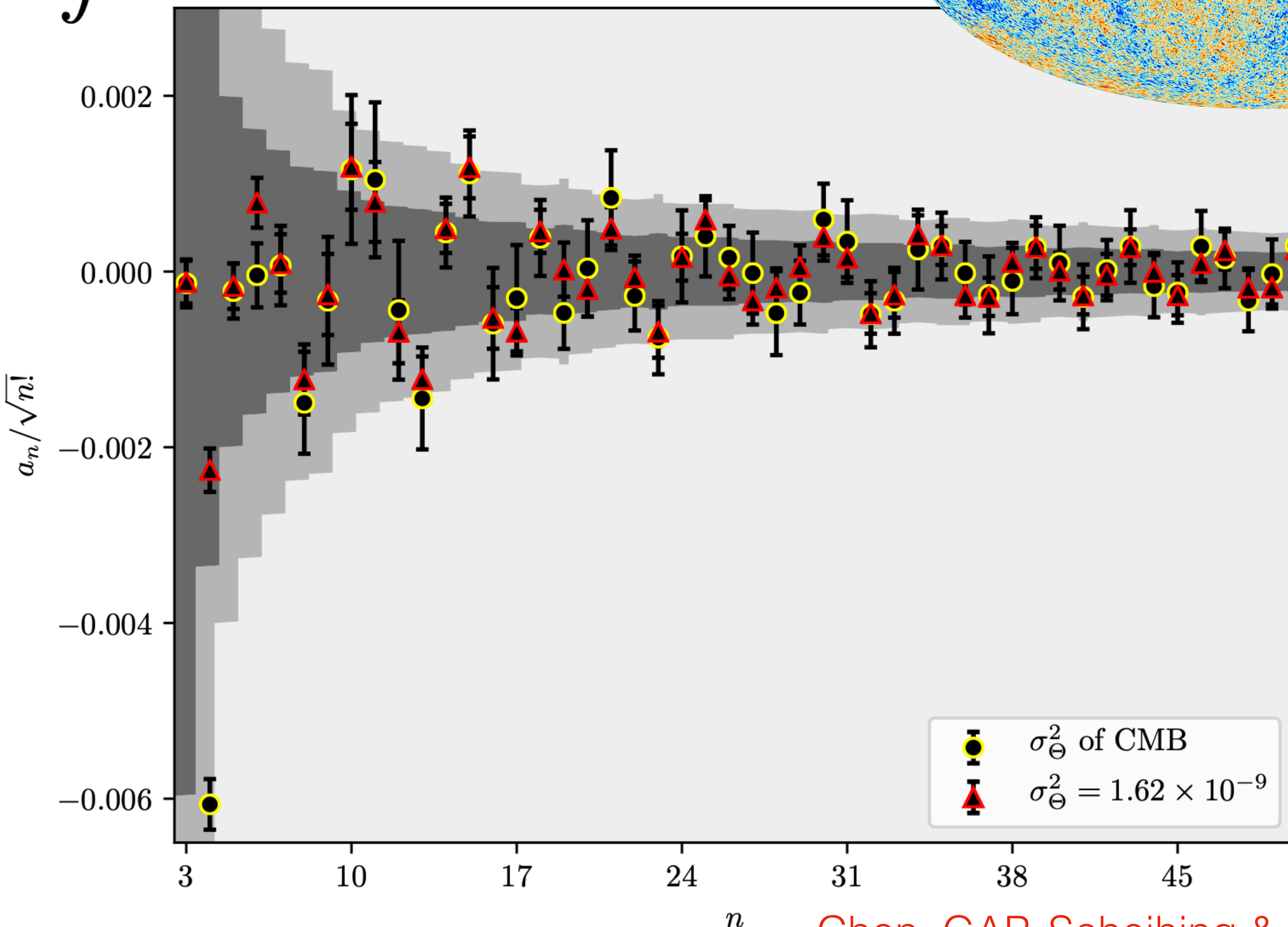
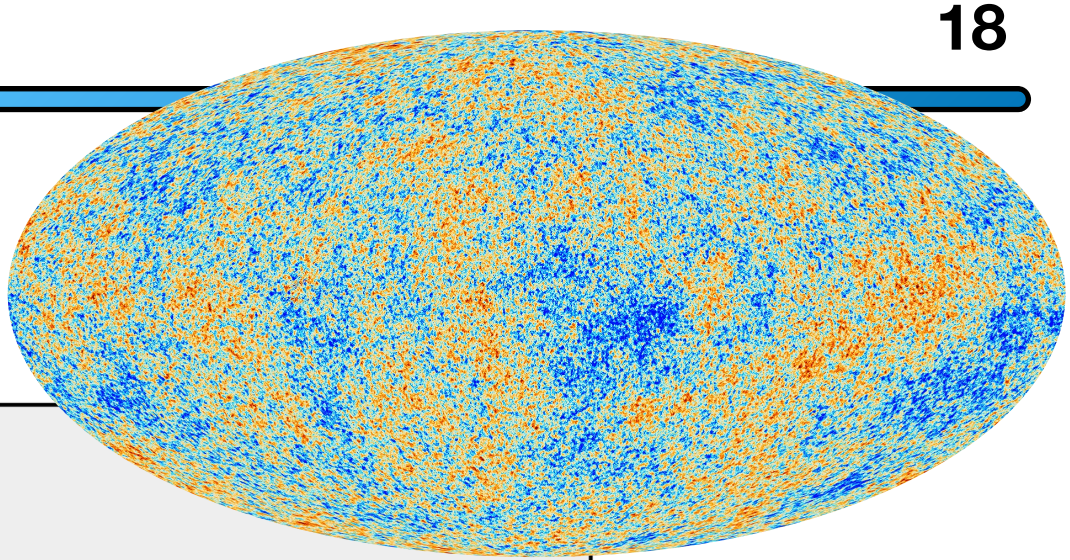
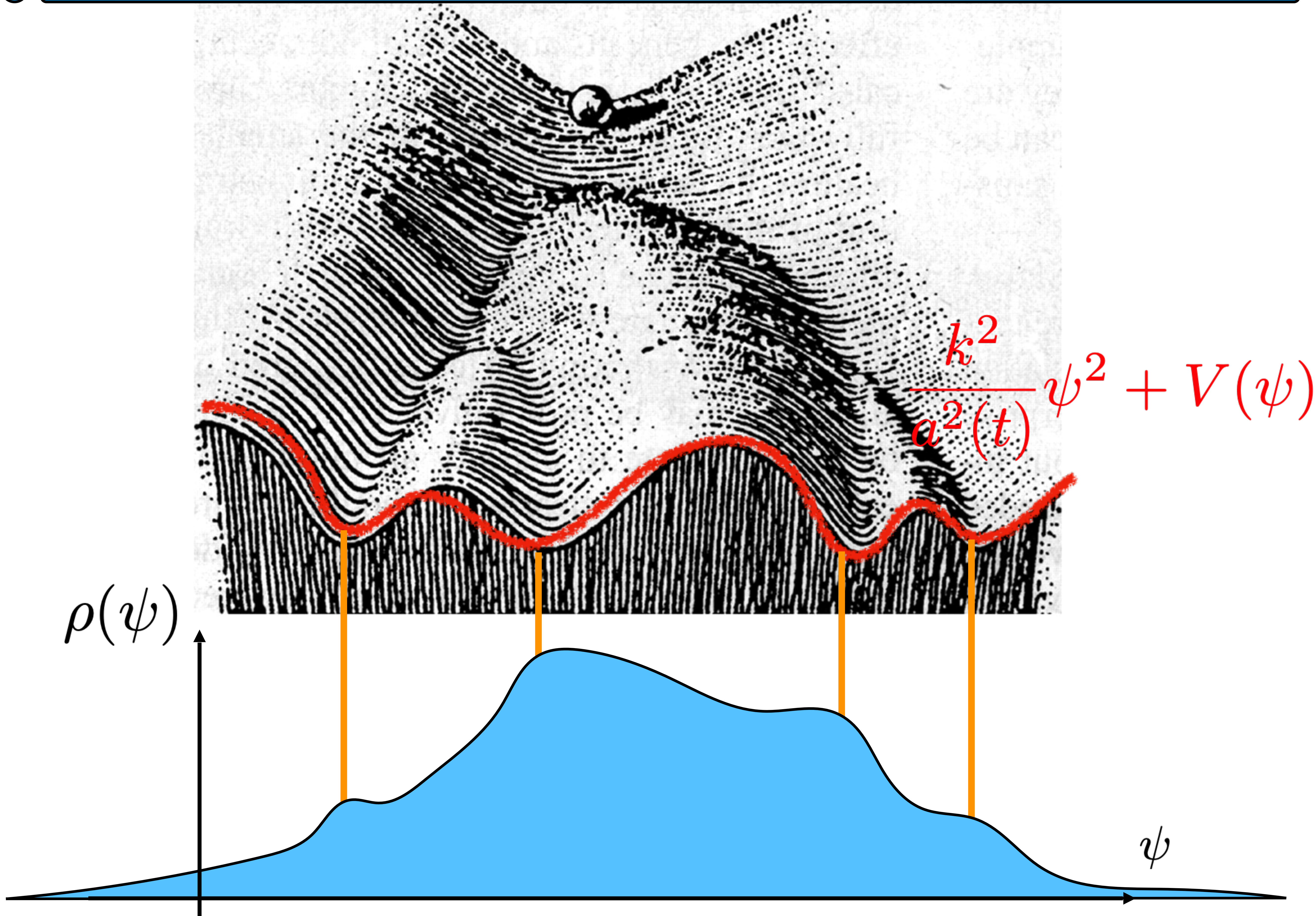




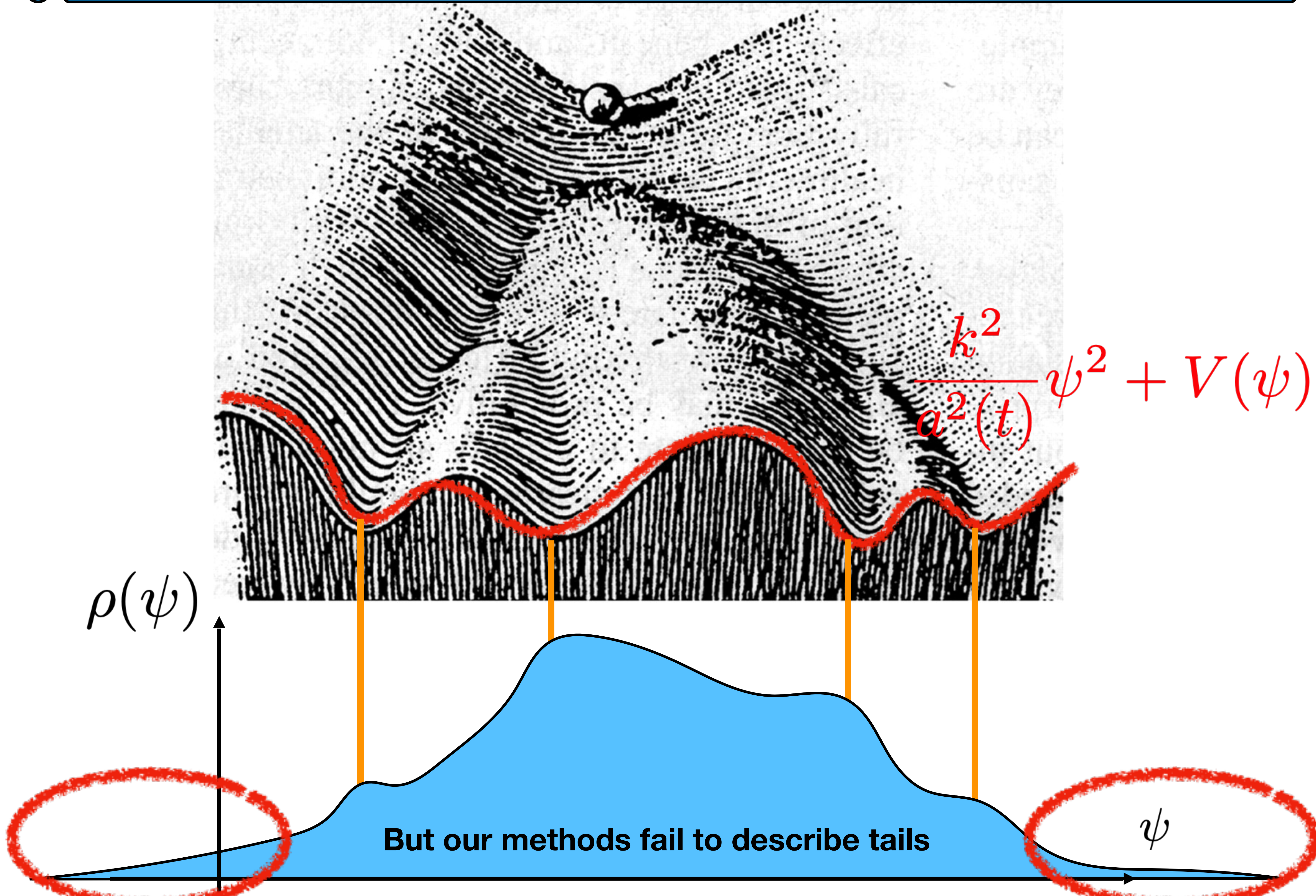
Chen, GAP, Scheihing & Sypsas (2018)

GAP, Sapone, Sohn, Sypsas (2023)

Beyond the bispectrum



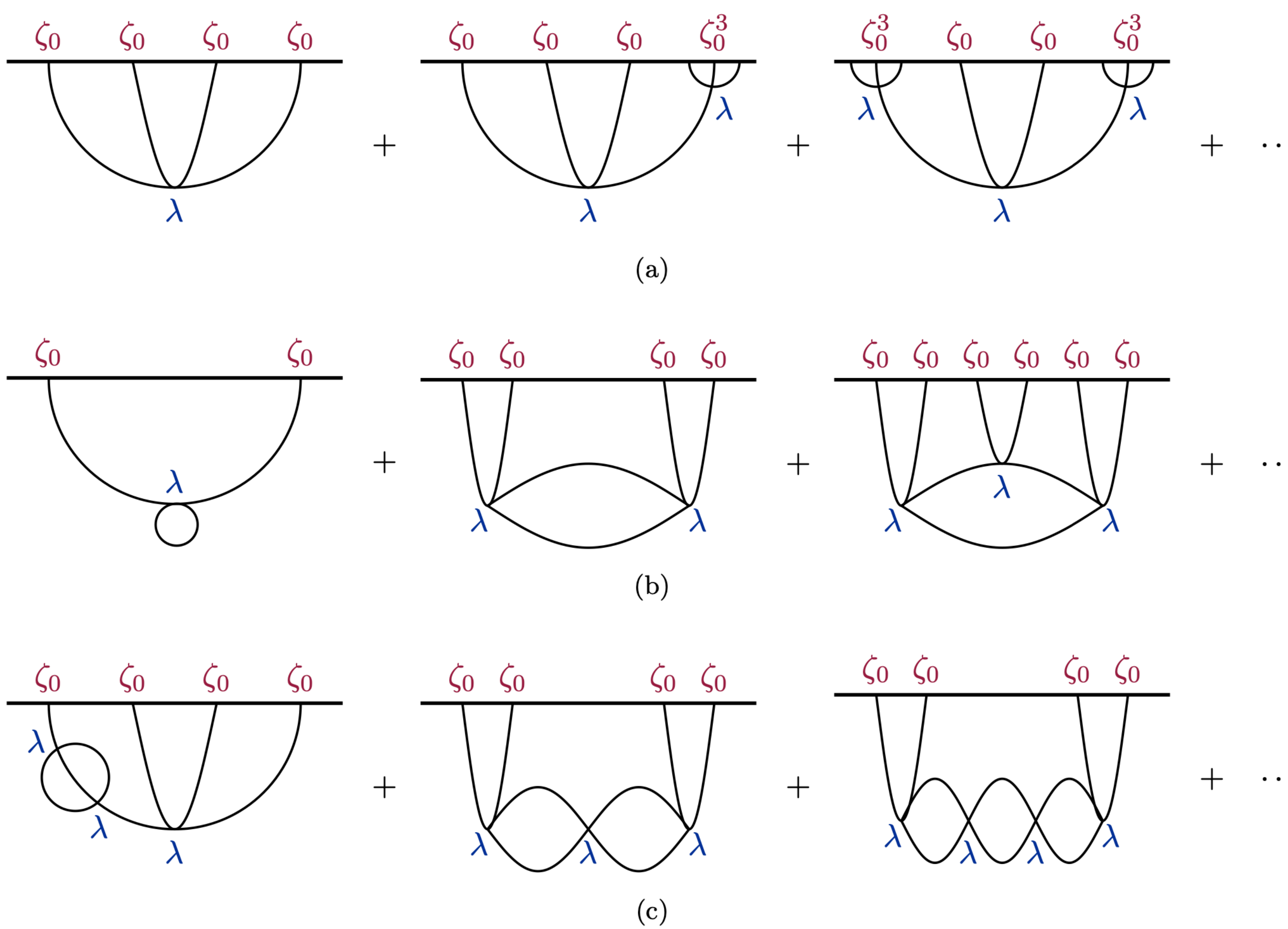
Beyond the bispectrum



Beyond the bispectrum

NG tails might be possible in single field inflation:

$$\mathcal{L} = \epsilon\left(\dot{\mathcal{R}}^2 - (\nabla\mathcal{R})^2 + \frac{\lambda}{4!H^2}\dot{\mathcal{R}}^4\right)$$



Celoria, Creminelli, Tambalo & Yingcharoenrat (2021)

Beyond the bispectrum

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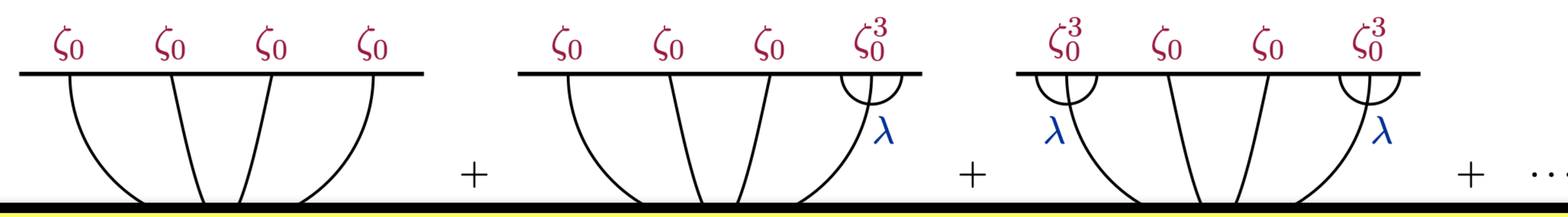
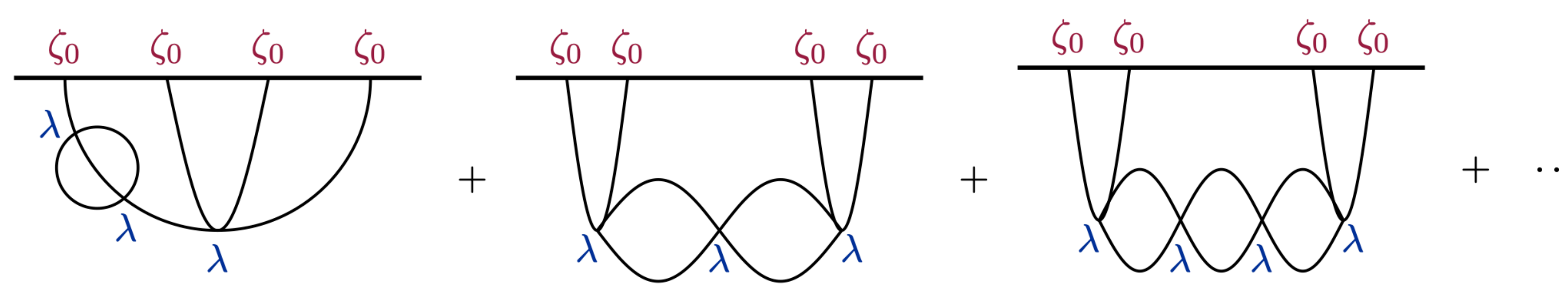


$$\rho(\mathcal{R}) \sim \exp\left[-\frac{\mathcal{R}^{3/2}}{\lambda^{1/4}}\right]$$

(b)



(c)

Celoria, Creminelli, Tambalo & Yingcharoenrat (2021)

Conclusions

- The primordial statistics may deviate significantly from Gaussianity in a way not parametrized by the bispectrum
- These effects could escape conventional data analysis
- New perturbative and nonperturbative techniques are necessary to uncover this type of NG

Conclusions

- The primordial statistics may deviate
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- Th …
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- Ne … ative
  tec … ver
  this type of NG

Thanks!