# A Fragmentation Approach to Jet Flavor 

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## Uses of Jet Flavor

## Whatever your taste, there's a jet flavor for you


2005.03016

NNLO predictions for heavy flavor jets

1706.00428

Correlation between jet flavor and initial state

2201.04206

New physics searches

## BSZ Flavor

Come on, just sum the flavors of partons in the jet
Unambiguous at LO


Unambiguous at NLO


Soft gluon splitting to quarks at NNLO spoils IR safety


## BSZ Flavor

Oh. What if you changed the clustering algorithm?
$y_{i j}^{(F)}=\frac{2\left(1-\cos \theta_{i j}\right)}{Q^{2}} \times \begin{cases}\max \left(E_{i}^{2}, E_{j}^{2}\right), & \text { softer of } i, j \text { is flavoured, } \\ \min \left(E_{i}^{2}, E_{j}^{2}\right), & \text { softer of } i, j \text { is flavourless }\end{cases}$
Modification to Durham $\mathrm{k}_{T}$ algorithm
Phys. Lett. B 269, 432 (1991)
Only cluster soft quarks together if they have equal and opposite flavor

- Pros
- It works and is IRC safe
- Can be easily implemented in partonic fixed order calculation
- Cons
- Modifies constituents of jets and no one uses $\mathrm{k}_{\mathrm{T}}$ to find jets anymore
anti-kT flavor algorithm, 2205.11879
- Complete non-starter in experiment; cannot begin to identify flavored jets


## What We Mean By "Jet Flavor"

Les Houches Study

## What is a Quark Jet?

## From lunch/dinner discussions



Well-Defined What we mean

A quark parton
A Born-level quark parton
The initiating quark parton in a final state shower
An eikonal line with baryon number I/3 and carrying triplet color charge

A quark operator appearing in a hard matrix element in the context of a factorization theorem

A parton-level jet object that has been quark-tagged using a soft-safe flavored jet algorithm (automatically collinear safe if you sum constituent flavors)

A phase space region (as defined by an unambiguous hadronic fiducial cross section measurement) that yields an enriched sample of quarks (as interpreted by some suitable, though fundamentally ambiguous, criterion)

## What We Mean By "Jet Flavor"

Focus on in-principle observables

- Many, many results in the literature about "quark vs. gluon discrimination"


De facto but flawed definition:
jet flavor defined by requested process in event simulation

## What We Mean By "Jet Flavor"

Focus on in-principle observables

- Many, many results in the literature about "quark vs. gluon discrimination"
- By asymptotic freedom, jet flavor is unambiguous in deep UV

User requests short distance process in event simulator

## What We Mean By "Jet Flavor"

Focus on in-principle observables

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- By asymptotic freedom, jet flavor is unambiguous in deep UV
- Measurements are performed in the deep IR


## Flow to IR governed by renormalization group/DGLAP

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Focus on in-principle observables

- Many, many results in the literature about "quark vs. gluon discrimination"
- By asymptotic freedom, jet flavor is unambiguous in deep UV
- Measurements are performed in the deep IR
- Flow from UV to IR is not invertible

Jet boundary destroys one-to-one UV to IR map

## What We Mean By "Jet Flavor"

Focus on in-principle observables

- Many, many results in the literature about "quark vs. gluon discrimination"
- By asymptotic freedom, jet flavor is unambiguous in deep UV
- Measurements are performed in the deep IR
- Flow from UV to IR is not invertible
- Give up on trying to get UV jet flavor; just focus on IR


# Desired Properties for Jet Flavor Your Tastes May Vary 

- Only returns a QCD parton flavor (up, down, strange, ...,gluon)

Simplifies classification

## Desired Properties for Jet Flavor Your Tastes May Vary

- Only returns a QCD parton flavor (up, down, strange,...,gluon)
- Can be applied to any set of partons

Does not require re-associating constituents of a jet

## Desired Properties for Jet Flavor Your Tastes May Vary

- Only returns a QCD parton flavor (up, down, strange,...,gluon)
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- IR safe, completely insensitive to soft particles

Ignores contribution to jet flavor from soft particles

## Desired Properties for Jet Flavor Your Tastes May Vary

- Only returns a QCD parton flavor (up, down, strange,...,gluon)
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- IR safe, completely insensitive to soft particles
- Inclusive over exactly collinear splittings

Absorb collinear divergences into fragmentation functions

## Desired Properties for Jet Flavor Your Tastes May Vary

- Only returns a QCD parton flavor (up, down, strange,...,gluon)
- Can be applied to any set of partons
- IR safe, completely insensitive to soft particles
- Inclusive over exactly collinear splittings
- Described by linear evolution equations

Use DGLAP as a guide and enables analytic solutions

## Definition of WTA Jet Flavor <br> Welcome to Flavortown!

The partonic flavor of a jet is defined to be the net flavor of the particle(s) whose momentum lies exactly along the WTA axis of the jet

Find closest pair of particles according to clustering metric $\mathrm{d}_{\mathrm{ij}}$


## Definition of WTA Jet Flavor Welcome to Flavortown!

The partonic flavor of a jet is defined to be the net flavor of the particle(s) whose momentum lies exactly along the WTA axis of the jet

Sum energies and new momentum points along direction of harder particle


## Definition of WTA Jet Flavor <br> Welcome to Flavortown!

The partonic flavor of a jet is defined to be the net flavor of the particle(s) whose momentum lies exactly along the WTA axis of the jet

Continue pairwise combination until one particle remains


## Definition of WTA Jet Flavor <br> Welcome to Flavortown!

The partonic flavor of a jet is defined to be the net flavor of the particle(s) whose momentum lies exactly along the WTA axis of the jet

Final momentum lies along direction of particle in jet


## Properties of Evolution Equations

Ain't they beautiful?

$$
Q^{2} \frac{d f_{q}\left(Q^{2}\right)}{d Q^{2}}=\frac{\alpha_{s}}{2 \pi}\left[-C_{F}\left(2 \log 2-\frac{5}{8}\right) f_{q}\left(Q^{2}\right)+\frac{1}{3} T_{R} f_{g}\left(Q^{2}\right)\right]
$$

$Q^{2} \frac{d f_{g}\left(Q^{2}\right)}{d Q^{2}}=\frac{\alpha_{s}}{2 \pi}\left[C_{F}\left(2 \log 2-\frac{5}{8}\right)-\left(C_{F}\left(2 \log 2-\frac{5}{8}\right)+\frac{2}{3} n_{f} T_{R}\right) f_{g}\left(Q^{2}\right)\right]$

- Linear, inhomogeneous evolution equations

Vastly simpler than nonlinear evolution for hardest subjet, NGLs, track functions,...

## Properties of Evolution Equations

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- Linear, inhomogeneous evolution equations
- Independent of the color of the gluon

Gluon emissions of gluons can't change WTA flavor

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- Linear, inhomogeneous evolution equations
- Independent of the color of the gluon
- Deep IR fixed points of jet flavor:

$$
\begin{aligned}
\lim _{Q_{0}^{2} \rightarrow \infty} f_{q}\left(Q^{2}\right) & =\frac{\frac{1}{3} T_{R}}{C_{F}\left(2 \log 2-\frac{5}{8}\right)+\frac{2}{3} n_{f} T_{R}} \approx 0.062149 \\
\lim _{Q_{0}^{2} \rightarrow \infty} f_{g}\left(Q^{2}\right) & =\frac{C_{F}\left(2 \log 2-\frac{5}{8}\right)}{C_{F}\left(2 \log 2-\frac{5}{8}\right)+\frac{2}{3} n_{f} T_{R}} \approx 0.37851
\end{aligned}
$$

## Comparison to Parton Shower

Evolution of WTA Gluon Flavor from Jet $p_{\text {t }}$ to IR

$$
\mathrm{pp} \rightarrow \mathrm{cc} \text { or } \mathrm{pp} \rightarrow \mathrm{gg} \text { events generated in Pythia8 }
$$

No hadronization; parton shower terminated at fixed IR scale Study evolution as a function of the UV scale; the jet $\mathrm{p}_{T}$



## Comparison to Parton Shower

Evolution of WTA Quark Flavor from Jet $p_{T}$ to IR

$$
\mathrm{pp} \rightarrow \mathrm{cc} \text { or } \mathrm{pp} \rightarrow \mathrm{gg} \text { events generated in Pythia8 }
$$

No hadronization; parton shower terminated at fixed IR scale
Study evolution as a function of the UV scale; the jet $\mathrm{p}_{T}$



## Summary

- Collinear divergences aren't that scary!
- Let fragmentation functions be your friend
- Other things:
- Can WTA flavor be embedded in a factorization theorem?
- FO matches to pdfs all the time; can they match to WTA flavor?
- WTA axis is extremely robust to contamination
- Observables measured about WTA axis may connect partonic flavor to realistic hadronic jets


## Bonus

## Derivation of Evolution Equations Just DGLAP + Hardest Energy Constraint

$$
Q^{2} \frac{d f_{q}\left(x, Q^{2}\right)}{d Q^{2}}=\frac{\alpha_{s}}{2 \pi} \int_{x}^{\min [1,2 x]} \frac{d z}{z}\left[P_{q g \leftarrow q}\left(\frac{x}{z}\right) f_{q}\left(z, Q^{2}\right)+P_{q \bar{q} \leftarrow g}\left(\frac{x}{z}\right) f_{g}\left(z, Q^{2}\right)\right]
$$

## Derivation of Evolution Equations Just DGLAP + Hardest Energy Constraint

$$
Q^{2} \frac{d f_{q}\left(x, Q^{2}\right)}{d Q^{2}}=\frac{\alpha_{s}}{2 \pi} \int_{x}^{\min [1,2 x]} \frac{d z}{z}\left[P_{q g \leftarrow q}\left(\frac{x}{z}\right) f_{q}\left(z, Q^{2}\right)+P_{q \bar{q} \leftarrow g}\left(\frac{x}{z}\right) f_{g}\left(z, Q^{2}\right)\right]
$$



$$
\&
$$

$$
x>z-x
$$

## Derivation of Evolution Equations

 Integrate Over Energy Fractions$$
\begin{gathered}
Q^{2} \frac{d f_{q}\left(x, Q^{2}\right)}{d Q^{2}}=\frac{\alpha_{s}}{2 \pi} \int_{x}^{\min [1,2 x]} \frac{d z}{z}\left[P_{q g \leftarrow q}\left(\frac{x}{z}\right) f_{q}\left(z, Q^{2}\right)+P_{q \bar{q} \leftarrow g}\left(\frac{x}{z}\right) f_{g}\left(z, Q^{2}\right)\right] \\
\int_{0}^{1} d x f_{i}\left(x, Q^{2}\right)=f_{i}\left(Q^{2}\right)
\end{gathered}
$$

Fraction of jets with parton $i$ along WTA axis

$$
\begin{gathered}
Q^{2} \frac{d f_{q}\left(Q^{2}\right)}{d Q^{2}}=\int_{1 / 2}^{1} d y\left[P_{q g \leftarrow q}(y) f_{q}\left(Q^{2}\right)+P_{q \bar{q} \leftarrow g}(y) f_{g}\left(Q^{2}\right)\right] \\
\text { Just need reduced moments of splitting functions }
\end{gathered}
$$

## Derivation of Evolution Equations

Final WTA Fraction Evolution Equations

$$
Q^{2} \frac{d f_{q}\left(Q^{2}\right)}{d Q^{2}}=\frac{\alpha_{s}}{2 \pi}\left[-C_{F}\left(2 \log 2-\frac{5}{8}\right) f_{q}\left(Q^{2}\right)+\frac{1}{3} T_{R} f_{g}\left(Q^{2}\right)\right]
$$

Quark flavor $q$ evolution equation

$$
Q^{2} \frac{d f_{g}\left(Q^{2}\right)}{d Q^{2}}=\frac{\alpha_{s}}{2 \pi}\left[C_{F}\left(2 \log 2-\frac{5}{8}\right)-\left(C_{F}\left(2 \log 2-\frac{5}{8}\right)+\frac{2}{3} n_{f} T_{R}\right) f_{g}\left(Q^{2}\right)\right]
$$

Gluon flavor evolution equation

## Flavored Observables Measuring WTA Axis

Energy fraction carried by WTA axis


$$
Q^{2} \frac{d f_{q_{i}}\left(x, Q^{2}\right)}{d Q^{2}}=\frac{\alpha_{s}}{2 \pi} \int_{x}^{\min [1,2 x]} \frac{d z}{z}\left[P_{q g \leftarrow q}\left(\frac{x}{z}\right) f_{q_{i}}\left(z, Q^{2}\right)+P_{q \bar{q} \leftarrow g}\left(\frac{x}{z}\right) f_{g}\left(z, Q^{2}\right)\right]
$$

$Q^{2} \frac{d f_{g}\left(x, Q^{2}\right)}{d Q^{2}}=\frac{\alpha_{s}}{2 \pi} \int_{x}^{\min [1,2 x]} \frac{d z}{z}\left[P_{g q \leftarrow q}\left(\frac{x}{z}\right) \sum_{i=1}^{n_{f}}\left(f_{q_{i}}\left(z, Q^{2}\right)+f_{\bar{q}_{i}}\left(z, Q^{2}\right)\right)+P_{g g \leftarrow g}\left(\frac{x}{z}\right) f_{g}\left(z, Q^{2}\right)\right]$
Solve $2 \mathrm{n}_{\mathrm{f}}+1$ coupled integro-differential equations
Just as complicated as DGLAP; solve in conjugate space and invert

# Flavored Observables Measuring WTA Axis 

Simplest Solution: Parton Shower Monte Carlo



## Flavored Observables Measuring WTA Axis

Simplest Solution: Parton Shower Monte Carlo


Flavor change = need two hard emissions
UV universality of IR gluons

# Flavored Observables Measuring about WTA Axis 

Generalized Angularities

$$
\lambda_{\beta}^{\kappa}=\sum_{i \in J} z_{i}^{\kappa} \theta_{i}^{\beta}
$$

$$
\mid \lambda_{\beta}^{\kappa}
$$

Flavor-Inclusive Distribution

$$
p\left(\lambda_{\beta}\right)=\frac{d \sigma^{\mathrm{fo}}}{d \lambda_{\beta}} \Delta_{\mathrm{Sud}}\left(\lambda_{\beta}\right)
$$

Ansatz Flavor-Sensitive Distribution

$$
p_{i}\left(\lambda_{\beta}\right)=\frac{d \sigma_{i}^{\mathrm{fo}}}{d \lambda_{\beta}} \Delta_{\mathrm{Sud}}\left(\lambda_{\beta}\right)
$$



## Flavored Observables

## Measuring about WTA Axis

## WTA Flavor Quark Jets

Pythia


Flavor change starts at order- $a_{s}^{\log _{10} \lambda_{0.5}}$

Analytic


Energy is clustered close to WTA axis

## Flavored Observables

## Measuring about WTA Axis

## WTA Flavor Gluon Jets




Good qualitative agreement
Can this be systematic? Is there a factorization that enables calculation?

