Power corrections for non-local subtraction methods

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Introduction

- When computing higher order corrections in QCD, **IR divergent phase-space integrals** of scattering amplitudes appear in the intermediate steps of the calculation.

- In order to perform numerical integration a regularization procedure is necessary.

- There are two techniques for removing IR divergences: **local subtraction** and **slicing/non-local subtraction**.

- This talk will focus on slicing for processes with jets.

- The idea of the slicing method at N^kLO is to define a **resolution variable** X such that:

1. In the region $X > 0$ there is **1-resolved emission**, there are only $N^{k-1}LO$ types singularities.

2. The N^kLO unresolved limits occur **only at** $X = 0$.

Slicing formalism

- The resolution variable can be used to split the cross-section as:

$$\int d\sigma_{N^kLO} = \int d\sigma_{N^kLO}\theta(r_{cut} - X) + \int d\sigma^R_{N^kLO}\theta(X - r_{cut})$$

- We can approximate the integral in the unresolved region by taking the soft and collinear limits:

$$\int d\sigma_{N^kLO}\theta(r_{cut} - X) = \int d\sigma^{sing}_{N^kLO}\theta(r_{cut} - X) + \mathcal{O}(r^{\ell}_{cut}) = \int H \otimes d\sigma_{LO} - \int d\sigma^{CT}_{N^kLO}\theta(X - r_{cut}) + \mathcal{O}(r^{\ell}_{cut})$$

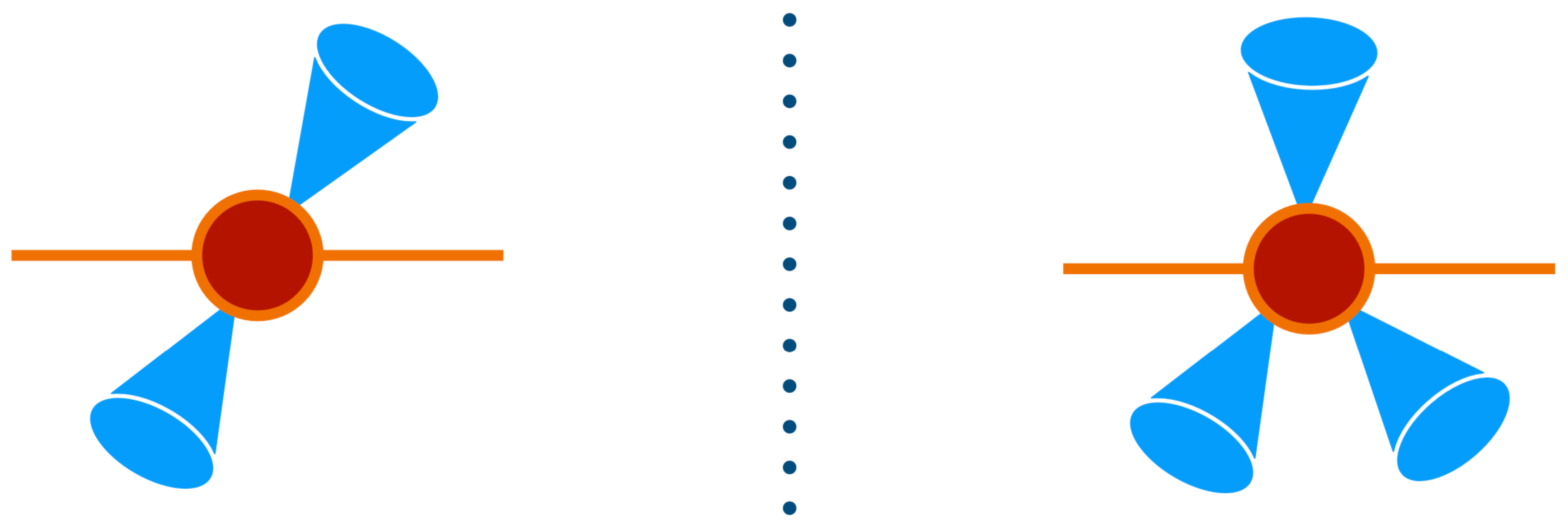
- The N^kLO cross-section is then:

$$\int d\sigma_{N^kLO} = \int H \otimes d\sigma_{LO} + \int \left[d\sigma^R_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{X>r_{cut}} + \mathcal{O}(r^{\ell}_{cut})$$

- The computation is performed by using a **small but finite** value of r_{cut}. This means that the final result will be affected by some missing power corrections.

N-jet resolution variable

- We want to apply the slicing formalism to processes with jets in the final state. It is then necessary to use a resolution variable that captures the transition from N to $N+1$ jets.



$X < r_{cut}$: 2 Jet configuration $X > r_{cut}$: 3 Jet configuration

- The first proposal for an N-Jet resolution variable was ***N*-Jettiness** (τ_N) [Stewart, Tackmann, Waalewijn (2010)], that, in the context of jet processes, has been applied in the NNLO computation for the production of Higgs or vector boson with 1 jet [Boughezal, Focke, Giele, Liu, Petriello (2015)][Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello (2016)].

- N-Jettiness exhibits **linear logarithmically enhanced** power corrections, $\mathcal{O}(r_{cut} \log r_{cut})$.

- A new variable, k_T^{ness}, has been proposed and a complete formulation of NLO k_T^{ness}-slicing has been provided [Buonocore, Grazzini, Haag, Rottoli, Savoini (2022)]. k_T^{ness} exhibits **purely linear** power corrections, $\mathcal{O}(r_{cut})$.

Exploring jet resolution variables

- While power corrections for N-Jettiness slicing have been studied in detail [Moult, Rothen, Stewart, Tackmann, Zhu (2016)][Ebert, Multi Stewart, Tackmann, Vita, Zhu (2018)][Boughezal, Isgrò, Petriello, (2018)][Ebert, Tackmann (2020)][Boughezal, Isgrò, Petriello (2020)], the considerations on power corrections made on $k_T^{ness}$are mainly based on empirical evidences.

- We would like to explicitly compute the power corrections and understand the origin of the differences in the scaling among different variables.

- We will focus on $e^+e^- \to 2j$ at NLO: in this case we do not have QCD initial-state singularities and the results obtained can be reused for hadronic collisions.

- In this talk we will discuss and compare three jet resolution variables: $y_{N,N+1}$, k_T^{ness}, k_T^{FSR}.

$y_{N,N+1}$: definition of the variable

- For leptonic collisions, we can consider the distance among partons from the k_T algorithm:

$$d_{ij} = \frac{2\ \min(E_i^2, E_j^2)}{Q^2}(1 - \cos\theta_{ij})$$

- Let us consider a final state with $M > N$ QCD partons. We run the k_T clustering algorithm until $N + 1$ protojets are left. $y_{N,N+1}$ is defined as the minimum among the d_{ij} of the $N + 1$ protojets.

$$y_{N,N+1} = \min\{d_{ij}\}$$

- The limit $y_{N,N+1} \to 0$ corresponds to the kinematical configuration in which one of the $N + 1$ partons is unresolved and thus there is an N-jet configuration.

Slicing formalism using $y_{2,3}$

- The counterterm corresponding to this variable can be obtained by integrating the collinear approximation of the real matrix element "below the cut":

$$8\pi\alpha_s\mu^{2\epsilon}|\mathscr{M}_B|^2\int d\phi_{rad}\frac{1}{s_{ij}}P_{qq}(z,\epsilon)\theta(r_{cut}^2-y_{2,3}) = -\,d\sigma^{CT} + \text{finite term} + \epsilon\text{-poles} + \mathcal{O}(r_{cut})$$

$$d\sigma^{CT} = d\sigma_{LO}\frac{\alpha_s}{2\pi}C_F\left(2\log^2 r_{cut} + 3\log r_{cut}\right)$$

- To compute the inclusive cross-section for $e^+e^- \to q\bar{q}$ the contribution coming from the soft wide-angle has to be considered too.

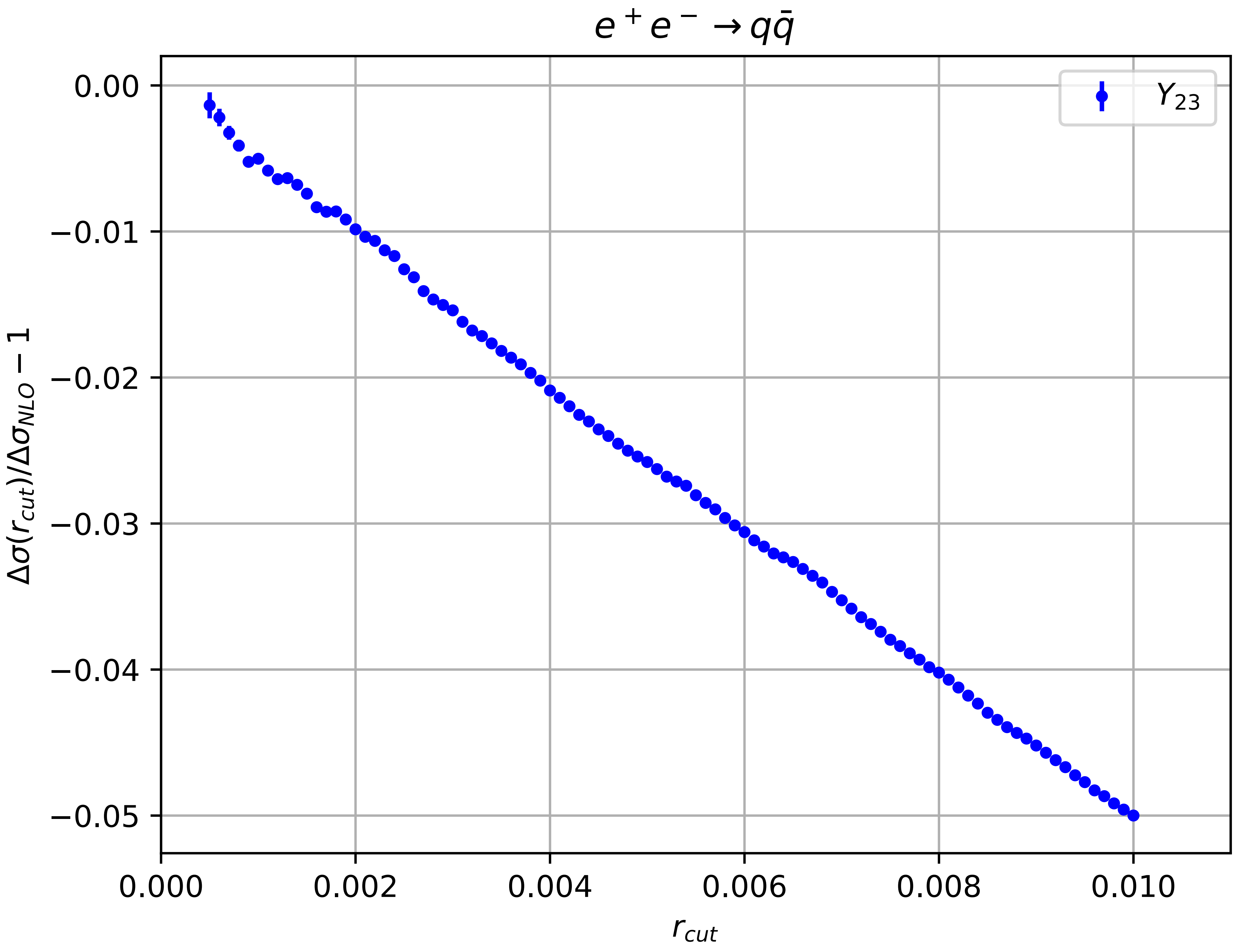
$$8\pi\alpha_s C_F|\mathscr{M}_B|^2\int d\phi_{rad}[(-T_1\cdot T_2)\omega_{12} - T_1^2\,\omega_1 - T_2^2\,\omega_2]\theta(r_{cut}^2-y_{2,3}) = |\mathscr{M}_B|^2\,C_F\frac{\alpha_S}{2\pi}\frac{\pi^2}{6}$$

$$\omega_1 = \frac{p_1\cdot p_2}{(p_1\cdot k)(p_1+p_2)\cdot k} \quad \omega_2 = \frac{p_1\cdot p_2}{(p_2\cdot k)(p_1+p_2)\cdot k} \quad \omega_{12} = \frac{p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}$$

$y_{2,3}$ power corrections

- We compared the cross-section computed using $y_{2,3}$ slicing (that depends on r_{cut}) with the exact analytical one.

- Since the soft wide-angle contribution is not vanishing we expect a linear scaling, similar to what is observed in heavy-quark production. This is confirmed by the numerical computation.

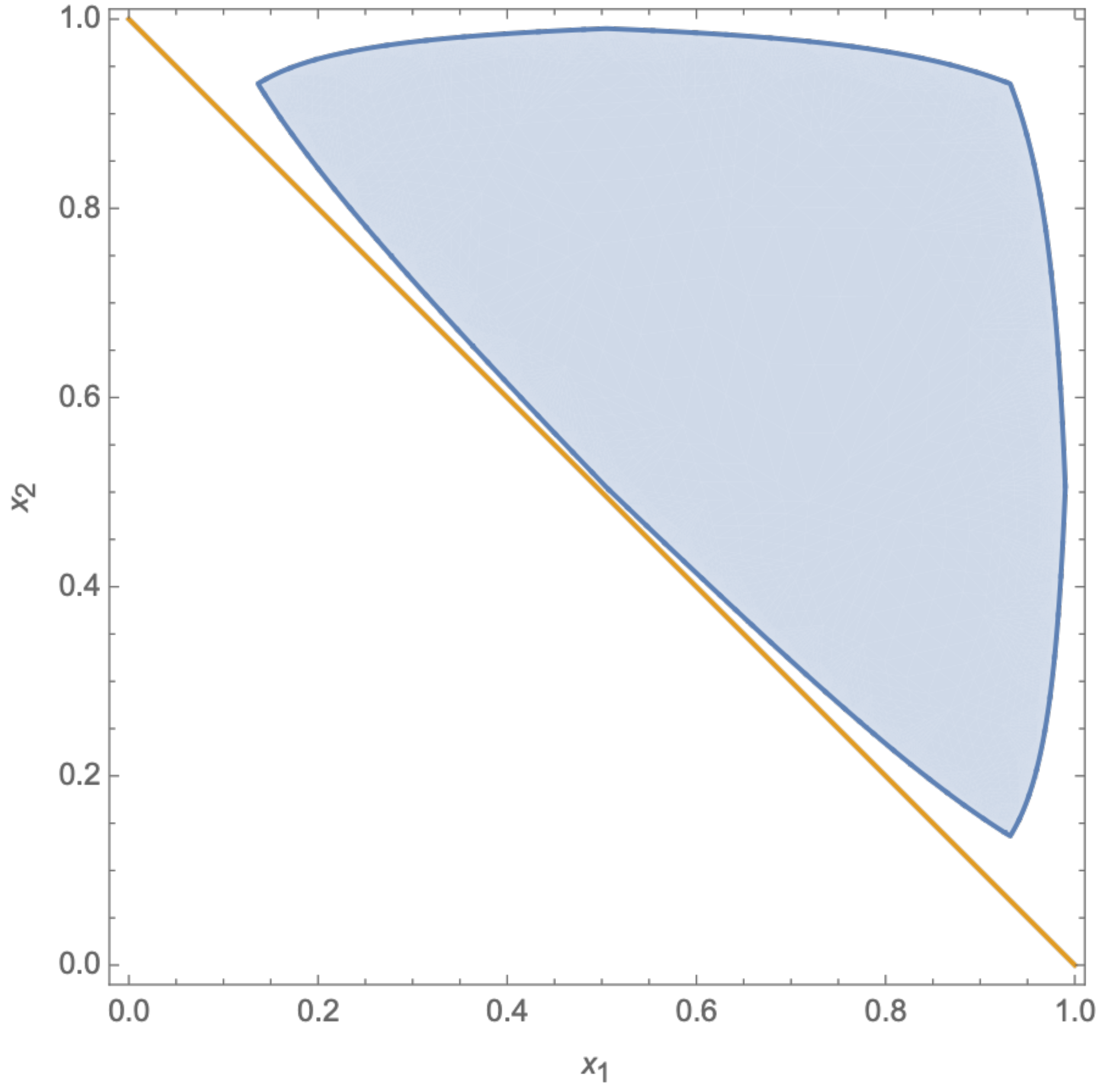


$y_{2,3}$ power corrections

- We analytically computed the missing power corrections, integrating the real matrix element (without approximation) above the cut:

$$d\sigma_{LO} C_F \frac{\alpha_s}{2\pi} \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \theta(y_{2,3} - r_{cut})$$

$$x_i = \frac{2E_i}{Q} \qquad x_1 + x_2 + x_3 = 2$$



- In the x_1-x_2 plane, the collinear limits correspond to the lines $x_1 = 1$ and $x_2 = 1$, while the soft limit occurs in the point $(x_1, x_2) = (1{,}1)$

- The result of the latter integral is a large expression, function of r_{cut}. This result, up to $\mathcal{O}(r_{cut})$, is:

$$d\sigma_{LO} C_F \frac{\alpha_s}{2\pi} \left(\frac{5}{4} - \frac{\pi^2}{12} + 3\log 2 + \underbrace{3\log r_{cut} + 2\log^2 r_{cut}}_{\text{Cancelled by the counterterm}} + \underbrace{(2\log(1+\sqrt{2}) - 4\sqrt{2}) r_{cut}}_{\text{Linear power correction}} + \mathcal{O}(r_{cut}^2) \right)$$

k_T^{ness} resolution variable

- The definition of k_T^{ness} is the same as the one of $y_{2,3}$, but uses the distance among partons:

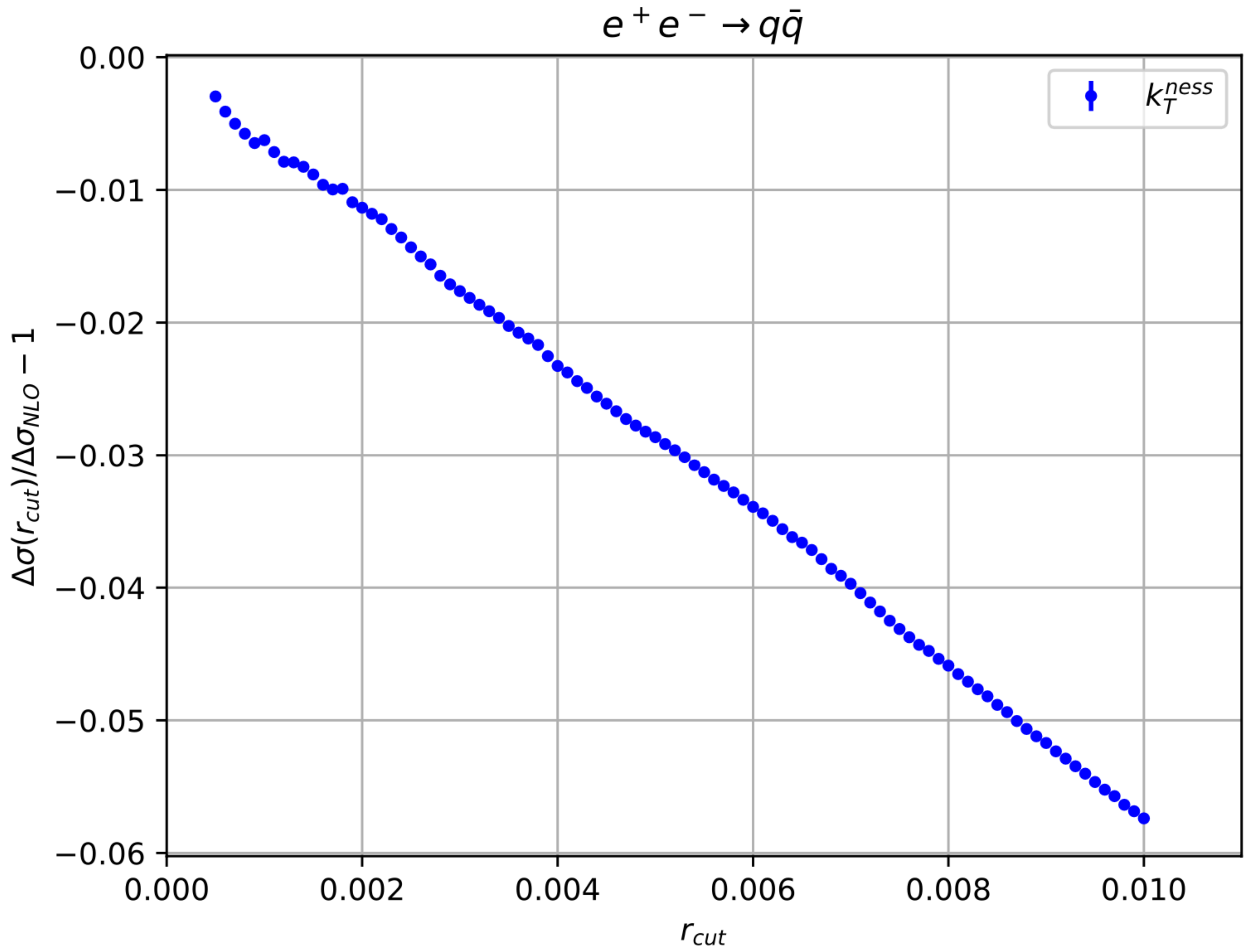
$$d_{ij} = \min(k_{i,T}^2, k_{j,T}^2)\frac{\Delta R_{ij}^2}{Q^2} \qquad \Delta R_{ij}^2 = \Delta\eta_{ij}^2 + \Delta\phi_{ij}^2$$

- The counterterm is the same as the one of $y_{2,3}$, while the non-vanishing soft wide-angle contribution is different, that has been computed numerically as a two-folded integral.

$$\int d\phi_{rad}\left\{\omega_{12}[\theta(\Delta R_{1k}^2 - \Delta R_{2k}^2)\theta(r_{cut}^2 - d_{2k}) + \theta(\Delta R_{2k}^2 - \Delta R_{1k}^2)\theta(r_{cut}^2 - d_{1k})] - \omega_1\theta(r_{cut}^2 - d_{1k}^{||}) - \omega_2\theta(r_{cut}^2 - d_{2k})\right\}$$

$$= 2\int d\phi_{rad}\left\{\omega_1\theta(\Delta R_{1k}^2 - \Delta R_{2k}^2)[\theta(r_{cut}^2 - d_{2k}) - \theta(r_{cut}^2 - d_{1k})] + \omega_1[\theta(r_{cut}^2 - d_{1k}) - \theta(r_{cut}^2 - d_{1k}^{||})]\right\} \longrightarrow$$

Finite in $d = 4$ dimensions

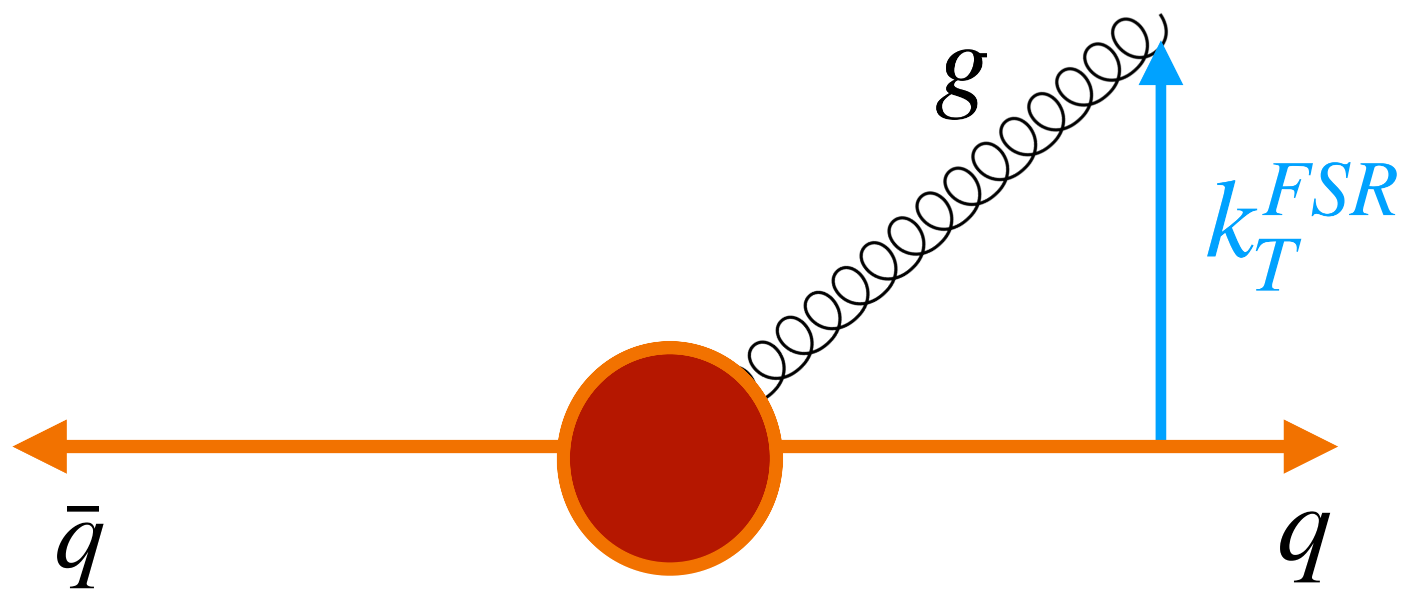


- The numerical application of the slicing method shows that the leading missing power corrections are linear.

k_T^{FSR} resolution variable

- We would like to understand if it is possible to define a variable that has a quadratic leading missing power correction, like q_T for initial-state singularities, that has a vanishing soft wide-angle contribution.

- We can consider the transverse momentum of the real radiation with respect to the $q\bar{q}$ axis, in the frame in which q and $\bar{q}$ are back-to-back. We denote this variable as k_T^{FSR}.



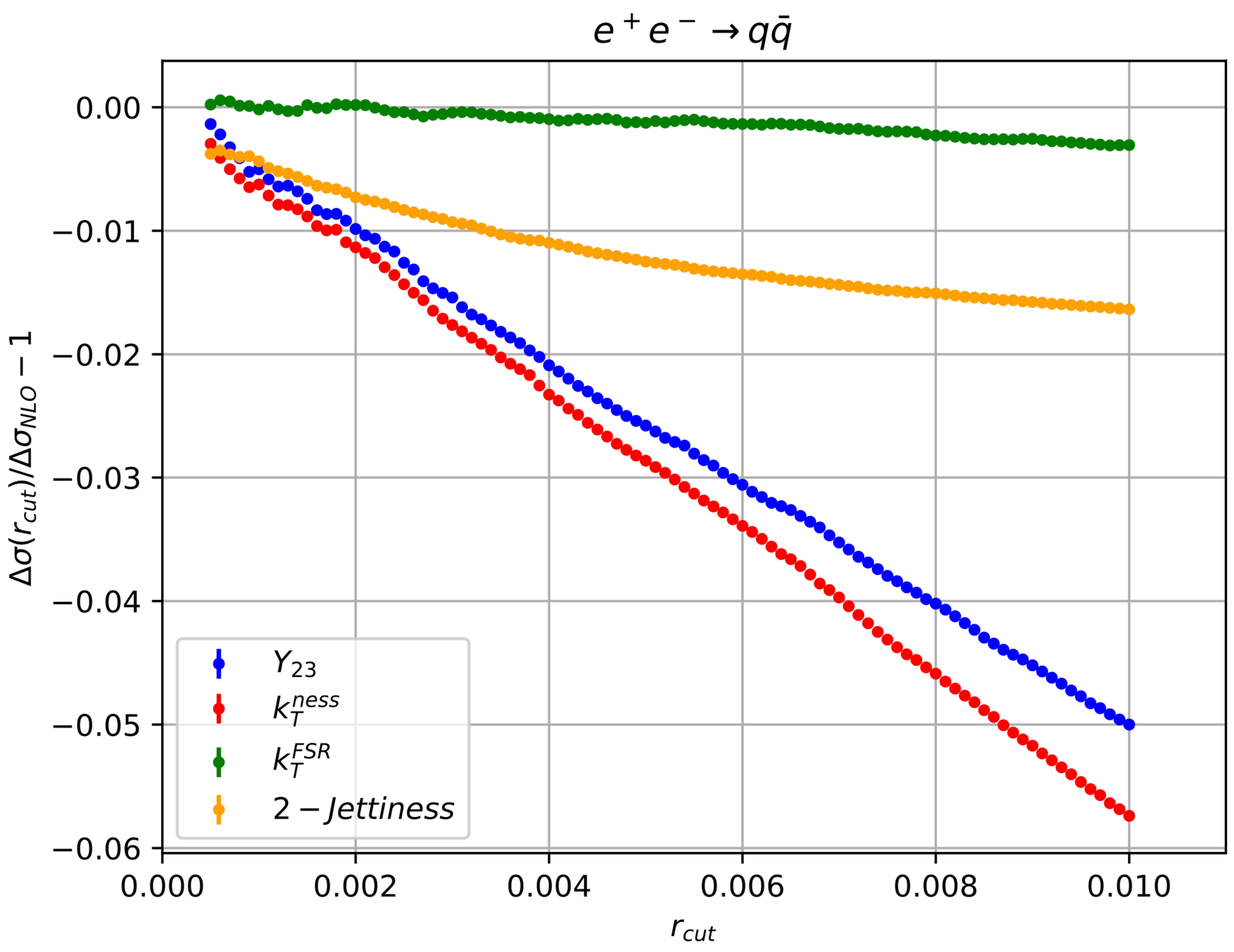
$$k_T^{FSR} = \sqrt{\frac{2(p_1 \cdot k)(p_2 \cdot k)}{p_1 \cdot p_2}}$$

- In this case, the soft wide-angle contribution vanishes:

$$2C_F \int d\phi_{rad}\omega_1[\theta(r_{cut}^2 - k_T^{FSR}) - \theta(r_{cut}^2 - k_T^{FSR,\|})] = 0, \text{ since } k_T^{FSR} = k_T^{FSR,\|}$$

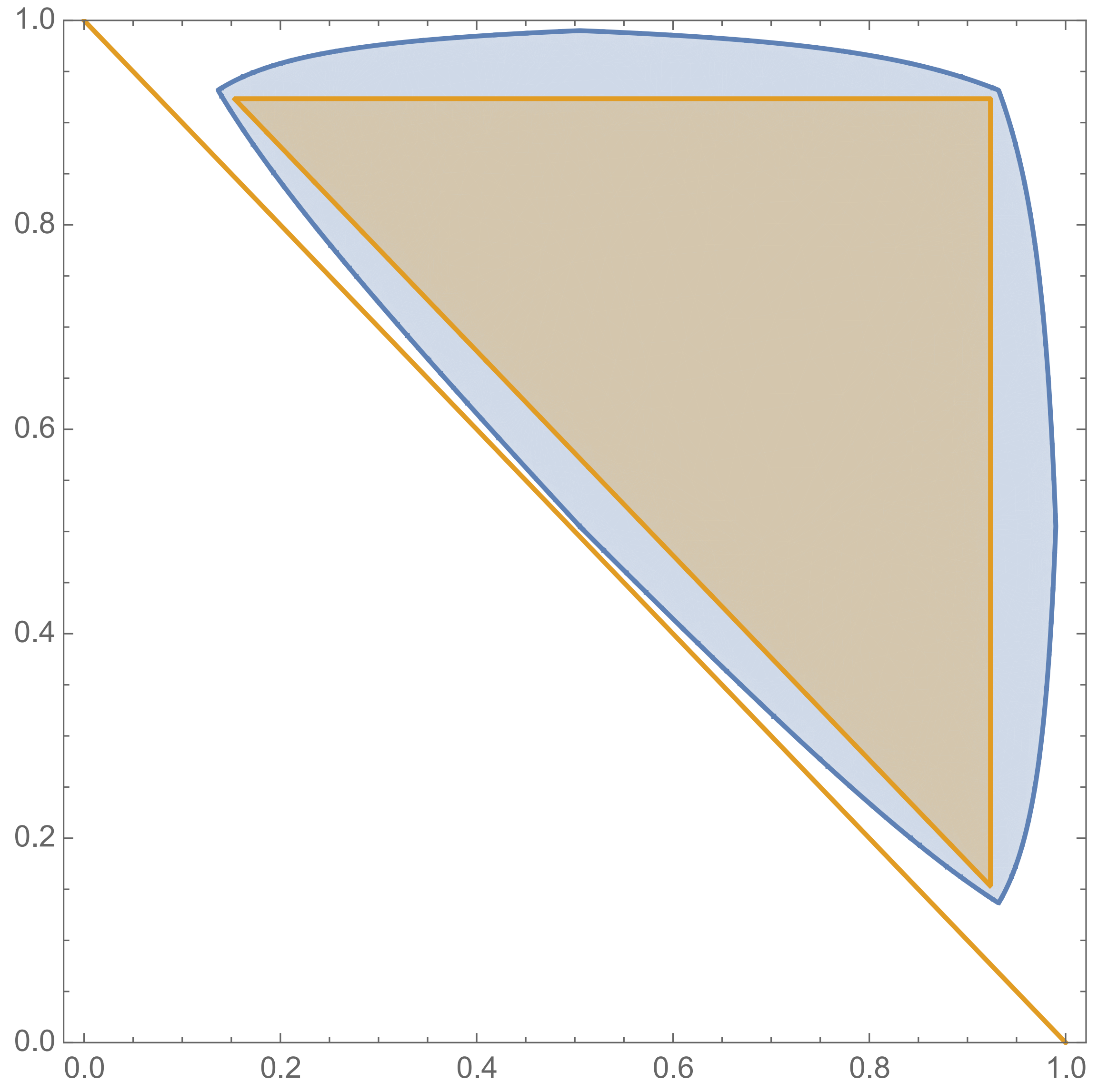
Comparison among the variables

- As we can see, N-Jettiness has a logarithmic enhanced scaling, $y_{2,3}$ and k_T^{ness} have linear scaling, k_T^{FSR} has quadratic scaling.



Comparison between 2-Jettiness and $y_{2,3}$

- We investigated the origin of the logarithmic enhanced contribution that is absent in $y_{2,3}$ slicing. Here we can see the different cuts imposed on the phase-space by the step function $\theta(X - r_{cut})$.



Cut imposed by $y_{2,3}$ (blue) and 2-Jettiness (orange)

- In the collinear limit, we have $\tau_2 \to k_T\, e^{-|\eta|}$, $y_{2,3} \to k_T^2$.

- Both the variables impose a cut on a non-singular part of the phase-space close to the line $x_2 = 1 - x_1$. That part is the origin of the logarithmic enhanced term, as we verified with an analytical analysis.

- 2-Jettiness suppresses a larger region of the phase-space, since it cuts out also the regions at high rapidity, that are not singular.

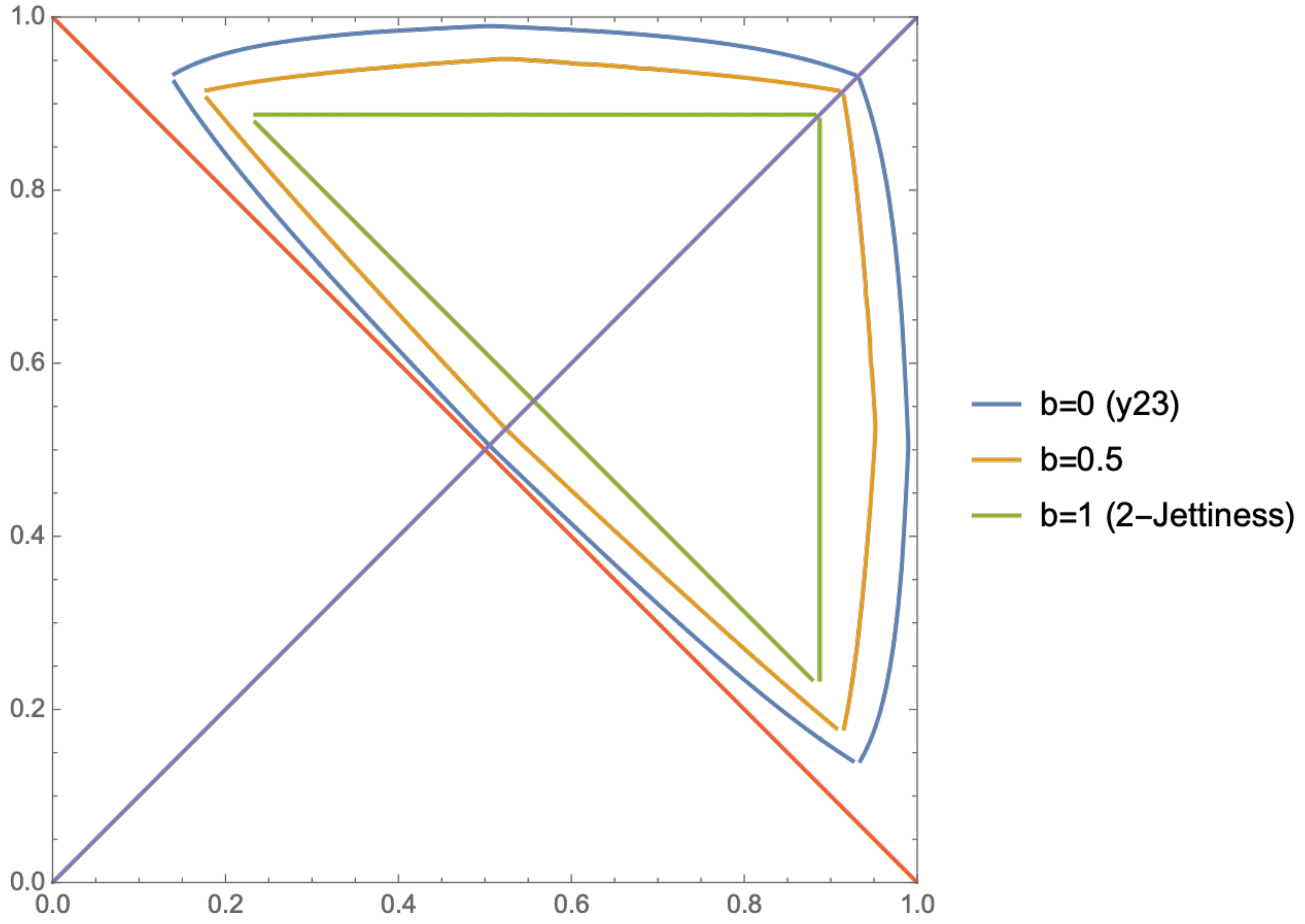
Variable X_b

- We can define a family of resolution variables that depends on a parameter $b \in [0,1]$

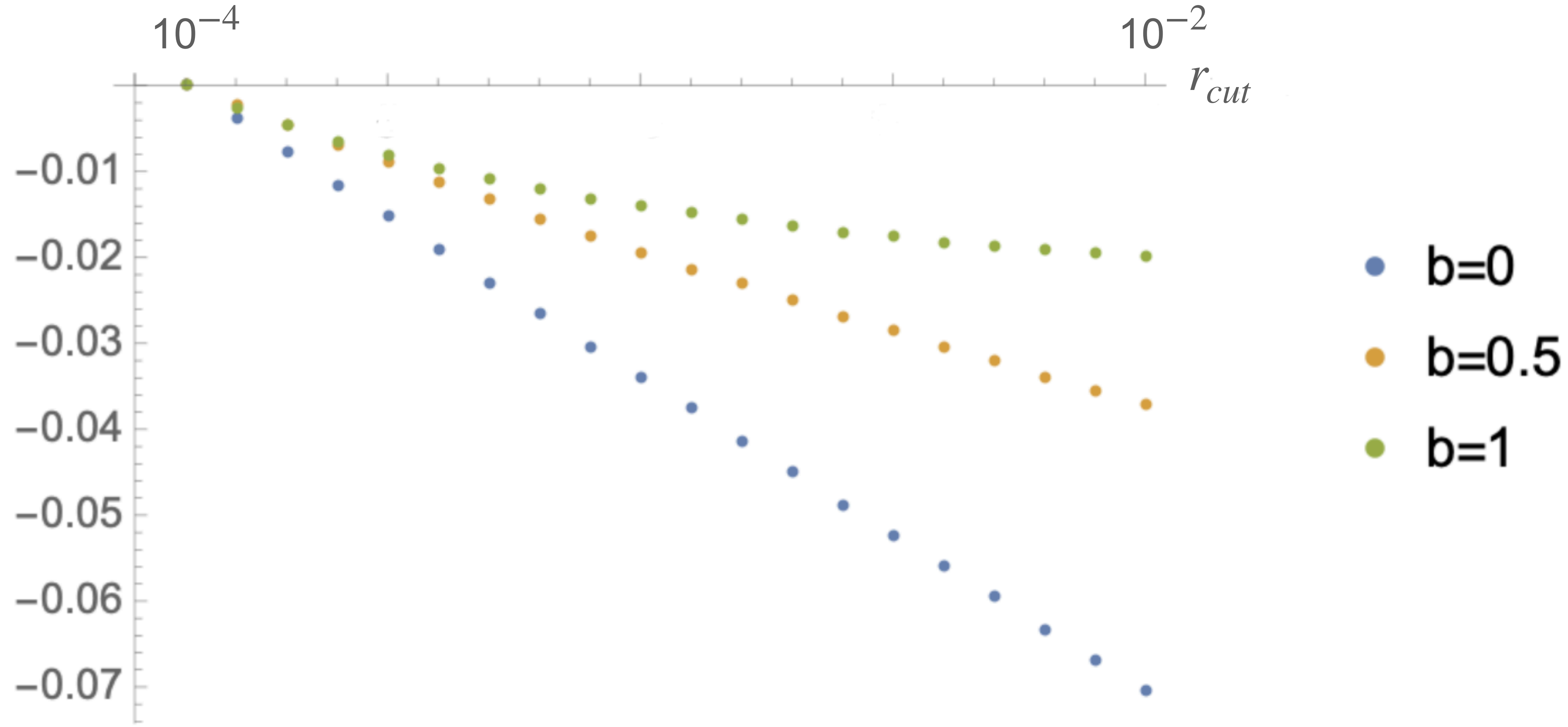
$$X_b = \tau_2^b y_{2,3}^{(1-b)/2}$$

- The parameter b controls the dependence on the rapidity of the resolution variable, since in the collinear limit $X_b \sim k_T e^{-b|\eta|}$. We have $X_0 = \sqrt{y_{2,3}}$ and $X_1 = \tau_2$.

- The counterterm associated to X_b is: $d\sigma^{CT} = d\sigma_{LO} \frac{\alpha_s}{2\pi} C_F \frac{1}{1+b} (2 \log^2 r_{cut} + 3 \log r_{cut})$



Cut imposed by $X_b > r_{cut}$ for different values of b.



Power correction scaling for different values of b.

Conclusions and outlook

- We analytically computed the power corrections of different resolution variables for $e^+e^- \to q\bar{q}$.
- We have shown that the dependence on the rapidity of the slicing variable is responsible for the different scaling of the power corrections.
- Our future goal is to extend k_T^{ness} slicing at NNLO.
- The study of k_T^{ness} slicing for leptonic collisions is only the first step toward the formulation of the method for hadronic collisions, since many ingredients that will be computed for the e^+e^- case can be reused for $p\,p$ scattering.

Backup Slides

2-Jettiness power corrections

- The expansion of the 2-Jettiness power correction, up to the linear term, is:

$$2\, r_{cut} \log r_{cut} + 7\, r_{cut} + \mathcal{O}(r_{cut}^2)$$