WWU MÜNSTER

Graduiertenkolleg 2149 Research Training Group

nCTEQ nuclear parton distribution functions

Fast evaluation of heavy quark contributions to DIS in APFEL++

DIS 2023 – Michigan State University

Peter Risse (risse.p@uni-muenster.de)

living.knowledge

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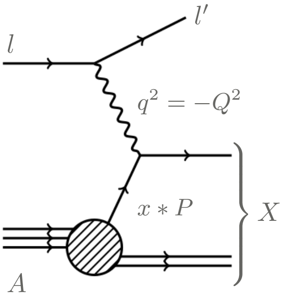
Introduction to heavy quark schemes in DIS

Factorization in DIS structure functions

$$F_\lambda(x, Q^2) = \sum_k C_k^\lambda \otimes f_k = \sum_k \int_\chi^1 \frac{\mathrm{d}\xi}{\xi} C_k^\lambda \left(\frac{\chi}{\xi}, \frac{Q}{\mu}, \frac{M_i}{\mu}, \alpha_s(\mu) \right) f_k(\xi, \mu)$$



Introduction to heavy quark schemes in DIS

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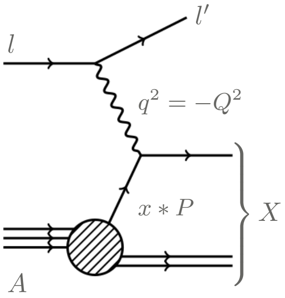

- Wilson coefficients have a **complicated α_s** expansion
  - these are the hard scattering amplitudes
- **heavy quark mass effects** sizable at $Q \sim M_H$
- extremely **important for global PDF** fits
  - bulk of experimental data from DIS
  - need fast theory predictions
- older implementations are **not well-optimized**

DIS mass schemes

Zero Mass Variable Flavor Number Scheme (ZMVFNS)

- consider only quarks below threshold: $M_H < Q$
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- do not take phase space constraints into account

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Fixed Flavor Number Scheme (FFNS)

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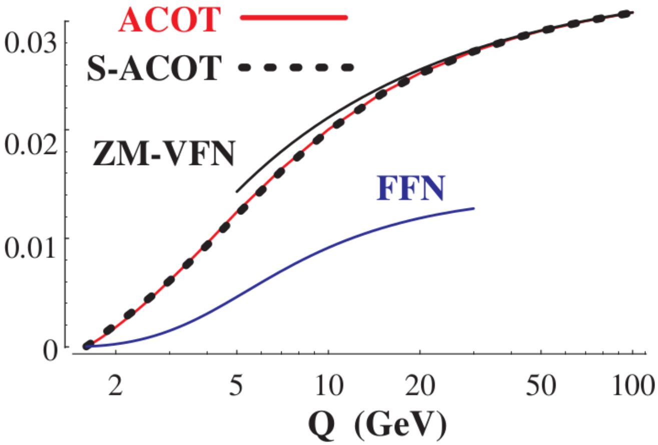
Fixed Flavor Number Scheme (FFNS)

- treat all quarks as massless except for the heaviest M_H
- this mass appears explicitly in the Wilson coefficients
- **good results for $\mathbf{Q \sim M_H}$ unreliable as Q becomes large**

General Mass Variable Flavor Number Schemes (GMVFNS)

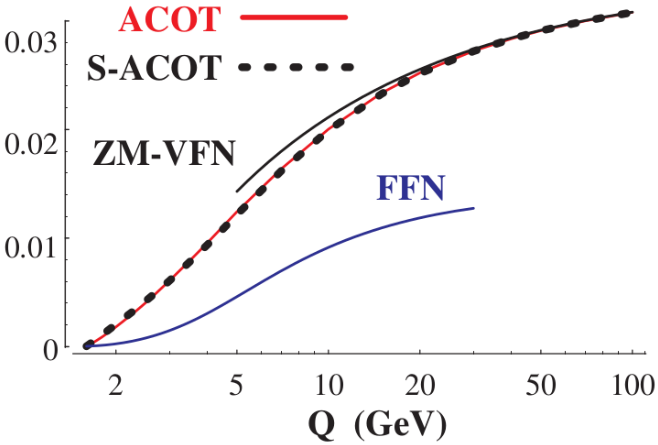
- 'interpolating' between FFNS and ZMVFNS



T. Stavreva et al., arXiv: 1203.0282

General Mass Variable Flavor Number Schemes (GMVFNS)

- 'interpolating' between FFNS and ZMVFNS
- several choices can be made, resulting in different schemes:
  - **ACOT**: minimal extension of the $\overline{\mathrm{MS}}$ renormalisation scheme
  - **FONLL**: interpolating between schemes with a damping function
  - **TR-method**: requiring smooth transition at $Q = M_H$



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Overview: Heavy Quark schemes in DIS

- Factorization
- Zero Mass Variable Flavor Number Scheme
- Fixed Flavor Number Scheme
- General Mass Variable Flavor Number Scheme

The approximate sACOT-χ scheme at NNLO

- neutral current (Stavreva *et al*, arXiv:1203.0282)
- charged current

Numerical implementation

- APFEL++
- results

approximate sACOT-χ at NNLO

Stavreva *et al*, arXiv:1203.0282

approximate simplified ACOT-χ

approximate: ACOT$\left[\mathcal{O}(\alpha_s^0) + \mathcal{O}(\alpha_s^1)\right]$ + ZM-VFNS$\left[\mathcal{O}(\alpha_s^2)\Big|_\chi\right]$

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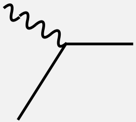
simplified: zero mass coefficients for

- incoming heavy quarks
- internal on-shell cuts on a heavy quark line



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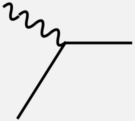
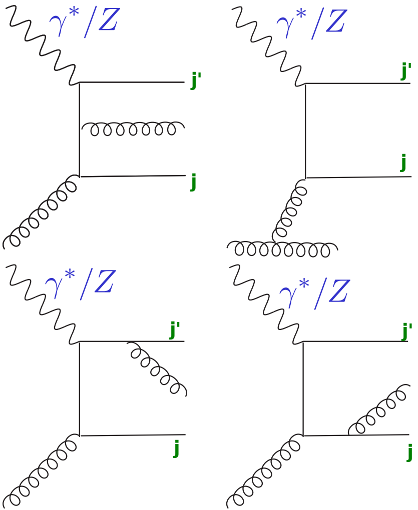


χ-prescription: $\chi = \chi(n) = x\left[1 + \left(\frac{nM_H}{Q}\right)^2\right] \qquad n = 0, 1, 2, 3$

sACOT-χ – NNLO gluon

Stavreva *et al*, arXiv:1203.0282



Neutral current

- filter out the individual quark flavor contribution
- hit coefficient with $\chi(n)$-prescription
  - $\chi(n) = x\left[1 + \left(\frac{nM_j}{Q}\right)^2\right] \qquad n = 0, 1, 2, 3$
- repeat for all flavors

sACOT-χ – NNLO gluon

Charged current

- filter out the individual quark flavor contribution
  - take care of the flavor change at the $W^{\pm}$ vertex
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  - use heaviest quark mass: $M_H = \text{Heaviest}(M_j, M_k)$
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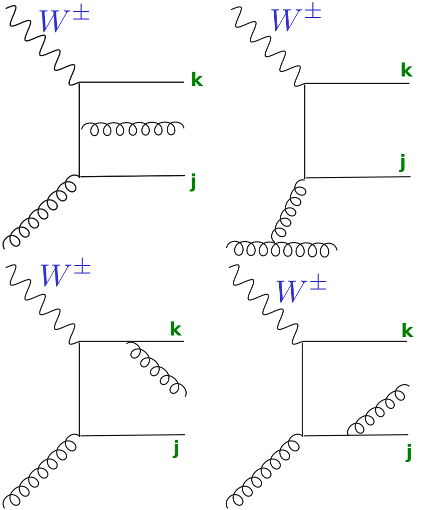

sACOT-χ – NNLO non-singlet

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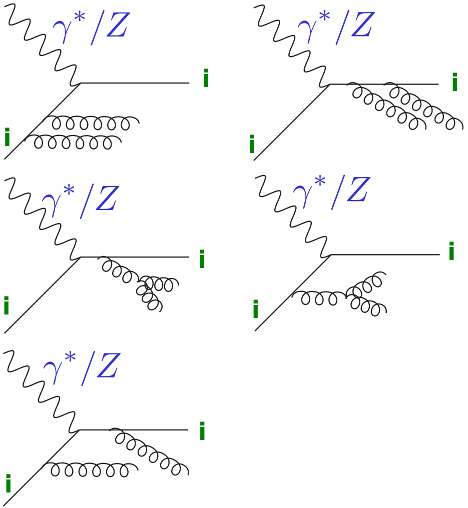
Non-singlet combination

$$q^{\mathrm{NS}} = \sum_{i=1}^{n_f} \left(\hat{e}_i^2 - \langle \hat{e}^2 \rangle\right)(q_i + \bar{q}_i) \quad \text{with} \quad \langle \hat{e}^2 \rangle = \frac{1}{n_f} \sum_{i=1}^{n_f} \hat{e}_i^2$$

"whenever the incoming parton couples to the EW-boson"



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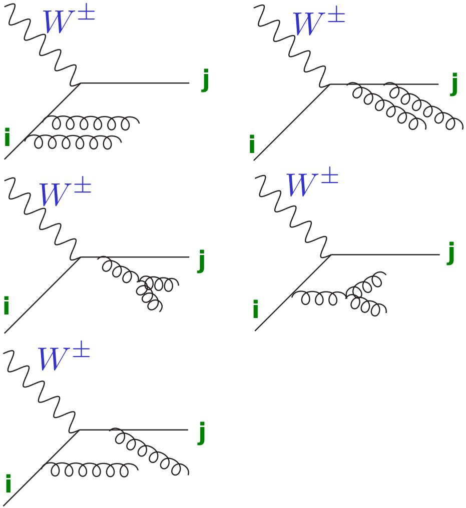

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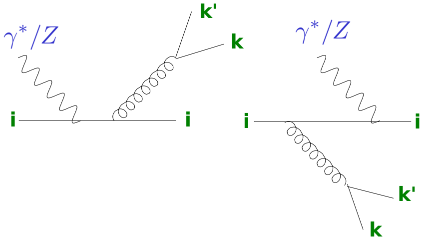

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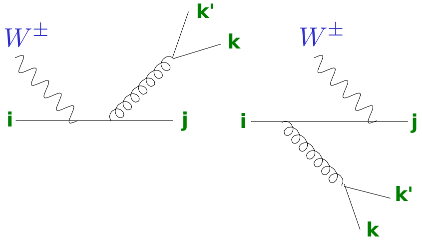

sACOT-χ – NNLO purely singlet

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Purly singlet contribution

$$F_{a,q_i} \simeq C^{\mathbf{PS}}_{a,q} \otimes \langle \hat{e}^2 \rangle (q_i + \bar{q}_i)$$

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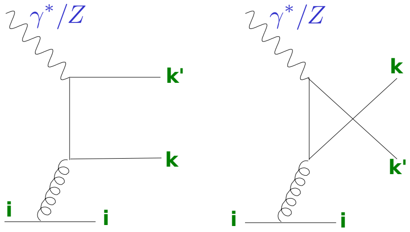

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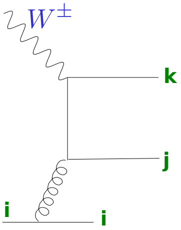

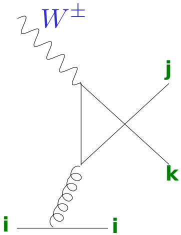

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APFEL++ – A PDF evolution library in C++

Bertone, arXiv:1708.00911

- main author: **V. Bertone**
- rewrite of the Fortran APFEL code
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Precompute observables

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Replace with interpolating functions: $\Uparrow$

$$\sum_\alpha^{N_\xi} w_\alpha(\xi) f_k(\xi_\alpha, \mu)$$

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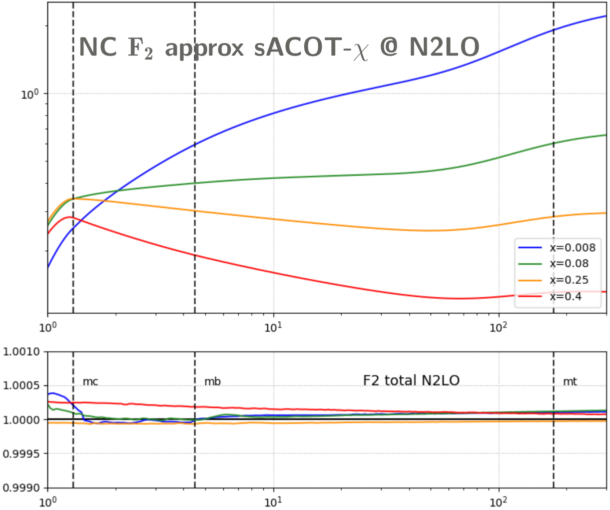
Available schemes in APFEL++

scheme	$\mathcal{O}(\alpha_s)$		NC: F_2	NC: F_3	NC: F_L		CC: F_2	CC: F_3	CC: F_L
ZM	N2LO		✓	✓	✓		✓	✓	✓
FONLL-C	N2LO		✓	✗	✓		✗	✗	✗
ACOT	NLO		✓	✓	✓		✗	✗	✗
sACOT-χ	NLO		✓	✓	✓		✓	✓	✓
approx. sACOT-χ	N2LO		✓	✓	✓		✓	✓	✓

Neutral Current F_2 at NNLO in APFEL++

- **very good agreement** with old implementation in all kinematic regions
  - compared to the nCTEQ++ code
- speed-up to current implementation: $\mathcal{O}(100)$



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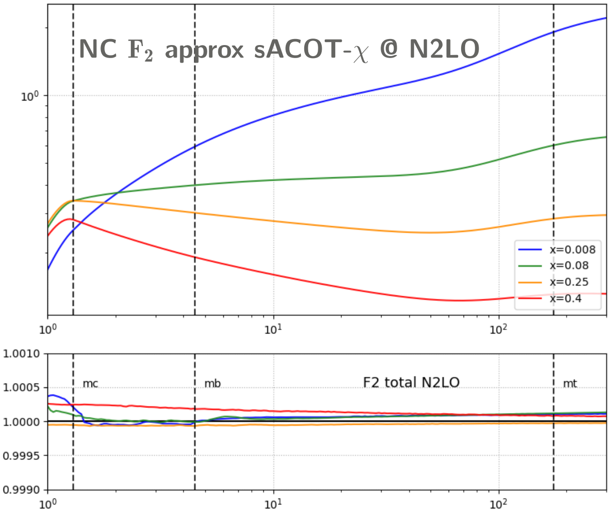
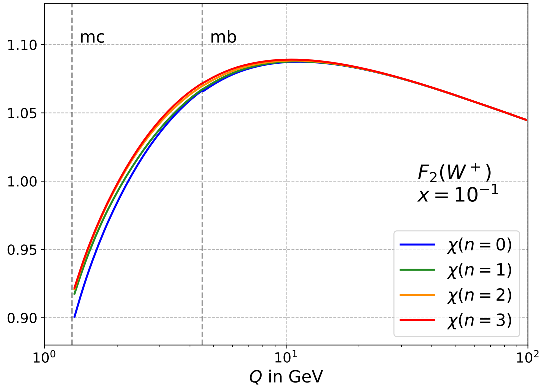
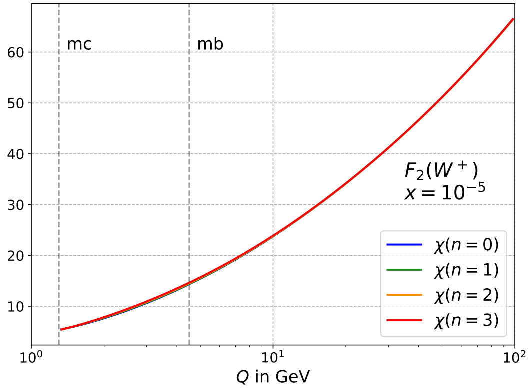
- speed-up to current implementation: $\mathcal{O}(100)$
- planned: make this available also via the xFitter code



n-scaling dependence for $F_2(W^+)$

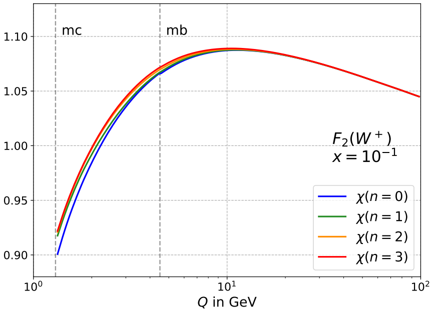




- **very small dependence on the *n*-scaling** (between $n = 1$ and $n = 3$)

Conclusion

- the approximate sACOT-χ extends the **sACOT scheme to NNLO**
  - NNLO contributions by including ZM-coefficients
  - approximate **phase-space constraints**



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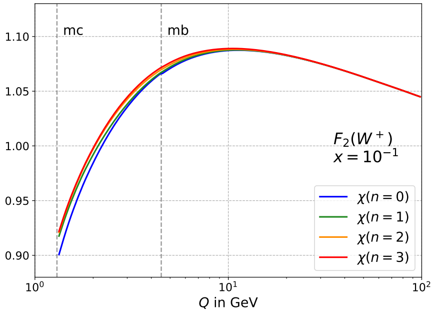
- scheme has been extended to the **charged current**
  - small dependence on the n-scaling variable
  - approximate sACOT-χ now available for all structure functions



Conclusion

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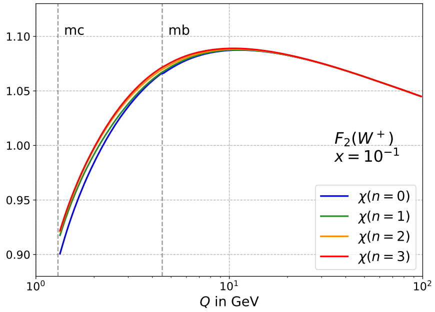
  - NNLO contributions by including ZM-coefficients
  - approximate **phase-space constraints**
- scheme has been extended to the **charged current**
  - small dependence on the n-scaling variable
  - approximate sACOT-χ now available for all structure functions
- fast implementation in the `apfel++` code
  - soon to be made public
  - all six structure functions available
  - evaluation **speed up:** $\mathcal{O}(100)$



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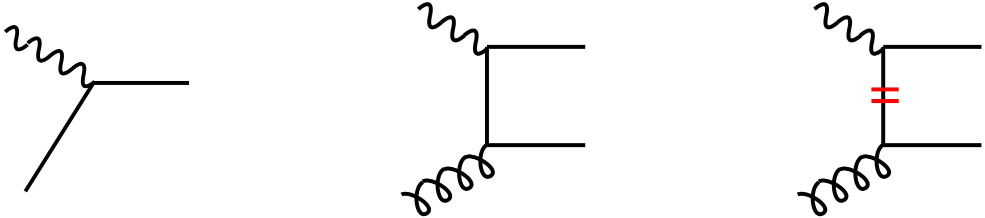
ACOT at NLO

Aivazis *et al*, arXiv:9312318 and arXiv:9312319

LO quark-boson scattering $+$ NLO gluon-boson scattering $-$ subtraction term



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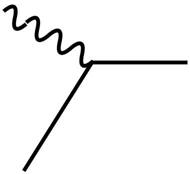
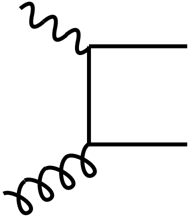
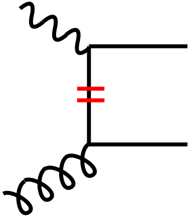
LO quark-boson scattering + **NLO gluon-boson scattering** − **subtraction term**







- $Q \gtrsim M_H$: removed by subtraction
- $Q \gtrsim M_H$: dominant contribution
- $Q \gtrsim M_H$: approx. equal to quark-boson scattering

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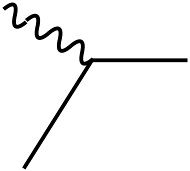

- $Q \gtrsim M_H$: removed by subtraction
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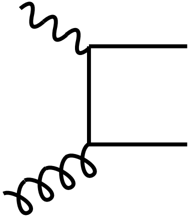

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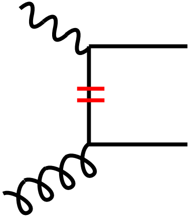

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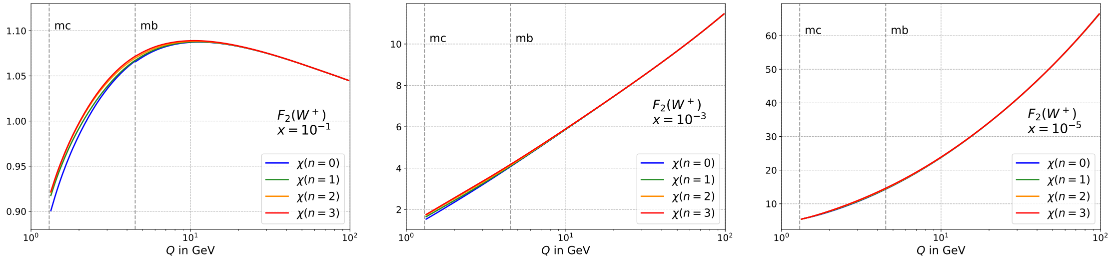
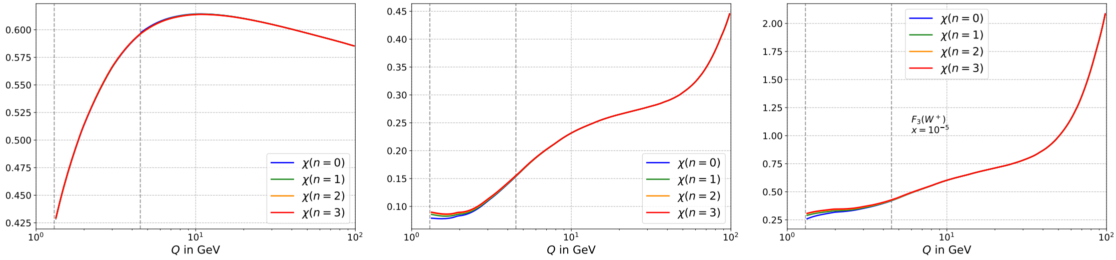
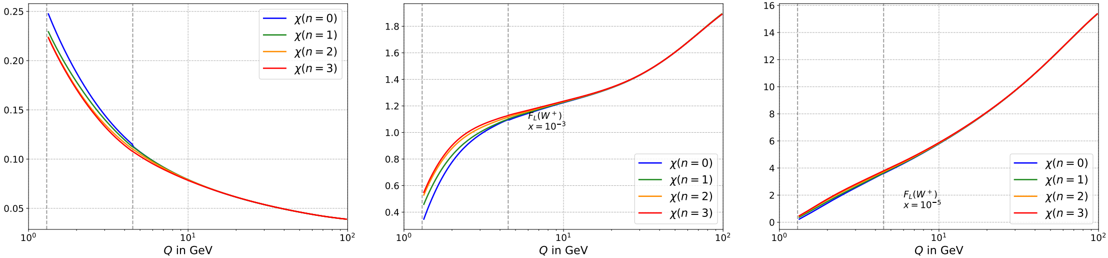
$F_2(W^+)$


mc
mb
$F_2(W^+)$
$x = 10^{-1}$
$\chi(n=0)$
$\chi(n=1)$
$\chi(n=2)$
$\chi(n=3)$
Q in GeV
$F_2(W^+)$
$x = 10^{-3}$
$F_2(W^+)$
$x = 10^{-5}$

$F_3(W^+)$


0.600
0.575
0.550
0.525
0.500
0.475
0.450
0.425
10^0
10^1
10^2
Q in GeV
$\chi(n=0)$
$\chi(n=1)$
$\chi(n=2)$
$\chi(n=3)$
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
$F_3(W^+)$
$x = 10^{-5}$

$F_L(W^+)$



0.25
0.20
0.15
0.10
0.05
χ(n = 0)
χ(n = 1)
χ(n = 2)
χ(n = 3)
10^0
10^1
10^2
Q in GeV
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
F_L(W^+)
x = 10^-3
χ(n = 0)
χ(n = 1)
χ(n = 2)
χ(n = 3)
10^0
10^1
10^2
Q in GeV
16
14
12
10
8
6
4
2
0
F_L(W^+)
x = 10^-5
χ(n = 0)
χ(n = 1)
χ(n = 2)
χ(n = 3)
10^0
10^1
10^2
Q in GeV