A renormalization group analysis of medium-modified fragmentation in SIDIS

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In collaboration with Ivan Vitev, *2301.11940 and works in preparation*

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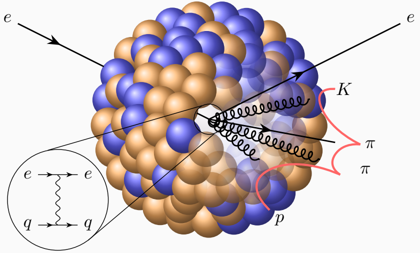
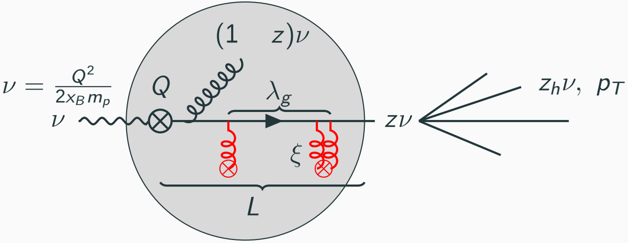
Fragmentation in cold nuclear matter via semi-inclusive DIS



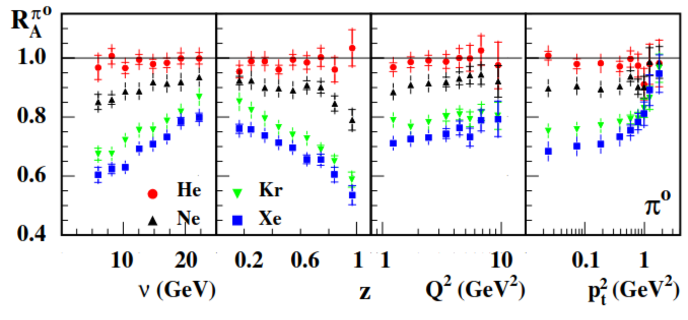
Primary hard process



SIDIS with nuclear targets probes parton dynamics in cold nuclear matter. An interplay of jet energy scale ($Q, E = z\nu$) and multiple medium scales:

- In-medium path length L.
- Mean free path of parton-medium rescattering λ_g.
- Inverse scattering range of rescattering ξ & $\Lambda_{\rm QCD}$.

Sizeable nuclear modifications in SIDIS observed at EMC, HERMES, CLAS



$$R_A^h = \frac{N_{eA \to \pi^0}(z_h, p_T^2; \nu, Q^2)}{N_{ed \to \pi^0}(z_h, p_T^2; \nu, Q^2)}$$

$$N_{eX \to h} = \frac{d\sigma_{eX \to h}}{d\nu dQ^2 dz_h dp_T^2} \Big/ \frac{d\sigma_{eX}}{d\nu dQ^2}$$

EMC ZPC52(1991)1–11

◁ HERMES NPB780(2007)1-27

CLAS PRC105(2022)015201

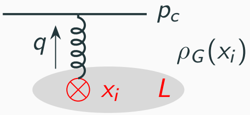
- Are these modifications (at least partly) perturbatively calculable?
- What are the NP inputs to understand data and to characterize the cold nuclear matter?

An EFT approach to in-medium parton dynamics: SCET$_G$

Soft-Collinear-Eective-Theory with Glauber gluon [A. Idilbi, A. Majumder PRD80(2009)054022, G. Ovanesyan, I. Vitev, JHEP06(2011)080] .

- Collinear mode $p_c \quad (1, \lambda^2, \lambda)\nu$ and soft mode $p_s \quad (\lambda^2, \lambda^2, \lambda^2)\nu$ from SCET.
- Glauber gluon $q \quad (\lambda^2, \lambda^2, \lambda)\nu$. Background field from medium sources $(x_1, x_2, \quad x_i, \quad)$

$$A_G^{\mu,a}(q) = \sum_i \frac{g_s e^{\ \ iq \ \ x_i^+}}{q_?^2 + \xi^2} hXjJ^{\mu,a}jii$$



- Medium-size sensitive modes have $p \qquad \frac{1}{L} =) \quad \lambda = p\frac{1}{\nu L}$.
  - $p_c^2 \quad q^2 \quad \nu \ \frac{1}{L}$ a semi-hard scale for thin medium!
  - $p_s^2 \quad 1/L^2$, non-perturbative.

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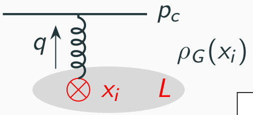

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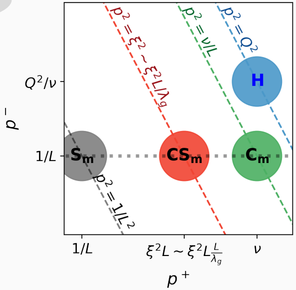
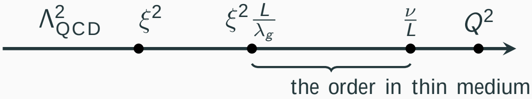
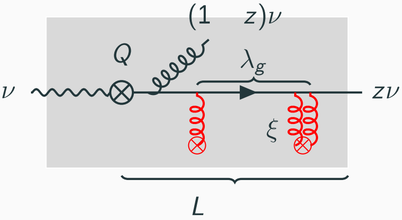


Scale separation in a thin/dilute medium

- Consider eA DIS at moderately large x_B (x_B & 0.1) such that $\frac{\nu}{L}$ $\frac{Q^2}{10x_B A^{1/3}} < Q^2$.
- “The semi-hard scale $\frac{\nu}{L}$” “the average q_T^2 transfer $\xi^2 \frac{L}{\lambda_g}$”.
- This work further assumes $L/\lambda_g = O(1)$.





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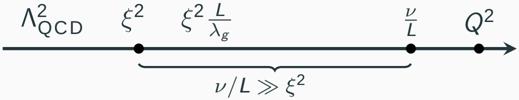
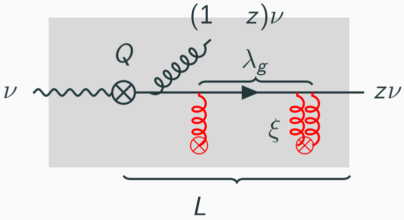
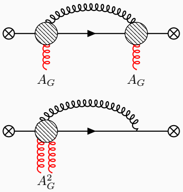




The single-hadron SIDIS cross section with $\nu/L \quad \xi^2$

$$\frac{d\sigma_{ep\,!\,h}}{dx_B\, dQ^2\, dz_h} = \frac{2\pi\alpha_e^2}{Q^4}\sum_{i,j}\underbrace{e_q^2 f_{i/A}(x_B) \quad C_{ij}^h(x,z)}_{F_{ij}(z)} \quad d_{h/j}(z_h)$$

$$\frac{d\sigma_{eA\,!\,h}}{dx_B\, dQ^2\, dz_h} = \sum_{i,j}\frac{2\pi\alpha_e^2}{Q^4}\left[F_{ij}(z) + \Delta F_{ij}^{\mathrm{med}}(z)\right] \quad d_{h/j}(z_h)$$



- $\Delta F_{ij}^{\mathrm{med}}(z) = F_{ik}^{(0)} \quad P_{kj}^{\mathrm{med}(1)}$ are corrections from medium-induced parton splittings.
- $P_{kj}^{\mathrm{med}(1)}$ are complicated, and past studies often rely on numerical approach/MC.
- We use analytic approach to gain understanding & insights for in-medium factorization.

The key observation: $P_{ij}^{\mathrm{med}(1)}$ constains endpoint divergences

- Endpoint divergences appears because all masses (ξ^2, etc) are dropped according to collinear power counting $\xi^2 \quad \nu/L$. For example, the flavor non-singlet spectrum

$$\Delta F_{\mathrm{NS}}^{\mathrm{med}}(z) = \int_z^1 \frac{dx}{x} F_{\mathrm{NS}}(\frac{z}{x}) P_{qq}^{\mathrm{med}(1)}(x) \quad + \quad \text{virtual term.}$$

$$P_{qq}^{\mathrm{med}(1)}(x) = A(\alpha_s, \quad) \quad \frac{P_{qq}^{vac(0)}(x)}{[x(1 \quad x)]^{1+2\epsilon}} \quad \left[\frac{\mu^2 L}{\chi z \nu}\right]^{2\epsilon} \quad C_n \Delta_n(x)$$

- They can be regulated using dimension regularization ($d = 4 \quad 2\epsilon$),

$$\Delta F_{\mathrm{NS}}(z) = A(\alpha_s, \quad) \left(\frac{1}{2\epsilon} + \ln\frac{\mu^2 L}{\chi z \nu}\right) 2C_F [\underbrace{2C_A \left(\quad \frac{d}{dz} + \frac{1}{z}\right)}_{\text{from } x\,!\,1} + \underbrace{\frac{C_F}{z}}_{x\,!\,0}] F_{\mathrm{NS}}(z) + \mathrm{F.O.}$$

- Absorb divergence with an in-medium renormalization $F_{ij} \quad ! \quad \left(M_{ik}^{(0)} + \frac{1}{\epsilon} M_{ik}^{(1)}\right) \quad F_{kj}$. It suggests another relevant sector (collinear-soft) as μ^2 approaches ξ^2 (or $\xi^2 L/\lambda_g$).

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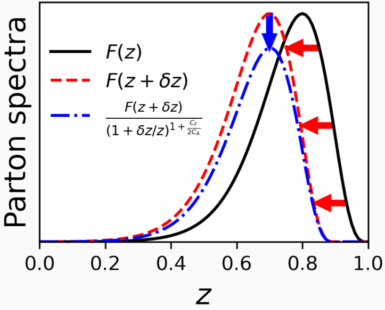
RG equations for the collinear sector

- Define $\tau(\mu^2) = \frac{\rho_G L^2}{\nu} \frac{\pi B}{2\beta_0} \left[\alpha_s(\mu^2) \quad \alpha_s\left(\chi \frac{z\nu}{L}\right)\right]$ evolving from $\mu^2 = \chi \frac{z\nu}{L}$ down to ξ^2. Depend on Q^2 only through coefficients B and $\chi > 3.0$.

$$\frac{\partial F_{\mathrm{NS}}(\tau, z)}{\partial \tau} = \left(4 C_F C_A \frac{\partial}{\partial z} \quad \frac{4 C_F C_A + 2 C_F^2}{z}\right) F_{\mathrm{NS}}$$



A “traveling wave” solution for F_{NS}

$$F_{\mathrm{NS}}(\tau, z) = \frac{F_{\mathrm{NS}}\left(0, z + 4 C_F C_A \tau\right)}{(1 + 4 C_F C_A \tau / z)^{1 + C_F/(2C_A)}}$$

The primary effect: shift spectra by $\delta z = \quad 4 C_F C_A \tau$.

The parton energy loss picture $\Delta E = \nu \delta z \ / \ \rho_G L^2$.

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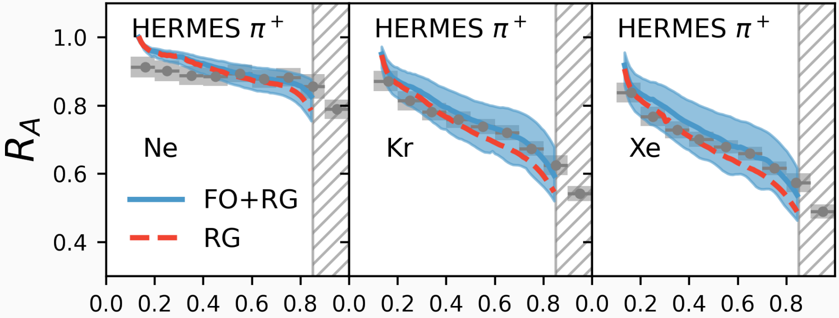
- Flavor singlets F_g and $F_f = F_q + F_{\bar{q}}$, for $f = u, d, s$.

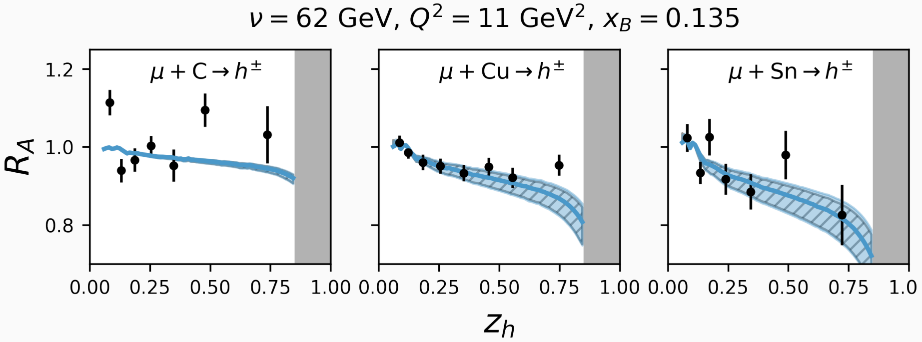
$$\begin{aligned} \frac{\partial F_f}{\partial \tau} &= \left(4 C_F C_A \frac{\partial}{\partial z} \quad \frac{4 C_F C_A + 2 C_F^2}{z}\right) F_f + 2 C_F T_F \frac{F_g}{z}, \\ \frac{\partial F_g}{\partial \tau} &= \left(4 C_A^2 \frac{\partial}{\partial z} \quad \frac{2 N_f C_F}{z}\right) F_g + 2 C_F^2 \sum_f \frac{F_f}{z}. \end{aligned}$$

Comparison with HERMES data [NPB780(2007)1-27]



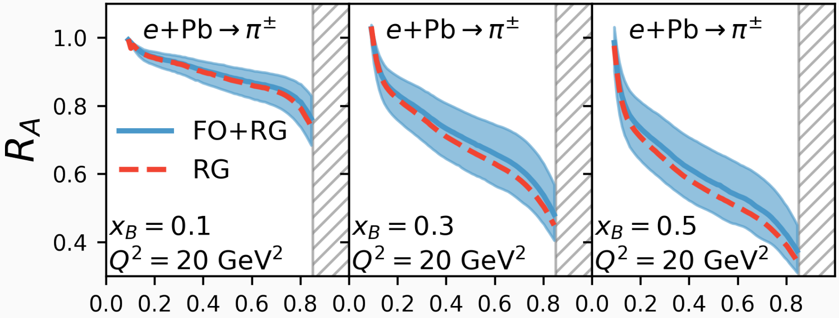
- Baseline: NLO DIS and SIDIS cross sections. NNFF1.0LO vacuum FF [EPJC77(2017)516] and nNNPDF3.0 nuclear PDF [EPJC82(2022)507] .
- Calculated with averaged HERMES $\langle Q^2\rangle \approx$ 2.25 GeV, $\langle \nu \rangle =$ 12 GeV.
- Central values tuned to $\xi =$ 0.35 GeV, $\rho_G =$ 0.4 fm^{-3}. Band: $\left(\frac{2}{3}, \frac{3}{2}\right) \rho_G$
- Good agreement except for the region $z_h \to 1$ (not dominated by collinear modes).

Comparison with SIDIS at EMC [ZPC52(1991)1–11]



- Such e□ects were observed at EMC at higher $\langle Q^2 \rangle = 11$ GeV^2 and $\langle \nu \rangle = 62$ GeV.
- **Same value of parameters** (ξ, ρ_G) **as used for HERMES.** Bands: $(\frac{2}{3}, \frac{3}{2})\xi^2$, $(\frac{2}{3}, \frac{3}{2})\rho_G$.

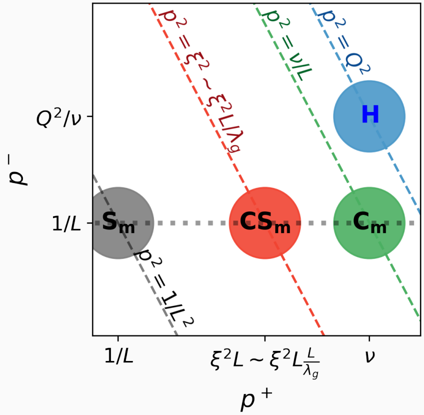
Projection for EIC: ePb versus ep



- Same parameters, but expects smaller e□ects at small x_B as parton too energetic in the nuclear rest frame. From left to right: $\nu = 107$ GeV, 36 GeV, 21 GeV
- EIC will enable a fully di□erential scan in a large range of ν, Q^2.

Towards a factorization formula for fragmentation in eA?

- Medium-size sensitive modes have p $1/L$.
- We have identified the semi-hard scale in the problem p^2 ν/L for a thin medium.
- *Ongoing & preliminary*: including medium-induced collinear-soft CS_m with p^2 & ξ^2. Relevant for the correct description as z_h ! 1.
- A formal definition of nuclear NP inputs.



Summary and Future

- In-medium fragmentation is a multi-scale problem and contains perturbative & NP physics.
- For thin medium, we identify the semi-hard scale ν/L for medium-induced collinear modes $\Longrightarrow$ a region for perturbative treatment.
- The first in-medium NLO calculation using the RG analysis.

  A partial-differential RG equation follows from endpoint divergences in the collinear sector.

  Simple RG solutions were obtained with a clear physical interpretation.
- Phenomenological parameters ξ^2, ρ_G tuned to HERMES SIDIS data

  $\Longrightarrow$ good descriptive power at both HERMES and EMC energy.
- Towards a factorization formulation and formal definition of NP nuclear parameters.

Questions?

Connection to the modified DGLAP equation

The medium-modified DGLAP are widely used phenomenology approach in both eA and AA

$$\frac{\partial}{\partial \ln \mu^2} D_{h/i} = [P_{ij}^{\mathrm{vac}} + \Delta P_{ij}^{\mathrm{med}}]_+ \quad D_{h/j}$$

- In numerical solver, all divergences in $\Delta P_{ij}^{\mathrm{med}}$ are screened by a mass

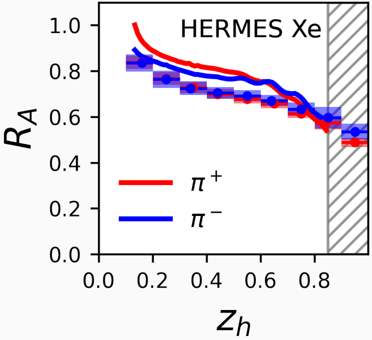
$$\mathrm{k}_?^2 > \xi^2 \qquad) \qquad x, (1 \quad x) > \frac{\xi^2}{\mu^2}$$

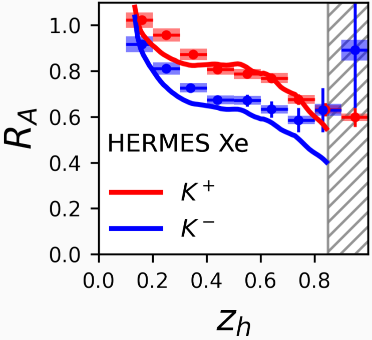
- The mDGLAP (a simplified version) can be Taylor expanded around, e.g., $x = 1$,

$$\begin{aligned} \frac{\partial F_{\mathrm{NS}}}{\partial \ln \mu^2} &= 4 C_F C_A A_0 \int_{\frac{\mu_D^2}{\mu^2}}^{1} \frac{4}{\pi} \frac{\Phi(\frac{\mu^2 L}{2E})}{\frac{\mu^2 L}{2E}} \frac{\left(\frac{x}{z}\right) F_{\mathrm{NS}}\left(\frac{z}{x}\right) \quad \frac{F_{\mathrm{NS}}(z)}{z}}{(1 \quad x)^2} dx \\ &= \frac{4}{\pi} \frac{\Phi(\frac{\mu^2 L}{2E})}{\frac{\mu^2 L}{2E}} \quad 4 C_F C_A A_0 \ln \frac{\mu^2}{\mu_D^2} \left[\frac{\partial F_{\mathrm{NS}}}{\partial z} \quad \frac{F_{\mathrm{NS}}}{z} \right] + \text{non-log-enhanced terms} \end{aligned}$$

- Same leading-log physics as the RG approach (if one chooses $\mu^2 = \mathrm{k}_?^2 / [x(1 \quad x)]$).

Pion vs Kaon





- Change to DSS parametization for π and K fragmentation function. D. Florian et al. PRD75(2007)114010 and PRD91(2015)014035

Ongoing: higher-order in opacity?

- Complexity of in-medium splitting function blows up with opacity $N = 1, 2$.
- Assume the leading contribution still comes from the endpoint region, especially near $x = 1$.
- The opacity $N = 2$ contributions leads to two types of corrections:

$$
\begin{aligned}
&\alpha_s C_R \frac{\mu_G^2}{E/L} \, [\underbrace{a_1 \partial_z + \quad}_{N=1} + \underbrace{a_2 \frac{\mu_G^2}{\xi^2} \partial_z + b_2 \frac{\mu_G^2}{E/L} \partial_z^2 + \quad}_{N=2}], \qquad \mu_G^2 = \alpha_s \rho_G L \\
= \quad &\alpha_s C_R \frac{\mu_G^2}{E/L} \left[\left(a_1 + a_2 \frac{\mu_G^2}{\xi^2} \right) \partial_z + b_2 \frac{\mu_G^2}{E/L} \partial_z^2 + \quad \right]
\end{aligned}
$$

It is interesting to investigate whether the opacity expansion leads to a gradient expansion of the evolution equation.