Definition and Evolution of Di-Hadron Fragmentation Functions

(Andreas Metz, Temple University)

- Motivation
- Definition and interpretation of di-hadron FFs (DiFFs)
- Definition and interpretation of extended DiFFs
- Evolution of DiFFs
- DiFFs for gluons
- Summary

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Motivation

- In spin physics, DiFFs relevant for extraction of transversity h_1^q
- Access to transversity using chiral-odd spin-dependent FFs
  - Single-hadron fragmentation (Collins effect) (Collins, 1992)

    $$h_1^q \otimes H_1^{\perp\, h/q}$$

    correlation btw transverse quark spin and transverse momentum of hadron,
    TMD factorization

  - Di-hadron fragmentation (Collins, Heppelmann, Ladinsky, 1993)

    $$h_1^q \otimes H_1^{\sphericalangle\, h_1 h_2/q}$$

    correlation btw transverse quark spin and relative transverse momentum of (h_1, h_2),
    collinear factorization

  - Single-hadron fragmentation using collinear twist-3 factorization
    (Kang, Yuan, Zhou, 2010 / Metz, Pitonyak, 2012)

- Previous work on phenomenology of $H_1^{\sphericalangle\, h_1 h_2/q}$ almost exclusively by Pavia Group
- Extraction of h_1^q from global analysis of di-hadron data (Radici, Bacchetta, 2018)
  - resulting tensor charge

$$\delta q = \int_0^1 dx \left(h_1^q(x) - h_1^{\bar{q}}(x) \right) \qquad\qquad g_T = \delta u - \delta d$$

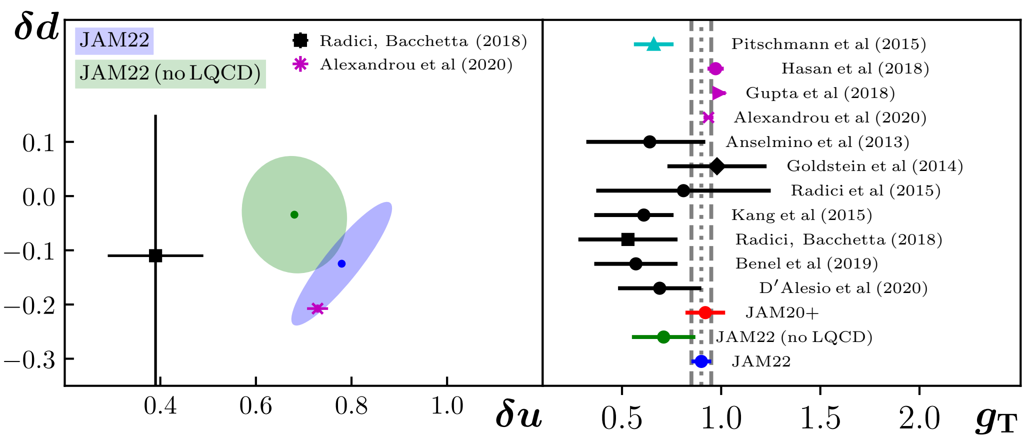


figure modified from arXiv:2205.00999 (JAM-3D)

for δu, some tension between di-hadron channel on the one hand, and single-hadron channel and lattice QCD on the other

- In present situation, independent numerical analysis of di-hadron channel well motivated $\longrightarrow$ talk by Chris Cocuzza
- We also revisited the definition, interpretation and evolution of DiFFs $\longrightarrow$ this talk

Lessons from Single-Hadron Fragmentation Functions

- Process and frames

$$q(k) \to h(P_h) + X \qquad\qquad P_h^- = z k^- \text{ large}$$

$$\text{hadron frame: } \vec{P}_{hT} = 0 \quad \vec{k}_T \neq 0$$
$$\text{parton frame: } \vec{P}_{h\perp} \neq 0 \quad \vec{k}_\perp = 0 \qquad\qquad (\vec{P}_{h\perp} = -z\vec{k}_T)$$

- Definition and interpretation

$$D_1^{h/q}(z, z^2\vec{k}_T^2) = \frac{1}{4z}\int \frac{d\xi^+ d^2\vec{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \mathrm{Tr}\Big[\langle 0|\psi_q(\xi)|h, X\rangle\langle h, X|\bar{\psi}_q(0)|0\rangle\, \gamma^-\Big]_{\xi^-=0}$$
$$= D_1^{h/q}(z, \vec{P}_{h\perp}^2)$$

  - $D_1^{h/q}(z, \vec{P}_{h\perp}^2)$ is number density (see, e.g., Collins, *Foundations of Perturbative QCD*)
  - $D_1^{h/q}(z, \vec{P}_{h\perp}^2)\, dz d^2\vec{P}_{h\perp}$ is number of hadrons h in $[z, z{+}dz]$, $[\vec{P}_{h\perp}, \vec{P}_{h\perp}{+}d^2\vec{P}_{h\perp}]$
  - factor $1/4z$ is crucial for interpretation as number density

- Integrating upon transverse momentum

$$D_1^{h/q}(z) = \int d^2\vec{P}_{h\perp}\, D_1^{h/q}(z, \vec{P}_{h\perp}^2)$$

- Number of hadrons in quark q

$$\sum_h \int dz\, D_1^{h/q}(z) = N^q$$

- Momentum sum rule (Collins, Soper, 1981 / Meissner, Metz, Pitonyak, 2010)

$$\sum_h \int dz\, z\, D_1^{h/q}(z) = 1$$

- Leading-order cross section for $e^- e^+ \to hX$

$$\frac{d\sigma}{dz} = \sum_{q,\bar{q}} \hat{\sigma}^q\, D_1^{h/q}(z) \quad \text{with} \quad \hat{\sigma}^q = \hat{\sigma}^{\bar{q}} = \hat{\sigma}(e^- e^+ \to \gamma^{(*)} \to q\bar{q}) = \frac{4\pi e_q^2 \alpha_{\text{em}}^2 N_c}{3Q^2}$$

- Number density interpretation also for transversely polarized quarks:

$$\text{(symbolic)} \qquad \text{Tr}\Big[\ldots i\, \sigma^{i-} \gamma_5 \Big] \;\to\; \text{pre-factor} \times H_1^{\perp\, h/q}(z, \vec{P}_{h\perp}^2)$$

Definition and Interpretation of DiFFs

- Process and frames

$$q(k) \to h_1(P_1) + h_2(P_2) + X \qquad\qquad P_h^- = z k^- \ \ \text{large}$$

$$P_h = P_1 + P_2 \qquad R = \frac{1}{2}(P_1 - P_2) \qquad P_{1,2}^- = z_{1,2}\, k^- \qquad z = z_1 + z_2$$

hadron frame: $\vec{P}_{hT} = 0 \quad \vec{k}_T \neq 0$

parton frame: $\vec{P}_{h\perp} \neq 0 \quad \vec{k}_\perp = 0 \qquad\qquad (\vec{P}_{h\perp} = -z\vec{k}_T)$

- Definition and interpretation

$$\frac{1}{64\pi^3 z_1 z_2} \int \frac{d\xi^+ d^2\vec{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \mathrm{Tr}\Big[\langle 0|\psi_q(\xi)|h_1, h_2, X\rangle\langle h_1, h_2, X|\bar{\psi}_q(0)|0\rangle\, \gamma^-\Big]_{\xi^-=0}$$

$$= D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) \equiv D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

  - $D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$ is number density for hadron pairs (h_1, h_2)
  - factor $1/64\pi^3 z_1 z_2$ is crucial for interpretation as number density
  - density interpretation also for polarized quarks $(H_1^{\sphericalangle\, h_1 h_2/q}, \ldots)$
  - previously defined/used DiFFs in spin physics have no number density interpretation

- Integrating upon transverse momentum (see also, Majumder, Wang, 2004)

$$D_1^{h_1 h_2/q}(z_1, z_2) = \int d^2\vec{P}_{1\perp}\, d^2\vec{P}_{2\perp}\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$$

- Number of hadron pairs in quark q

$$\sum_{h_1,h_2} \int dz_1 dz_2\, D_1^{h_1 h_2/q}(z_1, z_2) = N^q(N^q - 1)$$

- Momentum sum rule

$$\sum_{h_1} \int_0^{1-z_2} dz_1 \int d^2\vec{P}_{1\perp}\, z_1\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) = (1 - z_2)\, D_1^{h_2/q}(z_2, \vec{P}_{2\perp}^2)$$

  - sum rule after $\int d^2\vec{P}_{2\perp}$ in previous literature (de Florian, Vanni, 2003 / ...)
  - similar (integrated) sum rule for double-parton distributions (Gaunt, Stirling, 2009 / ...)

- Leading-order cross section for $e^- e^+ \to (h_1 h_2) X$

$$\frac{d\sigma}{dz_1 dz_2} = \sum_{q,\bar{q}} \hat{\sigma}^q\, D_1^{h_1 h_2/q}(z_1, z_2) \quad \text{with} \quad \hat{\sigma}^q = \frac{4\pi e_q^2 \alpha_{\text{em}}^2 N_c}{3Q^2}$$

Definition and Interpretation of Extended DiFFs

- More on kinematics
  - invariant mass of di-hadron pair, and alternative variable for longitudinal momentum

$$M_h^2 = P_h^2 = (P_1 + P_2)^2 \qquad \zeta = \frac{z_1 - z_2}{z}$$

  - momenta P_1 and P_2 in hadron frame ($\vec{P}_{hT} = 0$)

$$P_1 = \left(\frac{M_1^2 + \vec{R}_T^2}{(1+\zeta)P_h^-}, \frac{1+\zeta}{2} P_h^-, \vec{R}_T \right) \qquad P_2 = \left(\frac{M_2^2 + \vec{R}_T^2}{(1-\zeta)P_h^-}, \frac{1-\zeta}{2} P_h^-, -\vec{R}_T \right)$$

  - important relation

$$\vec{R}_T^2 = \frac{1-\zeta^2}{4} M_h^2 - \frac{1-\zeta}{2} M_1^2 - \frac{1+\zeta}{2} M_2^2$$

- Extended DiFFs (extDiFFs)
  - in contrast to $D_1^{h_1 h_2/q}(z_1, z_2)$, extDiFFs (also) depend on M_h (or $\vec{R}_T$)
  - extDiFFs appear in transversity-related observables

- Number density interpretation for (properly defined) extDiFFs
  - when changing variables, include Jacobian of transformation in definition of DiFFs
  - example

$$D_1^{h_1 h_2/q}(z, \zeta, \vec{k}_T, \vec{R}_T) = \frac{z^3}{2}\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$$

  - further extDiFFs

$$D_1^{h_1 h_2/q}(z, M_h) = \int d\zeta\, D_1^{h_1 h_2/q}(z, \zeta, M_h)$$
$$= \int d\zeta\, \frac{\pi}{2} M_h\, (1-\zeta^2)\, D_1^{h_1 h_2/q}(z, \zeta, \vec{R}_T^2)$$

  - experimental information on $D_1^{h_1 h_2/q}(z, M_h)$ from Belle (Seidl et al, 2017)

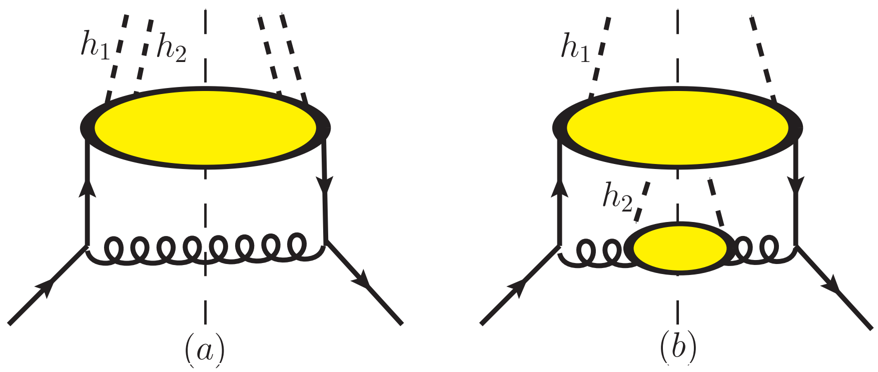
- Leading-order cross section for $e^- e^+ \to (h_1 h_2) X$ (example)

$$\frac{d\sigma}{dz\, dM_h} = \sum_{q,\bar{q}} \hat{\sigma}^q\, D_1^{h_1 h_2/q}(z, M_h) \quad \text{with} \quad \hat{\sigma}^q = \frac{4\pi e_q^2 \alpha_{\text{em}}^2 N_c}{3Q^2}$$

- Corresponding results also hold for extDiFF $H_1^{\sphericalangle\, h_1 h_2/q}$

Evolution of DiFFs

- Homogeneous and in-homogeneous contributions to evolution (sample diagrams)



- Evolution of extDiFFs (quark non-singlet)

$$\frac{\partial}{\partial \ln \mu^2} D_1^{h_1 h_2/q}(z, \zeta, \vec{R}_T^2; \mu) = \int_z^1 \frac{dw}{w} D_1^{h_1 h_2/q}\Big(\frac{z}{w}, \zeta, \vec{R}_T^2; \mu\Big) P_{q\to q}(w)$$

  - evolution of extDiFFs only contains homogeneous term (standard DGLAP)
    (see also Ceccopieri, Bacchetta, Radici, 2007)
  - corresponding evolution equation for $H_1^{\sphericalangle\, h_1 h_2/q}(z, \zeta, \vec{R}_T^2; \mu)$

- Upon $\int d^2\vec{R}_T$, we recover evolution of $D_1^{h_1 h_2/q}(z_1, z_2; \mu)$ where in-homogeneous term contributes as well (de Florian, Vanni, 2003 / ...)

DiFFs for Gluons

- Definition

$$\frac{z}{32\pi^3 z_1 z_2 P_h^-} \int \frac{d\xi^+ d^2\vec{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle 0|F^{+i}(\xi)|h_1,h_2,X\rangle \langle h_1,h_2,X|F^{+i}(0)|0\rangle\Big|_{\xi^-=0}$$

$$= D_1^{h_1 h_2/g}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$$

- Number density interpretation like in the quark case
  - momentum sum rule
  - holds also for extDiFFs, and different variables

- Leading-order cross section for $e^- e^+ \to (h_1 h_2) X$ using Higgs exchange

$$\frac{d\sigma^{H-\text{mediated}}}{dz_1 dz_2} = \hat{\sigma}^g\, 2\, D_1^{h_1 h_2/g}(z_1, z_2)$$

$$\text{with}\ \ \hat{\sigma}^g = \hat{\sigma}(e^- e^+ \to H^{(*)} \to gg) = \frac{\alpha_s\, G_F^2}{576\pi^3}\, \frac{m_e^2 Q^4 (N_c^2 - 1)}{(Q^2 - m_H^2)^2}$$

- We derived also evolution equations for gluon DiFFs

Summary

- DiFFs can be used for the extraction of transversity distribution
- For both quarks and gluons, we propose a field-theoretic definition of DiFFs which have an interpretation as number densities
- Number density interpretation can be established for all leading-twist DiFFs
- Number density interpretation can be obtained for different variables of interest (including extDiFFs)
- DiFFs used in spin physics for almost 25 years are not number densities
  (starting from pioneering work by Bianconi, Boffi, Jakob, Radici, 1999)
- Operator definition of DiFFs also allows one to "easily" obtain their evolution
- We are using the new definitions in our numerics $\longrightarrow$ talk by Chris Cocuzza