Unraveling anomalies in Deep Virtual Compton Scattering

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Based on: arXiv:2210.13419

DIS23 MICHIGAN STATE UNIVERSITY

East Lansing, MI

Chiral anomaly

Recap on chiral anomaly in QCD:

- **Lagrangian invariant under global chiral rotation** $\psi \to e^{i\alpha\gamma_5}\psi$
- **Axial-vector current:** $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$
- **But measure of the path integral is not invariant, which breaks the conservation of the axial current**

K. Fujikawa, PRL 1979

Chiral anomaly

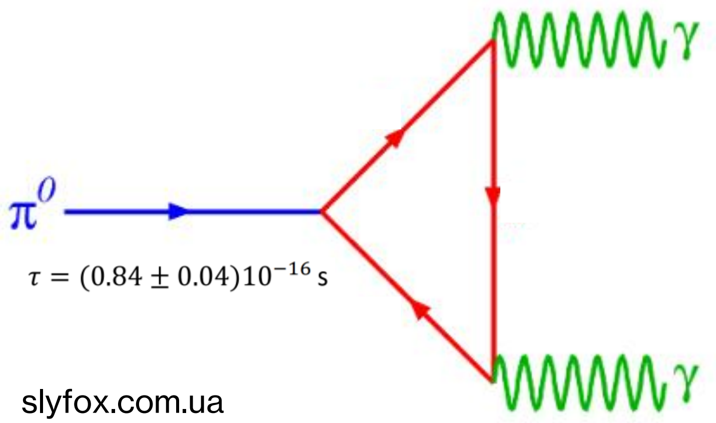
Anomaly equation:

$$\boxed{\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}} \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

A fundamental property of axial-vector current is the anomaly equation

Adler – Bell - Jackiw chiral anomaly

Famous example: ABJ anomaly contribution to $\pi^0 \to 2\gamma$



In the chiral limit, without the anomaly,

π^0 **does not decay!**

Chiral anomaly

Anomaly equation:

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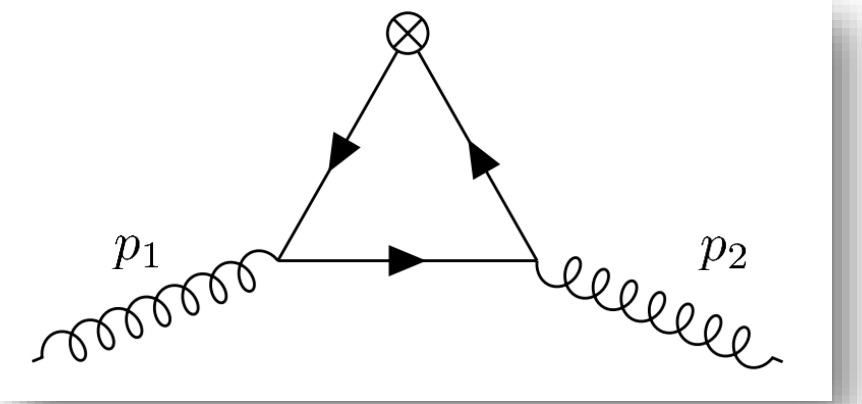
A fundamental property of axial-vector current is the anomaly equation

A perturbative solution to anomaly equation:



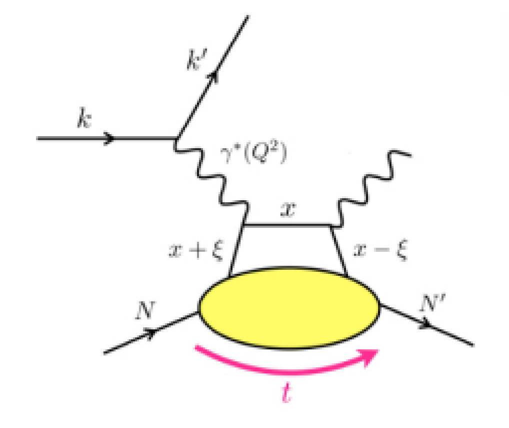
Calculation in off-forward kinematics $(l = p_2 - p_1)$**:**

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i l^\mu}{l^2} \langle p_2 | F_a^{\alpha\beta} \tilde{F}^a_{\alpha\beta} | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole

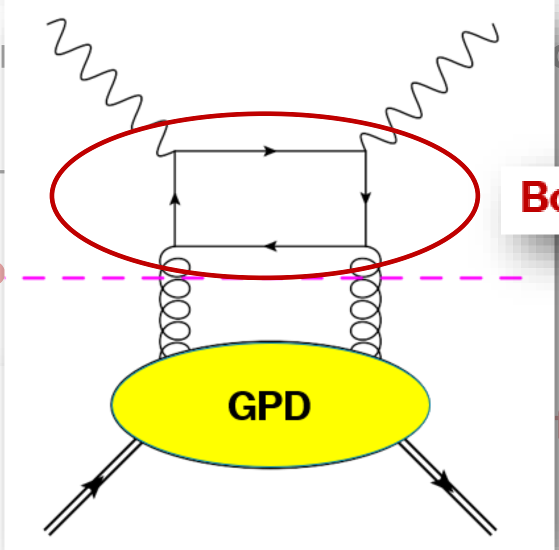
Imprint of Anomalies in QCD Compton scattering

QCD Compton Scattering



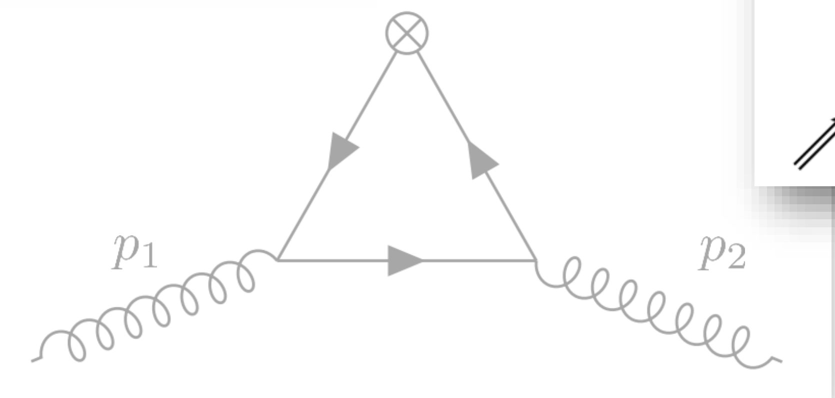
In QCD Compton scattering, box diagrams appear in perturbation theory at one-loop

y of axial-vector cu… on



Box diagram

to anomaly equatio



f-forward kinematics $(l = p_2 - p_1)$:

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{il^\mu}{l^2} \langle p_2 | F_a^{\alpha\beta} \tilde{F}^a_{\alpha\beta} | p_1 \rangle$$

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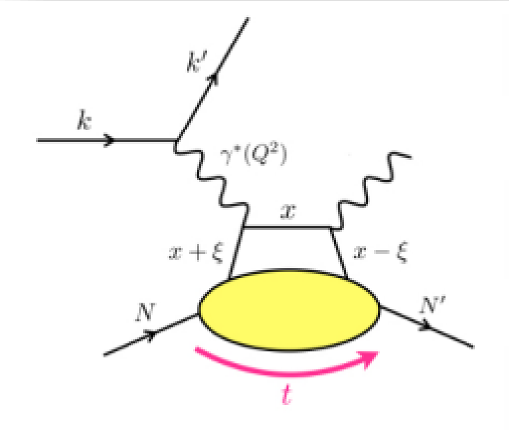
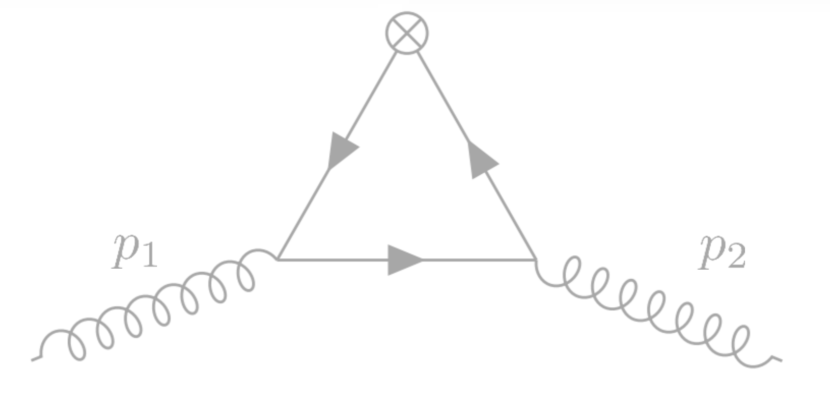


In QCD Compton scattering, box diagrams appear in perturbation theory at one-loop

Box diagram can be viewed as a non-local generalization of triangle diagram

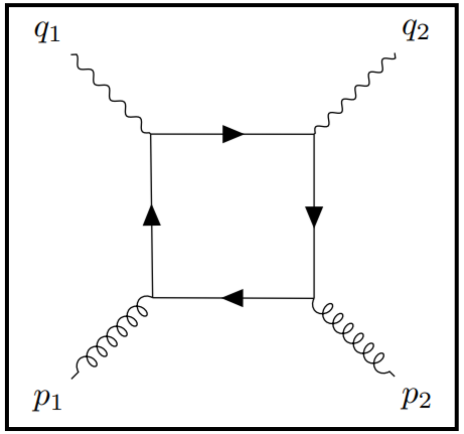
If triangle is dominated by anomaly pole, trace of that should be visible in box diagram



Calculation in off- ... $= p_2 - p_1$):

$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f C}{4\pi}$ ...

Triangle diagram ... ed pole



Box diagram

Imprint of Anomalies in QCD Compton scattering

First calculation of box diagram with $l^2 \neq 0$:

Anomaly equation:

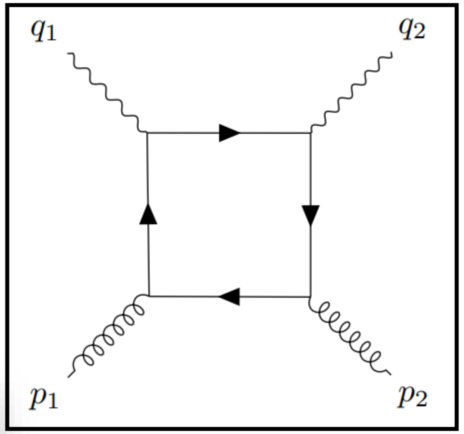
The role of the chiral anomaly in polarized deeply inelastic scattering I: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics

Andrey Tarasov[1,2] and Raju Venugopalan[3]

A fundamental prop

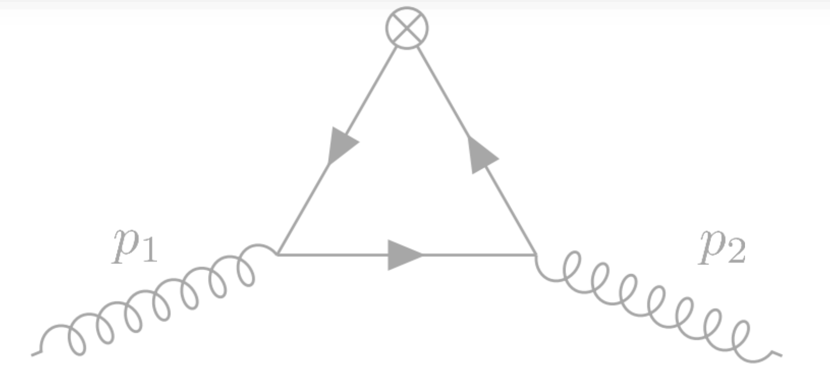
The role of the chiral anomaly in polarized deeply inelastic scattering II: Topological screening and transitions from emergent axion-like dynamics

Andrey Tarasov[1,2] and Raju Venugopalan[3]



Box diagram

Andrey & Raju demonstrated within world-line formalism that to capture the physics of anomaly we need to calculate everything in off-forward kinematics for polarized DIS



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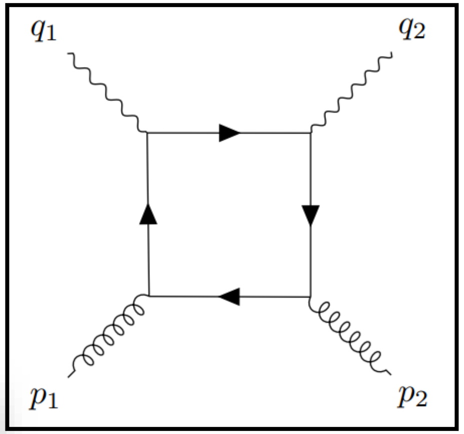

Box diagram

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$(l = p_2 - p_1)$:

p_1

arXiv: 2210.13419 (2022)

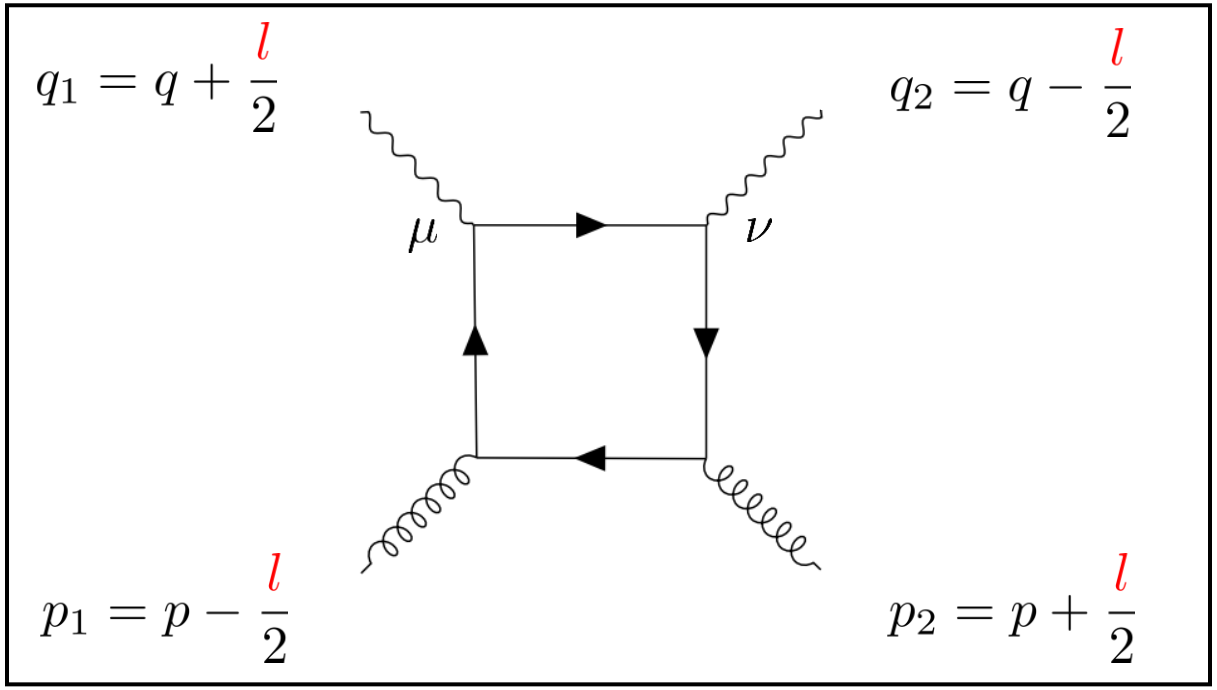
Chiral and trace anomalies in Deeply Virtual Compton Scattering

Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[1,2,†] and Werner Vogelsang[3,‡]

We explored the physics of anomaly in DVCS using Feynman-diagram approach

Imprint of Anomalies in QCD Compton scattering

Kinematics:

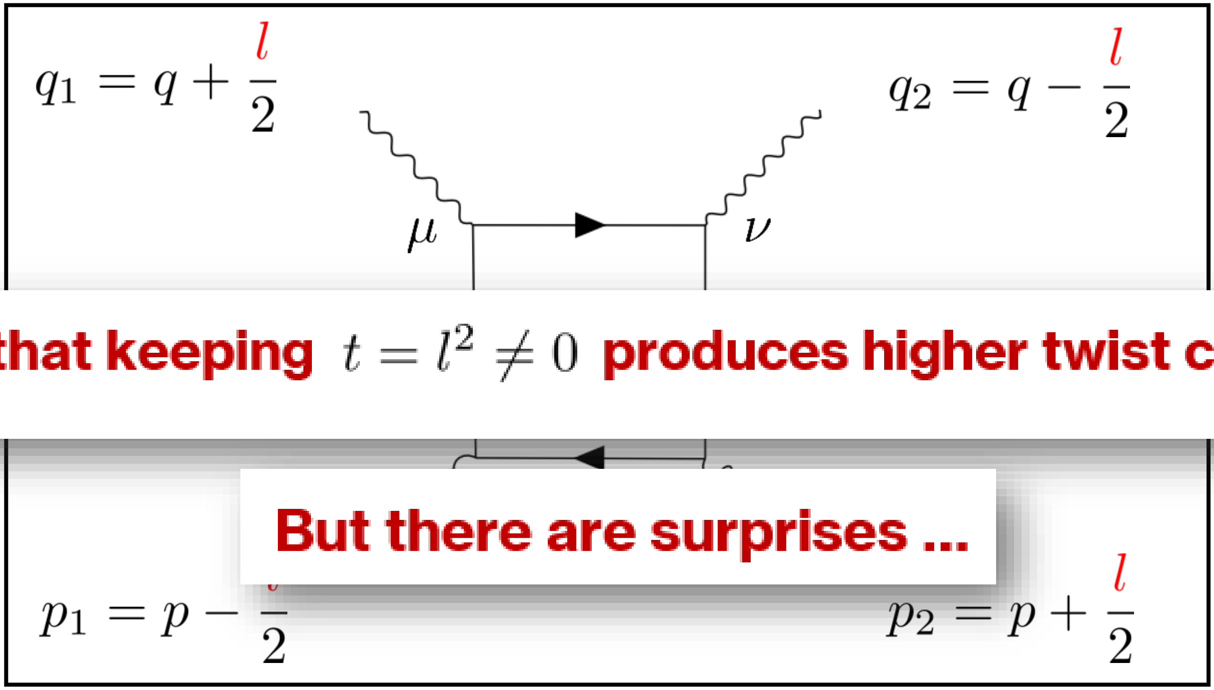


$$t = \boldsymbol{l^2 \neq 0}$$

Calculation of imaginary part of anti-symmetric/symmetric (μ, ν) of Compton amplitude with non-zero t

Imprint of Anomalies in QCD Compton scattering

Kinematics:

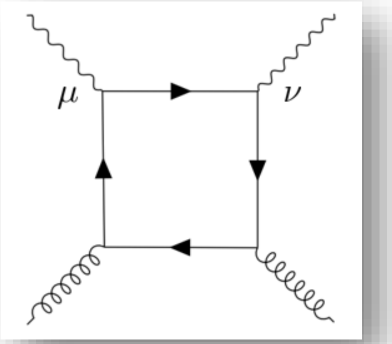


Usual rationale is that keeping $t = l^2 \neq 0$ produces higher twist corrections $\sim \frac{t}{Q^2}$

But there are surprises ...

Calculation of imaginary part of anti-symmetric/symmetric (μ, ν) of Compton amplitude with non-zero t

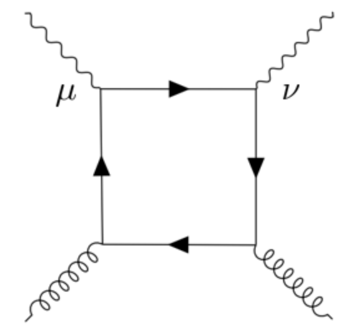
Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude $(\xi = 0)$

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \mathrm{Im} T^{\mathrm{asym}}_{\mu\nu}$$

Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude $(\xi = 0)$

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \mathrm{Im}T^{\mathrm{asym}}_{\mu\nu} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\bar{u}(P_2)\left[\left(\Delta P_{qg}\ln\frac{Q^2}{-l^2} + \delta C_g^{\mathrm{off}}\right)\otimes \Delta G(x_B)\gamma^\alpha\gamma_5 + \frac{l^\alpha}{l^2}\delta C_g^{\mathrm{anom}}\otimes\tilde{\mathcal{F}}(x_B)\gamma_5\right]u(P_1)$$

Polarized gluon distribution

Expected terms:

Splitting function $\Delta P_{qg}(\hat{x}) = 2T_R(2\hat{x}-1)$

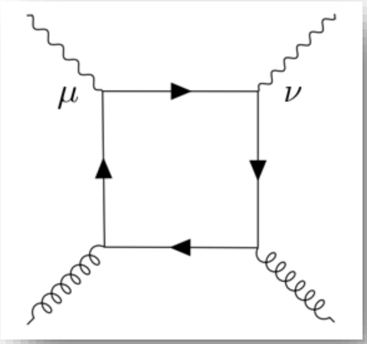
Coefficient function $\delta C_g^{\mathrm{off}}(\hat{x}) = 2T_R(2\hat{x}-1)\left(\ln\frac{1}{\hat{x}(1-\hat{x})} - 1\right)$

Recall: In DR, one obtains

$$\Delta P_{qg}\frac{-1}{\epsilon} + \delta C_g^{\overline{\mathrm{MS}}}$$

$$\delta C_g^{\overline{\mathrm{MS}}}(\hat{x}) = 2T_R(2\hat{x}-1)\left(\ln\frac{1-\hat{x}}{\hat{x}} - 1\right) + 4T_R(1-\hat{x})$$

Imprint of Anomalies in QCD Compton scattering



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Pole term
In agreement with Tarasov, Venugopalan

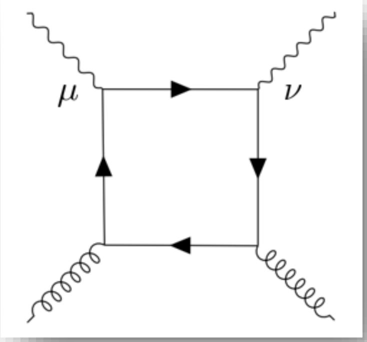
Coefficient function $\delta C_g^{\mathrm{anom}}(\hat{x}) = 4T_R(1-\hat{x})$

Twist-4 GPD:

$$\tilde{\mathcal{F}}(x,l^2) = \frac{iP^+}{\bar{u}(P_2)\gamma_5 u(P_1)}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle P_2|F_a^{\mu\nu}(-z^-/2)\tilde{F}^a_{\mu\nu}(z^-/2)|P_1\rangle$$

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude

Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude

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Twist-4 GPD

But no suppression in $1/Q^2$!

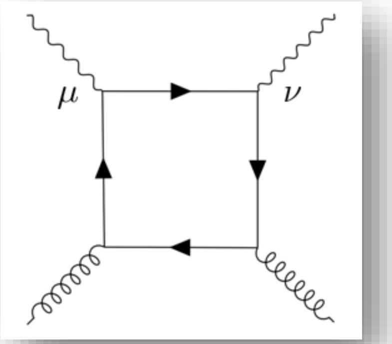
The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \mathrm{Im}T^{\mathrm{asym}}_{\mu\nu} = \frac{1}{2}\sum_f e_f^2\bar{u}(P_2)\left[\gamma^\alpha\gamma_5(\tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2))+\frac{l^\alpha\gamma_5}{2M}(\tilde{E}_f^{\mathrm{bare}}(x_B,\xi,l^2)+\tilde{E}_f^{\mathrm{bare}}(-x_B,\xi,l^2))\right]u(P_1)$$
$$+\mathcal{O}(\alpha_s)+\mathcal{O}(1/Q^2)$$

Twist-2 GPDs to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude possibly breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering



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Anomalous contribution to GPD $\tilde{E}$ at one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

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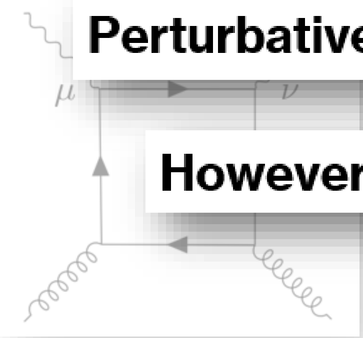
Twist-2 GPDs to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude

possibly breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering

Perturbative calculations suggest that massless poles are induced in GPD $\tilde{E}$



However, we know there are no massless poles in axial form factor (moment of GPD $\tilde{E}$)

$$g_P(l^2) = \int dx \tilde{E}(x) \sim \cancel{\frac{1}{l^2}}$$

ous contribution to GPD $\tilde{E}$ at one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

Deeply tied to the UA(1) problem: Why is the η' so massive (957 MeV!)?

$+\mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2)$

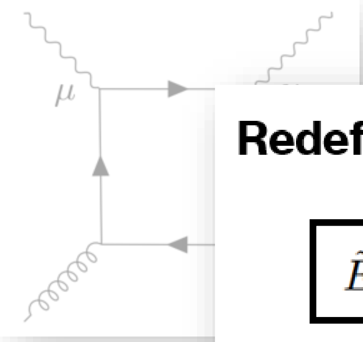
Twist-2 GPDs
to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude

possibly breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering

An attempt to rescue factorization



Antisymme

Redefine

$$\boxed{\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2)} = \boxed{\tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2)} + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

$\otimes \tilde{\mathcal{F}}(x_B)\gamma_5 \Big] u(P_1)$

one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$-\epsilon$

$\xi, l^2)) \Big] u(P_1)$

Postulate that the perturbative pole cancels the pre-existing pole in "bare" GPD:

$$\boxed{\tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2)} \approx -\frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2 = 0)$$

energy scattering amplitude

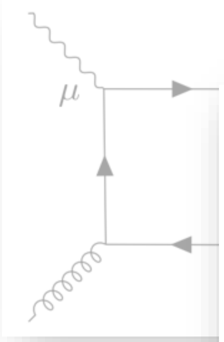
zation

Postulate that the "renormalized" GPD integrates to $g_P(l^2)$:

$$g_P(l^2) = \sum_f \int_{-1}^{1} dx \tilde{E}_f(x, \xi, l^2) = \sum_f \int_0^1 \boxed{dx(\tilde{E}_f(x, \xi, l^2) + \tilde{E}_f(-x, \xi, l^2))}$$

Imprint of Anomalies in QCD Compton scattering

An attempt to rescue factorization



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"Bare GPD" (tree level)

Perturbative pole (one loop)

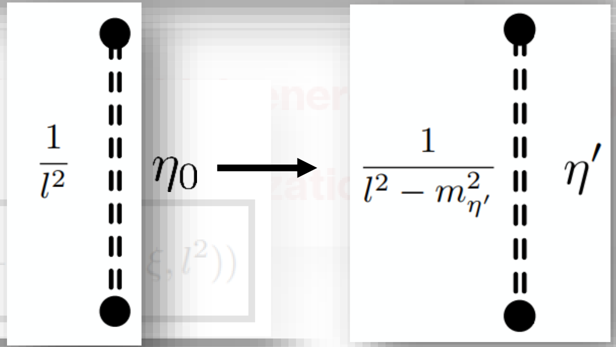
Pole cancellation at $\int dx$

We find:

$$\frac{g_P(l^2)}{2M} = -\frac{i}{l^2}\left(\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\bigg|_{l^2=0} - \frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\right) \sim \frac{1}{l^2 - m_{\eta'}^2}$$

"We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar $U_A(1)$ sector of QCD resolves both problems simultaneously: the lifting of the $\bar{\eta}$ pole by topological mass generation of the η' and the cancellation of the anomaly pole"

- Tarasov, Venugopalan



See also Jaffe Manohar, 1990

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It is highly non-trivial if a similar cancellation happens at the GPD (x-unintegrated) level which is what we need to justify factorization

Pole cancellation at $\int dx$

We find: $$\frac{g_P(l^2)}{2M} = -\frac{i}{l^2}\left(\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\bigg|_{l^2=0} - \frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\right) \sim \frac{1}{l^2-m_{\eta'}^2}$$

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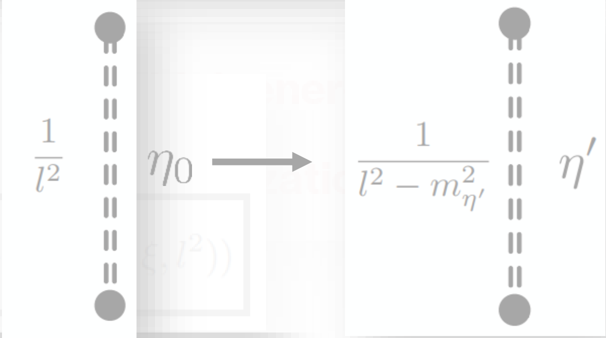

See also Jaffe Manohar, 1990

Trace anomaly

Recap on trace anomaly in QCD:

- **A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance**

Trace anomaly:

$$\boxed{\Theta^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}}$$

$\Theta^{\mu\nu}$ **: Energy Momentum Tensor (EMT)**

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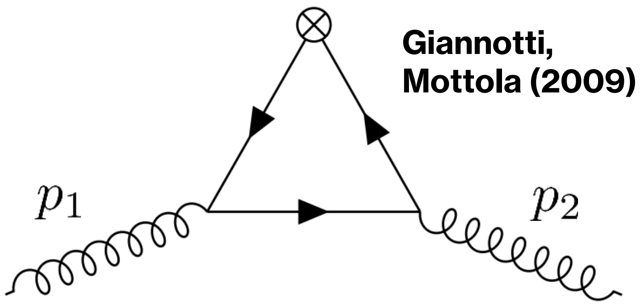


Giannotti, Mottola (2009)

Calculation in off-forward kinematics $(l = p_2 - p_1)$**:**

$$\langle p_2 | \Theta^{\mu\nu}_{\mathrm{QED}} | p_1 \rangle = -\frac{e^2}{24\pi^2 l^2} \left(p^\mu p^\nu + \frac{l^\mu l^\nu - l^2 g^{\mu\nu}}{4} \right) \langle p_2 | F^{\alpha\beta} F_{\alpha\beta} | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole

Trace anomaly

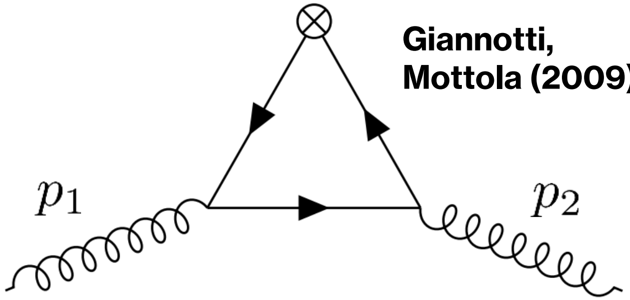
Gravitational Form Factors:

$$\boxed{\langle P_2|\Theta_f^{\mu\nu}|P_1\rangle = \frac{1}{M}\bar{u}(P_2)\left[P^\mu P^\nu A_f + (A_f+B_f)\frac{P^{(\mu}i\sigma^{\nu)\rho}l_\rho}{2} + \frac{D_f}{4}(l^\mu l^\nu - g^{\mu\nu}l^2) + M^2\bar{C}_f g^{\mu\nu}\right]u(P_1)}$$

Massless poles in Gravitational Form Factors?

$$A_f(l^2), B_f(l^2), D_f(l^2) \sim \frac{1}{l^2}$$

A perturbative solution to anomaly equation:



Giannotti, Mottola (2009)

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$$\langle p_2|\Theta^{\mu\nu}_{\rm QED}|p_1\rangle = -\frac{e^2}{24\pi^2 l^2}\left(p^\mu p^\nu + \frac{l^\mu l^\nu - l^2 g^{\mu\nu}}{4}\right)\langle p_2|F^{\alpha\beta}F_{\alpha\beta}|p_1\rangle$$

Triangle diagram is dominated by infra-red pole

Trace anomaly

Re

Gravitational Form Factors:

$$\boxed{\langle P_2|\Theta_f^{\mu\nu}|P_1\rangle = \frac{1}{M}\bar{u}(P_2)\left[P^\mu P^\nu A_f + (A_f+B_f)\frac{P^{(\mu}i\sigma^{\nu)\rho}l_\rho}{2} + \frac{D_f}{4}(l^\mu l^\nu - g^{\mu\nu}l^2) + M^2\bar{C}_f g^{\mu\nu}\right]u(P_1)}$$

omaly)

Massless poles in Gravitational Form Factors?

$$A_f(l^2), B_f(l^2), D_f(l^2) \sim \cancel{\frac{1}{l^2}}$$

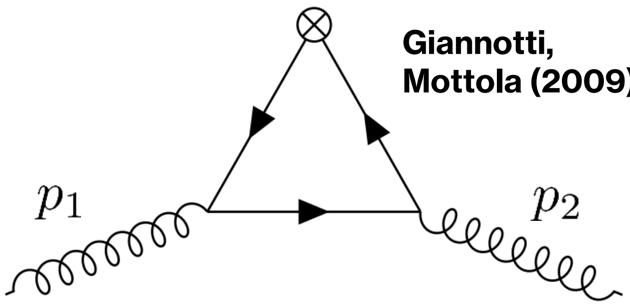
In QCD, we expect:

$$\frac{1}{l^2} \to \frac{1}{l^2 - m_G^2}$$

EMT)

glueball mass generations

A perturbative solution to anomaly equation:



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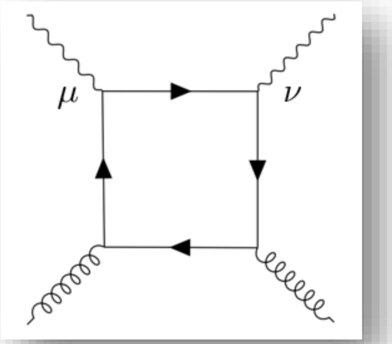
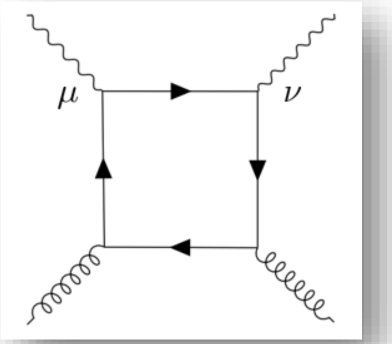
Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude $(\xi \neq 0)$

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude $(\xi \neq 0)$

Pole! (New result)

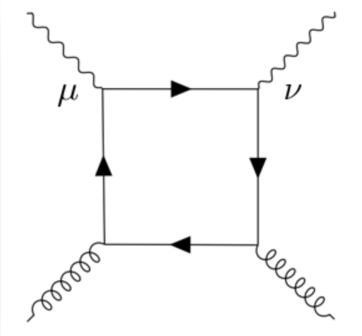
$$\boxed{(H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2))} = \boxed{(H_f^{\text{bare}}(x_B,\xi,l^2) - H_f^{\text{bare}}(-x_B,\xi,l^2))} + \frac{\alpha_s}{2\pi}\frac{1}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

$$\boxed{(E_f(x_B,\xi,l^2) - E_f(-x_B,\xi,l^2))} = \boxed{(E_f^{\text{bare}}(x_B,\xi,l^2) - E_f^{\text{bare}}(-x_B,\xi,l^2))} - \frac{\alpha_s}{2\pi}\frac{1}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

“Bare GPD” (tree level)

Perturbative pole (one loop)

Imprint of Anomalies in QCD Compton scattering



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$$\boxed{\left(H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2)\right)} = \boxed{\left(H_f^{\text{bare}}(x_B,\xi,l^2) - H_f^{\text{bare}}(-x_B,\xi,l^2)\right)} + \frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\text{anom}} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

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"Bare GPD" (tree level)

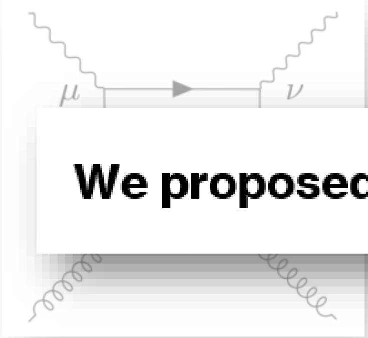
Perturbative pole (one loop)

Twist-4 GPD:

$$\mathcal{F}(x,\xi,l^2) = -4xP^+M\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)F_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)u(P_1)}$$

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & possibly breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering

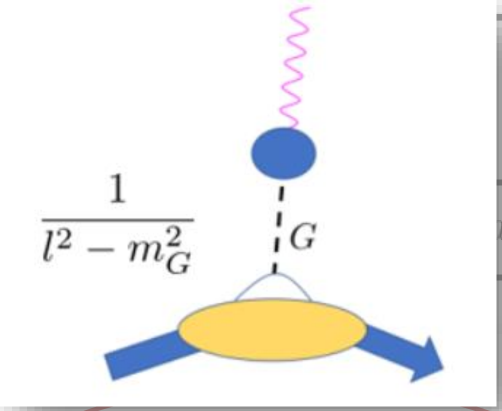


Symmetric part of Compton amplitude $(\xi \neq 0)$

We proposed a possible scenario of pole cancellation in an attempt to rescue QCD factorization

$$\left(H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2)\right) = \left(H_f^{\rm bare}(x_B,\xi,l^2) - H_f^{\rm bare}(-x_B,\xi,l^2)\right)$$

$$+ \frac{\alpha_s}{2\pi}\frac{1}{l^2} C^{\rm anom} \otimes' \mathcal{F}(x_B,\xi,l^2)$$



$$A(l^2),\ B(l^2),\ D(l^2) \sim \frac{1}{l^2 - m_G^2}$$

$(-x_B,\xi,l^2))$

glueball mass generations

$\mathcal{F}(x_B,\xi,l^2)$

Twist-4 G

$$\mathcal{F}(x,\xi,l^2) = -4xP^+M\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)F_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)u(P_1)}$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

(Non-local) trace anomaly manifests itself in high energy scattering amplitude &

possibly breaks QCD factorization

Summary

- Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC

- Importance to understand off-forward poles originating from chiral & trace anomalies

$$T^{\mu\nu} \sim \frac{\langle F\tilde{F}\rangle}{l^2}, \quad \frac{\langle FF\rangle}{l^2}$$

Unnoticed in literature, possible violation of factorization

Profound physical implications of these poles

Summary

Perturbative calculations suggest that massless poles are induced in GPDs $\tilde{E},\ H,\ E$

- Revisited QCD factorization for Compton scattering: Crucial topic for ongoing &

However, we know there are no massless poles in axial and gravitational form factors (moments of GPDs)

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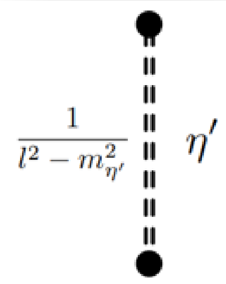
We proposed a possible scenario of pole cancellation

This has to do with eta-meson & glueball mass generations

- Importance to understand off-forward poles originating from chiral & trace anomalies

cf, the η' mass problem



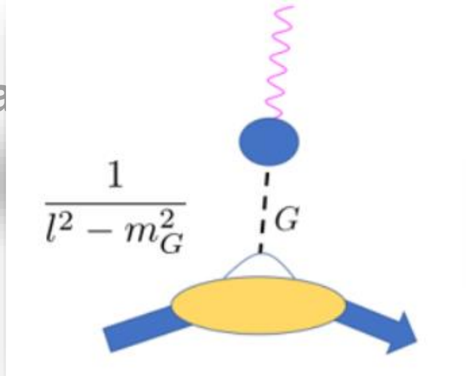
$$g_P \sim \int dx \tilde{E}(x) \sim \frac{1}{l^2 - m_{\eta'}^2}$$

Antisymmetric case:

Unnoticed in literatu

Profound physica ... se poles

Symmetric case:



$$A(l^2),\ B(l^2),\ D(l^2) \sim \frac{1}{l^2 - m_G^2}$$

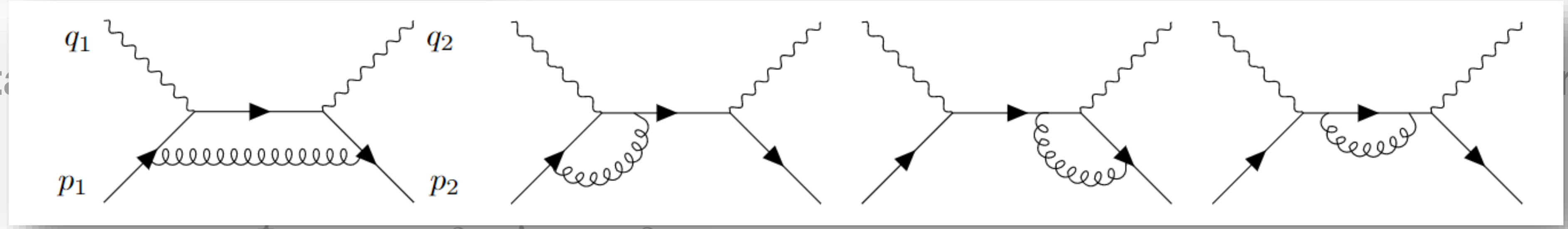
Summary & outlook

Novel connections between DVCS & chiral/trace anomalies:
This could be a new & potentially rich avenue for GPD research

rucial topic for ongoing &
future experiments including at EIC

Explore quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In preparation)

- Importa ... malies



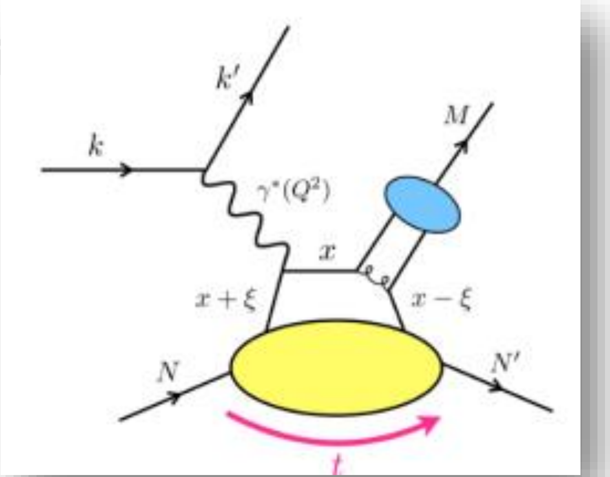
Calculate real part of Compton amplitude

Unnoticed in literature, possible violation of factor

Imprint of anomaly on other physical processes:

(Example: Deeply-virtual meson production)

les



Backup slides

Imprint of Anomalies in QCD Compton scattering

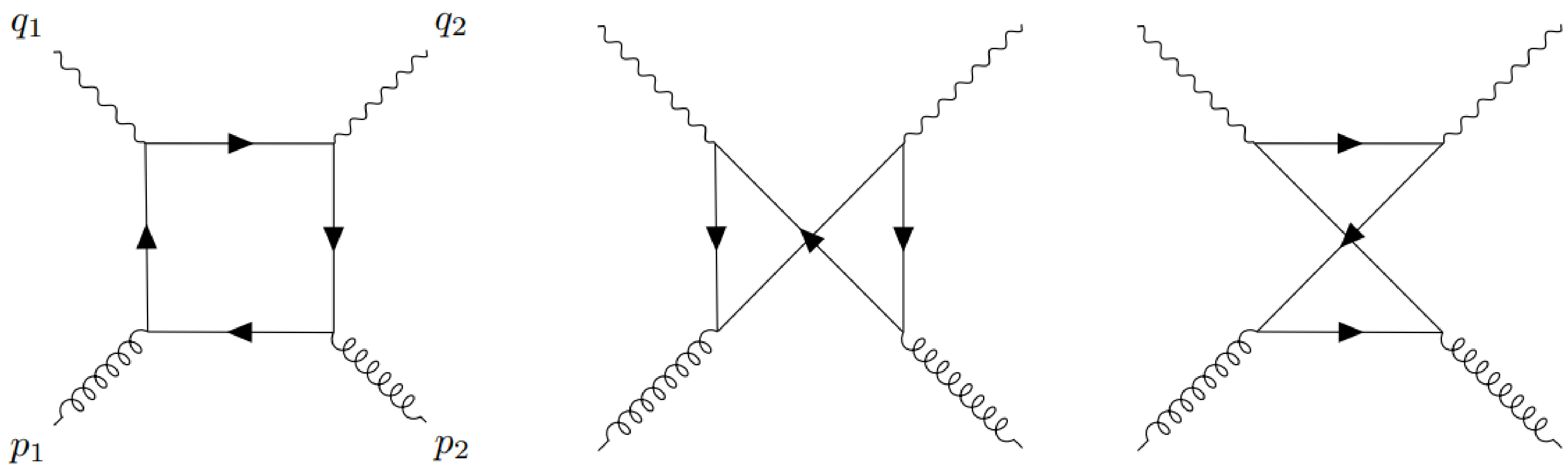


FIG. 1: Box diagrams for the Compton amplitude in off-forward kinematics.

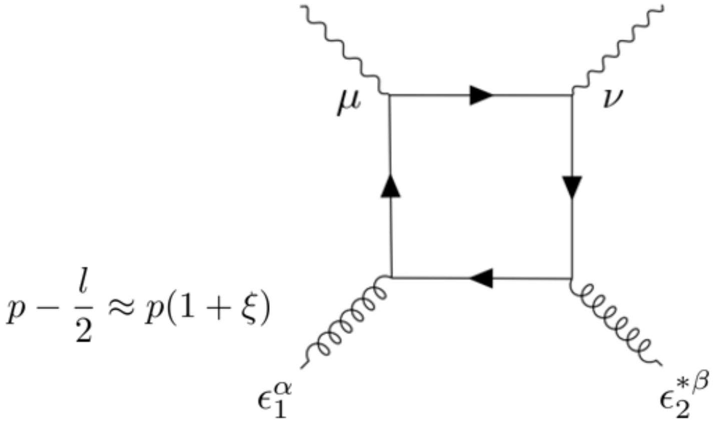
Imprint of Anomalies in QCD Compton scattering

Pole was unnoticed in the GPD literature because one typically assumes

$$l^\mu = -2\xi p^\mu \;\rightarrow\; t = l^2 = 0$$

before loop integration

Usual rationale: Corrections supposedly higher twist $\dfrac{t}{Q^2}$



The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

Ji, Osborne; Belitsky, Muller; Mankiewicz et al, Pire et al.

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \mathrm{Im}T^{\mathrm{asym}}_{\mu\nu} = \frac{1}{2}\sum e_f^2\bar{u}(P_2)\Big[\gamma^\alpha\gamma_5(\tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2)) + \frac{\ldots}{2M}\tilde{E}_f(x_B,\xi,l^2)+\tilde{E}_f(-x_B,\xi,l^2))\Big]u(P_1)$$

However, box diagram is power-divergent in the IR!

$$\frac{l^\alpha}{l^2}\delta C_g^{\mathrm{anom}} \otimes \tilde{\mathcal{F}}(x_B)\gamma_5$$

Chiral ... amplitude &

Still, pole was never seen before because:

$$\langle p_2|F^{\mu\nu}\tilde{F}_{\mu\nu}|p_1\rangle \propto \epsilon^{\mu\nu\alpha\beta}l_\mu p_\nu \epsilon_{1\alpha}\epsilon^*_{2\beta}$$

$\rightarrow 0$ **when** $l^\mu \propto p^\mu$

Imprint of Anomalies in QCD Compton scattering

An attempt to rescue factorization

Redefine

Form Factors (FF) of axial-vector operator:

$$\langle P_2|J_5^\mu|P_1\rangle = \bar u(P_2)\left[\gamma^\mu\gamma_5\, g_A(l^2) + \frac{l^\mu\gamma_5}{2M} g_P(l^2)\right]u(P_1)$$

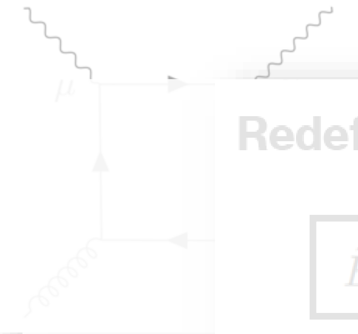
Postulate that the perturbative pole cancels the pre-existing pole in "bare" GPD:

$$\boxed{\tilde E_f^{\mathrm{bare}}(x_B,l^2) + \tilde E_f^{\mathrm{bare}}(-x_B,l^2)} \approx -\frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\mathrm{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2=0)$$

Postulate that the "renormalized" GPD integrates to $g_P(l^2)$:

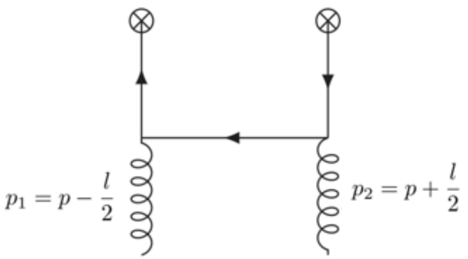
$$g_P(l^2) = \sum_f \int_{-1}^{1} dx \tilde E_f(x,\xi,l^2) = \sum_f \int_0^1 \boxed{dx(\tilde E_f(x,\xi,l^2) + \tilde E_f(-x,\xi,l^2))}$$

Imprint of Anomalies in QCD Compton scattering



An attempt to rescue factorization

Perhaps not an ad hoc argument?



Perturbative pole in GPD

$$\int \frac{dz^-}{4\pi} e^{i\hat{x}p^+z^-} \langle p_2|\bar{\psi}(-z^-/2)\gamma^+\gamma_5\psi(z^-/2)|p_1\rangle\Big|_{\text{pole}} \sim \frac{\alpha_s}{2\pi} T_R \frac{2il^+}{l^2}(1-\hat{x}) \otimes \delta(1-\hat{x})\epsilon^{\epsilon_1\epsilon_2^* lp}$$

Same pole in one-loop calculation!

Postulate that the perturbative pole cancels the pre-existing pole in “bare” GPD:

$$\boxed{\tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2)} \approx -\frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2 = 0)$$

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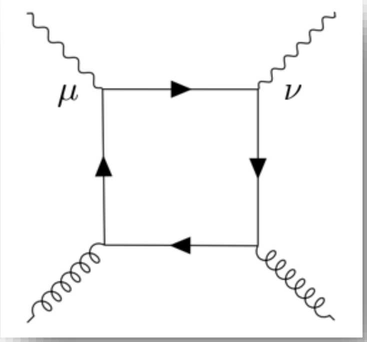
Imprint of Anomalies in QCD Compton scattering



Symmetric case:

Example: Antisymmetric part of Compton amplitude

$-\epsilon^{\alpha\beta\mu\nu} P_\beta$

$$\bar{F}_1^{\rm off}(x_B,l) \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\left[\left(P_{qg}\ln\frac{Q^2}{-l^2}+C_{1g}^{\rm off}\right)\otimes g(x_B)+\frac{1}{l^2}C^{\rm anom}\otimes'\mathcal{F}(x_B,\xi,l^2)\frac{\bar{u}(P_2)u(P_1)}{2M}\right],$$

$$\bar{F}_2^{\rm off}(x_B,l) \approx x_B\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\left[\left(P_{qg}\ln\frac{Q^2}{-l^2}+C_{2g}^{\rm off}\right)\otimes g(x_B)+\frac{1}{l^2}C^{\rm anom}\otimes'\mathcal{F}(x_B,\xi,l^2)\frac{\bar{u}(P_2)u(P_1)}{2M}\right]. \quad (31)$$

We recognize the expected structure of the one-loop corrections associated with the unpolarized gluon PDF $g(x)$, with the splitting function $P_{qg}(\hat{x}) = 2T_R((1-\hat{x})^2+\hat{x}^2)$. The coefficient functions are given by

$$C_{1g}^{\rm off}(\hat{x}) = 2T_R((1-\hat{x})^2+\hat{x}^2)\left(\ln\frac{1}{\hat{x}(1-\hat{x})}-1\right),$$

$$C_{2g}^{\rm off}(\hat{x}) = 2T_R((1-\hat{x})^2+\hat{x}^2)\left(\ln\frac{1}{\hat{x}(1-\hat{x})}-1\right)+8T_R\hat{x}(1-\hat{x}). \quad (32)$$

In addition, we find a pole $1/l^2$ in both $\bar{F}_1^{\rm off}$ and $\bar{F}_2^{\rm off}$ (but not in the difference $\bar{F}_2^{\rm off}-2x_B\bar{F}_1^{\rm off}$ relevant to the longitudinal structure function), with the following convolution formula

$$C^{\rm anom}\otimes'\mathcal{F}(x_B,\xi,l^2) \equiv \int_{x_B}^1\frac{dx}{x}K(\hat{x},\hat{\xi})\mathcal{F}(x,\xi,l^2)-\frac{\theta(\xi-x_B)}{2}\int_{-1}^1\frac{dx}{x}L(\hat{x},\hat{\xi})\mathcal{F}(x,\xi,l^2), \quad (33)$$

where

$$K(\hat{x},\hat{\xi}) = 2T_R\frac{\hat{x}(1-\hat{x})}{1-\hat{\xi}^2}, \qquad L(\hat{x},\hat{\xi}) = 2T_R\frac{\hat{x}(\hat{\xi}-\hat{x})}{1-\hat{\xi}^2}. \quad (34)$$

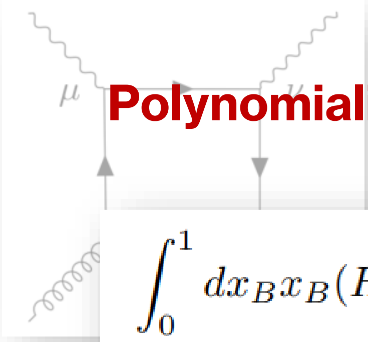
Antisymmetric case:

$$A\otimes B(x_B) \equiv \int_{x_B}^1\frac{dx}{x}A\left(\frac{x_B}{x}\right)B(x)$$

$-\epsilon^{\alpha\beta\mu\nu}P_\beta {\rm Im}T_{\mu\nu}^{\rm asym} =$

Twist-2 G

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude $(\xi \neq 0)$

Polynomiality:

Pole! (New result)

$$\int_0^1 dx_B x_B (H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2)) = \int_{-1}^1 dx_B x_B H_f(x_B,\xi,l^2) = A_f(l^2) + \xi^2 D_f(l^2)\,,$$

$$\int_0^1 dx_B x_B (E_f(x_B,\xi,l^2) - E_f(-x_B,\xi,l^2)) = \int_{-1}^1 dx_B x_B E_f(x,\xi,l^2) = B_f(l^2) - \xi^2 D_f(l^2)\,,$$

$-x_B,\xi,l^2))$

$C^{\text{anom}} \otimes' \mathcal{F}(x_B,\xi,l^2)$

$$\sum_f e_f^2 (A_f^{\text{bare}}(l^2) + \xi^2 D_f^{\text{bare}}(l^2)) \approx \frac{T_R \alpha_s}{12\pi l^2} \left(\sum_f e_f^2\right) \left(\frac{\langle P|F^{\alpha\beta}(i\overleftrightarrow{D}^+)^2 F_{\alpha\beta}|P\rangle}{(P^+)^2} + \xi^2 \langle P|F^2|P\rangle\right),$$

$$\sum_f e_f^2 (B_f^{\text{bare}}(l^2) - \xi^2 D_f^{\text{bare}}(l^2)) \approx -\frac{T_R \alpha_s}{12\pi l^2} \left(\sum_f e_f^2\right) \left(\frac{\langle P|F^{\alpha\beta}(i\overleftrightarrow{D}^+)^2 F_{\alpha\beta}|P\rangle}{(P^+)^2} + \xi^2 \langle P|F^2|P\rangle\right).$$

$\xi, l^2)$

$\mathcal{F}(x,\xi,l^2) = -4x$

Perturbative pole (one loop)

(Non-local) trace anomaly manifests itself in high energy scattering amplitude &

possibly breaks QCD factorization