Small-x behavior of the gluon GPD $E_g(x)$

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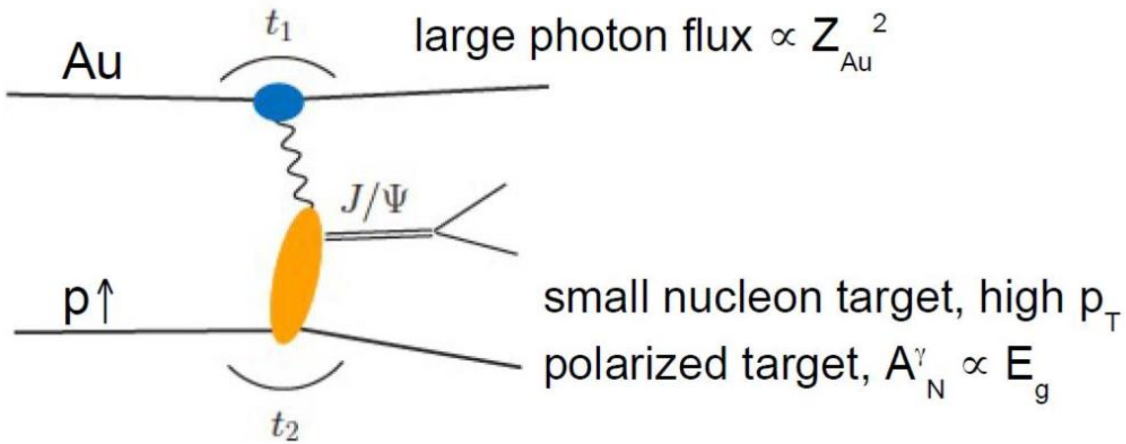
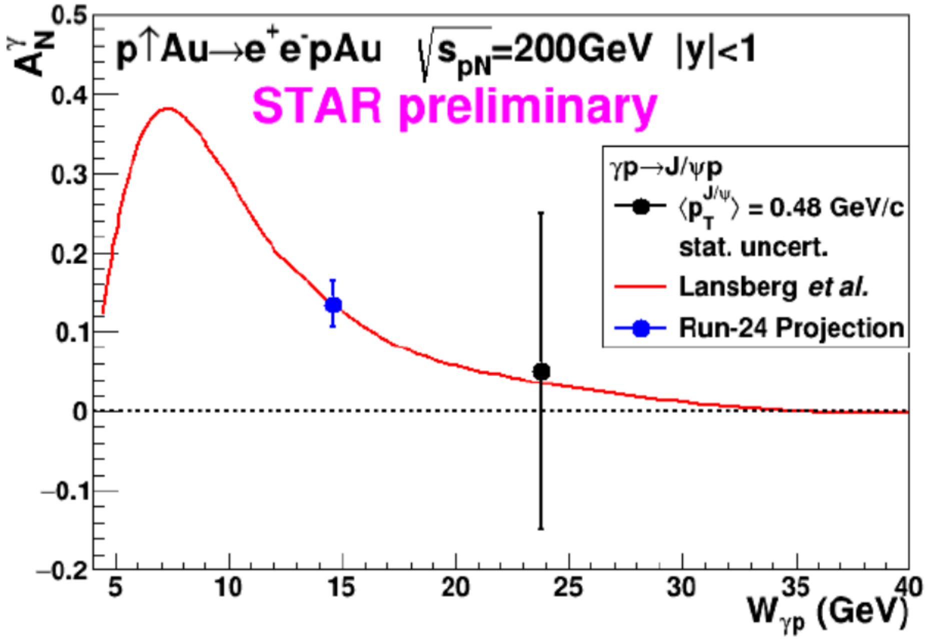
with Jian Zhou, PRL129 (2022) 252002

Single spin asymmetry in J/ψ exclusive production

STAR collaboration plans to measure SSA of J/ψ in ultraperipheral pA collisions at RHIC in order to access the gluon GPD $E_g(x)$



$$A_N \sim \frac{Im(\mathcal{H}_g^* \mathcal{E}_g)}{|\mathcal{H}_g|^2}$$



Koempel, Kroll, Metz, Zhou (2012)
Lansberg, Massacrier, Szymanowski, Wagner (2018)

Gluon GPDs

$$\delta_{ij} \int \frac{dz^-}{2\pi \bar{P}^+} e^{ix\bar{P}^+ z^-} \langle P' | F_a^{+i}(-z/2) F_a^{+j}(z/2) | P \rangle = \frac{1}{2\bar{P}^+} \bar{u}(P') \left(H_g \gamma^+ + E_g \frac{i\sigma^{+\nu}\Delta_\nu}{2m_N} \right) u(P)$$

Enter exclusive amplitudes as higher order corrections in general, difficult to access in experiments

Nothing is known about $E_g(x)$ experimentally, no theory guidance to model it.

Yet gluon GPDs are essential to understand spin sum rules

Ji sum rule

$$J_g = \frac{1}{2} \int_0^1 dx \; x \left[H_g(x,\xi) + E_g(x,\xi) \right]$$

OAM density in Jaffe-Manohar sum rule

$$\mathcal{L}_g(x) = x \int_x^1 \frac{dx'}{x'} (H_g(x') + E_g(x')) + \cdots$$

GPDs at small-x

Is there a significant contribution from small-x in spin sum rules?

$$J_g = \frac{1}{2}\int_0^1 dx\, x\,[H_g(x,\xi) + E_g(x,\xi)]$$

Integral independent of skewness ξ

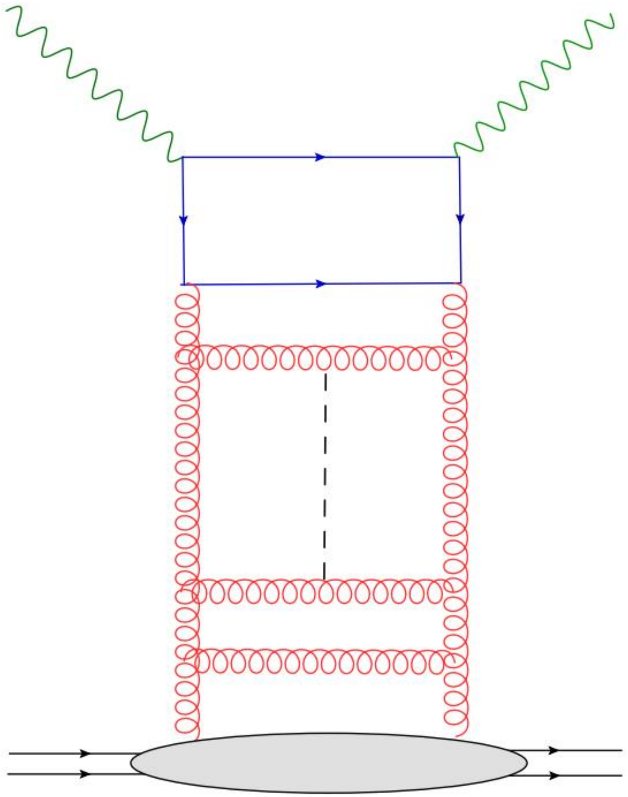
$$H_g(x,0) = G(x) \sim \frac{1}{x^{1+\alpha(Q^2)}} \qquad \alpha(Q^2) \sim 0.3$$ in the pQCD regime (from HERA)

Small-x region likely important for H_g $\quad \int_{0.0001}^{0.1} dx x H_g(x) \sim 0.3 \quad Q^2 = 10\,\mathrm{GeV}^2$

What about E_g ?

Prejudice: nucleon helicity-flip amplitudes are suppressed at high energy (small-x)

Helicity non-flip → BFKL



pQCD resummation known for almost a half century!

Introduce TMD

$$xH_g(x) = xG(x) = \int d^2k_\perp \mathcal{G}(x, k_\perp)$$

Balitsky-Lipatov-Kuraev-Fadin (BFKL) equation

$$\frac{d}{d\ln\frac{1}{x}}\mathcal{G}(x, k_\perp) = \alpha_s K \otimes \mathcal{G}(x, k_\perp)$$

$$xG(x) \sim \left(\frac{1}{x}\right)^{4\ln 2\bar{\alpha}_s}$$

At even smaller-x, expect gluon saturation → Balitsky-Kovchegov (BK) equation

Gluon GPD $E_g(x)$ at small-x

Nucleon helicity non-flip

$$xH_g(x) = xG(x) = \int d^2k_\perp \mathcal{G}(x, k_\perp)$$

BFKL equation

Nucleon helicity flip

$$xE_g(x) = \int d^2k_\perp \mathcal{E}(x, k_\perp)$$

$$\sim \left(\frac{1}{x}\right)^{??}$$

??

Introduce $k_\perp$ dependence in GPD → GTMD

Recent developments in GTMD help us to solve the problem

Outline of the derivation

Gluon Wigner at small-x

$$xW(x,\vec{k}_\perp,\vec{b}_\perp) \approx \frac{2N_c}{\alpha_s}\int \frac{d^2\vec{r}_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot\vec{r}_\perp}\left(\frac{1}{4}\vec{\nabla}_b^2 - \vec{\nabla}_r^2\right) S_x(\vec{b}_\perp,\vec{r}_\perp)$$

YH, Xiao, Yuan (2016)

Dipole S-matrix

$$S_x(\vec{b}_\perp,\vec{r}_\perp) = \left\langle \frac{1}{N_c}\mathrm{Tr}\, U\left(\vec{b}_\perp - \frac{\vec{r}_\perp}{2}\right) U^\dagger\left(\vec{b}_\perp + \frac{\vec{r}_\perp}{2}\right)\right\rangle_x$$

F.T.

assume transverse polarization

$$\frac{\pi g^2}{2N_c k_\perp^2}\left[f_{1,1} - i\frac{k_\perp \times S_\perp}{M^2}\left(\frac{k_\perp\cdot\Delta_\perp}{M^2} f_{1,2} + i g_{1,2}\right) + i\frac{\Delta_\perp \times S_\perp}{2M^2}(2f_{1,3} - f_{1,1})\right]$$

$$xE_g(x) = \int d^2k_\perp\left[-f_{1,1}(k_\perp) + 2f_{1,3}(k_\perp) + \frac{k_\perp^2}{M^2} f_{1,2}(k_\perp)\right]$$

The dipole S-matrix satisfies the BK equation at the operator level→ coupled equations for GTMDs $f_{1,i}$

The result

$$Y = \ln 1/x$$

$$\partial_Y \mathcal{E}(k_\perp) = \underbrace{\frac{\bar{\alpha}_s}{\pi}\int \frac{d^2k'_\perp}{(k_\perp - k'_\perp)^2}\left[\mathcal{E}(k'_\perp) - \frac{k_\perp^2}{2k'^2_\perp}\mathcal{E}(k_\perp)\right]}_{\text{BFKL equation}} - 4\pi^2\alpha_s^2\overline{\mathcal{F}}_{1,1}(k_\perp)\mathcal{E}(k_\perp)$$

coupling with H_g

$$xE_g(x) \sim xG(x) \propto \left(\frac{1}{x}\right)^{\bar{\alpha}_s 4\ln 2}$$

BFKL Pomeron behavior, the same as unpol gluon PDF

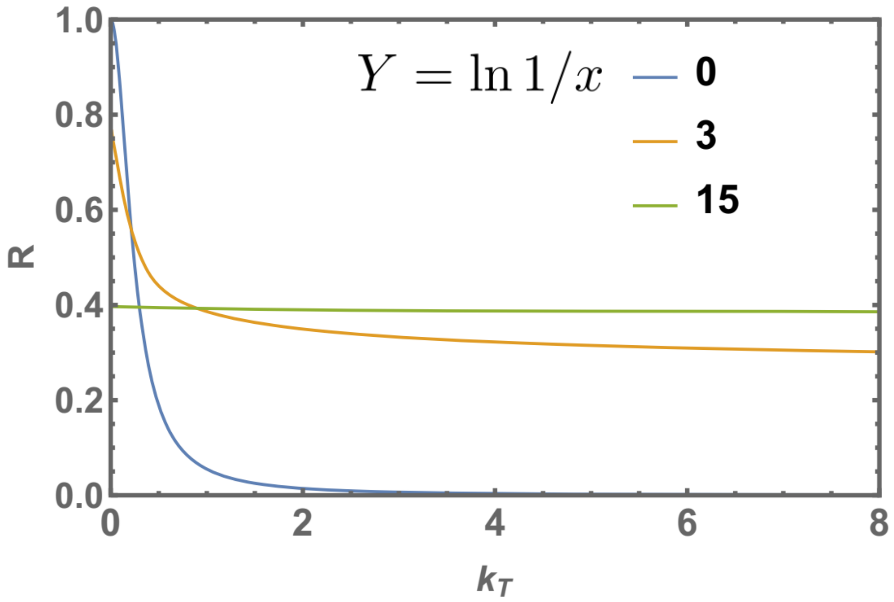
Gluon saturation of $E_g(x)$

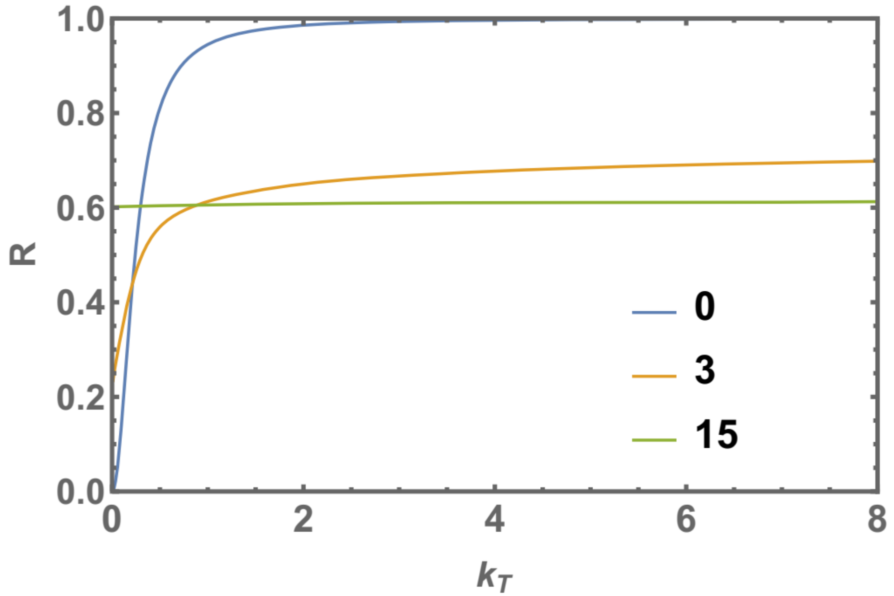
The ratio

$$R \equiv \frac{\mathcal{E}(x, k_\perp)}{\mathcal{F}_{1,1}(x, k_\perp)} \sim \frac{E_g(x)}{H_g(x)}$$

becomes constant at small-x.

$xE_g(x)$ gets saturated in the same way as $xG(x)$





Applications: SSA of J/ψ in UPC at LHC and RHIC

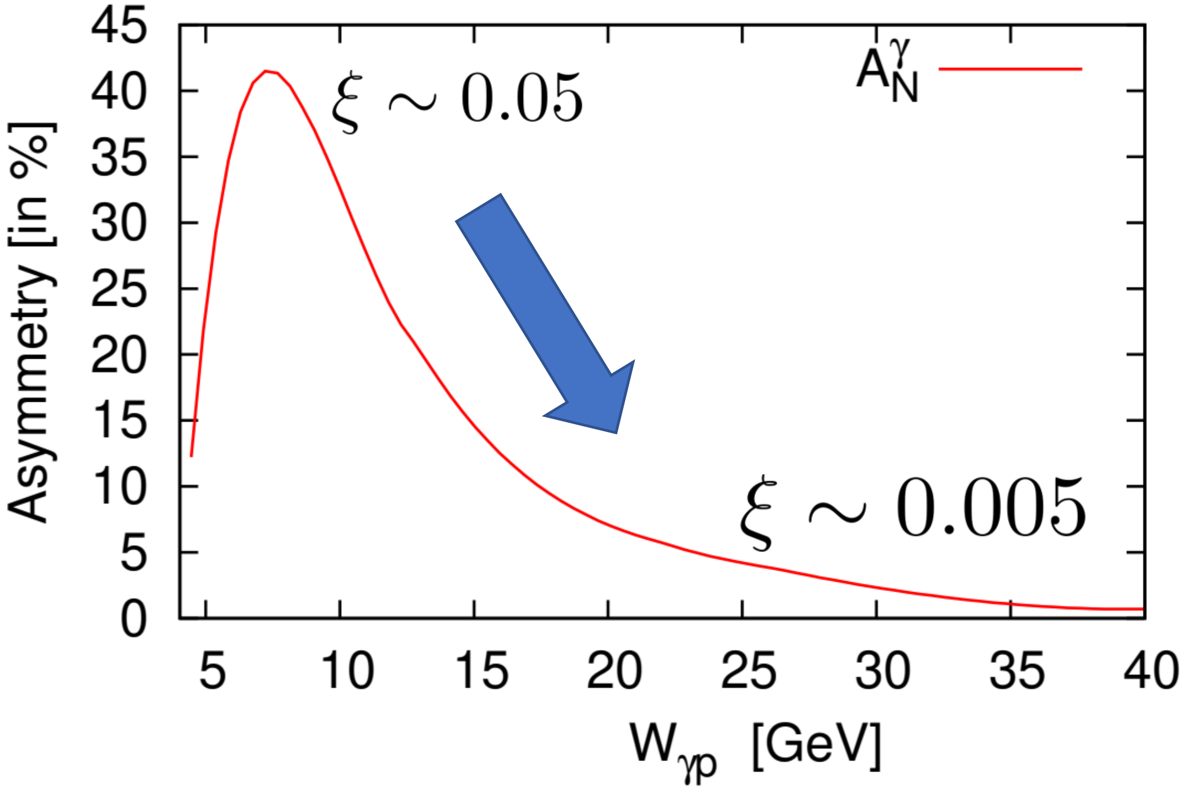
Koempel, Kroll, Metz, Zhou (2012)
Lansberg, Massacrier, Szymanowski, Wagner (2018)

$$A_N \sim \frac{Im(\mathcal{H}_g^* \mathcal{E}_g)}{|\mathcal{H}_g|^2} \sim \frac{Re\mathcal{E}_g}{Im\mathcal{H}_g} \sim \xi \sim \frac{M_{J/\psi}^2}{2W_{\gamma p}^2}$$

$$E_g(x) \propto H_g(x)$$

Small-ξ $\sim$ small-x

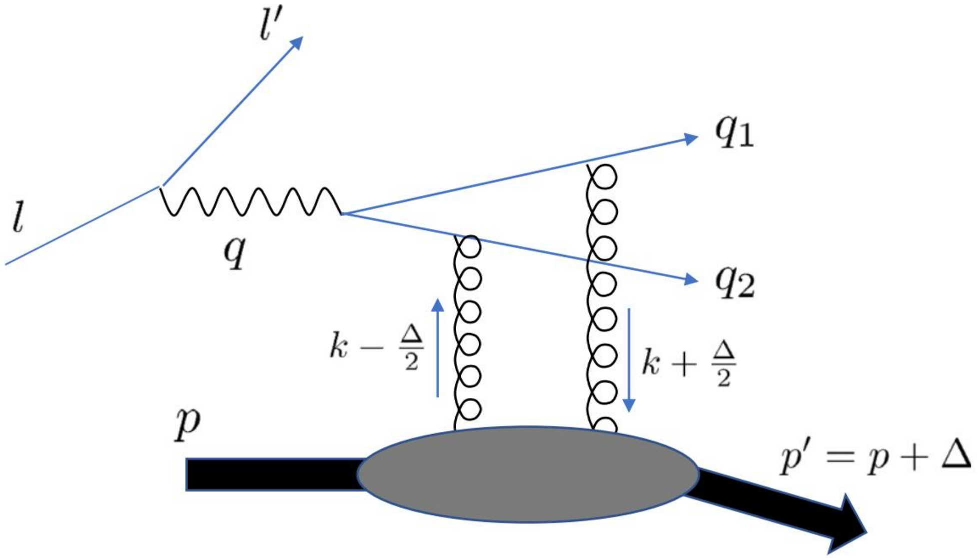
Shape of A_N at high-$W_{\gamma p}$ can constrain the x-dependence of $E_g(x)$



Applications: Double spin asymmetry in dijet production at EIC

Bhattacharya, Boussarie, YH (2022)

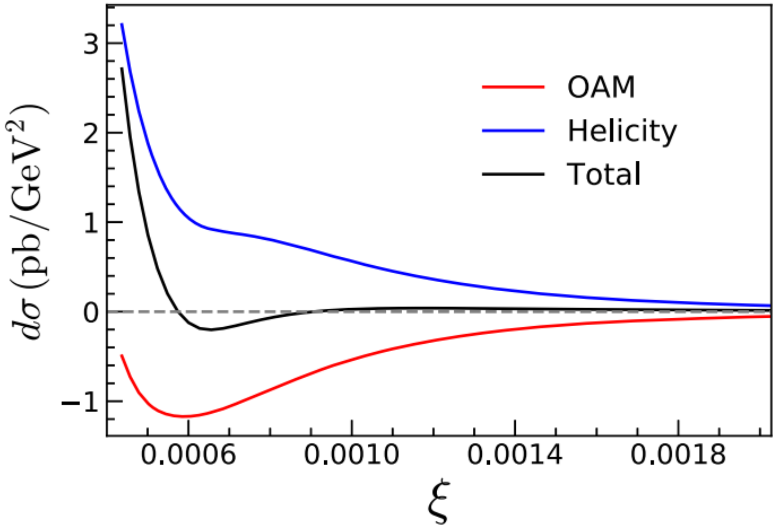
Proposed as a signal of parton OAM at the EIC



In WW approximation

$$\mathcal{L}_g(x) = x \int_x^1 \frac{dx'}{x'} (H_g(x') + E_g(x')) - 2x \int_x^1 \frac{dx'}{x'^2} \Delta G(x')$$

Asymmetry sensitive to the relative strengths of OAM and helicity.



In double logarithmic approximation $\mathcal{L}_g(x) \approx -\Delta G(x)$

Inclusion of single logarithmic terms H_g, E_g can tip the balance

Conclusions

- Small-x behavior of $E_g(x)$ systematically derived

$$xE_g(x) \sim xG(x) \propto \left(\frac{1}{x}\right)^{\bar{\alpha}_s 4 \ln 2}$$

- Strong growth towards small-x, despite its association with spin-flip (transversely polarized) processes
- $E_g(x)$ exhibits gluon saturation
- Useful input to the modeling of $E_g(x)$
- Connections to the small-x community.