Quarkonia pair production as a tool for study of gluon GPDs

Marat Siddikov

In collaboration with Ivan Schmidt

FEDERICO SANTA MARIA
TECHNICAL UNIVERSITY

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Foreword

Hadrons in QCD:

–Sophisticated strongly interacting dynamical systems

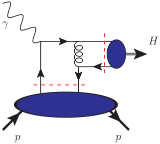
–Theoretical description challenging:

* *Many nontrivial nonperturbative phenomena (chiral symmetry breaking, dynamical masses and interaction vertices ...)
* *Can't evaluate everything from the first principles, have to rely on phenomenological inputs ...

Phenomenological approach:

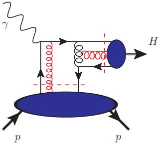
–Based on factorization (separation of amplitude or cross-section) onto:

* *soft hadron-dependent correlators, and
* *perturbative process-dependent parts



–Requires high energies, large invariant masses

* *suppress soft final-state interactions
* *suppress contributions of multiparton states, higher twists



–Light-cone description (quantization), effectively $P \to \infty$ frame

(Generalized) parton distributions: theoretical aspects

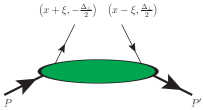
–Nonperturbative objects which encode information about 2-parton correlators.

Might be reinterpreted in terms of hadron-parton amplitudes in helicity basis

*GPDs are different for each flavour, depend on 4 variables: x, ξ, t, μ^2

**Dependence on μ^2 ⇒DGLAP

**Dependence on x, ξ ⇒positivity, polynomiality constraints



⇒Challenge for modelling ("dimensionality curse")

–Classification standardized since ∼2010 [PDG 2022, Sec 18.6]

– Leading twist-2 (dominant in many high-energy processes):

$$\int \frac{dz}{2\pi} e^{ix\bar{P}^+ z} \left\langle P' \left| \bar{\psi}\left(-\frac{z}{2}\right) \Gamma\, e^{i\int d\zeta\, n\cdot A} \psi\left(\frac{z}{2}\right) \psi \right| P \right\rangle = \bar{U}(P')\, \mathcal{F}^{(\Gamma)} U(P)$$

Γ	$\mathcal{F}^{(\Gamma)}$
γ^+	$H\gamma^+ + E\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}$
$\gamma^+\gamma_5$	$\tilde{H}\gamma^+\gamma_5 + \tilde{E}\frac{\gamma_5\Delta^+}{2m}$

Γ	$\mathcal{F}^{(\Gamma)}$
$i\sigma^{+i}$	$H_T i\sigma^{+i} + \tilde{H}_T \frac{\bar{P}^+\Delta^i - \bar{P}^i\Delta^+}{m^2} + E_T\frac{\gamma^+\Delta^i - \gamma^i\Delta^+}{2m} + \tilde{E}_T\frac{\gamma^+\bar{P}^i - \gamma^i\bar{P}^+}{m}$

$\bar{P} \equiv (P+P')/2$ $\Delta \equiv P' - P$

*For gluons use operators $G^{+\alpha}G^+_{\ \alpha}$, $G^{+\alpha}\tilde{G}^+_{\ \alpha}$, $\mathbb{S}G^{+i}G^{+j}$ in left-hand side

Why do GPDs matter ?

Many physical observables are constructed from bilinear partonic operators:
−Energy-momentum tensor (≈energy density, distribution of forces, ...):

$$T^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + \frac{1}{2}\bar{\psi}\gamma^{\{\mu}iD^{\nu\}}\psi + \eta^{\mu\nu}\bar{\psi}\left(i\hat{D} - m\right)\psi$$

−Angular momentum density:

$$M^{\mu\nu\rho} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}\gamma_{\sigma}\gamma_5\psi + \frac{1}{2}\bar{\psi}\gamma^{\mu}x^{[\nu}iD^{\nu]}\psi - 2\mathrm{Tr}\left[F^{\mu\alpha}x^{[\nu}F^{\rho]}_{\alpha}\right] - x^{[\nu}g^{\rho]\mu}\mathcal{L}_{\mathrm{QCD}}$$

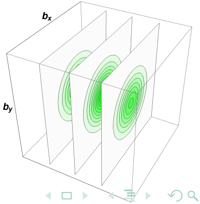
−Baryonic/electromagentic currents:

$$J^{\mu}_{\mathrm{baryonic}} = \bar{\psi}\gamma^{\mu}\psi, \qquad J^{\mu}_{\mathrm{em}} = \bar{\psi}\gamma^{\mu}\hat{Q}\psi$$

⇒GPDs contain information about contribution of each parton flavour to local energy/charge density, distribution of forces/pressure, etc.

$$\Downarrow$$

Study of GPDs ≈ "3D tomography" of the hadron.



How can we study GPD experimentally ?

– **Experimental constraints on GPDs:**

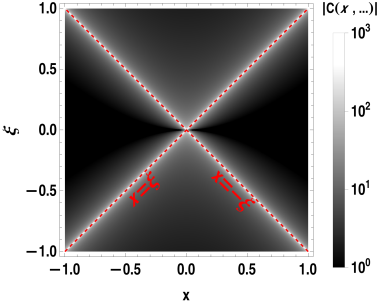
*Special limits (PDF, form factors)

*2 → 2 **processes** (DVCS, DVMP, TCS, WACS, ...)

**Amplitude is a convolution of GPD with process-dependent coef. function:
$\mathcal{A} = \int dx \, C(x, \xi) \, H(x, \xi, \ldots)$

**Predominantly sensitive to GPDs at $x = \pm\xi$ boundary

**Deconvolution is impossible, Compton FFs don't fix uniquely the GPDs [PRD 103, 114019 (2021)]



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Extraction of GPDs inevitably relies on modelling (and requires multichannel analysis)

What do we know about GPDs now ?

Quark sector:

–There is some qualitative understanding, phenomenological parametrizations (GK, KM, GUMP, ...)

Gluon sector:

–Uncertanties are much larger:

*Gluons don't interact directly with leptons.

*Gluons show up only via higher order (NLO) corrections in many observables

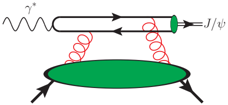
*6 of 8 GPDs are poorly known, yet contribute to physical observables, e.g.:

$$J_g = \frac{1}{2}\int_0^1 dx\, x\,(H_g(x,\xi)+E_g(x,\xi))$$

Best constraints from exclusive quarkonia production:

*No sizeable "intrinsic" charm, bottom GPDs

*Light quark GPDs only via NLO, strongly suppressed



*As for DVMP, coef. function sensitive to GPDs on $x = \pm\xi$ line.

New tool for tomography: $2 \to 3$ processes

Process:

$$\gamma^{(*)} + p \to h_1 + h_2 + p$$

States h_1, h_2 are light hadrons or photons, many possibilities studied in the literature:

$\gamma\pi$, $\gamma\rho$	[2212.00655, 2212.01034, JHEP 11 (2018) 179; 02 (2017) 054]
$\gamma\gamma$	[JHEP 08 (2022) 103; PRD **101**, 114027; **96**, 074008]
$\gamma\gamma^* \to \gamma\bar{\ell}\ell$	[Phys. Rev. D 103 (2021) 114002]
$\pi\rho$	[Phys.Lett.B 688 (2010) 154-167]

Main benefit:

– Can vary independently kinematics of h_1, h_2 to probe GPDs at $x \neq \xi$

Challenge:

– Cross-section significantly smaller than for $2 \to 2$ processes, especially for states with additional γ in final state. Need high luminosity collider (EIC)

Our suggestion:

– Exclusive photoproduction of quarkonia pairs ($\gamma + p \to M_1 + M_2 + p$)

– Focus on quarkonia with opposite C-parity (e.g. $J/\psi\ \eta_c$), largest cross-section

** Pairs with the same C-parity (e.g. $J/\psi\ J/\psi$) require C-odd exchange in t-channel (γ or 3-gluon), not related to twist-2 GPDs.*

*In $m_Q \to \infty$ limit, the LDMEs of η_c and J/ψ are proportional to each other

– Predominantly sensitive to gluon GPDs H_g, E_g, no direct (LO) contributions from light quarks *(compare $\pi\rho$ production: dominant contribution from H_T^q)*

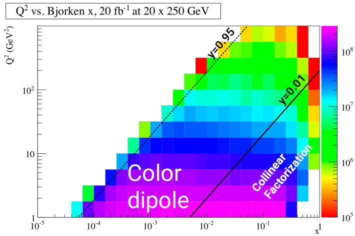
Kinematics choice: Electron Ion Collider

Typical values of variables ξ, x_B

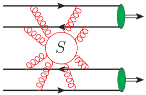
$$x_B \approx \frac{Q^2 + M_{12}^2}{Q^2 + W^2}, \qquad \xi = \frac{x_B}{2 - x_B}.$$

▷Accessible kinematics (x_B, Q^2) depends on choice of electron-proton energy E_e, E_p

▷ Dominant: $Q^2 \approx 0$, $x_B, \xi \in (10^{-4}, 1)$

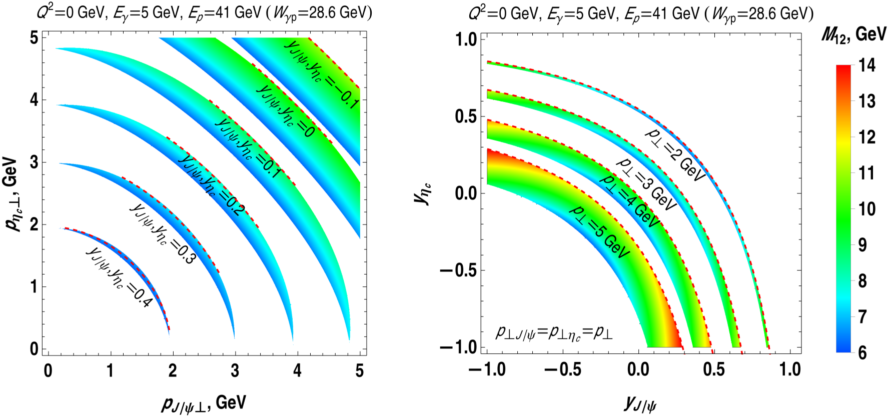


- Low-energy EIC runs to avoid $x_B, \xi \ll 1$ region (large NLO, saturation)
- We consider that $Q \sim M_{J/\psi} \sim M_{\eta_c} \sim W_{\gamma p}$ are large scales
  - Since $M_{12}^2 \gtrsim (M_{J/\psi} + M_{\eta_c})^2 \sim 36\,\mathrm{GeV}^2$ and cross-section is suppressed at large Q as $\lesssim 1/Q^6$, "classical" Bjorken limit $Q \gg M_{J/\psi}, M_{\eta_c}$ is difficult to study experimentally
  - Production at central rapidities, rapidity gaps from γ^*, p
  - Constraint on relative momentum of quarkonia $p_{\mathrm{rel}} \gtrsim 1\,\mathrm{GeV}$, to exclude possible soft final state interactions



Comment on kinematics

- ►Production at fixed Q^2, W of $\gamma^* p$ (fixed x_B) not very convenient:
  - ▷Sophisticated kinematic constraints on $y_1, p_{\perp 1}, y_2, p_{\perp 2}$, only certain domains (bands) are allowed:



- ►Alterative choice: work with $Q^2, y_1, \boldsymbol{p}_{1\perp}, y_2, \boldsymbol{p}_{2\perp}$
  - ▷No kinematic constraints on $y_1, \boldsymbol{p}_{1\perp}, y_2, \boldsymbol{p}_{2\perp}$, explicit symmetry w.r.t. permutation of quarkonia $1 \leftrightarrow 2$

Evaluations in collinear factorization framework

Evaluation is straightforward, amplitude (squared):

$$\sum_{\text{spins}} \left| \mathcal{A}^{(\mathfrak{a})}_{\gamma p \to M_1 M_2 p} \right|^2 = \frac{1}{(2-x_B)^2} \left[4(1-x_B)\left(\mathcal{H}_{\mathfrak{a}} \mathcal{H}^*_{\mathfrak{a}} + \tilde{\mathcal{H}}_{\mathfrak{a}} \tilde{\mathcal{H}}^*_{\mathfrak{a}} \right) - x_B^2 \left(\mathcal{H}_{\mathfrak{a}} \mathcal{E}^*_{\mathfrak{a}} + \mathcal{E}_{\mathfrak{a}} \mathcal{H}^*_{\mathfrak{a}} + \right.\right.$$

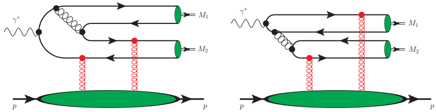
$$\left.\left. + \tilde{\mathcal{H}}_{\mathfrak{a}} \tilde{\mathcal{E}}^*_{\mathfrak{a}} + \tilde{\mathcal{E}}_{\mathfrak{a}} \tilde{\mathcal{H}}^*_{\mathfrak{a}} \right) - \left(x_B^2 + (2-x_B)^2 \frac{t}{4m_N^2} \right) \mathcal{E}_{\mathfrak{a}} \mathcal{E}^*_{\mathfrak{a}} - x_B^2 \frac{t}{4m_N^2} \tilde{\mathcal{E}}_{\mathfrak{a}} \tilde{\mathcal{E}}^*_{\mathfrak{a}} \right],$$

$$\{\mathcal{H}_{\mathfrak{a}}, \mathcal{E}_{\mathfrak{a}}\} = \int dx\, dz_1\, dz_2\, C_{\mathfrak{a}}(x, z_1, z_2, y_1, y_2) \{H_g, E_g\} \Phi_\eta(z_1) \Phi_{J/\psi}(z_2),$$

$$\left\{\tilde{\mathcal{H}}_{\mathfrak{a}}, \tilde{\mathcal{E}}_{\mathfrak{a}}\right\} = \int dx\, dz_1\, dz_2\, \tilde{C}_{\mathfrak{a}}(x, z_1, z_2, y_1, y_2) \left\{\tilde{H}_g, \tilde{E}_g\right\} \Phi_\eta(z_1) \Phi_{J/\psi}(z_2),$$

- Disregard transversity gluon GPDs (not known, should be small)
- Disregard light quarks (higher order corrections)

Evaluation of coefficient functions $C_{\mathfrak{a}}$, $\tilde{C}_{\mathfrak{a}}$:

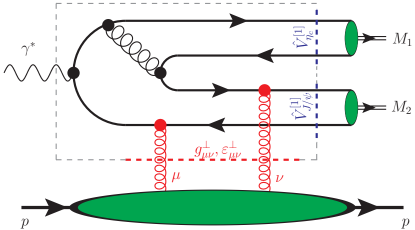


Summation over all possible gluon attachments is implied

— Two production mechanisms for $J/\psi\,\eta_c$

— Need to factorize out carefully nonperturbative blocks which correspond to initial/final state hadrons

Evaluations of the coefficient function



– Nucleon is described by leading-twist GPDs $H_g, E_g, \tilde{H}_g, \tilde{E}_g$

* Use light-cone gauge $n \cdot A = 0$

* Contract Lorentz indices of t-channel gluons with $g^{\perp}_{\mu\nu}, \varepsilon^{\perp}_{\mu\nu}$ to extract $C_{\mathfrak{a}}, \tilde{C}_{\mathfrak{a}}$

– Use NRQCD projectors $\hat{V}_{J/\psi}, \hat{V}_{\eta_c}$ to project out contributions of $\bar{Q}Q$ pairs with proper quantum numbers. Expect dominant contribution from color singlet

$$\left(\hat{V}^{[1]}_{\eta_c}\right)_{ij} \approx -\sqrt{\frac{\left\langle \mathcal{O}_{\eta_c}\left({}^1S_0^{[1]}\right)\right\rangle}{m_Q}} \frac{\delta_{ij}}{4N_c}\left(\frac{\hat{p}}{2} - m_Q\right)\gamma_5$$

$$\left(\hat{V}^{[1]}_{J/\psi}\right)_{ij} \approx \sqrt{\frac{\left\langle \mathcal{O}_{J/\psi}\left({}^3S_1^{[1]}\right)\right\rangle}{m_Q}} \frac{\delta_{ij}}{4N_c}\hat{\varepsilon}^*_{J/\psi}(p)\left(\frac{\hat{p}}{2} + m_Q\right)$$

– Virtuality of (black) gluon is large in heavy quark mass limit ($\gtrsim \mathcal{M}^2_{12}/4$), so for the coef. function (dashed square box) the perturbative treatment is justified.

See Appendix of this talk for more details

Results for coefficient function

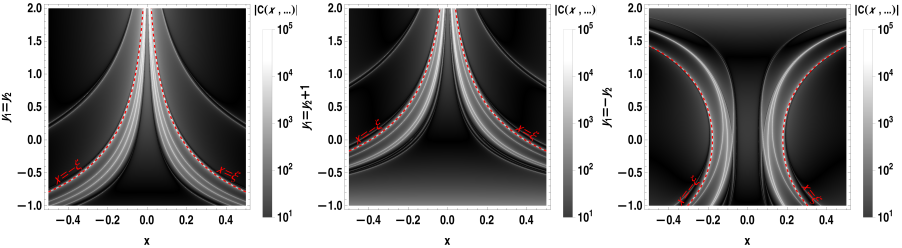
$$\{\mathcal{H}_{\mathfrak{a}},\, \mathcal{E}_{\mathfrak{a}}\} \sim \int dx\, C_{\mathfrak{a}}\,(x,\, y_1,\, y_2)\, \{H_g,\, E_g\},$$

- Structure function $C_{\mathfrak{a}}(x)$:
  $C_{\mathfrak{a}}\,(x,\, y_1,\, y_2) \sim$
  $$\sim \sum_{\ell} \frac{\mathcal{P}_\ell\,(x)}{\prod_{k=1}^{n_\ell}\left(x - x_k^{(\ell)} + i0\right)}$$
  where $\mathcal{P}_\ell\,(x)$ are finite for $|x| < 1$

– Each term might have up to 3 poles $x_k^{(\ell)}$ in the integration region $|x| < 1$

– Position of poles depends on kinematics $(y_1,\, y_2,\, Q^2/m_Q^2)$

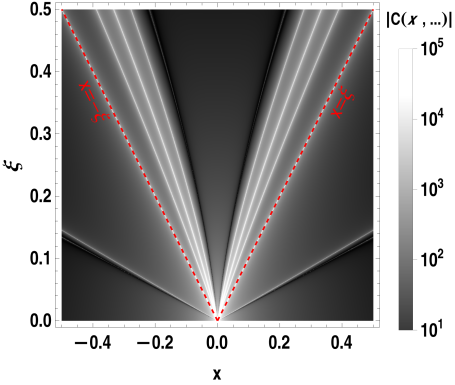
– Poles do NOT overlap for $m_Q \neq 0$, so integrals exist in Principal Value sense



- Density plot of coefficient function. Regions near poles (white lines) give the dominant contribution in convolution

Coefficient function in terms of x, ξ variables

– Consider $y_1 = y_2$, $Q = 0$



Density plot of coefficient function. Regions near poles (white lines) give the dominant contribution in convolution

► Location of poles for $Q = 0$, $y_1 = y_2$:

$$|x_k| = \left\{ \xi\left(1 - \frac{2}{3}\frac{1}{1+\xi}\right), \right.$$
$$\xi\left(1 - \frac{1}{2}\frac{1}{1+\xi}\right),$$
$$\xi\left(1 - \frac{1}{3}\frac{1}{1+\xi}\right),$$
$$\left. \xi,\ 3\xi\left(1 + \frac{1}{6}\frac{1}{1+\xi}\right)\right\}$$

Compare DVCS, DVMP: dominant contribution from $|x_k| = \xi$.

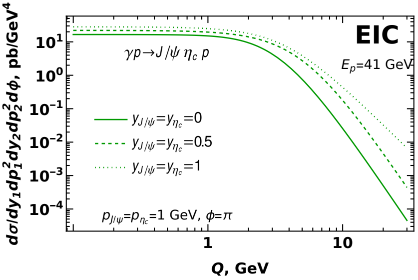
► In general expression for $C_a(x, y_1, y_2)$ is lengthy, deconvolution is impossible

– Coeff. function sensitive to behaviour of GPDs outside "classical" $|x| \approx \xi$ line, might be used to test/constrain existing phenomenological models of gluon GPDs

Results for Q^2, t-dependence

- Use Kroll-Goloskokov GPD for gluons, $\gamma^* p \to p\, \eta_c\, J/\psi$ subprocess

– Q^2 dependence:



- The Q^2-dependence is controlled by

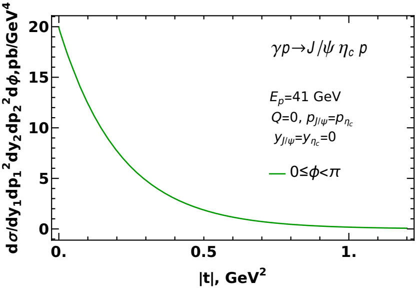
$$\mathcal{M}_{12} = \sqrt{\left(p_{J/\psi} + p_{\eta_c}\right)^2} \gtrsim \left(M_{J/\psi} + M_{\eta_c}\right)$$

– very mild dependence for $Q^2 \lesssim \mathcal{M}_{12}^2$

– $d\sigma \sim 1/Q^6$ for $Q^2 \gg \mathcal{M}_{12}^2$

– Transition scale largely independent on W

– t-dependence of the cross-section largely reflects dependence of GPD



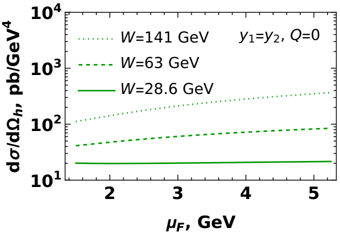
$$t = \Delta^2 = -\frac{4\xi^2 m_N^2 + \left(\boldsymbol{p}_1^\perp + \boldsymbol{p}_2^\perp\right)^2}{1-\xi^2}$$

– Predominantly $J/\psi\, \eta_c$ pairs are produced in back-to-back kinematics

Dependence on factorization scale $\mu_F = \mu_r = \mu$

– Physical observables should not depend on μ, yet when we cut pert. series, such dependence appears due to omitted higher order terms

*At LO dependence on μ due to $\alpha_s(\mu)$, DGLAP evolution of GPDs

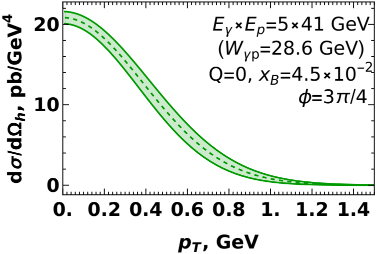


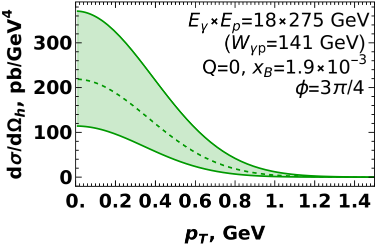
– At small W (large x_B) dependence is mild

– At large W (small x_B) dependence is gets more and more pronounced

– At smaller x_B the omitted higher order loop corrections become more pronounced, so μ-dependence is stronger

► Typical uncertainty due to scale dependence ($M_{J/\psi}/2 \lesssim \mu_F \lesssim 2M_{J/\psi}$) :



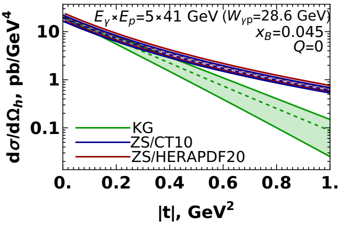


Dependence on choice of GPD

– Compare Kroll-Goloskokov (KG) and Zero Skewness (ZS) parametrizations

$$H_g(x,\xi,t) = g(x)\,F_N(t)$$

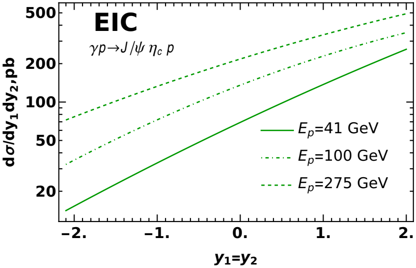
* Makes sense since $\xi \ll 1$ for *photo*production @EIC
* Gluon PDF $g\left(x,\,\mu^2\right)$ is taken from CT10 and HERAPDF20 fits

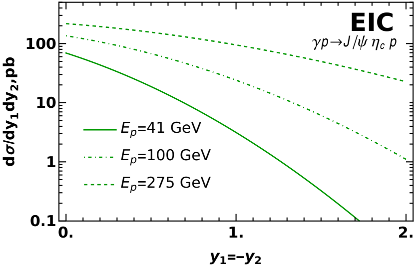


– At small t results agree within theoretical uncertainty ($\pm 20\%$)
– At larger t results differ quite significantly due to shapes of KG and ZS parametrizations.
  * The shape of GPD in KG is affected by $\sim x^{\alpha' t}$ factors even for $\xi \approx 0$
– Similar behaviour is observed for other energies

$\Rightarrow$ The process might be used to distinguish the gluon GPD models

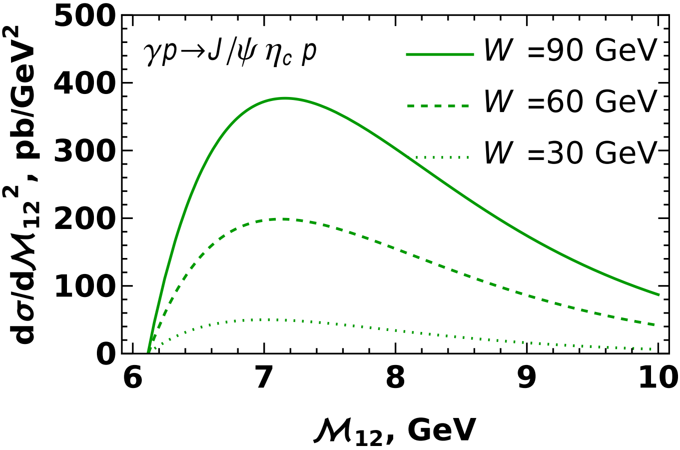
Results for rapidity dependence





- For $y_1 = y_2$ increase of rapidity implies:
  - *Larger invariant energy W
  - *Smaller x_B, ξ
  - *Larger cross-section due to growth of $H_g(x, \xi, t)$ at small x
- For $y_1 = -y_2$ increase of rapidity implies:
  - *Larger longitudinal recoil to proton Δ_L
  - *Larger values of $|t_{\min}|$, $|t| = |\Delta^2|$
  - *Suppression of cross-section is due to t-dependence of $H_g(x, \xi, t)$

Results for invariant mass dependence



–Pronounced peak at $M_{12} \approx 7\,\mathrm{GeV}$

**Small relative momentum of quarkonia, $p_{\rm rel} \lesssim 2-3\,\mathrm{GeV}$

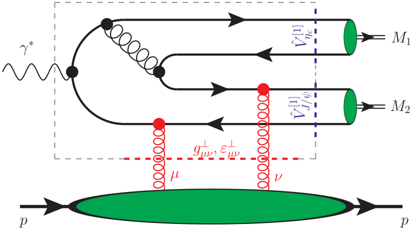
Summary

Exclusive production of heavy quarkonia pairs might be used as a new probe of the gluon GPDs:

- Unpolarized cross-section gets dominant contribution from GPD H_g, E_g
  * Sensitive to behaviour outside $x = \pm\xi$ line
  * Can vary independently rapidities of produced quarkonia to extract x, ξ dependence

- The cross-section is large enough for experimental studies, at least for charmonia
  * On par with $\gamma^{(*)} p \to \gamma\pi^0\, p$, $\gamma^{(*)} p \to \gamma\rho^0\, p$ suggested by other authors

Appendix: Evaluations of the coefficient function (II)



– Need to factorize out nonperturbative blocks which correspond to initial/final state hadrons

* Use light-cone gauge $n \cdot A = 0$
* For proton, at leading twist encode everything in terms of GPDs

$$\frac{1}{\bar{P}^+} \int \frac{dz}{2\pi} e^{ix\bar{P}^+} \cdot \left\langle P' \left| A^a_\mu \left(-\frac{z}{2}n\right) A^b_\nu \left(\frac{z}{2}n\right) \right| P \right\rangle \Big|_{A^+=0\ \mathrm{gauge}} =$$

$$= \frac{\delta^{ab}}{N_c^2 - 1} \left(\frac{-g^{\perp}_{\mu\nu} F^g(x,\xi,t) - \varepsilon^{\perp}_{\mu\nu} \tilde{F}^g(x,\xi,t)}{2\,(x-\xi+i0)(x+\xi-i0)} \right)$$

* Use $g^{\perp}_{\mu\nu}$, $\varepsilon^{\perp}_{\mu\nu}$ to take out contributions of leading twist GPDs

$$F^g(x,\xi,t) = \frac{1}{\bar{P}^+} \int \frac{dz}{2\pi} e^{ix\bar{P}^+} \left\langle P' \left| G^{+\mu\, a}\left(-\frac{z}{2}n\right) \mathcal{L}\left(-\frac{z}{2}, \frac{z}{2}\right) G^{+\, a}_{\mu}\left(\frac{z}{2}n\right) \right| P \right\rangle$$

$$\tilde{F}^g(x,\xi,t) = \frac{-i}{\bar{P}^+} \int \frac{dz}{2\pi} e^{ix\bar{P}^+} \left\langle P' \left| G^{+\mu\, a}\left(-\frac{z}{2}n\right) \mathcal{L}\left(-\frac{z}{2}, \frac{z}{2}\right) \tilde{G}^{+\, a}_{\mu}\left(\frac{z}{2}n\right) \right| P \right\rangle$$

Quarkonia structure: Two complementary approaches

NRQCD :Matrix elements (LDMEs)

$$\left\langle 0 \left| \hat{\mathcal{O}}_M \right| M(p) \right\rangle$$

–Ordering by $v \sim \alpha_s(m_Q)$

–Need a set of operator $\hat{\mathcal{O}}_M$ to describe each quarkonium.

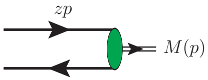
–Structure of $\hat{O}$ depends on quantum numbers of quarkonia.

. –Dominant contributions for J/ψ, η_c: $\mathcal{O}_{\eta_c}\left({}^1S_0^{[1]}\right)$, $\mathcal{O}_{J/\psi}\left({}^3S_1^{[1]}\right)$

LCDA :Description in terms of light-cone distribution amplitudes

$$\Phi_M(z) = \int \frac{d\eta}{2\pi} e^{izp^+\eta} \left\langle 0 \left| \bar{\psi}\left(-\frac{\eta}{2}\right) \Gamma_M \mathcal{L}\left(-\frac{\eta}{2}, \frac{\eta}{2}\right) \psi\left(\frac{\eta}{2}\right) \right| M(p) \right\rangle,$$

–z is the fraction of the quarkonium momentum carried by the c-quark:



–Straightforward extension from light quarks

–Structure of Γ_M depends on quantum numbers of quarkonium

–All DAs for a given quarkonium are ordered according to twist of operator

*Dominant (leading twist) DAs from $\Gamma_{\eta_c} = \gamma^+\gamma_5$, $\Gamma^{\perp}_{J/\psi} = -i\sigma^{+\mu}\varepsilon^*_{J/\psi,\,\mu}(p)$

Appendix: Relation of LCDA and NRQCD

* Dependence on z in LCDA is due to internal motion of quarks, formally $\mathcal{O}(\alpha_s(m_Q)) \ll 1$

$$\hat{\Phi}_\eta(z) \sim \hat{\Phi}_{J/\psi}(z) \sim \delta\left(z - \frac{1}{2}\right)$$

* Can relate DAs and NRQCD LDMEs:

$$\Phi_M(z) = \hat{\Phi}_M(z) \frac{\left\langle 0 \left| \hat{\mathcal{O}}_M \right| M(p) \right\rangle}{2\sqrt{m_Q}} \left(1 + \mathcal{O}\left(v^2\right)\right),$$

$$\left\langle 0 \left| \hat{\mathcal{O}}_M \right| M(p) \right\rangle \sim \int dz\, z^{n-1}\, \Phi_M(z)$$

See [PLB 647 (2007), 419; JHEP 06 (2014), 121; JHEP 12 (2017), 012]

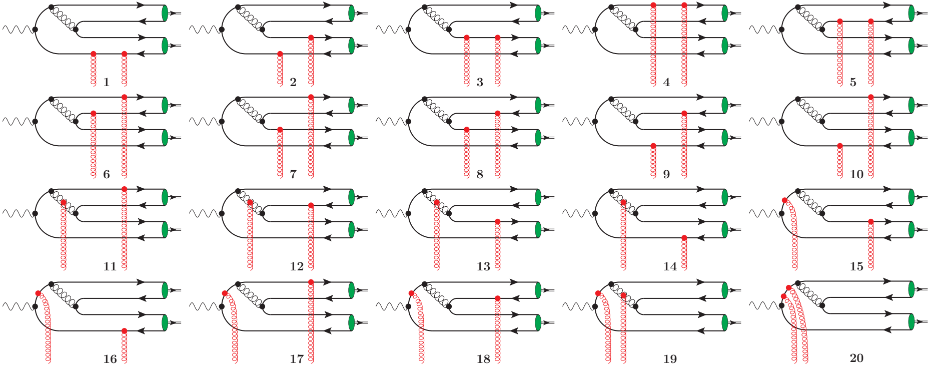
* May observe that for quarkonia with large $p^+ \to \infty$, neglecting m_Q(higher twist), obtain exactly the same operators
  ** LCDA: $\Gamma_{\eta_c} = \gamma^+\gamma_5$, $\Gamma^{\perp}_{J/\psi} = -i\sigma^{+\mu}\varepsilon^*_{J/\psi,\,\mu}(p)$
  ** NRQCD:

$$\hat{V}^{[1]}_{\eta_c} \sim \hat{p}\gamma_5 \sim \gamma^+\gamma_5, \quad \hat{V}^{[1]}_{J/\psi} \sim \hat{\varepsilon}^*_{J/\psi}(p)\hat{p}, \qquad p \cdot \varepsilon^*_{J/\psi} = 0$$

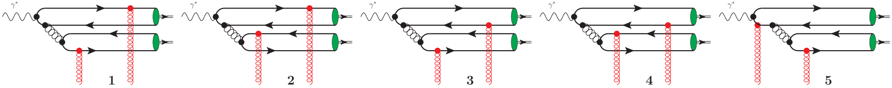
$\Rightarrow$The two approaches will give the same results

Appendix: Feynman diagrams for coef. function

– Example of diagrams with single quark loop:



– Example of diagrams with two quark loops:



* For color singlet contributions, *t*-channel gluons should be connected to *different* quark loops

– Use FeynCalc for evaluations (Dirac/color algebra)

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