TMD Factorization for NLP TMDs

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With L. Gamberg[2], ZB. Kang[3], DY. Shao[4], F. Zhao[3]: ArXiv 2211.13209

With L. Gamberg[2], ZB. Kang[3], DY. Shao[4], F. Zhao[3]: In preparation

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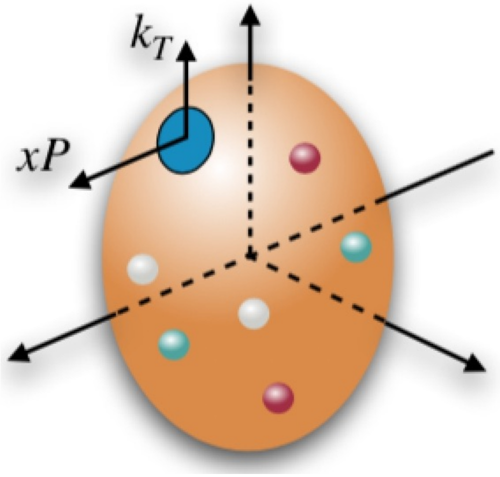
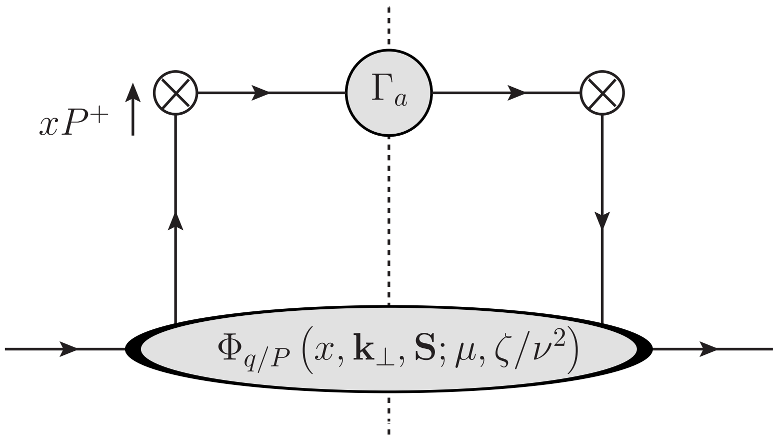
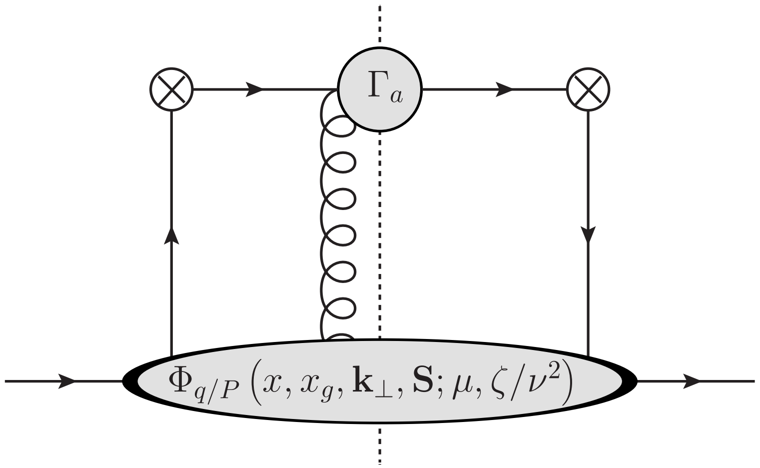
Why do we care about NLP?

LP factorization only allow imaging at trivial order in the power counting $\Lambda_{\text{QCD}} \lesssim q_\perp \ll Q \;\; q_\perp \sim M$





LP



NLP

Power corrections are a new frontier for increasing the perturbative precision

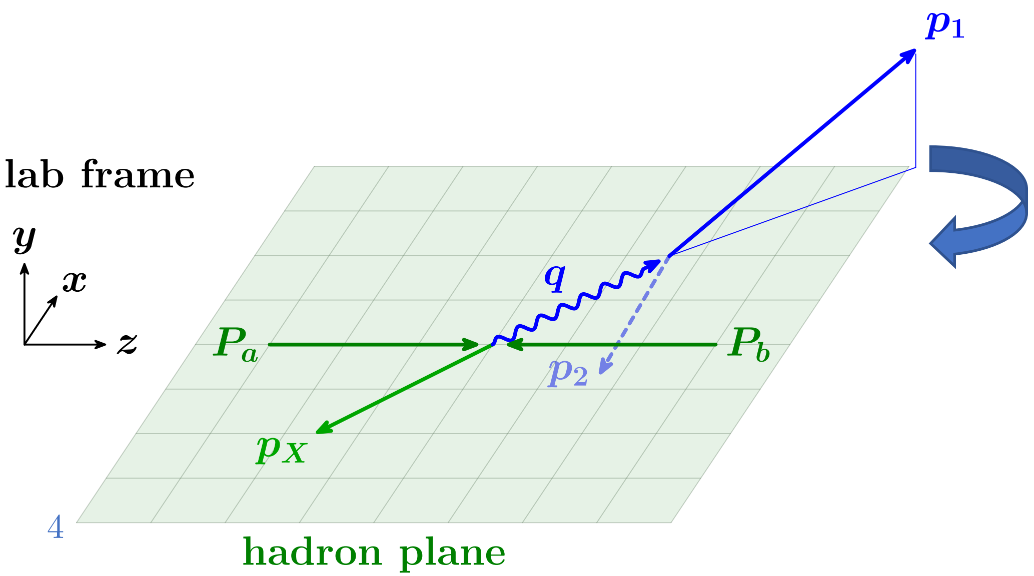
[1,2]

Accuracy	$H, \mathcal{J}$	$\Gamma_{\text{cusp}}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop			1-loop
NLL	Tree level	2-loop	1-loop	1-loop	2-loop
NLL′	1-loop	2-loop	1-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	2-loop	3-loop
NNLL′	2-loop	3-loop	2-loop	2-loop	3-loop
N^3LL	2-loop	4-loop	3-loop	3-loop	4-loop
$\text{N}^3\text{LL}'$	3-loop	4-loop	3-loop	3-loop	4-loop
N^4LL	3-loop	5-loop	4-loop	4-loop	5-loop
$\text{N}^4\text{LL}'$	4-loop	5-loop	4-loop	4-loop	5-loop

[3] Expect a renormalon turnaround, eventually.

[1]Duhr, Mistlberger, Vita 2022 [2]Moult, Zhu, Zhu 2022 [3]Beneke, Braun 2000

TMD angular correlations in Drell-Yan



[4]

Azimuthal angles of photon and lepton are correlated at NLP. For example, the Cahn effect[5]

$$\frac{d\sigma}{d^4q d\Omega} \sim \cos\left(\phi_q - \phi_p\right)$$

Both leptonic and hadronic tensors contain power corrections in the hadronic CM frame. The situation is simplified in the leptonic CM frame.

$$\frac{d\sigma}{d^4q\, d\Omega} = \frac{\alpha_{\rm em}^2}{4sQ^4}\left(L^0_{\mu\nu}W_0^{\mu\nu} + L^1_{\mu\nu}W_0^{\mu\nu} + L^0_{\mu\nu}W_1^{\mu\nu}\right) + \mathcal{O}\left(\lambda^2\right)$$

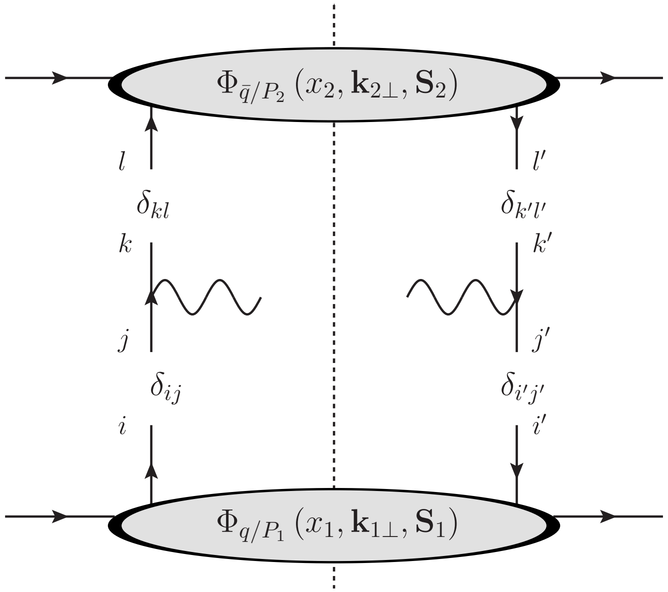
$W_0^{\mu\nu}$ contains leading power operators, while $W_1^{\mu\nu}$ contains sub-leading operators.

[4]Ebert, Michel, Stewart, Tackman 2022

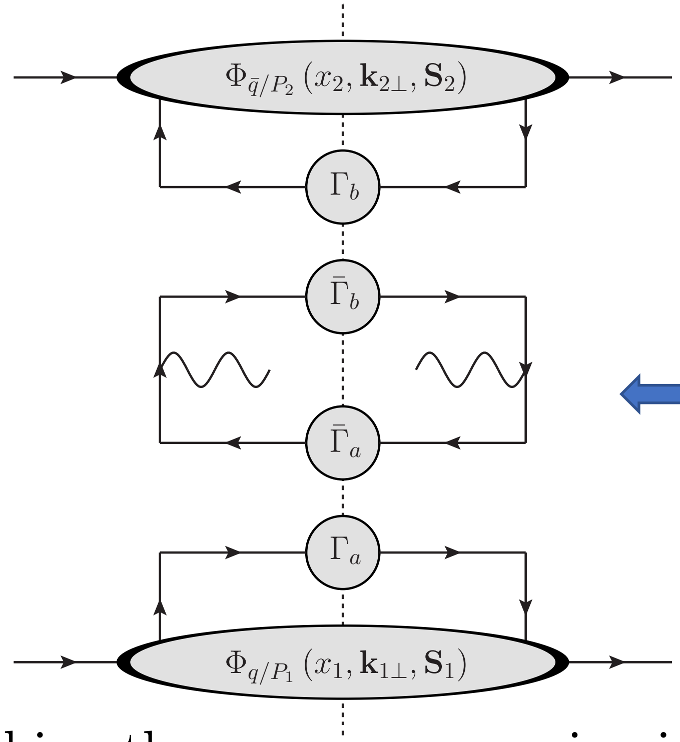
[5]Cahn 1978

Factorizing the cross section

The cross section is factorized through an OPE, or can be equivalently performing through a Fierz decomposition of the quark line[6-8]



$$\delta_{ij}\delta_{j'i'} = \sum_a \Gamma^a_{ii'}\bar{\Gamma}^a_{j'j}$$



Focus of this talk

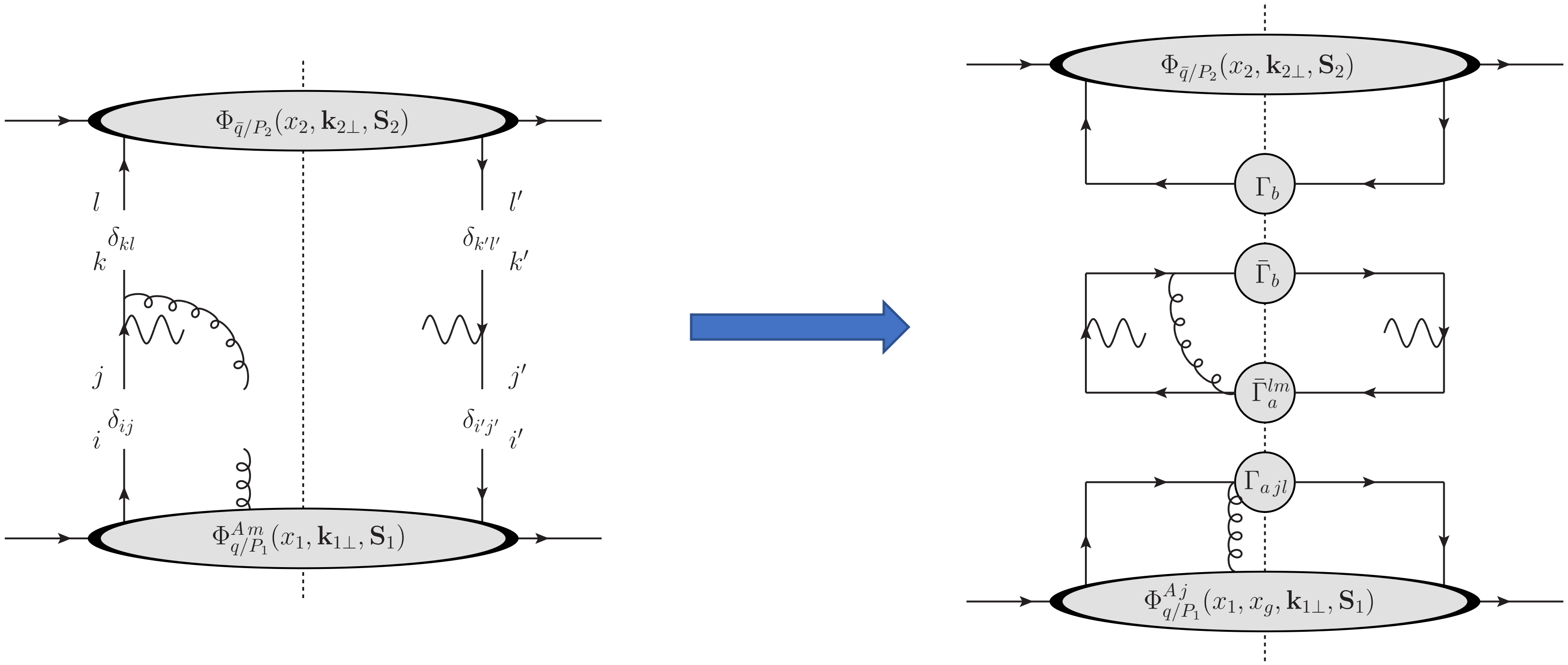
Equivalent to inserting the current operator and absorbing the power suppression into the collinear distributions

$$\Gamma_a \in \left\{ \underbrace{\frac{\not{n}}{4}, \frac{\not{n}\gamma^5}{4}, \frac{i}{4}\sigma^{i+}\gamma^5}_{\text{LP}}, \underbrace{\frac{1}{2}, \frac{\gamma^5}{2}, \frac{\gamma^i}{2}, \frac{\gamma^i\gamma^5}{2}, \frac{i}{2}\sigma^{ij}\gamma^5, \frac{i}{4}\sigma^{+-}\gamma^5}_{\text{NLP}} + \ldots.. \right\}$$

[6]Mulders, Tangerman 1995 [7]Bacchetta, Diehl, et al 2006 [8]Gamberg, Kang, Shao, Terry, Zhao 2022

Factorizing the cross section continued

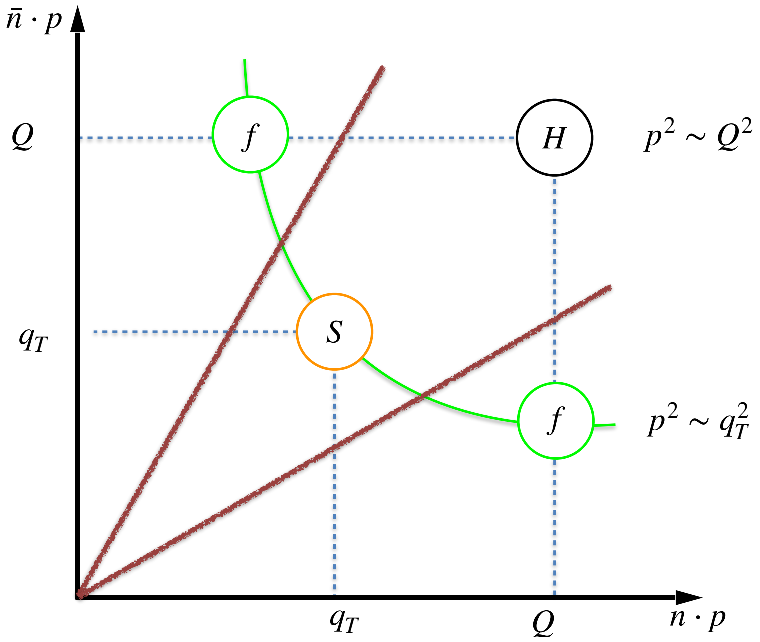
At NLP, we also need to consider the field strength tensor



[6]Mulders, Tangerman 1995 [7]Bacchetta, Diehl, et al 2006 [8]Gamberg, Kang, Shao, Terry, Zhao 2022

Renormalization group consistency

Hard and soft-collinear physics is done by introducing two scales: μ and ν



Consistent formalism should demonstrate invariance on the separation of physics at different scales

$$\frac{d\sigma}{d\mu} = 0 \qquad \frac{d\sigma}{d\nu} = 0$$

$$\Gamma^{\mu}_{H\,\mathrm{int}} + \Gamma^{\mu}_{\mathcal{S}\,\mathrm{int}} + \Gamma^{\mu}_{3\,\mathrm{int}} + \Gamma^{\mu}_{2\,\mathrm{mod}} = 0\,,$$
$$\Gamma^{\nu}_{\mathcal{S}\,\mathrm{int}} + \Gamma^{\nu}_{3\,\mathrm{int}} + \Gamma^{\nu}_{2\,\mathrm{mod}} = 0\,.$$

Sub-leading fields

Three possible sub-leading field configurations. They are related through the QCD EOM

$$\varphi^c(x) = \frac{\not{n}\bar{\not{n}}}{4}\psi^c(x) \qquad \chi^c(x) = \frac{\bar{\not{n}}\not{n}}{4}\psi^c(x) \qquad \varphi^c(x) = -\frac{\not{n}}{2}\frac{\not{D}_\perp}{n\cdot D}\chi^c(x)$$

Using properties of the Wilson lines, the relevant collinear functions are given by

$$\Phi^{\text{int}}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\langle P,\mathbf{S}\,|\bar{\chi}^c_{j'}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\varphi^c_j(\xi)|\,P,\mathbf{S}\rangle + \text{h.c.}\right]$$

$$\Phi^{\text{dyn}}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \frac{ig}{k^+}\int d\eta^- \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)$$
$$\times\left[\left\langle P,\mathbf{S}\left|\bar{\chi}^c(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\Gamma^a\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\eta)\,F^{i+}(\eta)\,\mathcal{U}^{\bar{n}}\left(\eta^-,\xi^-;\xi^+,\xi_\perp\right)\,\gamma_i\frac{\not{n}}{2}\chi^c(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$

$$\Phi^{\text{kin}}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\left\langle P,\mathbf{S}\left|\bar{\chi}^c_{j'}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\frac{i\partial^i_\perp}{in\cdot D}\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\frac{\bar{\not{n}}}{2}\gamma^\perp_i\chi^c_{\text{kin}\,\text{j}}(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$

All three distributions are not required to span the NLP cross section due to EOM

$$\Phi^{\text{int}}_{q/P\,jj'}\left(x,\mathbf{k}_\perp,\mathbf{S}\right) = \Phi^{\text{kin}}_{q/P\,jj'}\left(x,\mathbf{k}_\perp,\mathbf{S}\right) + \Phi^{\text{dyn}}_{q/P\,jj'}\left(x,\mathbf{k}_\perp,\mathbf{S}\right)$$

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$$\Phi^{\rm kin}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\left\langle P,\mathbf{S}\left|\bar{\chi}^c_{j'}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\frac{i\partial^i_\perp}{in\cdot D}\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\frac{\not{\bar{n}}}{2}\gamma^\perp_i\chi^c_{\rm kin\,j}(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$

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[8] Gamberg, Kang, Shao, Terry, Zhao 2022

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[9]

$$\Phi^{\rm dyn}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \frac{ig}{k^+}\int d\eta^- \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)$$
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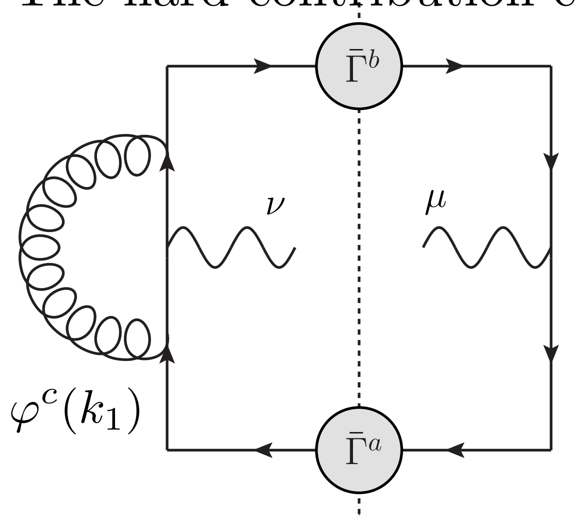
[9] Ebert, Gao, Stewart 2021

The hard region

The hard contribution can be obtained using the DY form factor $\gamma^\nu \to F^\nu(Q;\mu)$



$$
\begin{aligned}
F^\nu(Q;\mu) = {} & \gamma^\nu \left(1 + \frac{1}{2\epsilon} - L_Q\right) + \left(\frac{2}{\epsilon} - 4L_Q + 3\right) \frac{\not{n}\gamma^\nu \not{\bar{n}}}{4} \\
& + \left(-\frac{1}{\epsilon^2} - 2L_Q^2 + \frac{2}{\epsilon}L_Q - \frac{1}{\epsilon} + 2L_Q + \frac{\pi^2}{12} - 3\right) \frac{\not{\bar{n}}\gamma^\nu \not{n}}{4} + \left(2L_Q - \frac{1}{\epsilon} - 1\right) \frac{\not{\bar{n}}\bar{n}^\nu}{4} \\
& + \left(4L_Q - \frac{2}{\epsilon} - 3\right) \frac{\not{n}\bar{n}^\nu}{4} + \left(2L_Q - \frac{1}{\epsilon} - 1\right) \frac{\not{n}n^\nu}{Q^2} + \left(4L_Q - \frac{2}{\epsilon} - 3\right) \frac{\not{\bar{n}}n^\nu}{4},
\end{aligned}
$$

Double pole vanishes for one amplitude

The insertion of a sub-leading operator alters the divergences

$$
\hat{H}^{\mathrm{LP}}(Q;\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 4L_Q^2 + \frac{4L_Q}{\epsilon} + 6L_Q + \frac{7\pi^2}{6} - 8\right]
$$

$J^\nu = \bar{\chi}^{\bar{c}}(x)\gamma_\perp^\nu \chi^c(x) + \text{conjugate}$

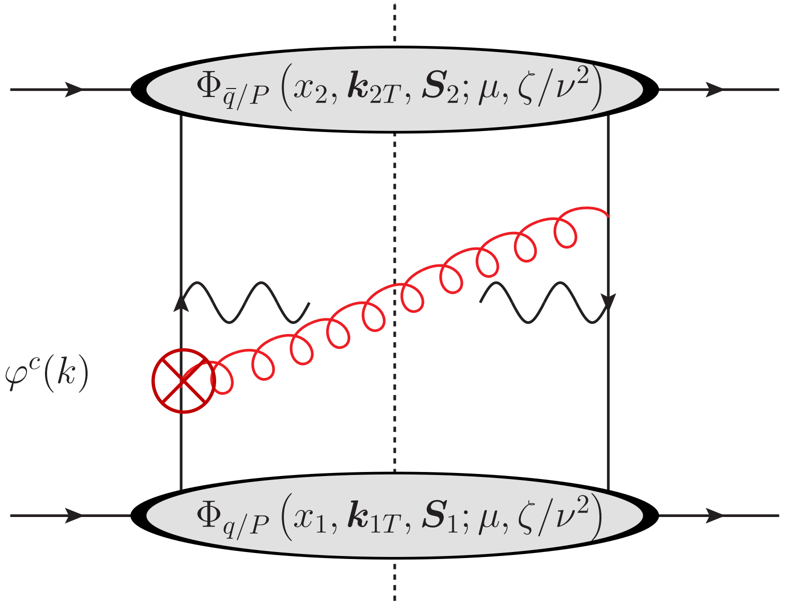
$$
\hat{H}^{\mathrm{NLP}}(Q;\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-\frac{1}{\epsilon^2} - \frac{2}{\epsilon} - 2L_Q^2 + \frac{2L_Q}{\epsilon} + 4L_Q + \frac{7\pi^2}{12} - 5\right]
$$

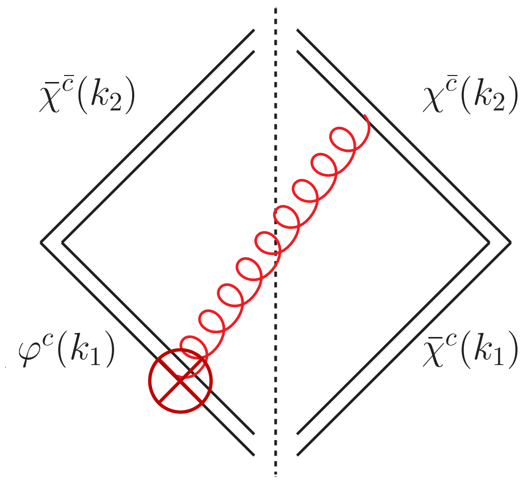
$J^\nu = \bar{\chi}^{\bar{c}}(x)\gamma^\nu \frac{\not{n}\not{\bar{n}}}{4} \varphi^c(x) + \text{perms}$

Double poles differ from those at LP. Issue enters due to current.

The soft region

The soft function is generated through the emissions of soft gluons in the partonic cross section





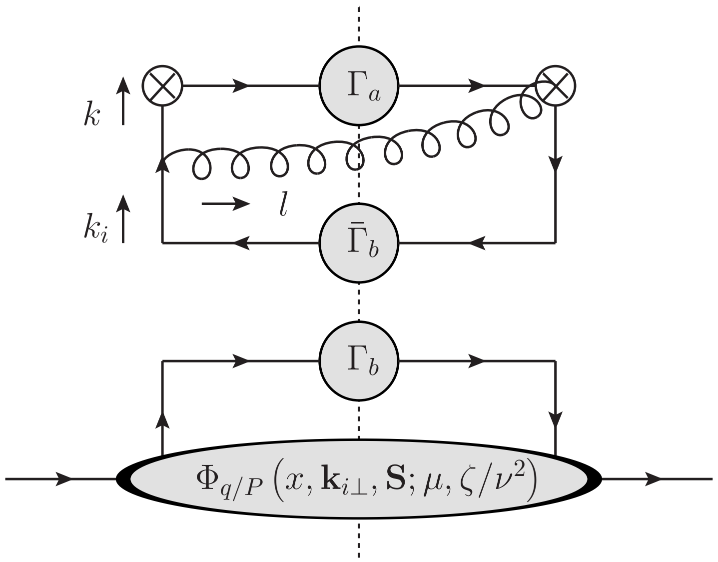
Soft emission from the sub-leading fields vanishes. NLO+NLP soft function is half the LP one

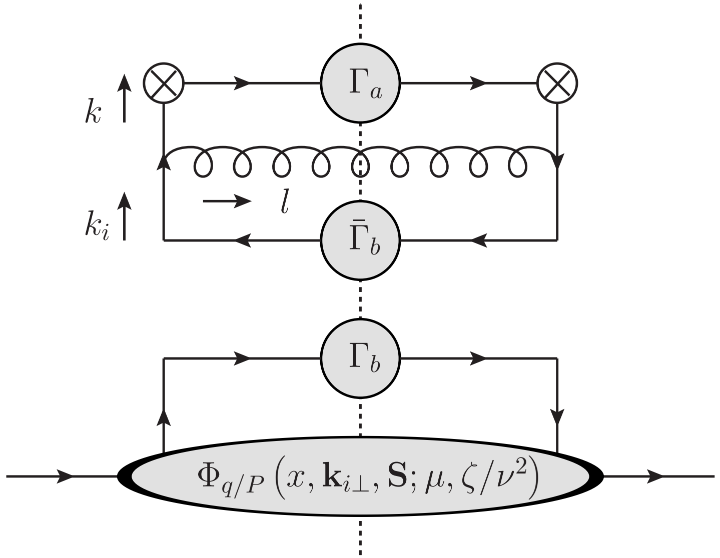
$$\Gamma^{\mu}_{\mathcal{S}\,\mathrm{int}} = \frac{1}{2}\Gamma^{\mu}_{\mathcal{S}\,\mathrm{LP}}\,, \qquad \Gamma^{\nu}_{\mathcal{S}\,\mathrm{int}} = \frac{1}{2}\Gamma^{\nu}_{\mathcal{S}\,\mathrm{LP}}$$

Double poles match what is required for RG consistency from the hard contribution.

The collinear region

Diagrams associated with the evolution of the collinear distributions





We break the integrands into the kinematic and the spinor pieces

$$\int d^2k_\perp\, e^{-i\mathbf{k}_\perp\cdot\mathbf{b}}\, \hat{\boldsymbol{\Phi}}^{[\Gamma^a]\,(1)}\left(x, \mathbf{k}_\perp, \mathbf{S}; \mu, \zeta/\nu^2\right) = \sum_b \int \frac{dx'}{x'} \int d^2k_{i\perp}\, e^{-i\mathbf{k}_{i\perp}\cdot\mathbf{b}}\, \boldsymbol{\Phi}^{[\Gamma^b]}\left(x', \mathbf{k}^i_\perp, \mathbf{S}\right)$$

$$\times \int d^2l_\perp\, e^{i\mathbf{l}_\perp\cdot\mathbf{b}} \left(\mathrm{I}_{\alpha\beta}\, \mathrm{Tr}\left[\bar{\Gamma}_b \gamma^\mu \gamma^\alpha \Gamma_a \gamma^\beta \gamma_\mu\right] + \mathrm{II}_\alpha\, \mathrm{Tr}\left[\bar{\Gamma}_b \not{l} \gamma^\alpha \Gamma_a\right]\right) ,$$

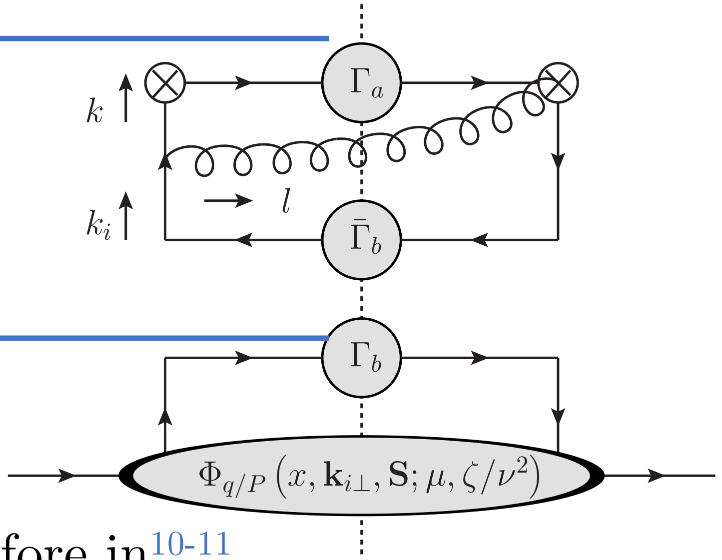
Kinematic only: Holds to all powers

Spinor only: Controls power of dist.

Anomalous dimension matrices

Evolution equations naturally enter as matrices due to mixing

$$\frac{\partial}{\partial \ln \mu}\begin{bmatrix}\Phi^{[\not n]}\\ \Phi^{[\not n\gamma^5]}\\ \Phi^{[i\sigma^{i+}\gamma^5]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{[\gamma^i]}\\ \Phi^{[\gamma^i\gamma^5]}\\ \Phi^{[i\sigma^{ij}\gamma^5]}\\ \Phi^{[i\sigma^{+-}\gamma^5]}\end{bmatrix} = \mathbf{\Gamma}^{\mu}\begin{bmatrix}\Phi^{[\not n]}\\ \Phi^{[\not n\gamma^5]}\\ \Phi^{[i\sigma^{l+}\gamma^5]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{[\gamma^l]}\\ \Phi^{[\gamma^l\gamma^5]}\\ \Phi^{[i\sigma^{lm}\gamma^5]}\\ \Phi^{[i\sigma^{+-}\gamma^5]}\end{bmatrix} \qquad \frac{\partial}{\partial \ln \nu}\begin{bmatrix}\Phi^{[\not n]}\\ \Phi^{[\not n\gamma^5]}\\ \Phi^{[i\sigma^{i+}\gamma^5]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{[\gamma^i]}\\ \Phi^{[\gamma^i\gamma^5]}\\ \Phi^{[i\sigma^{ij}\gamma^5]}\\ \Phi^{[i\sigma^{+-}\gamma^5]}\end{bmatrix} = \mathbf{\Gamma}^{\nu}\begin{bmatrix}\Phi^{[\not n]}\\ \Phi^{[\not n\gamma^5]}\\ \Phi^{[i\sigma^{l+}\gamma^5]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{[\gamma^l]}\\ \Phi^{[\gamma^l\gamma^5]}\\ \Phi^{[i\sigma^{lm}\gamma^5]}\\ \Phi^{[i\sigma^{+-}\gamma^5]}\end{bmatrix}.$$



We find operator mixing in the Collins-Soper equation. Seen before in[10-11]

$$\mathbf{\Gamma}^{\mu} = \begin{bmatrix}
\Gamma_2^{\mu} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \Gamma_2^{\mu} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \Gamma_2^{\mu}\delta_l^i & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \Gamma_3^{\mu} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \Gamma_3^{\mu} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \Gamma_3^{\mu}\delta_l^i & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^{\mu}\delta_l^i & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}\Gamma_3^{\mu}\left(\delta_l^i\delta_m^j - \delta_l^j\delta_m^i\right) & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^{\mu}
\end{bmatrix}$$

$$\mathbf{\Gamma}^{\nu} = \frac{\alpha_s C_F}{2\pi}\begin{bmatrix}
2L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 2L & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 2L\delta_l^i & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & L & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \frac{2ib_l}{xP^+}\frac{\partial L}{\partial b^2} & 0 & L & 0 & 0 & 0 & 0\\
\frac{2ib^i}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & L\delta_l^i & 0 & 0 & 0\\
0 & \frac{2ib^i}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & L\delta_l^i & 0 & 0\\
0 & 0 & \frac{i}{xP^+}\frac{\partial L}{\partial b^2}\left(b^j\delta_l^i - b^i\delta_l^j\right) & 0 & 0 & 0 & 0 & L\left(\delta_l^i\delta_m^j - \delta_l^j\delta_m^i\right) & 0\\
0 & 0 & \frac{2ib_l}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & 0 & L
\end{bmatrix}$$

LP to LP — LP to NLP — NLP to NLP

[10]Chen, Ma 2016 — [11]Rodini, Vladimirov 2022

Differences from LP TMDs

NLP anomalous divergences differ from LP ones consistent with results of[10,12]

$$
\boldsymbol{\Gamma}^{\mu} = \begin{bmatrix}
\Gamma_2^{\mu} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \Gamma_2^{\mu} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \Gamma_2^{\mu}\delta_l^i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \Gamma_3^{\mu} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Gamma_3^{\mu} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \Gamma_3^{\mu}\delta_l^i & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^{\mu}\delta_l^i & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}\Gamma_3^{\mu}\left(\delta_l^i\delta_m^j - \delta_l^j\delta_m^i\right) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^{\mu}
\end{bmatrix}.
$$

$$
\boldsymbol{\Gamma}^{\nu} = \frac{\alpha_s C_F}{2\pi}\begin{bmatrix}
2L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2L & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2L\delta_l^i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & L & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{2ib_l}{xP^+}\frac{\partial L}{\partial b^2} & 0 & L & 0 & 0 & 0 & 0 \\
\frac{2ib^i}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & L\delta_l^i & 0 & 0 & 0 \\
0 & \frac{2ib^i}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & L\delta_l^i & 0 & 0 \\
0 & 0 & \frac{i}{xP^+}\frac{\partial L}{\partial b^2}\left(b^j\delta_l^i - b^i\delta_l^j\right) & 0 & 0 & 0 & 0 & L\left(\delta_l^i\delta_m^j - \delta_l^j\delta_m^i\right) & 0 \\
0 & 0 & \frac{2ib_l}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & 0 & L
\end{bmatrix}
$$

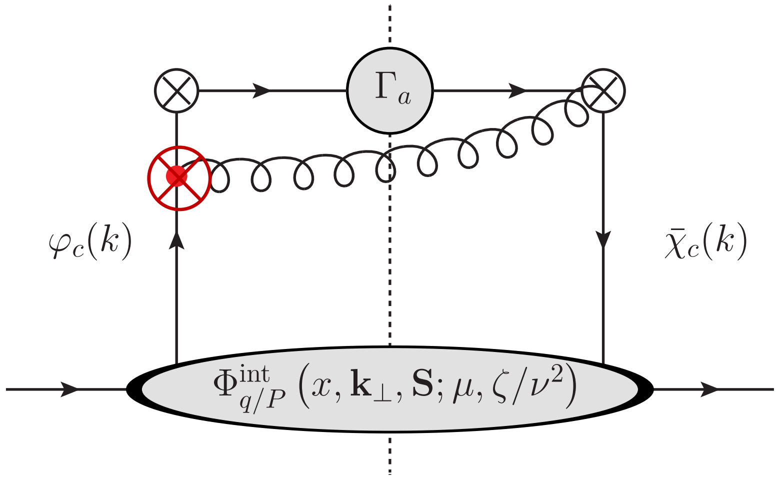
$$
\Gamma_2^{\mu} = \frac{\alpha_s C_F}{2\pi}\left[2\ln\left(\frac{\nu^2}{\zeta}\right) + 3\right] \quad \Gamma_3^{\mu} = \frac{\alpha_s C_F}{2\pi}\left[\ln\left(\frac{\nu^2}{\zeta}\right) + 1\right] \quad \Gamma_2^{\nu} = \frac{\alpha_s C_F}{\pi}\ln\left(\frac{\mu^2}{\mu_b^2}\right) \quad \Gamma_3^{\nu} = \frac{\alpha_s C_F}{2\pi}\ln\left(\frac{\mu^2}{\mu_b^2}\right)
$$

[10]Chen, Ma 2016

[12]Bacchetta, Boer, Diehl, Mulders 2008

Differences from LP TMDs

Study the interaction of the sub-leading fields with the Wilson lines



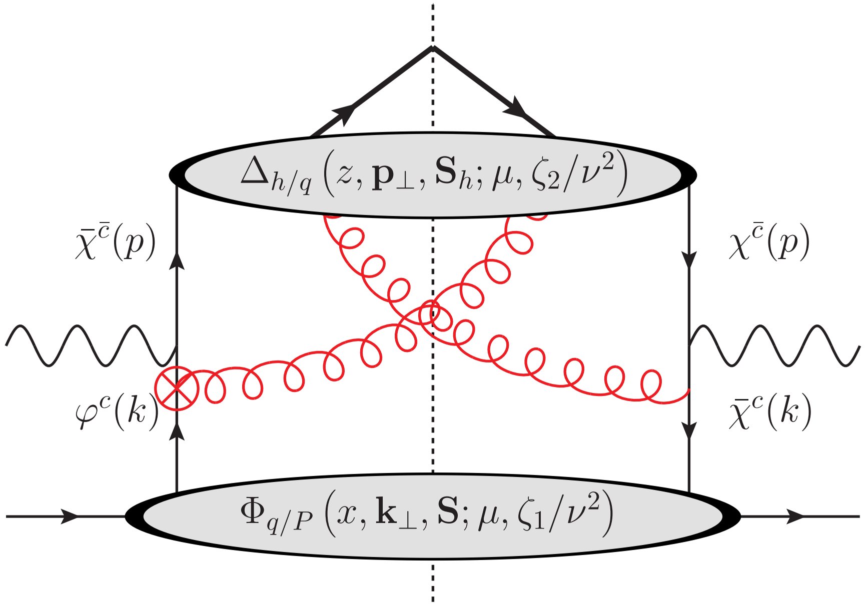
$$\frac{\not{n}}{2}\varphi_c(k) = 0$$

Can show that these interactions vanish trivially

[8]Gamberg, Kang, Shao, Terry, Zhao 2022

Modified Universality

The presence of the sub-leading fields alters the evolution in the LP distributions. The modification is universal for SIDIS, Drell-Yan, DIA.



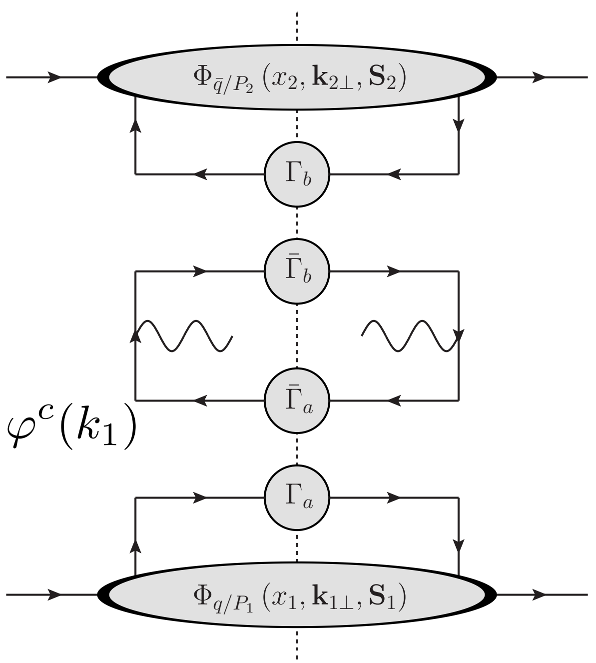
Wilson line interaction of the LP distribution is also altered.

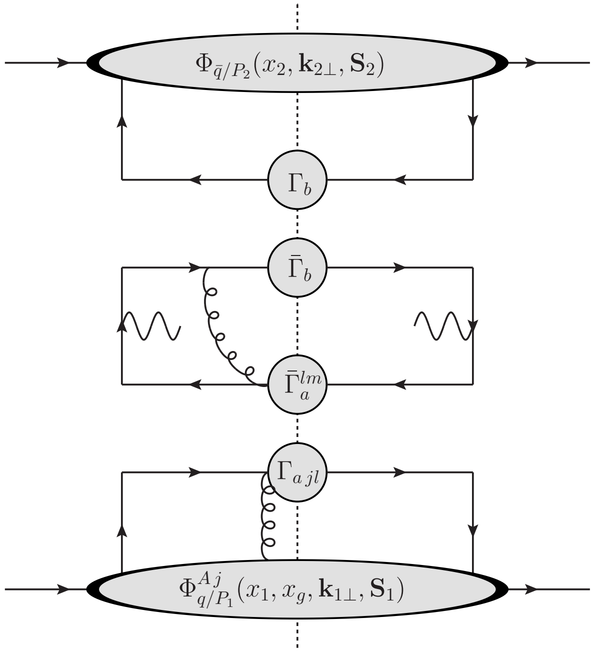
$$\Gamma_2^\mu = \frac{\alpha_s C_F}{2\pi}\left[2\ln\left(\frac{\nu^2}{\zeta}\right) + 3\right] \qquad \Gamma_2^\nu = \frac{\alpha_s C_F}{\pi}\ln\left(\frac{\mu^2}{\mu_b^2}\right)$$

$$\Gamma_{2\,\mathrm{mod}}^\mu = \frac{\alpha_s C_F}{2\pi}\left[\ln\left(\frac{\nu^2}{\zeta}\right) + 3\right] \qquad \Gamma_{2\,\mathrm{mod}}^\nu = \frac{\alpha_s C_F}{2\pi}\ln\left(\frac{\mu^2}{\mu_b^2}\right)$$

[8]Gamberg, Kang, Shao, Terry, Zhao 2022

Cross section at tree level int-dyn basis





Contribution from the leptonic tensor

$$d\sigma \sim W_0^{\mu\nu} L^1_{\mu\nu}$$

$$J^\mu = \bar{\chi}^{\bar{c}}(x)\gamma_\perp^\mu \chi^c(x) + \text{conjugate}$$

From the intrinsic TMDs

$$J^\mu = \bar{\chi}^{\bar{c}}(x)\gamma^\mu \varphi^c(x) + \text{perms}$$

From the dynamic TMDs

$$J^\mu = \bar{\chi}^{\bar{c}}(x)\gamma^\mu \frac{ig}{n \cdot D} F^{i+} \gamma_i \frac{\not{n}}{2} \chi^c(x) + \text{perms}$$

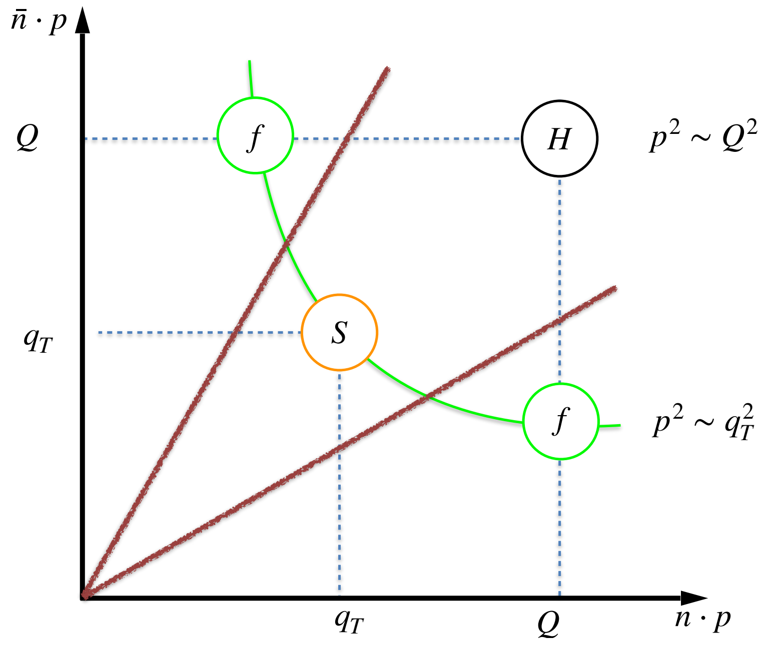
For the Cahn effect

$$\frac{d\sigma}{d^4q\, d\Omega} = -\frac{\alpha_{\text{em}}^2}{4sQ^2} \cos\phi \sin 2\theta \Bigg[-\frac{q_\perp}{Q} \mathcal{C}^{\text{DY}}[f\, f] + \mathcal{C}^{\text{DY}} \left[\left(x_1 \frac{\mathbf{k}_{1\perp} \cdot \hat{x}}{Q} f^\perp \right) f - f \left(x_2 \frac{\mathbf{k}_{2\perp} \cdot \hat{x}}{Q} f^\perp \right) \right]$$

$$+ \int \frac{dx_g}{x_g} \mathcal{C}^{\text{DY}}_{\text{dyn}\,1} \left[\left(x_1 \frac{\mathbf{k}_{1\perp} \cdot \hat{x}}{Q} \tilde{f}^\perp \right) f \right] - \int \frac{dx_g}{x_g} \mathcal{C}^{\text{DY}}_{\text{dyn}\,2} \left[f \left(x_2 \frac{\mathbf{k}_{2\perp} \cdot \hat{x}}{Q} \tilde{f}^\perp \right) \right] \Bigg]$$

Renormalization group consistency

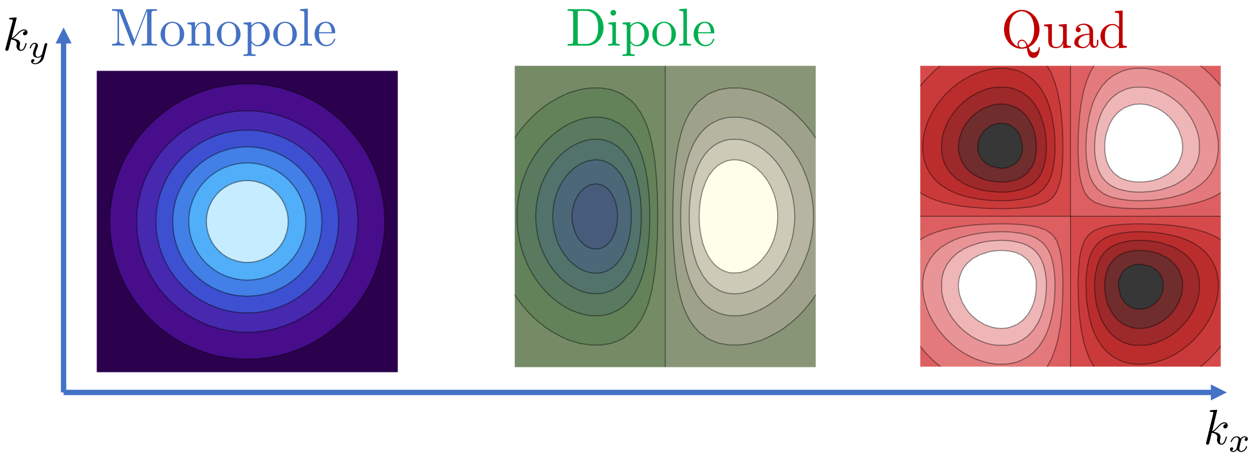
Cross section should be invariant under changes in the rapidity and renormalization scales



We have explicitly verified that the consistency of the anomalous dimensions

$$\Gamma^{\mu}_{H\,\mathrm{int}} + \Gamma^{\mu}_{\mathcal{S}\,\mathrm{int}} + \Gamma^{\mu}_{3\,\mathrm{int}} + \Gamma^{\mu}_{2\,\mathrm{mod}} = 0\,,$$

$$\Gamma^{\nu}_{\mathcal{S}\,\mathrm{int}} + \Gamma^{\nu}_{3\,\mathrm{int}} + \Gamma^{\nu}_{2\,\mathrm{mod}} = 0\,.$$



The LP PDFs

	U	L	T
U	f_1		
L		g_1	
T			h_1

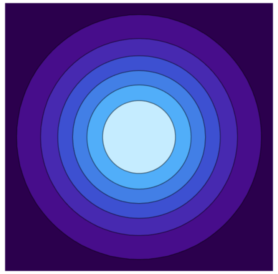
The LP TMDs

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

NLP PDFs

	U	L	T
U			$h_{1\,F}^\perp$
L			$h_{1L\,F}^\perp$ $h_{1L\,D}^\perp$
T	$f_{1T\,F}^\perp$	$g_{1T\,F}$ $g_{1T\,D}$	

Matching the LP TMDs



Monopole LP TMDs to LP PDFs

The LP TMDs

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

The LP PDFs

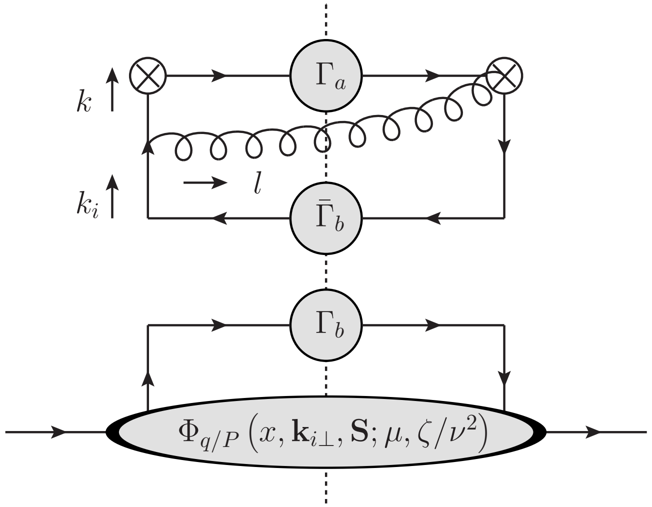
	U	L	T
U	f_1		
L		g_1	
T			h_1

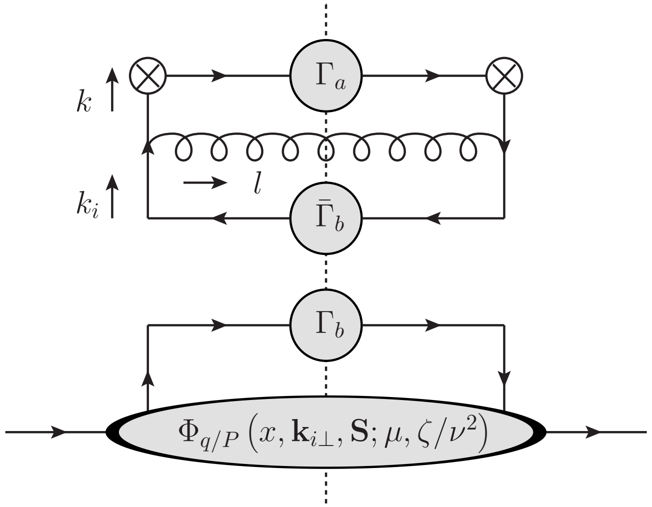
NLP PDFs

	U	L	T
U			$h_{1\ F}^\perp$
L			$h_{1L\ F}^\perp$ $h_{1L\ D}^\perp$
T	$f_{1T\ F}^\perp$	$g_{1T\ F}$ $g_{1T\ D}$	

Matching coefficients for NLP monopole TMDs

We can obtain the matching coefficients from the finite parts of the loop integrals from before



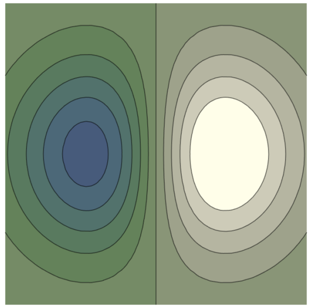


Example matching of the chiral odd TMD. We can calculate all monopole matching coefficients

$$C^{ee}_{q/q}\left(\hat{x}, b; \mu, \zeta/\nu^2\right) = \delta(1-\hat{x}) + \frac{\alpha_s C_F}{2\pi}\left[1 - L P^{ee}_{q/q}(\hat{x}) + \frac{1}{2} L\delta\left(1-\hat{x}\right)\left(L_\nu + 1\right)\right]$$

[13]Gamberg, Kang, Shao, Terry, Zhao In preparation

Matching the LP TMDs continued



The LP PDFs

	U	L	T
U	f_1		
L		g_1	
T			h_1

The LP TMDs

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

NLP PDFs

	U	L	T
U			$h_{1\,F}^\perp$
L			$h_{1L\,F}^\perp$ $h_{1L\,D}^\perp$
T	$f_{1T\,F}^\perp$	$g_{1T\,F}$ $g_{1T\,D}$	

Dipole LP TMDs to NLP PDFs

Matching the dipole distributions at all twists

Dipole TMDs enter the cross section as, for example, the Sivers function

$$\frac{d\Delta\sigma}{d\mathcal{PS}} = \sigma_0 H\left(Q;\mu\right) \int d^2k_\perp d^2p_\perp \delta^2\left(\mathbf{q}_\perp - \mathbf{k}_\perp - \mathbf{p}_\perp\right) \frac{\hat{\mathbf{q}}_\perp \cdot \mathbf{k}_\perp}{M} f_{1T\,q/P}^{\perp}\left(x, k_\perp;\mu,\zeta/\nu^2\right) D_{h/q}\left(z, p_\perp;\mu,\zeta/\nu^2\right)$$

$$T_{F\,q/P}(x,x;\mu) = \int d^2k_\perp \frac{k_\perp^2}{M} f_{1T\,q/P}^{\perp}(x, k_\perp;\mu,\zeta/\nu^2)$$

Expand in $k_\perp$

LO matching for $k_\perp$ odd TMDs can be demonstrated trivially[13-16]

$$k_\perp^j \Phi_{q/P}^{\alpha\alpha'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\Big[\Big\langle P,\mathbf{S}\Big|\bar{\psi}^{\alpha'}(0)\,\mathcal{U}_\llcorner^{\bar{n}}(0)\,\mathcal{U}_\llcorner^{\bar{n}\,\dagger}(\xi)\, iD_\perp^j(\xi)\psi^\alpha(\xi)\Big|P,\mathbf{S}\Big\rangle$$

$$+\, ig \int d\eta^- \Big\langle P,\mathbf{S}\Big|\bar{\psi}^{\alpha'}(0)\,\mathcal{U}_\llcorner^{\bar{n}}(0)\,\mathcal{U}_\llcorner^{\bar{n}\,\dagger}(\xi)\,\mathcal{U}_\llcorner^{\bar{n}\,\dagger}(\eta)\, F^{+j}(\eta)\,\mathcal{U}^{\bar{n}}\left(\eta^-,\xi^-;\xi^+,\xi_\perp\right)\,\psi(\xi)\Big|P,\mathbf{S}\Big\rangle\Big]\,.$$

Integrating in the transverse momentum, you obtain the collinear distributions

$$\Phi_{q/P}^{\alpha\alpha'\,j}(x,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\,\delta^2\left(\xi_\perp\right)\Big[\Big\langle P,\mathbf{S}\Big|\bar{\psi}^{\alpha'}(0)\,\mathcal{U}^{\bar{n}}\left(0^-,\xi^-\right)\, iD_\perp^j(\xi)\psi^\alpha(\xi)\Big|P,\mathbf{S}\Big\rangle$$

$$+\, ig\int d\eta^- \Big\langle P,\mathbf{S}\Big|\bar{\psi}^{\alpha'}(0)\,\mathcal{U}^{\bar{n}}\left(0^-,\eta^-\right)\, F^{+j}(\eta)\,\mathcal{U}^{\bar{n}}\left(\eta^-,\xi^-\right)\,\psi(\xi)\Big|P,\mathbf{S}\Big\rangle\Big]$$

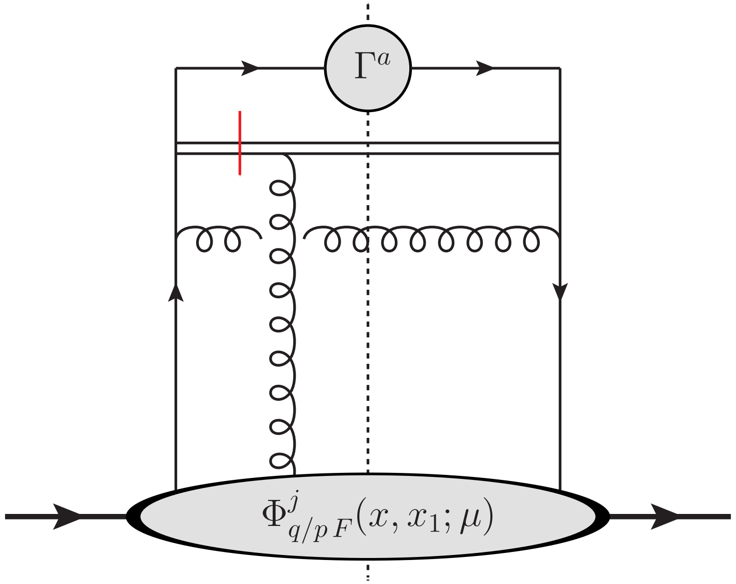
T-even

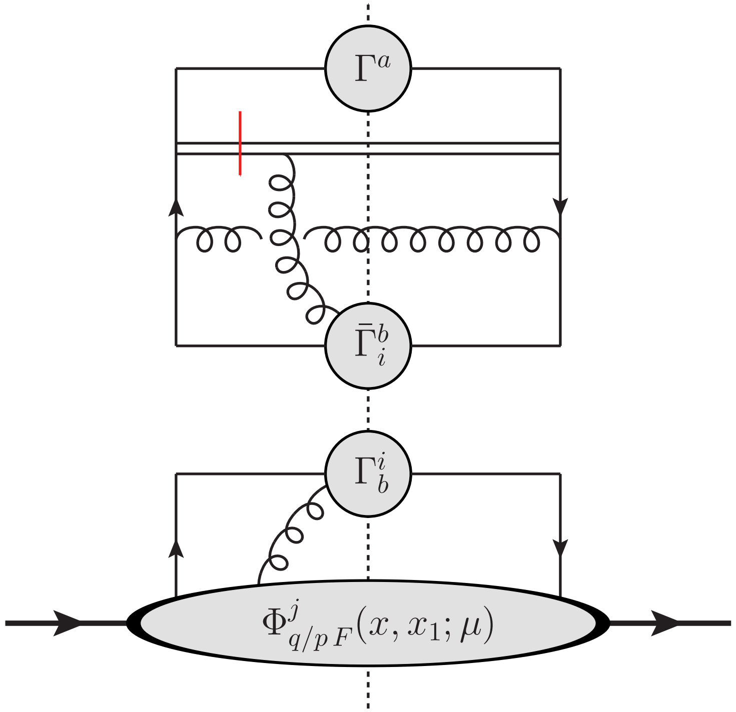
T-odd

[13]Gamberg, et al: In prep [14]Ji, Qiu, Vegelsang, Yuan 2006 [15]Yuan, Zhou 2008 [16]Liang, Yuan, Zhou 2008

Matching for the dipole distributions at one loop

Example factorization for the dipole distributions. Calculation ongoing, 16 graphs in total





Conclusion

We have demonstrated that the hard and soft anomalous dimensions associated with the intrinsic sub-leading distributions is modified.

We have calculated the anomalous dimension of all two parton NLP TMDs and demonstrated that they are modified relative to the LP ones.

We have clarified why the two parton NLP TMDs differ from the LP ones by considering the properties of the sub-leading fields.

We demonstrated renormalization group consistency for these terms in the cross section.

We have calculated the one loop matching coefficients for the two parton NLP TMDs.

We have demonstrated that the $k_\perp$-odd $\mathrm{N^nLP}$ match onto collinear distributions of power $\mathrm{N^{n+1}LP}$ at tree level.

Our ongoing research is to calculate matching of the $k_\perp$-odd, T-odd TMDs at one loop.