3D Imaging of the Nucleon from Lattice QCD

DIS2023: XXX International Workshop on Deep-Inelastic Scattering and Related Subjects

Michigan State University, East Lansing, MI, March 27-31, 2023

YONG ZHAO

MARCH 30, 2023

Argonne NATIONAL LABORATORY

Outline

- **TMDs from experiments**

- **TMDs from large-momentum effective theory (LaMET)**

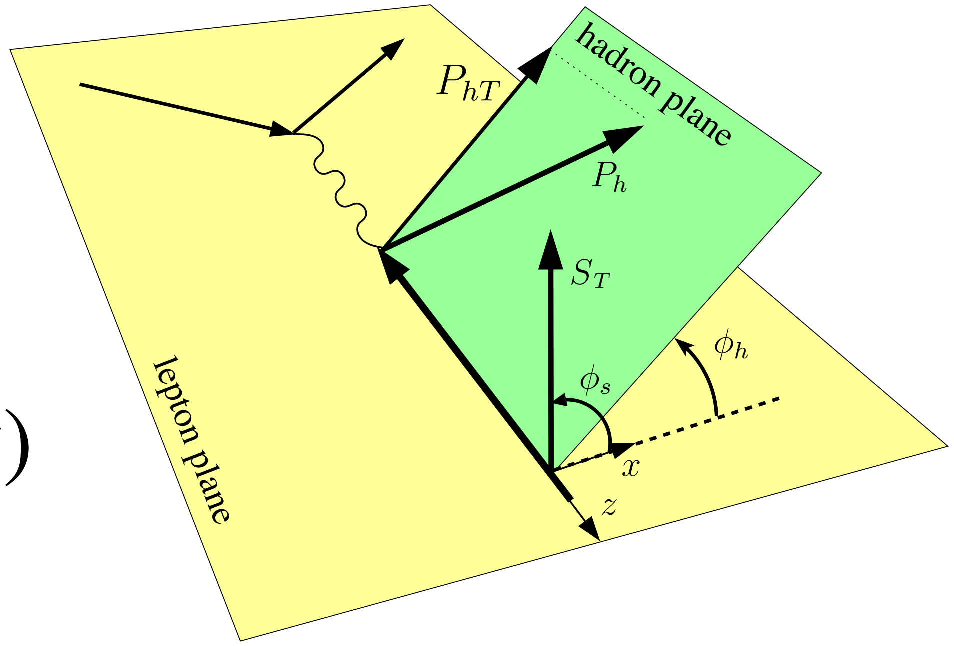
- **Results from lattice QCD**

TMDs from global analyses

e.g., semi-inclusive deep inelastic scattering: $l + p \longrightarrow l + h(P_h) + X$

$$\frac{d\sigma^W}{dx dy dz_h d^2\mathbf{P}_{hT}} \sim \int d^2\mathbf{b}_T \, e^{i\mathbf{b}_T\cdot\mathbf{P}_{hT}/z}$$

$$\times f_{i/p}(x, \mathbf{b}_T, Q, Q^2) \, D_{h/i}(z_h, \mathbf{b}_T, Q, Q^2)$$



Kang, Prokudin, Sun and Yuan, PRD **93** (2016)

$$f_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = f^{\text{pert}}_{i/p}(x, b^*(b_T), \mu, \zeta)$$

$$\times \left(\frac{\zeta}{Q_0^2}\right)^{g_K(b_T)/2} \quad f^{\text{NP}}_{i/p}(x, b_T)$$

$g_K(b_T)/2$ ⟶ **Collins-Soper kernel (NP part)**

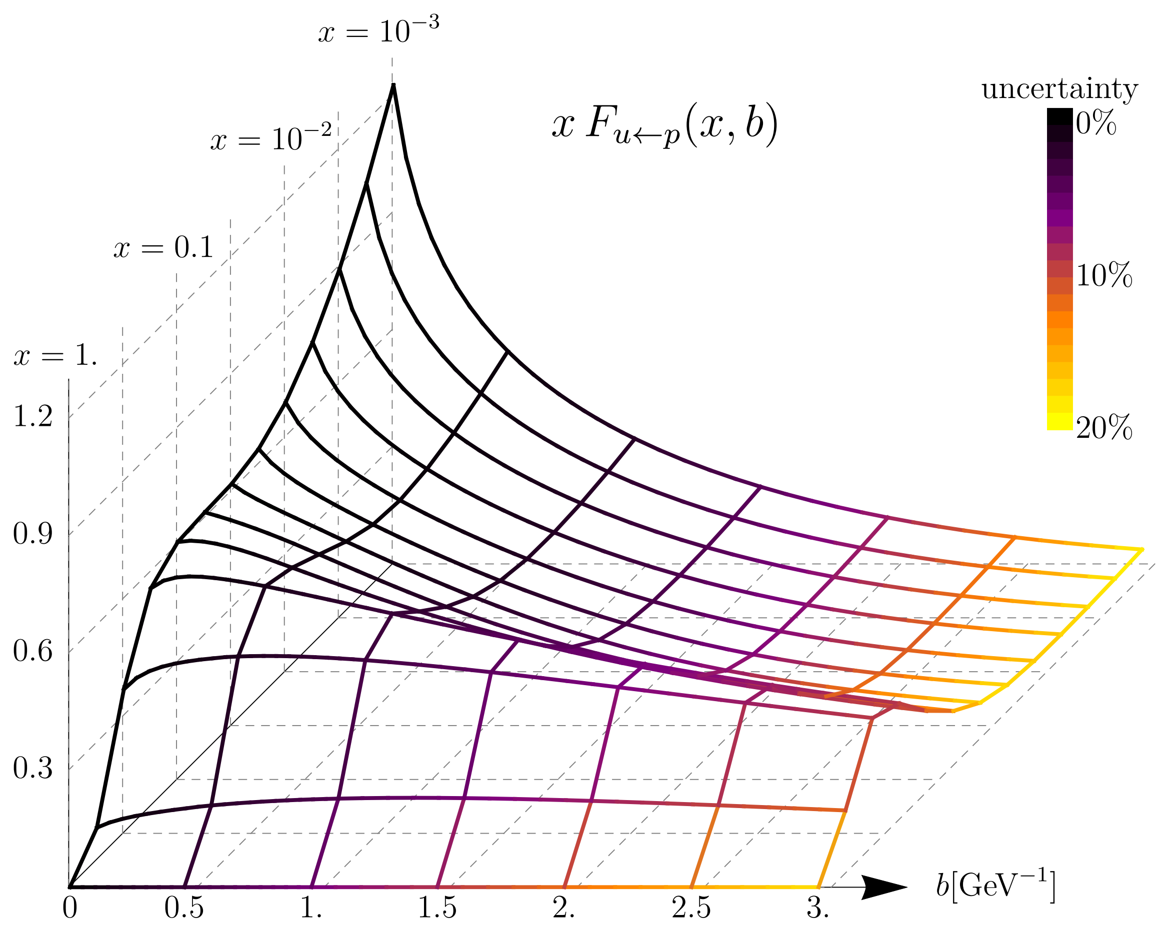
$f^{\text{NP}}_{i/p}(x, b_T)$ ⟶ **Intrinsic TMD**

$Q_0 \sim 1$ GeV

Non-perturbative when $b_T \sim 1/\Lambda_{\text{QCD}}$!

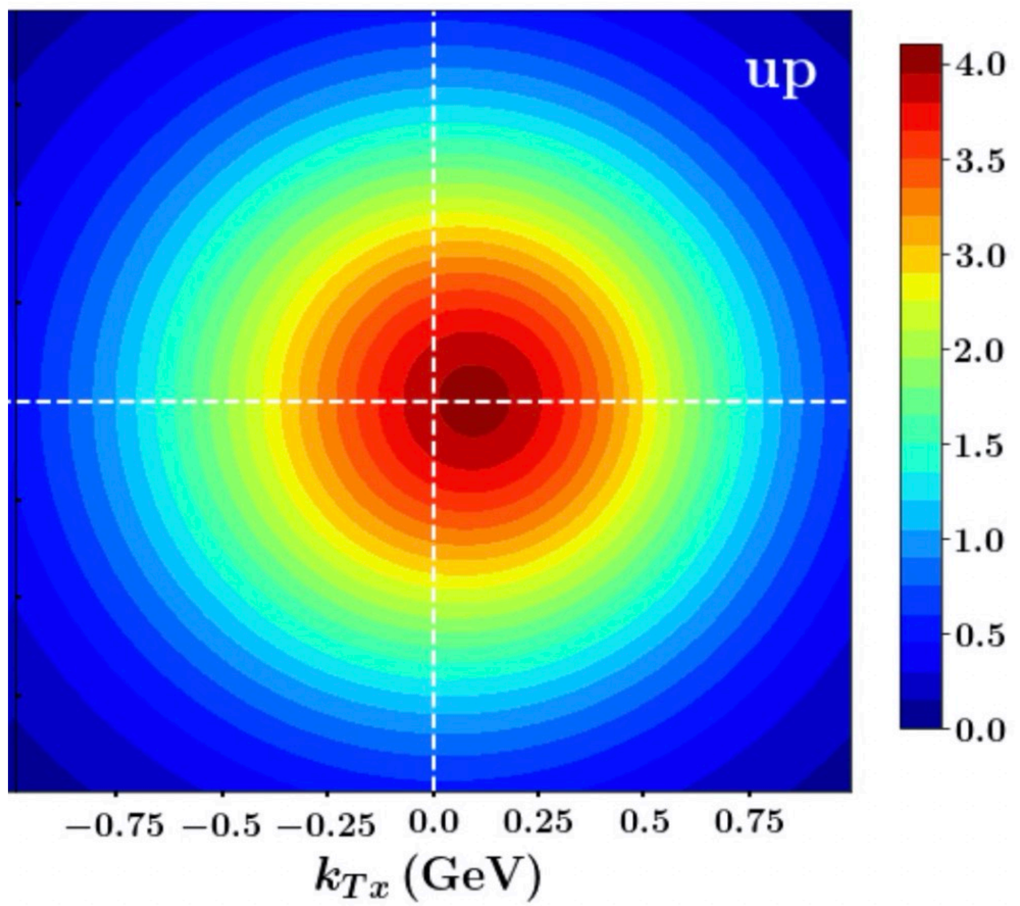
TMDs from global analyses

Unpolarized quark TMD



Scimemi and Vladimirov, JHEP **06** (2020).

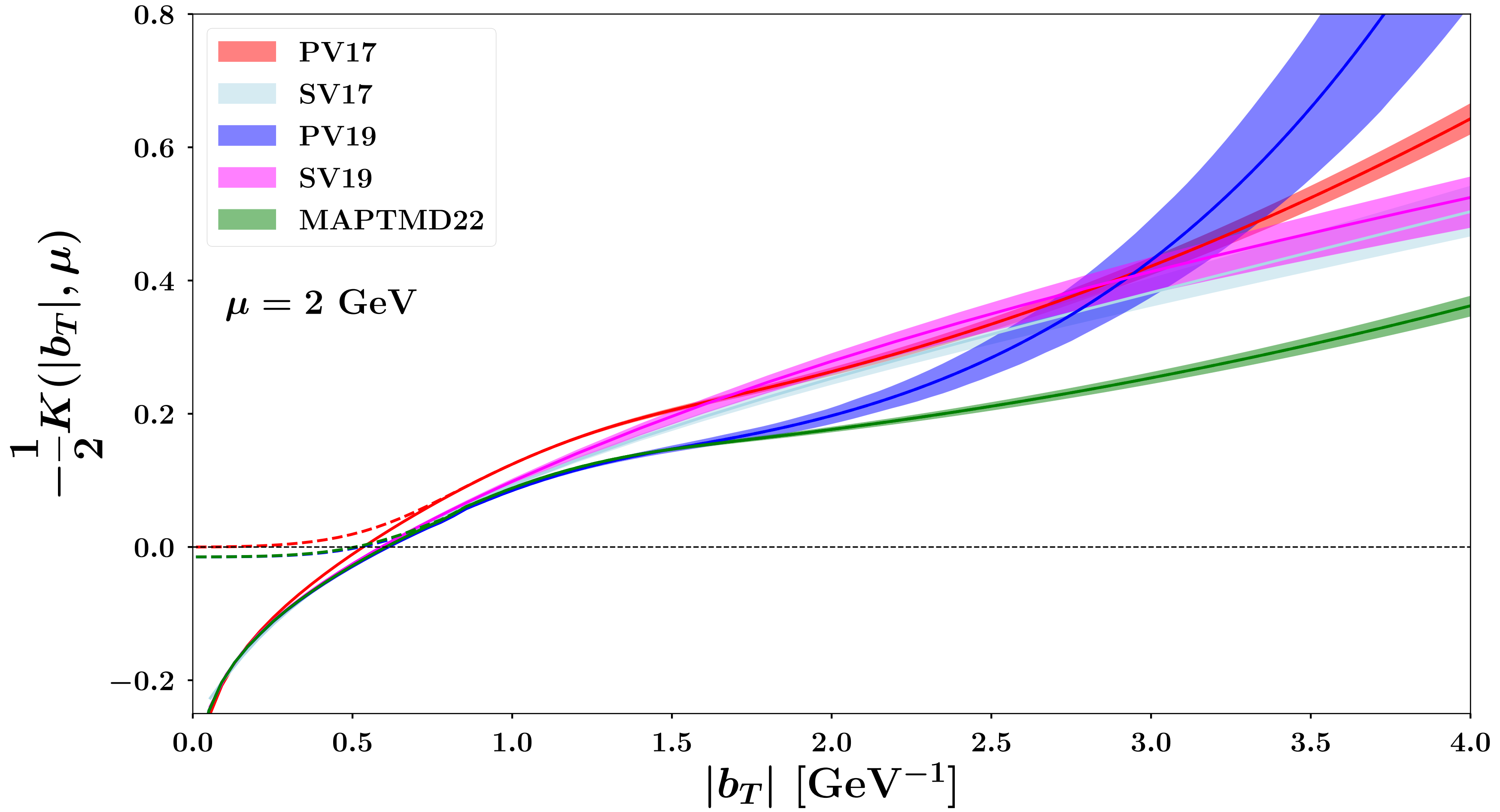
Quark Sivers function



Cammarota, Gamberg, Kang et al. (JAM Collaboration), PRD **102** (2020).

TMDs from global analyses

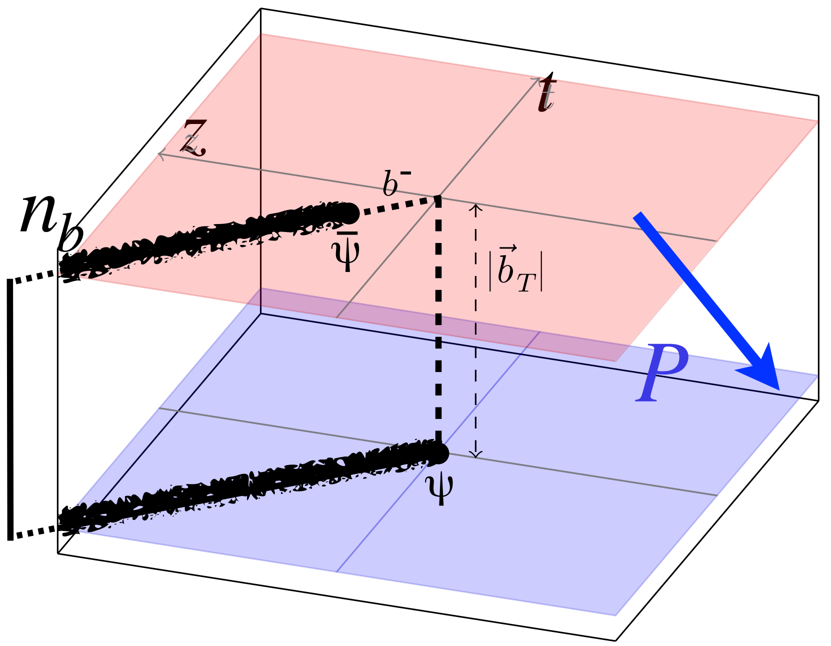
Collins-Soper Kernel $K(b_T, \mu)$ **or** $\gamma_\zeta(b_T, \mu)$ $\quad K(b_T, \mu) = K^{\text{pert}}(b_T, \mu) + g_K(b_T)$



Bacchetta, Bertone, Bissolotti, et al., MAP Collaboration, JHEP **10** (2022).

TMD definition

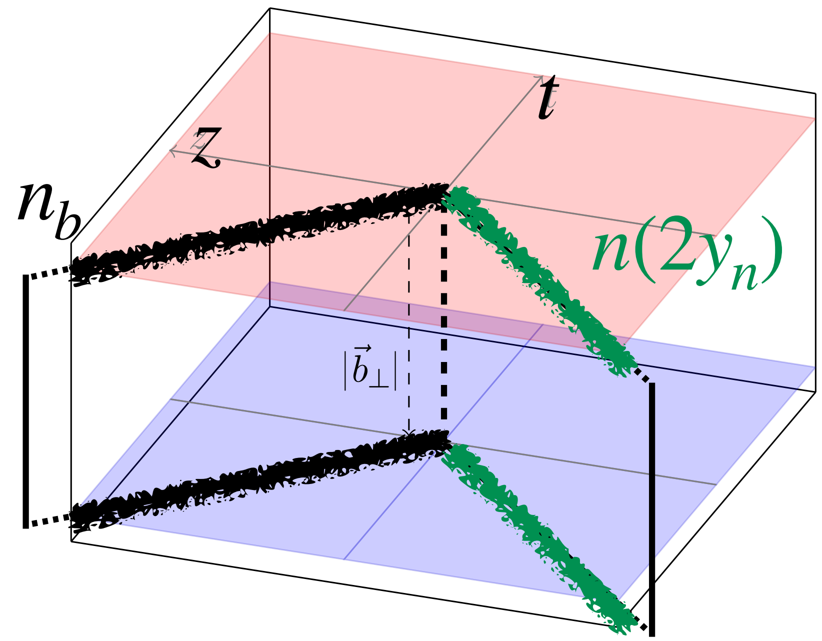
- Beam function:



Hadronic matrix element

$n_b^2 = 0$

- Soft function :



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\rm UV} \lim_{\tau \to 0} \frac{B_i}{\sqrt{S^q}}$$

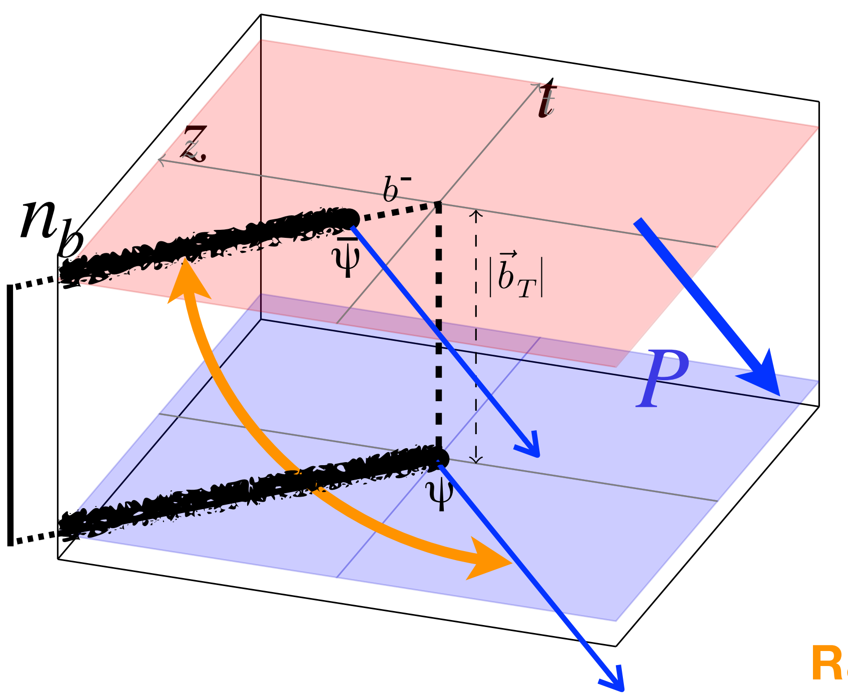
Collins-Soper scale: $\zeta = 2(xP^+ e^{-y_n})^2$

Rapidity divergence regulator

First principles calculation of TMDs from the above matrix elements would greatly complement global analyses!

TMD definition

- Beam function:

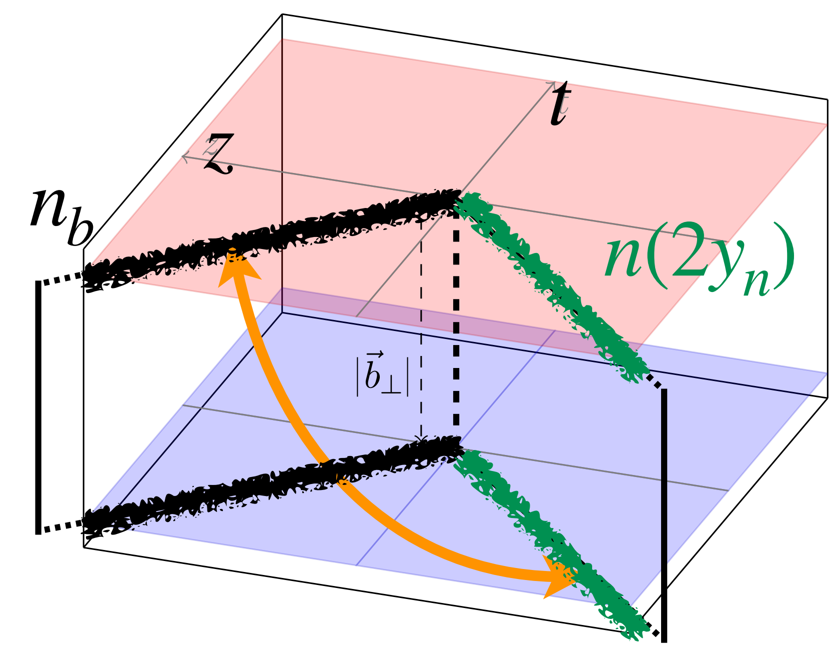


Hadronic matrix element

$$n_b^2 = 0$$

Rapidity divergences

- Soft function :



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\rm UV} \lim_{\tau \to 0} \frac{B_i}{\sqrt{S^q}}$$

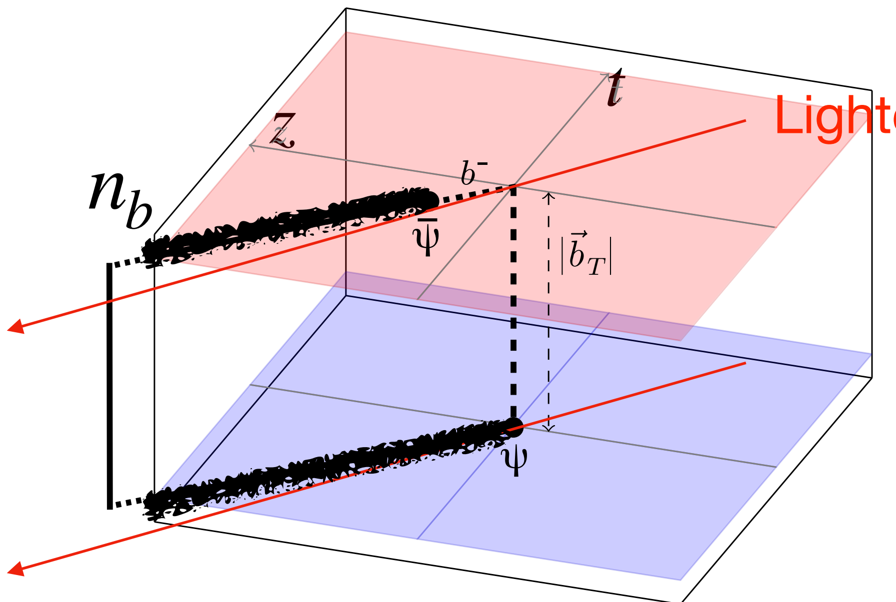
Collins-Soper scale: $\zeta = 2(xP^+ e^{-y_n})^2$

Rapidity divergence regulator

First principles calculation of TMDs from the above matrix elements would greatly complement global analyses!

Quasi TMD in LaMET

- **Beam function in Collins scheme:**

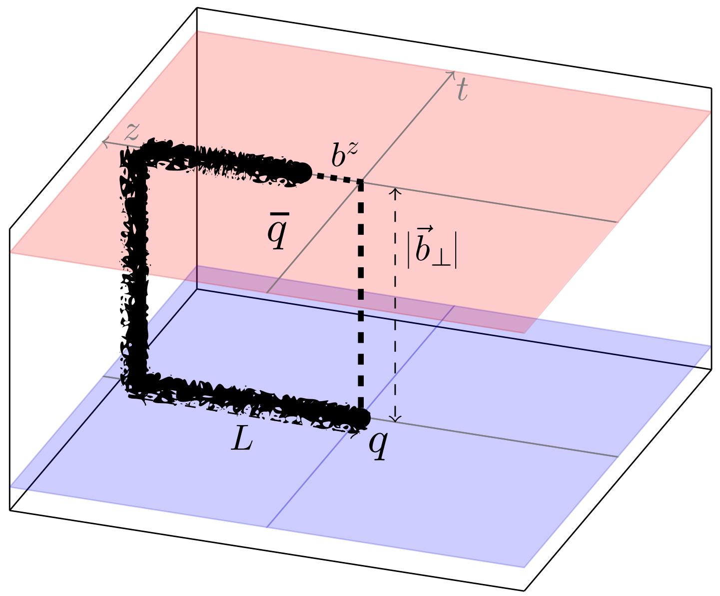


$n_b^\mu(y_B) \equiv (-e^{2y_B}, 1, 0_\perp)$

Spacelike but close-to-lightcone ($y_B \to -\infty$) Wilson lines, not calculable on the lattice ☹️

Lorentz boost and $L \to \infty$

- **Quasi beam function :**

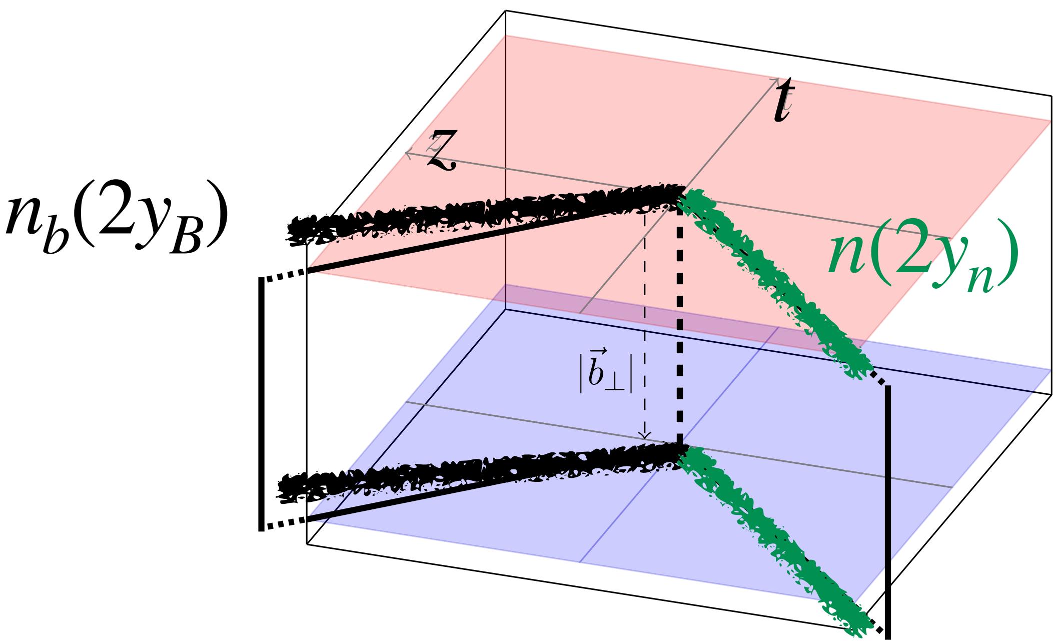


Equal-time Wilson lines, directly calculable on the lattice🙂

Related by Lorentz invariance, equivalent in the large $\tilde{P}^z$ or $(-y_B)$ expansion.

Ebert, Schindler, Stewart and YZ, JHEP 04 (2022).

Soft factor



$$= S_r(b_T,\mu)\ e^{-2(y_B-y_n)\gamma_\zeta(b_T,\mu)}$$

Collins-Soper kernel ($\gamma_\zeta(b_T,\mu)$)

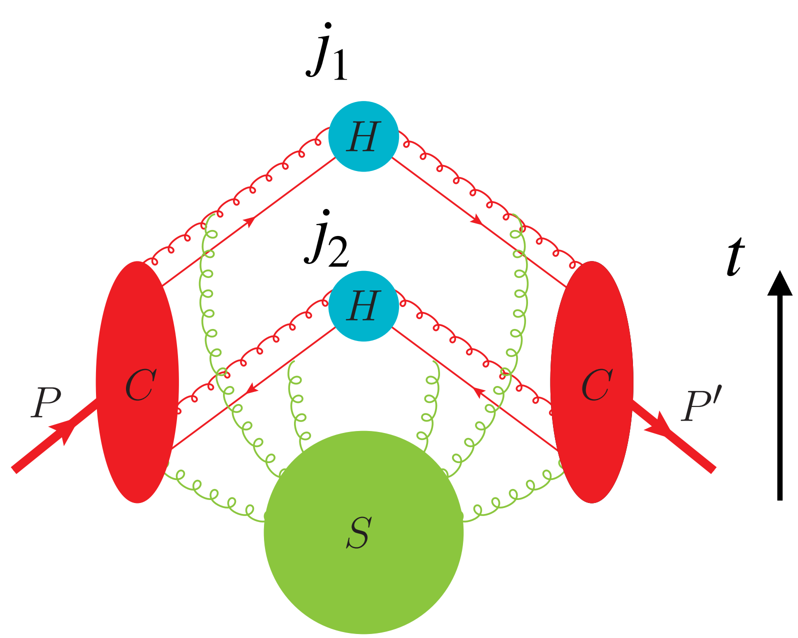
Reduced soft factor ($S_r(b_T,\mu)$)

Light-meson form factor:

$$F(b_T,P^z) = \langle \pi(-P)\,|\,j_1(b_T)j_2(0)\,|\,\pi(P)\rangle$$

$$\stackrel{P^z \gg m_N}{=} S_r(b_T,\mu)\int dx dx'\ H(x,x',\mu)$$

$$\times\, \Phi^\dagger(x,b_T,P^z,\mu)\Phi(x',b_T,P^z,\mu)$$



$\Phi(x,b_T,P^z,\mu)$: **quasi-TMD wave function**

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ji and Liu, PRD **105** (2022);
- Deng, Wang and Zeng, JHEP **09** (2022).

Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}_{i/p}^{\mathrm{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C\big(\mu, x\tilde{P}^z\big) \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP 04, 178 (2022).

Matching coefficient:

- Independent of spin;
  - Vladimirov and Schäfer, PRD 101 (2020);
  - Ebert, Schindler, Stewart and YZ, JHEP 09 (2020);
  - Ji, Liu, Schäfer and Yuan, PRD 103 (2021).
- No quark-gluon or flavor mixing, which makes gluon calculation much easier.

One-loop matching for gluon TMDs:

Schindler, Stewart and YZ, JHEP **08** (2022);

Zhu, Ji, Zhang and Zhao, JHEP **02** (2023).

Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f^{[s]}_{i/p}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda^2_{\text{QCD}}}{(x\tilde{P}^z)^2}\right]\right\}$$

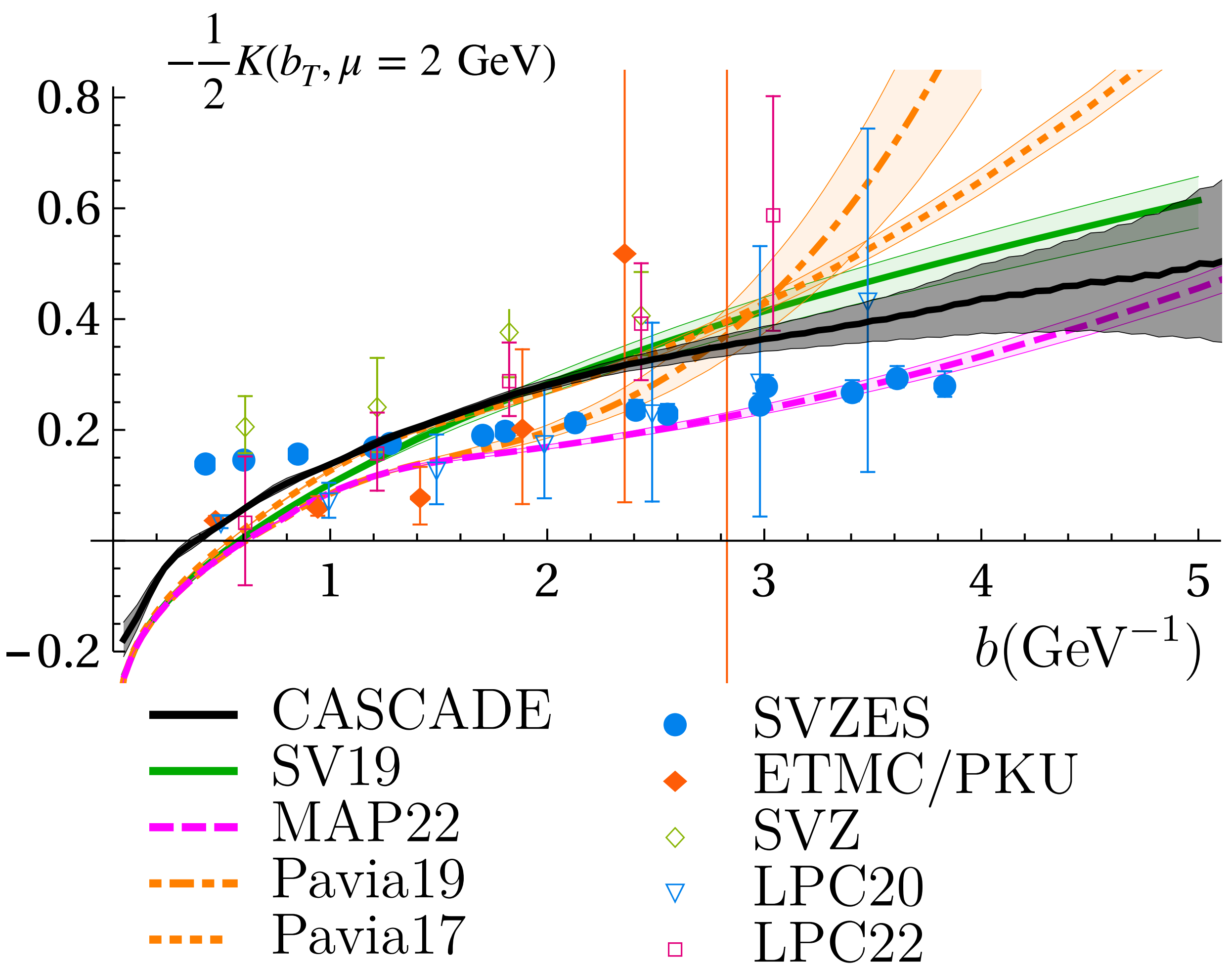
- ✳ Collins-Soper kernel;

$$\gamma_\zeta(\mu, b_T) = \frac{d}{d\ln\tilde{P}^z}\ln\frac{\tilde{f}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$$

- ✳ Flavor separation; $\dfrac{f^{[s]}_{i/p}(x, \mathbf{b}_T)}{f^{[s']}_{j/p}(x, \mathbf{b}_T)} = \dfrac{\tilde{f}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T)}{\tilde{f}^{\text{naive}[s']}_{j/p}(x, \mathbf{b}_T)}$
- ✳ Spin-dependence, e.g., Sivers function (single-spin asymmetry);
- ✳ Full TMD kinematic dependence.
- ✳ Twist-3 PDFs from small b_T expansion of TMDs. Ji, Liu, Schäfer and Yuan, PRD 103 (2021).
- ✳ Higher-twist TMDs. Rodini and Vladimirov, JHEP 08 (2022).

Collins Soper kernel from Lattice QCD

Comparison between lattice results and global fits



MAP22: Bacchetta, Bertone, Bissolotti, et al., JHEP **10** (2022)
SV19: I. Scimemi and A. Vladimirov, JHEP **06** (2020)
Pavia19: A. Bacchetta et al., JHEP **07** (2020)
Pavia 17: A. Bacchetta et al., JHEP **06** (2017)
CASCADE: Martinez and Vladimirov, PRD **106** (2022)

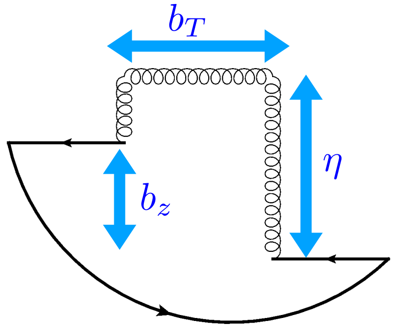
Approach	Collaboration
Quasi beam functions	P. Shanahan, M. Wagman and YZ (SWZ21), PRD **104** (2021)
Quasi TMD wavefunctions	Q.-A. Zhang, et al. (LPC20), PRL **125** (2020).
	Y. Li et al. (ETMC/PKU 21), PRL **128** (2022).
	M.-H. Chu et al. (LPC22), PRD 106 (2022)
Moments of quasi TMDs	Schäfer, Vladmirov et al. (SVZES21), JHEP **08** (2021), 2302.06502

Current status for the Collins-Soper kernel

	Lattice setup	Renormalization	Operator mixing	Fourier transform	Matching	x-plateau search
SWZ20 PRD 102 (2020) Quenched	$a = 0.06$ fm, $m_\pi = 1.2$ GeV, $P^z_{\max} = 2.6$ GeV	Yes	Yes	Yes	LO	Yes
LPC20 PRL 125 (2020)	$a = 0.10$ fm, $m_\pi = 547$ MeV, $P^z_{\max} = 2.11$ GeV	N/A	No	N/A	LO	N/A
SVZES 21 JHEP08 (2021), 2302.06502	$a = 0.09$ fm, $m_\pi = 422$ MeV, $P^+_{\max} = 2.27$ GeV	N/A	No	N/A	NLO	N/A
PKU/ETMC 21 PRL 128 (2022)	$a = 0.09$ fm, $m_\pi = 827$ MeV, $P^z_{\max} = 3.3$ GeV	N/A	No	N/A	LO	N/A
SWZ21 PRD 106 (2022)	$a = 0.12$ fm, $m_\pi = 580$ MeV, $P^z_{\max} = 1.5$ GeV	Yes	Yes	Yes	NLO	Yes
LPC22 PRD 106 (2022)	$a = 0.12$ fm, $m_\pi = 670$ MeV, $P^z_{\max} = 2.58$ GeV	Yes	No	Yes	NLO	Yes

Improved calculation with TMD wave function

Φ: Quasi-TMD wave function

$\tilde{\Phi} = \langle 0 |$  $| \pi(P) \rangle$

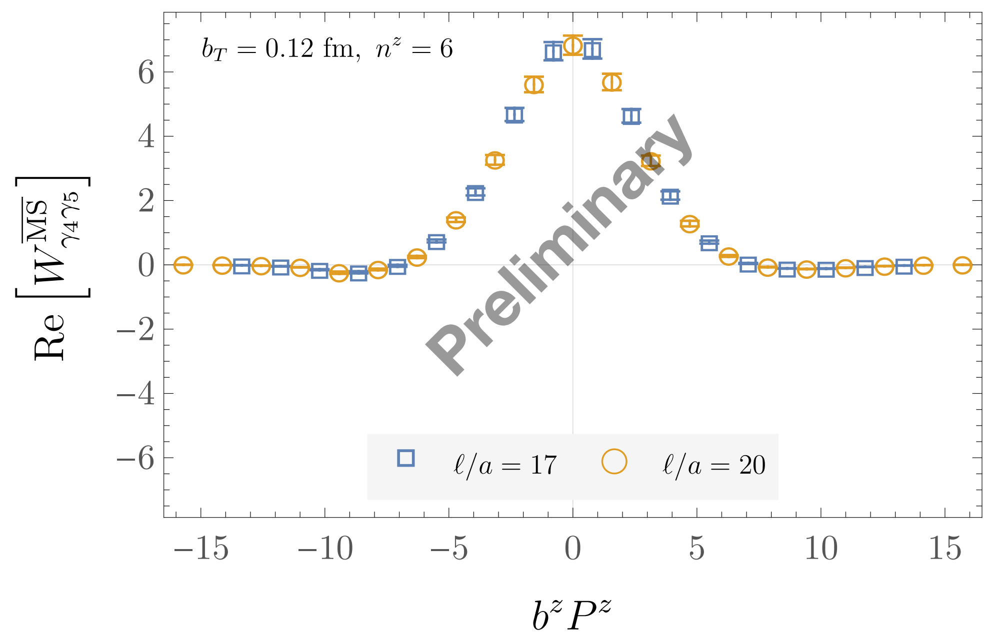
Q.-A. Zhang, et al. (LPC), PRL **125** (2020);
Y. Li et al., PRL **128** (2022);
M.-H. Chu et al. (LPC22), PRD **106** (2022).

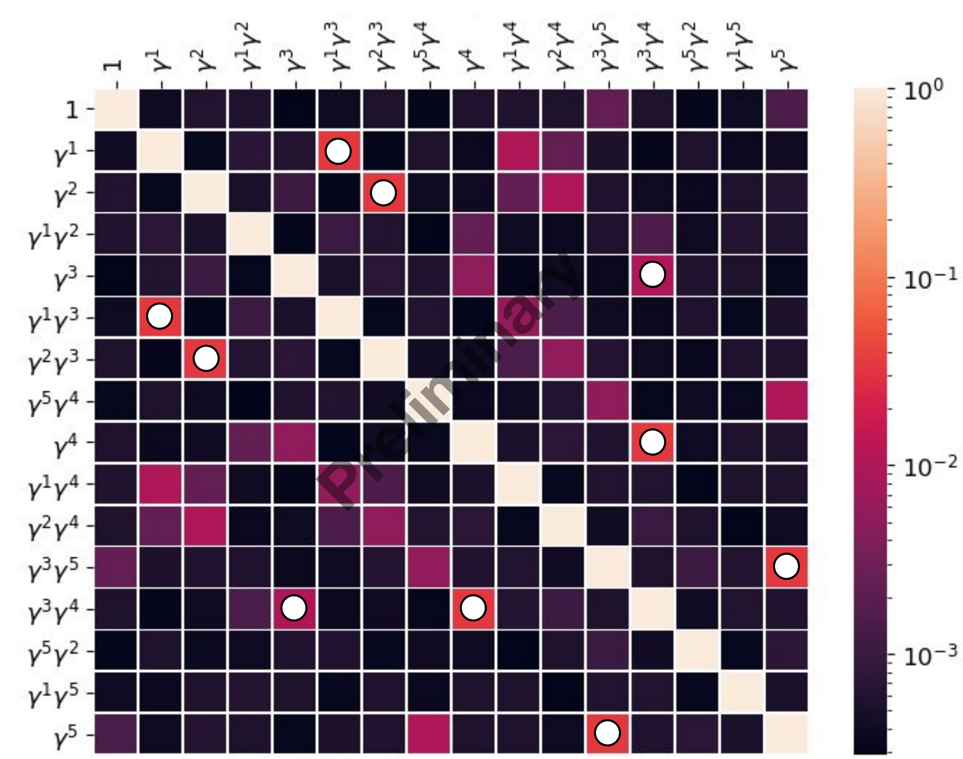
$$a = 0.12 \text{ fm}, \quad m_\pi = 140 \text{ MeV}, \quad P^z_{\text{max}} = 2.15 \text{ GeV}$$

- **Physical pion mass**
    - Better suppressed power corrections
    - More stable Fourier transform
- **Renormalization of nonlocal operator**
    - Systematic treatment of operator mixing using the RI-xMOM scheme

- Green, Jansen and Steffens, PRL **121** (2018) and PRD **101** (2020).
- Constantinou, Panagopoulos, and Spanoudes, PRD **99** (2019).

A. Avkhadiev, P. Shanahan, M. Wagman and YZ, work in progress.

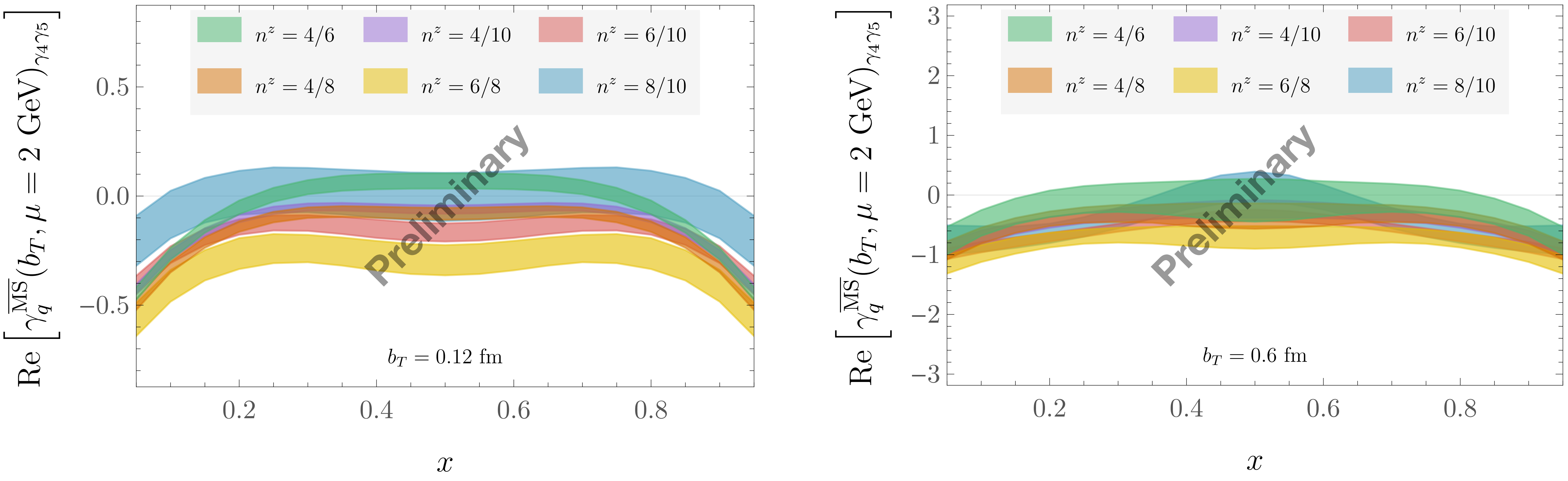




Improved calculation with TMD wave function

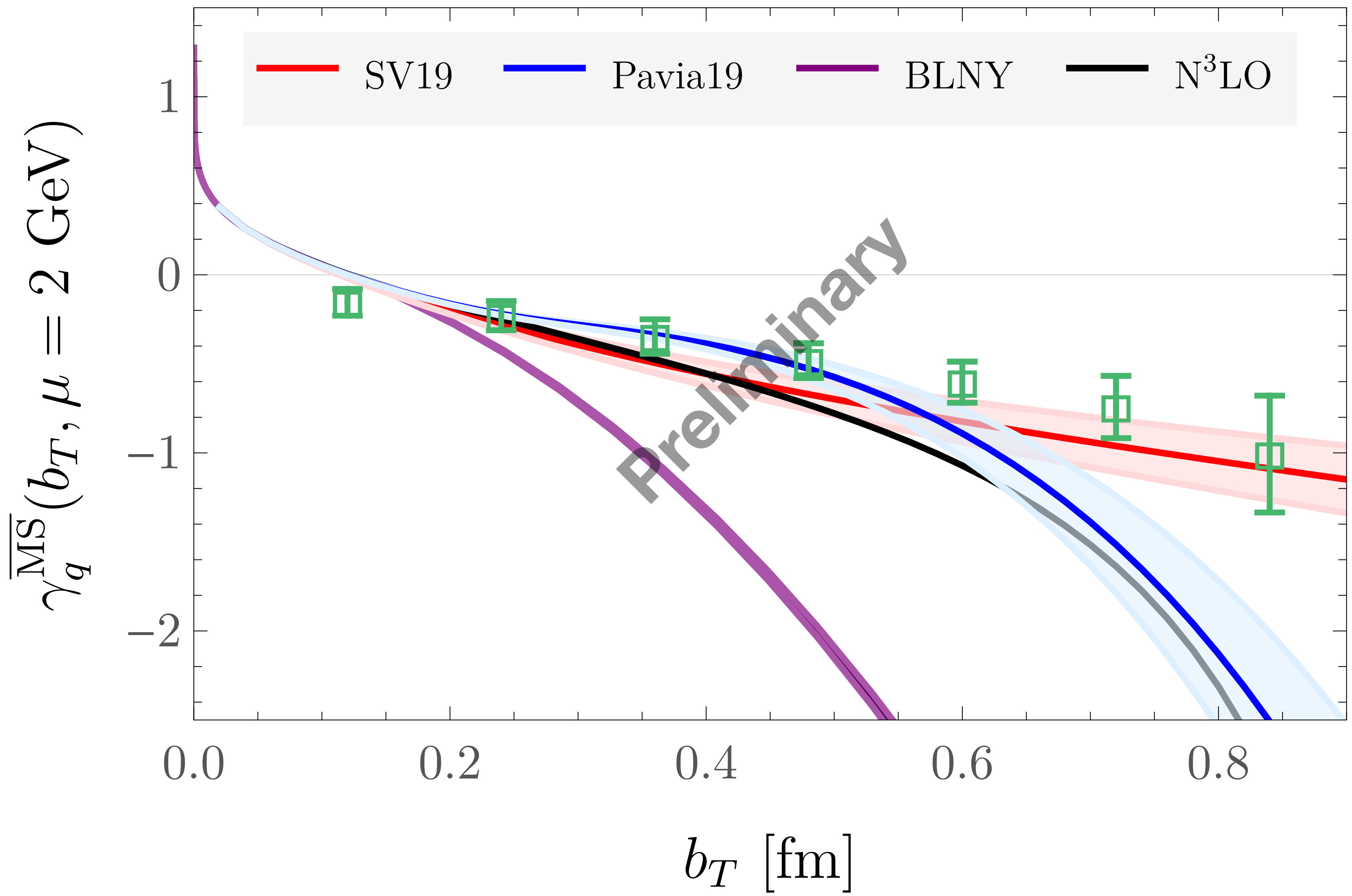
- Collins-Soper kernel extraction in x-space

$$\gamma_\zeta(\mu, b_T) = \frac{1}{\ln(\tilde{P}^z_1/\tilde{P}^z_2)} \ln \frac{\tilde{\Phi}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T, \mu, \tilde{P}^z_1)}{C\left(\mu, x\tilde{P}^z_1\right)} \Bigg/ \frac{\tilde{\Phi}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T, \mu, \tilde{P}^z_2)}{C\left(\mu, x\tilde{P}^z_2\right)}$$



Improved calculation with TMD wave function

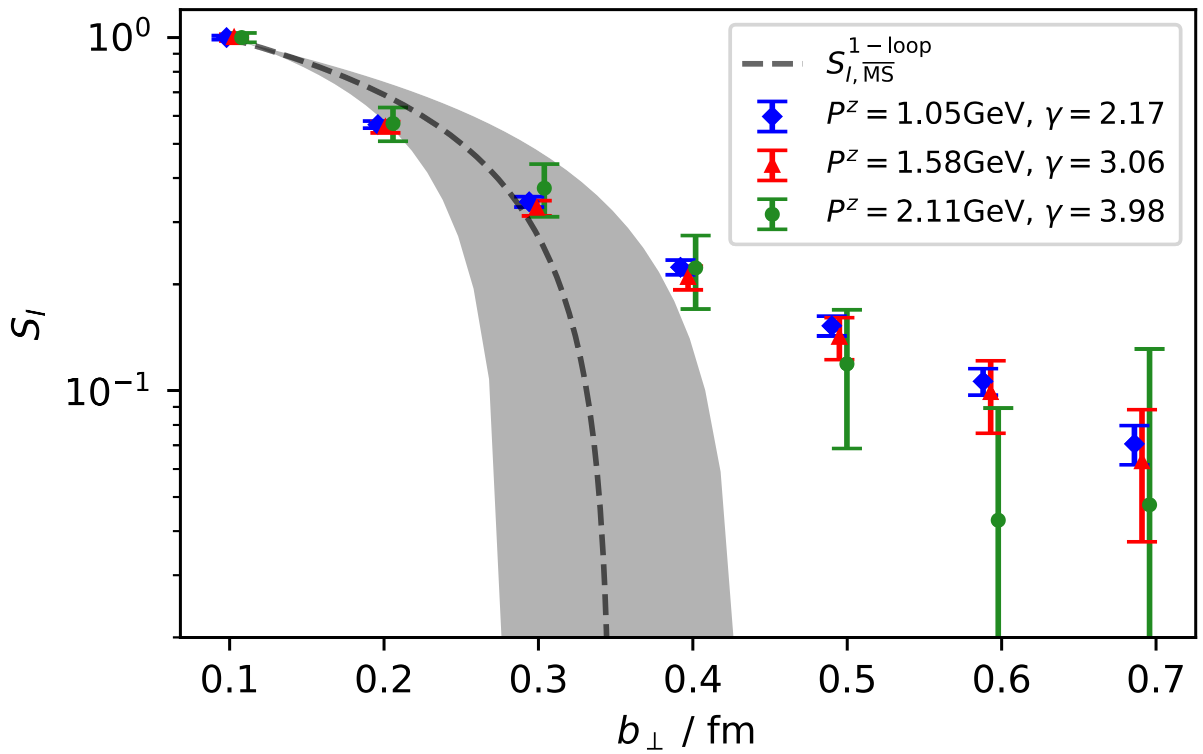
- Final result in comparison with global fits and perturbative QCD



SV19: I. Scimemi and A. Vladimirov, JHEP **06** (2020)
Pavia19: A. Bacchetta et al., JHEP **07** (2020)
BLNY: Landry, Brock, Nadolsky and Yuan, PRD 67 (2003)

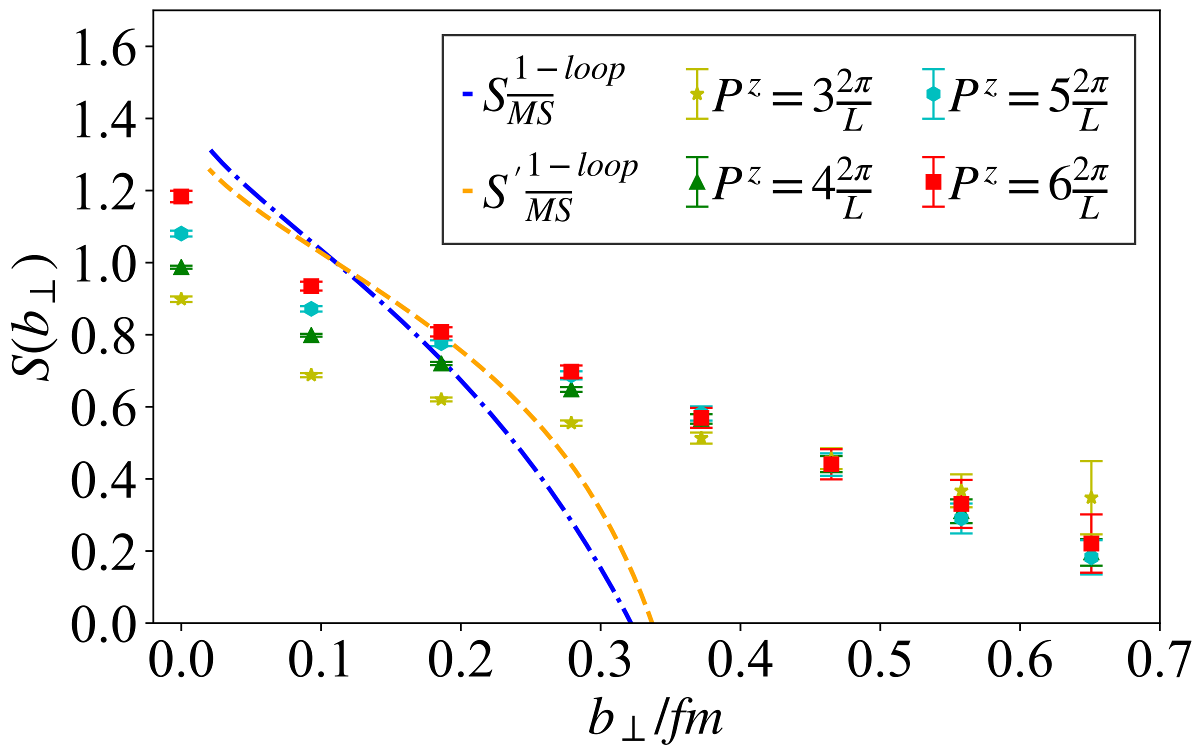
Lattice result of the reduced soft factor

$a = 0.10$ fm,
$m_\pi = 547$ MeV,
$P^z_{\max} = 2.11$ GeV



Q.-A. Zhang, et al. (LPC), PRL 125 (2020).

$a = 0.09$ fm,
$m_\pi = 827$ MeV,
$P^z_{\max} = 3.3$ GeV



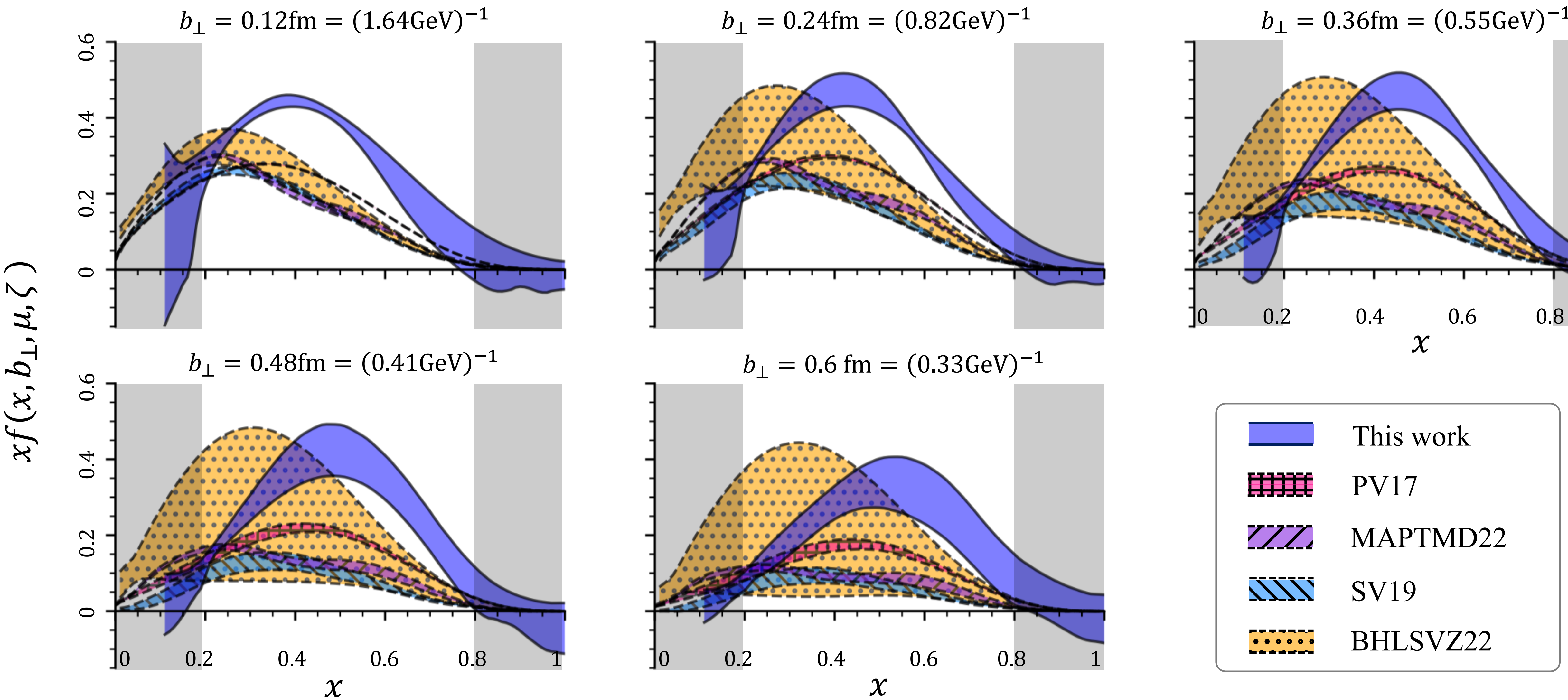
Y. Li et al., PRL 128 (2022).

Tree-level approximation:

$$H(x, x', \mu) = 1 + \mathcal{O}(\alpha_s) \quad \Rightarrow S^r_q(b_T) = \frac{F(b_T, P^z)}{[\tilde{\Phi}(b^z = 0, b_T, P^z)]^2}$$

(x, b_T) dependence of the unpolarized proton TMD

J.-C. He, M.-H. Chu, J. Hua et al., (LPC), arXiv: 2211.02340.



$a = 0.12$ fm,
$m_\pi = \{310, 220\}$ MeV,
$P^z_{\rm max} = 2.58$ GeV

SV19: Scimemi and Vladimirov, JHEP **06** (2020)
Pavia19: Bacchetta et al., JHEP **07** (2020).
MAPTMD22: Bacchetta et al., JHEP **10** (2022).
BHLSVZ22: Bury et al., JHEP **10** (2022).

Outlook

Observables	Status
Non-perturbative Collins-Soper kernel	✔, keep improving the systematics
Soft factor	✔, to be under systematic control
Info on spin-dependent TMDs (in ratios)	In progress
Proton v.s. pion TMDs, (x, b_T) (in ratios)	In progress
Flavor dependence of TMDs, (x, b_T) (in ratios)	to be studied
TMDs and TMD wave functions, (x, b_T)	✔, to be under systematic control
Gluon TMDs (x, b_T)	to be studied
Wigner distributions/GTMDs (x, b_T)	to be studied