# Frame-independent methods to access GPDs from lattice QCD

**Martha Constantinou**

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In collaboration with:

S. Bhattacharya, K. Cichy, J. Dodson, X. Gao, A. Metz,
J. Miller, A. Scapellato, F. Steffens, S. Mukherjee, Y. Zhao

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★ Crucial in understanding hadron tomography

★ Correlation between transverse position and longitudinal momentum of the quarks in the hadron and its mechanical properties

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- with realistic computational resources
- for a broad range of their variables
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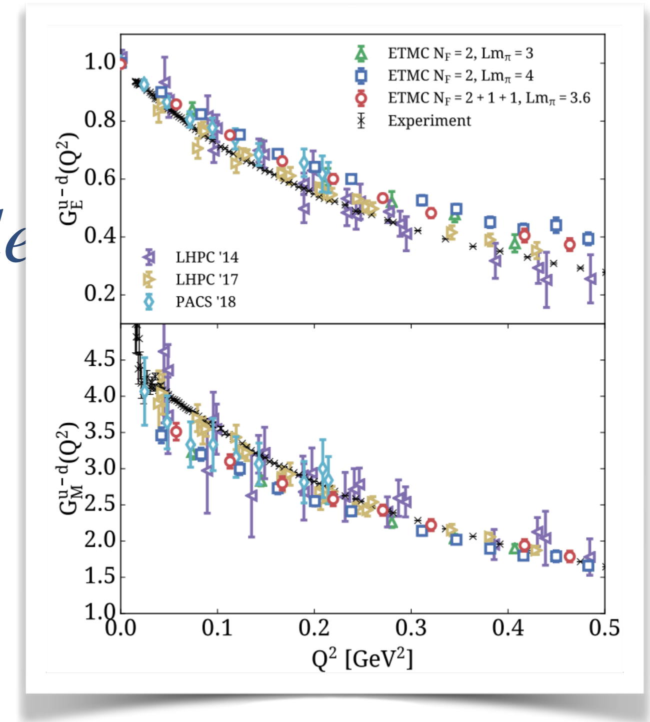

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PHYSICAL REVIEW D **106,** 114512 (2022)

**Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks**

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

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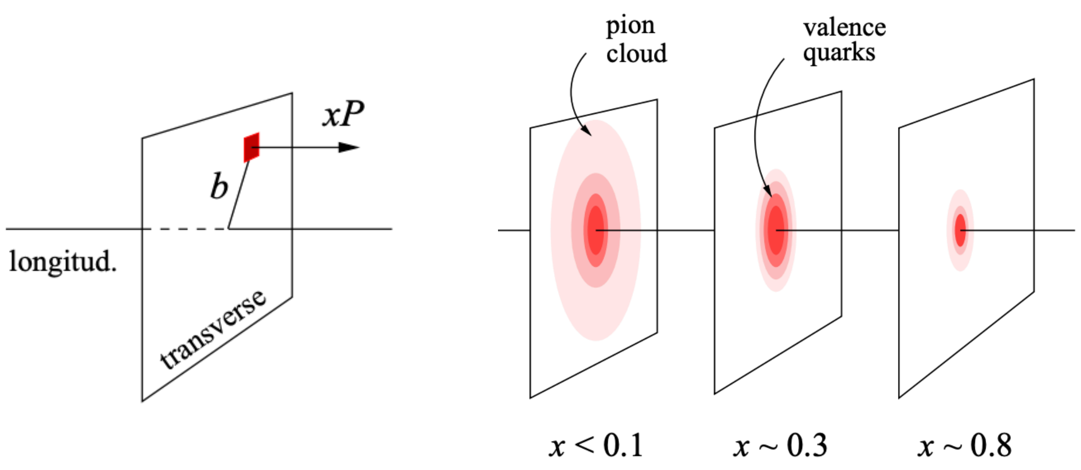


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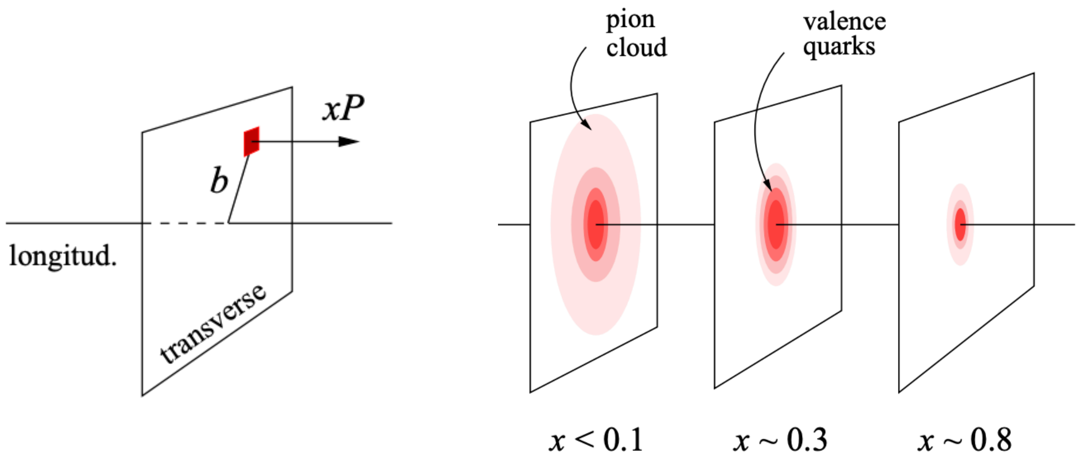


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- x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

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- GPDs phenomenology more complicated than PDFs (multi-dimensionality)

- and more challenges …

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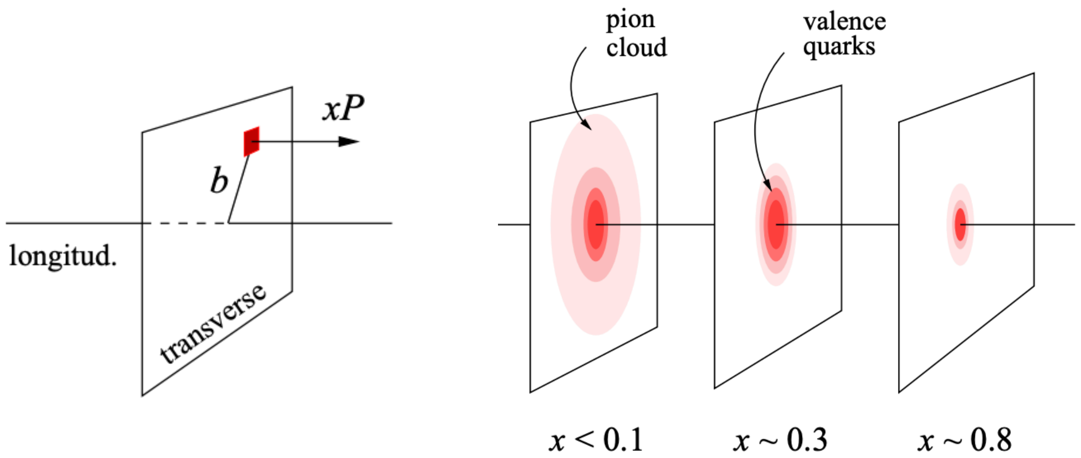


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- and more challenges …

*Essential to complement the knowledge on GPD from lattice QCD*

# GPDs

## Through non-local matrix elements

## of fast-moving hadrons

# GPDs on the lattice

★ GPDs: off-forward matrix elements of non-local light-cone operators

★ Off-forward correlators with nonlocal (equal-time) operators [Ji, PRL 110 (2013) 262002]

$$\tilde{q}_{\mu}^{\mathrm{GPD}}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\, \langle N(P_f)\,|\,\bar{\Psi}(z)\,\gamma^{\mu}\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_{\mu}$$

$$\Delta = P_f - P_i$$
$$t = \Delta^2 = -Q^2$$
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→ reduction of power corrections in fwd limit [Radyushkin, PLB 767, 314, 2017]

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(typically symmetric frame to extract the "standard" GPDs)

★ Symmetric frame ( $\vec{p}_f^{\,s} = \vec{P} + \vec{Q}/2\,,\, \vec{p}_i^{\,s} = \vec{P} - \vec{Q}/2$ ) requires separate calculations at each $t$

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*Light-cone GPDs using lattice correlators in non-symmetric frames*

# Theoretical setup

★ Parametrization of matrix elements in Lorentz invariant amplitudes

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu M A_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Advantages

- Applicable to any kinematic frame and $A_i$ have definite symmetries
- Lorentz invariant amplitudes $A_i$ can be related to the standard $H, E$ GPDs
- Quasi $H, E$ may be redefined (Lorentz covariant):

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Proof-of-concept calculation (zero quasi-skewness):

- symmetric frame: $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}$, $\quad \vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2} \quad t^s = -\vec{Q}^2$

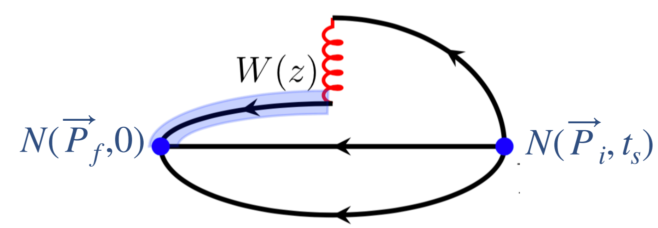
- asymmetric frame: $\vec{p}^{\,a}_f = \vec{P}$, $\quad \vec{p}^{\,a}_i = \vec{P} - \vec{Q} \quad t^a = -\vec{Q}^2 + (E_f - E_i)^2$

# Parameters of calculation

★ Nf=2+1+1 twisted mass (TM) fermions & clover improvement

| | |
|---|---|
| Pion mass: | 260 MeV |
| Lattice spacing: | 0.093 fm |
| Volume: | $32^3$ x 64 |
| Spatial extent: | 3 fm |

★ Calculation:
- isovector combination
- zero skewness
- $T_{sink}$=1 fm




| frame | $P_3$ [GeV] | $\mathbf{Q}$ $[\frac{2\pi}{L}]$ | $-t$ [GeV$^2$] | $\xi$ | $N_{\rm ME}$ | $N_{\rm confs}$ | $N_{\rm src}$ | $N_{\rm tot}$ |
|---|---|---|---|---|---|---|---|---|
| symm | 1.25 | (±2,0,0), (0,±2,0) | 0.69 | 0 | 8 | 249 | 8 | 15936 |
| non-symm | 1.25 | (±2,0,0), (0,±2,0) | 0.64 | 0 | 8 | 269 | 8 | 17216 |

★ Computational cost:
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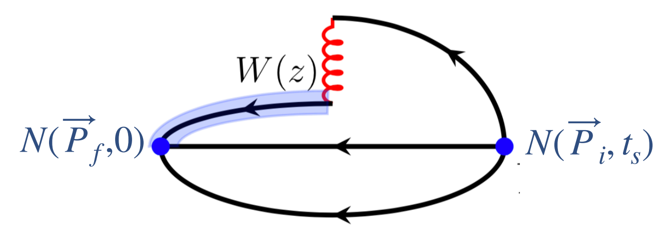


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Small difference: $t^s = -\vec{Q}^2$ $\quad$ $t^a = -\vec{Q}^2 + (E_f - E_i)^2$

$$A(-0.64\text{GeV}^2) \sim A(-0.69\text{GeV}^2)$$

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# Results: matrix elements

★ Eight independent matrix elements needed to disentangle the $A_i$

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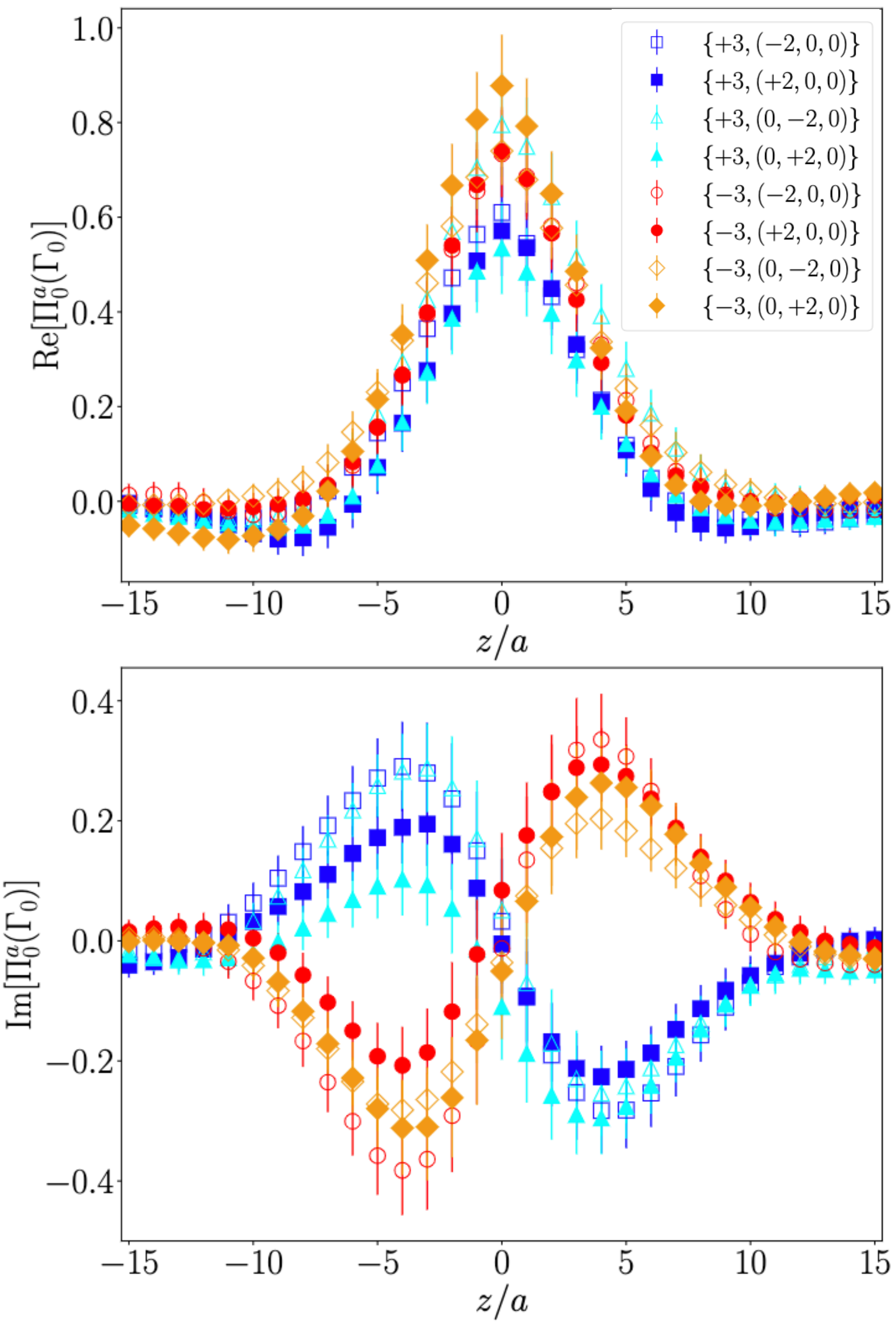

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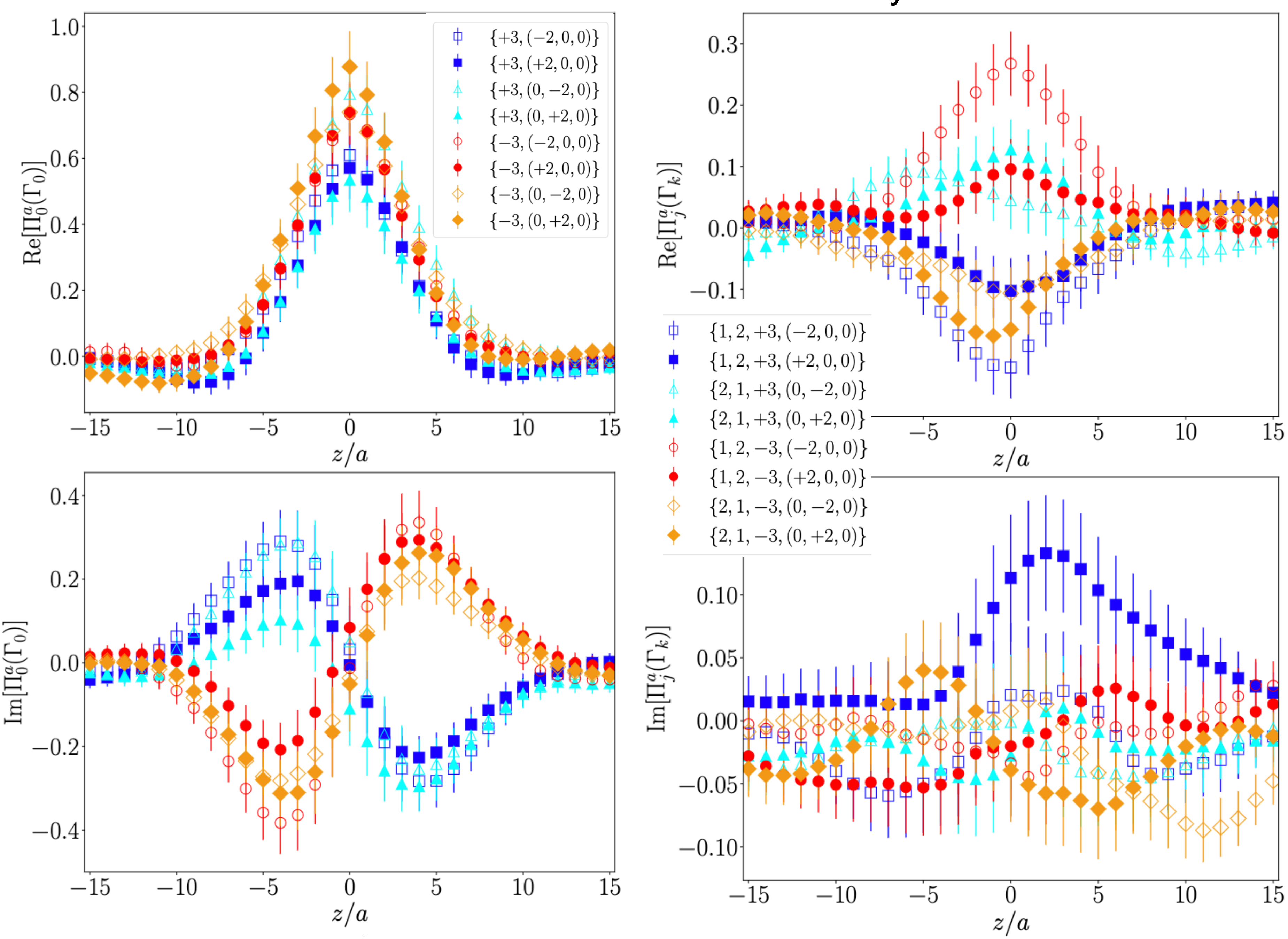

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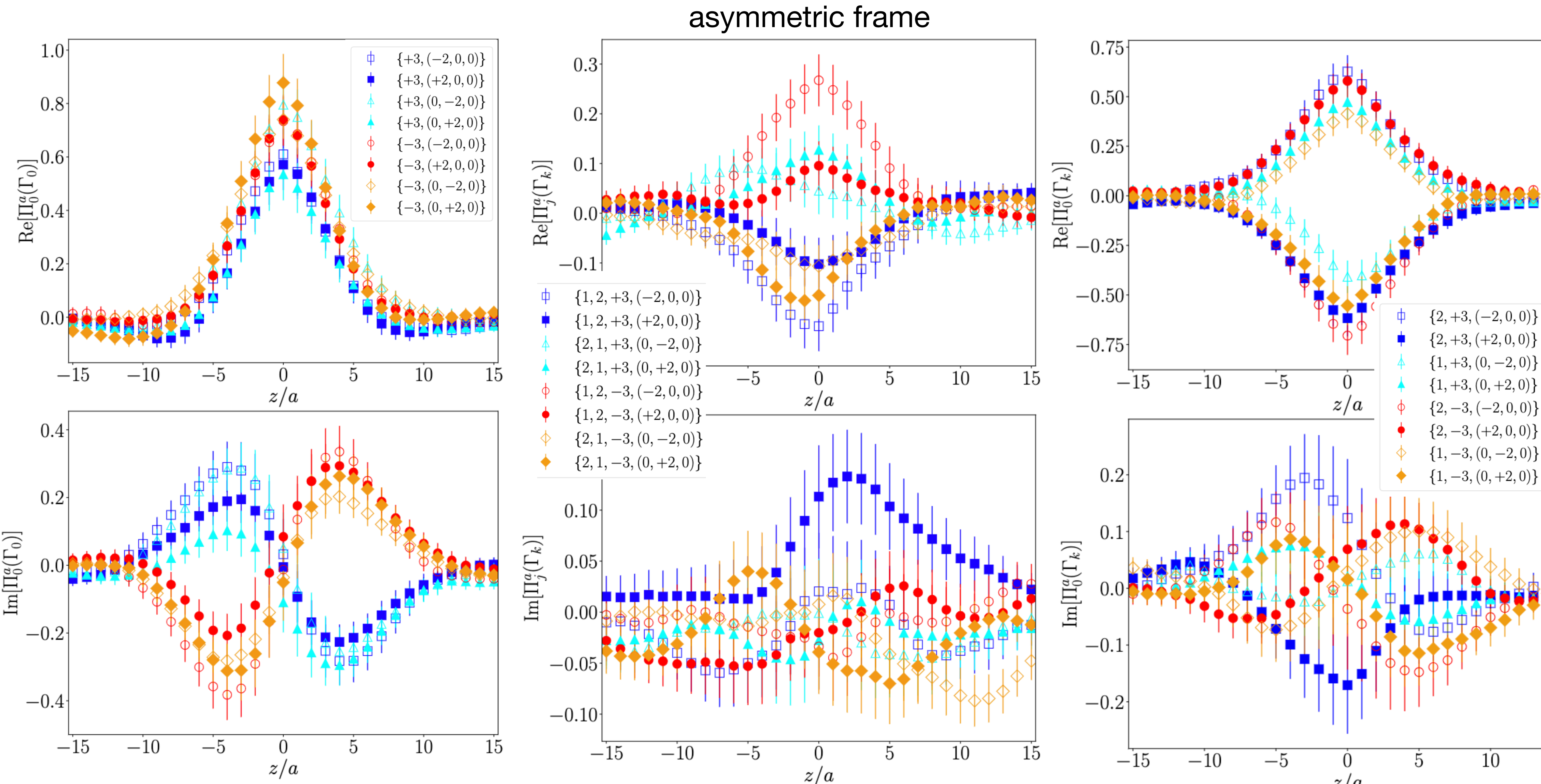

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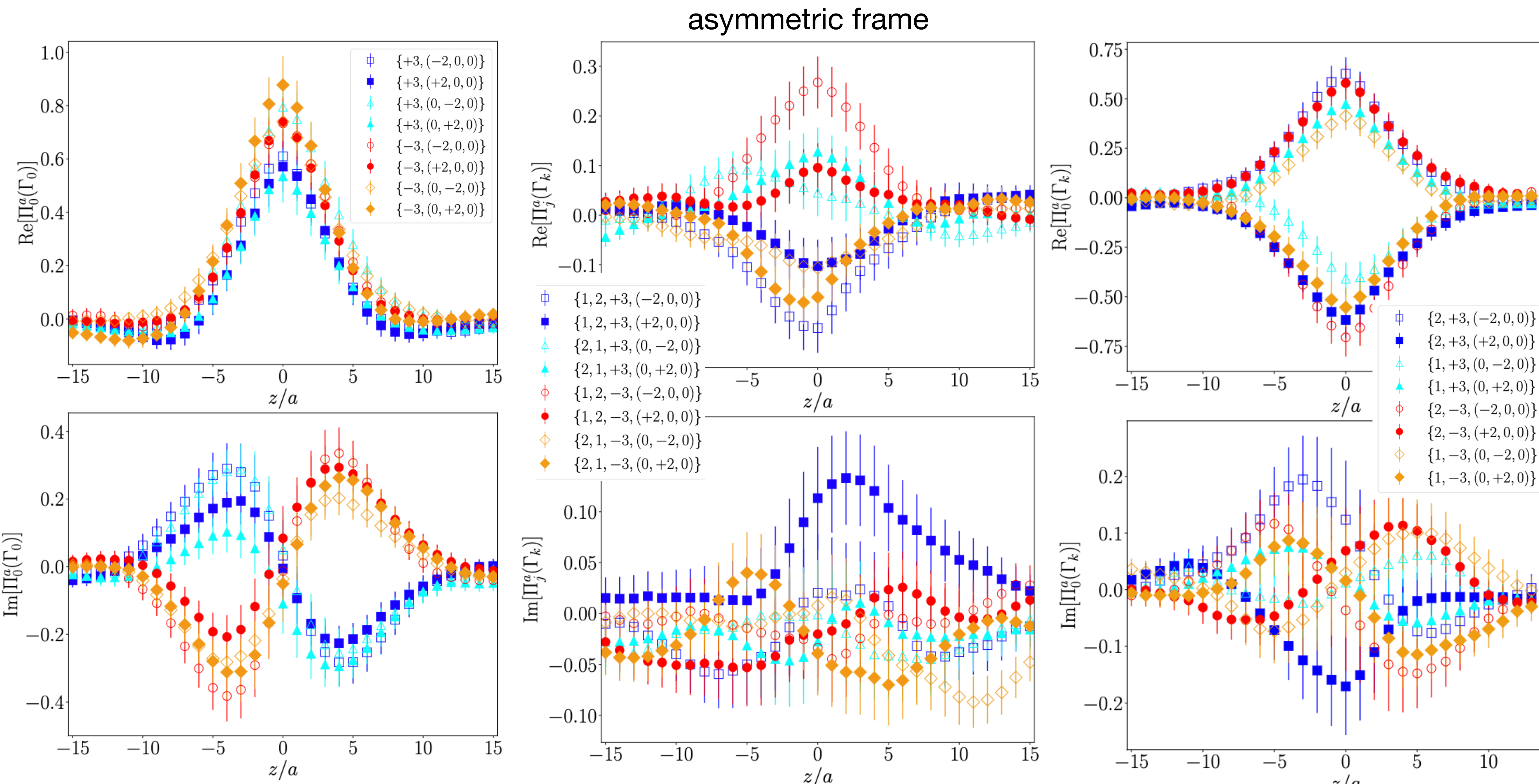


★ Asymmetric frame: ME do not have definite symmetries in $\pm P_3, \pm Q, \pm z$

★ Noisy ME lead to challenges in extracting $A_i$ of sub-leading magnitude

# *How do the $A_i$ compare between frames?*

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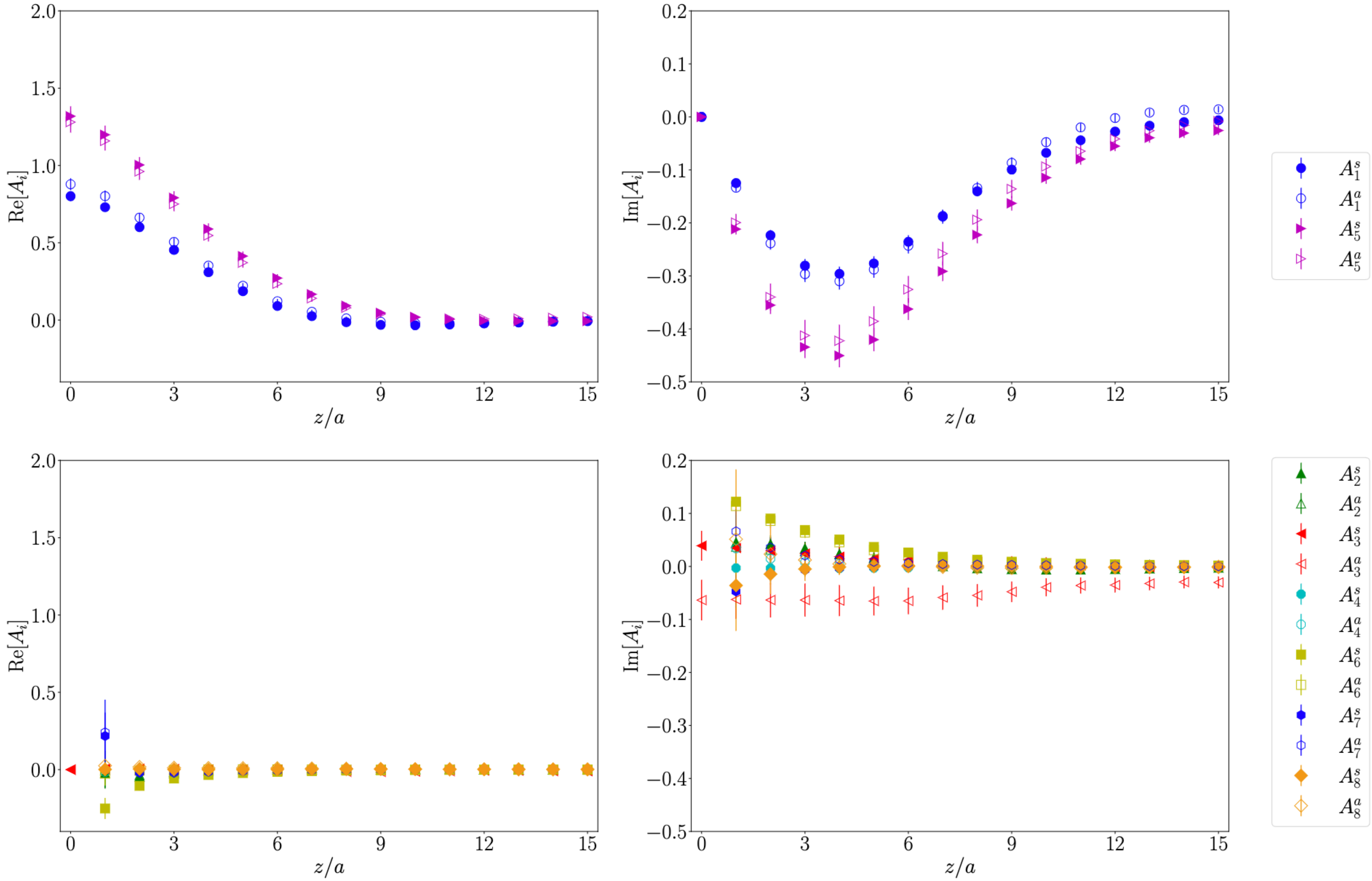


- ★ $A_1, A_5$ dominant contributions
- ★ Full agreement in two frames for both Re and Im parts of $A_1, A_5$
- ★ Remaining $A_i$ suppressed (at least for this kinematic setup and $\xi = 0$)

# quasi-GPDs in terms of $A_i$

★ The mapping of $A_i$ to the quasi-GPDs is not unique

★ Construction of a Lorentz invariant definition may be beneficial

$(\xi = 0)$

$$\Pi_H^{\text{impr}} = A_1$$

$$\Pi_E^{\text{impr}} = -A_1 + 2A_5 + 2zP_3A_6$$

★ All quasi-GPDs definitions converge to the same light-cone GPDs (up to systematic effects)

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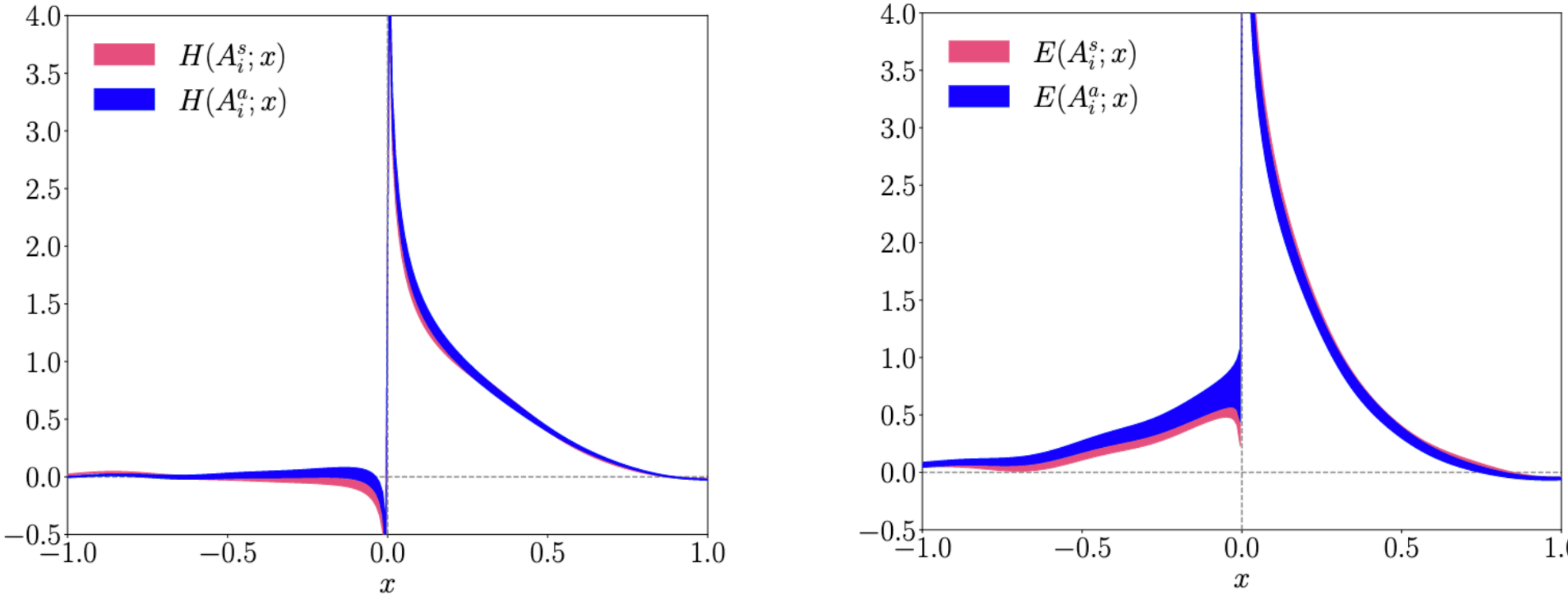



Agreement between frames for both quasi-GPDs (by definition)

# Beyond exploration

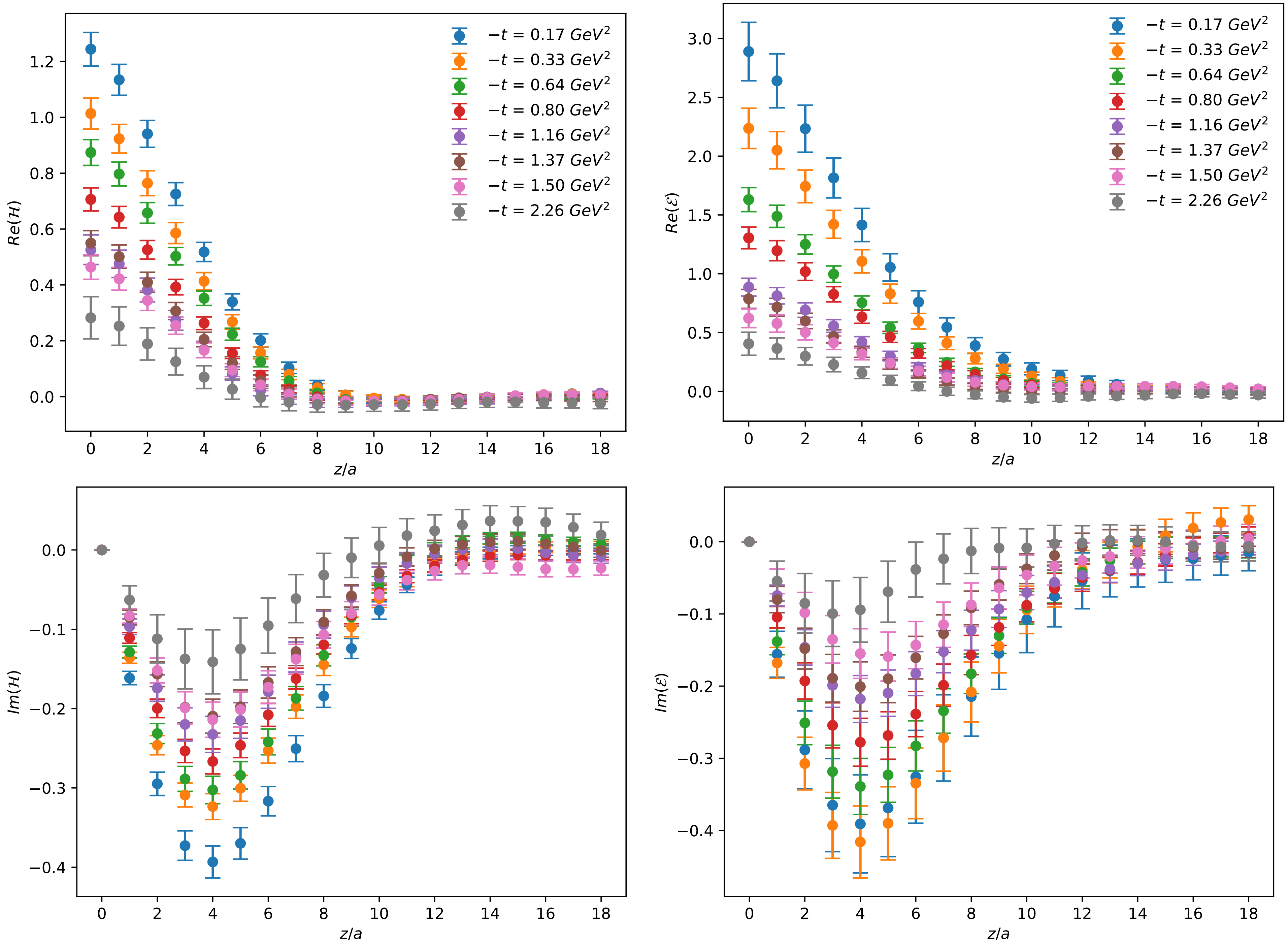
- 11 values of $-t$ (3 in symm. frame and 8 in asymm. frame)
- Separate calculation for each $-t$ value in symmetric frame
- Two groups of $-t$ value in asymmetric frame: $\vec{Q} = (Q_x,0,0), (Q_x, Q_y,0)$

| frame | $P_3$ [GeV] | $\mathbf{\Delta}$ $[\frac{2\pi}{L}]$ | $-t$ [GeV$^2$] | $\xi$ | $N_{\rm ME}$ | $N_{\rm confs}$ | $N_{\rm src}$ | $N_{\rm tot}$ |
|---|---|---|---|---|---|---|---|---|
| N/A | $\pm1.25$ | (0,0,0) | 0 | 0 | 2 | 731 | 16 | 23392 |
| symm | $\pm0.83$ | $(\pm2,0,0)$, $(0,\pm2,0)$ | 0.69 | 0 | 8 | 67 | 8 | 4288 |
| symm | $\pm1.25$ | $(\pm2,0,0)$, $(0,\pm2,0)$ | 0.69 | 0 | 8 | 249 | 8 | 15936 |
| symm | $\pm1.67$ | $(\pm2,0,0)$, $(0,\pm2,0)$ | 0.69 | 0 | 8 | 294 | 32 | 75264 |
| symm | $\pm1.25$ | $(\pm2,\pm2,0)$ | 1.39 | 0 | 16 | 224 | 8 | 28672 |
| symm | $\pm1.25$ | $(\pm4,0,0)$, $(0,\pm4,0)$ | 2.76 | 0 | 8 | 329 | 32 | 84224 |
| asymm | $\pm1.25$ | $(\pm1,0,0)$, $(0,\pm1,0)$ | 0.17 | 0 | 8 | 429 | 8 | 27456 |
| asymm | $\pm1.25$ | $(\pm1,\pm1,0)$ | 0.33 | 0 | 16 | 194 | 8 | 12416 |
| asymm | $\pm1.25$ | $(\pm2,0,0)$, $(0,\pm2,0)$ | 0.64 | 0 | 8 | 429 | 8 | 27456 |
| asymm | $\pm1.25$ | $(\pm1,\pm2,0)$, $(\pm2,\pm1,0)$ | 0.80 | 0 | 16 | 194 | 8 | 12416 |
| asymm | $\pm1.25$ | $(\pm2,\pm2,0)$ | 1.16 | 0 | 16 | 194 | 8 | 24832 |
| asymm | $\pm1.25$ | $(\pm3,0,0)$, $(0,\pm3,0)$ | 1.37 | 0 | 8 | 429 | 8 | 27456 |
| asymm | $\pm1.25$ | $(\pm1,\pm3,0)$, $(\pm3,\pm1,0)$ | 1.50 | 0 | 16 | 194 | 8 | 12416 |
| asymm | $\pm1.25$ | $(\pm4,0,0)$, $(0,\pm4,0)$ | 2.26 | 0 | 8 | 429 | 8 | 27456 |

- Momentum transfer range is very optimistic
  (some values have enhanced systematic uncertainties)

# Unpolarized quasi-GPDs

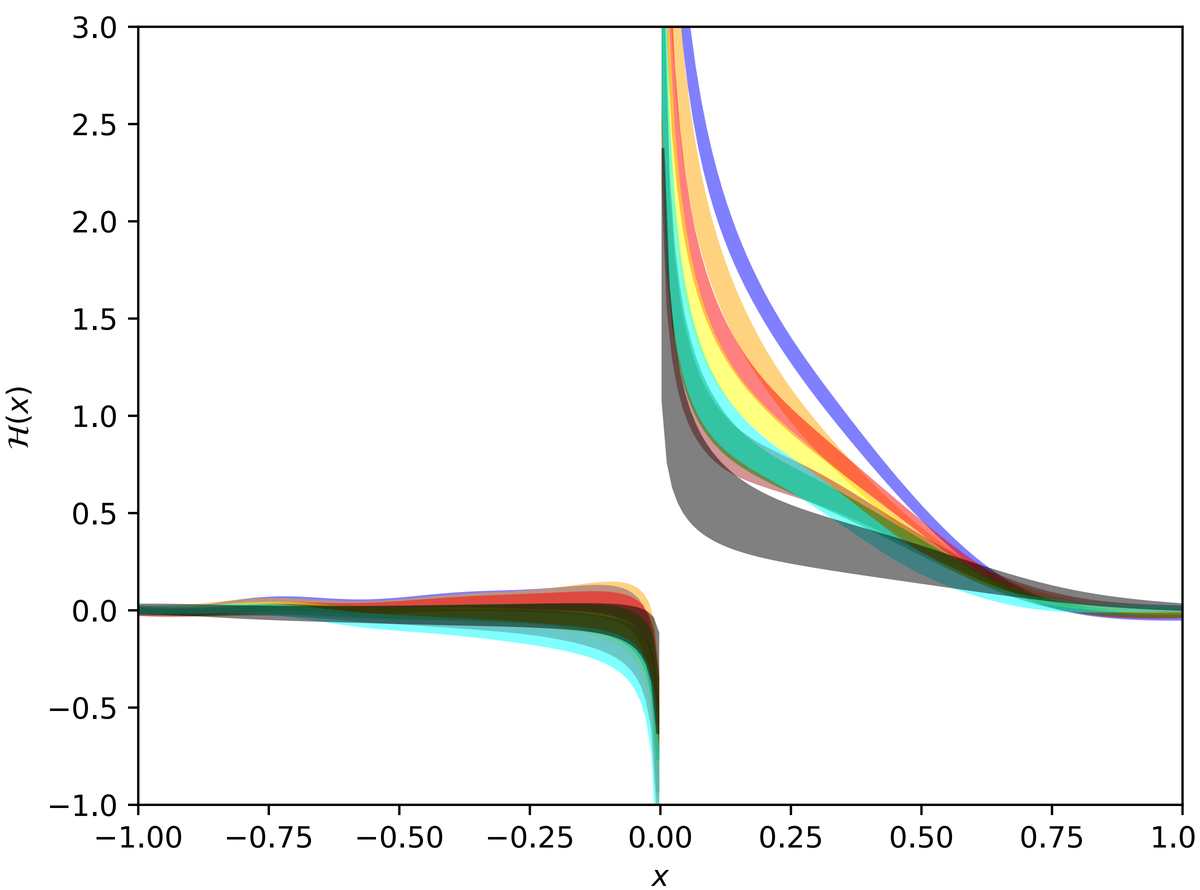
asymmetric frame

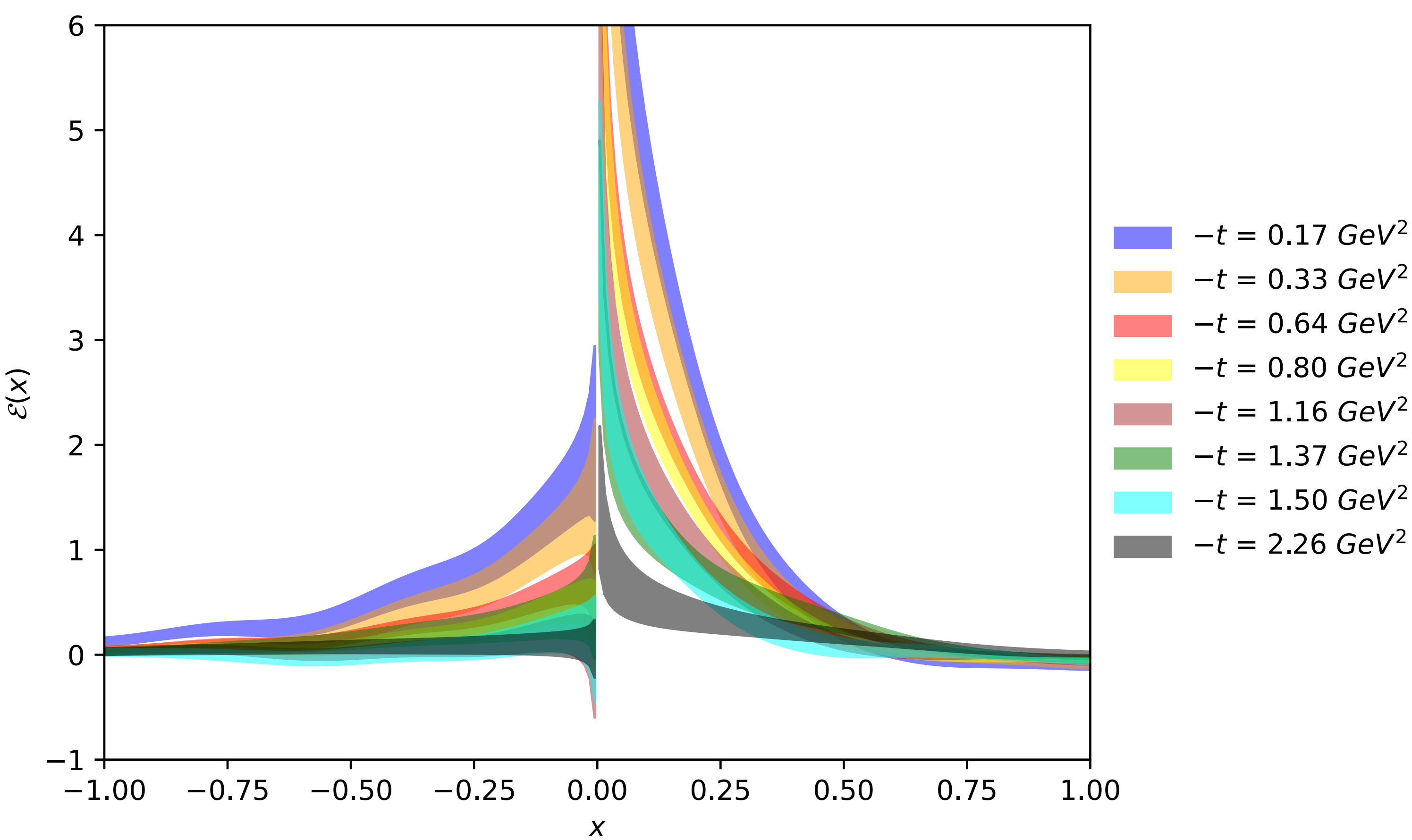



- ★ Impressive quality of signal quality
- ★ Behavior with increasing $-t$ as “expected” qualitatively

# Unpolarized light-cone GPDs

- ★ quasi-GPDs transformed to momentum space
- ★ Matching formalism to 1 loop accuracy level
- ★ +/-x correspond to quark and anti-quark region
- ★ Anti-quark region susceptible to systematic uncertainties.



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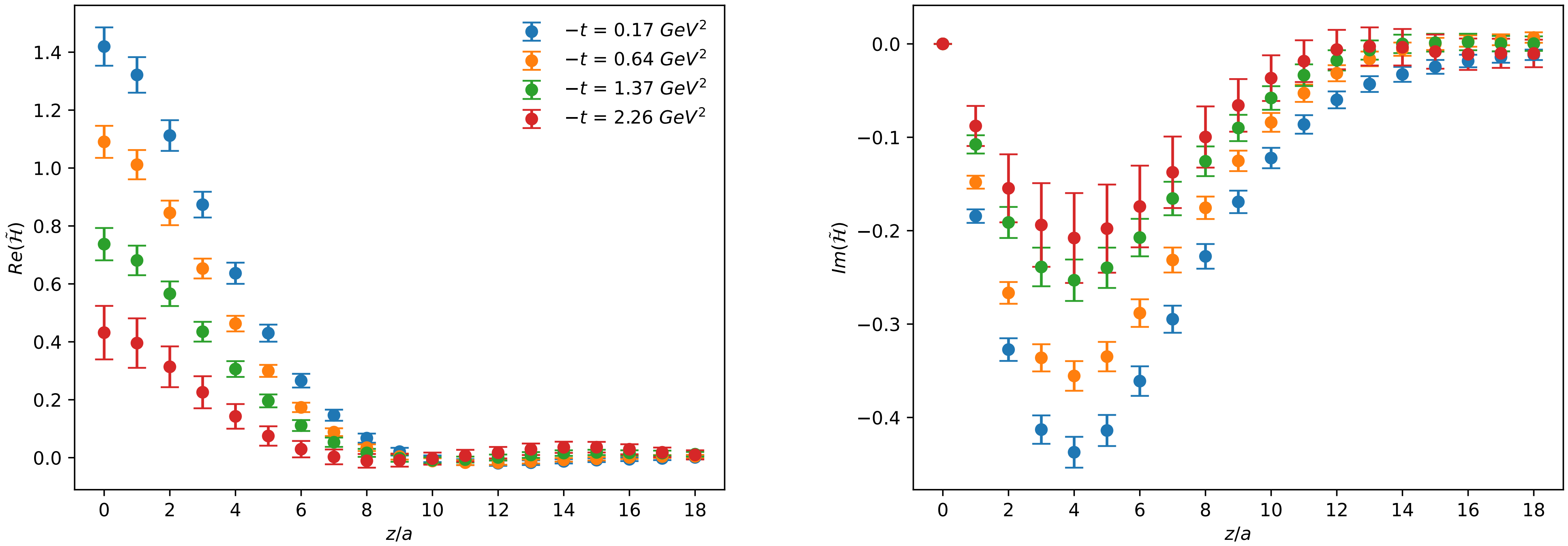

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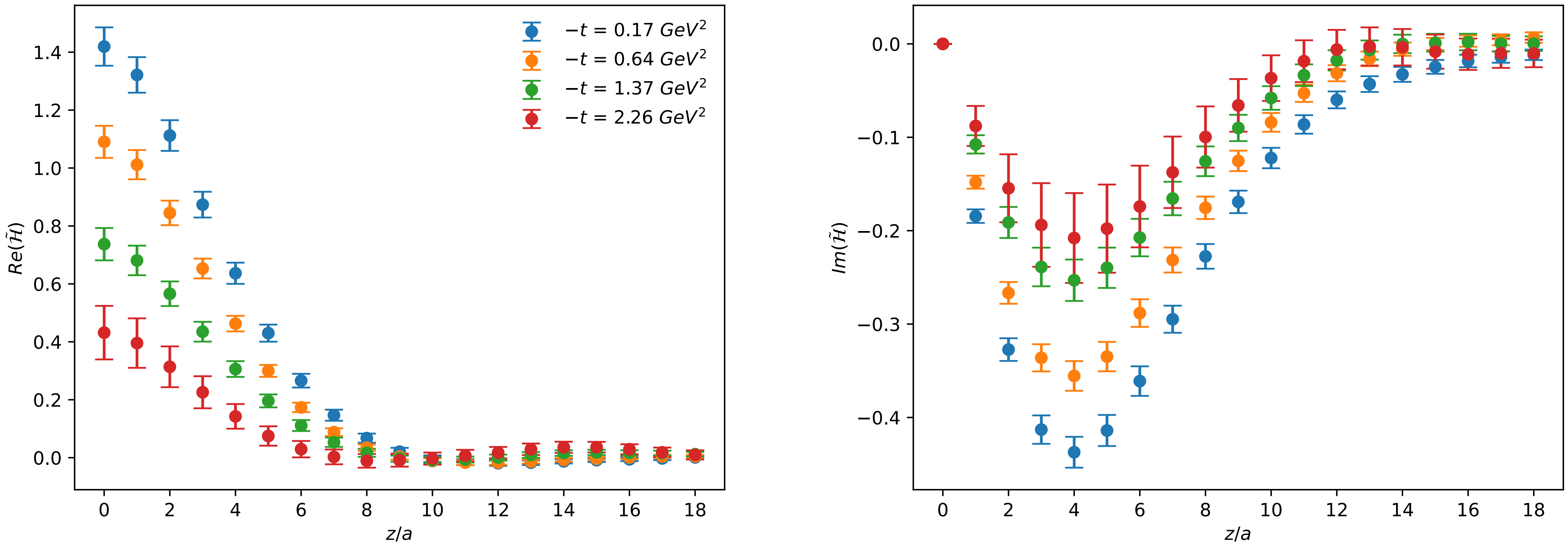


★ All values of $t$ obtained at the cost of one

★ Preliminary analysis very encouraging!

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QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

**Award Number:**
DE-SC0023646

1. **Theoretical studies** of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties

2. **Lattice QCD** calculations of GPDs and related structures

3. **Global analysis** of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

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*Thank you*

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