Deeply-virtual and photoproduction of pseudoscalar mesons at higher-order and higher-twist

Kornelija Passek-K.

Rudjer Bošković Institute, Croatia

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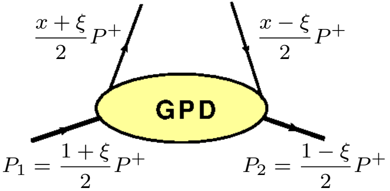
Outline

Generalized Parton Distributions



$P = P_1 + P_2 \qquad \Delta = P_2 - P_1$

$\Delta^2 = t$ momentum transfer

$\xi = -\frac{\Delta^+}{P^+}$ longitudinal momentum transfer (skewness)

$$F^a(x,\xi,t;\mu) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\mathcal{O}^a(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$a \in \{q,g\}$, $\mu\ldots$ factorization scale

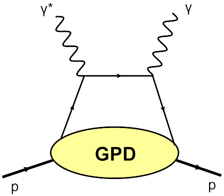
- vector (H^a, E^a) and axial-vector GPDs ($\widetilde{H}^a$, $\widetilde{E}^a$)
  $\rightarrow$ chiral-even $\qquad \mathcal{O}^q = \bar{q}(z)\gamma^+(\gamma^+\gamma_5)q(-z)$

- transversity GPDs (H_T^a, E_T^a, $\widetilde{H}_T^a$, $\widetilde{E}_T^a$)
  $\rightarrow$ chiral-odd $\qquad \mathcal{O}^q = \bar{q}(z)i\sigma^{+i}q(-z)$

$H^a,\ \widetilde{H}^a\ ,\ H_T^q \xrightarrow{\xi=0,t=0}$ PDFs

Selected exclusive processes

DVCS



$$\boxed{\gamma^* N \to \gamma N}$$

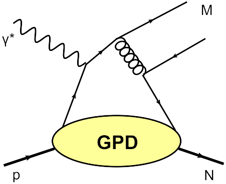
factorization:

[Collins, Freund '99]

H^q, E^q, $\widetilde{H}^q$, $\widetilde{E}^q$

H^g, E^g $\widetilde{H}^g$, $\widetilde{E}^g$ (NLO)

DVMP



$$\boxed{\gamma^* N \to M N'}$$

factorization:

[Collins, Frankfurt, Strikman '97]

H^{q_i}, E^{q_i}; H^g, E^g (V_L)

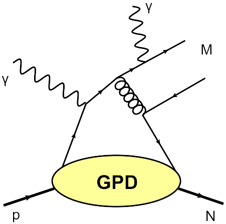
$\widetilde{H}^{q_i}$, $\widetilde{E}^{q_i}$ (PS)

[Collins, Diehl '99]

$$(\cancel{\gamma^* N \to V_T N'}) \Rightarrow (\cancel{F_T^q})$$

$$\boxed{M_{\text{twist-3}} \Rightarrow F_T^q}$$

$(\gamma M)P$



$$\boxed{\gamma N \to \gamma M N'}$$

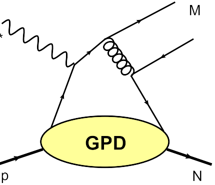
factorization:

[Qiu, Yu '22]

H^a, E^a, $\widetilde{H}^a$, $\widetilde{E}^a$

H_T^a, E_T^a $\widetilde{H}_T^a$, $\widetilde{E}_T^a$

Meson Production: handbag factorization



$\boxed{\gamma^{(*)} N \to M N'}$

DEEPLY VIRTUAL	WIDE ANGLE
$Q^2 >>,\ -t <<$	$-t,\ -u,\ s >>$
DVMP	WAMP
[Collins, Frankfurt, Strikman '97]	[Huang, Kroll '00]

DEEPLY VIRTUAL:

- factorization $\boxed{\mathcal{H}^a \otimes GPD}$
- GPDs at small $(-t)$

WIDE ANGLE:

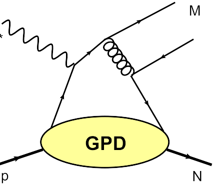
- arguments for factorization $\boxed{\mathcal{H}^a (1/x \otimes GPD(\xi = 0))}$
- GPDs at large $(-t)$

$\mathcal{H}^a$ ... parton subprocess helicity amplitudes

$\Rightarrow \mathcal{M}$ ... hadron helicity amplitudes

$\Rightarrow$ observables (cross sections, asymmetries)

Meson Production: handbag factorization



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DEEPLY VIRTUAL	WIDE ANGLE
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DVMP	WAMP
[Collins, Frankfurt, Strikman '97]	[Huang, Kroll '00]

DVMP

- factorization $\boxed{\mathcal{H}^a \otimes GPD}$
- GPDs at small $(-t)$
- tw2: γ_L^*, tw3: γ_T^*

WAMP

- arguments for factorization $\boxed{\mathcal{H}^a(1/x \otimes GPD(\xi = 0))}$
- GPDs at large $(-t)$

large scale $\mathcal{Q}^2$ (Q^2, s or ...)

- twist expansion: $< \mathcal{H} >^{tw2} + \dfrac{< \mathcal{H} >^{tw3}}{\mathcal{Q}} + \dots$
- α_S expansion for each twist: $\alpha_S(\mathcal{Q}) < \mathcal{H} >^{LO} + \alpha_S^2(\mathcal{Q}) < \mathcal{H} >^{NLO} + \dots$

Status and motivation

- DV (PS) P:
  - tw-2 predictions $(\gamma_L^* N \to \pi N')$ bellow the data [HERMES '09] [JLab '12,'16, '20] [COMPAS '19] $\Rightarrow$ importance of $\gamma_T^* N \to \pi N'$

$\Rightarrow$ tw-3 calculations (WW approximation, i.e., just 2-body tw-3 in PS), with transversity GPDs F_T^q [Goloskokov, Kroll '10] [Ahmad, Goldstein Liuti '09, Goldstein, Hernandez, Liuti '13]

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- WA (PS) P:
  - tw-2 predictions [Huang, Kroll '00] bellow the data [SLAC '76], [JLab '05, '18] for photoproduction ($Q^2 = 0$)
  - tw-3 2-body π photoproduction vanishes [Huang, Jakob, Kroll, P-K '03]

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  $\Rightarrow$ tw-3 (2- and 3-body) prediction for π_0 photoproduction [Kroll, P-K '18] fitted to CLAS data [CLAS '18]; photoproduction of η, η' mesons [Kroll, P-K. '22] [preliminary GlueX '20]

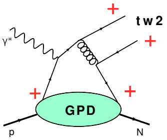
  $\Rightarrow$ tw-3 prediction for $\pi^\pm, \pi^0$ photo- and electroproduction ($Q^2 < -t$) [Kroll, P-K. '21]; extension to DV (PS) P

Wide-angle meson production at twist-3

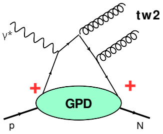
PS meson production to twist-3: subprocess amplitudes $\mathcal{H}$

μ photon helicity, $\lambda\ldots$ quark helicities, $P \in \{\pi^{\pm}, \pi^{0}, \eta_8, \eta_1, \eta, \eta'\}$

$\mathcal{H}^{P}_{0\lambda,\mu\lambda}$ ... non-flip subprocess amplitudes (twist-2)

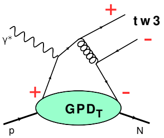


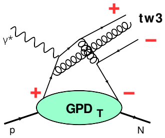
and



for η_1

$\mathcal{H}^{P}_{0-\lambda,\mu\lambda}$ ... flip subprocess amplitudes (twist-3)





$\rightarrow$ just meson DA tw-3 contributions ($\sim \mu_\pi = 2$ GeV $\Rightarrow$ large parameter)

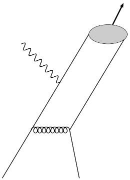
distribution amplitudes (DAs):

twist-2 ($q\bar{q}$) : ϕ_P

2-body ($q\bar{q}$) twist-3 ϕ_{Pp}, $\phi_{P\sigma}$ 3-body ($q\bar{q}g$) twist-3 ϕ_{3P}

$\rightarrow$ connected by equations of motion (EOMs)

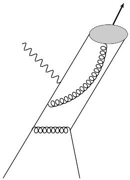
Subprocess amplitudes $\mathcal{H}$: projectors



$q\bar{q} \to \pi$ projector [Beneke, Feldmann '00]

$$(\tau q' + k_\perp) + (\bar{\tau} q' - k_\perp) = q'$$

$$\begin{aligned}
\mathcal{P}_2^\pi \quad \sim \quad & f_\pi \Big\{ \gamma_5 \, \not{q}' \phi_\pi(\tau, \mu_F) \\
& + \mu_\pi(\mu_F) \Big[\gamma_5 \, \phi_{\pi p}(\tau, \mu_F) \\
& - \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \, \frac{q'^\mu n^\nu}{q' \cdot n} \, \phi'_{\pi\sigma}(\tau, \mu_F) \\
& + \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \, q'^\mu \phi_{\pi\sigma}(\tau, \mu_F) \frac{\partial}{\partial k_{\perp\nu}} \Big] \Big\}_{k_\perp \to 0}
\end{aligned}$$



$q\bar{q}g \to \pi$ projector [Kroll, P-K '18]

$$\tau_a q' + \tau_b q' + \tau_g q' = q'$$

$$\mathcal{P}_3^\pi \sim f_{3\pi}(\mu_F) \, \frac{i}{g} \, \gamma_5 \, \sigma_{\mu\nu} q'^\mu g_\perp^{\nu\rho} \, \frac{\phi_{3\pi}(\tau_a, \tau_b, \tau_g, \mu_F)}{\tau_g}$$

$\mu_\pi = m_\pi^2/(m_u + m_d) \cong 2$ GeV, $f_{3\pi} \sim \mu_\pi$

Subprocess amplitudes: twist-3

General structure:

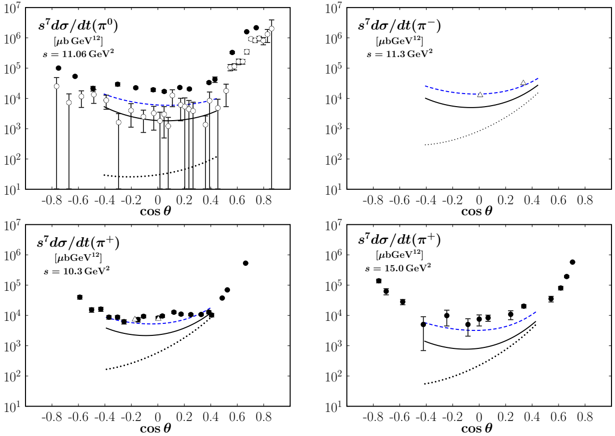
$$
\begin{aligned}
\mathcal{H}^{P,tw3} &= \mathcal{H}^{P,tw3,q\bar{q}} + \mathcal{H}^{P,tw3,q\bar{q}g} \\
&= \left(\mathcal{H}^{P,\phi_{Pp}} + \underbrace{\mathcal{H}^{P,\phi_{P2}^{EOM}}\right) + \left(\mathcal{H}^{P,q\bar{q}g,C_F}}_{} + \mathcal{H}^{P,q\bar{q}g,C_G}\right) \\
&= \mathcal{H}^{P,\phi_{Pp}} \quad + \qquad \mathcal{H}^{P,\phi_{3P},C_F} \qquad + \mathcal{H}^{P,\phi_{3P},C_G}
\end{aligned}
$$

- 2- and 3-body contributions necessary for gauge invariance
- photoproduction ($Q \to 0$): $\mathcal{H}^{P,\phi_{Pp}} = 0$ [Kroll, P-K '18]
- no end-point singularities for $\hat{t} \neq 0$!

Photoproduction (π)

- complete tw-3 prediction for π_0 photoproduction fitted to CLAS data and obtained predictions for $\pi^{\pm}$

[Kroll, P-K '21]



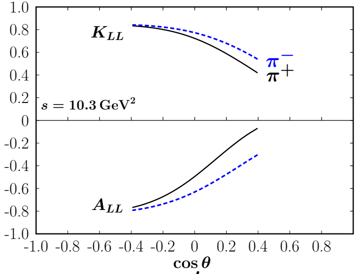
solid curves:
complete twist-3
dotted curves: twist-2

exp data:
full circles [SLAC '76]
open circles [CLAS '17]
triangles [JLab, Hall A '05]

- twist-2 prediction well below the data

Spin effects - photoproduction

[Kroll, P-K '21]: $\pi^{\pm}$



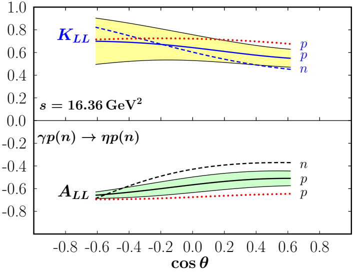
$A_{LL}(K_{LL})\ldots$ correlation of the helicities of the photon and incoming (outgoing) nucleon

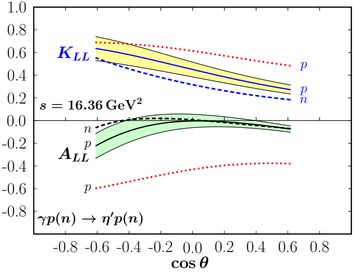
$$A_{LL}^{P,tw2} = K_{LL}^{P,tw2}$$

$$A_{LL}^{P,tw3} = -K_{LL}^{P,tw3}$$

$\rightarrow$ characteristic signature for dominance of twist-3 (like $\sigma_T \gg \sigma_L$ in DVMP)

[Kroll, P-K '22]: η, η'





$\rightarrow$ in contrast to π and η, for η' dominance of twist-2 and sensitivity to gluons

DVMP

Subprocess amplitudes for electroproduction: twist-2

Transverse photon polarization ($\mu = \pm 1$)   T

$$\mathcal{H}^{\pi,tw2}_{0\lambda,\,\mu\lambda} \sim f_\pi\, C_F\, \alpha_s(\mu_R)\, \frac{\sqrt{-\hat{t}}}{\hat{s}+Q^2} \int_0^1 d\tau\, \phi_\pi(\tau) \Bigg[(2\lambda\mu+1)\left(\frac{(\hat{s}\tau+Q^2)(\hat{s}+Q^2)-\hat{u}Q^2\bar{\tau}}{\hat{s}\bar{\tau}(Q^2\bar{\tau}-\hat{t}\tau)}\, e_a + \frac{(\hat{s}\tau-Q^2)(\hat{s}+Q^2)-\hat{u}Q^2\bar{\tau}}{\hat{u}\tau(Q^2\tau-\hat{t}\bar{\tau})}\, e_b\right) + (2\lambda\mu-1)\left(\frac{\hat{u}\, e_a}{(Q^2\bar{\tau}-\hat{t}\tau)} + \frac{\hat{s}\bar{\tau}\, e_b}{\tau(Q^2\tau-\hat{t}\bar{\tau})}\right)\Bigg]$$

Longitudinal photon polarization   L

$$\mathcal{H}^{\pi,tw2}_{0\lambda,\,0\lambda} \sim f_\pi\, C_F\, \alpha_s(\mu_R)\, \lambda\, \frac{Q\sqrt{-\hat{u}\hat{s}}}{\hat{s}+Q^2} \int_0^1 d\tau\, \phi_\pi(\tau) \left(\frac{\hat{u}\, e_a}{\hat{s}(Q^2\bar{\tau}-\hat{t}\tau)} - \frac{(\hat{t}+\tau\hat{u})\, e_b}{\tau\hat{u}(Q^2\tau-\hat{t}\bar{\tau})}\right)$$

$\rightarrow$ photoproduction ($Q \rightarrow 0$):

$$\boxed{\mathcal{H}^{\pi,tw2}_{T}\Big|_{Q\to 0} \sim f_\pi\, C_F\, \alpha_s(\mu_R)\, \frac{1}{\sqrt{-\hat{t}}} \int_0^1 \frac{d\tau}{\tau}\, \phi_\pi(\tau)\, \big((1+2\lambda\mu)\,\hat{s} - (1-2\lambda\mu)\,\hat{u}\big) \left(\frac{e_a}{\hat{s}} + \frac{e_b}{\hat{u}}\right)}$$

Subprocess amplitudes for electroproduction: twist-2

Transverse photon polarization ($\mu = \pm 1$) T

$$\mathcal{H}^{\pi,tw2}_{0\lambda,\,\mu\lambda} \sim f_\pi\, C_F\, \alpha_s(\mu_R)\, \frac{\sqrt{-\hat{t}}}{\hat{s}+Q^2} \int_0^1 d\tau\, \phi_\pi(\tau) \Bigg[(2\lambda\mu+1)\left(\frac{(\hat{s}\tau+Q^2)(\hat{s}+Q^2)-\hat{u}Q^2\bar{\tau}}{\hat{s}\bar{\tau}(Q^2\bar{\tau}-\hat{t}\tau)}\, e_a + \frac{(\hat{s}\tau-Q^2)(\hat{s}+Q^2)-\hat{u}Q^2\bar{\tau}}{\hat{u}\tau(Q^2\tau-\hat{t}\bar{\tau})}\, e_b\right) + (2\lambda\mu-1)\left(\frac{\hat{u}\, e_a}{(Q^2\bar{\tau}-\hat{t}\tau)} + \frac{\hat{s}\bar{\tau}\, e_b}{\tau(Q^2\tau-\hat{t}\bar{\tau})}\right)\Bigg]$$

Longitudinal photon polarization L

$$\mathcal{H}^{\pi,tw2}_{0\lambda,\,0\lambda} \sim f_\pi\, C_F\, \alpha_s(\mu_R)\, \lambda\, \frac{Q\sqrt{-\hat{u}\hat{s}}}{\hat{s}+Q^2} \int_0^1 d\tau\, \phi_\pi(\tau) \left(\frac{\hat{u}\, e_a}{\hat{s}(Q^2\bar{\tau}-\hat{t}\tau)} - \frac{(\hat{t}+\tau\hat{u})\, e_b}{\tau\hat{u}(Q^2\tau-\hat{t}\bar{\tau})}\right)$$

$\rightarrow$ photoproduction ($Q \rightarrow 0$):

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$\rightarrow$ DVMP ($\hat{t} \rightarrow 0$):

$\mathcal{H}^{\pi,tw2}_{L}\Big|_{\hat{t}\rightarrow 0}$: $\hat{s} = -\dfrac{\xi - x}{2\xi}\, Q^2\,,\ \hat{u} = -\dfrac{\xi + x}{2\xi}\, Q^2$ $\Rightarrow$ well known LO result for DVMP

Twist-2 NLO predictions for $\left\langle \mathcal{H}_L^{\mathsf{DV}\pi\mathsf{P},tw2} \right\rangle \to \sigma_L^{\mathsf{DV}\pi\mathsf{P}}$

[Belitsky, Müller '01], [Müller, Lautenschlager, P-K., Schäfer '14], [Duplančić, Müller, P-K., '17]

- large NLO corrections and model dependence

- LO evolution important
- NLO calculations should include NLO evolution
  (conformal momentum representation favorable)

- results sensitive to the choice of DA

Subprocess amplitudes: twist-3

General structure:

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\end{aligned}
$$

- DVMP ($\hat{t} \to 0$):
  - end-point singularities in $\mathcal{H}^{P,\phi_{Pp}}$ $\int_0^1 \frac{d\tau}{\bar{\tau}} \phi_{Pp}(\tau)$,

$$\underline{\phi_{Pp}(\tau) = 1 + a_{2p} C_2^{1/2}(2\tau - 1) + \ldots}$$

$\Rightarrow$ modified hard-scattering picture

(with $k_\perp$ quark transverse momenta) [Goloskov, Kroll, '10]

$\Rightarrow$ pure collinear picture with effective m_g^2 [Shuryak, Zahed '20]

Subprocess amplitudes: twist-3

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$$

- DVMP ($\hat{t} \to 0$): $\hat{s} = -\frac{\xi - x}{2\xi} Q^2, \hat{u} = -\frac{\xi + x}{2\xi} Q^2$

$$
\mathcal{H}^{\pi,\phi_{\pi p}}_{0-\lambda,\mu\lambda}|_{\hat{t}\to 0} \sim (2\lambda + \mu)\; f_\pi \mu_\pi C_F \alpha_S(\mu_R) \sqrt{-\hat{u}\hat{s}} \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2}\right) \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}} \phi_{\pi p}(\tau)}
$$

$$
\mathcal{H}^{\pi,\phi_{3\pi},C_F}_{0-\lambda,\mu\lambda}|_{\hat{t}\to 0} \sim -(2\lambda + \mu)\; f_{3\pi}\, C_F \alpha_S(\mu_R) \sqrt{-\hat{u}\hat{s}} \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2}\right) \times \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}^2} \int_0^{\bar{\tau}} \frac{d\tau_g}{\tau_g(\bar{\tau} - \tau_g)} \phi_{3\pi}(\tau, \bar{\tau} - \tau_g, \tau_g)}
$$

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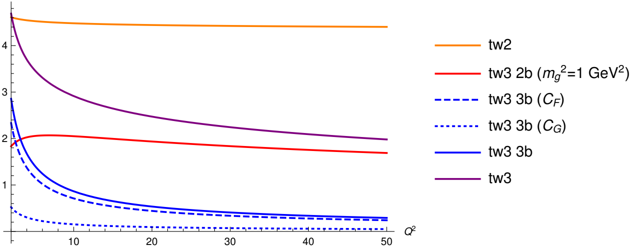
$$

Subprocess amplitudes: twist-3 2-body vs. 3-body

- tw3 3-body contributions are significant

  (60% at $Q^2 = 2$ GeV^2 and 20% at $Q^2 = 20$ GeV^2)



tw2 : $3\, C_F\, (1 + a_2 + a_4)$

tw3 2b : $\boxed{\dfrac{\mu_\pi}{Q}\, C_F \displaystyle\int_0^1 d\tau \frac{1}{\tau + m_g^2/Q^2}\, \phi_{Pp}}$

tw3 3b(C_F) : $\boxed{-\dfrac{\mu_\pi}{Q}\, C_F\, R \left(-20 + \dfrac{15}{4}\omega_{1,0} - \dfrac{24}{5}\omega_{2,0} + \dfrac{6}{5}\omega_{1,1}\right)}$

tw3 3b(C_G) : $\boxed{\dfrac{\mu_\pi}{Q}\, C_G\, R \left(-30 + 10\omega_{1,0} - 8\omega_{2,0} + \dfrac{1}{2}\omega_{1,1}\right)}$

- DVπ_0P numerical analysis underway [Duplančić, Kroll, P-K., Szymanowsky]

Summary

- complete (2- and 3-body) twist-3 prediction for PS electroproduction has been obtained

- 3-body tw3 contributions needed for the gauge invariance of the results but are also numerically important

- WA (PS) P / photoproduction:
  - meson's twist-3 contributions dominate for πs and η
  - possibility of extraction large $-t$ behaviour of transversity GPDs (F_T^q)

- DV (PS) P
  - similary twist-3 dominates (γ_T^*)
  - twist-2 (γ_L^*) NLO contributions available, possibly large and should be included
  - meson DA additional nontrivial nonperturbative input
  - complete numerical twist-3 analysis underway

Summary

- complete (2- and 3-body) twist-3 prediction for PS electroproduction has been obtained
- 3-body tw3 contributions needed for the gauge invariance of the results but are also numerically important
- WA (PS) P / photoproduction:
  - meson's twist-3 contributions dominate for πs and η
  - possibility of extraction large $-t$ behaviour of transversity GPDs (F_T^q)
- DV (PS) P
  - similary twist-3 dominates (γ_T^*)
  - twist-2 (γ_L^*) NLO contributions available, possibly large and should be included
  - meson DA additional nontrivial nonperturbative input
  - complete numerical twist-3 analysis underway

Thank you.

Helicity amplitudes $\mathcal{M}$ for WAMP

$$\mathcal{M}^P_{0+,\mu+} = \frac{e_0}{2}\sum_\lambda \Big[\mathcal{H}^P_{0\lambda,\mu\lambda}\left(R^P_V(t) + 2\lambda\, R^P_A(t)\right) \quad \to \text{twist-2}$$

$$-2\lambda\,\frac{\sqrt{-t}}{2m}\,\mathcal{H}^P_{0-\lambda,\mu\lambda}\,\bar{S}^P_T(t)\Big] \quad \to \text{twist-3}$$

$$\mathcal{M}^P_{0-,\mu+} = \frac{e_0}{2}\sum_\lambda \Big[\frac{\sqrt{-t}}{2m}\,\mathcal{H}^P_{0\lambda,\mu\lambda}\,R^P_T(t) \quad \to \text{twist-2}$$

$$-2\lambda\,\frac{t}{2m^2}\,\mathcal{H}^P_{0-\lambda,\mu\lambda}\,S^P_S(t)\Big] + e_0\mathcal{H}^P_{0-,\mu+}\,S^P_T(t) \quad \to \text{twist-3}$$

μ photon helicity, $\lambda\ldots$ quark helicities, $P \in \{\pi^\pm, \pi^0, \eta_8, \eta_1, \eta, \eta'\}$,

$$\boxed{R^a_V(t) = \int \frac{dx}{x}\, H^a(x, \xi = 0, t)}$$

. . . form factors

$$a \in \{u, d\} \Rightarrow R^{\pi^\pm}_V = R^u_V - R^d_V,\; R^{\pi^0}_V = \frac{1}{\sqrt{2}}\left(e_u R^u_V - e_d R^d_V\right)$$

$$R^{\eta_8}_V \approx \frac{1}{\sqrt{2}} R^{\eta_1}_V \approx \frac{1}{\sqrt{6}}\left(e_u R^u_V + e_d R^d_V\right)$$

$$(H, \tilde{H}, E) \to (R_V, R_A, R_T)$$

$$(H_T, \tilde{H}_T, \bar{E}_T) \to (S_T, S_S, \bar{S}_T)$$ transversity GPDs

DAs and EOMs

$$\tau\phi_{\pi p}(\tau)+\frac{\tau}{6}\,\phi'_{\pi\sigma}(\tau)-\frac{1}{3}\,\phi_{\pi\sigma}(\tau) \;=\; \phi^{EOM}_{\pi 2}(\bar\tau)$$

$$\bar\tau\phi_{\pi p}(\tau)-\frac{\bar\tau}{6}\phi'_{\pi\sigma}(\tau)-\frac{1}{3}\phi_{\pi\sigma}(\tau) \;=\; \phi^{EOM}_{\pi 2}(\tau)$$

$$\phi^{EOM}_{\pi 2}(\tau)=2\,\frac{f_{3\pi}}{f_\pi\mu_\pi}\int_0^{\bar\tau}\frac{d\tau_g}{\tau_g}\,\phi_{3\pi}(\tau,\bar\tau-\tau_g,\tau_g)$$

- EOMs and symmetry properties
  $\Rightarrow$ the subprocess amplitudes in terms of two twist-3 DAs
  and 2- and 3-body contributions combined
- combined EOMs $\rightarrow$ first order differential equation $\Rightarrow$ from known form of $\phi_{3\pi}$ [Braun, Filyanov '90] one determines $\phi_{\pi p}$ (and $\phi_{\pi\sigma}$)

Note: $q\bar{q}g$ projector and EOMs were derived using light-cone gauge for constituent gluon

Subprocess amplitudes: twist-3

General structure:

$$
\begin{aligned}
\mathcal{H}^{P,tw3} &= \mathcal{H}^{P,tw3,q\bar{q}} + \mathcal{H}^{P,tw3,q\bar{q}g} \\
&= \Big(\mathcal{H}^{P,\phi_{\pi p}} + \underbrace{\mathcal{H}^{P,\phi_{\pi 2}^{EOM}}\Big) + \Big(\mathcal{H}^{P,q\bar{q}g,C_F}} + \mathcal{H}^{P,q\bar{q}g,C_G}\Big) \\
&= \mathcal{H}^{P,\phi_{\pi p}} + \mathcal{H}^{P,\phi_{3\pi},C_F} + \mathcal{H}^{P,\phi_{3\pi},C_G}
\end{aligned}
$$

- 2-body twist-3 $\sim C_F$; 3-body C_F and C_G proportional parts
- C_G part is separately gauge invariant
- the sum of 2- and 3-body C_F parts is gauge invariant (QED and QCD)
- no end-point singularities for $\hat{t} \neq 0$!

Subprocess amplitudes: twist-3 at $Q <<$ or $\hat{t} <<$

General structure:

$$
\begin{aligned}
\mathcal{H}^{P,tw3} &= \mathcal{H}^{P,tw3,q\bar{q}} + \mathcal{H}^{P,tw3,q\bar{q}g} \\
&= \left(\mathcal{H}^{P,\phi_{\pi p}} + \underbrace{\mathcal{H}^{P,\phi_{\pi 2}^{EOM}}\right) + \left(\mathcal{H}^{P,q\bar{q}g,C_F}}\, + \mathcal{H}^{P,q\bar{q}g,C_G}\right) \\
&= \mathcal{H}^{P,\phi_{\pi p}} \quad + \qquad \mathcal{H}^{P,\phi_{3\pi},C_F} \qquad + \mathcal{H}^{P,\phi_{3\pi},C_G}
\end{aligned}
$$

- $\mathcal{H}_L^{P,tw3} \sim Q\sqrt{-t} \to 0$ both for $Q \to 0$ and $\hat{t} \to 0$
- photoproduction ($Q \to 0$):
  - $\mathcal{H}^{P,\phi_{\pi p}} = 0$ [Kroll, P-K '18]
- DVMP ($\hat{t} \to 0$):
  - end-point singularities in $\mathcal{H}^{P,\phi_{\pi p}}$ [Goloskokov, Kroll '10]
  - $\mathcal{H}^{P,\phi_{\pi 2}^{EOM}} = 0$

Subprocess amplitudes: twist-3 at $Q \to 0$, $t \to 0$

photoproduction

$$\mathcal{H}^{P,tw3}_{0-\lambda,\,\mu\lambda}|_{Q^2\to 0} \sim (2\lambda-\mu)\, f_{3\pi}\,\alpha_S(\mu_R)\sqrt{-\hat{u}\hat{s}}\int_0^1 d\tau \int_0^{\bar{\tau}} \frac{d\tau_g}{\tau_g}\,\phi_{3\pi}(\tau,\bar{\tau}-\tau_g,\tau_g)$$
$$\times\left[C_F\left(\frac{1}{\bar{\tau}^2}-\frac{1}{\bar{\tau}(\bar{\tau}-\tau_g)}\right)\left(\frac{e_a}{\hat{s}^2}+\frac{e_b}{\hat{u}^2}\right)+ -C_G\,\frac{2}{\tau\tau_g}\,\frac{\hat{t}}{\hat{s}\hat{u}}\left(\frac{e_a}{\hat{s}}+\frac{e_b}{\hat{u}}\right)\right]$$

DVMP

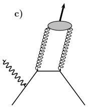
$$\mathcal{H}^{P,\phi_{\pi p}}_{0-\lambda,\mu\lambda}|_{\hat{t}\to 0} \sim (2\lambda+\mu)\; f_\pi\mu_\pi C_F\alpha_S(\mu_R)\sqrt{-\frac{\hat{u}}{\hat{s}}}\left[\frac{e_a}{\hat{s}}+\frac{\hat{s}}{\hat{u}}\frac{e_b}{\hat{u}}\right]\int_0^1\frac{d\tau}{\bar{\tau}}\phi_{\pi p}(\tau)$$

$$\mathcal{H}^{P,C_F,\phi_{3\pi}}_{0-\lambda,\mu\lambda}|_{\hat{t}\to 0} \sim -(2\lambda+\mu)\, f_{3\pi}\, C_F\alpha_S(\mu_R)\sqrt{-\frac{\hat{u}}{\hat{s}}}\left(\frac{e_a}{\hat{s}}+\frac{\hat{s}}{\hat{u}}\,\frac{e_b}{\hat{u}}\right)$$
$$\times\int_0^1\frac{d\tau}{\bar{\tau}^2}\int_0^{\bar{\tau}}\frac{d\tau_g}{\tau_g(\bar{\tau}-\tau_g)}\,\phi_{3\pi}(\tau,\bar{\tau}-\tau_g,\tau_g)$$

$$\mathcal{H}^{P,qqg,C_G}_{0-\lambda,\mu\lambda}|_{\hat{t}\to 0} \sim (2\lambda+\mu)\; f_{3\pi}\, C_G\alpha_S(\mu_R)\frac{Q^2}{\sqrt{-\hat{s}\hat{u}}}\left(\frac{e_a}{\hat{s}}+\frac{e_b}{\hat{u}}\right)$$
$$\times\int_0^1\frac{d\tau}{\bar{\tau}}\int_0^{\bar{\tau}}\frac{d\tau_g}{\tau_g(\bar{\tau}-\tau_g)}\,\phi_{3\pi}(\tau,\bar{\tau}-\tau_g,\tau_g)$$

Subprocess amplitudes $\mathcal{H}^{\eta_8,\eta_1} \to \mathcal{H}^{\eta,\eta'}$

Novel features:

1. 

$|gg\rangle$ states contribute to twist-2
   - $\mathcal{H}^{\pi,tw2} \Rightarrow \mathcal{H}^{\eta_8,tw2}, \mathcal{H}^{\eta_{1,q},tw2}$ $\quad (\phi_\pi, f_\pi) \to (\phi_{\eta_8}, f_{\eta_8}), (\phi^q_{\eta_1}, f_{\eta_1})$

$\mathcal{H}^{\eta_1} = \mathcal{H}^{\eta_{1q},tw2} + \mathcal{H}^{\eta_{1g},tw2}$ $\quad \phi^q_{\eta_1}$ and $\phi^g_{\eta_1}$ mix under evolution

   - $\mathcal{H}^{\pi,tw3} \Rightarrow \mathcal{H}^{P,tw3}$ $\quad (\phi_{3\pi}, f_\pi, f_{3\pi}) \to (\phi_{3P}, f_P, f_{3P})$

2. flavour-mixing:
   - simplest: flavour-mixing embedded in the decay constants

$$f^8_\eta = f_8 \cos\theta_8 \qquad f^1_\eta = -f_1 \sin\theta_1$$

$$f^8_{\eta'} = f_8 \sin\theta_8 \qquad f^1_{\eta'} = f_1 \cos\theta_1$$

[review Feldmann '00]

Pion distribution amplitudes

Twist-2 DA: $\boxed{\phi_\pi(\tau,\mu_F) = 6\tau\bar{\tau}\left[1 + a_2(\mu_F)\, C_2^{3/2}(2\tau-1)\right]}$

Twist-3 DAs:

$$\begin{aligned}
\phi_{3\pi}(\tau_a,\tau_b,\tau_g,\mu_F) &= 360\tau_a\tau_b\tau_g^2\Big[1 + \omega_{1,0}(\mu_F)\,\frac{1}{2}(7\tau_g - 3) \\
&+ \omega_{2,0}(\mu_F)\,(2 - 4\tau_a\tau_b - 8\tau_g + 8\tau_g^2) \\
&+ \omega_{1,1}(\mu_F)\,(3\tau_a\tau_b - 2\tau_g + 3\tau_g^2)\Big]
\end{aligned}$$
[Braun, Filyanov '90]

using EOMs [Kroll, P-K '18]:

$$\begin{aligned}
\phi_{\pi p}(\tau,\mu_F) &= 1 + \frac{1}{7}\frac{f_{3\pi}(\mu_F)}{f_\pi\mu_\pi(\mu_F)}\Big(7\,\omega_{1,0}(\mu_F) - 2\,\omega_{2,0}(\mu_F) - \omega_{1,1}(\mu_F)\Big) \\
&\times\Big(10\,C_2^{1/2}(2\tau-1) - 3\,C_4^{1/2}(2\tau-1)\Big), \quad \phi_{\pi\sigma}(\tau) = \dots
\end{aligned}$$

Parameters:

- $a_2(\mu_0) = 0.1364 \pm 0.0213$ at $\mu_0 = 2$ GeV [Braun et al '15] (lattice)
- $\omega_{10}(\mu_0) = -2.55\,, \omega_{10}(\mu_0) = 0.0$ and $f_{3\pi}(\mu_0) = 0.004\ \mathrm{GeV}^2$. [Ball '99]
- $\omega_{20}(\mu_0) = 8.0$ [Kroll, P-K '18] fit to π^0 photoproduction data [CLAS '17]

Evolution of the decay constants and DA parameters taken into account.
Choice of scales: ${\mu_R}^2 = {\mu_F}^2 = \hat{t}\hat{u}/\hat{s}$

η, η' distribution amplitudes

Twist-2 DA:

$$\boxed{\phi_8(\tau,\mu_F) \;=\; 6\tau\bar{\tau}\left[1 + a_2^8(\mu_F)\, C_2^{3/2}(2\tau-1)\right]}$$

$$\boxed{\begin{aligned} \phi_{1,q}(\tau,\mu_F) &= 6\tau\bar{\tau}\left[1 + a_2^1(\mu_F)\, C_2^{3/2}(2\tau-1)\right] \\ \phi_{1,g}(\tau,\mu_F) &= 30\tau^2\bar{\tau}^2\left[1 + a_2^g(\mu_F)\, C_1^{5/2}(2\tau-1)\right] \end{aligned}}$$

Twist-3 DAs:

assumption

$$\boxed{\phi_{38}(\tau_a,\tau_b,\tau_g,\mu_F) = \phi_{31}(\tau_a,\tau_b,\tau_g,\mu_F) \approx \phi_{3\pi}(\tau_a,\tau_b,\tau_g,\mu_F)}$$

Parameters:

- $a_2^8(\mu_0) = -0.039$, $a_2^1(\mu_0) = -0.057$, $a_2^g(\mu_0) = 0.038$ [Kroll, KPK '13], and other choices tested
- $f_{38}(\mu_0) = 0.86 f_{3\pi}(\mu_0) \Leftarrow$ [Ball '99; Braun, Filyanov '90]
- $f_{31}(\mu_0) = 0.86 f_{3\pi}(\mu_0) \Leftarrow \eta$ exp: [GlueX preliminary '20]
- mixing parameters from [Feldmann, Kroll, Stech '98]

Form factors and GPDs

$R_i \ldots 1/x$ moment of $\xi = 0$ GPD (K_i)

- $R_V(\leftarrow H)$, $R_T(\leftarrow E)$ from nucleon form factor analysis [Diehl, Kroll '13]
- $R_A(\leftarrow \tilde{H})$ form factor analysis and WACS KLL asymmetry [Kroll '17]
- $S_T(\leftarrow H_T)$, $\bar{S}_T(\leftarrow \bar{E}_T)$ low $-t$ from DVMP analysis [Goloskokov, Kroll '11]
- $S_S(\leftarrow \tilde{H}_T) \cong \bar{S}_T/2$ ($\bar{E}_T = 2\tilde{H}_T + E_T$)

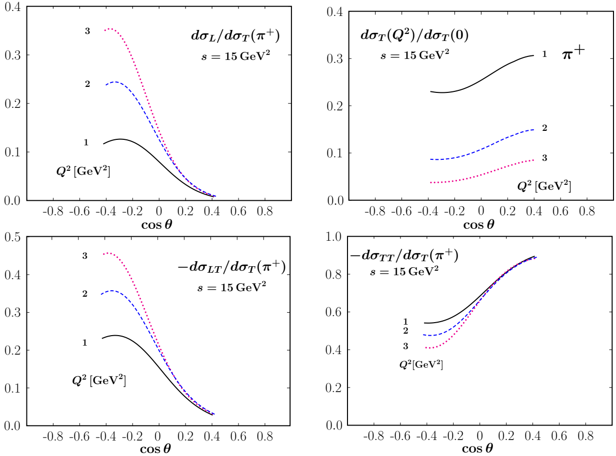
GPD parameterization [Diehl, Feldmann, Jakob, Kroll '04, Diehl, Kroll '13]

$$K_i^a = k_i^a(x) \exp\left[t f_i^a(x)\right], \; f_i^a(x) = \big(B_i^a - \alpha_i'^a \ln x\big)(1-x)^3 + A_i^a x (1-x)^2$$

- strong $x - t$ correlation
- power behaviour for large $(-t)$
- choice for transversity GPDs $A = 0.5$ GeV^{-2}

Electroproduction (π): $Q^2 < -t$

[Kroll, P-K '21]



- both for σ_L and σ_{LT} no twist-2 and twist-3 interference $\Rightarrow$ information on H_T
- σ_{TT} $\Rightarrow$ information on $\tilde{H}_T$ (suppressed for DVMP)

DVMP

Transition form factors

$$\boxed{{}^a\mathcal{T}(\xi,t,Q^2) = \int \mathrm{d}x \int \mathrm{d}u\, T^a(x,\xi,u,\mu_\varphi,\mu_F)\; F^a(x,\xi,t,\mu_\varphi)\; \phi(u,\mu_F)}$$

$a = q, g$ or NS,S(Σ, g)

hard-scattering amplitude (known up to NLO)

$$\begin{aligned} T^a(x,\xi,u,\mu_\varphi,\mu_F) \quad = \quad & \frac{\alpha_s(\mu_R)}{4\pi} T^{a(1)}(x,\xi,u) \\ & + \frac{\alpha_s^2(\mu_R)}{(4\pi)^2} T^{a(2)}(x,\xi,u,\mu_R,\mu_\varphi,\mu_F) + \cdots \end{aligned}$$

distribution amplitude (DA) evolution, similary GPD (F^a) evolution (known up to NNLO)

$$\begin{aligned} \phi(x;\mu_F,\mu_0) \quad = \quad & \phi^{(0)}(u,\mu_F,\mu_0) + \frac{\alpha_s(\mu_F)}{4\pi}\phi^{(1)}(u,\mu_F,\mu_0) \\ & + \frac{\alpha_s^2(\mu_F)}{(4\pi)^2}\phi^{(2)}(u,\mu_F,\mu_0) + \cdots \end{aligned}$$

$\rightarrow$ evolution simpler to implement in conformal momentum representation [Müller '98]

From x space to conformal momentum space

$$\boxed{{}^a\mathcal{T}(\xi, t, Q^2) = \int \mathrm{d}x \int \mathrm{d}u \, T^a(x, \xi, y, \mu^2)) \, F^a(x, \xi, t, \mu^2) \, \phi(u, \mu^2)}$$

F...GPDs,a=q,g or NS,S(Σ,g)

conformal moments (analogous to Mellin moments in DIS $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$)

[Müller, Lautenschläger, P-K., Schäfer 2014] [Duplančić, Müller, P-K. 2017]

$$\begin{aligned} {}^a\mathcal{T}(\xi, t, \mathcal{Q}^2) &= \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \left[i \pm \left\{ \begin{array}{c} \tan \\ \cot \end{array} \right\} \left(\frac{\pi j}{2} \right) \right] \xi^{-j-1} \\ &\quad \times \left[\mathrm{T}_{jk}(\mathcal{Q}^2/\mu^2) \overset{k}{\otimes} \phi_{\mathrm{M},k}(\mu^2) \right] F_j^{\mathsf{a}}(\xi, t, \mu^2) \end{aligned}$$

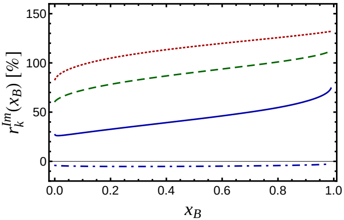
all channels calculated to NLO :

$\mathcal{H}_M^{q^{(+)}}, \mathcal{E}_M^{q^{(+)}}, \mathcal{H}_M^{\mathsf{g}}, \mathcal{E}_M^{\mathsf{g}}$ $\widetilde{\mathcal{H}}_M^{q^{(-)}}, \widetilde{\mathcal{E}}_M^{q^{(-)}}$	1_L^{--}=V$_L$ 0^{-+}=PS	$\mathcal{H}_M^{q^{(-)}}, \mathcal{E}_M^{q^{(-)}}$ $\widetilde{\mathcal{H}}_M^{q^{(+)}}, \widetilde{\mathcal{E}}_M^{q^{(+)}}, \widetilde{\mathcal{H}}_M^{\mathsf{g}}, \widetilde{\mathcal{E}}_M^{\mathsf{g}}$	0^{++}=S 1_L^{+-}=PV$_L$

(x-space, conformal mom. space, imaginary parts for disp. relations)

Twist-2 NLO predictions for $\left\langle \mathcal{H}_L^{\mathrm{DV}\pi\mathrm{P},tw2} \right\rangle$

[Duplančić, Müller, P-K., '17]



Relative NLO corrections to Im $< \mathcal{H}_L^{tw2} >$ for different DA conf. moments:

solid: $k = 0$ (asymptotic)

dashed: $k = 2$

dotted: $k = 4$

- NLO corrections higher for higher DA conformal moments

  $\Rightarrow$ important for non-asymptotic DAs