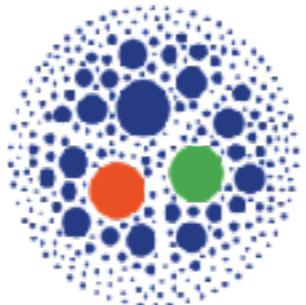




Istituto Nazionale di Fisica Nucleare



HAS QCD
HADRONIC STRUCTURE AND
QUANTUM CHROMODYNAMICS

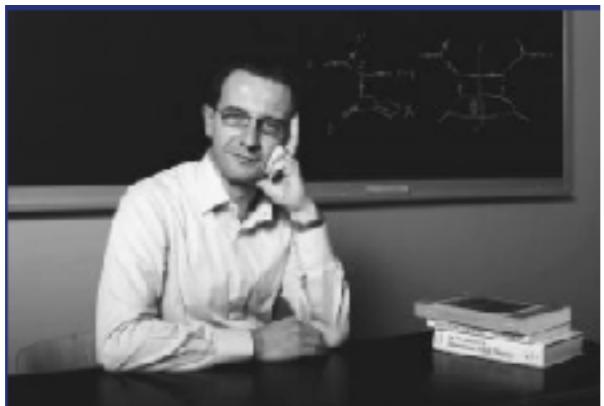


UNIVERSITÀ
DI PAVIA



TMDs global extractions by the MAP Collaboration

Alessandro Bacchetta



Marco Radici



Valerio Bertone



Andrea Signori



Chiara Bissolotti



Giuseppe Bozzi



Lorenzo Rossi



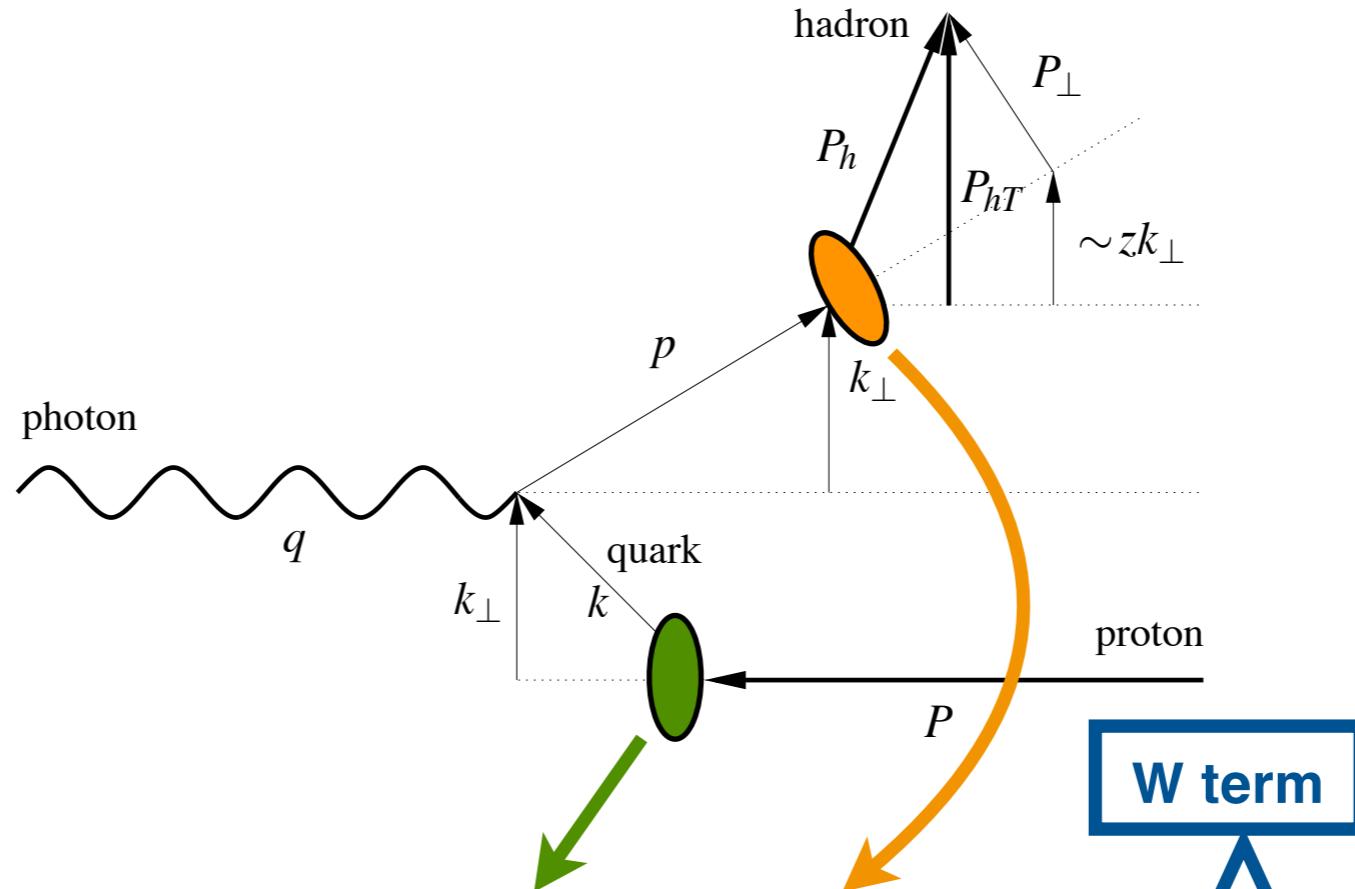
Simone Venturini



Fulvio Piacenza



TMD Factorization - SIDIS



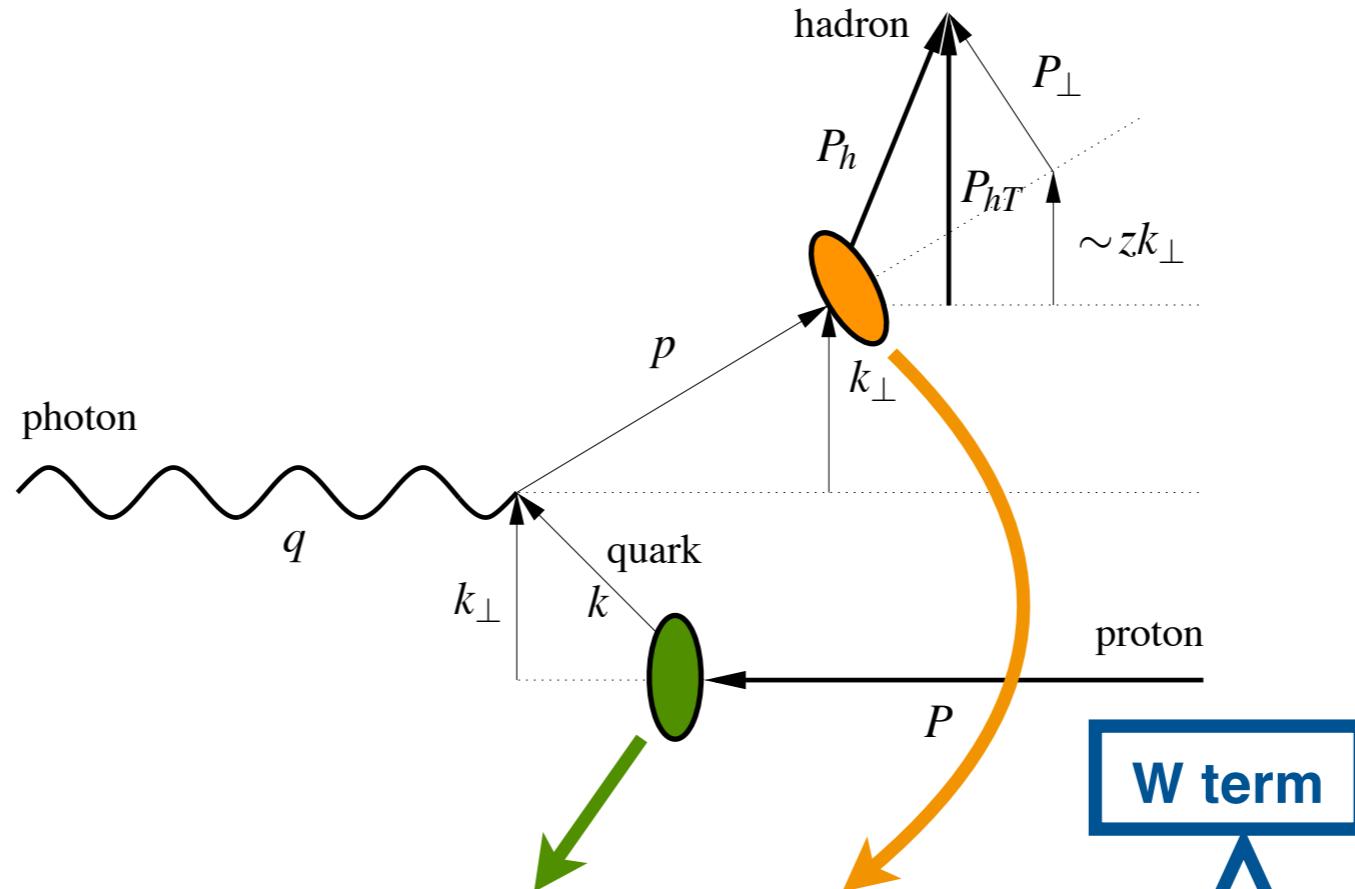
$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2)$$

$$= x \sum_a \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)$$

$$= x \sum_a \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_\perp^2; \mu^2)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

TMD Factorization - SIDIS



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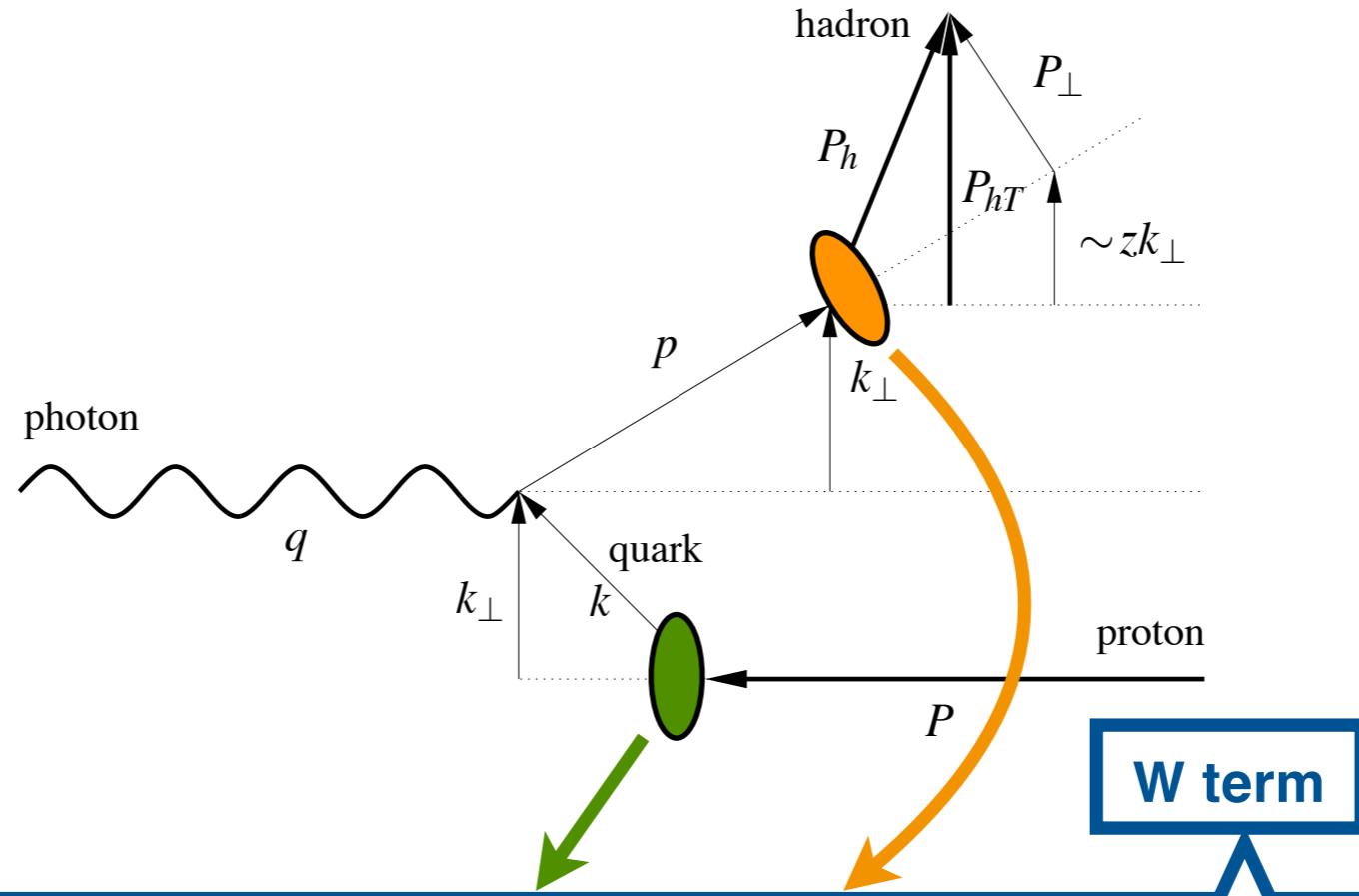
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The W term dominates in the region where $qT \ll Q$

TMD Factorization - SIDIS



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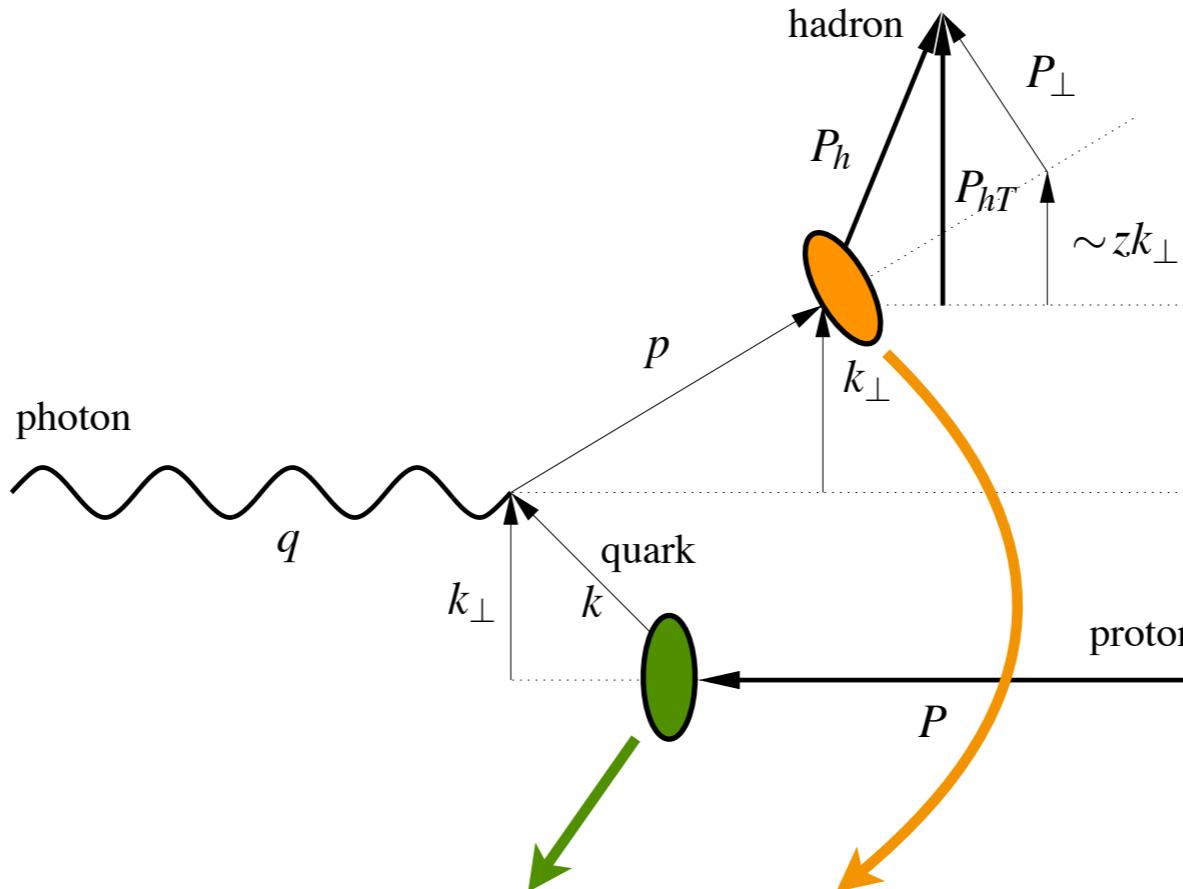
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Bacchetta, Diehl, et al., JHEP 02 (2007)

- The W term dominates in the region where $qT \ll Q$
- Y term has been excluded in the Pavia analyses

TMD Factorization - SIDIS



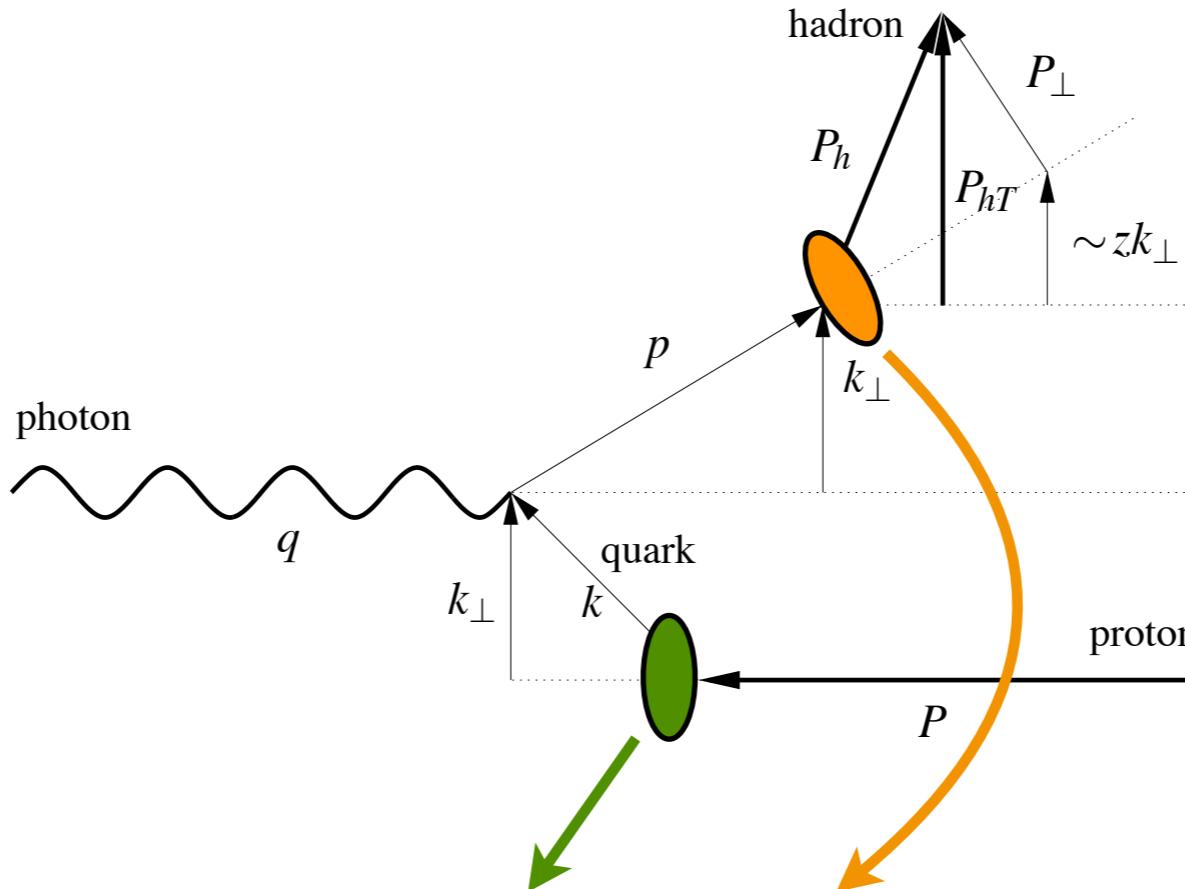
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TMD Factorization - SIDIS



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Bacchetta, Diehl, et al., JHEP 02 (2007)

- ➊ Fourier-transformed space to avoid convolutions
- ➋ TMDs formally depend on two scales, but we set them equal

TMD Factorization - Drell—Yan

The diagram illustrates the TMD factorization of the Drell-Yan process. A nucleon with momentum P_B emits a quark. This quark annihilates with an antiquark from another nucleon with momentum P_A . The resulting photon has momentum q and transverse momentum q_T . The quark and antiquark momenta are k_B and k_A , respectively, with transverse components $k_{\perp B}$ and $k_{\perp A}$.

$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2 k_{\perp A} d^2 k_{\perp B} f_1^a(x_A, k_{\perp A}^2; \mu^2) f_1^{\bar{a}}(x_B, k_{\perp B}^2; \mu^2) \delta^{(2)}(k_{\perp A} - \mathbf{q}_T + k_{\perp B})$$

$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

Arnold, Metz and Schlegel, Phys.Rev.D 79 (2009)

Structure of a TMD

Fourier Transform in b_T -space

$$\tilde{f}_1(x, b_T^2; Q^2) = \int \frac{d^2 k_T}{(2\pi)^2} e^{ib_T \cdot k_T} f_1(x, k_T^2; Q^2)$$

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How to model a TMD distribution?

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How to model a TMD distribution?

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_i (C_{qi} \otimes f_1^i)(x, b_*; \mu_{b_*}) e^{S_{\text{pert}}(b_*; \mu_{b_*}, \mu, \zeta)} e^{S_{\text{NP}}(b_T^2; \zeta)} \hat{f}_{NP}^q(x, b_T^2)$$

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Matching coefficients
(perturbatively
calculable)

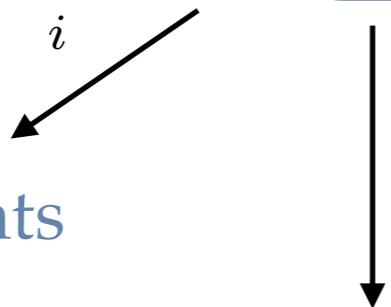
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Matching coefficients
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Collinear PDF

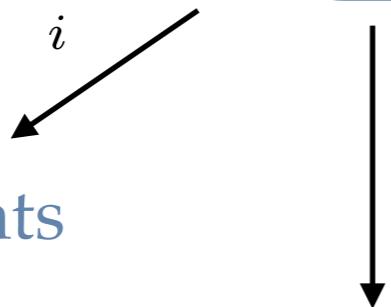
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Collinear PDF

LHAPDF

<https://lhapdf.hepforge.org>

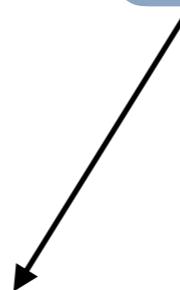
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Sudakov form factor
(Perturbatively calculable)

Structure of a TMD

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Accuracy	H and C	K and γ_F	γ_K	PDF and a_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
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Collinear fragmentation functions not fully available beyond NLO!!

Structure of a TMD

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$$\mu_{b_*} = \frac{2e^{-\gamma_E}}{b_*}$$

Model-dependent function

Structure of a TMD

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Model-dependent function



FIT to data

Available global fits of PROTON TMDs

	Accuracy	SIDIS	DY	Z production	N of points	χ^2/N_{data}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✓	✓	✓	1039	1.06
MAPTMD22	N^3LL^-	✓	✓	✓	2031	1.06

MAP Collaboration, JHEP 10 (2022), 127

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MAP Collaboration, JHEP 10 (2022), 127

- Best theoretical accuracy reached
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- Overcoming the normalization problem of SIDIS data

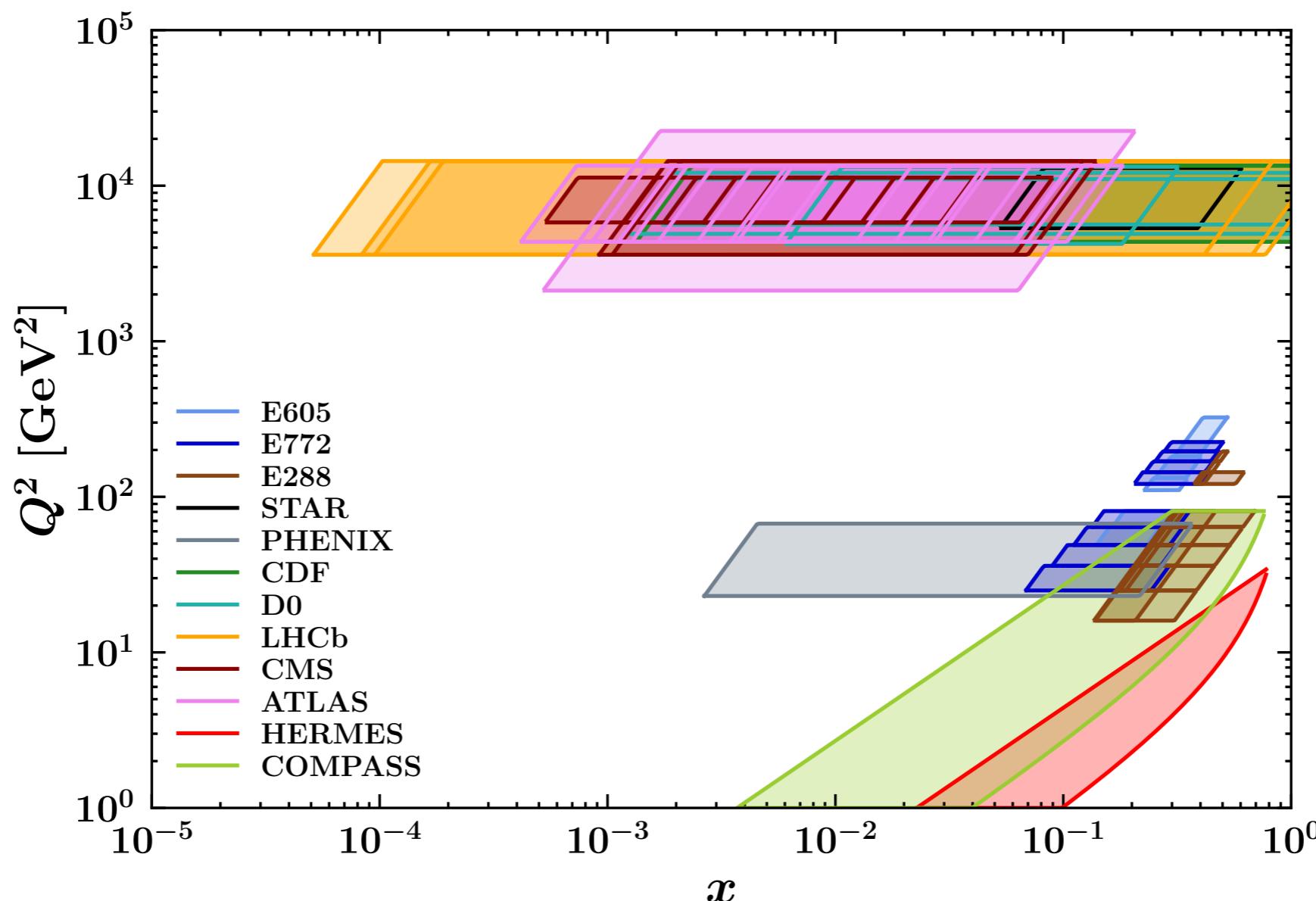
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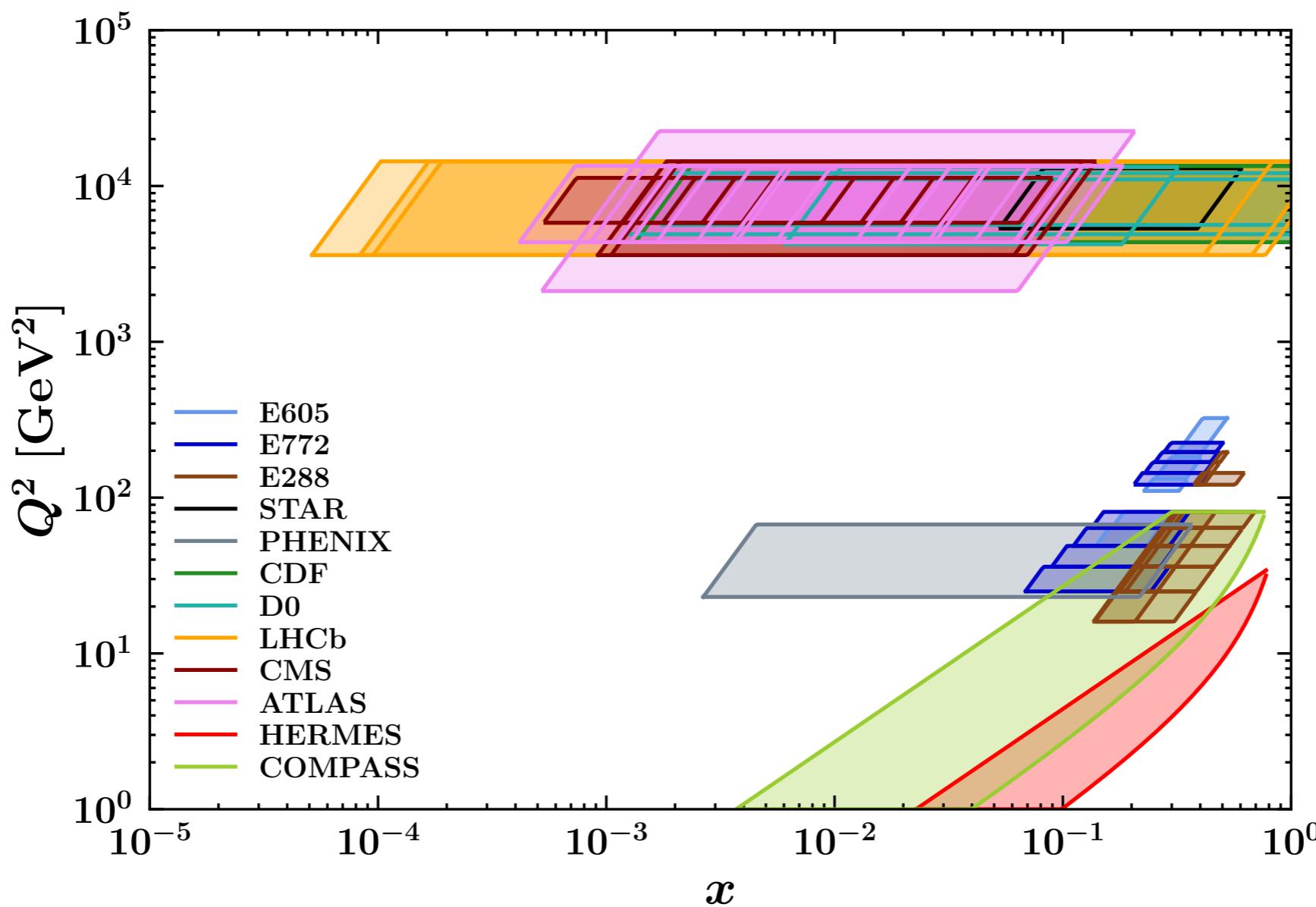
MAP Collaboration, JHEP 10 (2022), 127

- Best theoretical accuracy reached
- Number of included data of the SV19 fit doubled
- Overcoming the normalization problem of SIDIS data
- Very good global description obtained

MAPTMD22 - Included data sets



MAPTMD22 - Included data sets



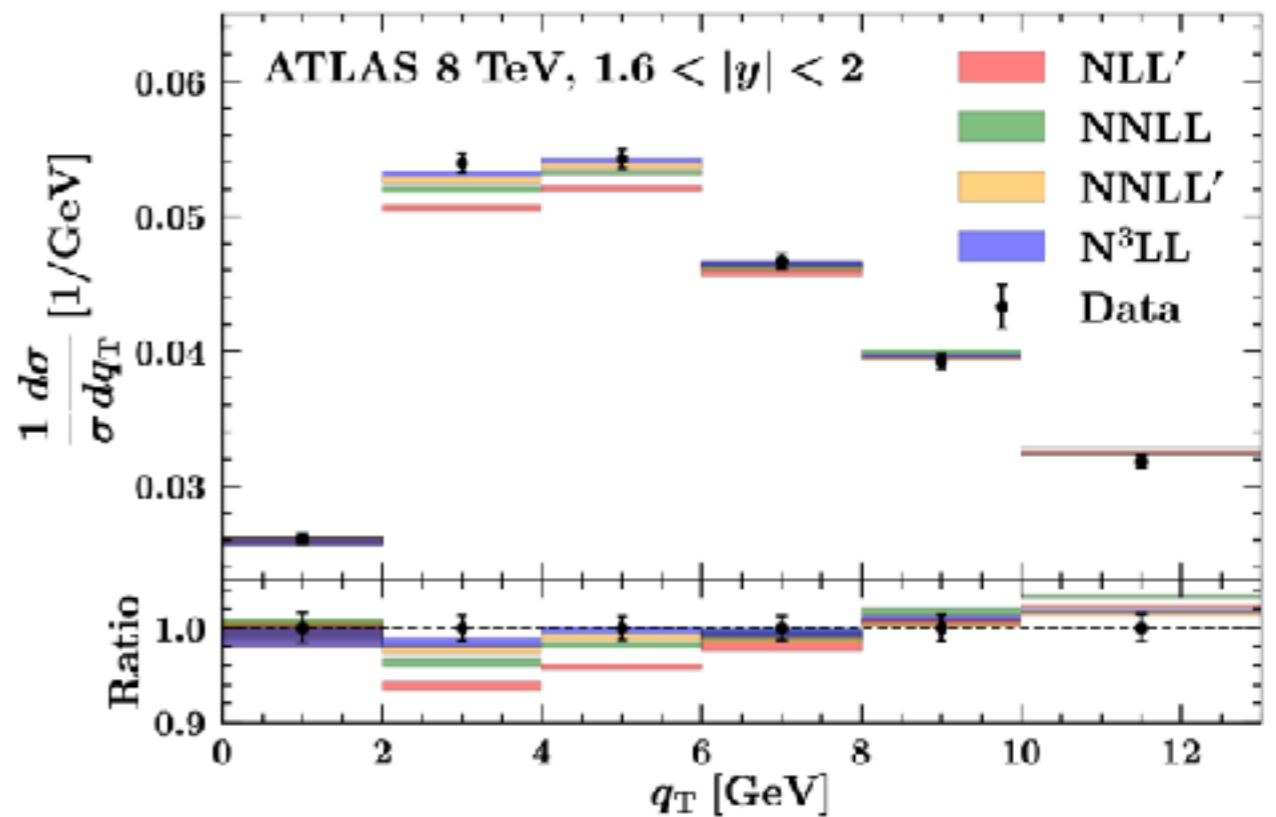
484 (DY) + 1547 (SIDIS)
2031 fitted experimental points

MAPTMD22 – Normalization of SIDIS

MAPTMD22 – Normalization of SIDIS

High-Energy Drell-Yan beyond NLL

$$Q \sim 100 \text{ GeV}$$

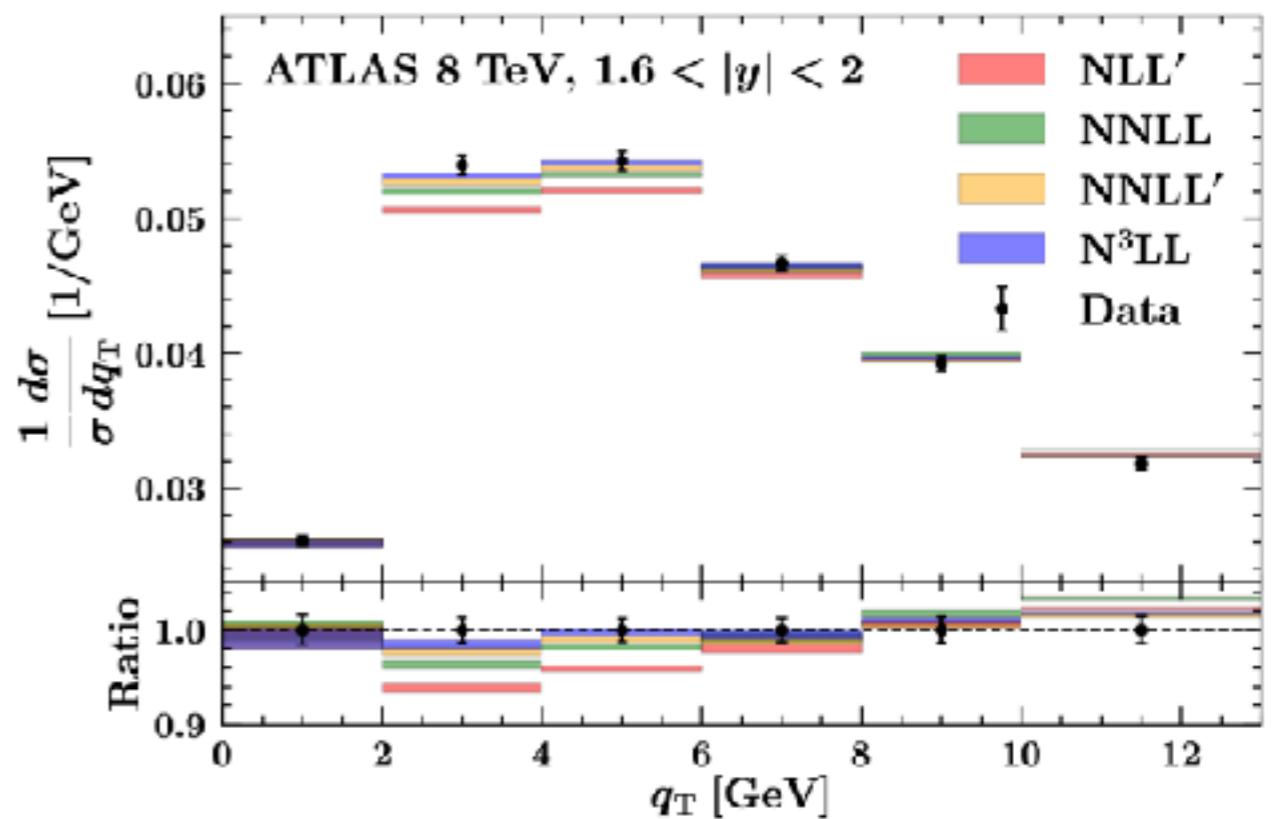


MAPTMD22 – Normalization of SIDIS

SIDIS multiplicities beyond NLL

High-Energy Drell-Yan beyond NLL

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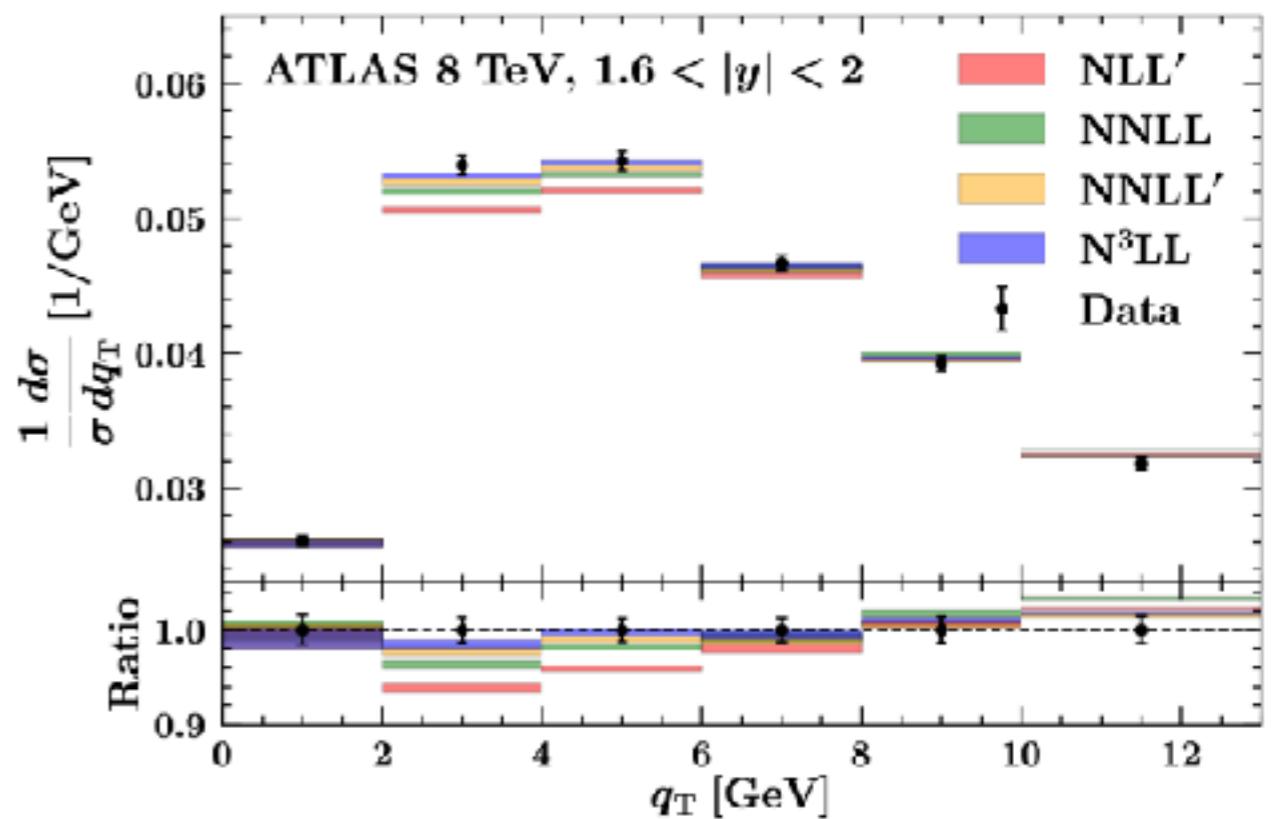
MAPTMD22 – Normalization of SIDIS

SIDIS multiplicities beyond NLL

$Q \sim 2 \text{ GeV}$

High-Energy Drell-Yan beyond NLL

$Q \sim 100 \text{ GeV}$

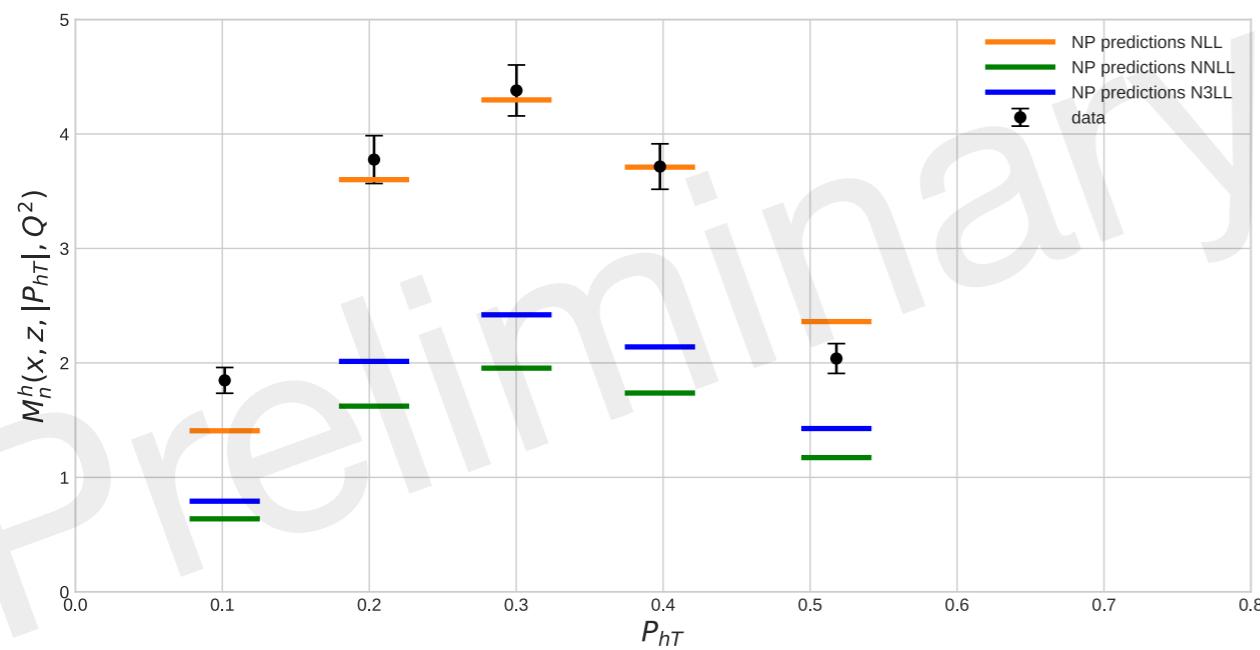


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SIDIS multiplicities beyond NLL

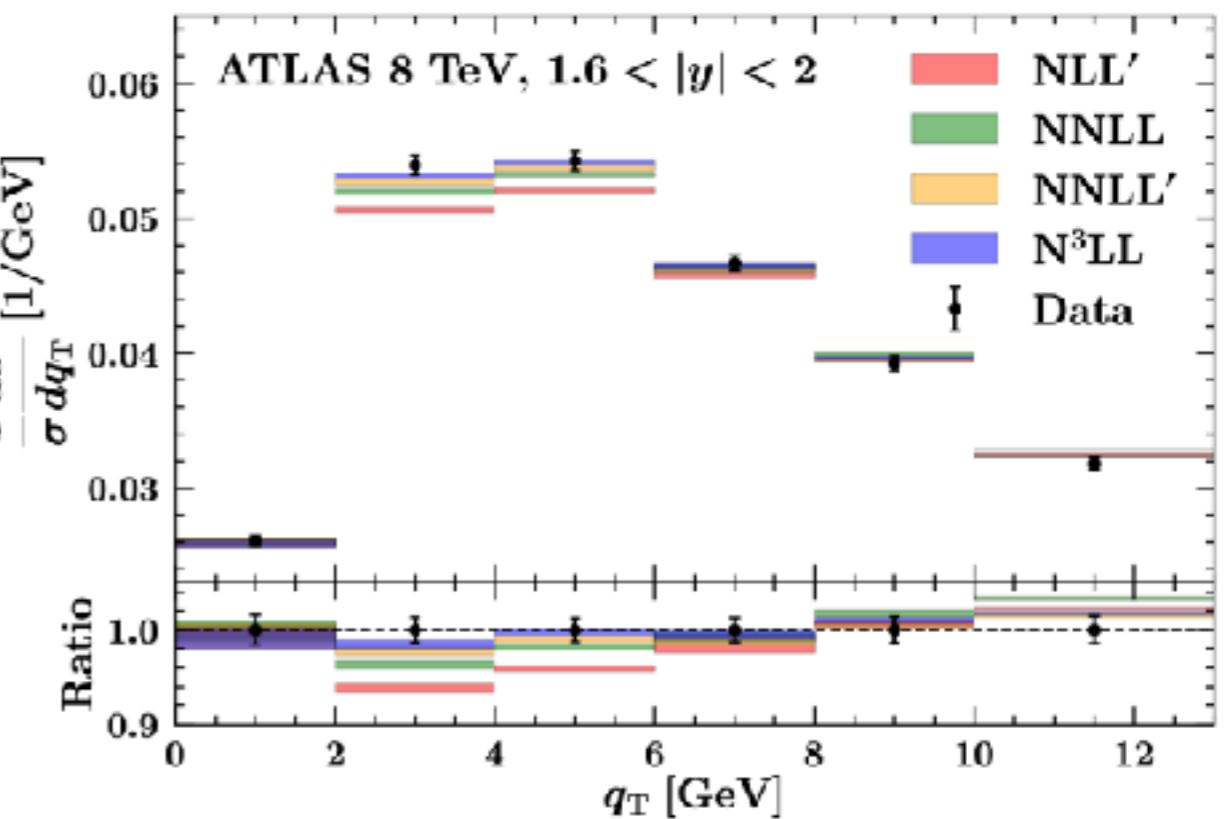
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HERMES



High-Energy Drell-Yan beyond NLL

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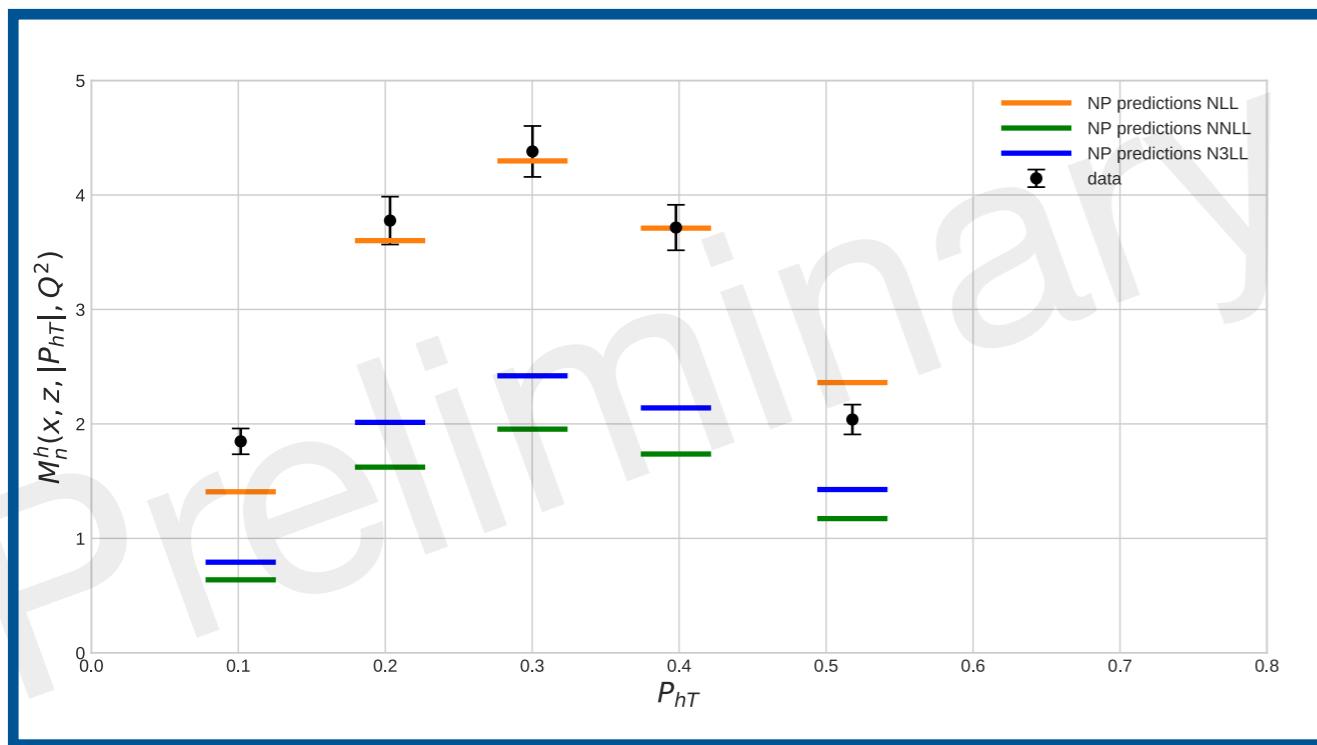


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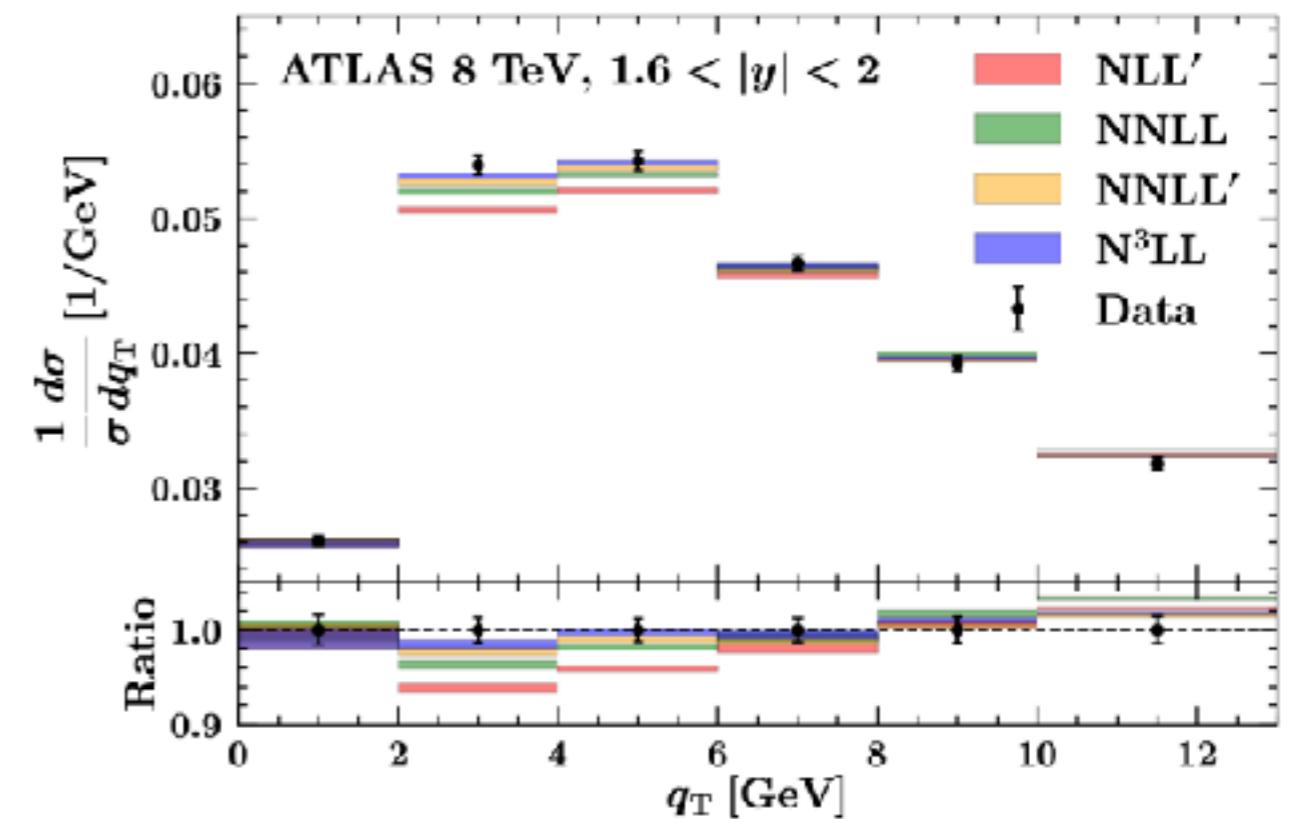
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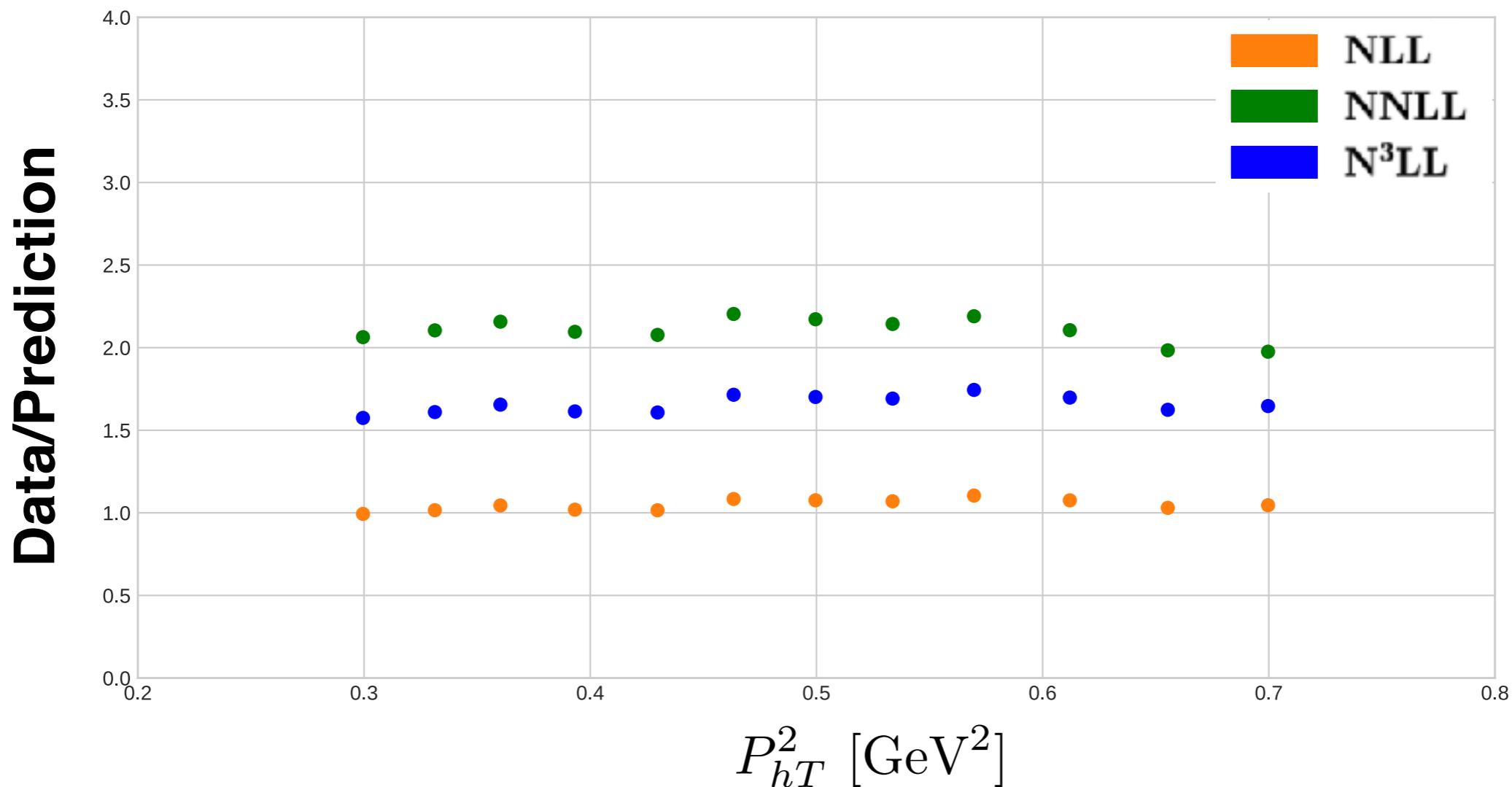
$Q \sim 100 \text{ GeV}$



The description considerably worsens at higher orders!!

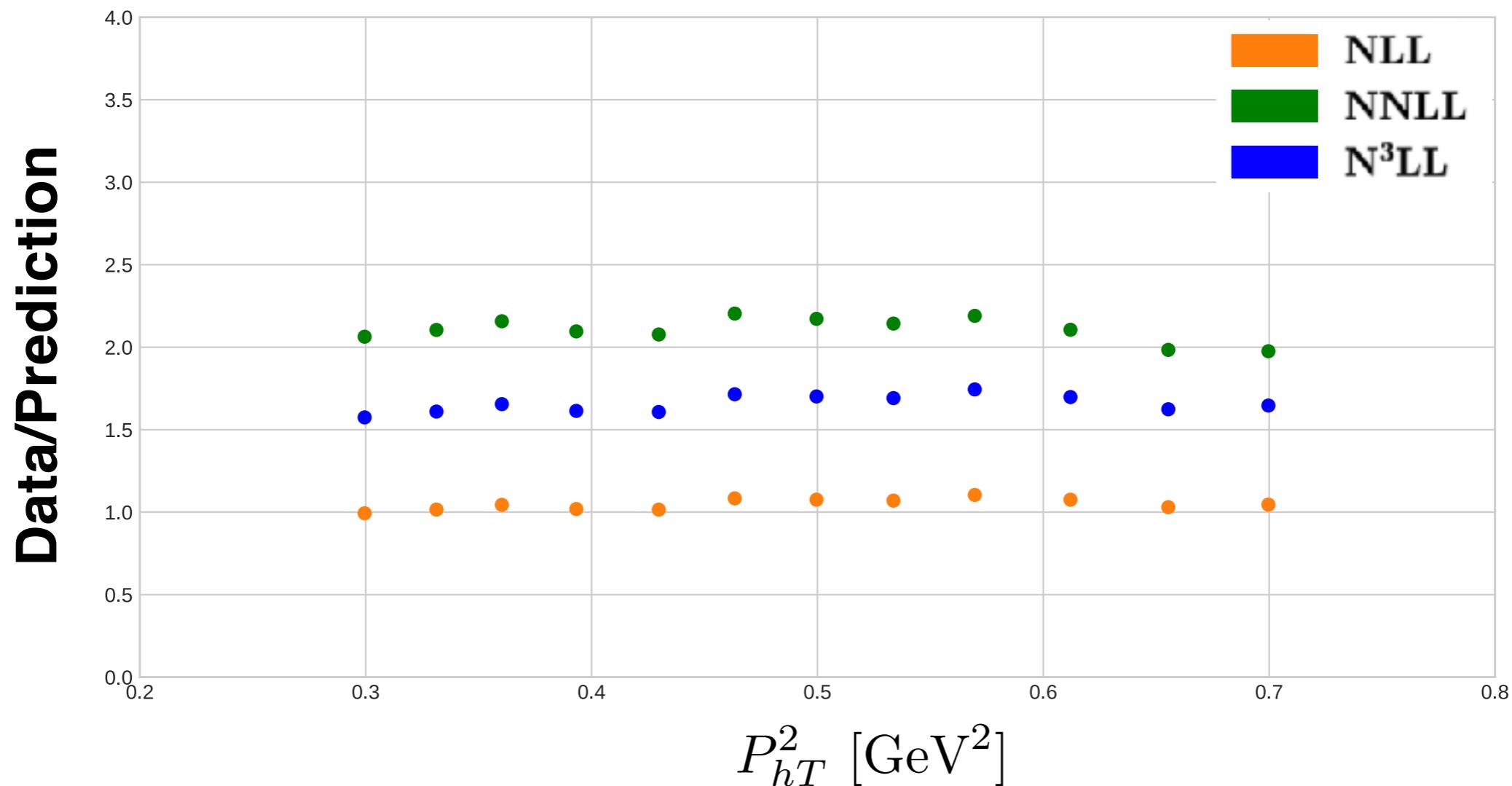
MAPTMD22 – Normalization of SIDIS

COMPASS multiplicities (one of many bins)



MAPTMD22 – Normalization of SIDIS

COMPASS multiplicities (one of many bins)



The discrepancy amounts to an almost constant factor

MAPTMD22 – Normalization prefactor

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SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dxdQ \cancel{dz} \cancel{dP_{hT}}} \Bigg/ \frac{d\sigma}{dxdQ}$$

MAPTMD22 – Normalization prefactor

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Collinear SIDIS cross section

$$\frac{d\sigma}{dxdQ \cancel{dz}}$$

MAPTMD22 – Normalization prefactor

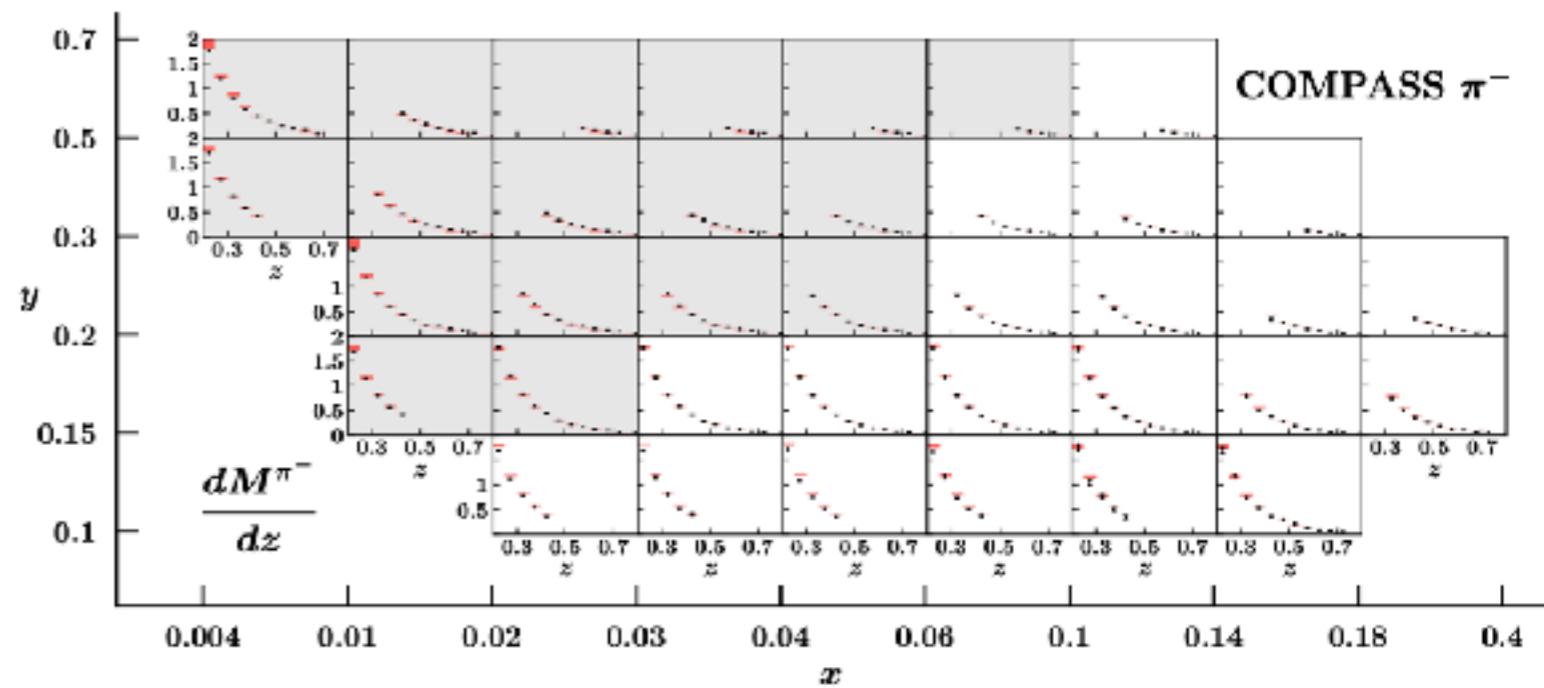
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Collinear SIDIS cross section

$$\frac{d\sigma}{dxdQ \cancel{dz}}$$

No problems of normalization!!



MAPTMD22 – Normalization prefactor

SIDIS multiplicity

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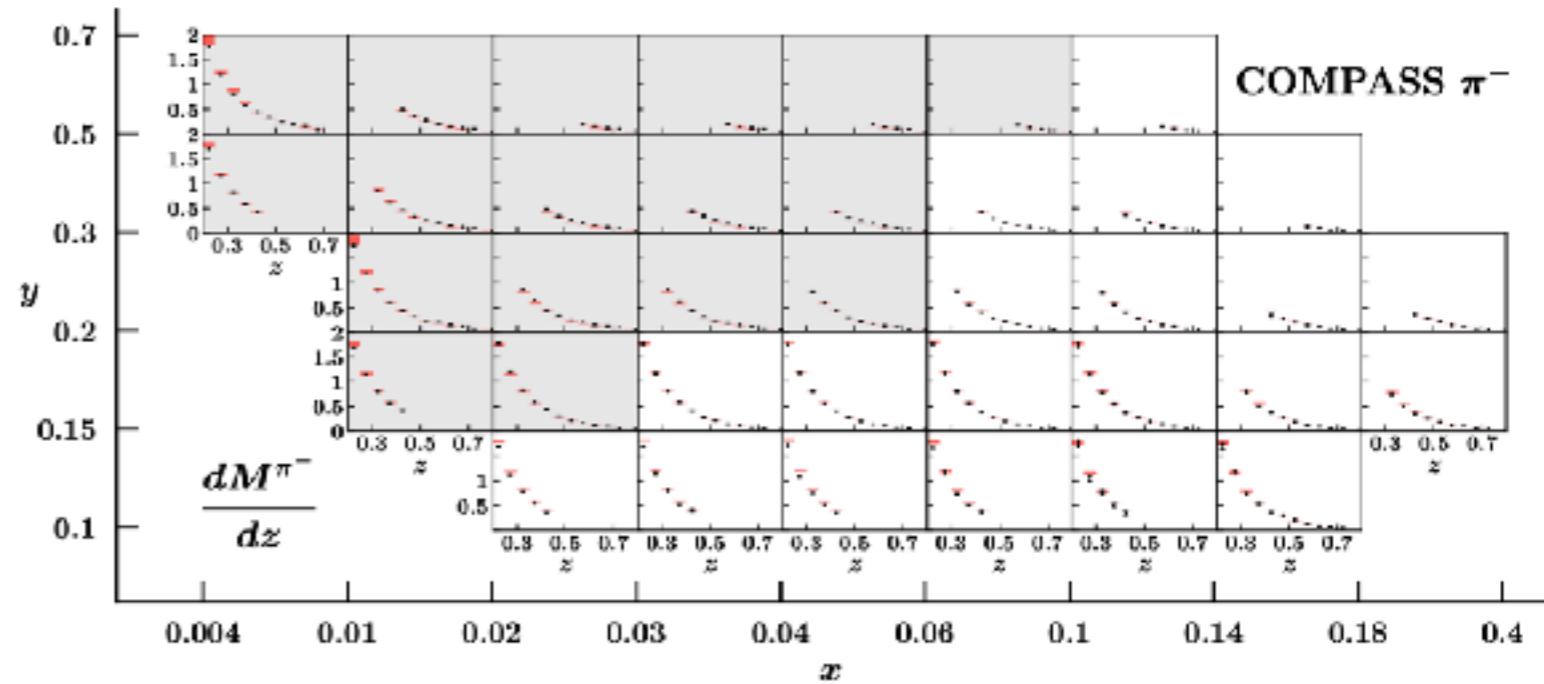
Collinear SIDIS cross section

$$\frac{d\sigma}{dx dQ \cancel{dz}}$$

Normalization of prediction such that

$$\int dP_{hT} \frac{d\sigma}{dx dQ \cancel{dz} dP_{hT}} = \frac{d\sigma}{dx dQ \cancel{dz}}$$

No problems of normalization!!



MAPTMD22 – Normalization prefactor

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \Bigg/ \frac{d\sigma}{dx dQ}$$

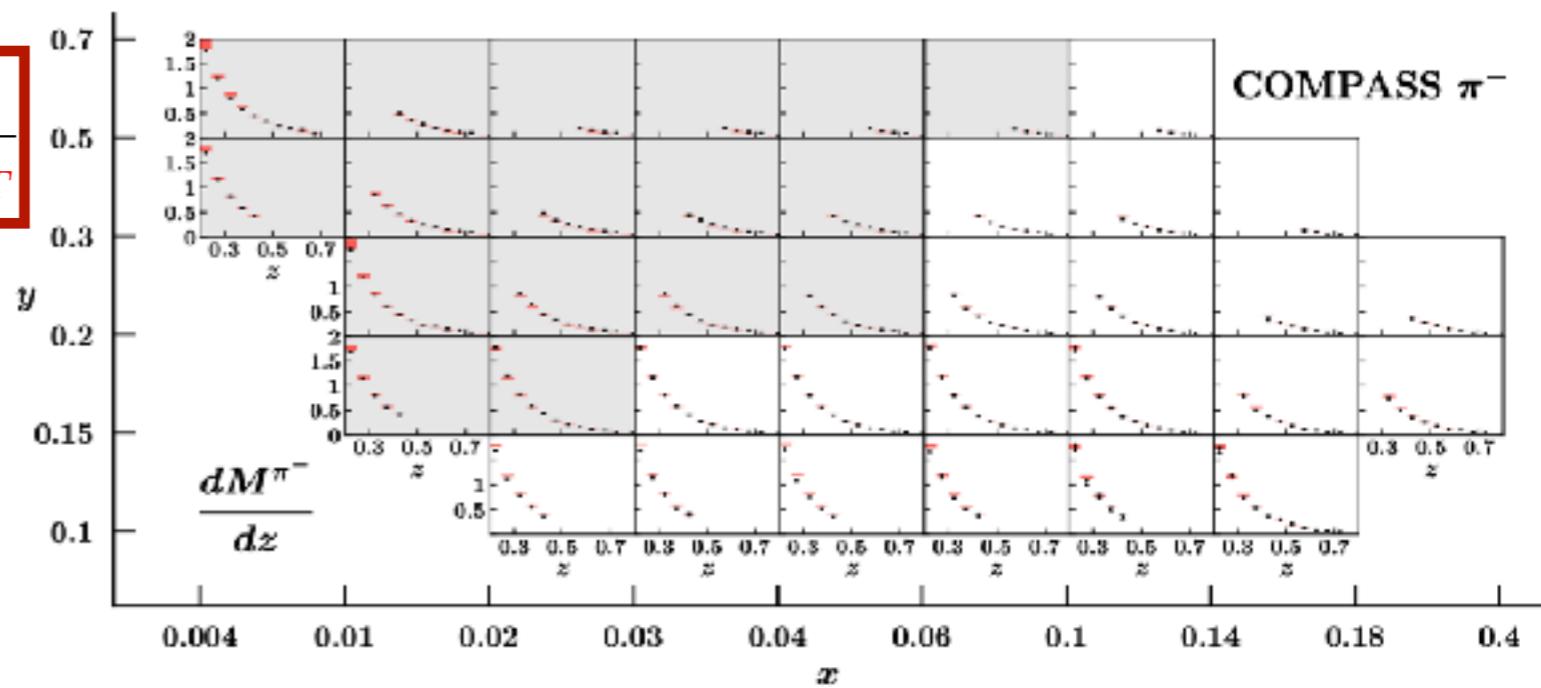
Collinear SIDIS cross section

$$\frac{d\sigma}{dx dQ dz}$$

Normalization of prediction such that

No problems of normalization!!

$$w(x, z, Q) = \frac{d\sigma}{dx dQ dz} \Bigg/ \int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}}$$



MAPTMD22 – Normalization prefactor

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \Bigg/ \frac{d\sigma}{dx dQ}$$

Collinear SIDIS cross section

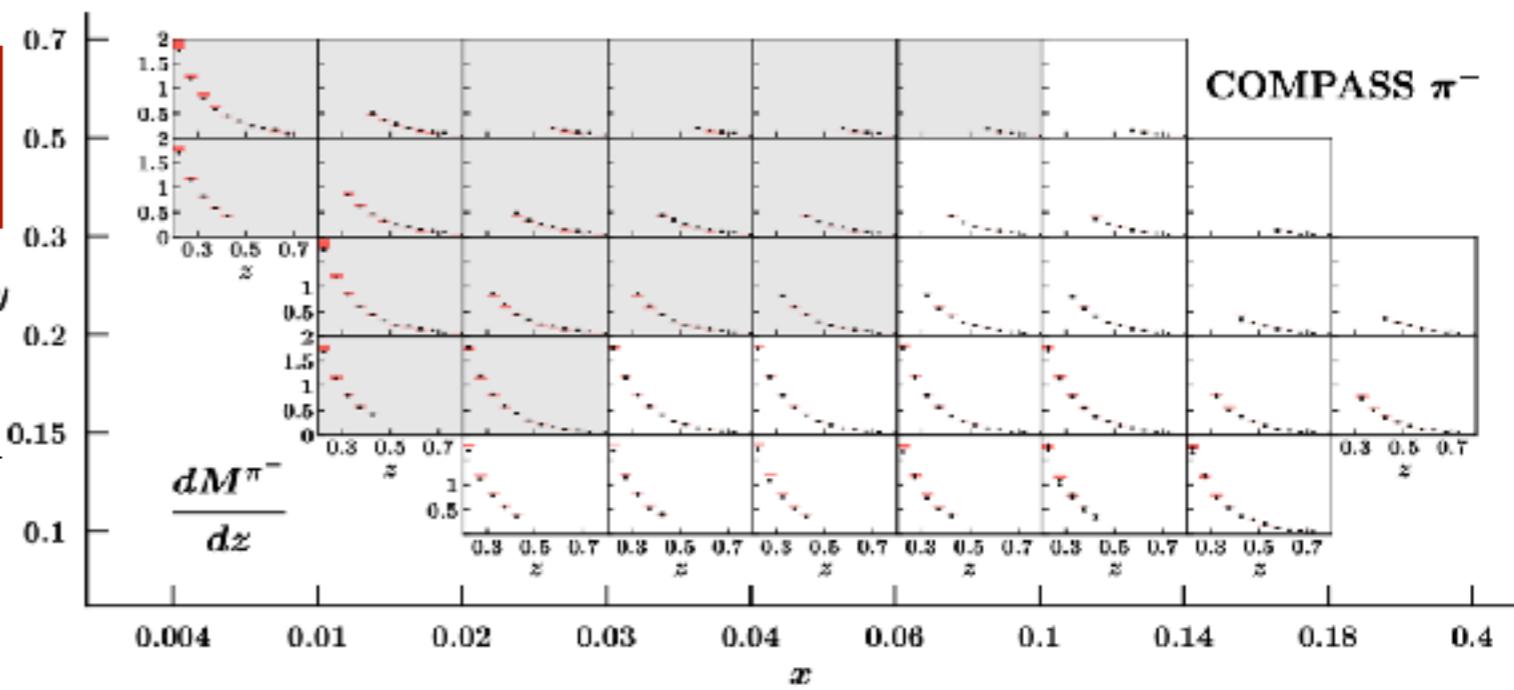
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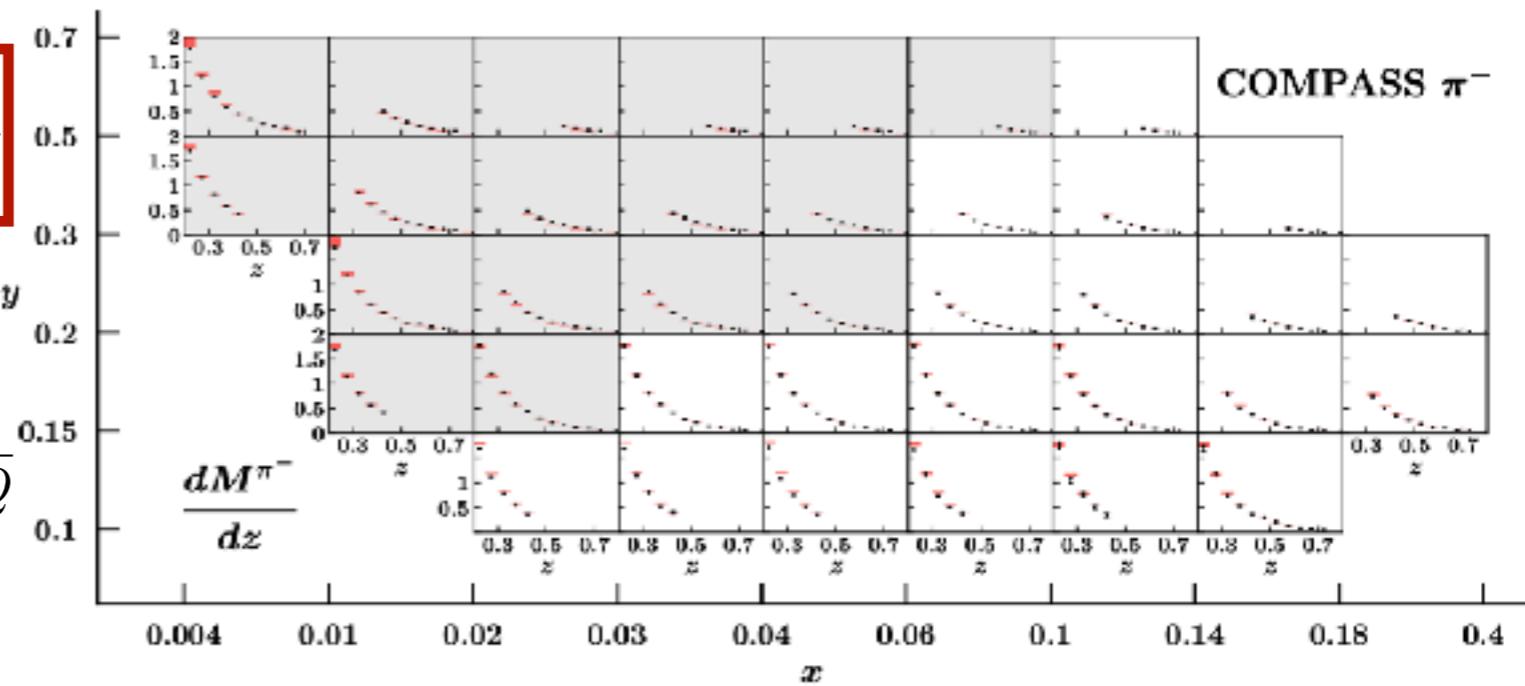
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Independent of the fitting parameters!!

MAPTMD22 – NP models

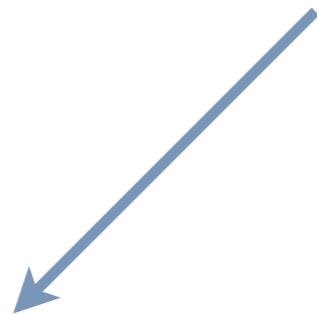
$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$

A. Bacchetta, F. Conti and M. Radici, Phys. Rev. D 78 (2008) 074010

A. Bacchetta, L.P. Gamberg, G.R. Goldstein and A. Mukherjee, Phys. Lett. B 659 (2008) 234

MAPTMD22 – NP models

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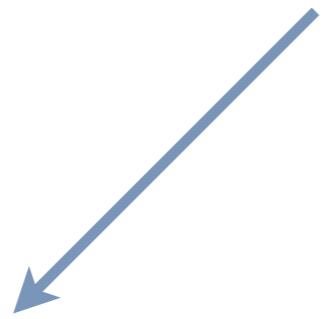
$$g_1(x) = N_1 \frac{(1-x)^\alpha \ x^\sigma}{(1-\hat{x})^\alpha \ \hat{x}^\sigma}$$

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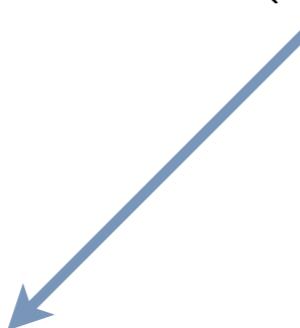
$$D_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

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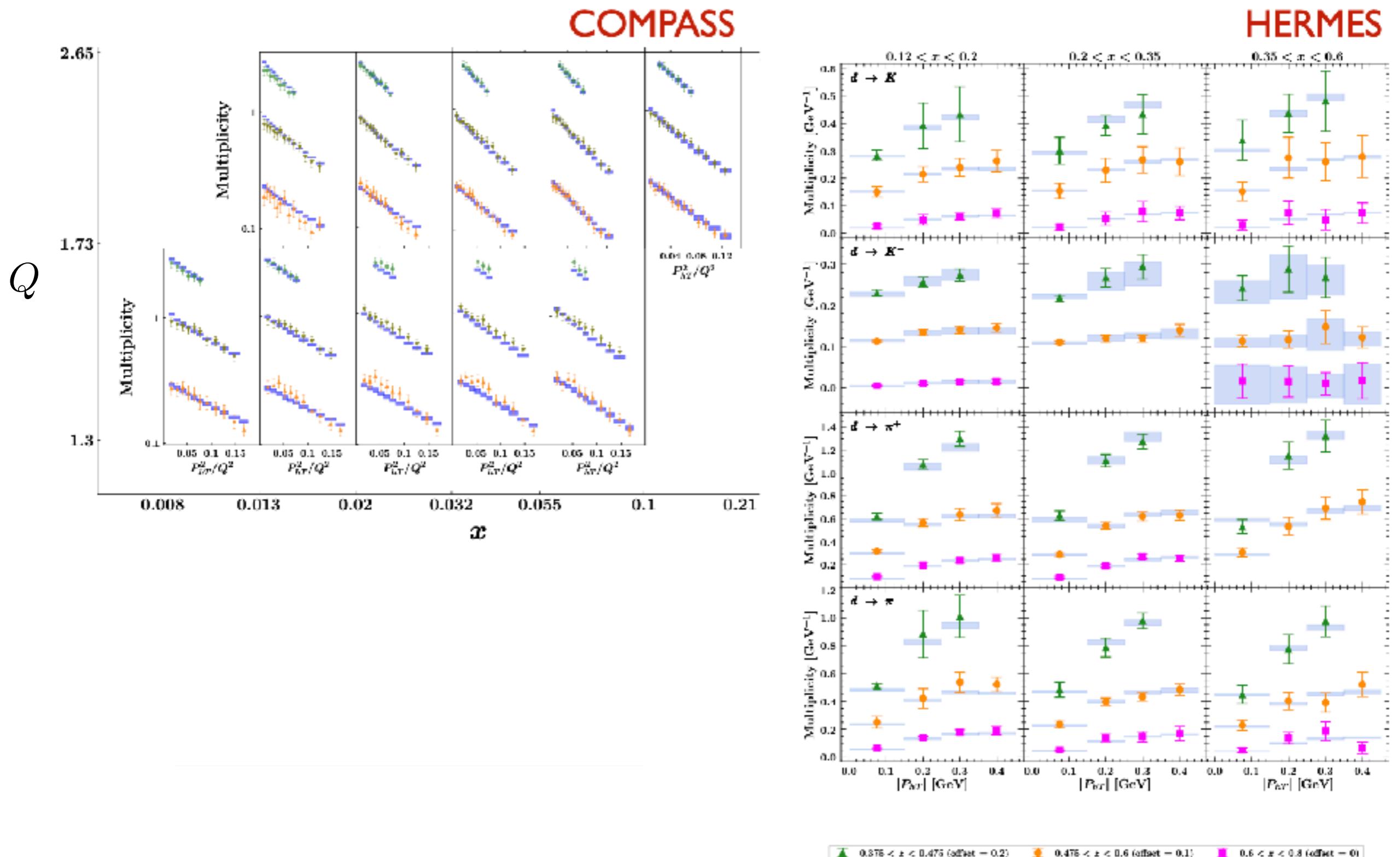
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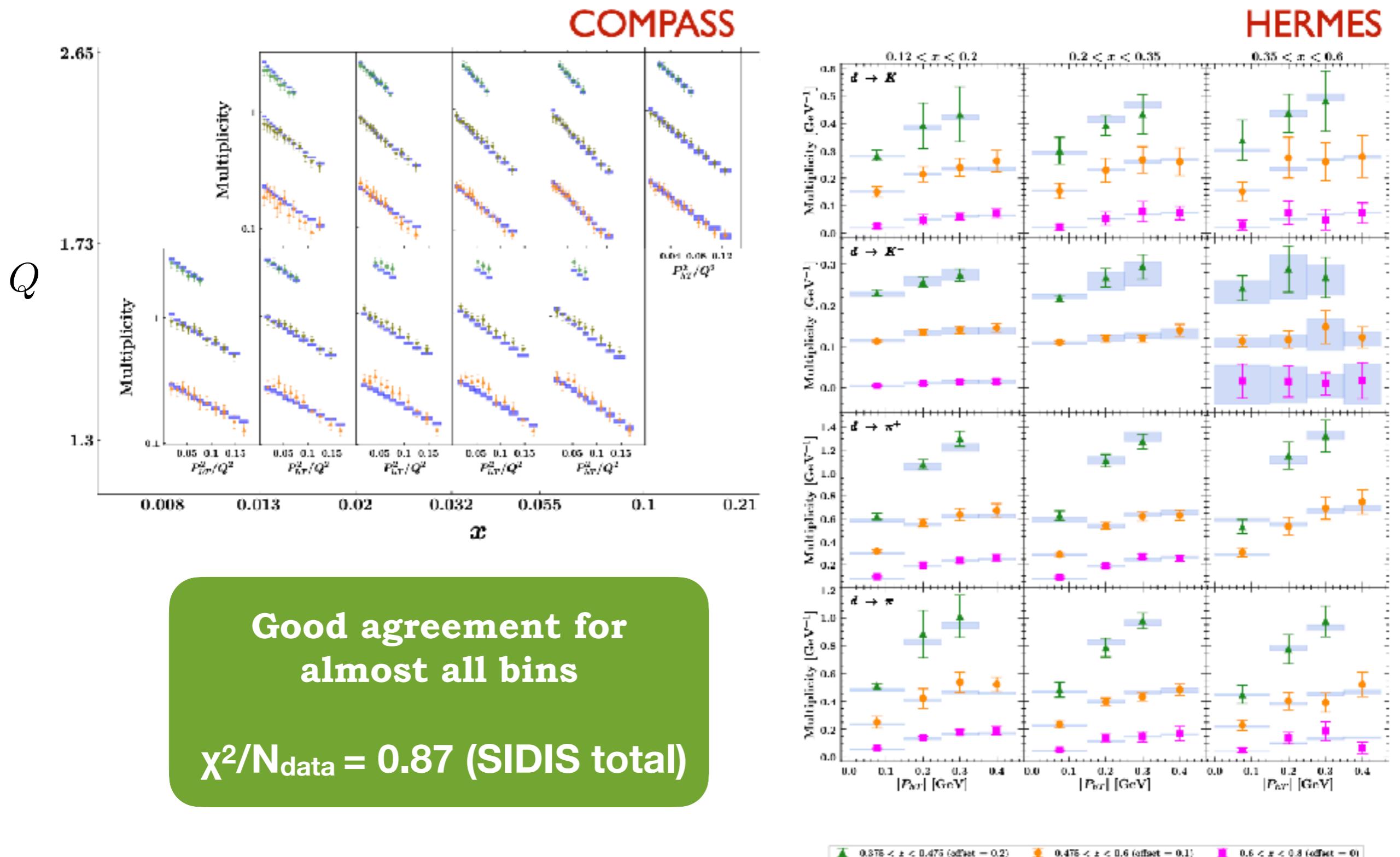
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**11 parameters for TMD PDF
+ 1 for NP evolution + 9 for TMD FF
= 21 free parameters**

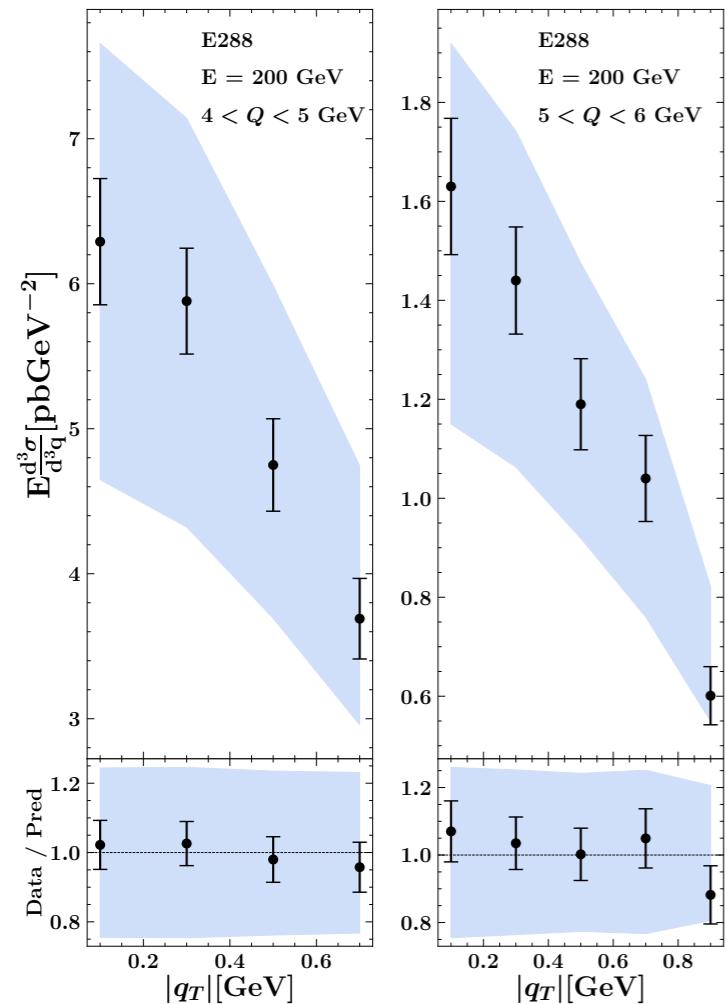
MAPTMD22 — Fit results



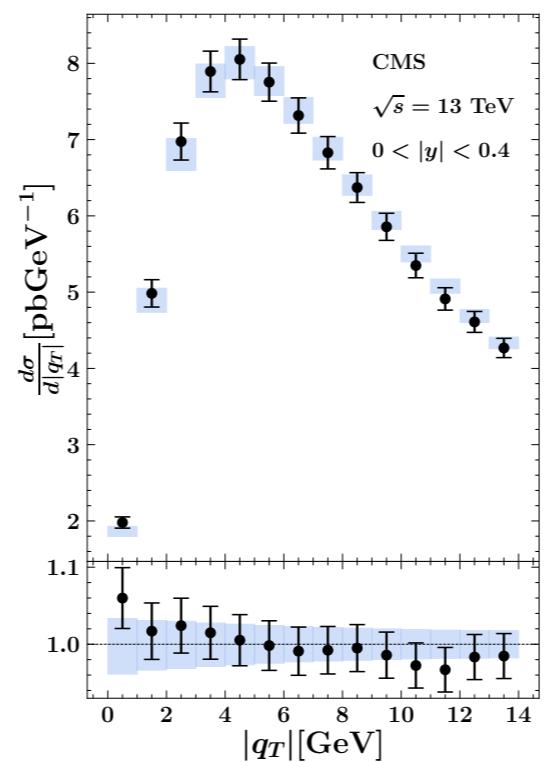
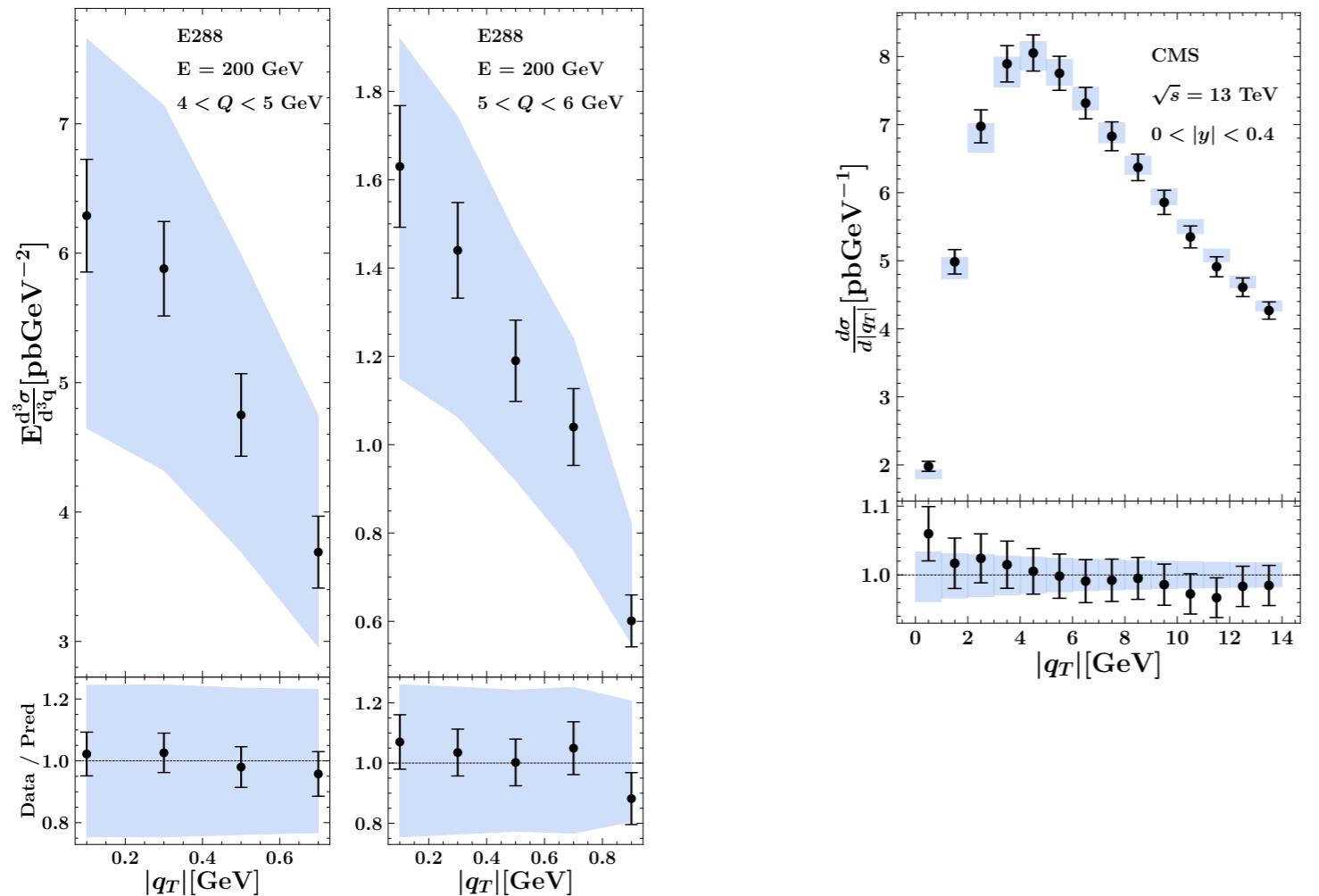
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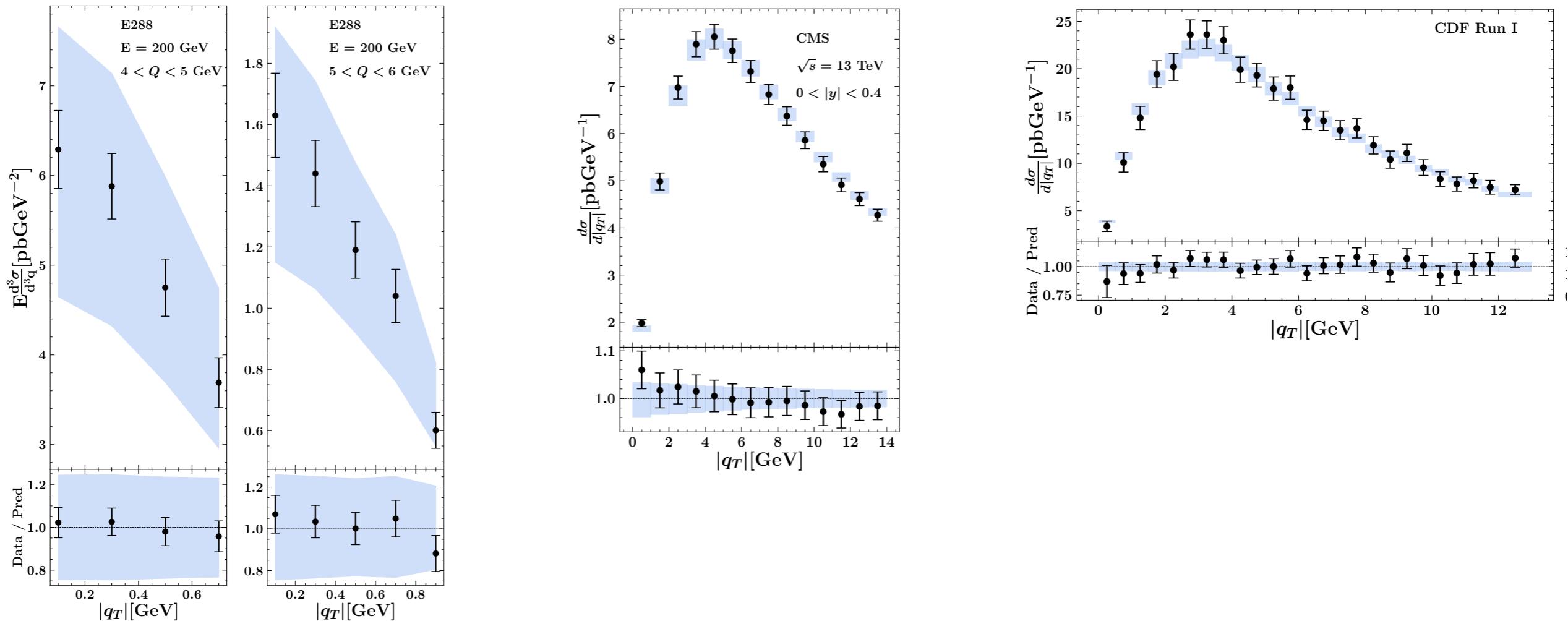
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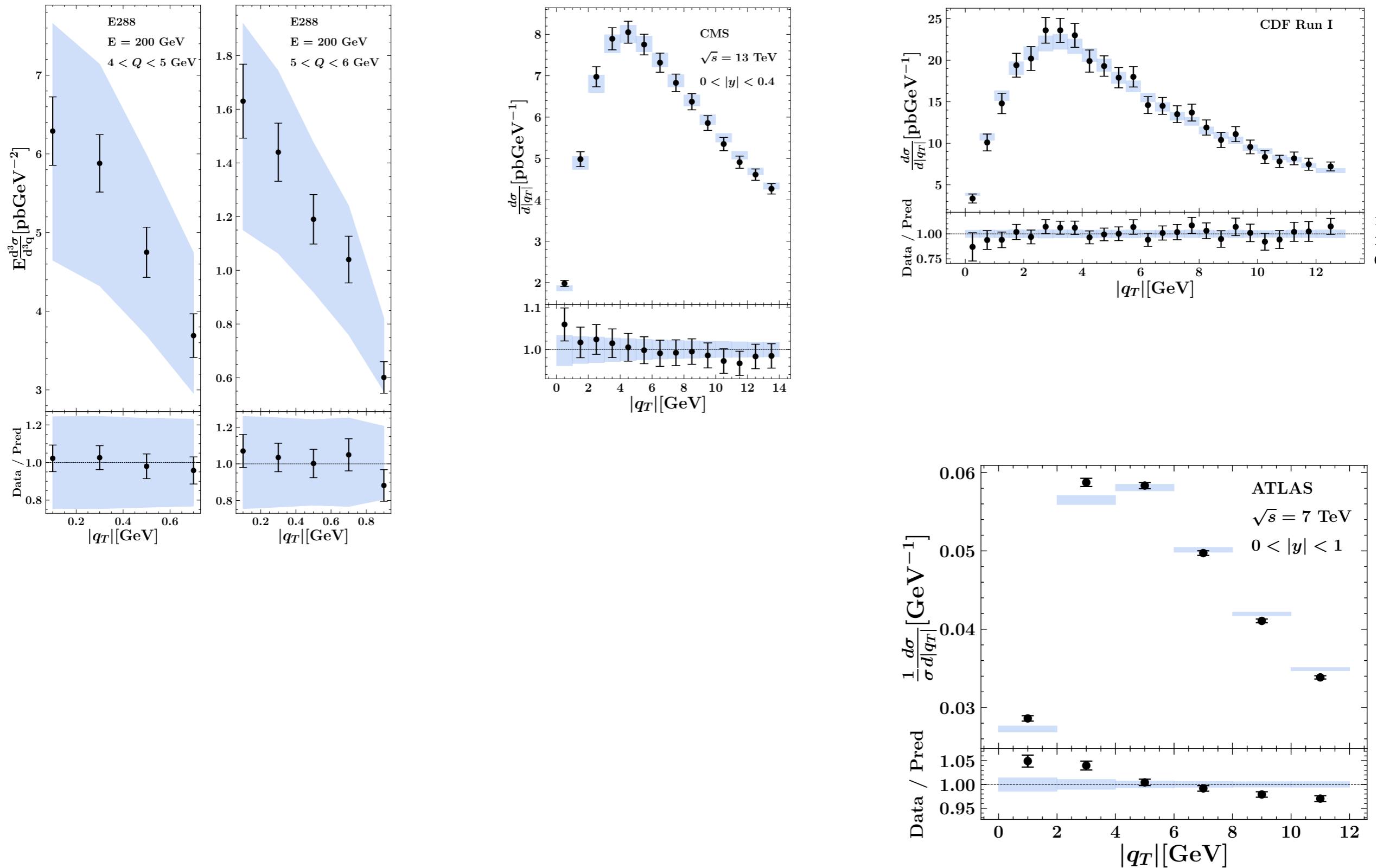
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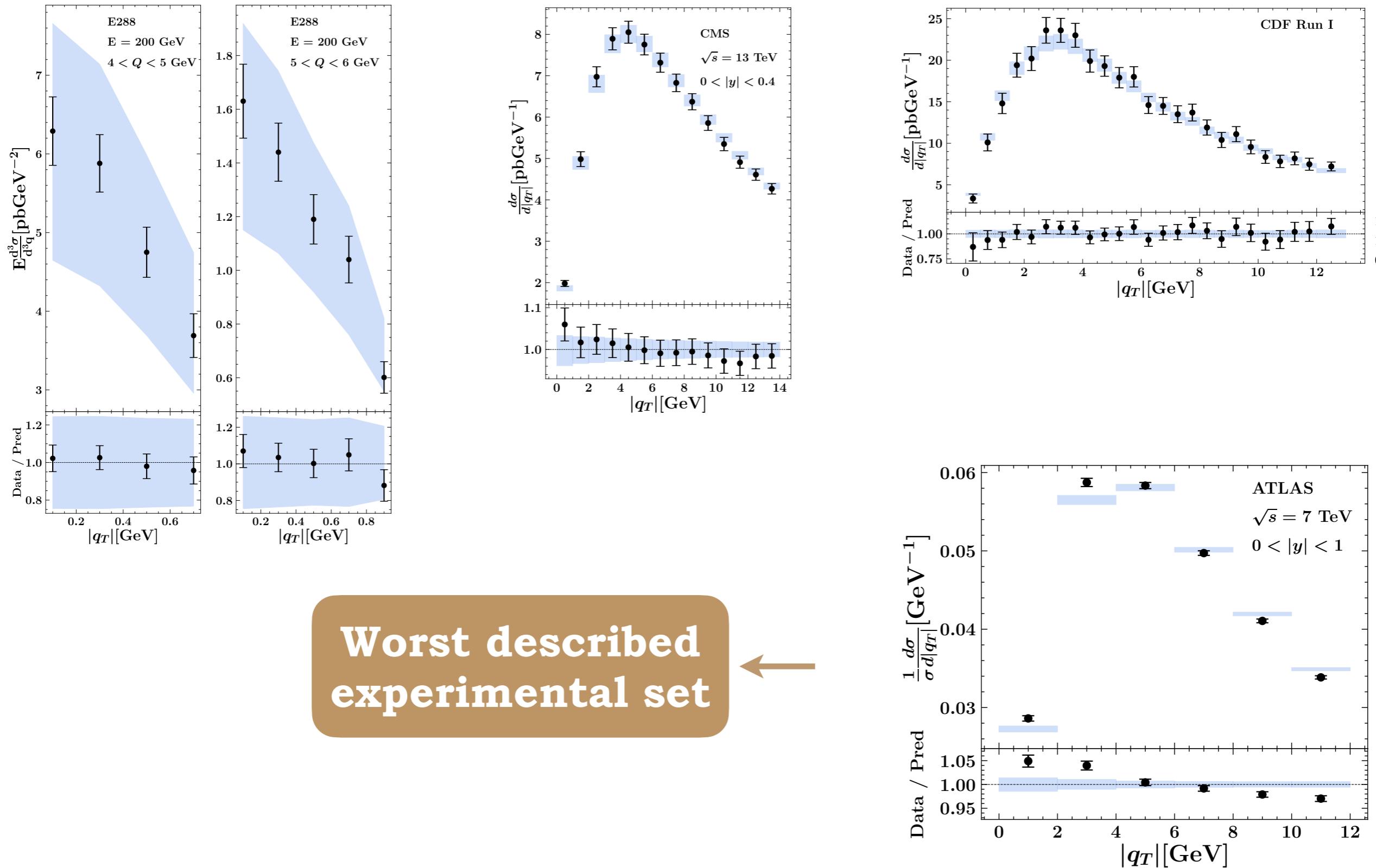
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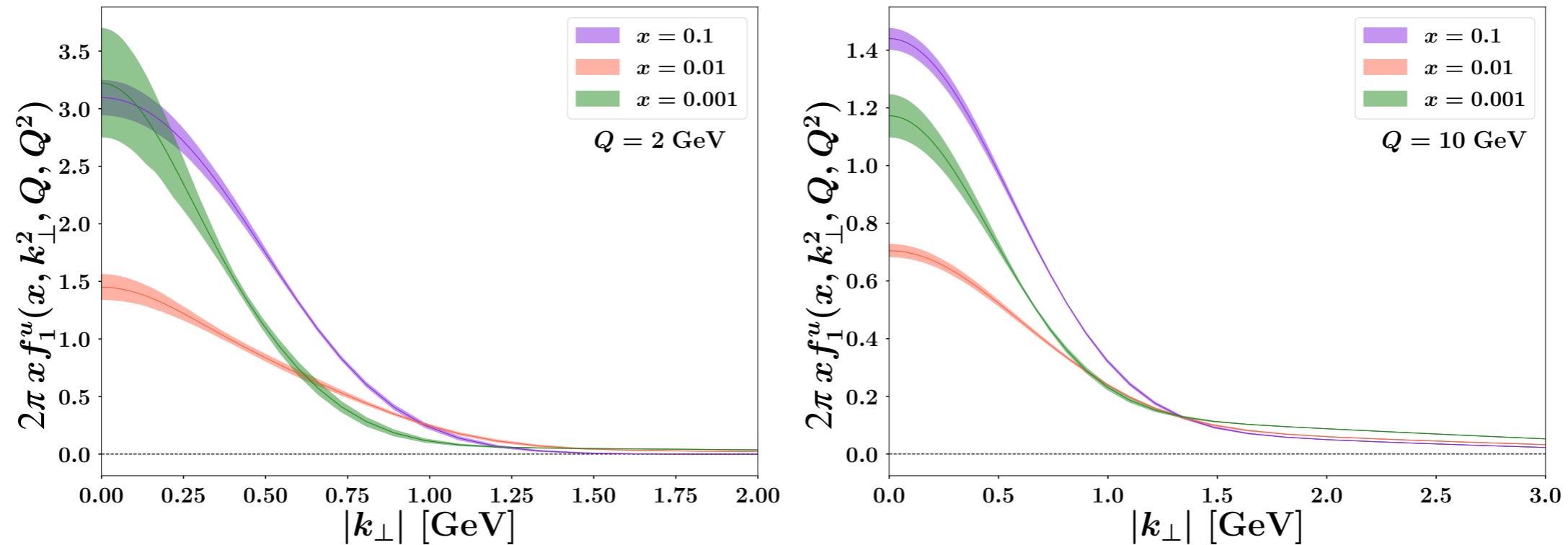


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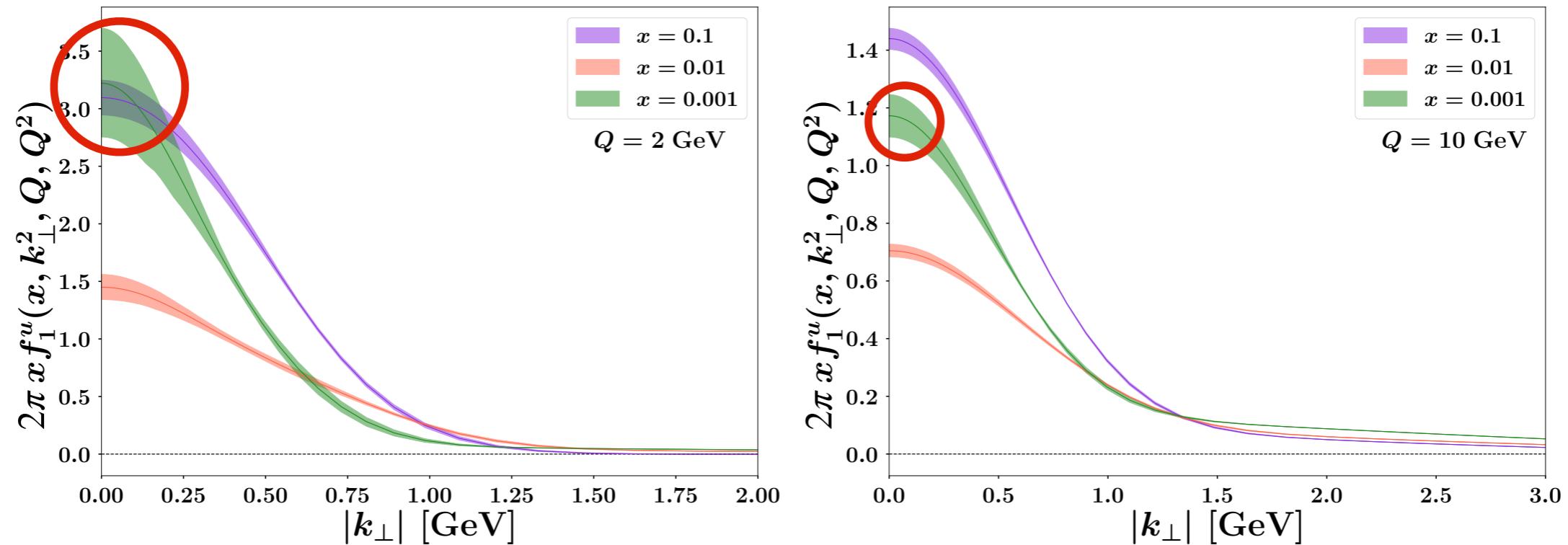


MAPTMD22 – Extracted TMDs

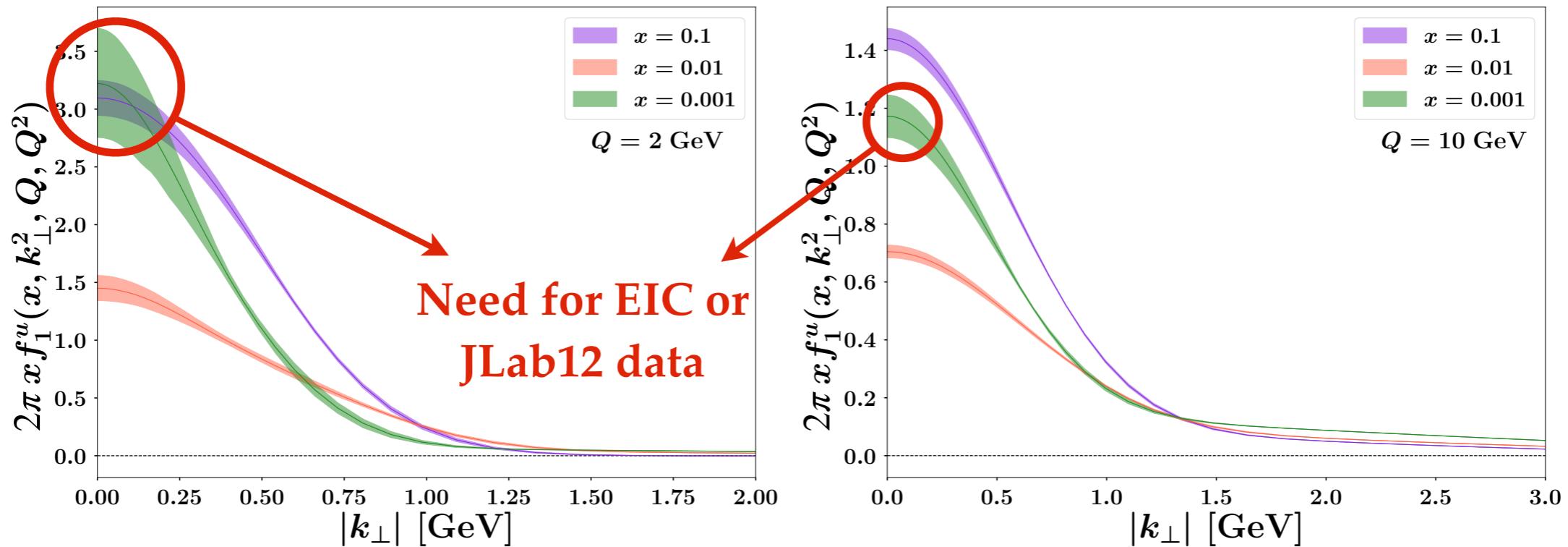
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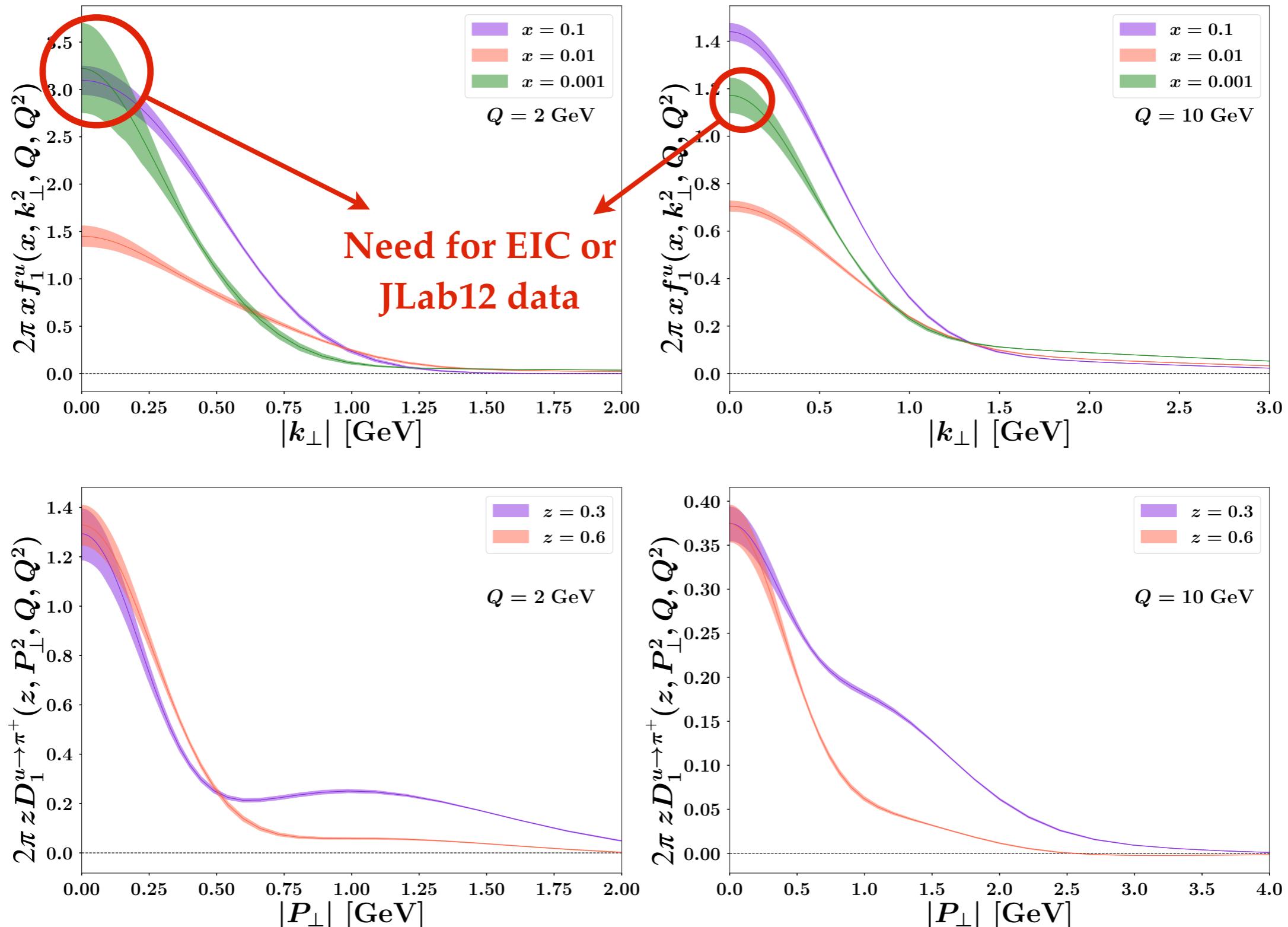
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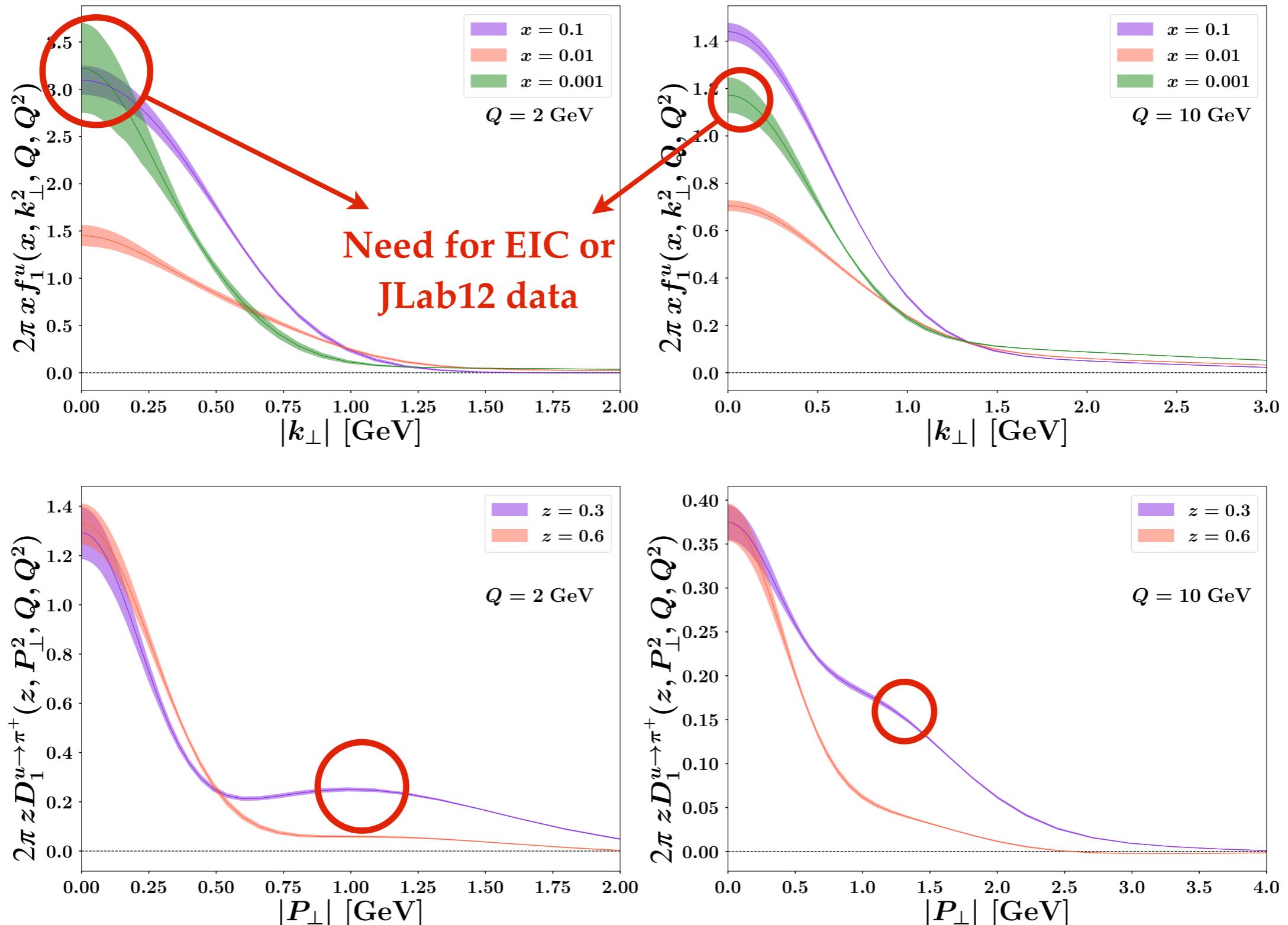
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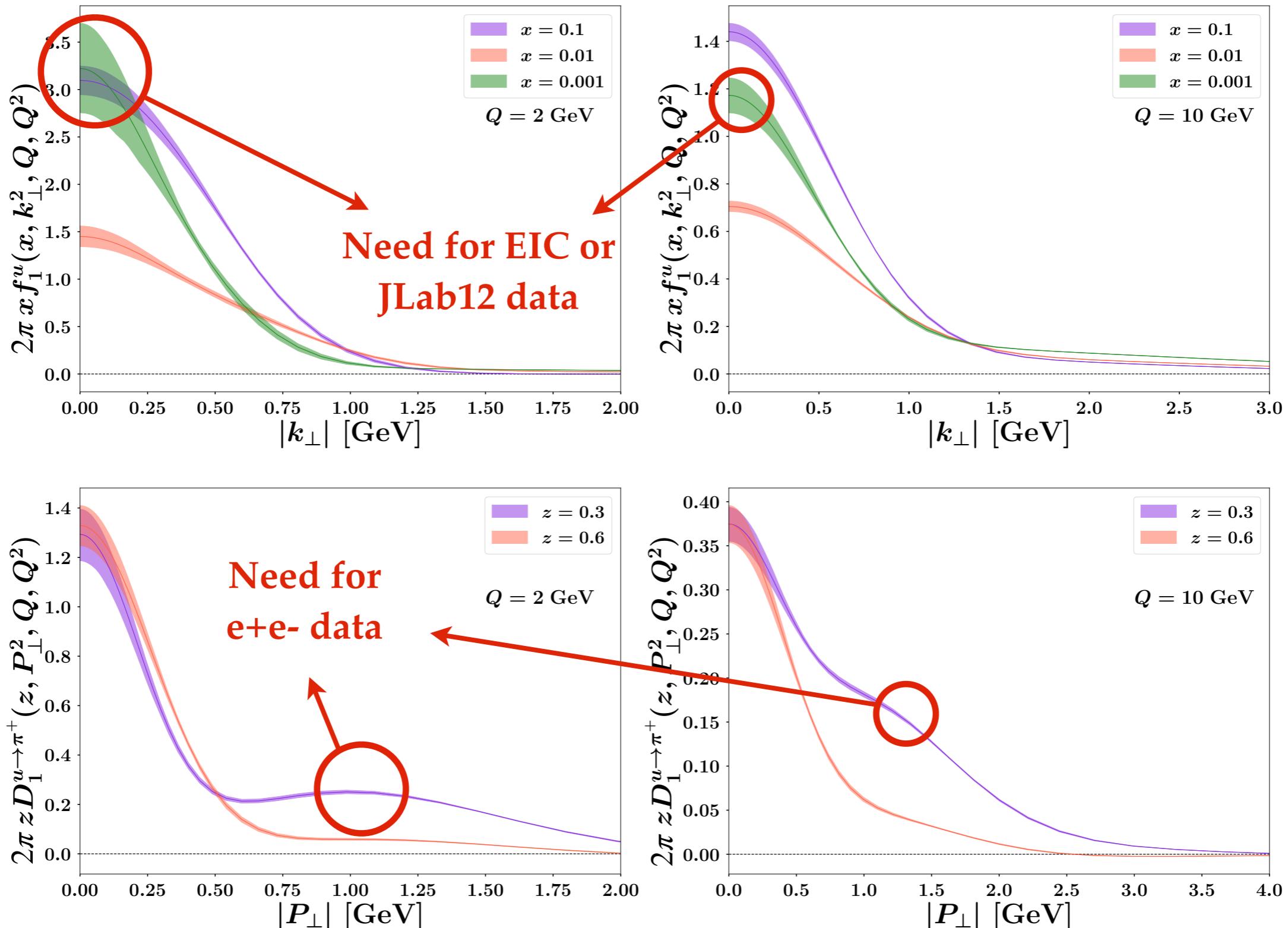
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Available fits of PION TMDs

	Accuracy	SIDIS	DY	N of points	χ^2/N_{data}
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MAP Collaboration, Phys. Rev. D 107, 014014

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MAP Collaboration, Phys. Rev. D 107, 014014

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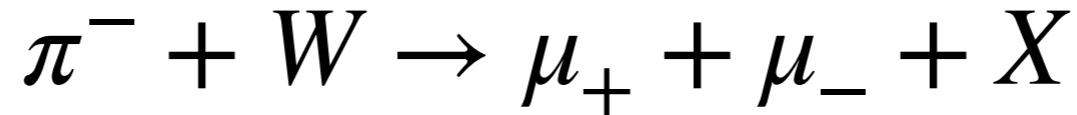
MAP Collaboration, Phys. Rev. D 107, 014014

- Best theoretical accuracy reached
- Other data sets included in the fit
- Good global description obtained

PionMAPTMD22 – Included data sets

$$\pi^- + W \rightarrow \mu_+ + \mu_- + X$$

PionMAPTMD22 – Included data sets

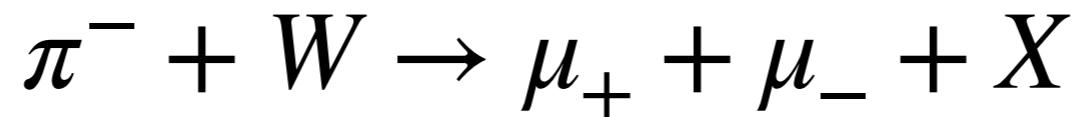


Experiment	\sqrt{s} [GeV]	Q [GeV]	N_{bins}	x_F
E615 (Q-diff)	21.8	$4.05 < Q < 13.05$	10 (8)	$0 < x_F < 1$
E537 (Q-diff)	15.3	$4.0 < Q < 9.0$	10	$-0.1 < x_F < 1$

E. Anassontzis et al. 1988

W. J. Stirling et al. 1993

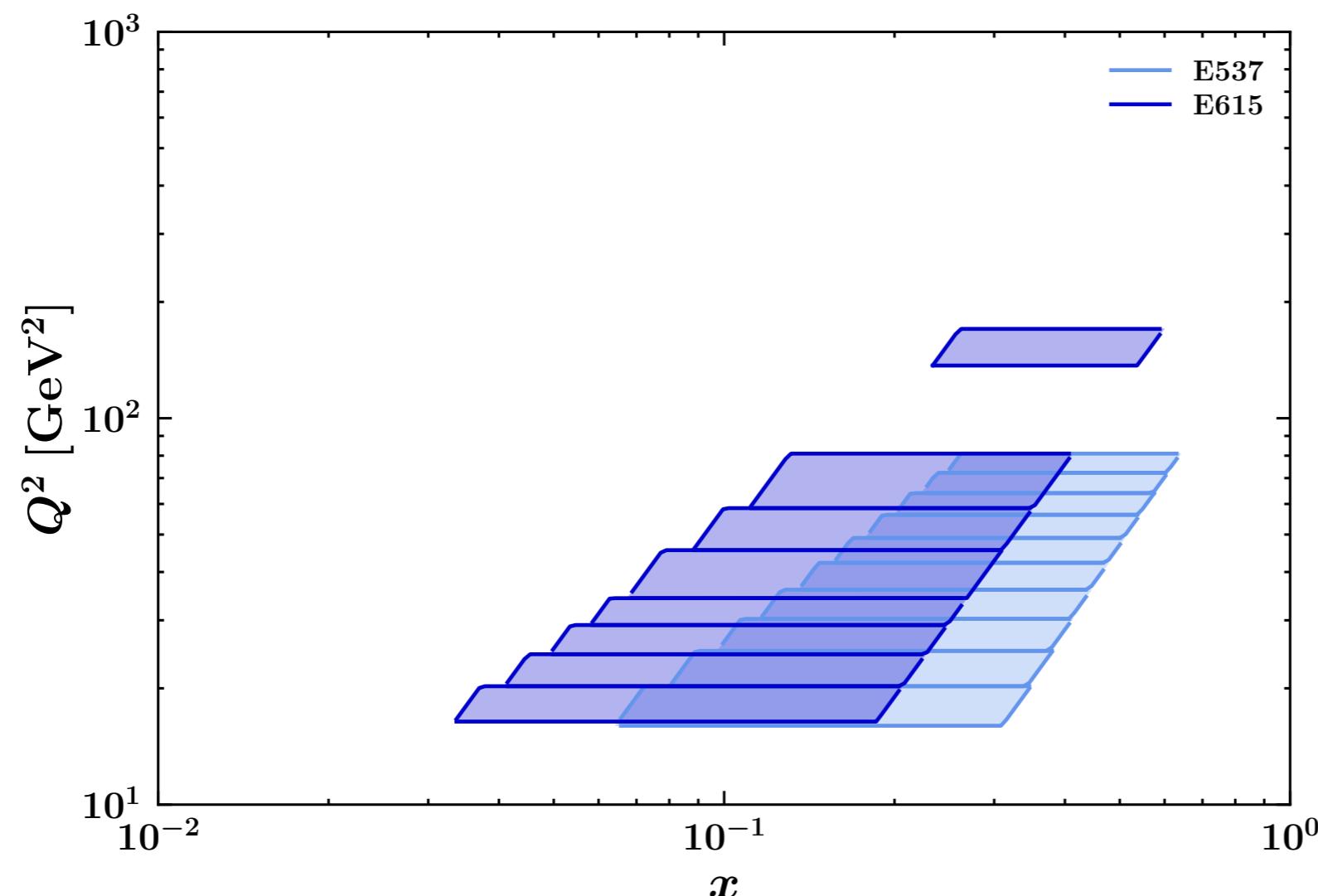
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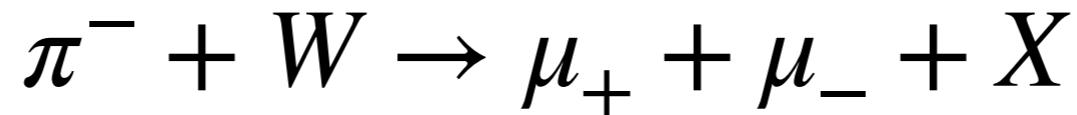
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Small phase-space
region covered by
present data sets

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Experiment	Number of points	Statistical errors	Systematic errors	Theoretical errors
E615 (Q-diff)	74/155	5%	16%	5-8%
E537 (Q-diff)	64/150	15-20%	8%	5-8%
Total	138/305	Large Uncertainties	Large Normalization Errors	Extra uncertainties

PionMAPTMD22 – NP Models

Proton TMD

Pion TMD

PionMAPTMD22 – NP Models

Proton TMD  **MAPTMD22**

Pion TMD

PionMAPTMD22 – NP Models

Proton TMD \longrightarrow **MAPTMD22**

Pion TMD \longrightarrow $f_{1,\pi}(x, \mathbf{b}_T) = e^{-g_{1\pi}(x) \frac{\mathbf{b}_T^2}{4}}$

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PionMAPTMD22 – NP Models

Proton TMD \longrightarrow **MAPTMD22**

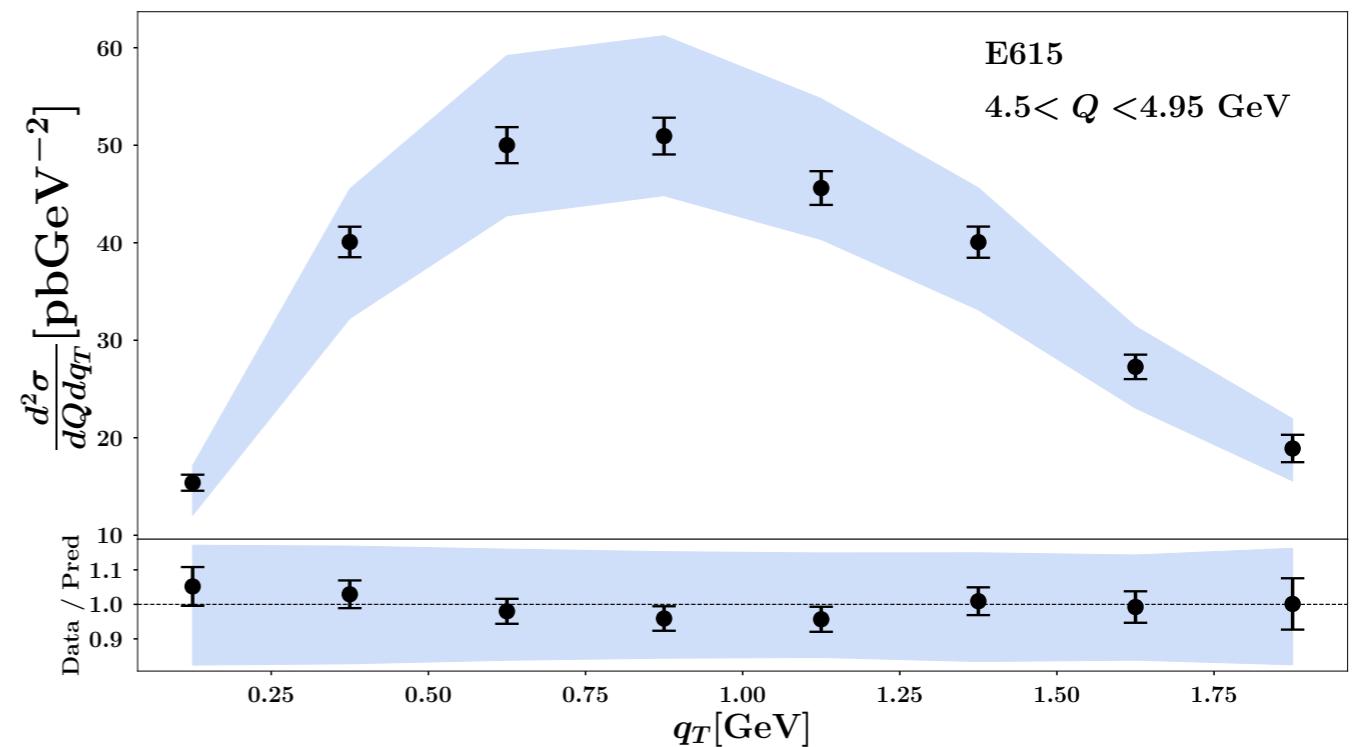
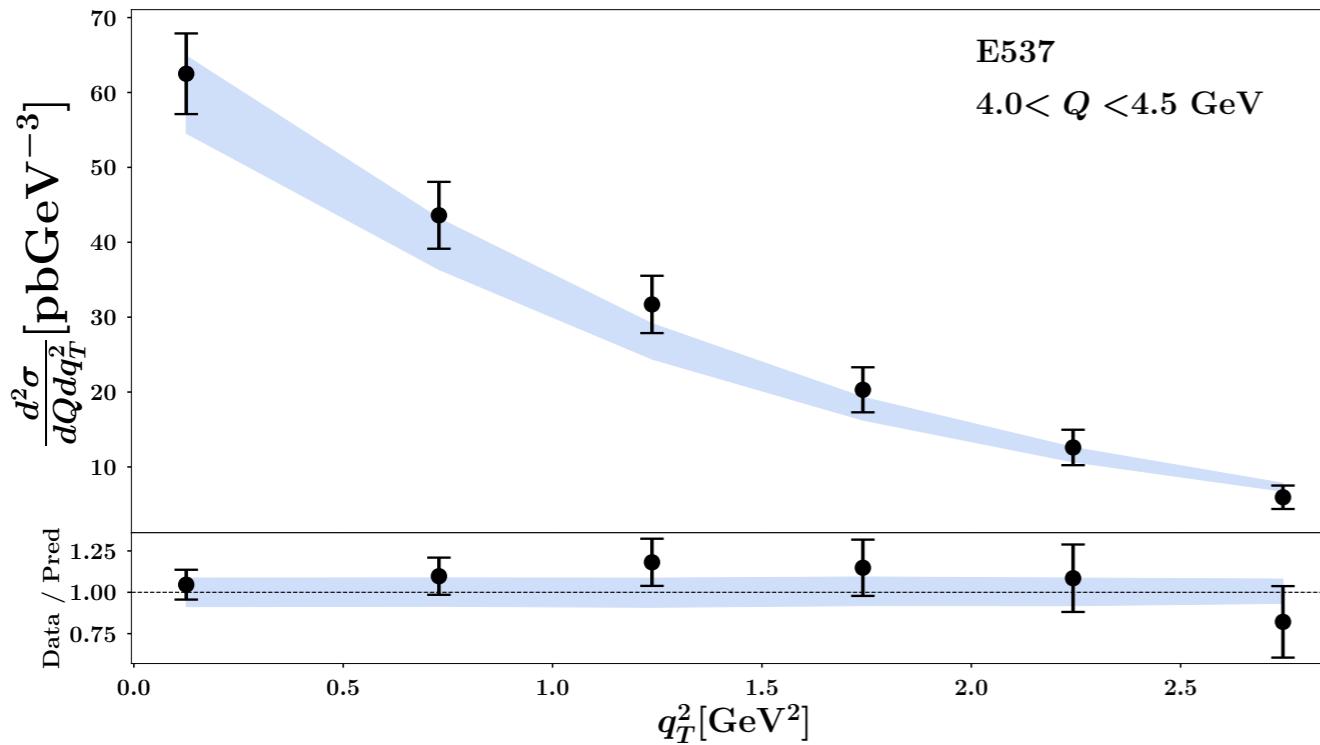
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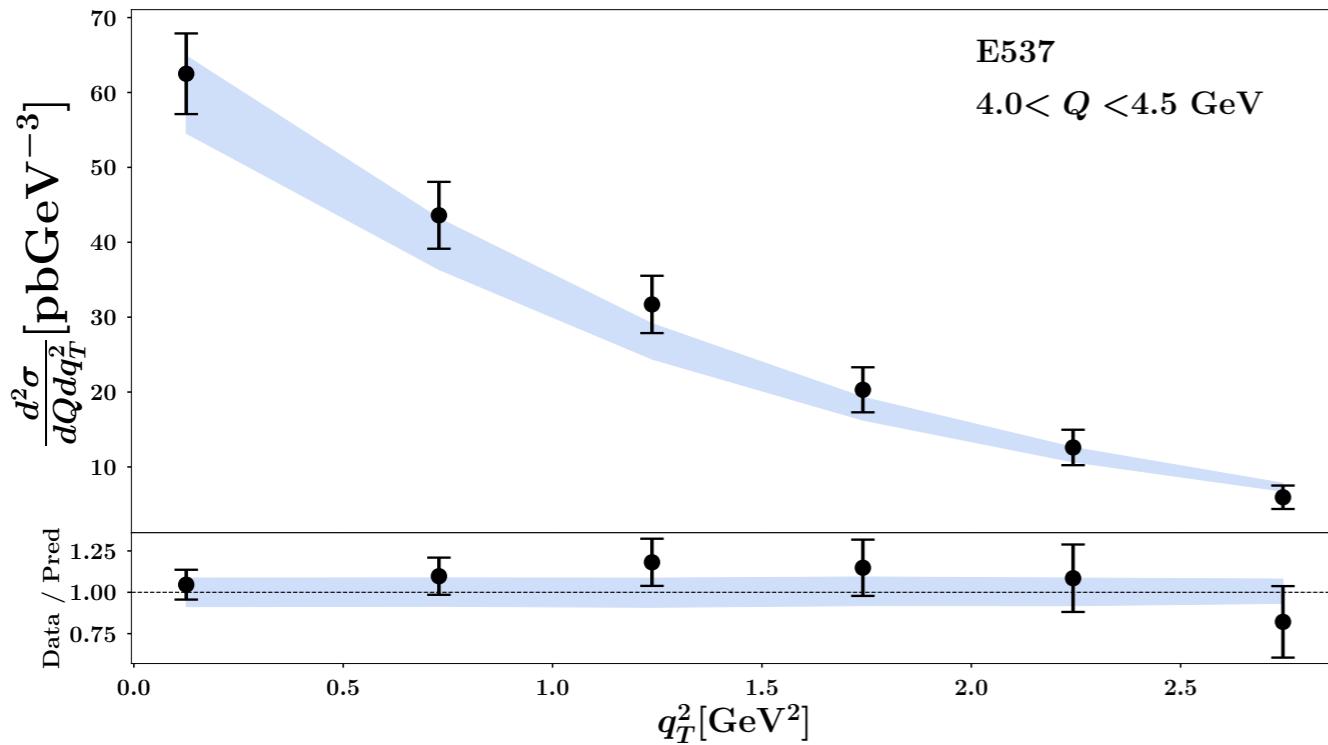
3 free parameters

Parameters for proton TMD and NP evolution are FIXED

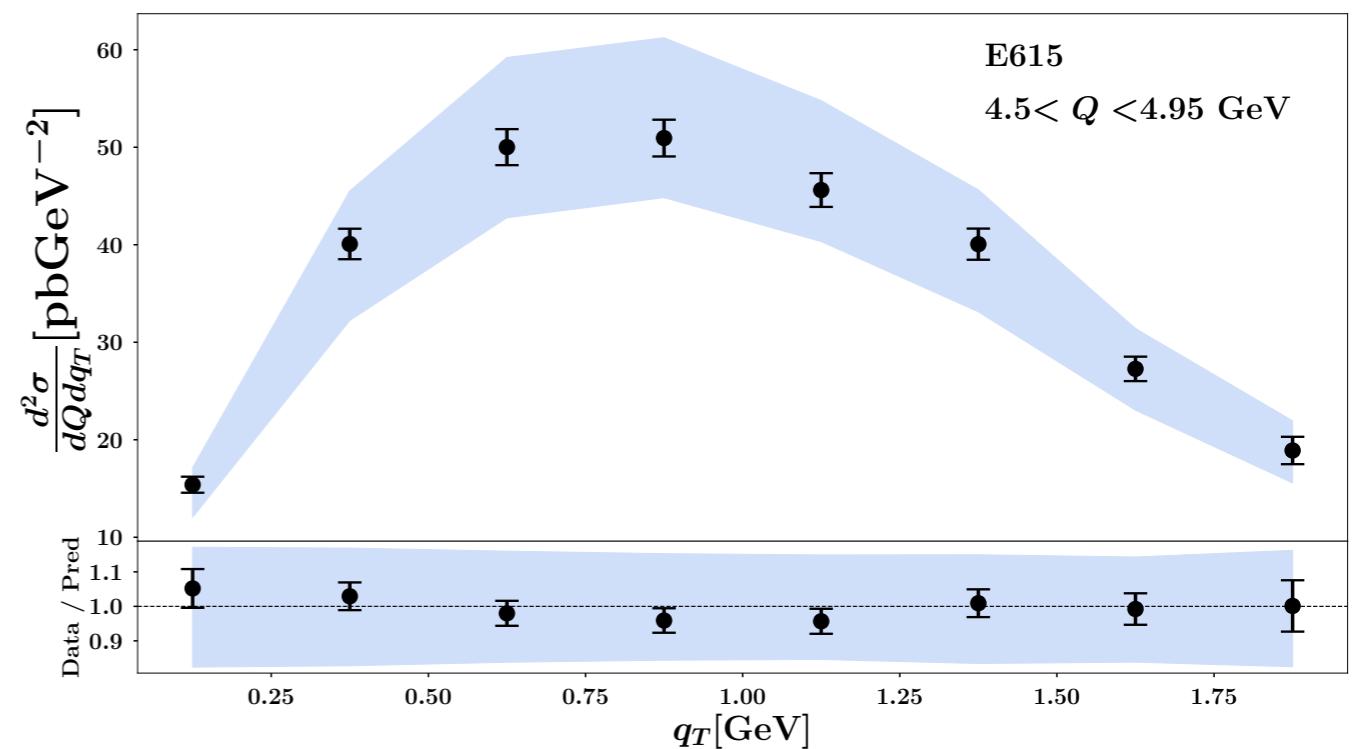
PionMAPTMD22 – Fit results



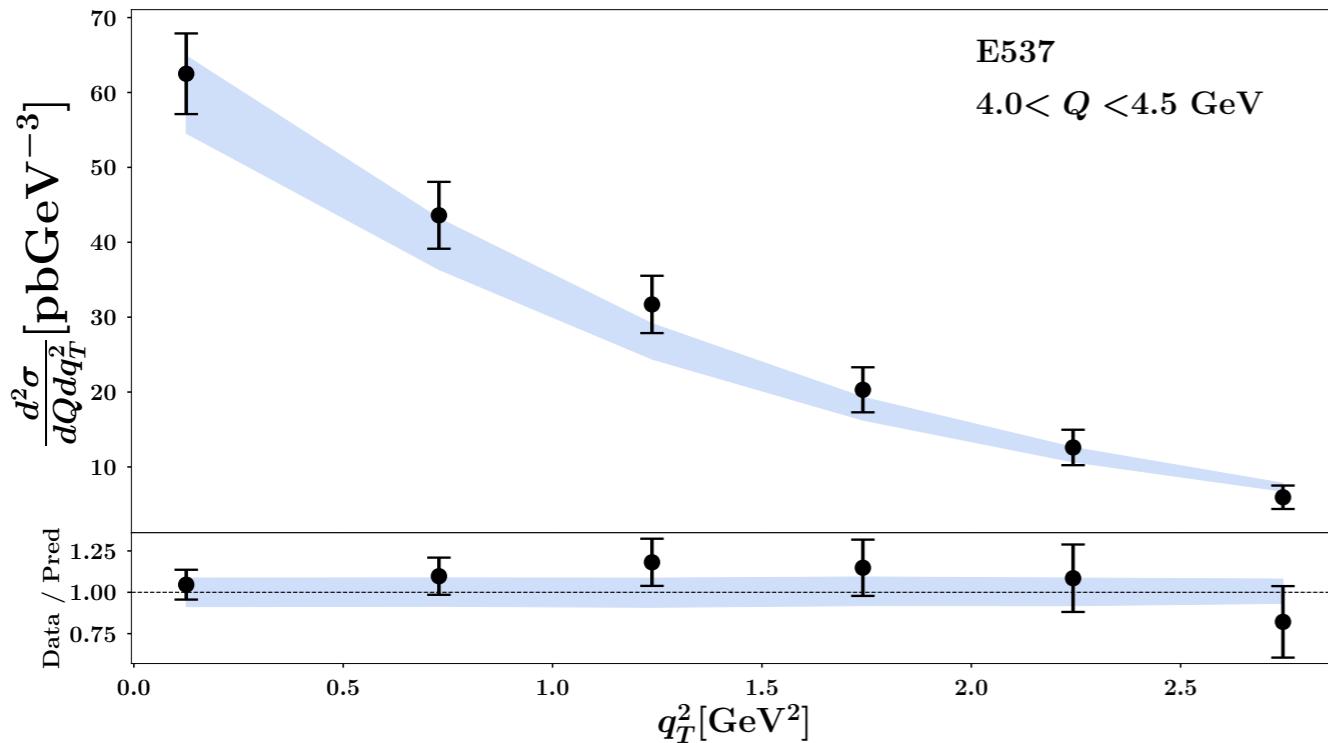
PionMAPTMD22 – Fit results



Good agreement
in the shape

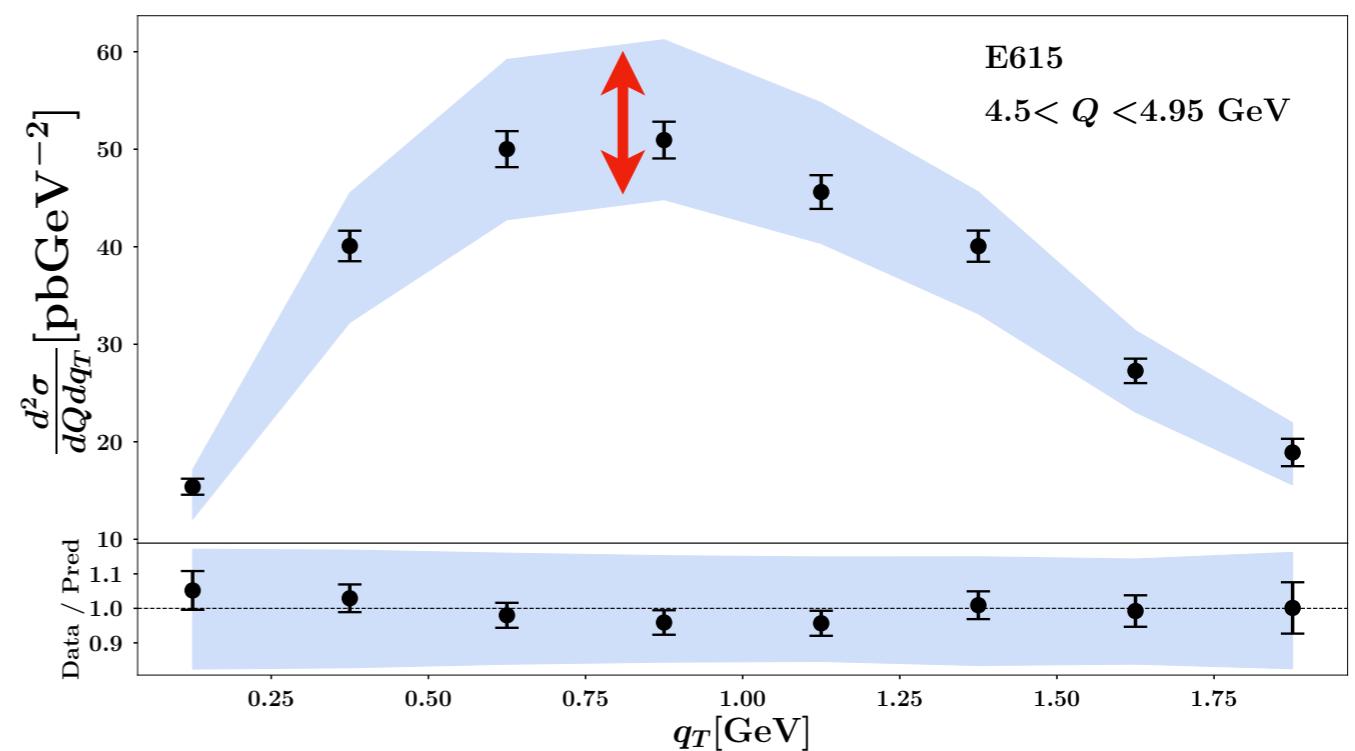


PionMAPTMD22 – Fit results



Quite big
normalization errors

Good agreement
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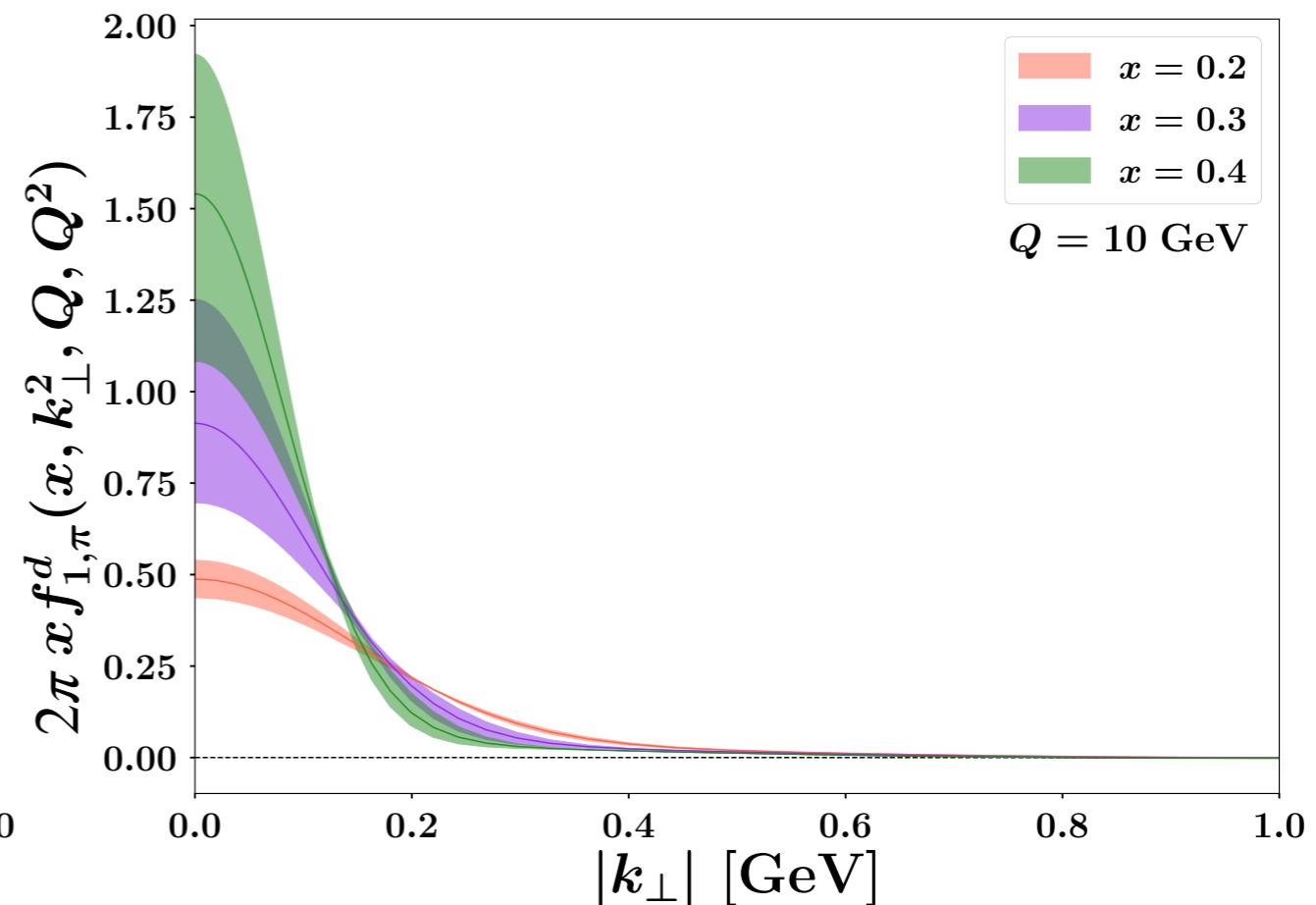
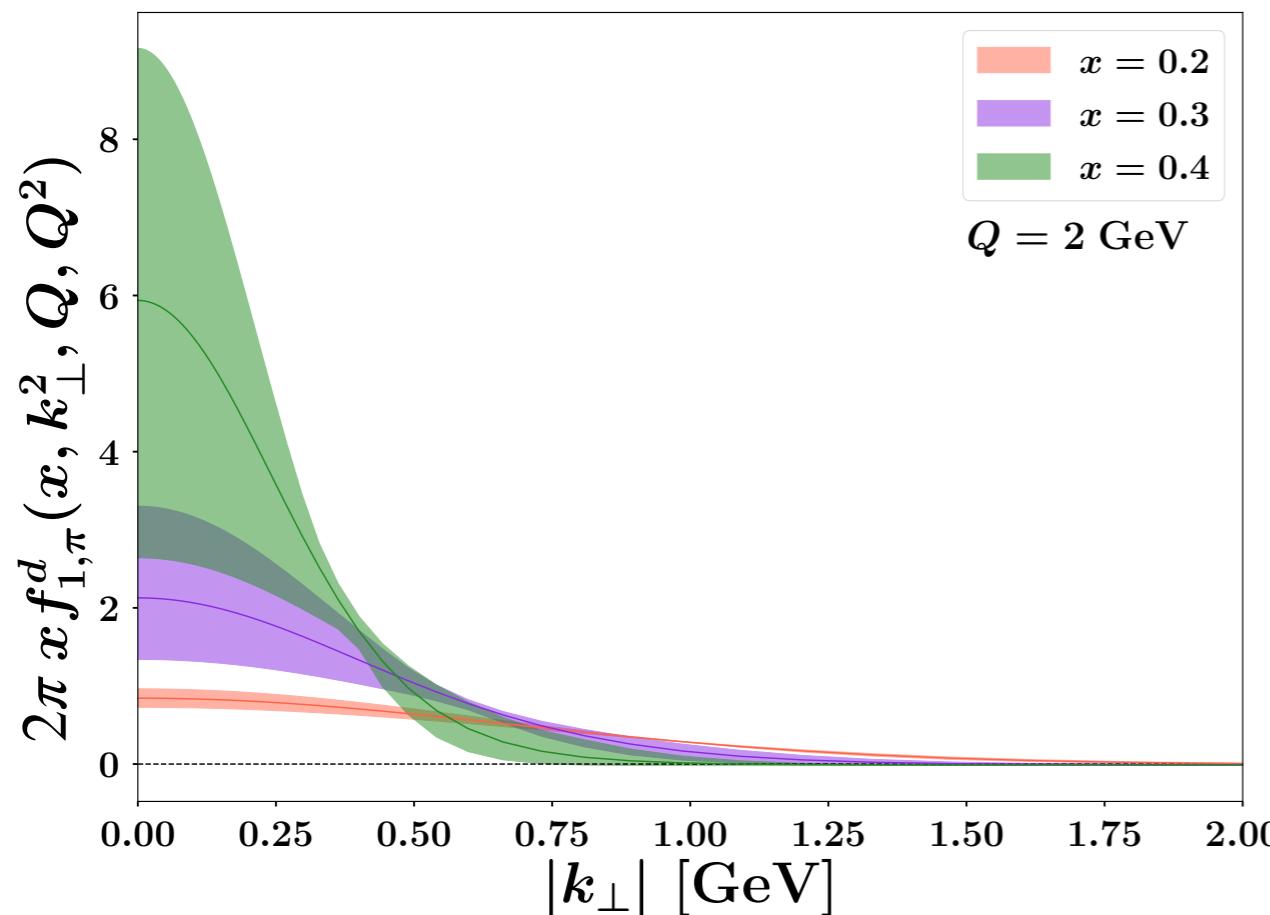


PionMAPTMD22 – Extracted TMDs

$$N_{1\pi}[\text{GeV}^2] = 0.47 \pm 0.12$$

$$\sigma_\pi = 4.50 \pm 2.25$$

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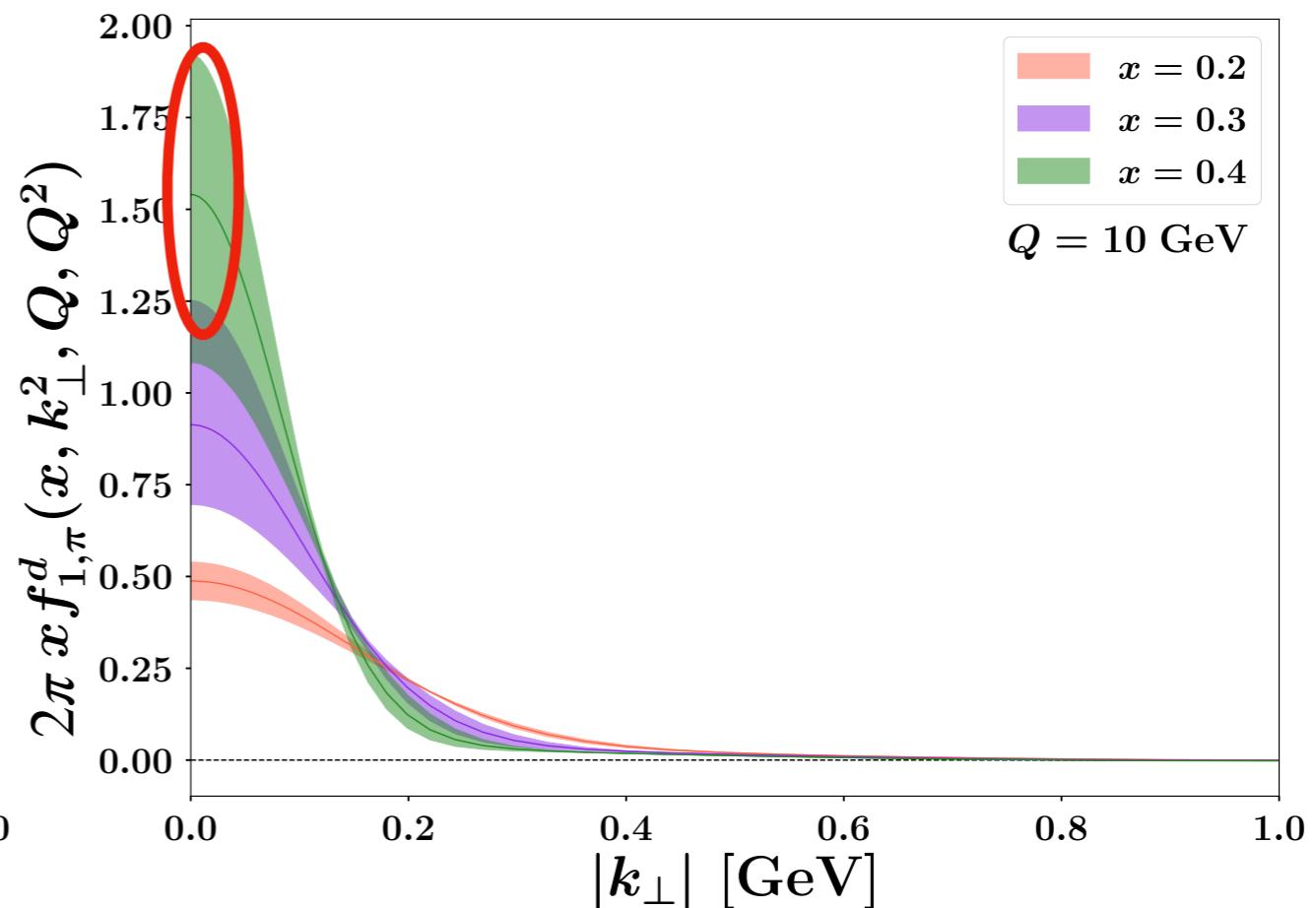
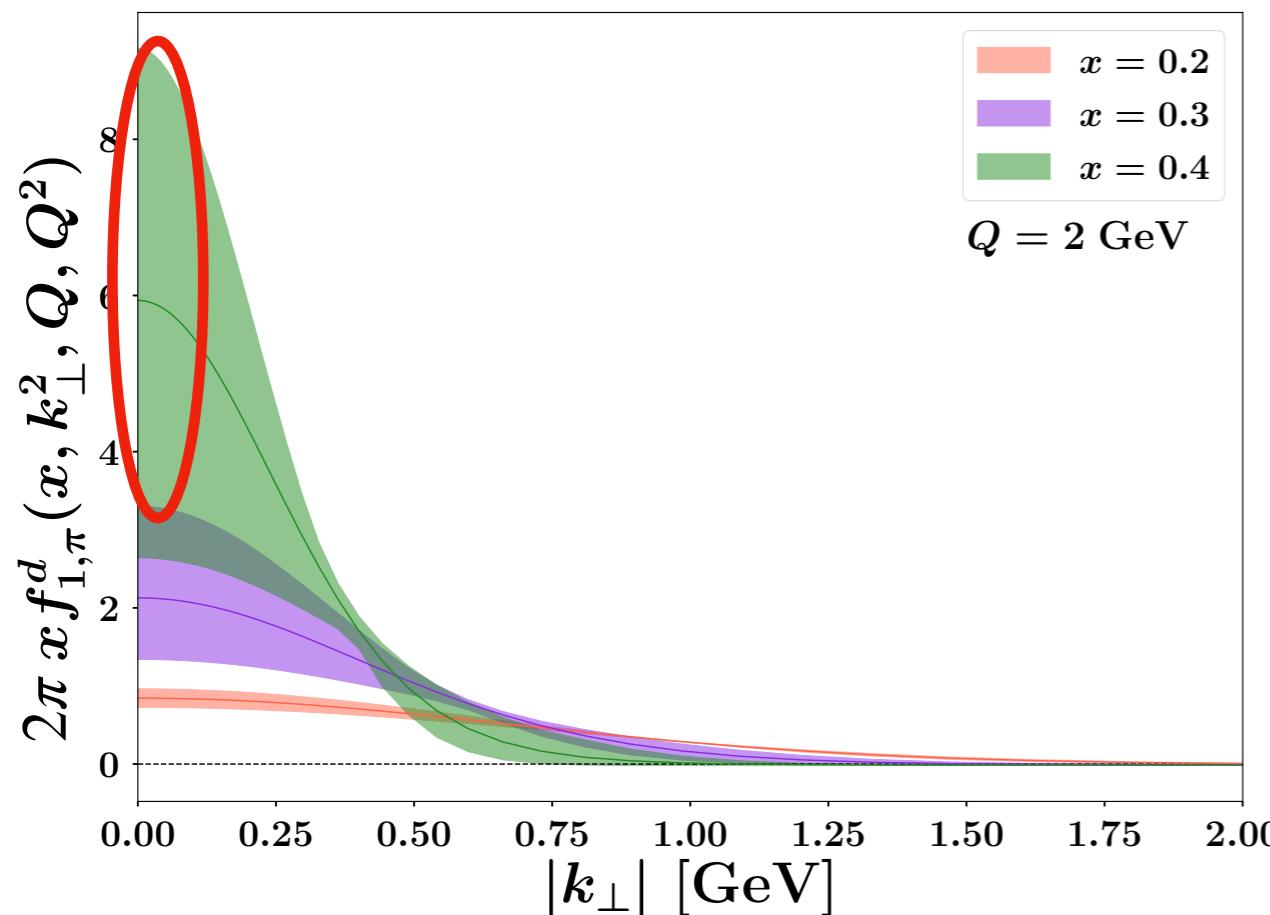


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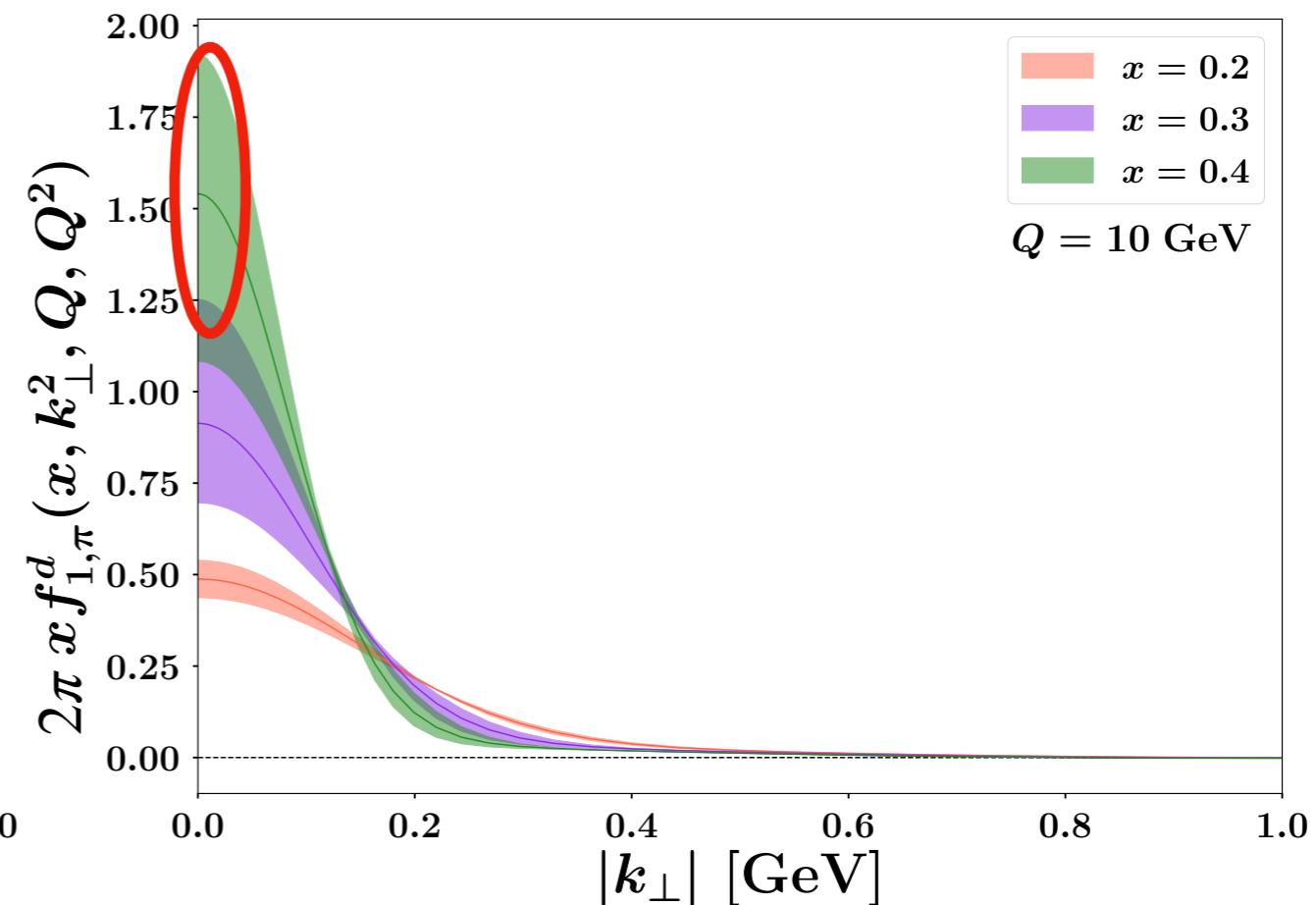
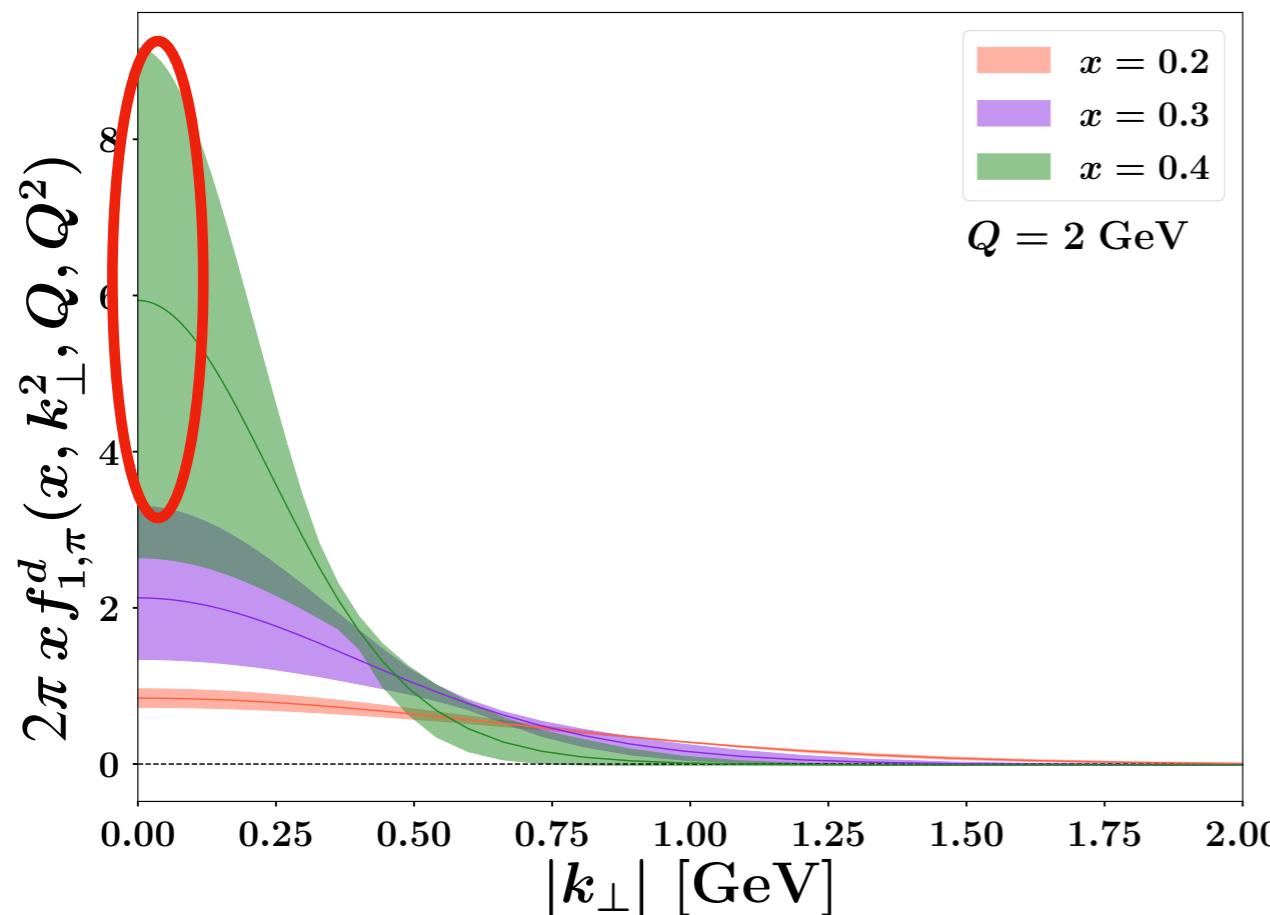


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Need of new data to better constrain the TMD

Conclusions and Outlook

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Still open issues

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Still open issues

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- *Normalization of SIDIS multiplicities beyond NLL*
- *Net effect of power corrections in SIDIS*

Conclusions and Outlook

- *MAPTMD22* is the most recent extraction of unpolarized quark TMDs in the PROTON through global fits
- *PionMAPTMD22* is the first extraction by the MAP Collaboration of unpolarized quark TMDs in the PION through a fit of pion-induced Drell-Yan

Still open issues

- *Region of applicability of TMD formalism*
- *Normalization of SIDIS multiplicities beyond NLL*
- *Net effect of power corrections in SIDIS*
- *Refined extraction of TMDs in the pion*

Backup

Structure of a TMD - TMD evolution

$$\mu_0 = \sqrt{\zeta_0} = \mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

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Stewart, Tackmann, Walsh and Zuberi, Phys. Rev. D89 (2014)
Ebert and Tackmann, JHEP 02 (2017)

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$$S_{\text{pert}}(\mu_b, Q) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(Q)}{4\pi} \right)^n \sum_{k=1}^{2n} L^{2n-k} R^{(n, 2n-k)} \quad L = \ln \left(\frac{Q^2}{\mu_b^2} \right)$$

Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)

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Accuracy	H and C	K and γ_F	γ_K	PDF and α_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
$N^3\text{LL}$	2	3	4	NNLO
$N^3\text{LL}'$	3	3	4	$N^3\text{LO}$

MAPTMD22 - Included data sets

Drell-Yan

Fixed-target low-energy DY

RHIC data

LHC and Tevatron data

$9 \lesssim Q \lesssim 11$ GeV excluded (Υ resonance)

$$q_T|_{\max} = 0.2Q$$

484 experimental points

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Drell-Yan

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$$q_T|_{\max} = 0.2Q$$

SIDIS

HERMES data

COMPASS data

$$Q > 1.3 \text{ GeV}$$

$$0.2 < z < 0.7$$

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

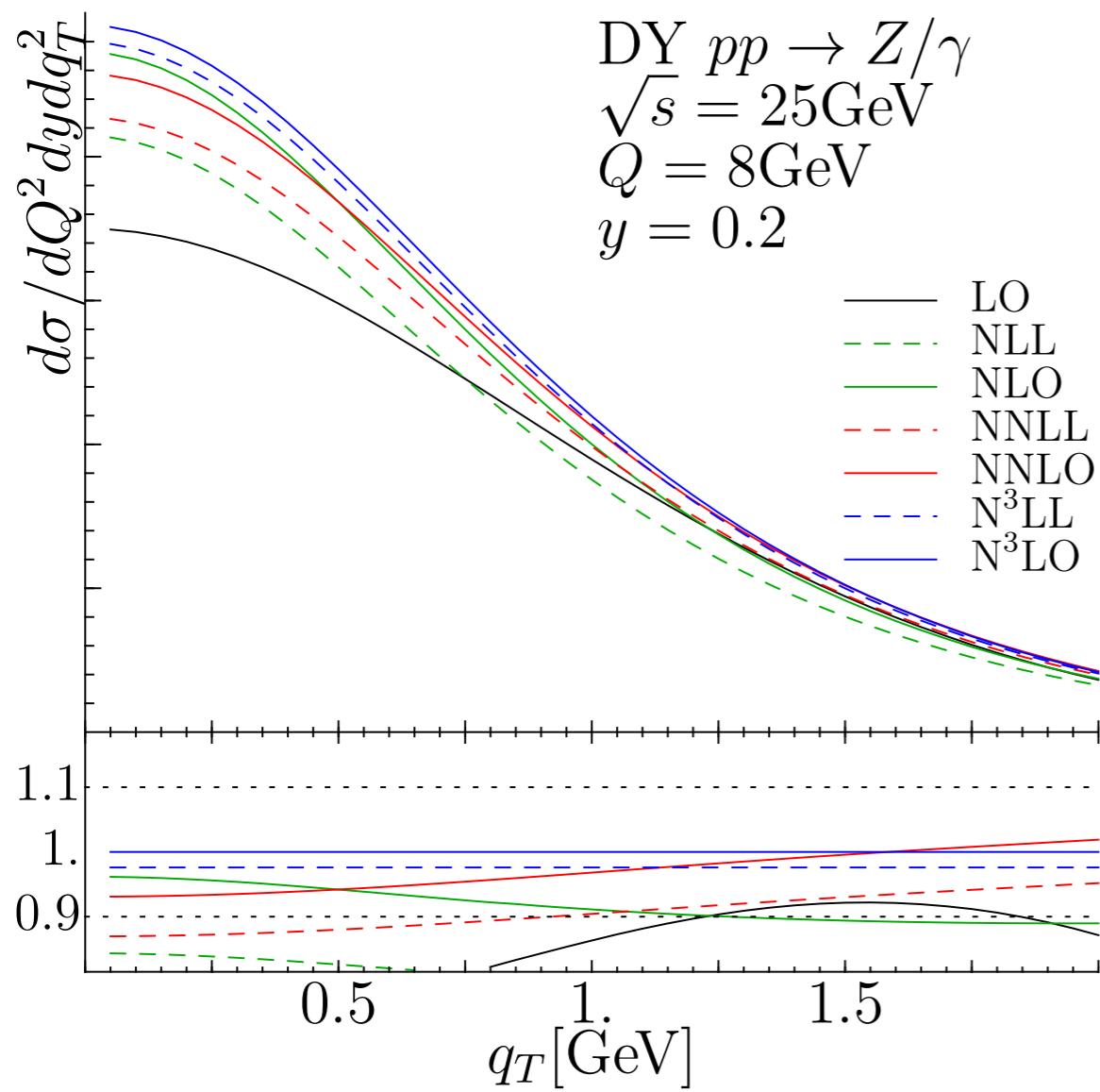
484 experimental points

1547 experimental points

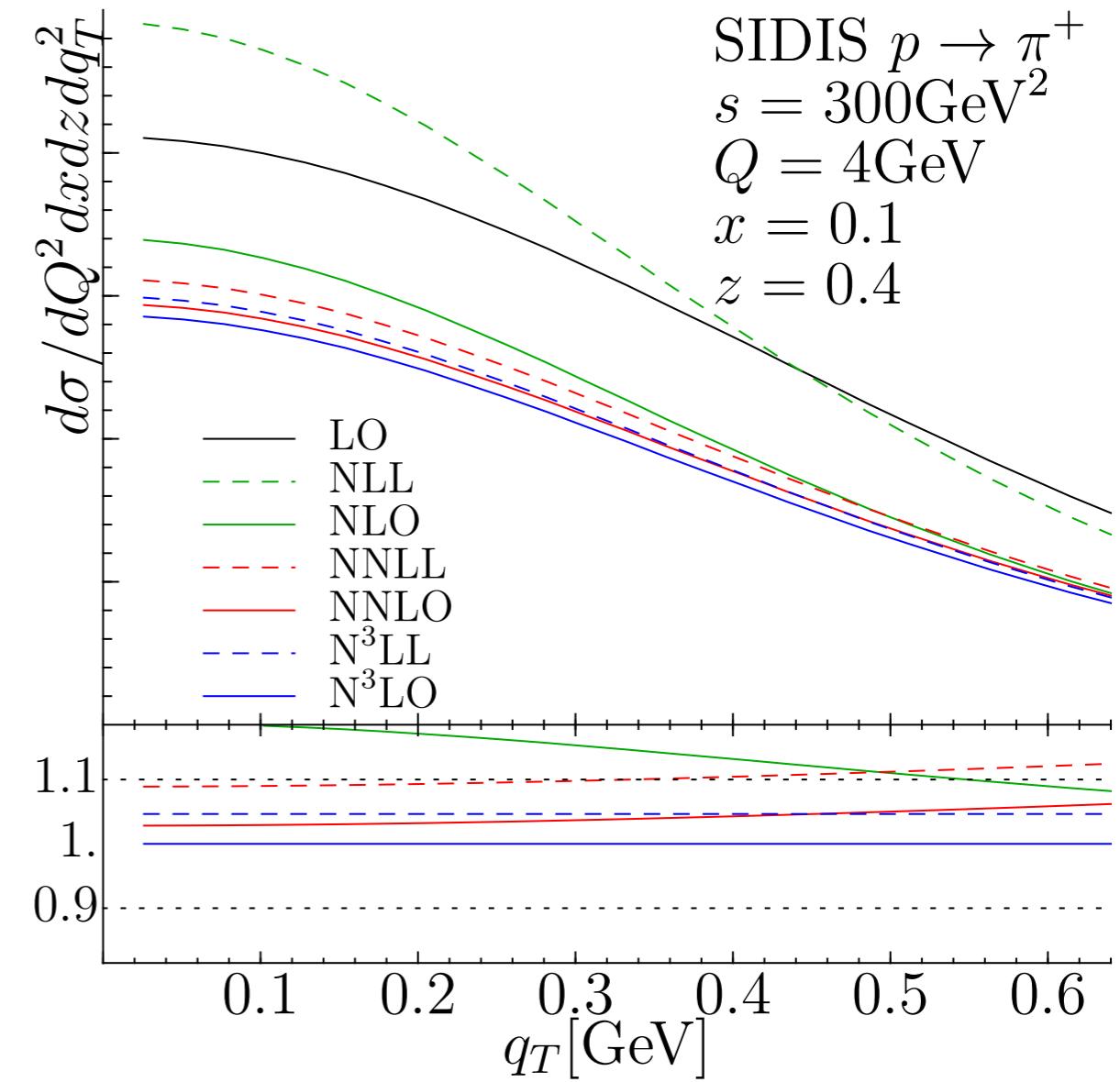
Comparison with SV19

Scimemi, Vladimirov, arXiv:1912.06532

Drell-Yan



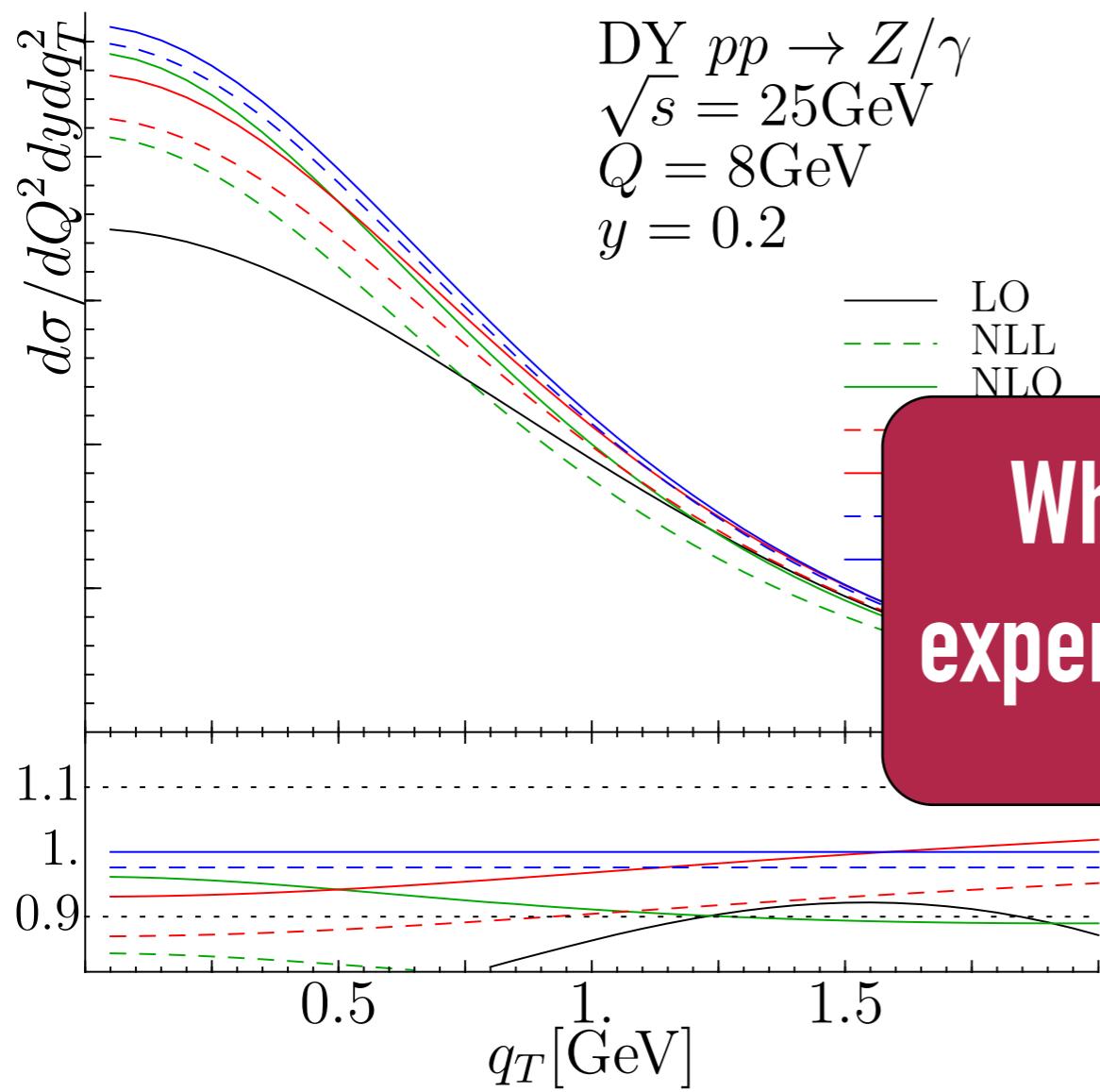
SIDIS



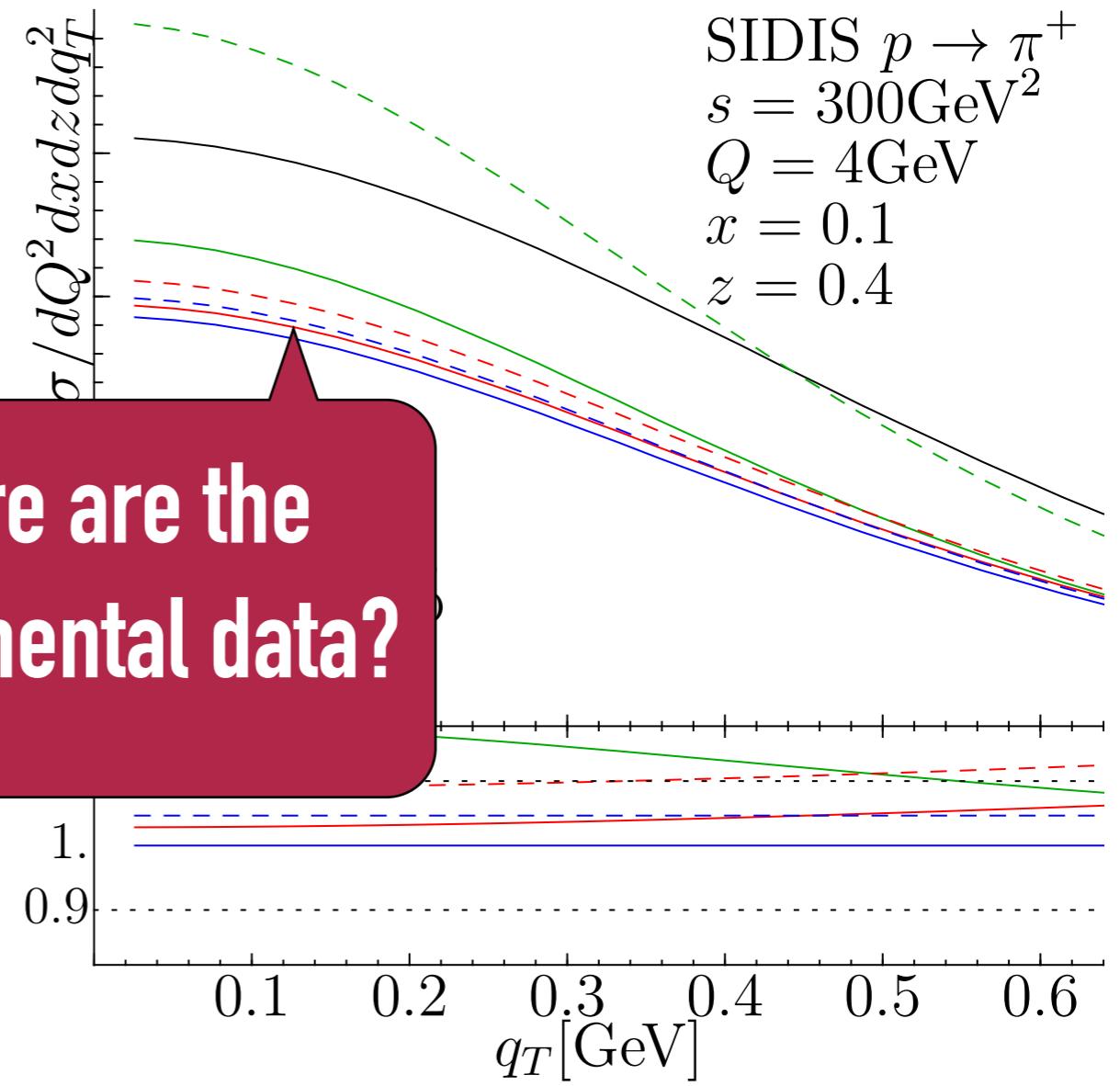
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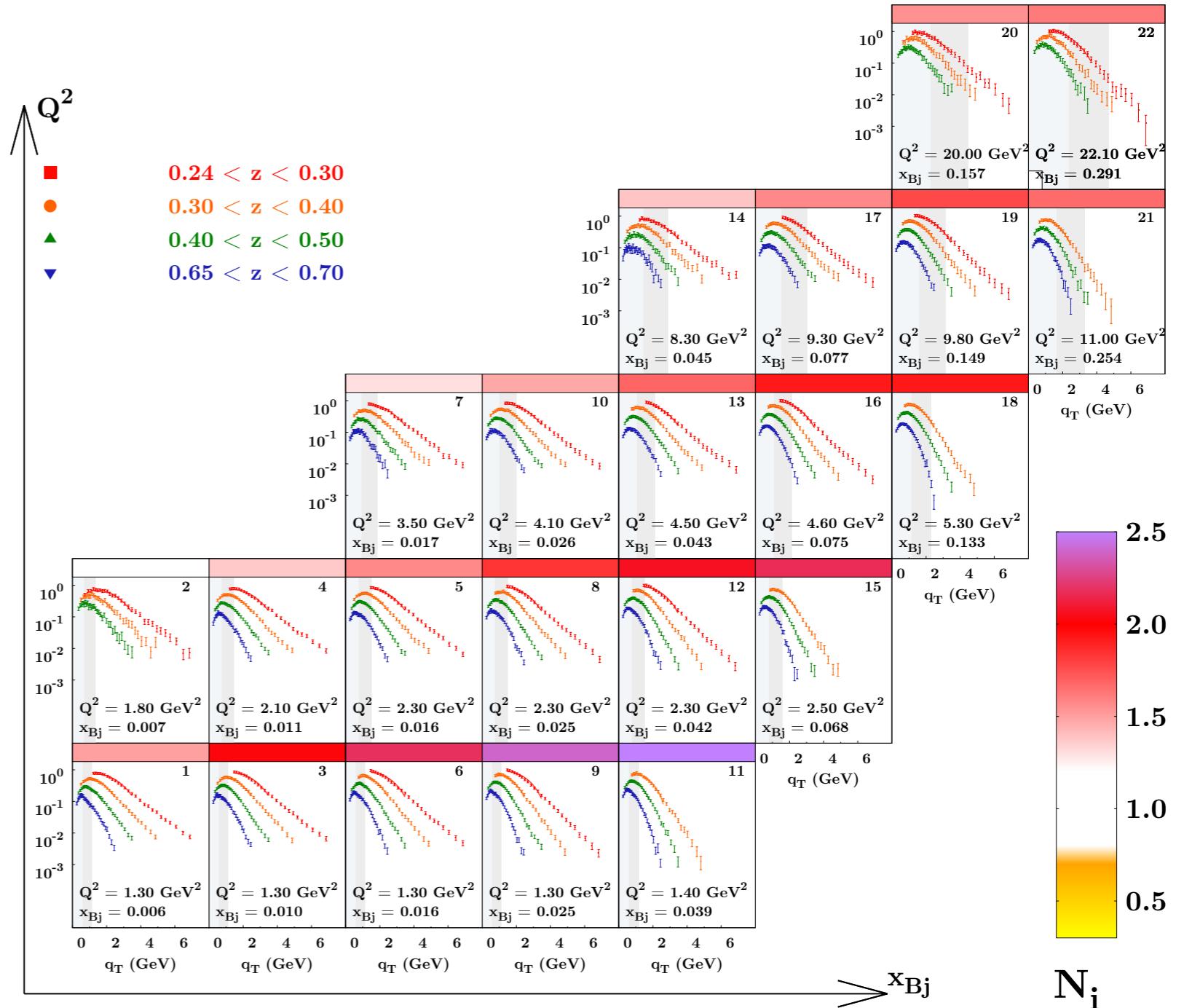
SIDIS



Where are the
experimental data?

MAPTMD22 – Normalization issue

Torino's group also confirmed that large normalisation factors have to be introduced to describe COMPASS data

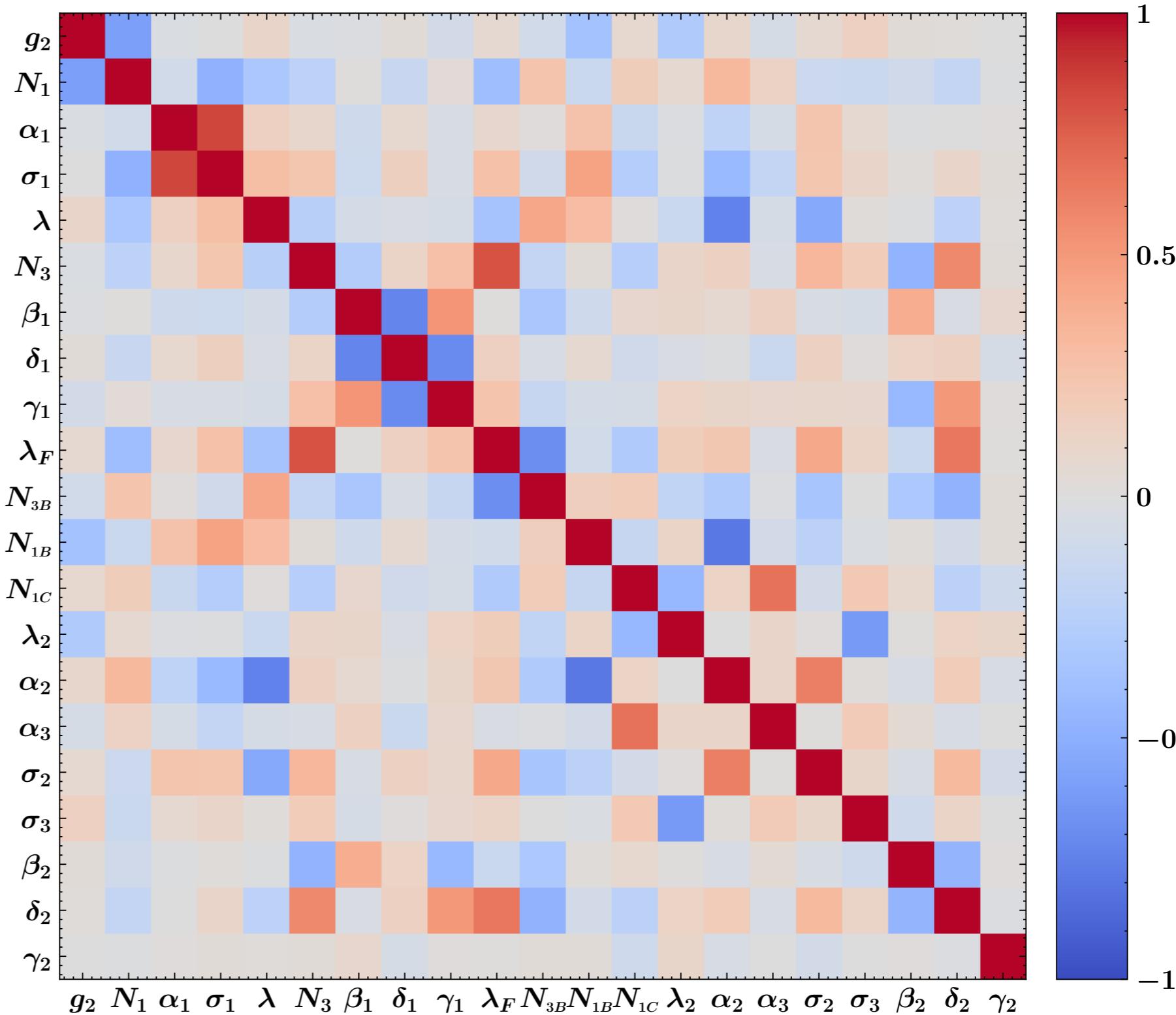


Fit error propagation

Error propagation



250 Montecarlo
replicas



Fit error propagation

Error propagation

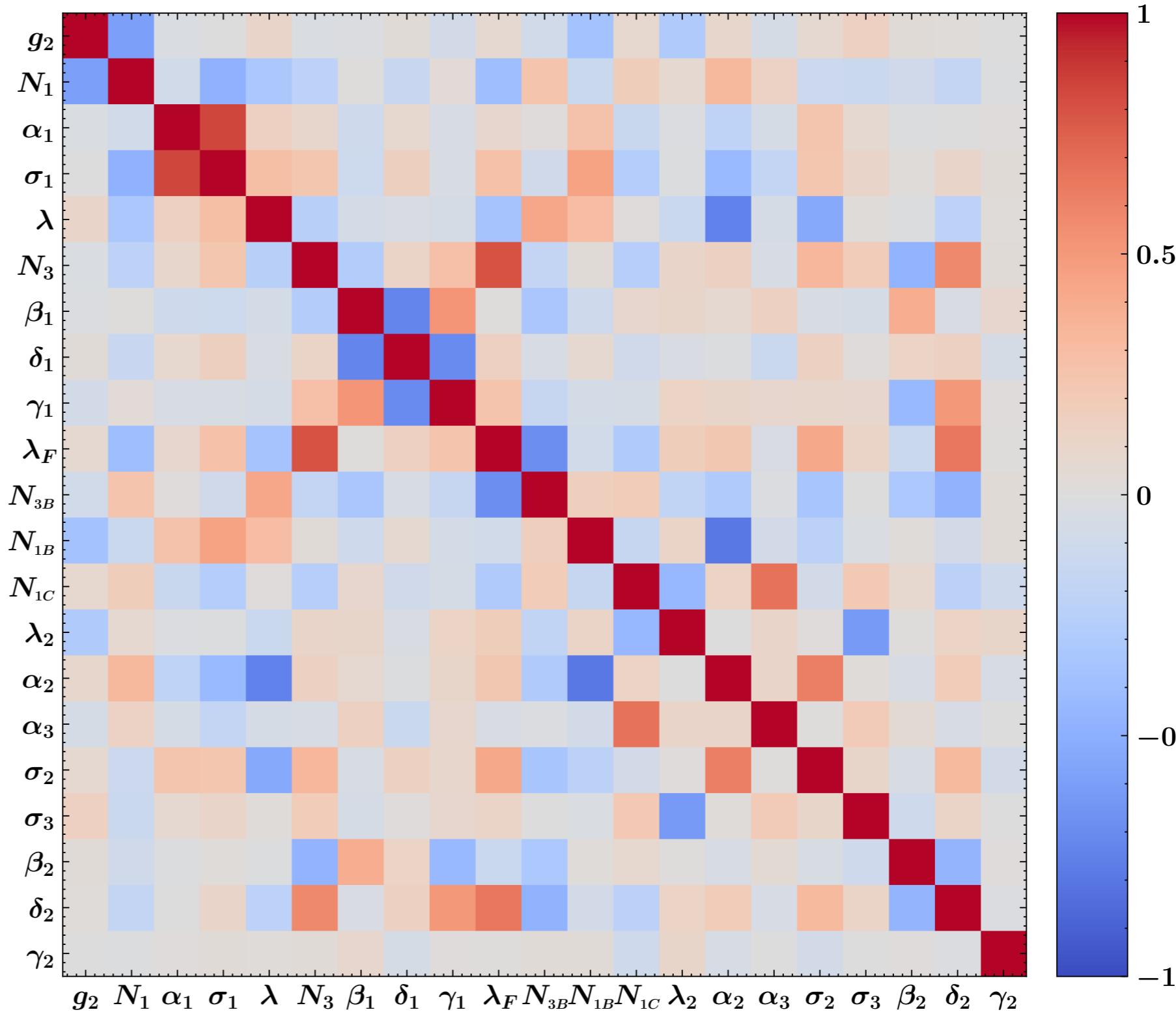


250 Montecarlo
replicas

Correlation matrix



Hints of the
appropriateness of
the chosen
functional form



MAPTMD22 results

Collins-Soper kernel

MAPTMD22 results

Collins-Soper kernel

Kernel of the rapidity evolution equation

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu)$$

MAPTMD22 results

Collins-Soper kernel

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