Hadronization dynamics from the spectral representation of the gauge invariant quark propagator

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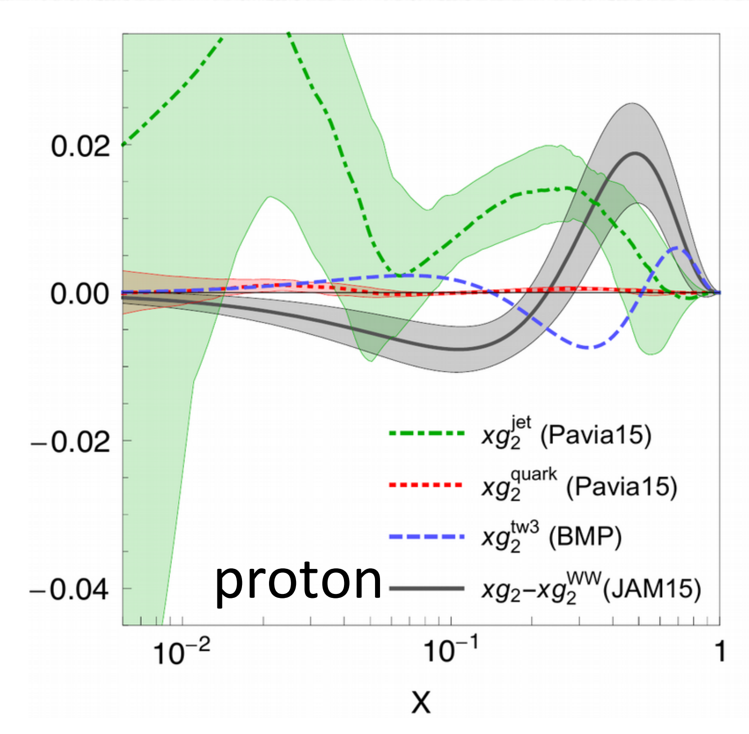
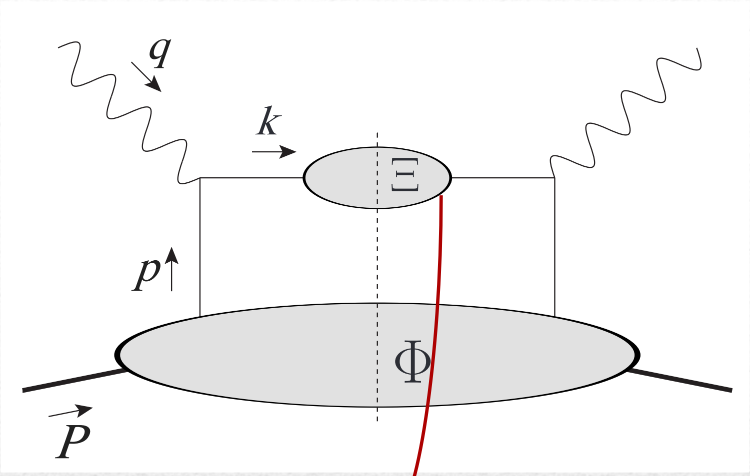
Exploring the Nature of Matter

- Quarks and gluons are the fundamental d.o.f of QCD, yet we lack understanding on how color neutral and massive hadrons emerge out of these colored and massless quarks and gluons
- Understanding hadronization remains elusive, but studying it will shed light on QCD dynamics and hadron formation





dynamically generated mass

$$g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2}\sum_a e_a^2 \left(g_2^{tw-3}(x_B) + \frac{m_q}{M}\left(\frac{h_1^q}{x}\right)^{\star}(x_B) + \frac{M_q - m_q}{M}\frac{h_1^q(x_B)}{x_B} \right)$$

AA, Bacchetta, AA (2017)

Gauge invariant quark propagator

AA, AS (2019)

$$\Xi_{ij}(k;w) = \mathrm{Disc} \int d^4p \frac{\mathrm{Tr}_c}{N_c} \langle \Omega | i\widetilde{S}_{ij}(p;v) \widetilde{W}(k-p;w,v) | \Omega \rangle$$

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$$i\widetilde{S}_{ij}(p,v) = \hat{s}_3(p^2, p\cdot v) \not{p}_{ij} + \sqrt{p^2}\, \hat{s}_1(p^2, p\cdot v)\, \mathbb{I}_{ij} + \hat{s}_0(p^2, p\cdot v) \not{v}_{ij}$$

(axial gauges)

$$\widetilde{W}(k-p;\, w,v) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\xi\cdot(k-p)}\, W(0,\xi;w,v)$$

- Gauge invariant and fully dressed quark propagator
- Color averaging mimics color neutralization

Gauge invariant quark propagator

$$\Xi_{ij}(k;w) = \text{Disc} \int d^4p \frac{\text{Tr}_c}{N_c} \langle \Omega | i\widetilde{S}_{ij}(p;v) \widetilde{W}(k-p;w,v) | \Omega \rangle$$

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$$i\widetilde{S}_{ij}(p,v) = \underbrace{\hat{s}_3(p^2, p\cdot v)}_{\hat{s}_3(p^2)} \not{p}_{ij} + \sqrt{p^2}\, \underbrace{\hat{s}_1(p^2, p\cdot v)}_{\hat{s}_2(p^2)} \mathbb{I}_{ij} + \underbrace{\hat{s}_0(p^2, p\cdot v)}_{\frac{p^2}{p\cdot v}\hat{s}_0(p^2)} \not{v}_{ij}$$

(lightlike axial gauges)

$\hat{s}_3(p^2)$, $\hat{s}_2(p^2)$, $\hat{s}_0(p^2)$: spectral operators

$$\widetilde{W}(k-p;\, w,v) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\xi\cdot(k-p)}\, W(0,\xi;w,v)$$

Light-cone spectral representation

$$\frac{\mathrm{Tr}_c}{N_c}\langle\Omega|i\tilde{S}(p)|\Omega\rangle = \frac{1}{(2\pi)^4}\int_{-\infty}^{\infty} d\sigma^2 \rho(\sigma^2)\frac{i}{p^2-\sigma^2+i\epsilon}\theta(\sigma^2)$$

$$\rho(p^2) = \rho_3(p^2)\not{p} + \sqrt{p^2}\rho_1(p^2) + \frac{p^2}{p\cdot v}\rho_0(p^2)\not{v}$$

$$\mathrm{Disc}\frac{\mathrm{Tr}_c}{N_c}\langle\Omega|i\tilde{S}(p)|\Omega\rangle = \frac{1}{(2\pi)^3}\rho(p^2)\theta(p^2)\theta(p^-)$$

Integrated g.i. quark propagator

- Boost quark at large light-cone momentum: $k^- \sim Q$

$$k^- \gg |\mathbf{k}_\perp| \gg k^+$$

$\boxed{w = n^+}$

Integrate out the supressed component of the quark momentum:

$$J_{ij}(k^-, \vec{k}_\perp; n_+) \equiv \frac{1}{2} \int dk^+ \Xi_{ij}(k; n_+)$$

- Generalizes the perturbative quark propagator that appears in in Inclusive and semi-inclusive DIS

$$W_{\rm TMD}(\xi^+, \boldsymbol{\xi}_\perp) = \mathcal{U}_{n_+}[0^-, 0^+, \mathbf{0}_\perp; 0^-, \infty^+, \mathbf{0}_\perp]\,\mathcal{U}_{\boldsymbol{n}_\perp}[0^-, \infty^+, \mathbf{0}_\perp; 0^-, \infty^+, \boldsymbol{\xi}_\perp]\,\mathcal{U}_{n_+}[0^-, \infty^+, \boldsymbol{\xi}_\perp; 0^-, \xi^+, \boldsymbol{\xi}_\perp]$$

$$W_{\rm coll}(\xi^+) = \mathcal{U}_{n_+}[0^-, 0^+, \mathbf{0}_\perp; 0^-, \xi^+, \mathbf{0}_\perp]$$

Integrated g.i. quark propagator

- Expand in Dirac structures, in powers of $1/k^-$

$$J(k^-, \boldsymbol{k}_\perp; n_+) = \frac{1}{2}\alpha(k^-)\gamma^+ + \frac{\Lambda}{k^-}\left[\zeta(k^-)\mathbb{I} + \alpha(k^-)\frac{\not{\boldsymbol{k}}_\perp}{\Lambda}\right] + \frac{\Lambda^2}{2(k^-)^2}\omega(k^-, \boldsymbol{k}_\perp^2)\gamma^-$$

$$\alpha(k^-) = J^{[\gamma^-]}$$

$$\zeta(k^-) = \frac{k^-}{\Lambda}J^{[\mathbb{I}]}$$

$$\omega(k^-, \boldsymbol{k}_\perp^2) = \left(\frac{k^-}{\Lambda}\right)^2 J^{[\gamma^+]}$$

Spectral sum rule

- Based solely on the gauge invariance o J, we can obtain a new sum rule for the “light-cone spectral function”:

$$\int_0^\infty dp^2\, p^2\, \rho_0(p^2) = 0$$

- Rules out the contribution that would in principle be present at twist-4 due to $\not{\partial}$

Integrated g.i. quark propagator

$$J(k^-, \boldsymbol{k}_\perp; n_+) = \frac{1}{2}\alpha(k^-)\gamma^+ + \frac{\Lambda}{k^-}\left[\zeta(k^-)\mathbb{I} + \alpha(k^-)\frac{\not{\boldsymbol{k}}_\perp}{\Lambda}\right] + \frac{\Lambda^2}{2(k^-)^2}\omega(k^-, \boldsymbol{k}_\perp^2)\gamma^-$$

$$\not{k} + m = k^-\,\gamma^+ + \not{k}_\perp + m\mathbb{I} + \frac{m^2 + \boldsymbol{k}_\perp^2}{2k^-}\,\gamma^-$$

$$J(k^-, \boldsymbol{k}_T; n_+) = \frac{\theta(k^-)}{4(2\pi)^3\,k^-}\left\{k^-\,\gamma^+ + \not{k}_T + M_j\mathbb{I} + \frac{K_j^2 + \boldsymbol{k}_T^2}{2k^-}\gamma^-\right\}$$

Average mass of all the hadronization products produced during the fragmentation of a quark

Integrated g.i. quark propagator

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Average mass of all the hadronization products produced during the fragmentation of a quark

$$K_j^2 = \mu_j^2 + \tau_j^2$$

(Jet virtuality)

- In any gauge,

$$M_j = \int dp^2 \, \sqrt{p^2} \, \rho_1(p^2)$$

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Gauge invariant generalization of the gauge dependent dressed quark mass

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Gauge invariant generalization of the gauge dependent dressed quark mass

(l.c.g)

$$K_j^2 = \mu_j^2 + \cancel{\tau_j^2} = \int_0^\infty dp^2\, p^2\, \rho_3^{\mathrm{lcg}}(p^2)$$

Invariant mass of the particles produced in the quark's fragmentation process

- In other gauges,

$$K_j^2 = \mu_j^2 + \tau_j^2$$

Final state interactions

$$\tau_j^2 = (2\pi)^3 \int_0^\infty dp^2 \, \mathrm{Disc} \frac{\mathrm{Tr_c}}{\mathrm{N_c}} \langle \Omega | \hat{\sigma}_3(p^2) ig \, \boldsymbol{D}_\perp \left(\boldsymbol{A}^\perp(\boldsymbol{\xi}_\perp) + \mathcal{Z}^\perp(\boldsymbol{\xi}_\perp) \right)_{\boldsymbol{\xi}_\perp = 0} |\Omega\rangle$$

$$\mathcal{Z}^\perp(\boldsymbol{\xi}_\perp) = \int_0^{\infty^+} ds^+ \boldsymbol{D}_\perp \Big(U_{n_+}[0^-, 0^+, \boldsymbol{\xi}_\perp; 0^-, s^+, \boldsymbol{\xi}_\perp] G^{\perp -}(0^-, s^+, \boldsymbol{\xi}_\perp) U_{n^+}[0^-, s^+, \boldsymbol{\xi}_\perp; 0^-, \infty^+, \boldsymbol{\xi}_\perp] \Big) |\Omega\rangle$$

- M_j provides a gauge invariant generalization of the gauge dependent dressed quark mass
- Non-vanishing even in the chiral limit
- Provides a direct way to probe dynamical mass generation
- Not only of theoretical interest..
- It’s calculable, but moreover.. It can be measured!

Snowmass 2021 White Paper
Upgrading SuperKEKB with a Polarized Electron Beam: Discovery Potential and Proposed Implementation

April 13, 2022

US Belle II Group [1]
and
Belle II/SuperKEKB e- Polarization Upgrade Working Group [2]

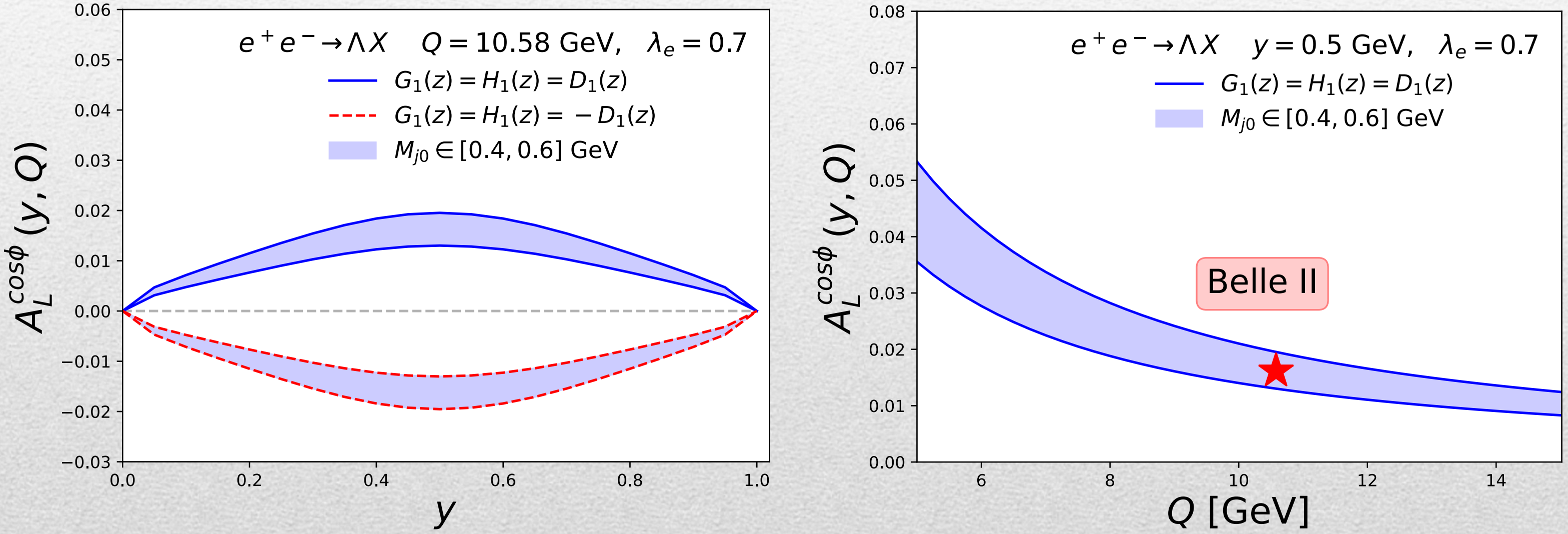

Figure 13: The Fourier components $A_L^1(y)$ and $A_L^{\cos\phi}(y, Q)$ of the longitudinal electron spin asymmetry as a function of y at the SuperKEKB nominal energy $Q = 10.58$ GeV. The band in the $\cos\phi$ modulation indicates the sensitivity of the measurement to $\pm 20\%$ variation in the jet mass at the initial scale. The rightmost panel shows the $A_L^{\cos\phi}$ modulation as a function of Q at fixed $y = 0.5$, along with its 20% sensitivity to M_j, which also slightly increases at lower energies due to QCD evolution.