

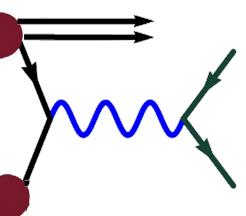


Probing the weak mixing angle at high energy at LHC

Clara Lavinia Del Pio, Simone Amoroso, Mauro Chiesa, Ekaterina Lipka, Fulvio Piccinini,
Federico Vazzoler, Alessandro Vicini

arXiv:2302.10782 [hep-ph]

DIS 2023 - 28 March 2023



The weak mixing angle in the Standard Model

Definitions of the weak mixing angle

$$\sin \theta_w = \frac{e}{g_2} = \frac{g_1}{\sqrt{g_1^2 + g_2^2}} \xrightarrow{\text{MS scheme}} \sin \theta_w^{\overline{\text{MS}}} (\mu) = \frac{e^{\overline{\text{MS}}}(\mu)}{g_2^{\overline{\text{MS}}}(\mu)}$$

Effective weak mixing angle defined at the Z pole

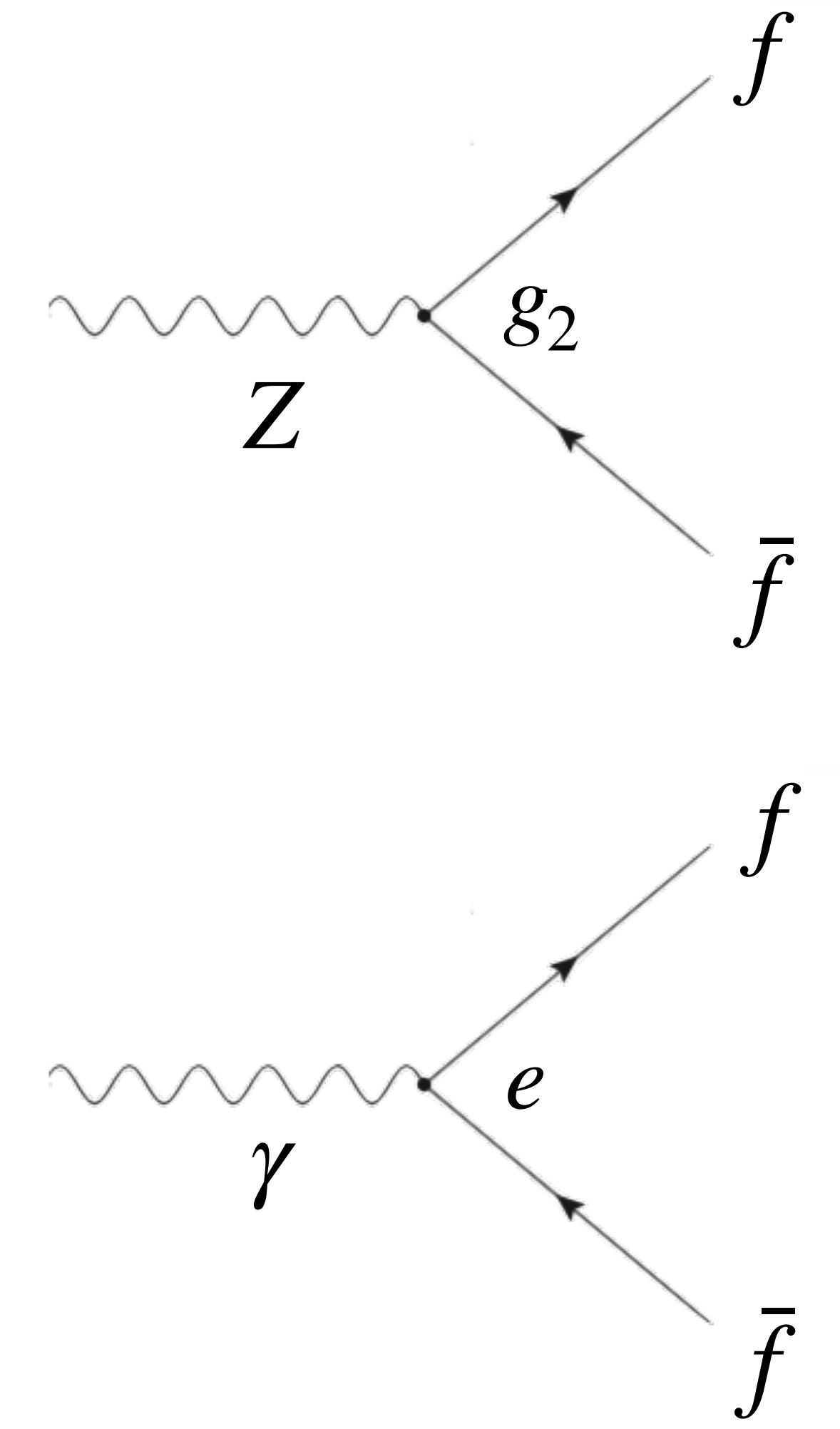
$$\sin^2 \theta_{eff} = \frac{1}{4|Q|} \left(1 - \frac{g_V}{g_A} \right)$$

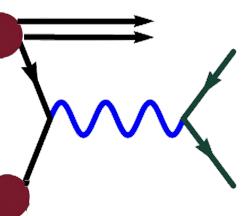
On-shell definition

$$\sin^2 \theta_{OS} = 1 - \frac{M_W^2}{M_Z^2}$$

| Scheme | Notation | Value | Uncertainty |
|--------------------------------|------------------|---------|---------------|
| On-shell | s_W^2 | 0.22339 | ± 0.00010 |
| $\overline{\text{MS}}$ Z pole | \hat{s}_Z^2 | 0.23122 | ± 0.00004 |
| $\overline{\text{MS}}$ $Q^2=0$ | \hat{s}_0^2 | 0.23863 | ± 0.00005 |
| Effective angle | \bar{s}_ℓ^2 | 0.23155 | ± 0.00004 |

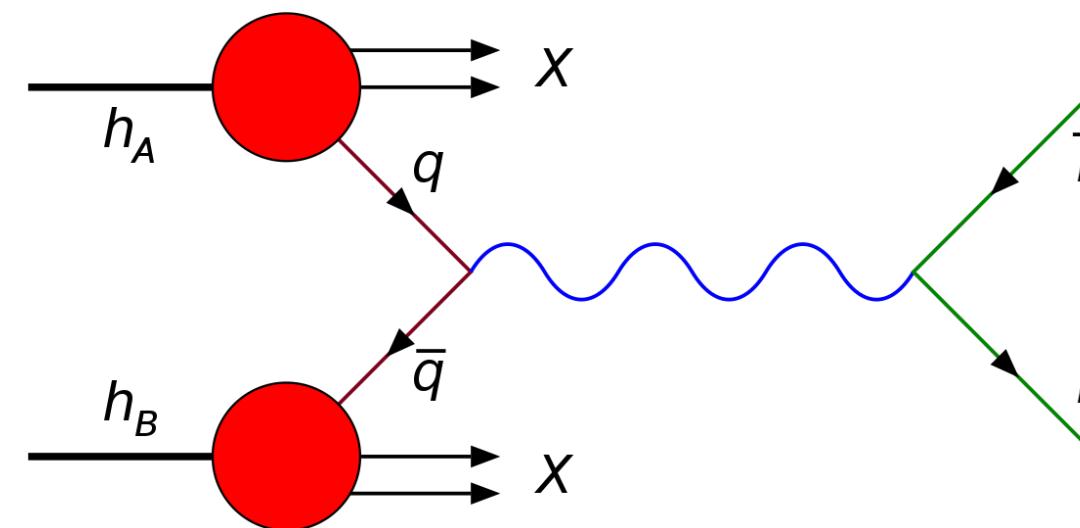
PDG, 2022





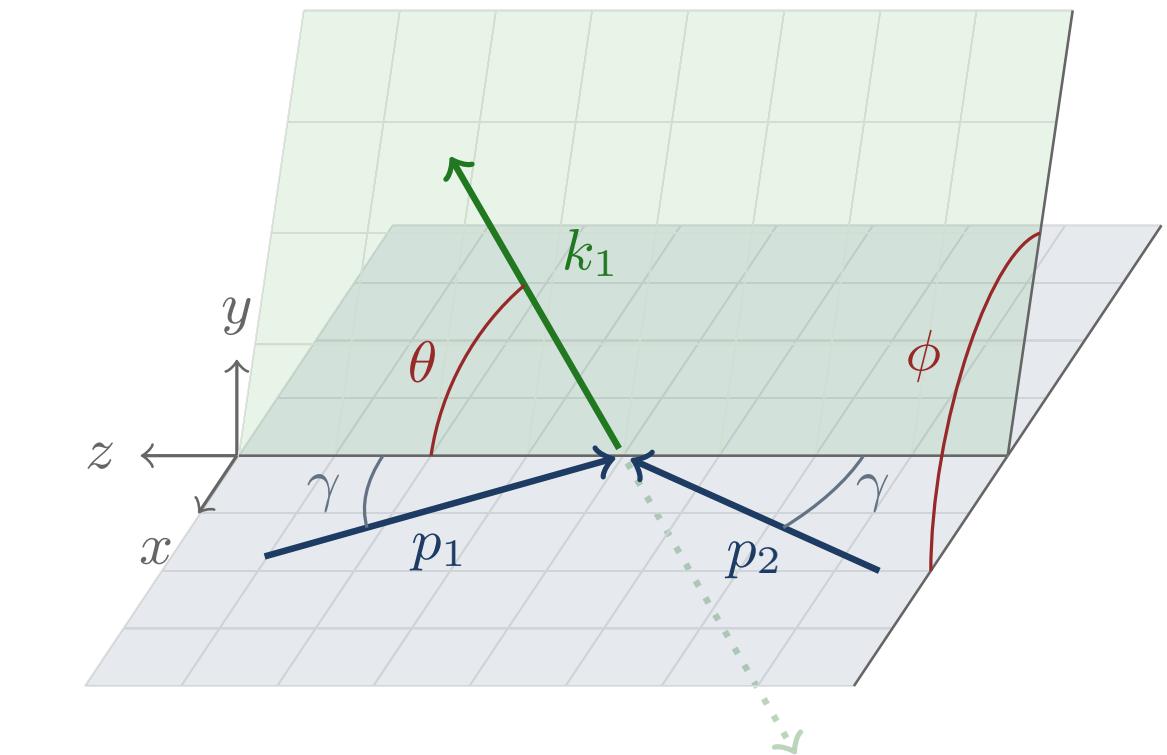
Direct determination of $\sin^2 \theta_{eff}^f$ around the Z pole

Hadronic colliders



$$\sim 1 + \cos^2 \theta_{\ell\bar{\ell}} + A_4 \cos \theta_{\ell\bar{\ell}}$$

$$A_4 \propto A_{FB}$$



Template fit procedure

$A_{FB}(m_{ll}, |y_{ll}|)$
data



MC samples with different
values of $\sin^2 \theta_{eff}^f$



Measured value = MC sample
which best fit the data

Dilution effect due to unknown direction of incoming quarks → enhanced sensitivity at high $|y_{\ell\bar{\ell}}|$
- Collins-Soper frame needed - careful treatment of PDFs (can be constrained in template fit)

→ Tevatron

CDF and DØ Collaborations, arXiv:1801.06283 [hep-ex], 2018

$$\sin^2 \theta_{eff}^\ell = 0.23148 \pm 0.00033$$

Comparable with
LEP!

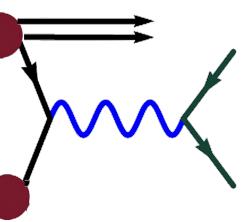
→ LHC

ATLAS Collaboration, Tech. Rep. ATLAS-CONF-2018-037, CERN, 2018

CMS Collaboration, arXiv:1806.00863 [hep-ex], 2018

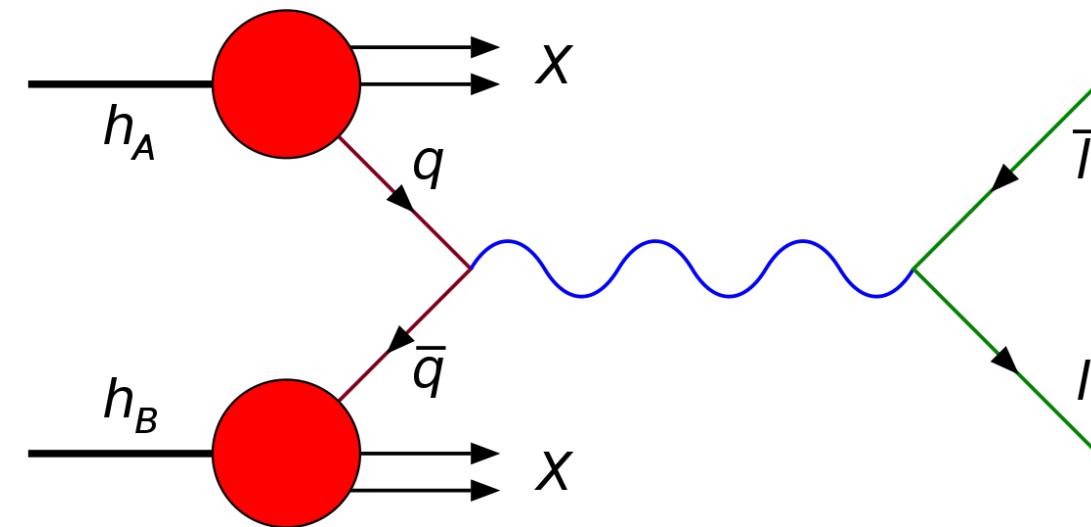
LHCb Collaboration, arXiv:1509.07645 [hep-ex], 2015

$$\sin^2 \theta_{eff}^\ell = 0.23129 \pm 0.00033$$



Direct determination of $\sin^2 \theta_{eff}^f$ around the Z pole

Hadronic colliders



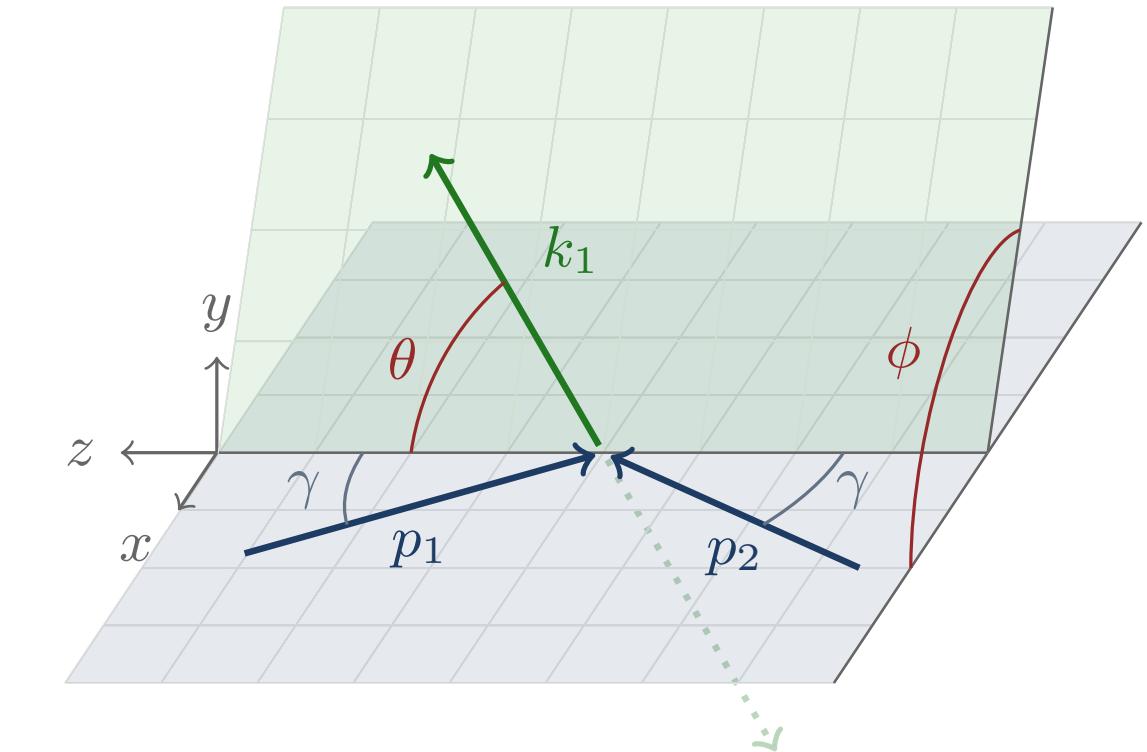
→ LHC

ATLAS Collaboration, Tech. Rep. ATLAS-CONF-2018-037, CERN, 2018

CMS Collaboration, arXiv:1806.00863 [hep-ex], 2018

LHCb Collaboration, arXiv:1509.07645 [hep-ex], 2015

$$\boxed{\sin^2 \theta_{eff}^f = 0.23129 \pm 0.00033}$$

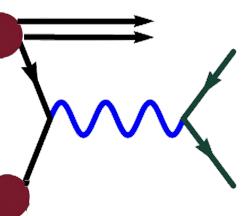


In the future expected big improvement in precision:

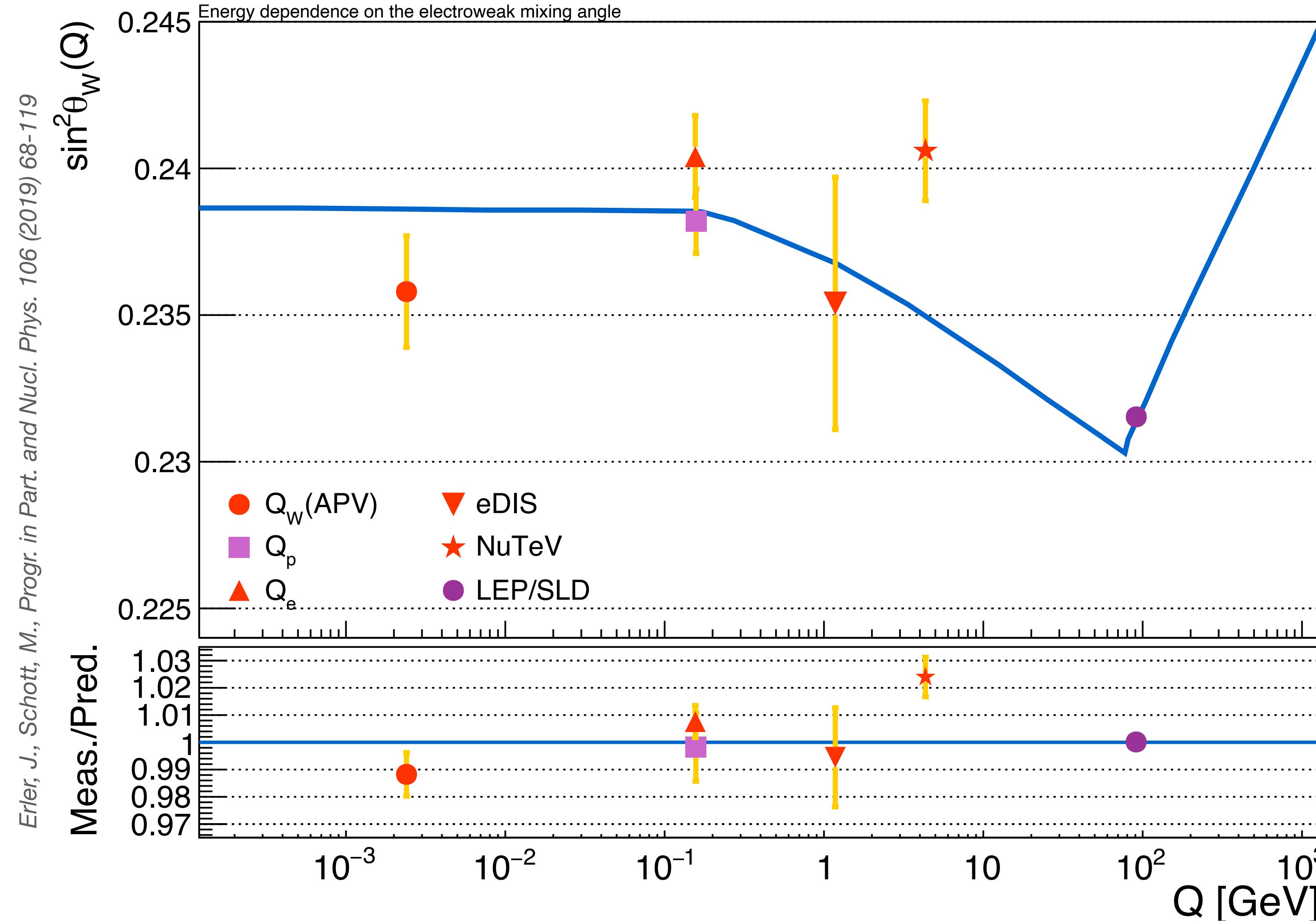
- **more statistics** available (Run 3 and HL-LHC)
- **improved analysis** techniques and **PDF constraining** methods



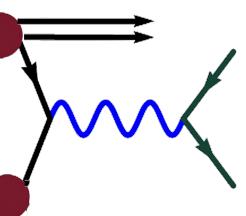
Ongoing work within the **LHC EWWG** to quantify **uncertainties** and **theoretical issues** in the extraction of $\sin^2 \theta_{eff}^f$ (EW precision measurement subgroup)



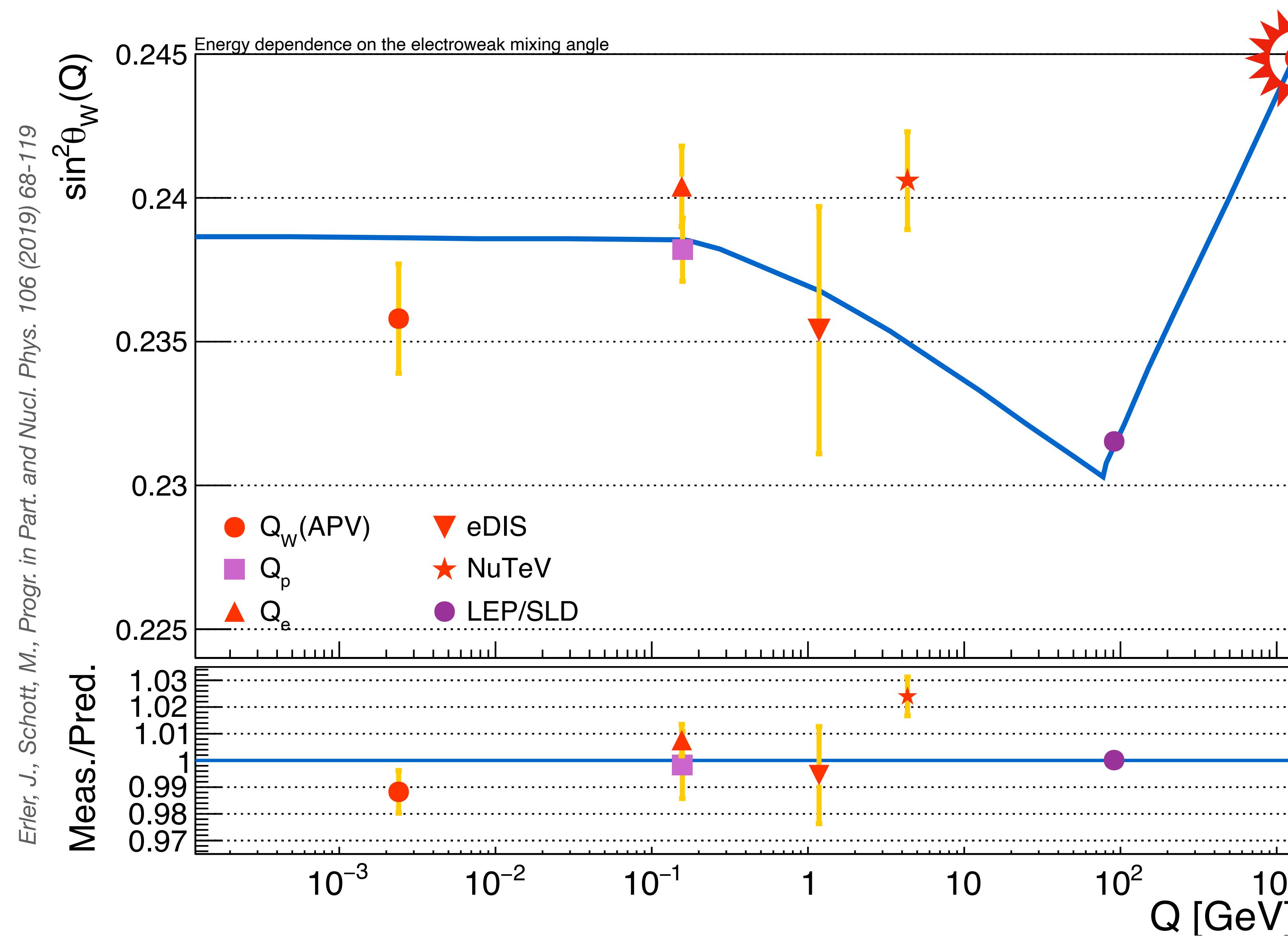
The running $\sin^2 \theta_w^{MS}(\mu)$



Several measurements at low Q^2
but **no experimental results**
of the running **at high energies!**

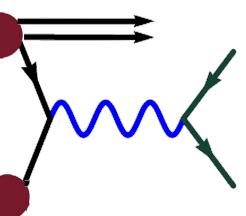


The running $\sin^2 \theta_w^{MS}(\mu)$



Will it be possible to test
the running at high energies
at the LHC?

Several measurements at low Q^2
but **no experimental results**
of the running at high energies!



Implementation of $\sin^2 \theta_w^{\overline{MS}}(\mu)$ in POWHEG-BOX

Chiesa, M., CD, Piccinini, F., *in preparation*

NLO implementation of the **running couplings** in the **Z_EW-BMNNPV** package of POWHEG-BOX-V2, allowing the production of LHE events

Barzé, L., et al., *Eur. Phys. J. C* 73 (6) (2013) 2474

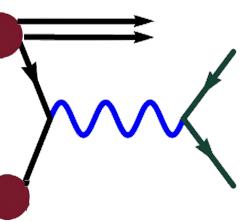
hybrid **renormalization scheme** used $(\alpha(\mu), \sin^2 \theta_w^{\overline{MS}}(\mu), M_Z)$

having $\sin^2 \theta_w^{\overline{MS}}(\mu)$ as input allows its **consistent direct determination** with template fit

h.o. corrections to the running couplings available as options

Erler, J., Ramsey-Musolf, M. J., *Phys. Rev. D* 72 (2005) 073003

possibility to decouple W boson and top quark and to switch on/off the threshold corrections at the W and top masses



Analysis strategy

Theoretical predictions

$$\frac{d^3\sigma}{dm_{\ell\ell}dy_{\ell\ell}d\cos\theta_{CS}} = \frac{\pi\alpha^2}{3m_{\ell\ell}s} \left((1 + \cos^2\theta_{CS}) \sum_q S_q [f_q(x_1, Q^2) f_{\bar{q}}(x_2, Q^2) \right.$$

$$+ f_q(x_2, Q^2) f_{\bar{q}}(x_1, Q^2)] + \cos\theta_{CS} \sum_q A_q \text{sign}(y_{\ell\ell})$$

$$\cdot [f_q(x_1, Q^2) f_{\bar{q}}(x_2, Q^2) - f_q(x_2, Q^2) f_{\bar{q}}(x_1, Q^2)] \right) \square$$

Sensitivities at high masses:

$$\left. \frac{\delta \frac{d^3\sigma}{dm_{\ell\ell}dy_{\ell\ell}d\cos\theta_{CS}}}{\frac{d^3\sigma}{dm_{\ell\ell}dy_{\ell\ell}d\cos\theta_{CS}}} \right|_{\mu=1 \text{ TeV}} \sim 0.9 \frac{\delta \sin^2 \theta_w^{\overline{MS}}(\mu)}{\sin^2 \theta_w^{\overline{MS}}(\mu)}$$

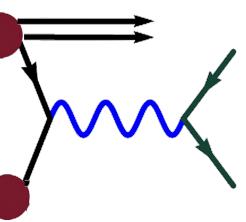
$$\left. \frac{\delta A_{FB}}{A_{FB}} \right|_{\mu=1 \text{ TeV}} \sim 0.3 \frac{\delta \sin^2 \theta_w^{\overline{MS}}(\mu)}{\sin^2 \theta_w^{\overline{MS}}(\mu)}$$

$$S_q = e_\ell^2 e_q^2 + P_{\gamma Z} \cdot e_\ell v_\ell e_q v_q + P_{ZZ} \cdot (v_\ell^2 + a_\ell^2)(v_q^2 + a_q^2)$$

$$A_q = P_{\gamma Z} \cdot 2e_\ell a_\ell e_q a_q + P_{ZZ} \cdot 8v_\ell a_\ell v_q a_q,$$

$$P_{\gamma Z}(m_{\ell\ell}) = \frac{2m_{\ell\ell}^2(m_{\ell\ell}^2 - m_Z^2)}{\sin^2 \theta_W \cos^2 \theta_W [(m_{\ell\ell}^2 - m_Z^2)^2 + \Gamma_Z^2 m_Z^2]}$$

$$P_{ZZ}(m_{\ell\ell}) = \frac{m_{\ell\ell}^4}{\sin^4 \theta_W \cos^4 \theta_W [(m_{\ell\ell}^2 - m_Z^2)^2 + \Gamma_Z^2 m_Z^2]}$$



Analysis strategy

- Two scenarios: **Run 3** (300 fb^{-1}) and **HL-LHC** (3000 fb^{-1})

- $m_{\ell\bar{\ell}}$: [116, 150, 200, 300, 500, 1500, 5000] GeV

$|y_{\ell\bar{\ell}}|$: [0.0, 0.4, 0.8, 1.2, 1.6, 2.0, 2.5]

2 bins in $\cos \theta_{CS}$ for forward/backward directions

- Fiducial selection for **realistic scenario** $p_T^\ell > 40$ (30) GeV, $|\eta_\ell| < 2.5$

CMS Collab., *Eur. Phys. J. C* 78 (9) (2018) 701

- Decoupling on, threshold corrections off for running couplings, complex mass scheme for resonance

Sjöstrand, T., et al., *Comput. Phys. Commun.* 191 (2015) 159-177

- **10^9 events** at **NLOQCD+NLOEW+PS** with PYTHIA8.307

Denner, A., et al., *Nucl. Phys. B Proc. Suppl.* 160 (2006) 22-26

- NPDF31_nnlo_as_0118_hessian PDF set with $\mu_R = \mu_F = m_{\ell\bar{\ell}}$

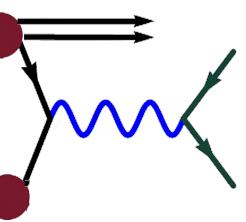
Ball, R. D., et al., *Eur. Phys. J. C* 77 (10) (2017) 663

- Simplified detector emulation with lepton efficiencies and resolutions evaluated with RIVET

ATLAS Collab., *Eur. Phys. J. C* 79 (8) (2019) 639

ATLAS Collab., <http://cds.cern.ch/record/2047831>

Buckley, A., et al., *Comput. Phys. Commun.* 184 (2013) 2803-2819



Fit strategy

- **Templates** generated by assuming SM running for $\alpha_{\text{EM}}^{\overline{\text{MS}}}(\mu)$ and varying $\sin^2 \theta_W^{\overline{\text{MS}}}(\mu = \hat{m}_{\ell\bar{\ell}}) \pm 0.01$
- $\delta \sin^2 \theta_W^{\overline{\text{MS}}}(\mu)$ in each $m_{\ell\bar{\ell}}$ determined at same time in the fit with xFitter using a linear approximation
- **uncertainties** included as **nuisance parameters**

Alekhin, S., et al., Eur. Phys. J. C 75 (7) (2015) 304

| $m_{\ell\bar{\ell}}^{\text{lo}}$ [GeV] | $m_{\ell\bar{\ell}}^{\text{hi}}$ [GeV] | $\hat{m}_{\ell\bar{\ell}}$ [GeV] | $(\alpha_{\text{EM}}^{\overline{\text{MS}}}(\hat{m}_{\ell\bar{\ell}}))^{-1}$ | $\sin^2 \theta_W^{\overline{\text{MS}}}(\hat{m}_{\ell\bar{\ell}})$ |
|--|--|----------------------------------|--|--|
| 66 | 116 | m_Z | 127.951 | 0.23122 |
| 116 | 150 | 133 | 127.838 | 0.23323 |
| 150 | 200 | 175 | 127.752 | 0.23468 |
| 200 | 300 | 250 | 127.544 | 0.23648 |
| 300 | 500 | 400 | 127.269 | 0.23885 |
| 500 | 1500 | 1000 | 126.735 | 0.24350 |
| 1500 | 5000 | 3250 | 126.047 | 0.24954 |

Uncertainties

- **Statistical** from predicted N_{events} in each bin
- **Lepton syst.** extrapol. to Run 3 (reduced of 2) and HL-LHC (4); uncorrelated in the fit

ATLAS Collab., JHEP 08 (2016) 009

- **Luminosity**: 1.5% for Run 3 and 1% for HL-LHC

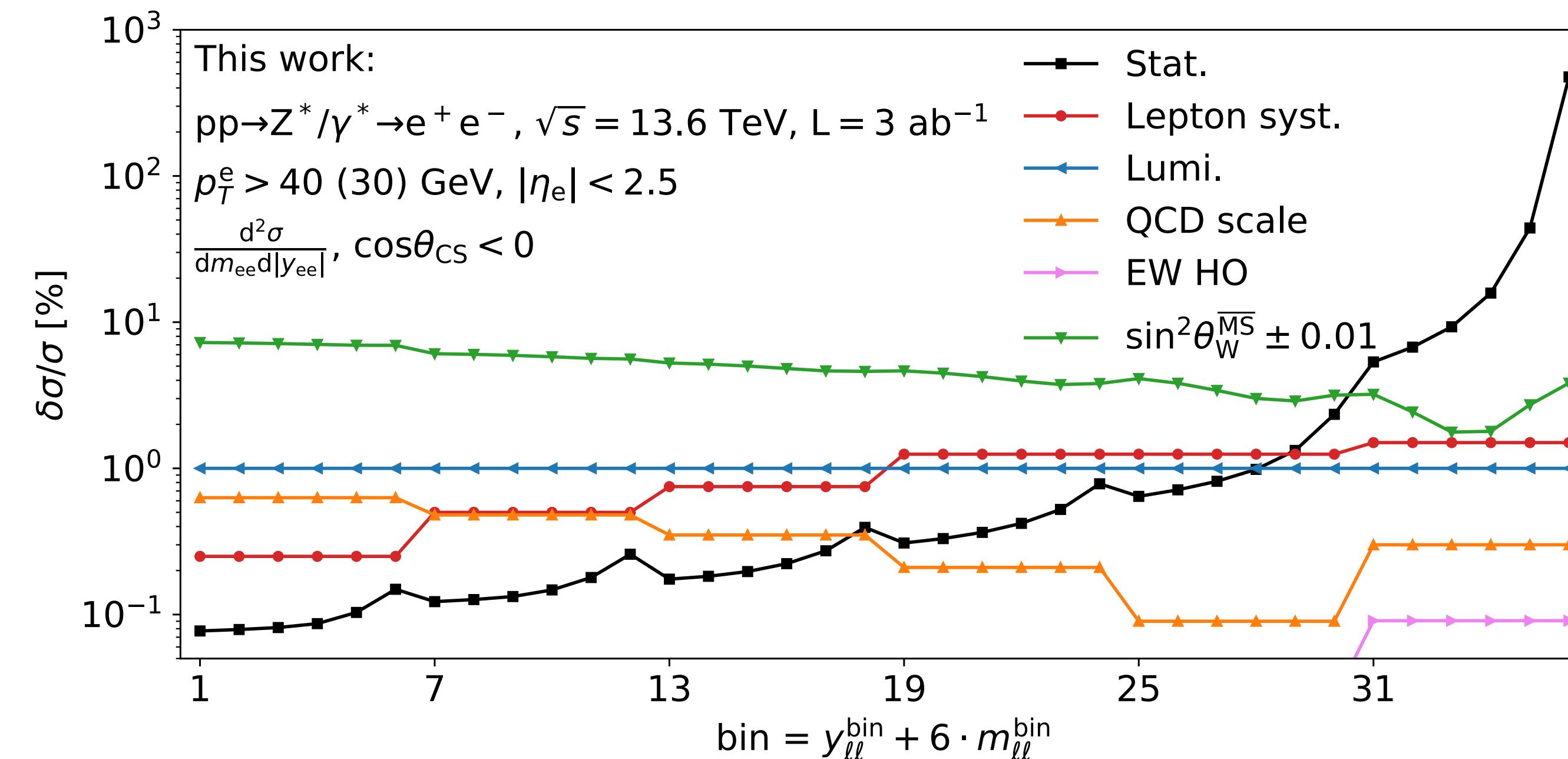
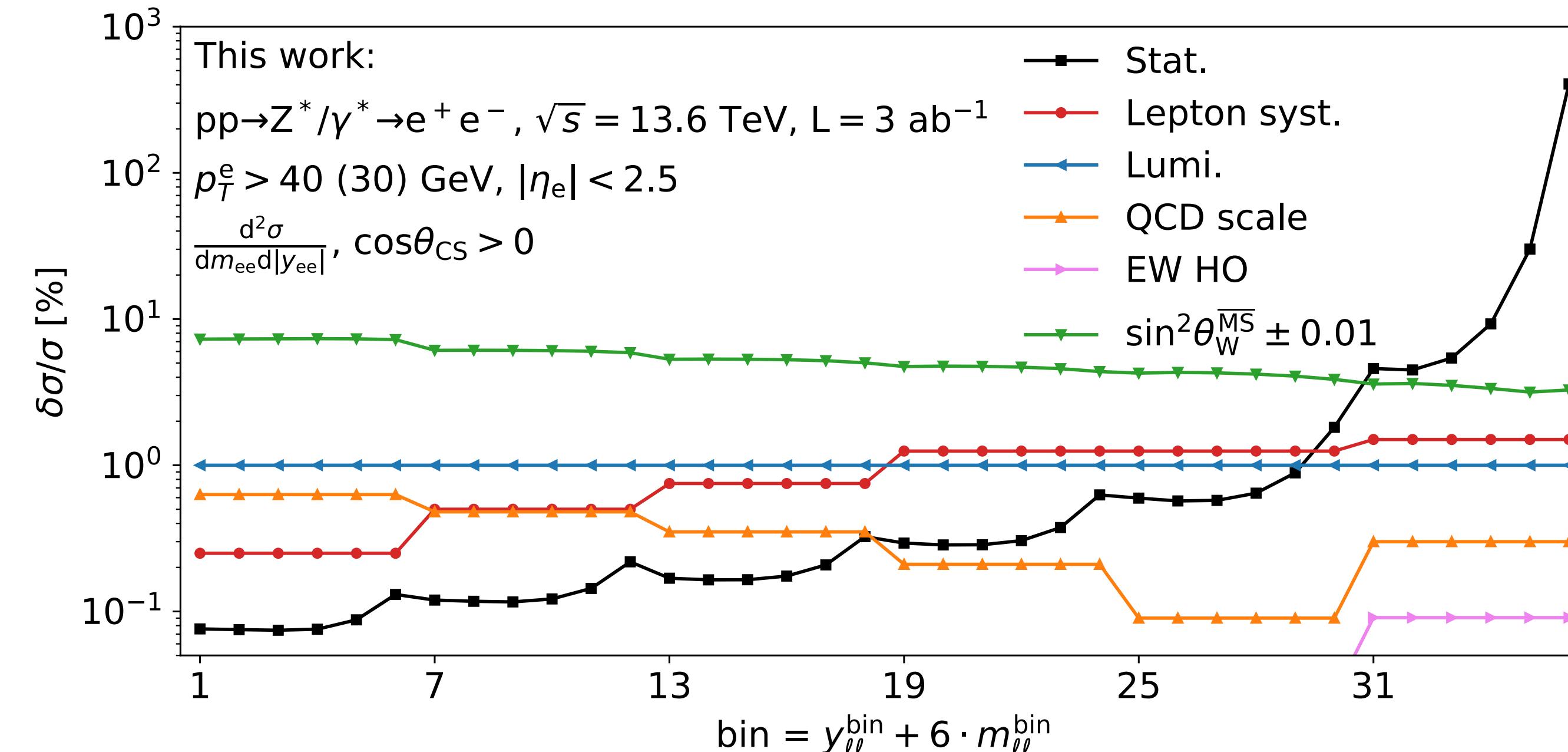
- **QCD scale**: n3loxs for cross-sections (2%) and 7-point variations of μ_R and μ_F (negligible) w.r.t. $m_{\ell\bar{\ell}}$ at N3LO
- PDF uncertainties** (not on plot)

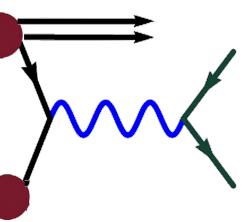
Bertone, V., et al., JHEP 08 (2014) 166

Alwall, J., et al., JHEP 07 (2014) 079

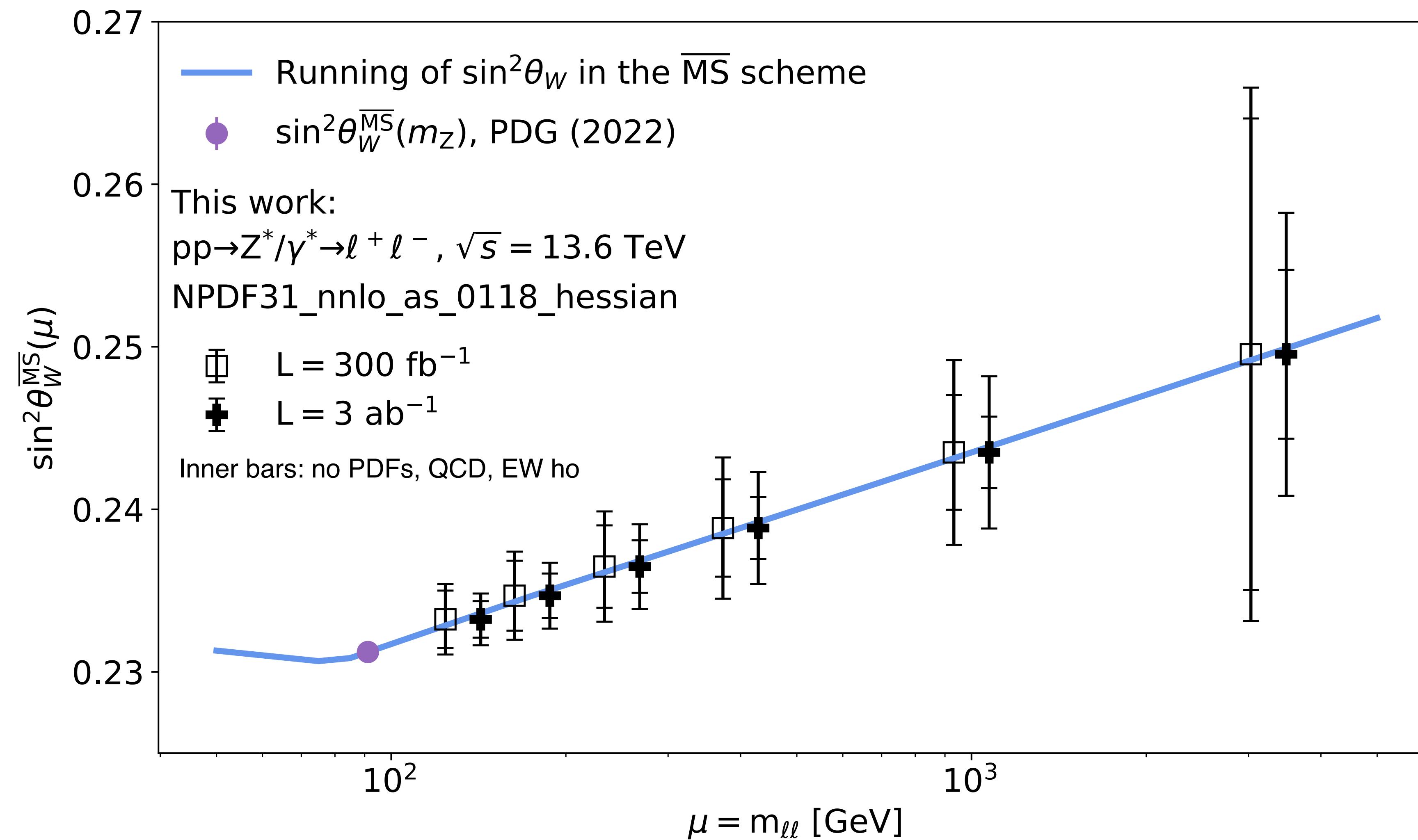
Baglio, J., et al., JHEP 12 (2022) 066

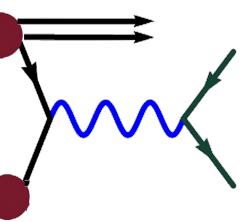
- **EW**: scale variations of $\mu = 2m_{\ell\bar{\ell}}$ or $m_{\ell\bar{\ell}}/2$ change cross section of % at LO and 0.1% at NLO - same variations of $\sin^2 \theta_w^{\overline{MS}}(\mu)$ at LO and NLO





Results



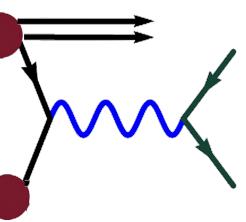


Results

| $\hat{m}_{\ell\ell}$ [GeV] | $\sin^2 \theta_W^{\overline{\text{MS}}}(\hat{m}_{\ell\ell})$ | Run 3 | | HL-LHC | |
|----------------------------|--|---|-----|---|-----|
| | | $\delta \sin^2 \theta_W^{\overline{\text{MS}}}(\hat{m}_{\ell\ell})$ | [%] | $\delta \sin^2 \theta_W^{\overline{\text{MS}}}(\hat{m}_{\ell\ell})$ | [%] |
| 133 | 0.23323 | 0.00216 | 0.9 | 0.00159 | 0.7 |
| 175 | 0.23468 | 0.00271 | 1.2 | 0.00202 | 0.9 |
| 250 | 0.23648 | 0.00339 | 1.4 | 0.00260 | 1.1 |
| 400 | 0.23885 | 0.00434 | 1.8 | 0.00345 | 1.4 |
| 1000 | 0.24350 | 0.00569 | 2.3 | 0.00468 | 1.9 |
| 3250 | 0.24954 | 0.01640 | 6.6 | 0.00870 | 3.5 |

Largest uncertainty due to PDFs (high x)
fit repeated with 5 different sets (1 including photon)
→ contribution to $\delta \sin^2 \theta_W^{\overline{\text{MS}}}(\mu)$ varies significantly

| $\hat{m}_{\ell\ell}$ [GeV] | $\delta \sin^2 \theta_W^{\overline{\text{MS}}}(\hat{m}_{\ell\ell})$ [%] | |
|----------------------------|---|---------|
| | NNPDF31 | NNPDF40 |
| 133 | 0.5 | 0.3 |
| 175 | 0.6 | 0.4 |
| 250 | 0.8 | 0.5 |
| 400 | 1.2 | 0.6 |
| 1000 | 1.6 | 0.8 |
| 3250 | 2.7 | 1.6 |



To summarise

First projections for the LHC **measurement of $\sin^2 \theta_w$ at NLO at high masses!**

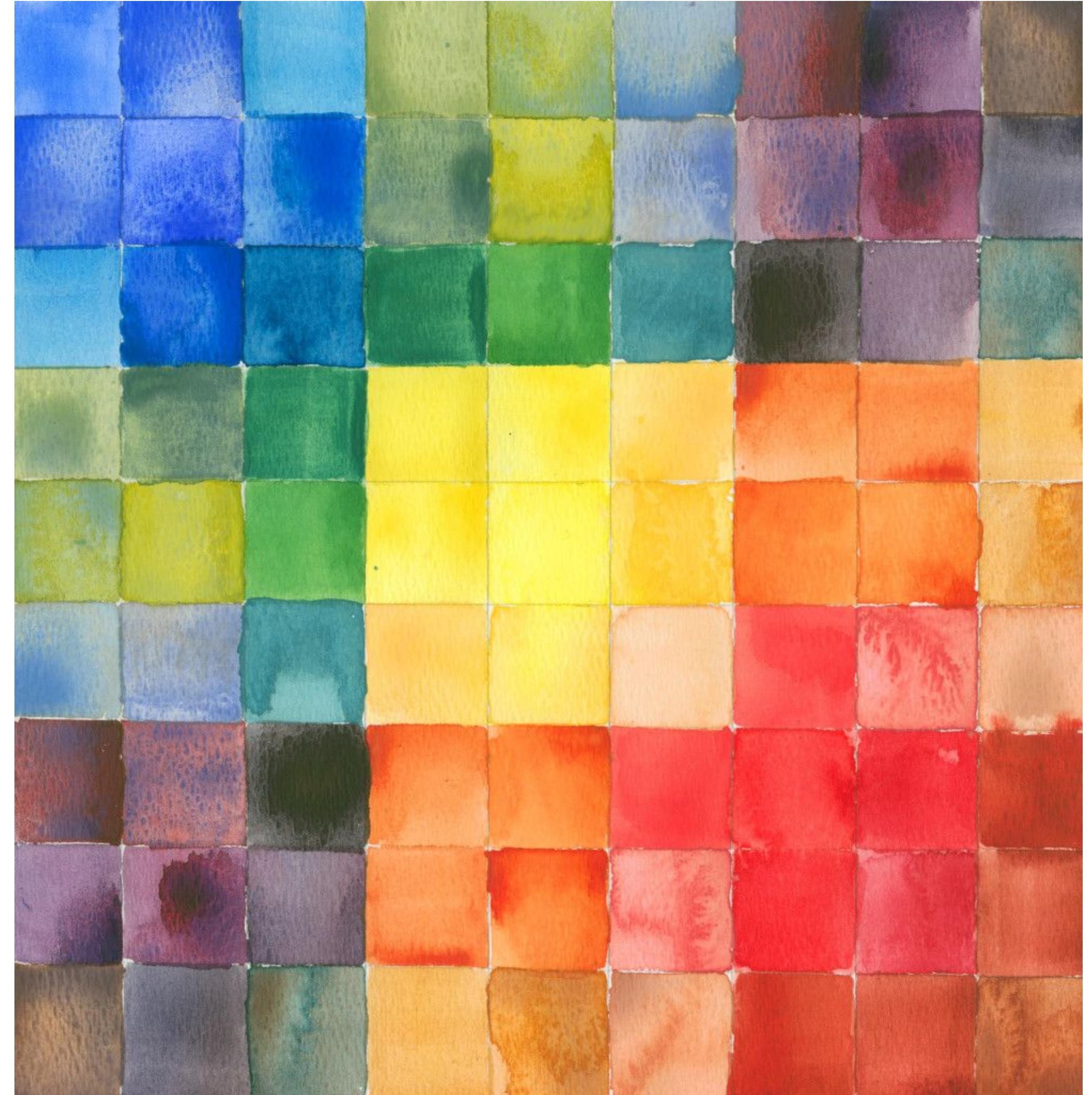
Monte Carlo code with $\sin^2 \theta_w^{\overline{MS}}(\mu)$ **as input** that allows its direct determination consistently at NLO at hadronic colliders

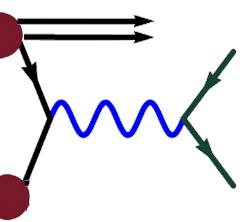
Sensitivity study shows that the LHC and the HL-LHC will measure the energy dependence of $\sin^2 \theta_w$ with a **precision of few %** up to 3 TeV, assuming SM running of $\alpha(\mu)$

Large uncertainty due to PDFs will be reduced by future PDFs fits

Future improvements: more refined analysis with additional observables

Backup





Low-energy determinations

- $6S \rightarrow 7S$ transitions in Cs at 539 nm

Guéna, J., et al., *Phys. Rev. A* 71 (2005)

- $^1S_0 \rightarrow ^3D_1$ transitions in Yb at 408 nm

Antipas, D., et al., *Nature Physics* 15 (2019) 120-123

- Qweak Collaboration @ JLAB

JLab Qweak Collaboration, *Nature* 557 (2018) 207-211

- PVDIS Collaboration @ JLAB

PVDIS Collaboration, *Nature* 506 (2014) 67-70

DIS of polarised e^- beams on deuterium

- SLAC E158 experiment

Scattering of polarised e^- beams on liquid H (electrons)

- NuTeV Collaboration @ Fermilab Tevatron

NuTeV Collaboration, *Phys. Rev. Lett.* 88 (2002) 901802

DIS of (anti)neutrinos on protons

Erler, J., Su, S., *Progr. in Part. and Nucl. Phys.* 71 (2013) 119-149

Ginges, J. S. M., Flambaum, V. V., *Phys. Rep.* 397 (2004) 63-154

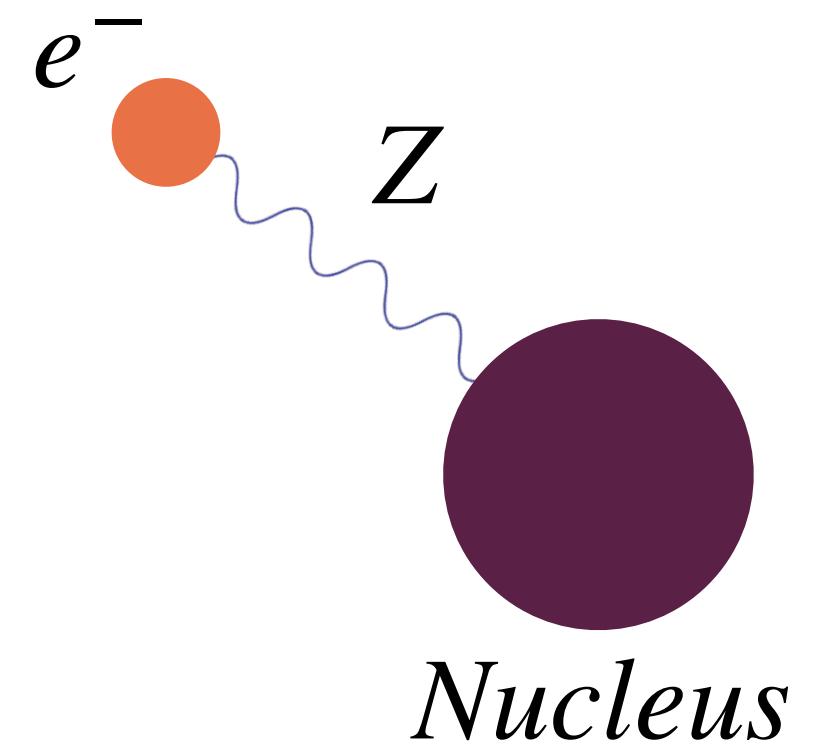
$$\sin^2 \theta^{\overline{MS}}(0) = 0.2383 \pm 0.0011$$

$$\sin^2 \theta^{\overline{MS}}(M_Z) = 0.2299 \pm 0.0043$$

$$\sin^2 \theta_w^{OS} = 0.2277 \pm 0.0016$$

PARITY TRANSFORMATION
 $(x, y, z) \rightarrow (-x, -y, -z)$

3σ tension with the SM



Low-energy determinations: electron-nucleus interaction

Erler, J., Su, S., arXiv:1303.5522 [hep-ph], 2013

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{e} \gamma^\mu \gamma_5 e \left[g_{AV}^{eu} \bar{u} \gamma_\mu u + g_{AV}^{ed} \bar{d} \gamma_\mu d \right]$$

PARITY TRANSFORMATION
 $(x, y, z) \rightarrow (-x, -y, -z)$

$$+ \frac{G_F}{\sqrt{2}} \bar{e} \gamma^\mu e \left[g_{VA}^{eu} \bar{u} \gamma_\mu \gamma_5 u + g_{VA}^{ed} \bar{d} \gamma_\mu \gamma_5 d \right]$$

violated by γ_5 terms

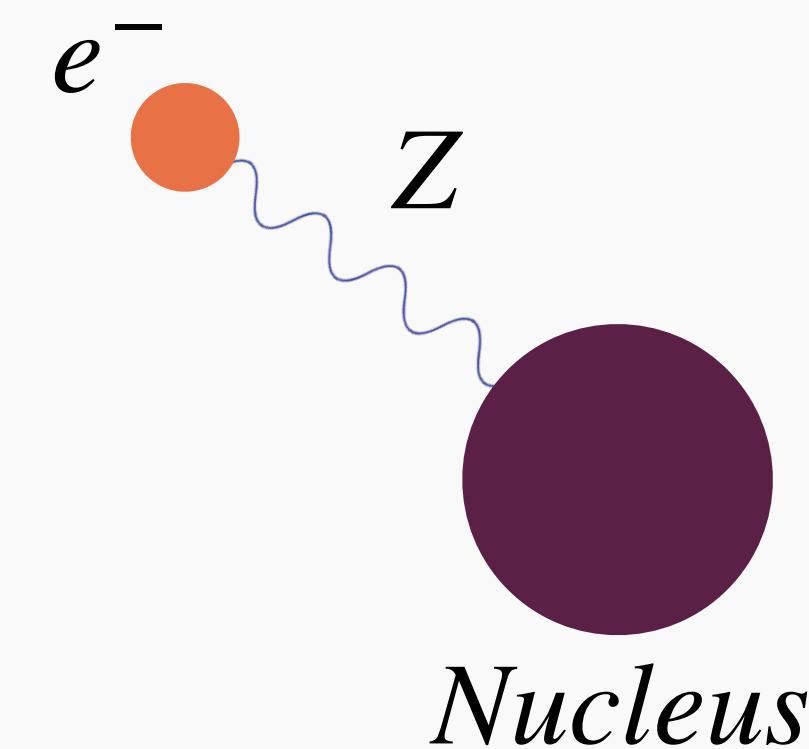
$$g_{AV}^{eu} = -\frac{1}{2} + \frac{4}{3} \sin^2 \theta_w$$

$$g_{AV}^{ed} = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w$$

$$g_{VA}^{eu} = -g_{VA}^{ed} = -\frac{1}{2} + 2 \sin^2 \theta_w$$

$$Q_w = -2 \left(Z g_{AV}^{ep} + N g_{AV}^{en} \right) = -N + Z (1 - 4 \sin^2 \theta_w)$$

$$-1/2 + 2 \sin^2 \theta_w \quad \quad \quad 1/2$$



Low-energy determinations: APV and proton charge

Ginges, J. S. M., Flambaum, V. V., arXiv:0309054 [physics], 2004

Erler, J., Su, S., arXiv:1303.5522 [hep-ph], 2013

Momentum transfer \sim MeV

→ $6S \rightarrow 7S$ transitions in Cs at 539 nm

Guéna, J., et al., arXiv:041207 [physics.atom-ph], 2003

→ $^1S_0 \rightarrow ^3D_1$ transitions in Yb at 408 nm

Antipas, D., et al., arXiv:1804.05747 [physics.atom-ph], 2018

→ Qweak Collaboration @ JLAB

Qweak Collaboration, <https://doi.org/10.1038/s41586-018-0096-0>, 2018

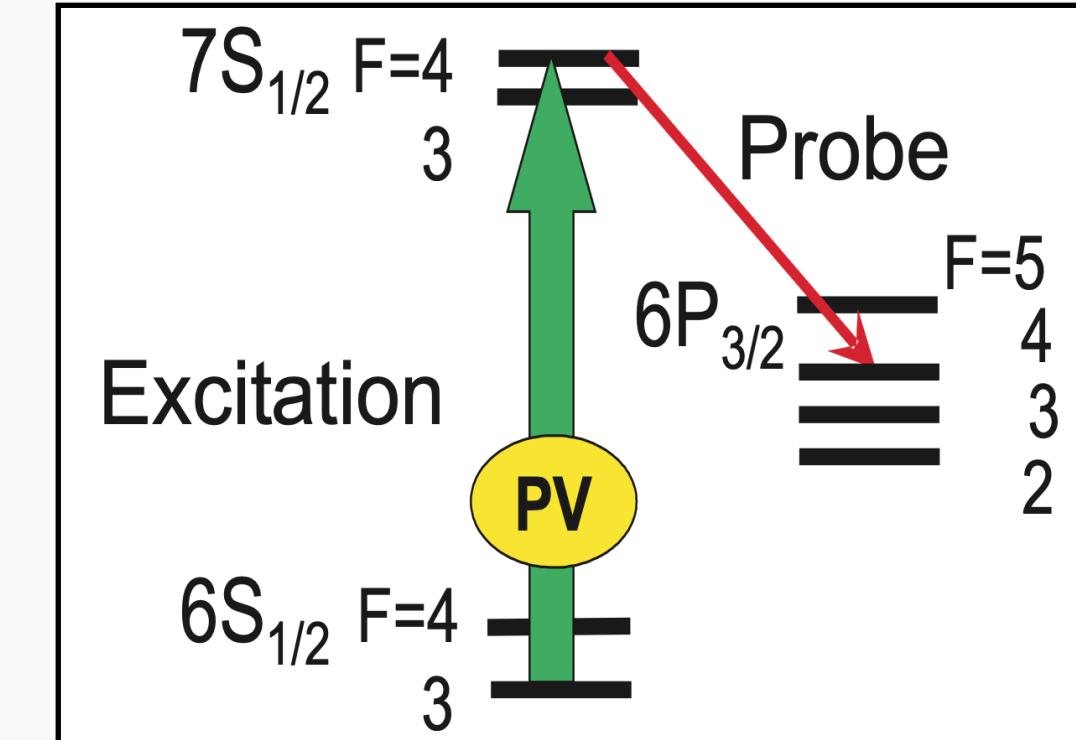
Polarised e^- beams on ℓH_2 (proton)

$$Q^2 = -q^2 = 0.158 \text{ GeV}^2$$

$$Q_w^P = -2(2g_{AV}^{eu} + g_{AV}^{ed})|_{Q^2=0} = 1 - 4 \sin^2 \theta_w^{\overline{MS}}(0)$$

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} [Q_w^P + Q^2 B(Q^2, \theta)]$$

$$\sin^2 \theta_w^{\overline{MS}}(0) = 0.2383 \pm 0.0011$$



Low-energy determinations: electron DIS

→ PVDIS Collaboration @ JLAB

PVDIS Collaboration, doi:10.1038/nature12964, 2014

DIS of polarised e^- beams on deuterium

$$A_{PV} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \left[a_1(x, Q^2) \underbrace{Y_1(x, y, Q^2)}_{\text{Kinematics}} + a_3(x, Q^2) \underbrace{Y_3(x, y, Q^2)}_{\text{Kinematics}} \right]$$

$$a_1 = \frac{6}{5} (2g_{AV}^{eu} - g_{AV}^{ed})$$

$$Q^2 = 1.085 \text{ GeV}^2$$

$$a_3 = \frac{6}{5} (2g_{VA}^{eu} - g_{VA}^{ed})$$

$$Q^2 = 1.901 \text{ GeV}^2$$

$$\sin^2 \theta^{\overline{MS}}(M_Z) = 0.2299 \pm 0.0043$$

→ NuTeV Collaboration @ Fermilab Tevatron

NuTeV Collaboration, arXiv:hep-ex/0110059, 2003

DIS of (anti)neutrinos on protons

$$r = \frac{\sigma(\bar{\nu}N \rightarrow \ell^+ X)}{\sigma(\nu N \rightarrow \ell^- X)}$$

$$R^\nu = \frac{\sigma(\nu N \rightarrow \nu X)}{\sigma(\nu N \rightarrow \ell^- X)} = g_L^2 + r g_R^2$$

$$\sin^2 \theta_w^{OS} = 0.2277 \pm 0.0016$$

3 σ tension with the SM

Low-energy determinations: electron Møller scattering

Erler, J., Su, S., arXiv:1303.5522 [hep-ph], 2013

SLAC E158 Collaboration, arXiv:hep-ex/0504049, 2005

$$A_{PV} = \frac{G_F}{2\sqrt{2}\pi\alpha} m_e E_e \left[1 - 4 \sin^2 \theta_{eff}(Q) \right] \underline{\mathcal{F}^{ee}(y, Q^2)}$$

Kinematics + radiation, $\gamma\gamma$ and γZ box corrections

→ SLAC E158 experiment

Scattering of polarised e^- beams on liquid hydrogen (electron)

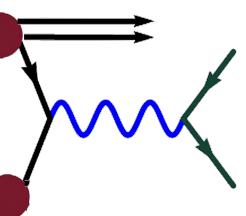
$$Q^2 = 0.026 \text{ GeV}^2$$

$$y = 0.6$$

$$A_{PV} = (-131 \pm 14 \text{ (stat.)} \pm 10 \text{ (syst.)}) \cdot 10^{-9}$$

$$\sin^2 \theta_{eff}(Q = 0.16 \text{ GeV}) = 0.2397 \pm 0.0013$$

$$\sin^2 \theta^{\overline{MS}}(M_Z) = 0.2330 \pm 0.0015$$

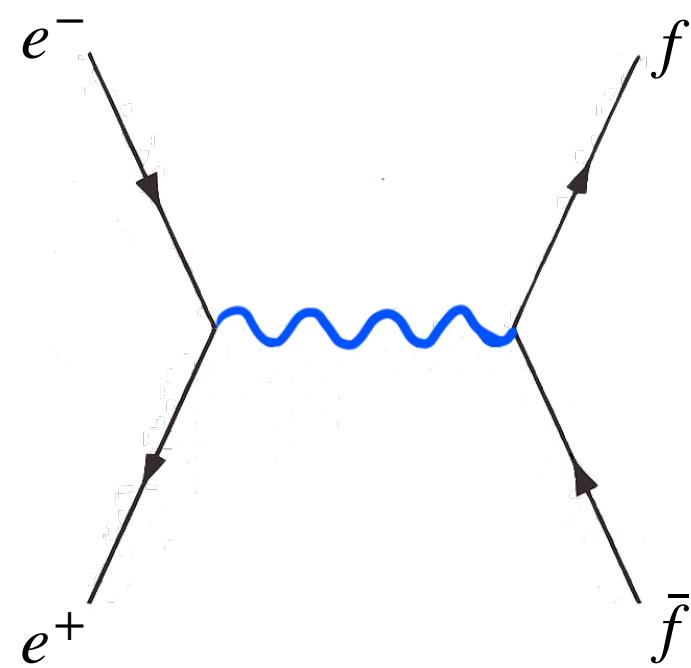


Direct determination of $\sin^2 \theta_{eff}^f$ around the Z pole

Leptonic colliders

LEP/SLD Collaboration, arXiv:hep-ex/0509008v3, 2006

At the Z pole:



A_f linked to $\sin^2 \theta_{eff}^f$

→ LEP: $e^+e^- \rightarrow f\bar{f}$ at $\sqrt{s} = M_Z$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \simeq \frac{3}{4} A_e A_f$$

→ SLC: same process with e^- with polarisation P_e

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \frac{1}{\langle |P_e| \rangle} \simeq A_e$$

$$\boxed{\sin^2 \theta_{eff}^\ell = 0.23153 \pm 0.00016}$$

High-energy determinations: LEP and SLD

LEP/SLD Collaboration, arXiv:hep-ex/0509008v3, 2006

MEASURED QUANTITIES

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \frac{1}{\langle |P_e| \rangle}$$

$$A_{LRFB} = \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R} \frac{1}{\langle |P_e| \rangle}$$

$$\langle P_f \rangle = \frac{\sigma_r - \sigma_l}{\sigma_r + \sigma_l}$$

$$A_{FB}^{pol} = \frac{(\sigma_r - \sigma_l)_F - (\sigma_r - \sigma_l)_B}{(\sigma_r + \sigma_l)_F + (\sigma_r + \sigma_l)_B}$$

PSEUDO-OBSERVABLES

$$A_{FB}^{0,f} = \frac{3}{4} A_e A_f$$

$$A_{LR}^0 = A_e$$

$$A_{LRFB}^0 = \frac{3}{4} A_f$$

$$\langle P_\tau^0 \rangle = -A_\tau$$

$$A_{FB}^{0,pol} = -\frac{3}{4} A_e$$

$$\boxed{\sin^2 \theta_{eff}^\ell = 0.23153 \pm 0.00016}$$

High-energy determinations: hadronic colliders

At LO

$$\frac{d\sigma}{dp_T^2 dy d\cos\theta d\phi} = \frac{d\sigma_{\text{unpol}}}{dp_T^2 dy} \left[(1 + \cos^2\theta) + \frac{A_0}{2}(1 - 3\cos^2\theta) + A_1 \sin 2\theta \cos \phi + \frac{A_2}{2} \sin^2\theta \cos 2\phi \right. \\ \left. + A_3 \sin \theta \cos \phi + A_4 \cos \theta + A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi \right]$$

$$A_4 \propto \frac{g_V^q g_A^q g_V^\ell g_A^\ell}{[(g_V^q)^2 + (g_A^q)^2][(g_V^\ell)^2 + (g_A^\ell)^2]} \quad \begin{array}{l} \xrightarrow{\hspace{1cm}} A_4 = \frac{8}{3} A_{FB} \\ \xrightarrow{\hspace{1cm}} A_4 = 4 \langle \cos \theta \rangle \\ \langle \cos \theta \rangle = \frac{\int \cos \theta d\sigma(\cos \theta, \phi) d\cos \theta d\phi}{\int d\sigma(\cos \theta, \phi) d\cos \theta d\phi} \end{array}$$

→ Tevatron

CDF and DØ Collaborations, arXiv:1801.06283 [hep-ex], 2018

$$\sin^2 \theta_{eff}^\ell = 0.23148 \pm 0.00033$$