Cross Sections for Polarized Lepton-Hadron Collisions

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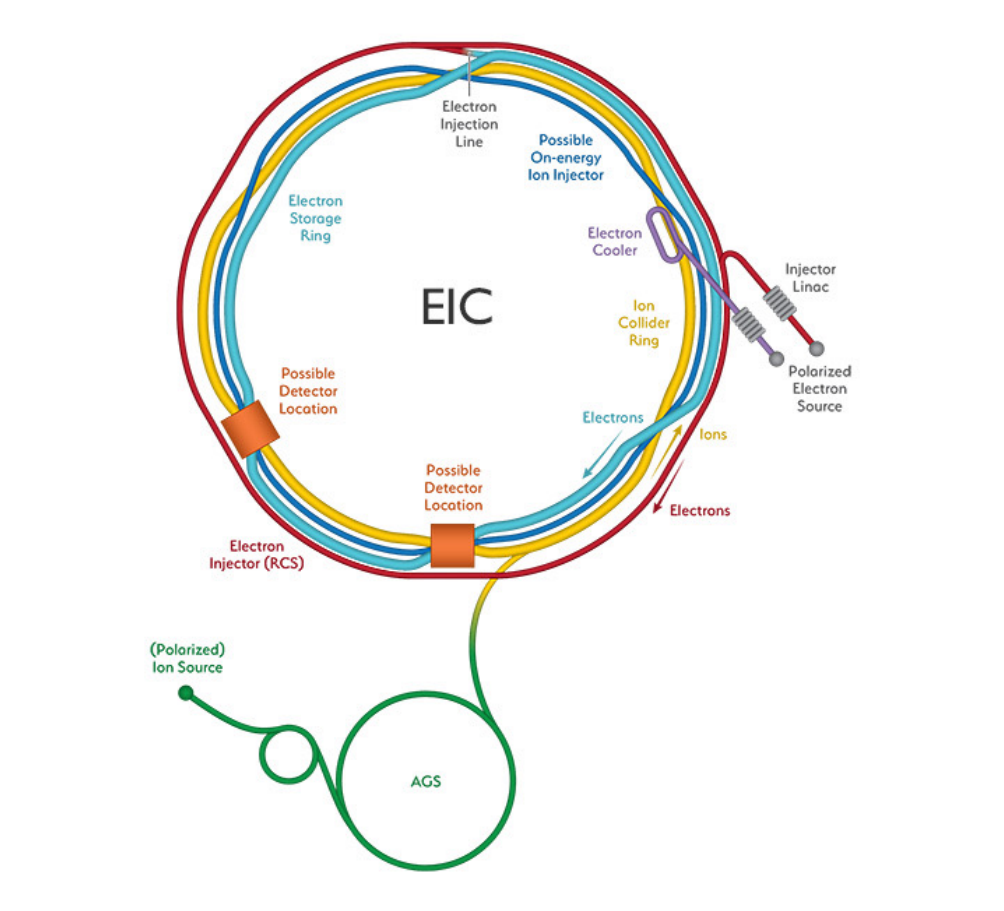
The Electron Ion Collider (EIC)

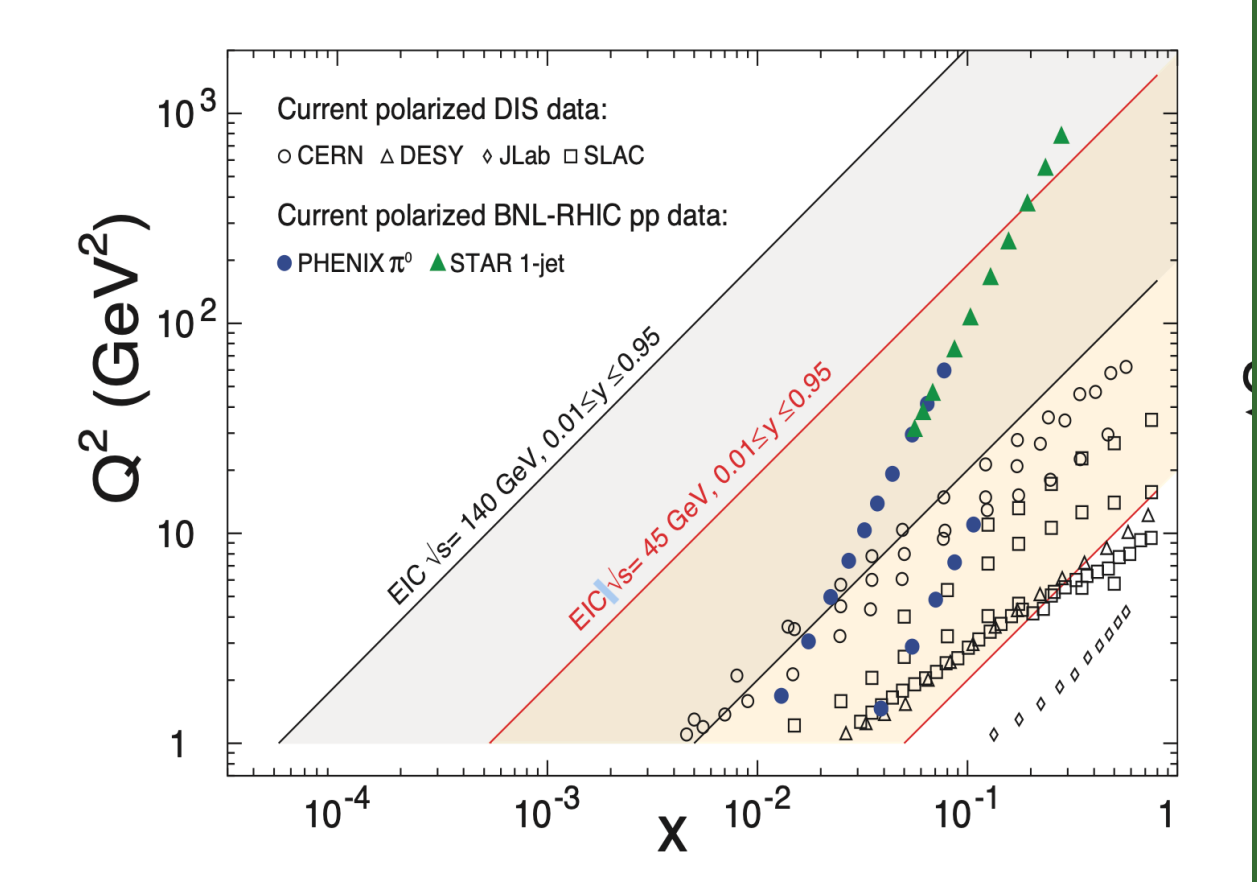


Schematic of Electron Ion Collider

Brookhaven National Laboratory's Relativistic Heavy Ion Collider

EIC Kinematics

- Will provide kinematic reach into the gluon-dominated regime.
- Will be the second ep (ion) collider facility, the first being HERA.
- The EIC will also increase the luminosity 1000x compared to HERA.
- Will be the first ep collider to have polarized proton/ion beam.



Accardi, Alberto et al. "Electron-Ion Collider: The next QCD frontier." *The European Physical Journal A* 52 (2012): 1-100.

Déjà vu?

Yes, calculations for DIS cross sections have been done. However,

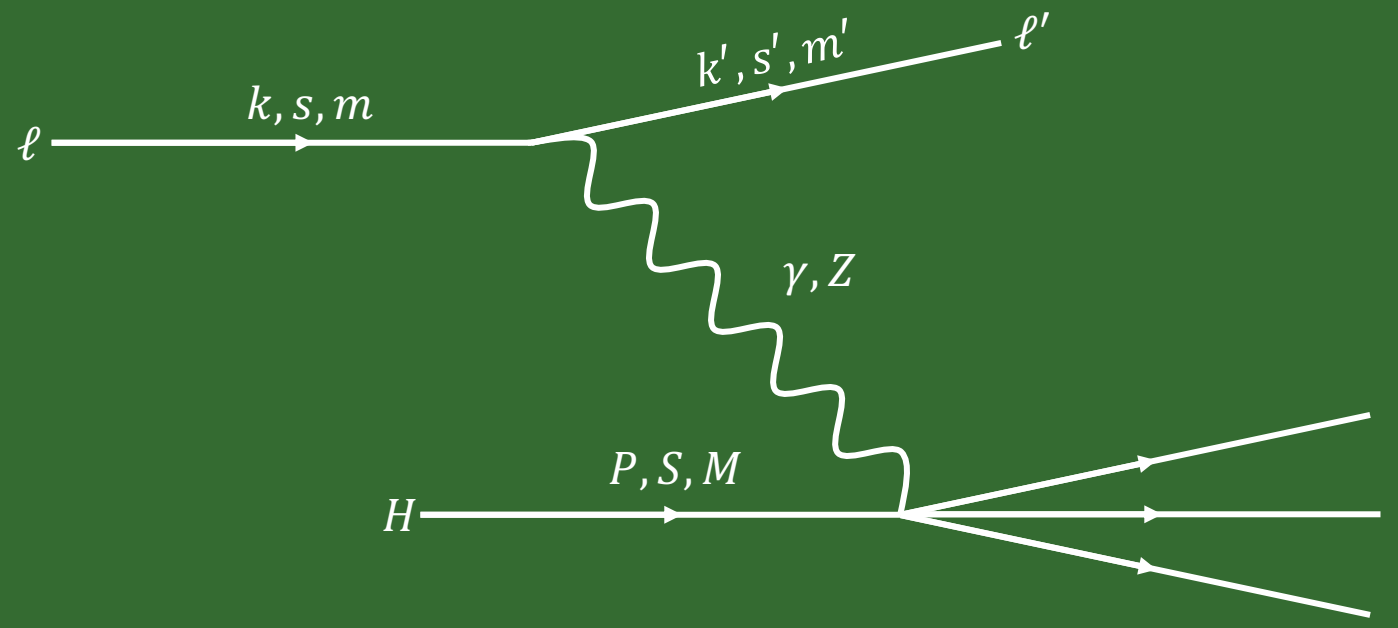
- For Jefferson Lab experiments with $Q^2 \ll M_Z^2$,
  - purely weak interactions are not significant
  - Does not include full nucleon polarization

This is not always the case at the EIC.

- Previous analysis by high-energy electron-proton colliders (e.g. HERA) typically ignored lepton and proton masses

- The formalism and analysis needs to be updated in preparation for simulation studies at the EIC.

DIS Scattering Problem: Definitions



k (k'): Initial (final) lepton momentum
s (s'): Initial (final) lepton spin
$m = m'$: lepton mass
P: Initial hadron momentum
S: Initial hadron spin
M: Initial hadron mass
γ (Z): Transferred photon (Z boson)

DIS variables

- $q = k - k'$: momentum transfer
- $Q^2 \coloneqq -q^2$: negative square momentum transfer
- $x \coloneqq \frac{Q^2}{2(P \cdot q)}$: Bjorken scaling variable
- $y \coloneqq \frac{P \cdot q}{P \cdot k}$: Inelasticity
- $\lambda_\ell(\lambda_H)$: Lepton (hadron) helicity
- $g_V(g_A)$: Vector (axial vector) coupling constant of lepton to Z-boson
- α: Fine structure constant
- G: Fermi coupling constant
- M_Z: Mass of Z boson

Spin Cases Completed

Lepton Spin	**Hadron Spin**	**Experimental Relevance**
Longitudinal	Longitudinal	EIC and Jefferson Lab
Longitudinal	Transverse	EIC and Jefferson Lab
Transverse	Unpolarized	Background for PVES at Jefferson Lab
Transverse	Longitudinal	Not Relevant
Transverse	Transverse	Not Relevant

Cross Section

$$\frac{\mathrm{d}^2\sigma_{nc}}{dxdy} = \frac{\pi y \alpha^2}{Q^4} \sum_{j=\gamma,\gamma Z,Z} \eta^j L^{(j)}_{\mu\nu} W^{\mu\nu(j)}$$

$$\eta^\gamma = 1; \qquad \eta^{\gamma Z} = \left(\frac{GM_Z^2}{2\sqrt{2}\pi\alpha}\right)\left(\frac{Q^2}{Q^2 + M_Z^2}\right); \qquad \eta^Z = \left(\eta^{\gamma Z}\right)^2$$

Initial Tensors

Leptonic Tensor: $L^{(\gamma)}_{\mu\nu} = 2\left[k_\mu k'_\nu + k'_\mu k_\nu - (k\cdot k')g_{\mu\nu} - 2i\lambda_\ell \varepsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta\right]$

Hadronic Tensor: $\frac{1}{2M}W^{\mu\nu(j)} = -\frac{g^{\mu\nu}}{M}F_1^j + \frac{P^\mu P^\nu}{M(P\cdot q)}F_2^j$

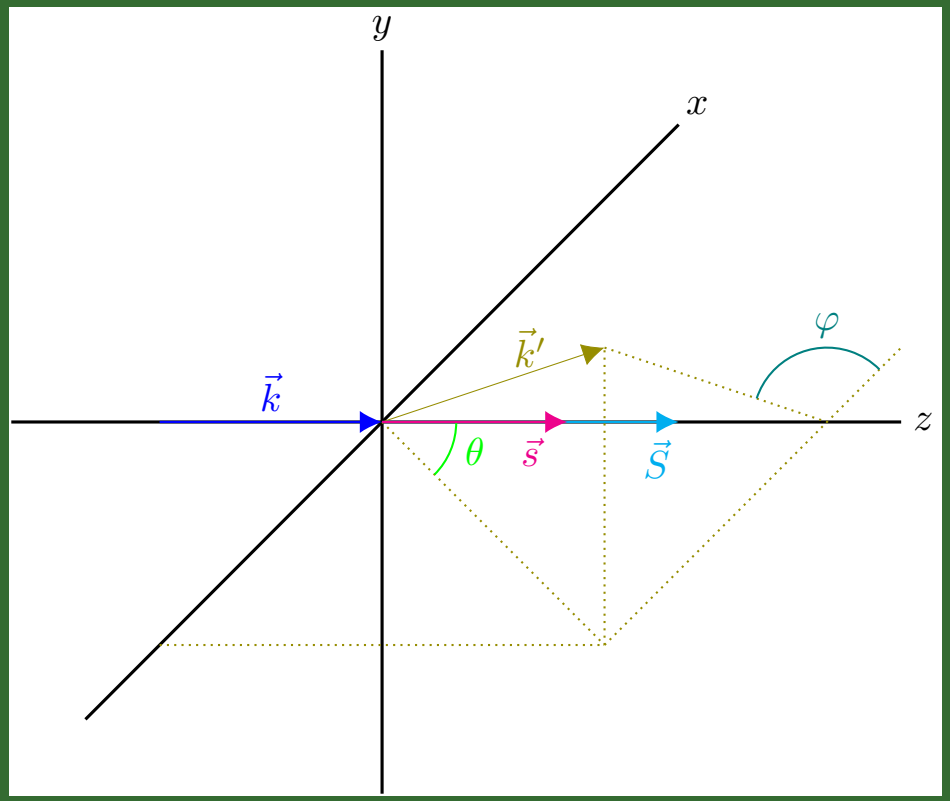
$j \in \{\gamma, \gamma Z, Z\}$

$$+ i\frac{\varepsilon^{\mu\nu\alpha\beta}}{2(P\cdot q)}\left(\frac{P_\alpha q_\beta}{M}F_3^j + 2q_\alpha S_\beta g_1^j - 4xP_\alpha S_\beta g_2^j\right)$$

$$- \frac{P^\mu S^\nu + S^\mu P^\nu}{2(P\cdot q)}g_3^j + \frac{S\cdot q}{(P\cdot q)^2}P^\mu P^\nu g_4^j + \frac{S\cdot q}{P\cdot q}g^{\mu\nu}g_5^j$$

M. Anselmino, A. Efremov and E. Leader, The Theory and phenomenology of polarized deep inelastic scattering, Phys. Rept. 261 (1995) 1–124, [hep-ph/9501369].

Longitudinal Lepton, Longitudinal Hadron



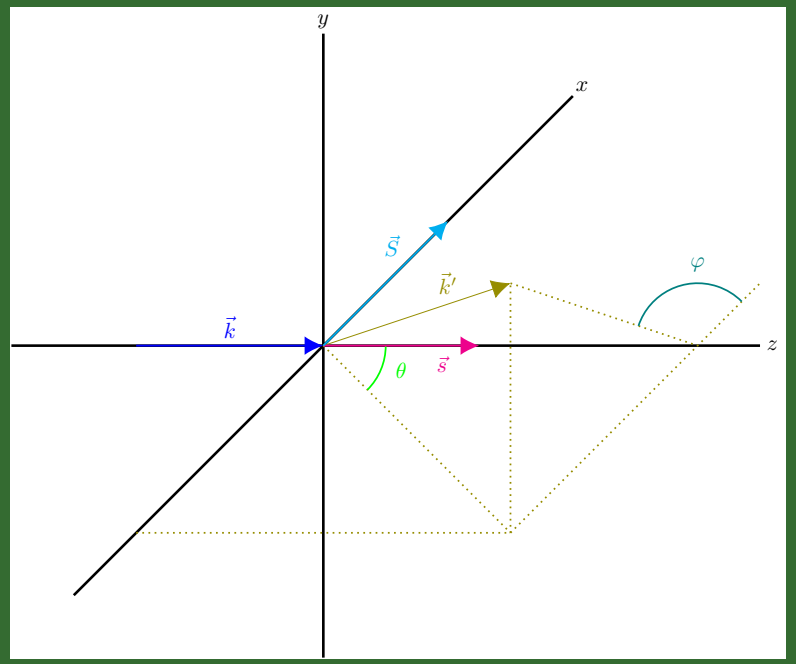
$$j \in \{\gamma, \gamma Z, Z\}$$

$$\begin{aligned}
L^{(\gamma)}_{\mu\nu} W^{\mu\nu(j)} = {} & 4Q^2 F_1^j + \frac{4Q^2}{xy^2}\left(1 - y - \frac{M^2x^2y^2}{Q^2}\right) F_2^j - \frac{4\lambda_\ell Q^2(2-y)}{y} F_3^j \\
& + \frac{8\lambda_\ell \lambda_H Q^2}{y}\left(2 - y - \frac{2M^2x^2y^2}{Q^2}\right) g_1^j - 32M^2x^2\lambda_\ell\lambda_H g_2^j \\
& - \frac{4Q^2\lambda_H}{xy^2}\left(1 - y - \frac{M^2x^2y^2}{Q^2}\right) g_3^j \\
& + \frac{4Q^2\lambda_H}{xy^2}\left(1 + \frac{2M^2x^2y}{Q^2}\right)\left(1 - y - \frac{M^2x^2y^2}{Q^2}\right) g_4^j \\
& - 4\lambda_H Q^2\left(1 + \frac{2M^2x^2y}{Q^2}\right) g_5^j
\end{aligned}$$

Almost consistent with PDG: extra term with g_3, up to convention on sign

Longitudinal Lepton, Transverse Hadron

$$L^{(\gamma)}_{\mu\nu}W^{\mu\nu(j)} = 4Q^2F_1^j + \frac{4Q^2}{xy^2}\left(1-y-\frac{M^2x^2y^2}{Q^2}\right)F_2^j - \frac{4\lambda_\ell Q^2(2-y)}{y}F_3^j$$
$$+16M\lambda_\ell x(k'\cdot S)g_1^j + \frac{32Mx\lambda_\ell}{y}(k'\cdot S)g_2^j - \frac{4M}{y}(k'\cdot S)g_3^j$$
$$-\frac{8M}{y^2}\left(1-y-\frac{M^2x^2y^2}{Q^2}\right)(k'\cdot S)g_4^j + 8Mx(k'\cdot S)g_5^j$$

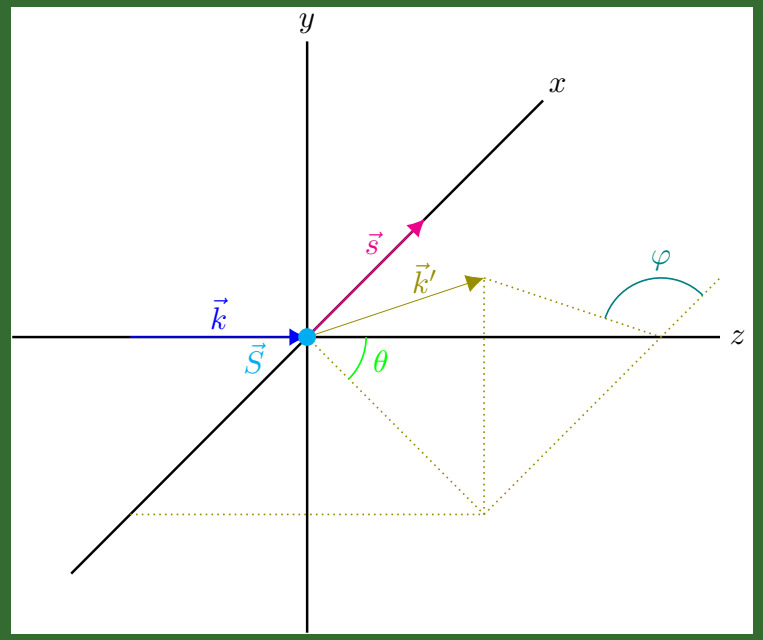


$$k' = \left(E', \left|\vec{k}'\right|\cos(\varphi)\sin(\theta), \left|\vec{k}'\right|\sin(\varphi)\sin(\theta), \left|\vec{k}'\right|\cos(\theta)\right)$$

$$S = (0,1,0,0)$$

Transverse Lepton, Unpolarized Hadron

$$
\begin{aligned}
\mathrm{d}\sigma_\ell = & \frac{16\pi y\alpha^2 m g_A (k'\cdot s)}{Q^4}\left(\eta^{\gamma Z}F_1^{\gamma Z} - g_V\eta^Z F_1^Z\right) \\
& - \frac{16\pi m M^2 x y \alpha^2 g_A (k'\cdot s)}{Q^6}\left(\eta^{\gamma Z}F_2^{\gamma Z} - g_V\eta^Z F_2^Z\right) \\
& + \frac{4\pi\alpha^2 m (k'\cdot s)}{Q^4}\left\{-2y g_V \eta^{\gamma Z}F_3^{\gamma Z} + \left[y\left(g_V^2+g_A^2\right) - 2g_A^2\right]\eta^Z F_3^Z\right\}
\end{aligned}
$$



$$k' = \left(E', \left|\vec{k}'\right|\cos(\varphi)\sin(\theta), \left|\vec{k}'\right|\sin(\varphi)\sin(\theta), \left|\vec{k}'\right|\cos(\theta)\right)$$

$$s = (0,1,0,0)$$

In the above we considered longitudinally polarized leptons. Note that it is impractical to deal with transversely polarized leptons. Because, for transversely polarized incoming leptons, that is leptons with spin perpendicular to the direction of motion, we have

$$s = (0, \hat{\boldsymbol{s}}), \tag{2.1.26}$$

with the unit vector $\hat{\boldsymbol{s}}$ orthogonal to $\hat{\boldsymbol{k}}$, $\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{k}} = 0$. In such a case, contrary to what happens for longitudinally polarized leptons, Eq. (2.1.20), there is no factor E/m to cancel the factor m/E which appears in the cross-section differences (2.1.17), and the latter turn out to be vanishingly small in the large energy limit ($m/E \to 0$).

The “Master” Equations - Arbitrary Lepton/Hadron Polarizations

$$L_{\mu\nu}^{(\gamma)} W^{\mu\nu(\gamma)} = -4\left(2m^2 - Q^2\right) F_1^\gamma + \frac{4Q^2}{xy^2}\left(1 - y - \frac{M^2x^2y^2}{Q^2}\right) F_2^\gamma$$
$$+ \frac{16Mmx}{Q^2}\left[Q^2(s\cdot S) - (q\cdot S)(k'\cdot s)\right] g_1^\gamma + \frac{16Mmx}{Q^2}\left[Q^2(s\cdot S) - 2x(q\cdot S)(P\cdot s)\right] g_2^\gamma$$

The “Master” Equations - Arbitrary Lepton/Hadron Polarizations

$$
\begin{aligned}
L_{\mu\nu}^{(\gamma Z)} W^{\mu\nu(\gamma Z)} = {} & 8\left[g_V\left(2m^2 - Q^2\right) + 2mg_A(k'\cdot S)\right] F_1^{\gamma Z} \\
& - \frac{8Q^2}{xy^2}\left\{g_V\left(1 - y - \frac{M^2x^2y^2}{Q^2}\right) + \frac{2mg_Axy}{Q^2}\left[\frac{M^2xy(k'\cdot s)}{Q^2} - (P\cdot s)(1-y)\right]\right\} F_2^{\gamma Z} \\
& + \frac{4}{y}\left\{2g_Vmy[2x(P\cdot s) - (k'\cdot s)] + g_AQ^2(y-2)\right\} F_3^{\gamma Z} \\
& + \frac{16Mx}{Q^2}\left\{2g_Vm\left[(q\cdot S)(k'\cdot s) - Q^2(s\cdot S)\right] + g_AQ^2[(k\cdot S) + (k'\cdot S)]\right\} g_1^{\gamma Z} \\
& - \frac{32Mx}{Q^2y}\left\{g_Vmy\left[Q^2(s\cdot S) - 2x(q\cdot S)(P\cdot s)\right] + g_AQ^2[(k\cdot S)(1-y) - (k'\cdot S)]\right\} g_2^{\gamma Z} \\
& + \frac{8M}{Q^2y}\left\{g_VQ^2[(k'\cdot S) + (k\cdot S)(1-y)] - mg_A\left[Q^2(1-y)(S\cdot s) + 2xy(P\cdot s)(k'\cdot S)\right]\right\} g_3^{\gamma Z} \\
& - \frac{16M(q\cdot S)}{y^2}\left\{g_V\left(1 - y - \frac{M^2x^2y^2}{Q^2}\right) + \frac{2mg_Axy}{Q^2}\left[\frac{M^2xy(k'\cdot s)}{Q^2} - (P\cdot s)(1-y)\right]\right\} g_4^{\gamma Z} \\
& - \frac{16Mx(S\cdot q)}{Q^2}\left[g_V\left(2m^2 - Q^2\right) + 2mg_A(k'\cdot s)\right] g_5^{\gamma Z}
\end{aligned}
$$

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The “Master” Equations - Arbitrary Lepton/Hadron Polarizations

$$
\begin{aligned}
L^{(Z)}_{\mu\nu}W^{\mu\nu(Z)} = & -4\left[\left(g_V^2+g_A^2\right)\left(2m^2-Q^2\right)-8g_A^2m^2+4mg_Vg_A(k'\cdot s)\right]F_1^Z \\
& +\frac{4Q^2}{xy^2}\Bigg\{\left(g_V^2+g_A^2\right)\left(1-y-\frac{M^2x^2y^2}{Q^2}\right)-\frac{4g_A^2x^2y^2m^2M^2}{Q^4} \\
& +\frac{4mg_Vg_Axy}{Q^2}\left[\frac{M^2xy(k'\cdot s)}{Q^2}-(1-y)(P\cdot s)\right]\Bigg\}F_2^Z \\
& -\frac{4}{y}\left\{my\left(g_V^2+g_A^2\right)\left[2x(P\cdot s)-(k'\cdot s)\right]+2g_A^2m[(k'\cdot s)-xy(P\cdot s)]+Q^2g_Vg_A(y-2)\right\}F_3^Z \\
& +\frac{16Mx}{Q^2}\Bigg\{m\left(g_V^2+g_A^2\right)\left[Q^2(s\cdot S)-(q\cdot S)(k'\cdot s)\right]+mg_A^2\left[2(k\cdot S)(k'\cdot s)-Q^2(s\cdot S)\right] \\
& -Q^2g_Vg_A[(k\cdot S)+(k'\cdot S)]\Bigg\}g_1^Z \\
& +\frac{16Mx}{Q^2y}\Bigg\{my\left(g_V^2+g_A^2\right)\left[Q^2(s\cdot S)-2x(q\cdot S)(P\cdot s)\right]-2g_A^2m\left[Q^2(s\cdot S)-2xy(k\cdot S)(P\cdot s)\right] \\
& +2Q^2g_Vg_A[(1-y)(k\cdot S)-(k'\cdot S)]\Bigg\}g_2^Z \\
& -\frac{4M}{yQ^2}\Bigg\{Q^2\left(g_V^2+g_A^2\right)\left[(S\cdot k')+(1-y)(k\cdot S)\right] \\
& -2mg_Vg_A\left[Q^2(1-y)(S\cdot s)+2xy(P\cdot s)(k'\cdot S)\right]\Bigg\}g_3^Z \\
& +\frac{8M(S\cdot q)}{y^2}\Bigg\{\left(g_V^2+g_A^2\right)\left(1-y-\frac{M^2x^2y^2}{Q^2}\right)-\frac{4g_A^2m^2M^2x^2y^2}{Q^4} \\
& +\frac{4mxyg_Vg_A}{Q^2}\left[\frac{M^2xy(k'\cdot s)}{Q^2}-(1-y)(P\cdot s)\right]\Bigg\}g_4^Z \\
& +\frac{8Mx(S\cdot q)}{Q^2}\left[\left(g_V^2+g_A^2\right)\left(2m^2-Q^2\right)-8g_A^2m^2+4mg_Vg_A(k'\cdot s)\right]g_5^Z
\end{aligned}
$$

Cross Checks

$$A_{RL}^{e^-} = \frac{|P_b|\eta_{\gamma Z}\left[g_A^e 2yF_1^{\gamma Z} + g_A^e F_2^{\gamma Z}\left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2xy}{Q^2}\right) + g_V^e(2-y)F_3^{\gamma Z}\right]}{2yF_1^{\gamma} + \left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2xy}{Q^2}\right)F_2^{\gamma} - \eta_{\gamma Z}\left[g_V^e 2yF_1^{\gamma Z} + g_V^e\left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2xy}{Q^2}\right)F_2^{\gamma Z} + g_A^e(2-y)F_3^{\gamma Z}\right]}$$

R. Boughezal, A. Emmert, T. Kutz, S. Mantry, M. Nycz, F. Petriello et al., Neutral-current electroweak physics and
SMEFT studies at the EIC, Phys. Rev. D 106 (2022) 016006, [2204.07557].

From Here …

- More cross checks
  - J. Blümlein and N. Kochelev, On the twist-2 and twist-3 contributions to the spin-dependent electroweak structure functions, Nuclear Physics B 498 (aug, 1997) 285–309.
  - Longitudinal lepton and hadron, Longitudinal lepton, transverse hadron
- Generalize calculations to positive lepton charge
- Order of magnitude estimates for Jefferson Lab and EIC kinematics

Questions?

Bonus Slide

our notation	[6]	[9]	[10]	[11]	[12]
g_1	g_1	g_1	g_1	$F_3 + F_4$	$2(F_1 + F_2)$
g_2	g_2	g_2	g_2	$-F_4$	$-2F_2$
g_3	$-g_3$	g_5	$(g_4 - g_5)/2$	$2F_8$	$4F_4$
g_4	$g_4 - g_3$	$g_4 + g_5$	g_4	$2F_8 + F_7$	$2(F_6 + 2F_4)$
g_5	$-g_5$	$-g_3$	g_3	$F_7/(2x) - F_6$	$F_7/(2x) - 2F_5$
	[13]	[15, 23]	[16]	[17]	[24]
g_1	$G_1 + G_2$	g_1	g_1	$\tilde{F}_1$	$M\nu \mathrm{Im}\ G_1/\pi$
g_2	$-G_2$	g_2	g_2	$\tilde{F}_2/(2x)$	$-\nu^2 \mathrm{Im}\ G_2/(M\pi)$
g_3	$2G_3$	$b_1 + b_2$	$(A_2 - A_3)/2$	$-2\tilde{F}_3$	
g_4	$2G_4 + G_5$	$a_2 + b_1 + b_2$	A_2		
g_5	$-G_4$	a_1	A_1		

Table 1: A comparison of different conventions to denote the polarized nucleon structure functions.