Forward photon+jet production in pA collisions at next-to-eikonal accuracy

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in collaboration with Tolga Altinoluk, Nestor Armesto and Guillaume Beuf

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CGC formalism used for dilute-dense high energy scattering so we apply Semi-classical approximation

- Dense target: classical background field $A_a^\mu(x)$
- Dilute projectile: virtual photon treated in perturbation theory

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We also adopt Eikonal approximation which amount to taking the high energy limit $s \to \infty$. Beyond eikonal limit give corrections of order $1/s$. We can obtain this limit boosting the target in following way:

$$A_a^\mu(x) \to \begin{cases} \gamma_t A_a^-\left(\gamma_t x^+, \frac{1}{\gamma_t} x^-, x_\perp\right) \\ \frac{1}{\gamma_t} A_a^+\left(\gamma_t x^+, \frac{1}{\gamma_t} x^-, x_\perp\right) \\ A_a^i\left(\gamma_t x^+, \frac{1}{\gamma_t} x^-, x_\perp\right) \end{cases}$$

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- Interactions with the perpendicular component of the field
  T. Altinoluk, G. Beuf. A.Czajka, A.Tymowska (2021) [arXiv:2012.03886] see also G.A.Chirilli [arXiv:1807.11435], [arxiv:2101.12744]

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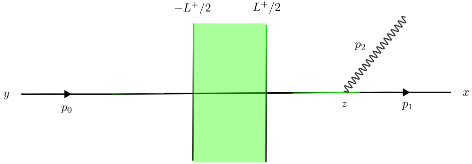
- Taking into account x^- -dependence
  T. Altinoluk, G. Beuf (2021) [arXiv:2109.01620] , T. Altinoluk, G. Beuf. A.Czajka, A.Tymowska (2022) [arXiv:2212.10484] and T. Altinoluk, N.Armesto, G. Beuf. and A.Tymowska (2023) [ongoing collaboration]

We want to calculate the following Photon-jet production at next-to eikonal order, we will have 3 possible diagrams.

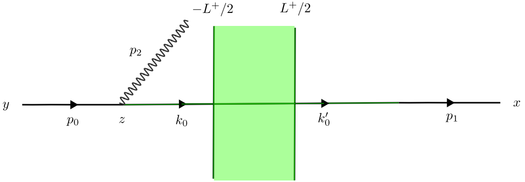


-Lower energies at RHIC compared to LHC $\rightarrow$ NEik corrections
-Establishing an important formalism for future EIC.
-The LSZ reduction formula for this process:

$$S^{\mathrm{LO}}_{q_1\,\gamma_2 \leftarrow q_0} = \lim_{y^+\to -\infty} \int d^2\mathbf{y} \int dy^- \, e^{-iy\cdot\check{p}_0} \lim_{x^+\to +\infty} \int d^2\mathbf{x} \int dx^- \, e^{+ix\cdot\check{p}_1}$$
$$\times \; (-i)e\,e_f \int d^4z \; e^{+iz\cdot\check{p}_2} \; \bar{u}(\underline{p_1}, h_1)\gamma^+ \left[S_F(x,z)\right]_{\alpha_1\beta} \; \not{\epsilon}_\lambda(\underline{p_2})^* \; \left[S_F(z,y)\right]_{\beta\alpha_0} \; \gamma^+ u(\underline{p_0}, h_0)$$

Photon emission before the medium

The first diagram contributing to photon-jet production at both eikonal and NEik order is:



Our final amplitude for this case is:

$$S'^{\,\mathrm{LO\text{-}bef}}_{q_1\,\gamma_2\leftarrow q_0} = (-i) e\, e_f (1)_{\beta\alpha_0}\, \theta(p_0^+)\theta(p_1^+) \int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot(\mathbf{p_1}-\mathbf{p_0}+\mathbf{p_2})}\, \bar{u}(1)(-i\bar{\Delta}')\Bigg\{$$

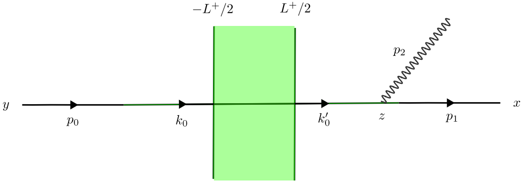
$$\times \int dv^-\, e^{iv^-(p_1^+-p_0^++p_2^+)}\, \mathcal{U}_F\left(\frac{L^+}{2}, -\frac{L^+}{2}; \mathbf{v}, v^-\right) + \frac{2\pi\delta(p_1^+-p_0^++p_2^+)}{2p_1^+}$$

$$\times \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\, \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathbf{v}\right) \left[\frac{[\gamma^i,\gamma^j]}{4} gt\cdot\mathcal{F}_{ij}(v) - \frac{(\mathbf{p}_1^j+\mathbf{p}_0^j-\mathbf{p}_2^j)}{2}\overleftrightarrow{\mathcal{D}_{\mathbf{v}^j}} - i\overleftarrow{\mathcal{D}_{\mathbf{v}^j}}\,\overrightarrow{\mathcal{D}_{\mathbf{v}^j}}\right] \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathbf{v}\right)\Bigg\}$$

$$\times\, \epsilon^k_\lambda(p_2)\gamma^+ \left(\left(p_2^l - \frac{p_2^+}{p_0^+}p_0^l\right)\left(2\delta^{kl} - \frac{p_2^+}{p_0^+}\gamma^k\gamma^l\right) + \left(\frac{p_2^+}{p_0^+}\right)^2 m\gamma^k\right) u(0)$$

Photon emission after the medium

The second diagram contributing to photon-jet production at both eikonal and NEik order is:



with the amplitude :

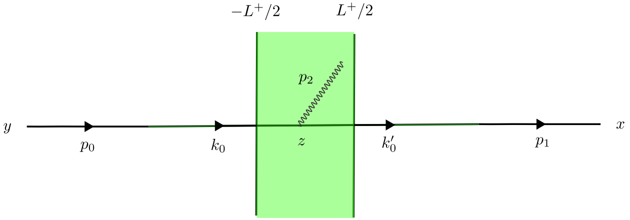
$$S'^{\text{LO-aft}}_{q_1\,\gamma_2 \leftarrow q_0} = (-i)e\, e_f\, \epsilon^k_\lambda(p_2)(1)_{\alpha_1\beta}\,\theta(p_1^+)\theta(p_0^+)\int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot(\mathbf{p_1}+\mathbf{p_2}-\mathbf{p_0})}\,\bar{u}(1)i\bar{\Delta}\gamma^+$$

$$\times\left(\left(p_2^l - \frac{p_2^+}{p_1^+}p_1^l\right)\left(2\delta^{kl} - \frac{p_2^+}{p_1^+}\gamma^l\gamma^k\right) - \left(\frac{p_2^+}{p_1^+}\right)^2 m\gamma^k\right)\left\{\int dv^-\, e^{iv^-(p_1^+ + p_2^+ - p_0^+)}\,\mathcal{U}_F\left(\frac{L^+}{2}, -\frac{L^+}{2}; \mathbf{v}, v^-\right)\right.$$

$$+\ \frac{2\pi\delta(p_1^+ + p_2^+ - p_0^+)}{(2p_0^+)}$$

$$\left.\times\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\,\mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathbf{v}\right)\left[\frac{[\gamma^i,\gamma^j]}{4}gt\cdot\mathcal{F}_{ij}(v) - \frac{(\mathbf{p}_0^j + \mathbf{p}_1^j + \mathbf{p}_2^j)}{2}\overleftrightarrow{\mathcal{D}_{\mathbf{v}^j}} - i\overleftarrow{\mathcal{D}_{\mathbf{v}^j}}\,\overrightarrow{\mathcal{D}_{\mathbf{v}^j}}\right]\mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathbf{v}\right)\right\}_{\beta\alpha_0} u(0)$$

The third and last diagram contributing to photon-jet production contributing only at NEik order is:



with the amplitude :

$$S'^{\text{LO-in}}_{q_1\,\gamma_2\leftarrow q_0} = (-i)e\,e_f\epsilon_{\lambda k}(\underline{p_2})^* \int d^2\mathbf{z}\; 2\pi\delta(p_1^+ + p_2^+ - p_0^+)\theta(p_0^+)\theta(p_1^+)e^{-i\mathbf{z}\cdot(\mathbf{p_1}+\mathbf{p_2}-\mathbf{p_0})}\bar{u}(\underline{p_1},h_1)$$
$$\times \int_{-L^+/2}^{L^+/2} dz^+ \mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\left[\frac{i\gamma^+\gamma^i\gamma^k}{2p_1^+} + \frac{i\gamma^k\gamma^i\gamma^+}{2p_0^+}\right]\frac{1}{2}\overleftrightarrow{\mathcal{D}_{\mathbf{z}^i}}\mathcal{U}_F\Big(z^+,\frac{-L^+}{2};\mathbf{z}\Big)u(\underline{p_0},h_0)$$

Cross-section

We have the following definitions for the Wilson lines appearing in our cross-section

$$\mathcal{U}^{(1)}_{F;j}(\mathrm{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \, \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathrm{v}\right) \overleftrightarrow{\mathcal{D}_{\mathrm{v}^j}} \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathrm{v}\right)$$

$$\mathcal{U}^{(2)}_{F;j}(\mathrm{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \, \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathrm{v}\right) \overleftarrow{\mathcal{D}_{\mathrm{v}^j}} \, \overrightarrow{\mathcal{D}_{\mathrm{v}^j}} \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathrm{v}\right)$$

$$\mathcal{U}^{(3)}_{F;ij}(\mathrm{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \, \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathrm{v}\right) gt \cdot \mathcal{F}_{ij}(\underline{v}) \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathrm{v}\right)$$

$$\mathcal{U}_F(\mathrm{v}) = \mathcal{U}_F\left(\frac{L^+}{2}, -\frac{L^+}{2}; \mathrm{v}\right)$$

All factors will be expressed in terms of:

$$\mathrm{k}_\perp = \mathrm{p}_1 + \mathrm{p}_2 - \mathrm{p}_0$$

$$\mathrm{P}_\perp = \frac{p_2^+ \mathrm{p}_1 - p_1^+ \mathrm{p}_2}{p_1^+ + p_2^+}$$

$$\mathrm{p}_0 = 0$$

We also have that:

$$\overrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\partial_{z^\mu}} + igt \cdot \mathcal{A}_\mu(z)$$

$$\overleftarrow{\mathcal{D}_{z^\mu}} \equiv \overleftarrow{\partial_{z^\mu}} - igt \cdot \mathcal{A}_\mu(z)$$

$$\overleftrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}} - \overleftarrow{\mathcal{D}_{z^\mu}} = \overleftrightarrow{\partial_{z^\mu}} + 2igt \cdot \mathcal{A}_\mu(z)$$

$$\mathcal{F}^a_{\mu\nu}(z) \equiv \partial_{z^\mu} \mathcal{A}^a_\nu(z) - \partial_{z^\nu} \mathcal{A}^a_\mu(z) - gf^{abc} \mathcal{A}^b_\mu(z) \mathcal{A}^c_\nu(z)$$

Cross-section (I)-Generalized eikonal contribution

$$\Bigg[S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger + S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger$$
$$+ S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger + S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger\Bigg]_{\text{G-Eik}} =$$
$$(e\,e_f)^2\int d^2\mathrm{v}\int d^2\mathrm{w}\int dv^-\int dw^-\, e^{-i(\mathrm{v}-\mathrm{w})\mathrm{k}}e^{i(v^- - w^-)(p_1^+ + p_2^+ - p_0^+)}$$
$$\times\Big[(\bar\Delta)^2\mathrm{F}_a[\mathrm{P},p_0^+,p_1^+,p_2^+] + \theta(p_0^+ - p_2^+)\,(\bar\Delta')^2\mathrm{F}_b[\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+]$$
$$+ 2\theta(p_0^+ - p_2^+)\bar\Delta'\bar\Delta\mathrm{F}_{a-b}[\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+]\Big]\frac{1}{N_c}tr\left[\mathcal{U}_F\left(\mathrm{v},v^-\right)\mathcal{U}_F^\dagger\left(\mathrm{w},w^-\right)\right]$$

with the following dependence:

$$\bar\Delta = \bar\Delta(\mathrm{P},p_1^+,p_2^+) = \left(\left(-\frac{(p_1^+ + p_2^+)}{p_1^+}\mathrm{P}\right)^2 + m^2\left(\frac{p_2^+}{p_1^+}\right)^2\right)^{-1}$$
$$\bar\Delta' = \bar\Delta'(\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+) = \left(\left(-\frac{(p_1^+ + p_2^+)}{p_0^+}\mathrm{P} + \frac{p_2^+}{p_0^+}\mathrm{k}\right)^2 + m^2\left(\frac{p_2^+}{p_0^+}\right)^2\right)^{-1}$$

Cross-section (II) Next-to-eikonal corrections

The first contribution at NEik accuracy.

$$\left[S^{\text{LO-all}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-all}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger\right]_{\text{NEik}-\mathcal{U}^{(1)}_{F;j}(\mathrm{v})} = (e\,e_f)^2\int d^2\mathrm{v}\int d^2\mathrm{w}\,e^{-i(\mathrm{v}-\mathrm{w})\cdot\mathrm{k}}(2\pi)^2\delta(p_1^+ + p_2^+ - p_0^+)$$

$$\times\Bigg\{(-1)(\bar\Delta)^2\frac{\mathrm{k}^j}{2}\frac{\mathrm{F}_a[\mathrm{P},p_0^+,p_1^+,p_2^+]}{2p_0^+} - (\bar\Delta')^2\frac{\left(\frac{(p_1^+-p_2^+)}{p_0^+}\mathrm{k}^j + 2\mathrm{P}^j\right)}{2}\frac{\mathrm{F}_b[\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+]}{2p_1^+}$$

$$-(\bar\Delta\bar\Delta')\left(\frac{(2p_1^+-p_2^+)}{4p_0^+p_1^+}\mathrm{k}^j + \frac{\mathrm{P}^j}{2p_1^+}\right)\mathrm{F}_{a-b}[\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+]$$

$$+\frac{1}{2}\left(\left(-\mathrm{P}^j + \frac{p_2^+}{p_0^+}\mathrm{k}^j\right)\bar\Delta'\left(\frac{p_1^+}{p_0^+}(p_0^+ + p_2^+) + p_0^+\right) - \left(\frac{p_0^+}{p_1^+}\mathrm{P}^j\right)\bar\Delta\left(\frac{p_0^+}{p_1^+}(p_1^+ + p_2^+) + p_1^+\right)\right)\Bigg\}$$

$$\times\frac{1}{N_c}tr\left[\mathcal{U}^{(1)}_{F;j}(\mathrm{v})\mathcal{U}^\dagger_F(\mathrm{w})\right]$$

The second contribution at NEik accuracy:

$$\left[S^{\text{LO-all}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-all}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger\right]_{\text{NEik}-\mathcal{U}^{(2)}_{F;j}(\mathrm{v})} = (e\,e_f)^2\int d^2\mathrm{v}\int d^2\mathrm{w}\, e^{-i(\mathrm{v}-\mathrm{w})\cdot(\mathrm{p}_1+\mathrm{p}_2-\mathrm{p}_0)}$$

$$\times(2\pi)^2\delta(p_1^+ + p_2^+ - p_0^+)\Bigg\{(-1)(\bar{\Delta})^2\frac{1}{2p_0^+}\mathrm{F}_a[\mathrm{P},p_0^+,p_1^+,p_2^+] - (\bar{\Delta}')^2\frac{1}{2p_1^+}\mathrm{F}_b[\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+]$$

$$+(\bar{\Delta}\bar{\Delta}')\left(\frac{1}{2p_0^+}+\frac{1}{2p_1^+}\right)\mathrm{F}_{a-b}[\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+]\Bigg\}i\frac{1}{N_c}tr\left[\mathcal{U}^{(2)}_{F;j}(\mathrm{v})\mathcal{U}^\dagger_F(\mathrm{w})\right]$$

Cross-section (IV) Next-to-eikonal corrections

The third and last contribution at NEik accuracy:

$$\left[S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger + S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger\right]_{\text{NEik-Fij}} = (e\,e_f)^2$$

$$\times \int d^2\mathrm{v}\, d^2\mathrm{w}\, e^{-i(\mathrm{v}-\mathrm{w})\cdot(\mathrm{p}_1+\mathrm{p}_2-\mathrm{p}_0)}(2\pi)^2\delta(p_1^+ + p_2^+ - p_0^+)$$

$$\times\ (8\bar{\Delta}\bar{\Delta}')\frac{(p_2^+)^3}{p_1^+}\left(-\mathrm{P}^i + \frac{p_2^+}{p_0^+}\mathrm{k}^i\right)\left(\frac{1}{p_1^+}\mathrm{P}^j\right)\frac{1}{N_c}tr\left[\mathcal{U}^{(3)}_{F;ij}(\mathrm{v})\mathcal{U}^\dagger_F(\mathrm{w})\right]$$

We would need to take also into account the conjugate of all these contributions.

Outlook

- We computed the cross section for the case of forward photon-jet production at full NEik order
- Next-to eikonal corrections include:
  - Relaxing the shockwave approximation $\rightarrow$ transverse motion through the target
  - Including interactions with transverse component of the background field
  - Taking into account z^{-}-dependence
- We plan on taking the back to back limit for this process.

Thank you for your attention