Small-x Helicity Phenomenology

Daniel Adamiak
The Ohio State University

In collaboration with Yuri Kovchegov, Dan Pitonyak, Matt Sievert, Nobuo Sato, Wally Melnitchouk, Josh Tawabutr, Andrey Tarasov and Nick Balsonado

Proton Spin Puzzle

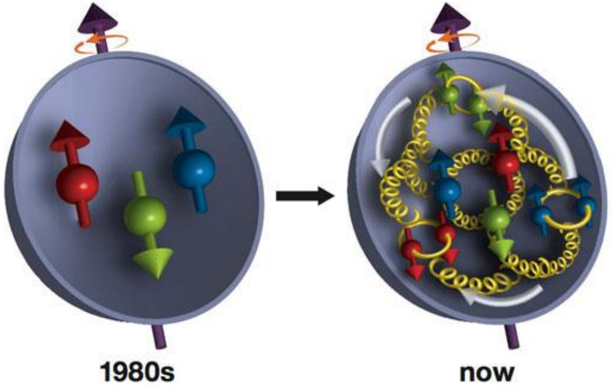
Jaffe-Manohar Spin Sum Rule:

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

$S_{q,g}$ = Helicity of quarks and gluons

$L_{q,g}$ = Orbital angular momentum

S_q ~ 30% of proton spin!

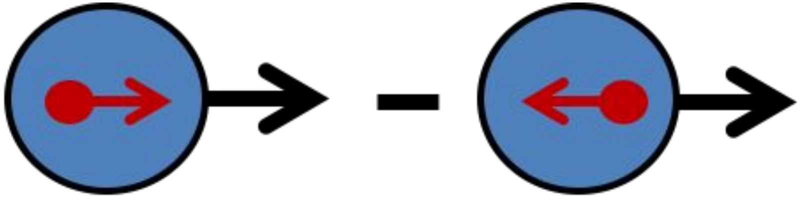


Quark Helicity Parton Distribution Functions

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx \sum_q (\Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2))$$

Helicity PDFs:

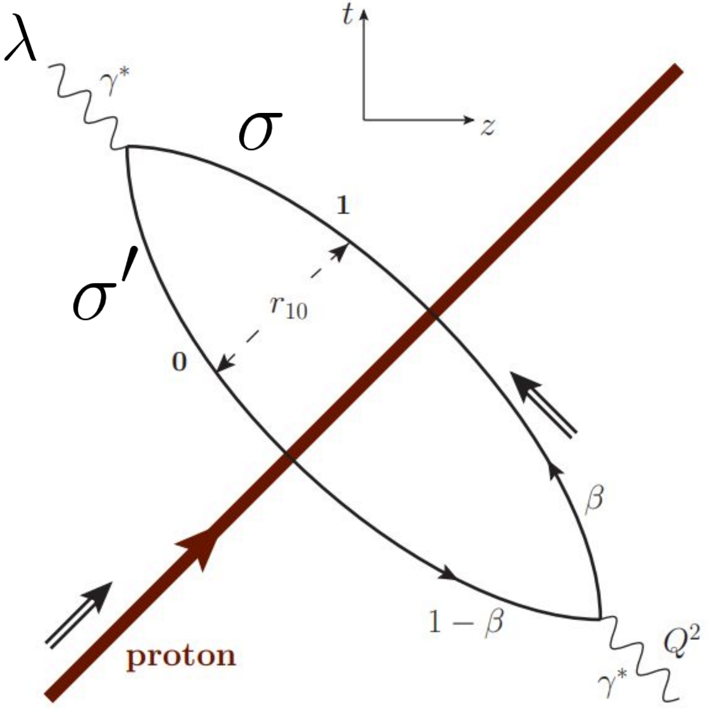
$$\Delta q =$$



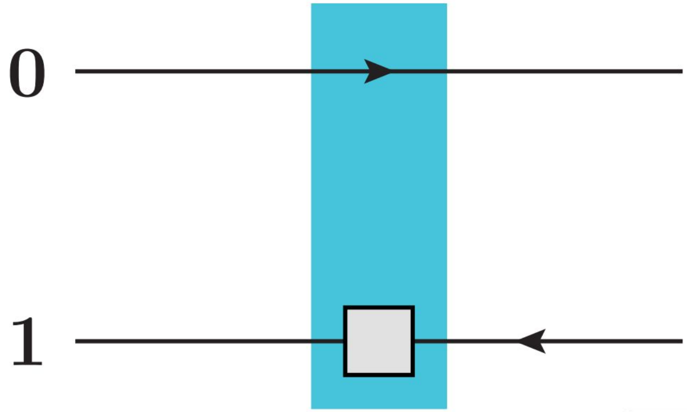
- Q^2 = resolution at which we probe the proton
- Bjorken $x \sim \frac{1}{s}$. We need theory to extrapolate to *x*=0

(Polarized) DIS in the (Polarized) Dipole Picture

$$g_1 \propto |\psi|^2 \otimes (Q + 2G_2)$$



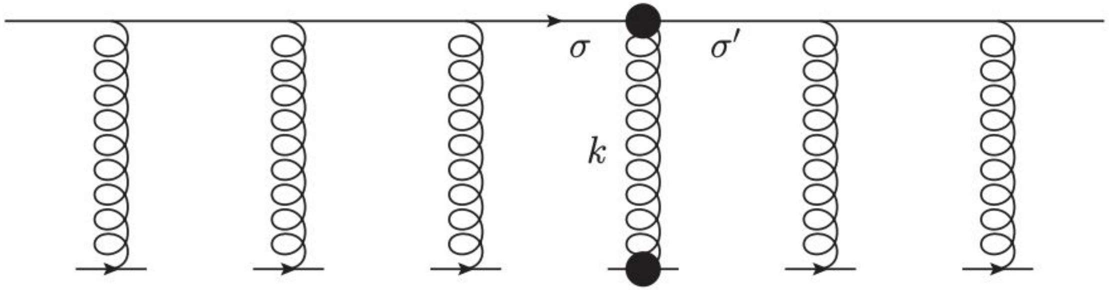
(Polarized) DIS in the (Polarized) Dipole Picture



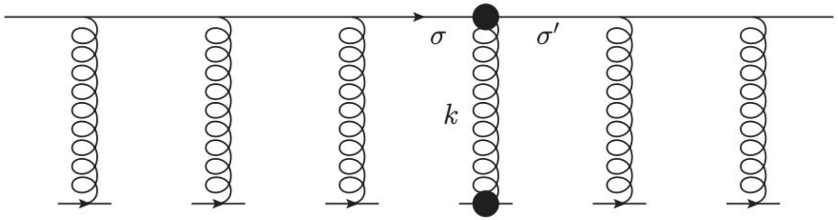
- In pDIS, the electron and proton have their helicity specified
- Cross-section now dependent on **Polarized Dipole Amplitudes**:

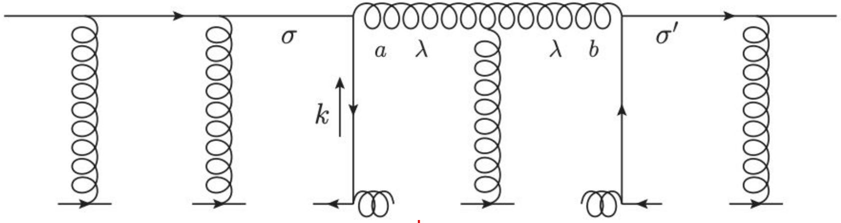
$$Q_q, G_2, \tilde{G}$$

- Quark line undergoes one extra helicity exchange, which is **sub-eikonal**



Polarized Wilson Lines





$$\vec{\mu} \cdot \vec{B}$$

- Chromo-magnetic field

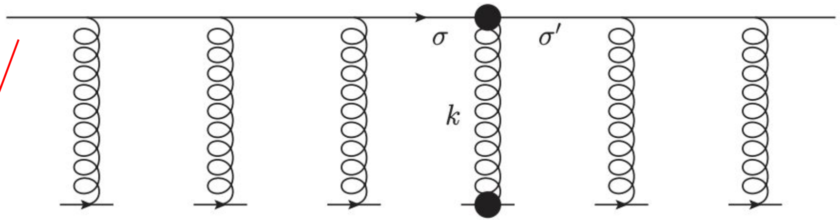
$$\mu B_z \quad \sim F_{12}$$

$$\bar{\psi}\gamma^{+}\gamma^{5}\psi$$

- Axial Current

Polarized Dipole Amplitudes: $``Q_q"$ , $``\tilde{G}"$

Polarized Wilson Lines



- Quark propagator

$$\int \frac{dk^+}{2\pi} e^{ikx} \frac{\not{k}}{2k^+k^- - k_\perp^2}$$

- Sub-eikonal phase expansion

$$A_\perp$$

- Polarized gluon vertex

$$e^{-ix^- \frac{k_\perp^2}{2k^-}} \approx 1 - ix^- \frac{k_\perp^2}{2k^-} \Rightarrow \partial_\perp^2$$

$$D_\perp^2$$

$$\text{“}G_2\text{”}$$

Calculating Helicity Distributions

$$\Delta q + \Delta \bar{q} = \frac{1}{N_c} \int_0^{\eta_{max}} d\eta \int_{s_{10}^{min}}^{\eta} ds_{10} \frac{1}{\alpha_s(s_{10})} \left(Q_q(s_{10}, \eta) + 2G_2(s_{10}, \eta) \right)$$

- We incorporate running coupling that runs with size of the dipole

- η ~ Longitudinal momentum
- s_{10} ~ Transverse separation of Dipole

Large Nc&Nf Helicity Evolution

In the large Nc&Nf, Nc/Nf fixed limit, the evolution equations for the polarized dipole amplitudes close:

$$Q_q(s_{10},\eta) = Q_q^{(0)}(s_{10},\eta) + \int_{s_{10}+y_0}^{\eta} d\eta' \int_{s_{10}}^{\eta'-y_0} ds_{21}\Big[Q_q(s_{21},\eta') + 2\tilde{G}(s_{21},\eta') + 2\tilde{\Gamma}s_{10},s_{21},\eta') - \bar{\Gamma}_f(s_{10},s_{21},\eta') + 2G_2(s_{21},\eta') + 2\Gamma_2(s_{10},s_{21},\eta')\Big]$$

$$+\frac{1}{2}\int_{y_0}^{\eta} d\eta' \int_{\max\{0,s_{10}+\eta'-\eta\}}^{\eta'-y_0} ds_{21}\Big[Q_q(s_{21},\eta') + 2G_2(s_{21},\eta')\Big]$$

+ 9 more

- 5 Polarized dipole amplitudes mix under evolution: $Q_{u,d,s}, \tilde{G}, G_2$
- With 5 auxiliary dipoles: $\Gamma_{u,d,s}, \tilde{\Gamma}, \Gamma_2$ - which impose lifetime ordering
- Small-*x* cutoff, $y_0 \propto \ln 1/x_0$

Large Nc&Nf Helicity Evolution

- 5 **Polarized dipole amplitudes** mix under evolution: $Q_{u,d,s}, \tilde{G}, G_2$
- With 5 auxiliary dipoles: $\Gamma_{u,d,s}, \tilde{\Gamma}, \Gamma_2$
- For a total of 10 equations that form a **closed system**
- Undetermined initial conditions: $Q^{(0)}_{u,d,s}, \tilde{G}^{(0)}, G_2^{(0)}$

Recap:

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx \sum_q (\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2))$$

$$S_g(Q^2) = \int_0^1 dx \Delta G(x,Q^2)$$

$$\Delta q + \Delta\bar{q} = \frac{1}{N_c}\int_0^{\eta_{max}} d\eta \int_{s_{10}^{min}}^{\eta} ds_{10} \frac{1}{\alpha_s(s_{10})}\left(Q_q(s_{10},\eta) + 2G_2(s_{10},\eta)\right)$$

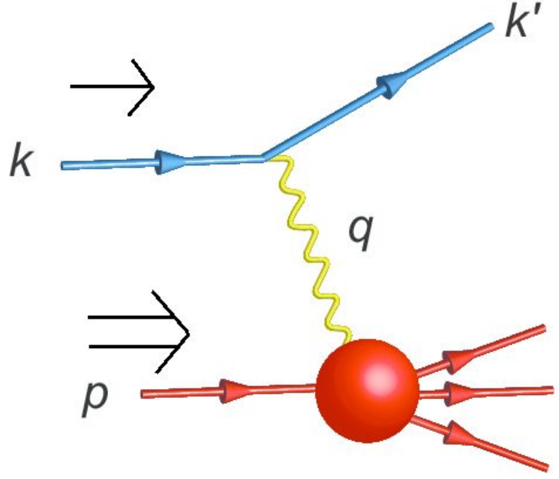
$$\Delta G(x,Q^2) = \frac{2N_c}{\alpha_s(Q^2)} G_2\left(\sqrt{\frac{N_c}{2\pi}}\ln\frac{Q^2}{\Lambda^2}, \sqrt{\frac{N_c}{2\pi}}\ln\frac{Q^2}{x\Lambda^2}\right)$$

Large $N_c \& N_f$ Helicity Evolution

$$Q_q^{(0)}, \tilde{G}^{(0)}, G_2^{(0)}$$

Observables - Double Spin Asymmetries in DIS

$$A_{||} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} \propto A_1 \propto g_1^{p,n}$$



- $\uparrow (\downarrow)$ is positive (negative) helicity electron
- $\Uparrow (\Downarrow)$ is positive (negative) helicity proton
- A_1 is virtual photoproduction asymmetry

Describing Observables - pDIS

What enters into observables are linear combinations of hPDFs

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$
$$\Delta q^- = \Delta q - \Delta \bar{q}$$

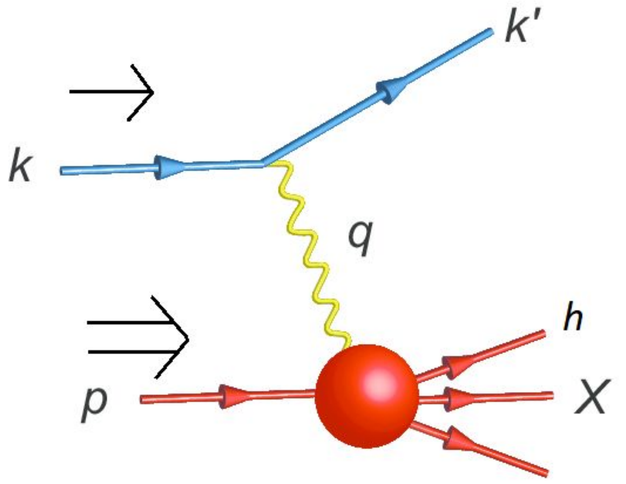
- **Three** relevant hPDFs in DIS: $\Delta u^+, \Delta d^+, \Delta s^+$, involving **five** amplitudes
- Data exist for **two** observables that contain these hPDFs in linearly independent combinations: g_1^p and g_1^n

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_q Z_q^2 \Delta q^+(x, Q^2)$$

- Z_q is the quark charge fraction

Observables - Double Spin Asymmetries in SIDIS

$$A_{||}(z) = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} \propto g_1^h(z)$$



- *h* is the tagged hadron
- *z* is the momentum fraction of the virtual photon carried by the tagged hadron

Describing Observables - pSIDIS

- 2 observables are not enough to describe 3 hPDFs.
- Expand our horizons to Semi-Inclusive DIS - all hPDFs are relevant here, both singlet, Δq^+ and non-singlet, Δq^-
- **Non-singlet distributions obey their own small-*x* evolution that has been solved**

$$\Delta q^- = \frac{N_c}{2\pi^3} \int d\eta \int ds_{10} Q_q^{NS}(s_{10}, \eta)$$

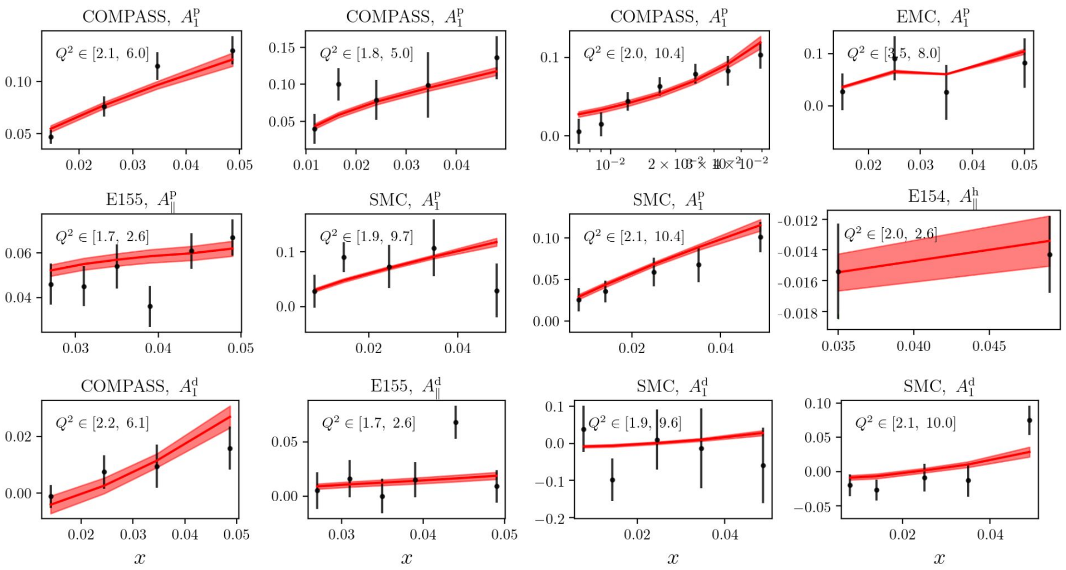
- Q_q^{NS} is the non-singlet Polarized Dipole Amplitude - obeys its own evolution equation
- pSIDIS grants us access to the semi-inclusive, spin dependent structure functions g_1^h

g_1^h Structure Functions

$$g_1^h(x,z,Q^2) = \frac{1}{2}\sum_q Z_q^2 \Delta q(x,z,Q^2) D_q^h(z,Q^2)$$

- D_q^h are fragmentation functions - giving the probability quark *q* fragments into hadron *h*
- z Is the fraction of the virtual photons momentum carried by the hadron
- The flavour hPDF is obtained via $\Delta q = \frac{1}{2}(\Delta q^+ + \Delta q^-)$
- In pSIDIS, we are able to scatter on 2 targets (proton, neutron), tag 2 outgoing hadrons (pion, kaon) that each have 2 charges - 2x2x2=8 new observables

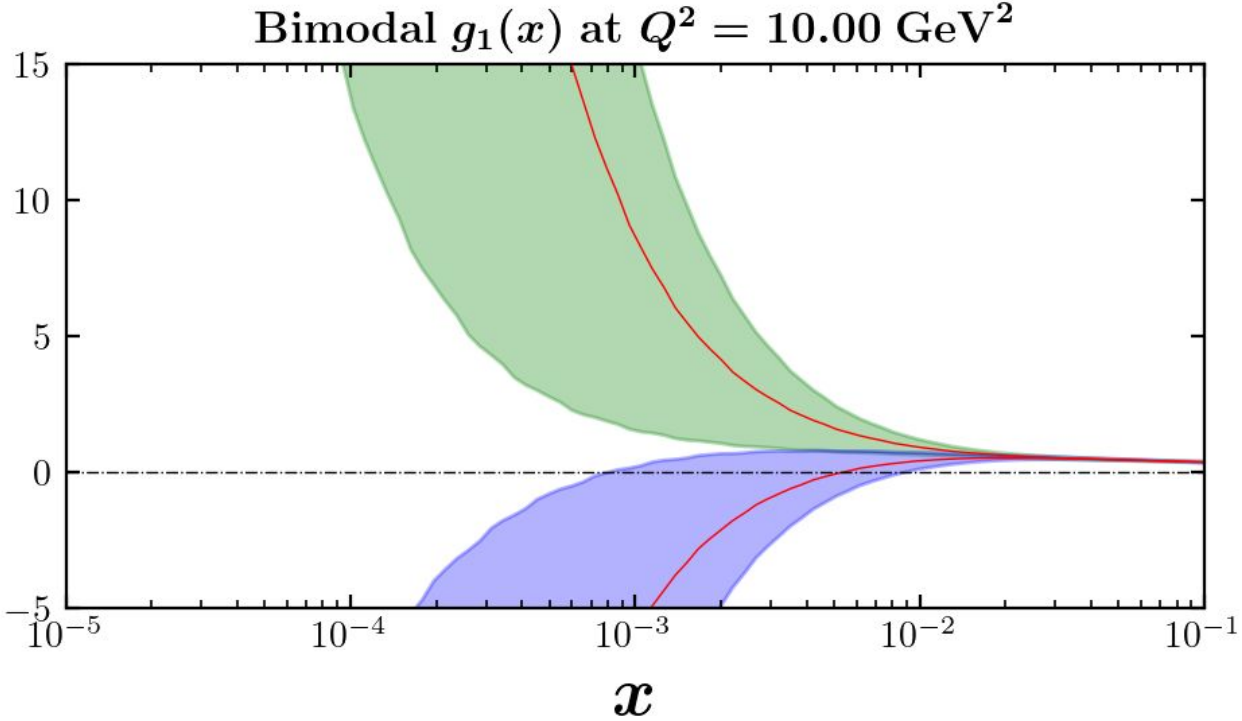
Global fit of DIS - Data vs Theory



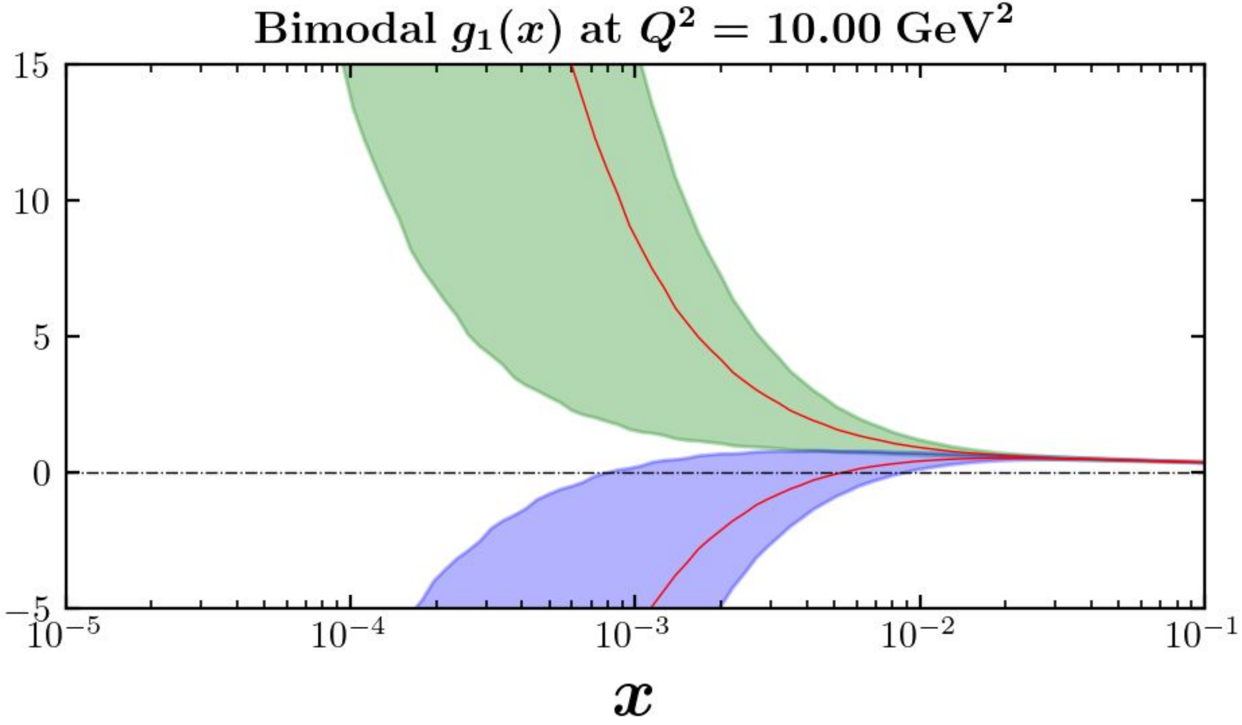
- Red curves - our theory
- Black dots - data
  - COMPASS
  - EMC
  - SMC
  - SLAC
  - HERMES
- Preliminary results

- Cut of 0.005< x < 0.1
- Cut of $1.69\ GeV^2 < Q^2 < 10.4\ GeV^2$
- Cut of 0.2 < z <1.0
- Describing 234 data points
- With a $\chi^2/npts$ = 1.04

(Preliminary) Extraction of g_1^p



(Preliminary) Extraction of g_1^p



$\tilde{G}$ not sensitive to large-*x* data, but responsible for small-*x* behaviour

Constraining the rest of the Polarized Dipole Amplitudes

$$g_1^{p,n} \sim Q_u, Q_d, Q_s, G_2$$

- 2 observables, 4 polarized dipole amplitudes. Under constrained system

$$g_1^h \sim Q_q, G_2, Q_q^{NS}$$

- 8 new observables, 3 new polarized dipole amplitudes. Exactly constrained - but $\tilde{G}$ does not enter directly into observables

$$pp \to jets \sim G_2, \tilde{G}$$

- Particle production might provide final constraints

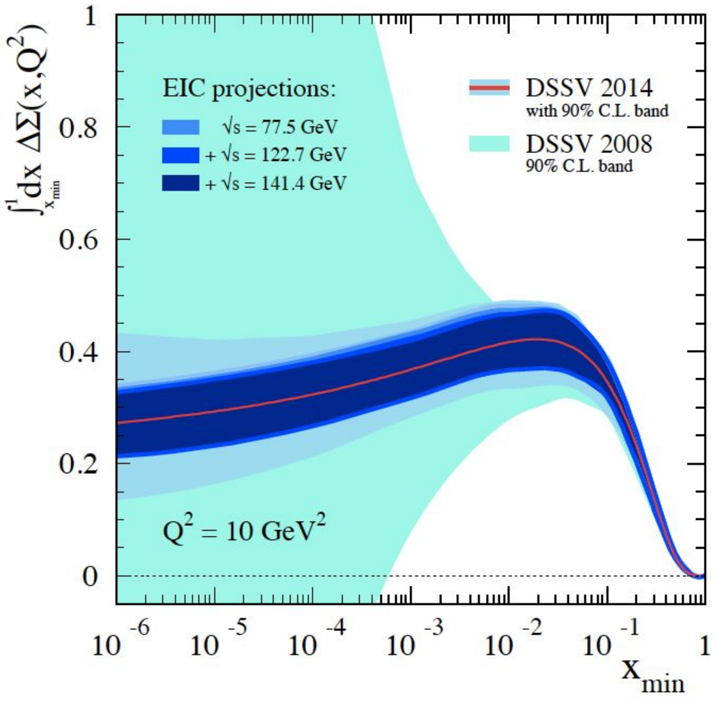
Conclusions

- In order to resolve the spin puzzle, the small-*x* behaviour of the hPDFs need to be understood
- This is accomplished using small-*x* evolution
- Along with fitting to data
- Potentially a significant amount if spin is hiding in the small-*x* region
- More work needs to be done to constrain small-*x* behavior of the various polarized dipoles - especially G_2 and $\tilde{G}$
- Could be constrained by studying particle production in *pp* collisions as well as smaller-*x* EIC data

Backup Slides

Quark hPDF - DGLAP extraction

2 x (quark spin)

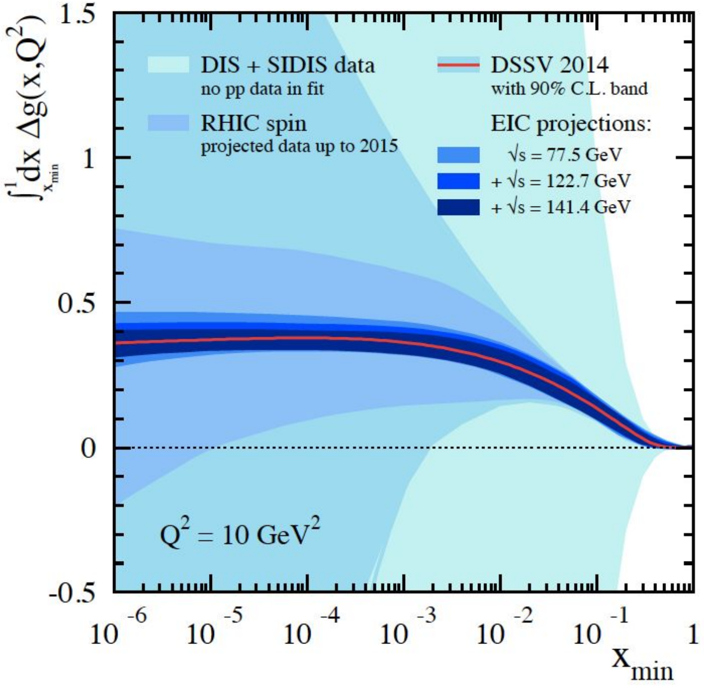


$$\Delta\Sigma = \sum_q (\Delta q + \Delta\bar{q})$$

- E. Aschenauer et al, **arXiv:1509.06489 [hep-ph]**, (DSSV = de Florian, Sassot, Stratmann, Vogelsang, DGLAP-based helicity PDF extraction from data)
- Large uncertainty at small-x!

Gluon Helicity Parton Distributions Function

gluon
spin



$$S_g(Q^2) = \int_0^1 dx \Delta G(x, Q^2)$$

ΔG = Gluon Helicity PDF

- Uncertainty consistently blows up when extrapolating beyond data

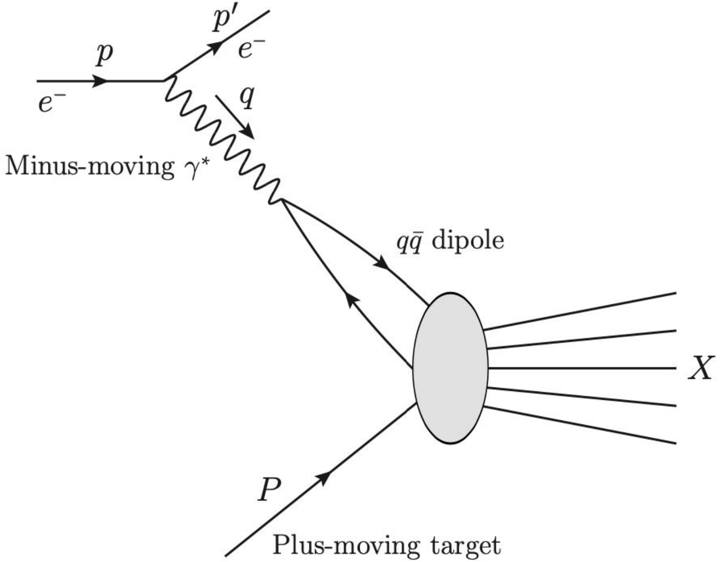
The Plan

Any complete description of quark and gluon helicity needs to

- Describe existing data $(5 \times 10^{-3} < x < 0.7)$
- Predict future, e.g EIC, data $(4 \times 10^{-3} < x < 5 \times 10^{-3})$
- Compare with said data
- Extrapolate down to $x = 0$
- While maintaining good control over theoretical uncertainty

Deep-Inelastic Scattering (DIS)

Probing the proton at small *x*



- Electron of momentum *p* scatters off proton of momentum *P*
- Transverse size given by virtuality of photon:

$$\frac{1}{x_\perp^2} \propto Q^2 = -q^2$$

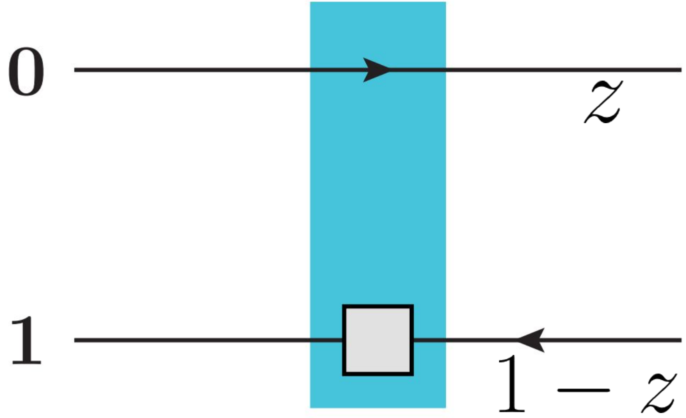
- Bjorken-*x*: $$x = \frac{Q^2}{2P \cdot q} \approx \frac{Q^2}{s}$$

Calculating Helicity Distributions

$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s(Q^2)} G_2 \left(\sqrt{\frac{N_c}{2\pi}} \ln \frac{Q^2}{\Lambda^2}, \sqrt{\frac{N_c}{2\pi}} \ln \frac{Q^2}{x\Lambda^2} \right)$$

- Jaffe-Manohar Gluon Helicity Distribution
- Λ^2 Infrared cutoff

Polarized Dipole Amplitude - Degrees of Freedom



$$Q_q(s_{10}, \eta)$$

Polarized Dipole Amplitudes are functions of

- Transverse separation:

$$x_{10}^2 = (\underline{x}_1 - \underline{x}_0)^2$$

- Momentum Fraction times center of mass energy: zs
- Rescaled variables:

$$\eta = \sqrt{\frac{N_c}{2\pi}} \ln \frac{zs}{\Lambda^2} \qquad s_{10} = \sqrt{\frac{N_c}{2\pi}} \ln \frac{1}{x_{10}^2 \Lambda^2}$$

Helicity Evolution

Using Light-Cone Operator Treatment, we need to resum all gluon exchanges that exchange helicity information

$\Rightarrow$ Resumming all terms containing: $$\alpha_s \int\limits_x^1 \frac{dz}{z} \int\limits_{1/s}^{1/Q^2} \frac{d^2 x_{21}}{x_{21}^2}$$

Longitudinal part. Present in un-polarized evolution

Transverse part. UV exactly cancelled in un-polarized evolution

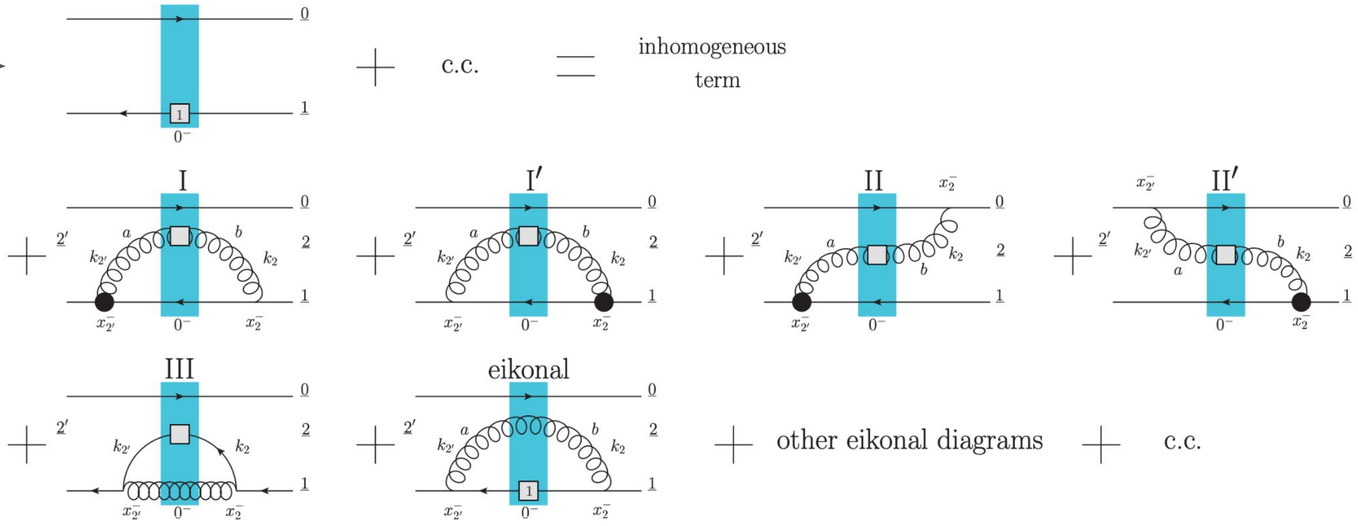
Resum double log (DLA) terms:

$$\alpha_s \ln^2(1/x)$$

Helicity Evolution

- Relate Polarized Dipole Amplitude to themselves at higher energies by resumming emission diagrams - resumming Double Log (DLA) contributions: $\alpha_s \ln^2(1/x)$

$Q_q(x_{10}^2, zs) \longrightarrow$



Sub-eikonal Expansion

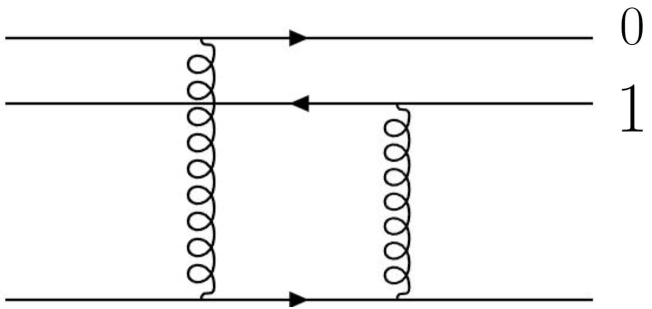
- Expansion in energy or in *x*

$1/x,$	$x^0,$	x^1
Eikonal	Sub-Eikonal	Sub-Sub-Eikonal
F_1, F_2	$g_1^{p,n}, \Delta q, \Delta \bar{q}$	Transversity

- No eikonal terms contain any helicity information - Wilson lines are helicity independent
- Must calculate sub-eikonal terms to access helicity

Inhomogeneous term

The inhomogeneous term is given by a Born-inspired ansatz:

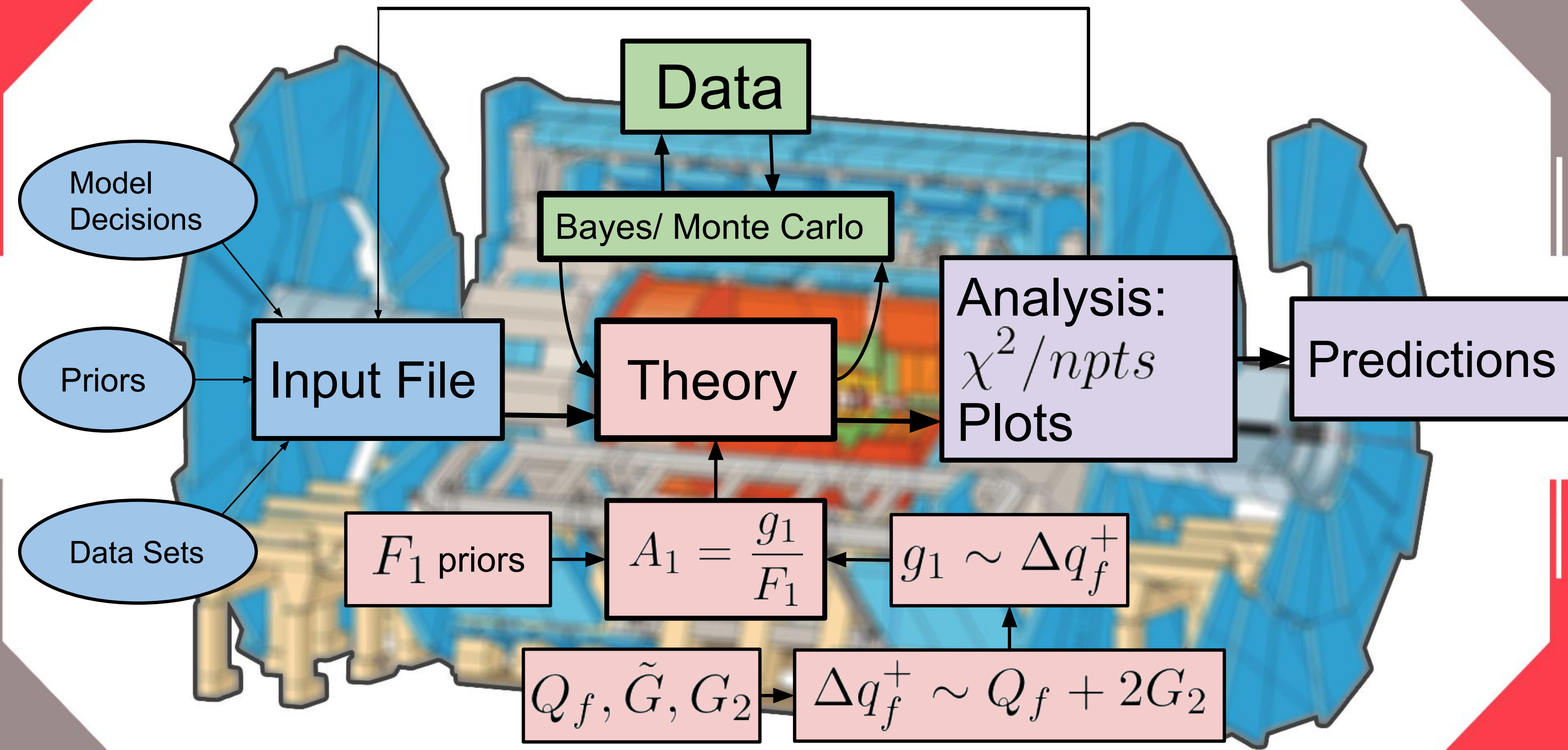


$$\propto \int_0^s \frac{dk_\perp^2}{k_\perp^2}\left(1 - e^{-\underline{k}\cdot\underline{x}_{10}}\right) = \pi \ln(s x_{10}^2)$$

$$\propto \eta - s_{10}$$

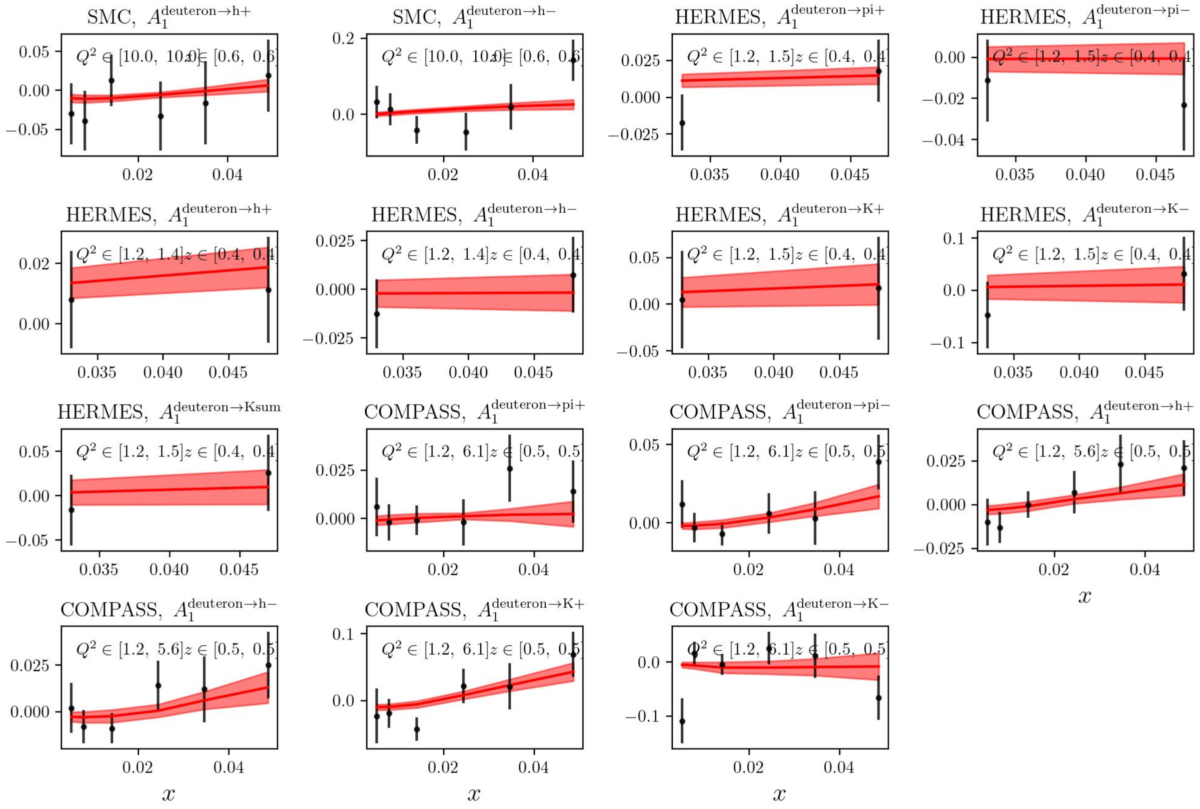
$$\Gamma_q^{(0)} = Q_q^{(0)} = a\eta + b s_{10} + c$$

- Same form of the other Dipole Amplitudes
- Parameters a,b,c need to be extracted from data

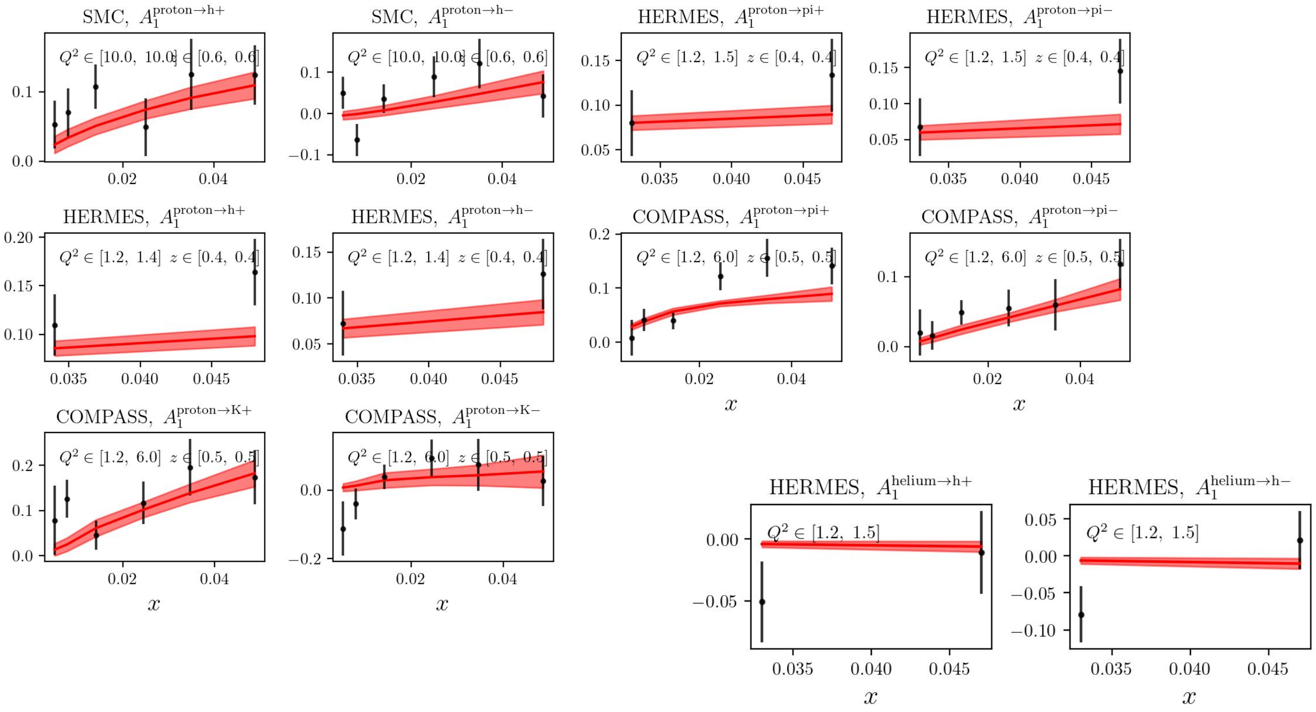


Data
Bayes/ Monte Carlo
Model Decisions
Priors
Data Sets
Input File
Theory
Analysis:
$\chi^2/npts$
Plots
Predictions
F_1 priors
$A_1 = \frac{g_1}{F_1}$
$g_1 \sim \Delta q_f^+$
$Q_f, \tilde{G}, G_2$
$\Delta q_f^+ \sim Q_f + 2G_2$

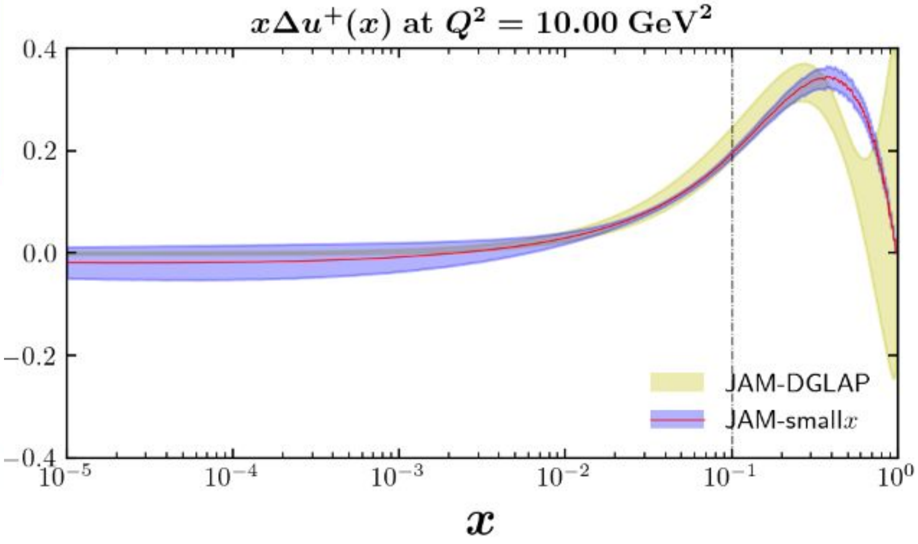
Global fit of SIDIS - Data vs Theory

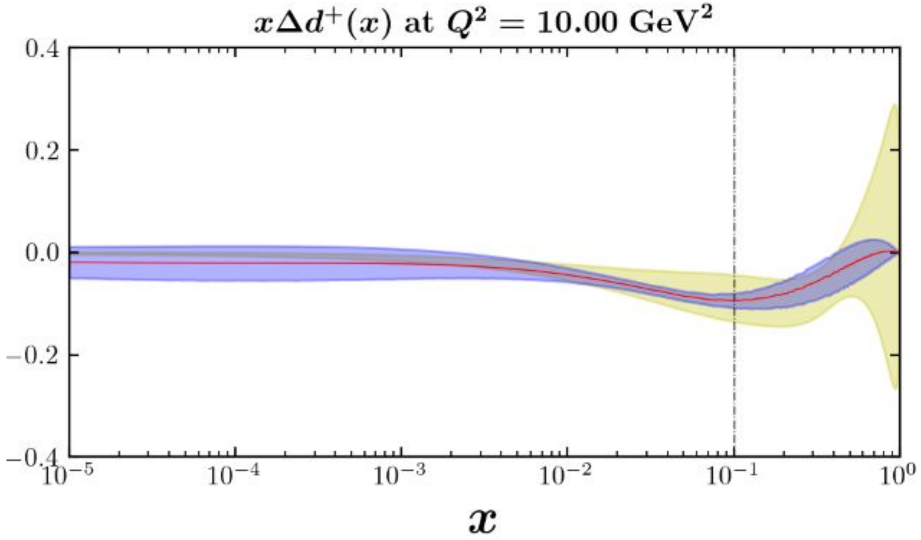


Fitting SIDIS - Data vs Theory



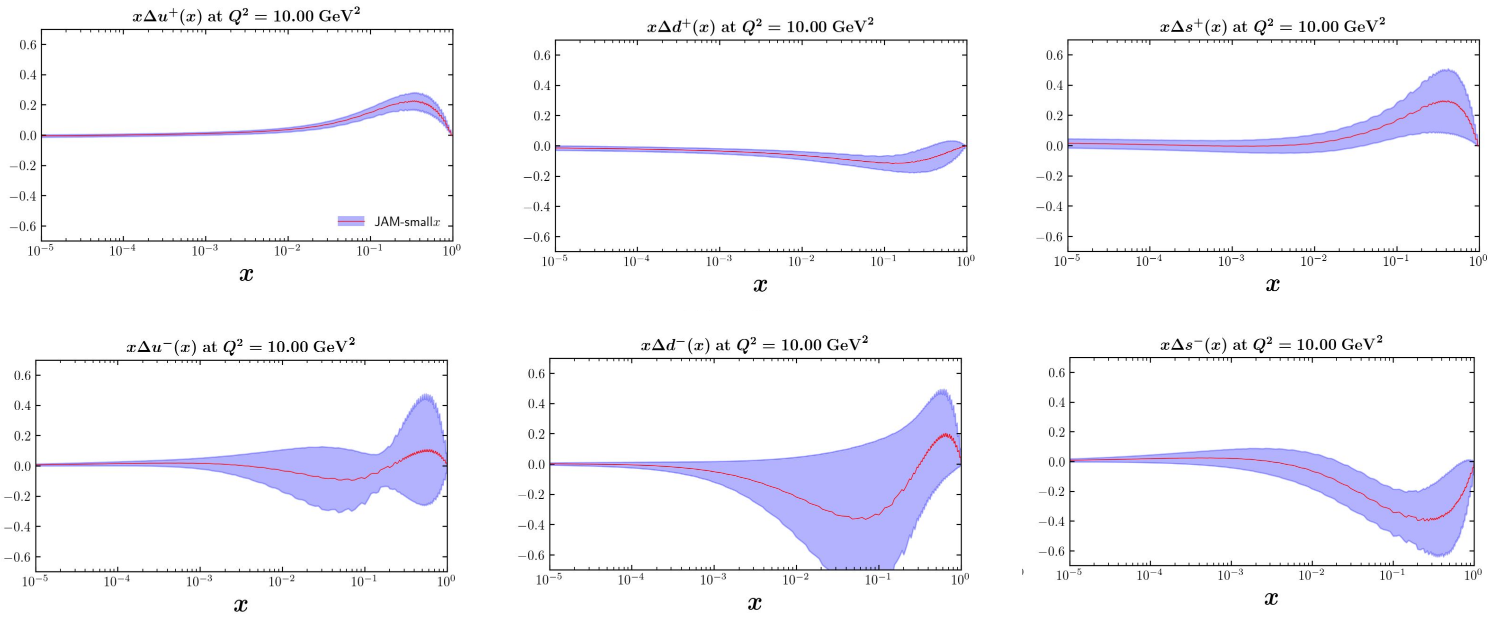
hPDFs - (Preliminary)





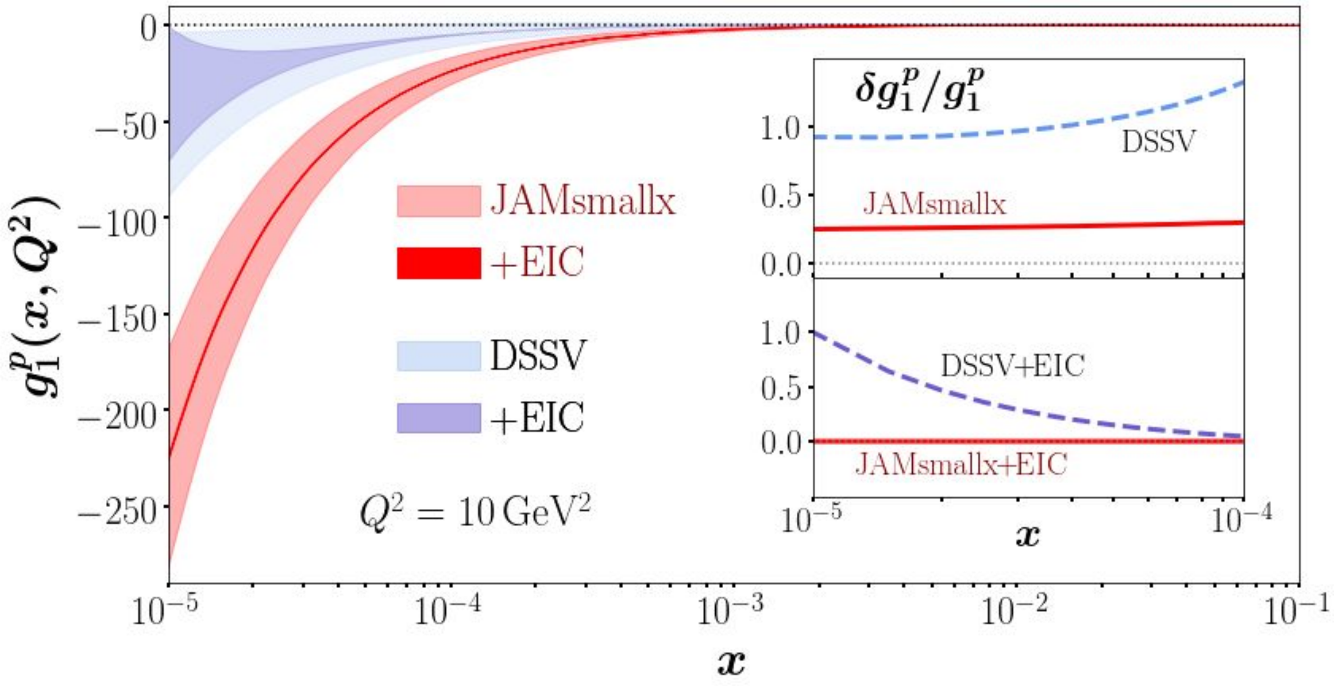
- DIS only: Strange distribution set to zero

hPDFs - Preliminary



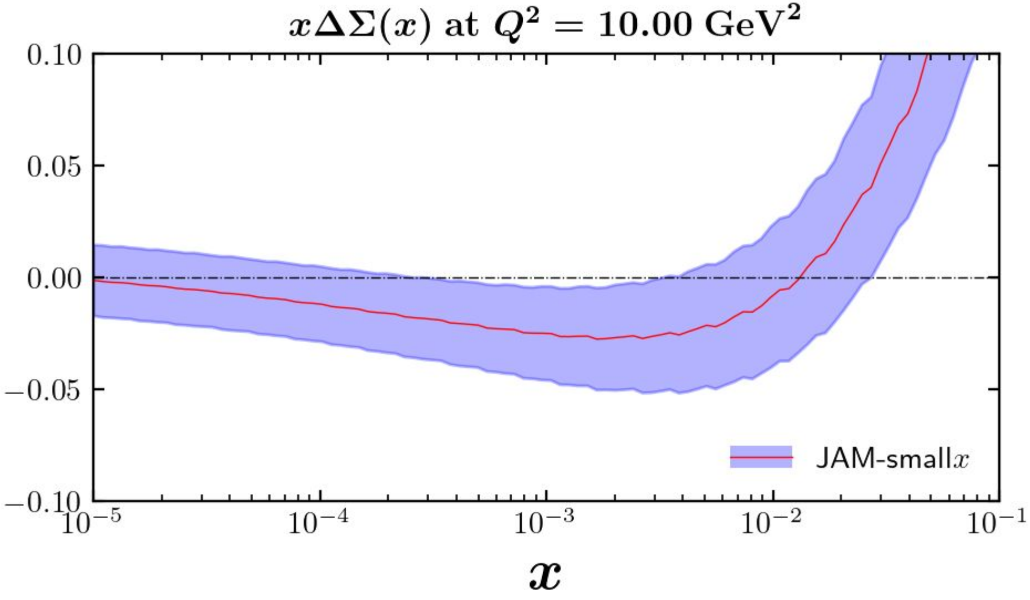
- Old version of evolution

(Preliminary) Extraction of g_1^p



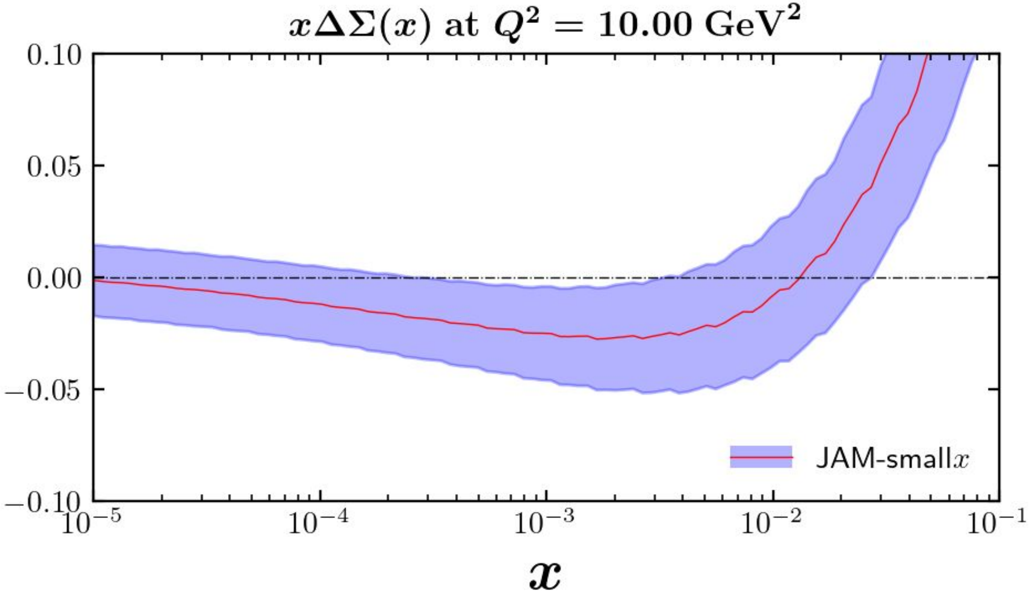
- DSSV uses DGLAP - rational function extrapolation of *x*
- We use small-*x* helicity evolution to predict the *x* behaviour
- Leads to control over uncertainty

Contribution from Quark Spin $\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$



Contribution from Quark Spin

$$\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$$

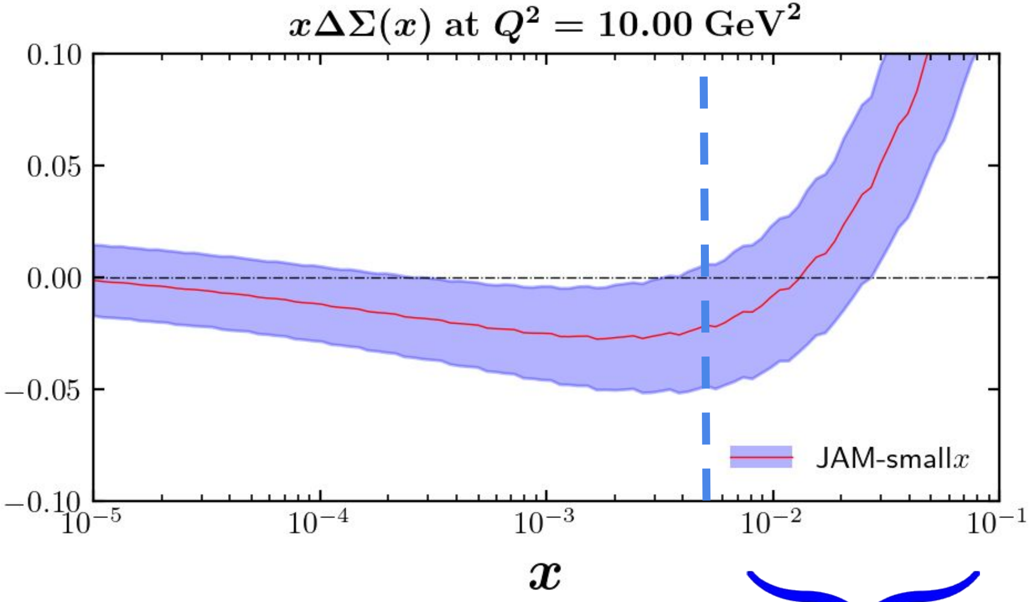


Large-*x* region

$$\int_{0.01}^{0.7} dx \Delta\Sigma(x) = \pm 0.36$$

Contribution from Quark Spin

$$\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$$



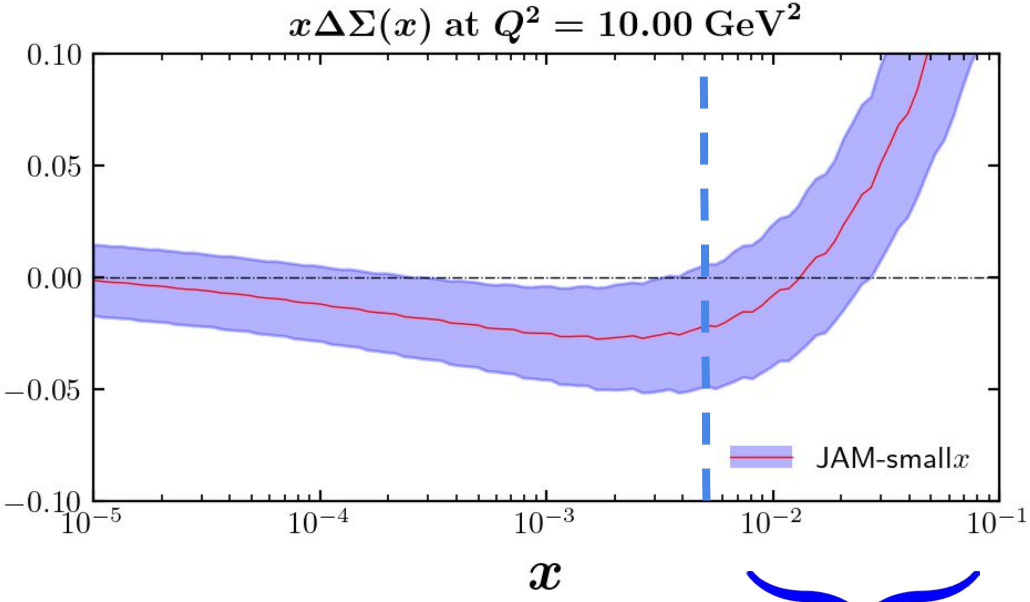
Large-*x* region

$$\int_{0.01}^{0.7} dx\Delta\Sigma(x) = \pm 0.36$$

Existing small-*x* data

Contribution from Quark Spin $\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$

Saturation region



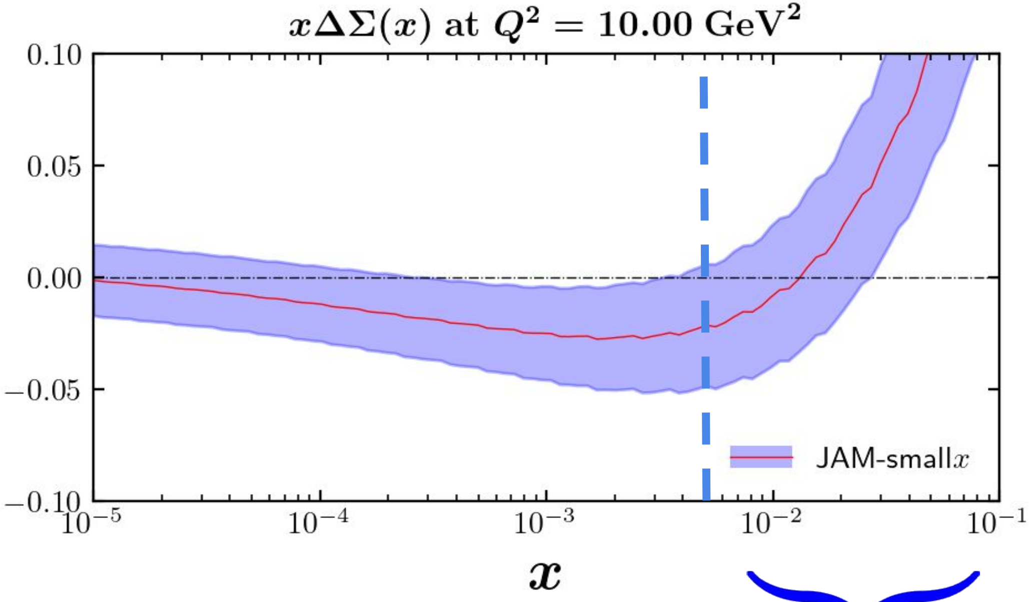
Existing small-*x* data

Large-*x* region

$$\int_{0.01}^{0.7} dx\Delta\Sigma(x) = \pm 0.36$$

Contribution from Quark Spin $\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$

Saturation region



Existing small-x data

Large-x region

$$\int_{0.01}^{0.7} dx\Delta\Sigma(x) = \pm 0.36$$

Compare with:

$$\int_{10^{-5}}^{10^{-3}} dx\Delta\Sigma(x) = -0.1 \pm 0.1$$