Diffractive structure functions at NLO in the dipole picture

Jani Penttala

In collaboration with Guillaume Beuf, Tuomas Lappi, Heikki Mäntysaari and Risto Paatelainen

University of Jyväskylä

Centre of Excellence in Quark Matter

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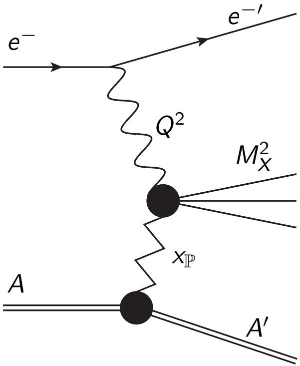
JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Diffractive structure functions

- Definition of diffraction
  - Experimental: A rapidity gap in the final state
  - Theoretical: No color exchange

    $\Rightarrow$ Both the target and projectile remain in color singlet state
- HERA: Almost 10% of DIS events diffractive
- Sensitive to gluon structure at high energy: $\sigma^D \sim [xg(x)]^2$
- Inclusive diffraction: any final state with a definite invariant mass M_X^2



Diffractive structure functions

$$x_{\mathbb{P}} F_\lambda^{D(3)}(x_{\mathbb{P}}, Q^2, M_X^2) = \frac{Q^2}{(2\pi)^2 \alpha_{\mathrm{em}}} \frac{Q^2}{\beta} \frac{\mathrm{d}\sigma_\lambda^D}{\mathrm{d}M_X^2}$$

$$x_{\mathbb{P}} \approx \frac{M_X^2 + Q^2}{W^2 + Q^2}, \quad \beta \approx \frac{Q^2}{Q^2 + M_X^2}, \quad Q^2 = \text{ photon virtuality}, \quad \lambda = \text{ photon polarization } (L \text{ or } T)$$

Inclusive diffraction in the high-energy limit

High-energy limit leads to factorization:

Inclusive diffraction cross section at LO

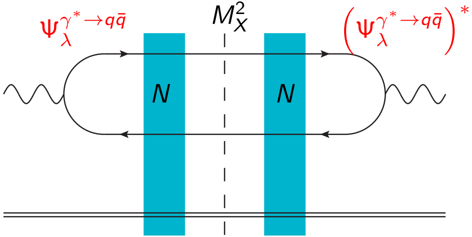
$$\frac{\mathrm{d}\sigma_\lambda^D}{\mathrm{d}M_X^2} = \frac{N_C}{(4\pi)^2} \int \mathrm{d}z\, \mathrm{d}^2\mathbf{r}\, \mathrm{d}^2\bar{\mathbf{r}}\, \mathrm{d}^2\mathbf{b}\, J_0\left(M_X|\Delta\mathbf{r}|\sqrt{z(1-z)}\right) N(\mathbf{r},\mathbf{b}) N(\bar{\mathbf{r}},\mathbf{b}) \Psi_\lambda^{\gamma^*\to q\bar{q}}(\mathbf{r},z) \left(\Psi_\lambda^{\gamma^*\to q\bar{q}}(\bar{\mathbf{r}},z)\right)^*$$

- $\Psi_\lambda^{\gamma^*\to q\bar{q}}$: Photon wave function for the $q\bar{q}$ state

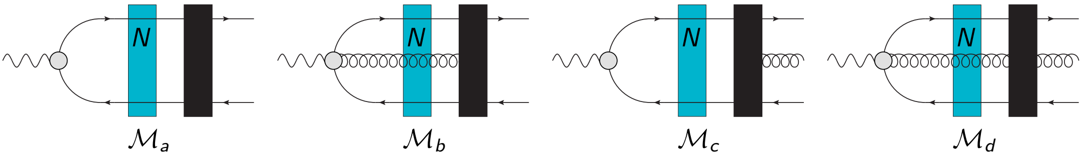
  Calculable perturbatively
- N: Dipole-target scattering amplitude

  Energy dependence by the JIMWLK equation
- Eikonal interaction with target:

  Convenient to work in the mixed space $(\mathbf{r}, z)$



Next-to-leading order: different contributions



Inclusive diffraction cross section at NLO

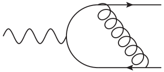
$$\frac{\mathrm{d}\sigma_\lambda^D}{\mathrm{d}M_X^2} = \sum_n \int \mathrm{d}[\mathrm{PS}]_n \, 2q^+ (2\pi)\delta(q^+ - p_n^+)\delta(M_X^2 - M_n^2)|\mathcal{M}_n|^2$$

where at NLO we need $\mathcal{M}_{q\bar{q}} = \mathcal{M}_a + \mathcal{M}_b$ and $\mathcal{M}_{q\bar{q}g} = \mathcal{M}_c + \mathcal{M}_d$.

- The finite contribution $|\mathcal{M}_d|^2$ already calculated Hänninen et al. 2206.13161 – previous talk by Henri

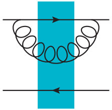
Divergences at NLO

Regularization scheme: dim. reg. for transverse coordinates, cutoff α for longitudinal momentum

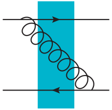


1. Corrections to the initial state: UV and $\log^2 \alpha$ divergences

   Included in the photon wave function $\Psi_\lambda^{\gamma^* \to q\bar{q}}$

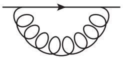


2. UV divergences from gluon loops over the shock wave



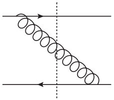
3. Rapidity divergence for gluons with small plus momentum over the shock wave:

   $\log \alpha$ divergence regularized by JIMWLK (BK) equation



4. Self-energy diagrams: IR and UV divergences

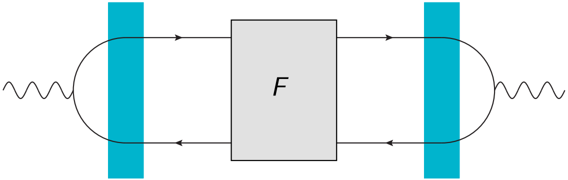
   Cancel in dimensional regularization with one ε



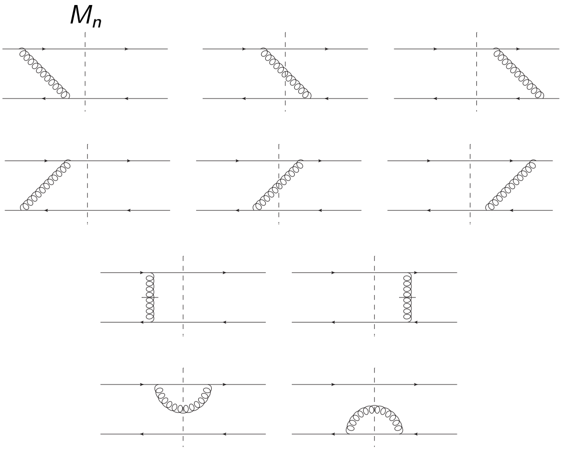
5. Corrections to the final state: IR and $\log^2 \alpha$ divergences

   Complicated!

Final-state corrections

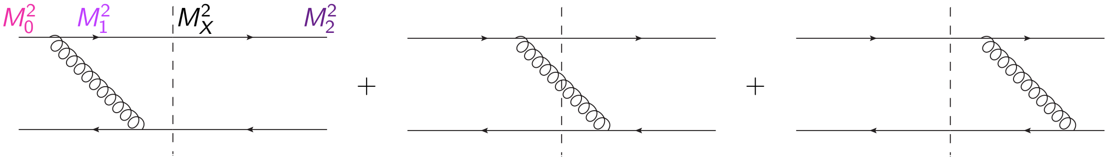


F = sum of final-state corrections



- Lots of pesky diagrams to calculate
- Cut introduces a delta function $\delta(M_X^2 - M_n^2)$
  – Hard to integrate!
- Strategy:
  1. Sum the diagrams in momentum space
  2. Fourier transform to mixed space

Final state: Regular gluon exchange



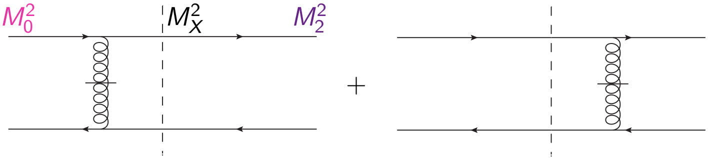
Sum these diagrams together and use the following trick:

$$\frac{\delta(M_0^2 - M_X^2)}{(M_0^2 - M_1^2 - i\delta)(M_0^2 - M_2^2 - i\delta)} + \frac{\delta(M_1^2 - M_X^2)}{(M_1^2 - M_0^2 + i\delta)(M_1^2 - M_2^2 - i\delta)} + \frac{\delta(M_2^2 - M_X^2)}{(M_2^2 - M_0^2 + i\delta)(M_2^2 - M_1^2 + i\delta)}$$

$$= \frac{1}{2\pi i}\left[\frac{1}{(M_X^2 - M_0^2 - i\delta)(M_X^2 - M_1^2 - i\delta)(M_X^2 - M_2^2 - i\delta)} - \frac{1}{(M_X^2 - M_0^2 + i\delta)(M_X^2 - M_1^2 + i\delta)(M_X^2 - M_2^2 + i\delta)}\right]$$

- This combines divergences from different graphs
- Note: the signs of the infinitesimals $i\delta$ important!
- This expression also simpler to Fourier transform
  - We can use Feynman or Schwinger parametrization to calculate the integrals

Final state: Instantaneous gluon exchange

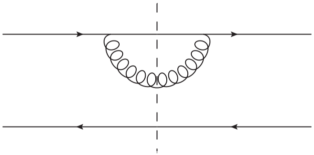


A similar trick can be used to combine these:

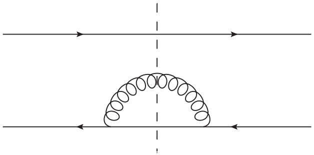
$$\frac{\delta(M_X^2 - M_2^2)}{M_0^2 - M_2^2 - i\delta} + \frac{\delta(M_X^2 - M_0^2)}{M_2^2 - M_0^2 + i\delta}$$
$$= \frac{1}{2\pi i}\left[\frac{1}{(M_X^2 - M_0^2 + i\delta)\,(M_X^2 - M_2^2 + i\delta)} - \frac{1}{(M_X^2 - M_0^2 - i\delta)\,(M_X^2 - M_2^2 - i\delta)}\right]$$

- Combined with the previous diagrams, these cancel $\frac{\log \alpha}{\alpha}$ divergences
- Some $\log^2 \alpha$ divergences still left

Final state: Cut gluon loops



and

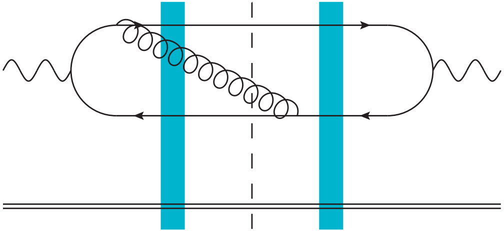


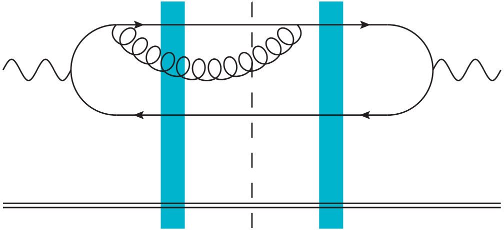
- Can be calculated analytically without additional Feynman/Schwinger integrals
- Contain IR and $\log^2 \alpha$ divergence
- Self-energy diagrams: IR $\to$ UV divergences
- UV divergences: cancel with divergences from initial state + gluon loop over shock wave
- Remaining $\log^2 \alpha$ divergence cancels with the other final-state diagrams

$\Rightarrow$ All of the divergences cancelled: finite result!

Remaining finite pieces

- Cross-terms with gluon emission from initial and final states
- Finite:
  - Cut regulates UV region
  - Shock wave regulates IR region
- Work-in-progress





Conclusions and future considerations

- Diffractive structure functions are a good probe for saturation effects
- Previously: leading log Q^2 calculated at NLO
- We are completing the full NLO calculation
  - Explicit cancellation of divergences
  - One finite contribution still being calculated
- Future:
  - Numerical implementation of the full NLO result
  - Comparisons to the existing HERA data and predictions for the EIC