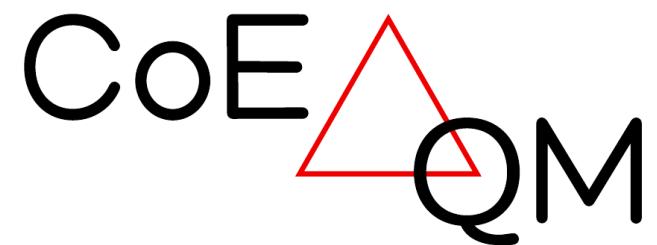


Single inclusive diffraction in DIS: From HERA to EIC

Anh-Dung Le

(Based on the work in progress with T. Lappi and H. Mäntysaari)

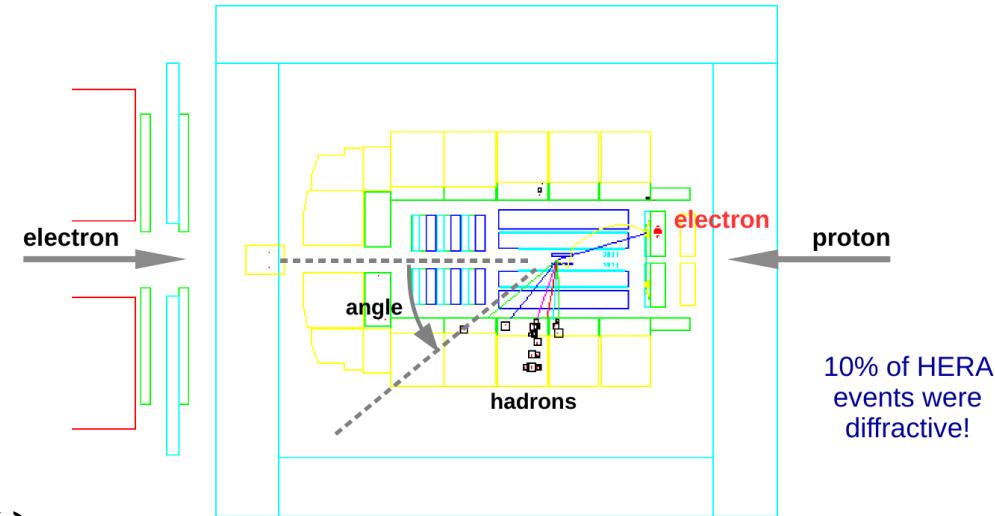
DIS2023, Michigan State University



Diffractive dissociation in DIS

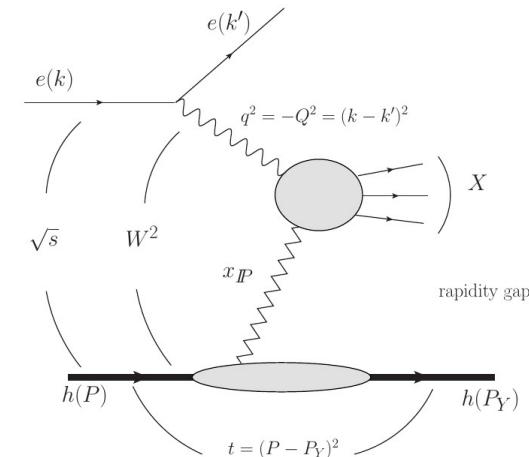
▷ Single inclusive diffraction:

- Hadron (p/A) intact
- Dissociated inclusive system X
- Large rapidity gap



▷ Striking observation at HERA ($\sim 10\%$)

- Various available data



▷ One of key measurements at EIC

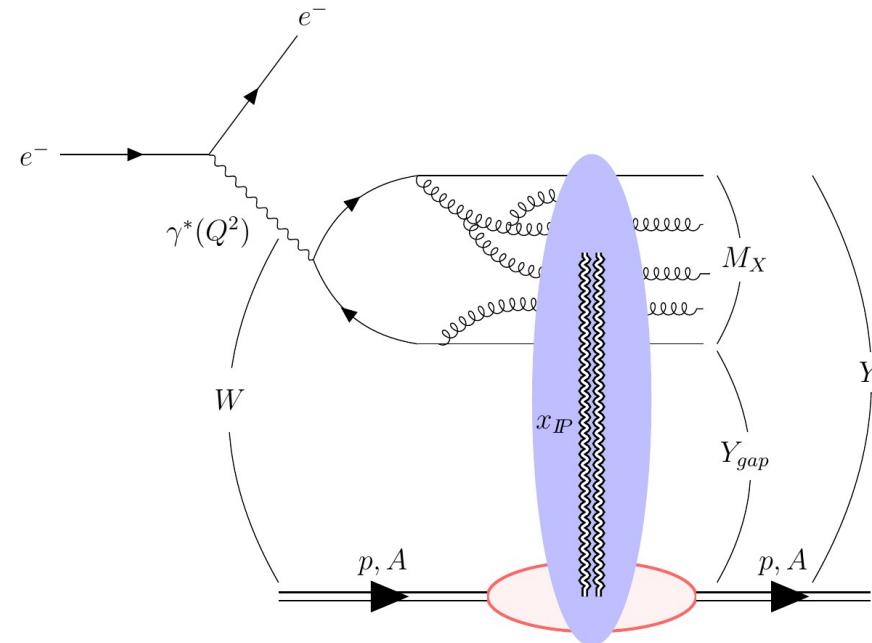
- Sensitive to gluon saturation

(EIC White Paper (2012))

Outlines

- **Dipole picture of diffraction**
 - Kovchegov - Levin evolution
- **Scattering off proton: comparison to HERA data**
- **Scattering off nucleus: predictions for EIC**

Dipole picture



$$x_{IP} = \frac{Q^2 + M_X^2}{Q^2 + W^2}, \quad \beta = \frac{Q^2}{Q^2 + M_X^2}$$

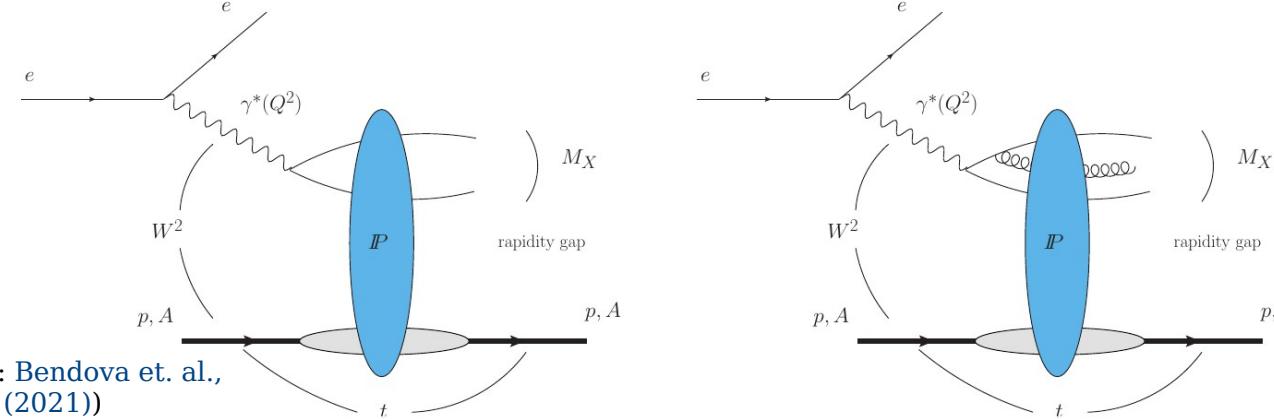
$$x = x_{IP} \beta$$

- ▷ M_X : dissociation of the quantum Fock state of the virtual photon ($q\bar{q} + q\bar{q}g + \dots$)
- ▷ Dipole factorization:

$$\sigma_{tot(T/L)}^{y^* h}(x, Q^2) = |\Psi_{T/L}^{y^* \rightarrow q\bar{q}}|^2 \otimes \sigma_{tot}^{q\bar{q} h}(x, r)$$

$$\frac{d \sigma_{D,(T/L)}^{y^* h}(x_{IP}, \beta, Q^2)}{d \ln(1/\beta)} = |\Psi_{T/L}^{y^* \rightarrow q\bar{q}}|^2 \otimes \frac{d \sigma_D^{q\bar{q} h}(x_{IP}, \beta, r)}{d \ln(1/\beta)}$$

Dipole picture: Golec-Biernat-Wusthoff (GBW) result



e. g.,
 Wusthoff, PRD 56, 4311 (1997);
 Golec-Biernat and Wusthoff, PRD 60, 114023 (1999);
 Munier and Shoshi, PRD 69, 074022 (2004);
 Kowalski et. al., PRC 78, 045201 (2008);
 ...

- ▷ Gluon emissions are suppressed by power of α_s
 $\Rightarrow (\mathbf{q}\mathbf{q} + \mathbf{q}\mathbf{q}\mathbf{g})$ contributions are dominant at medium β .
- ▷ **qq** component $\sim N^2$ (N : forward dipole-hadron elastic amplitude)
- ▷ **qqg** component:
 - Known at exact kinematics (NLO) (G. Beuf et. al., PRD 106, 094014 (2022))
 - Well-known, and widely used, result at large Q^2 (Wusthoff's limit):

$$F_{q\bar{q},T}^{D(3)} \sim (2N - N^2)^2 \quad (\text{gluon dipole-hadron elastic cross section})$$

$N(\mathbf{r}, x, \mathbf{b})$: solution to Balitsky-Kovchegov (BK) evolution (**b**: impact parameter)

$$\frac{\partial N(\mathbf{r}, x, \mathbf{b})}{\partial \ln(1/x)} = K(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) \otimes [N(\mathbf{r}_1, x, \mathbf{b}_1) + N(\mathbf{r}_2, x, \mathbf{b}_2) - N(\mathbf{r}, x, \mathbf{b}) - N(\mathbf{r}_1, x, \mathbf{b}_1)N(\mathbf{r}_2, x, \mathbf{b}_2)]$$

➡ Dipole evolution kernel

Balitsky, NPB 463, 99 (1996)
 Kovchegov, PRD 60, 034008 (1999)

Dipole picture: Kovchegov - Levin (KL) evolution

- At small β , need to resum soft gluon contributions \rightarrow KL evolution:

$$\frac{\partial N_I(\mathbf{r}, Y, Y_0, \mathbf{b})}{\partial Y} = K(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) \otimes [N_I(\mathbf{r}_1, Y, Y_0, \mathbf{b}_1) + N_I(\mathbf{r}_2, Y, Y_0, \mathbf{b}_2) - N_I(\mathbf{r}, Y, Y_0, \mathbf{b}) - N_I(\mathbf{r}_1, Y, Y_0, \mathbf{b}_1)N_I(\mathbf{r}_2, Y, Y_0, \mathbf{b}_2)]$$

Kovchegov and Levin, NPB 577, 221 (2000)

Kovchegov, PLB 710, 192 (2012)

- $N_D(\mathbf{r}, Y, Y_0, \mathbf{b}) = 2N(\mathbf{r}, Y, \mathbf{b}) - N_I(\mathbf{r}, Y, Y_0, \mathbf{b})$: diffractive dipole-target cross section with a minimal gap Y_0 (per impact parameter).
- Initial condition: $N_D(\mathbf{r}, Y=Y_0, Y_0, \mathbf{b}) = N^2(\mathbf{r}, Y_0, \mathbf{b})$.
- One needs to BK evolve the forward elastic amplitude N to Y_0 from $Y = 0$ (or $x = x_{init} = 0,01$).

- Progress:

- Analytical: ADL et. al., PRD 104, 034026 (2021); Mueller and Munier, PRL 121, 082001 (2018) & PRD 98, 034021 (2018); Contreras et. al., EPJC 78, 699 (2018).
- Numerical: ADL, PRD 104, 014014 (2021); Levin and Lublinsky, PLB 521, 233 (2001), EPJC 22 (2002) & NPA 712, 95 (2002).
- Confronting experimental data: this study !!

- Running coupling: only known NLO effect for KL up-to-date

$$K(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) \rightarrow K_{rc}^{Bal}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \frac{N_C \alpha_s(r^2)}{2\pi^2} \left[\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right],$$

Balitsky, PRD 75, 014001 (2007)

$$\alpha_s(r^2) = \frac{12\pi}{(33-2N_f)\ln\frac{4C^2}{r^2\Lambda_{QCD}^2}}$$

Diffractive observables

- ▷ Diffractive cross section

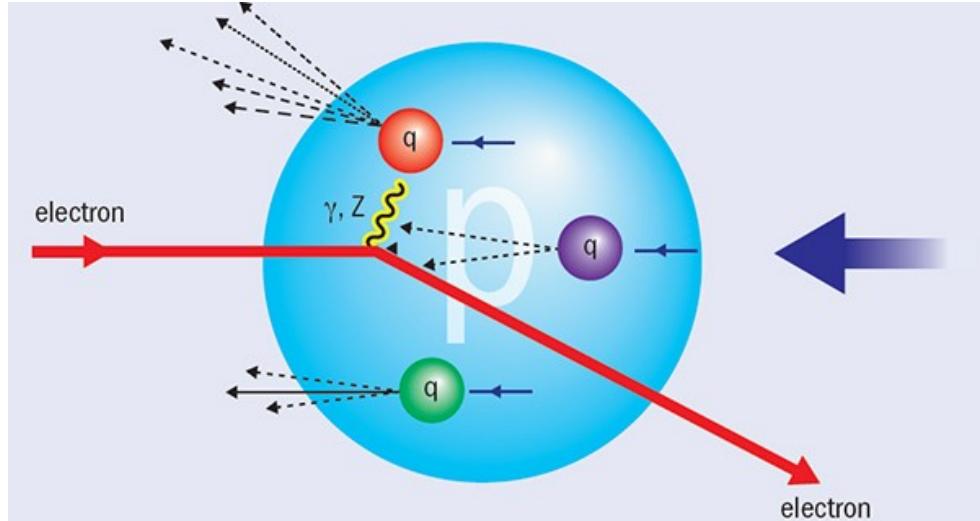
$$\frac{d \sigma_D^{y^* h}}{d M_x} = \frac{2 M_X}{Q^2 + M_X^2} \left[\frac{d \sigma_{D(T)}^{y^* h}}{d \ln(1/\beta)} + \frac{d \sigma_{D(L)}^{y^* h}}{d \ln(1/\beta)} \right]$$

- ▷ Diffractive structure function

$$x_{IP} F_2^{D(3)} = x_{IP} F_L^{D(3)} + x_{IP} F_T^{D(3)}, \quad x_{IP} F_{L/T}^{D(3)} = \frac{Q^2}{4 \pi^2 \alpha_{em}} \frac{d \sigma_{D(L/T)}^{y^* h}}{d \ln(1/\beta)}$$

- ▷ Diffractive reduced cross section ($y = Q^2/(xs)$): inelasticity)

$$\sigma_{red}^{D(3)} = F_2^{D(3)} - \frac{y^2}{1 + (1-y)^2} F_2^{D(3)}$$



Scattering off proton: Confronting the HERA data

Impact parameter dependency

- ▷ \mathbf{b} -dependence is factorized in both total and diffractive cross-sections:

$$\sigma_{tot}^{q\bar{q}h} = 2 \int d^2 \mathbf{b} N(\mathbf{r}, Y, \mathbf{b}) = 2 \int d^2 \mathbf{b} N(r, Y) T_p(\mathbf{b}) = \sigma_0 N(r, Y)$$

$$\sigma_D^{q\bar{q}h} = \int d^2 \mathbf{b} N_D(\mathbf{r}, Y, Y_0 \mathbf{b}) = N_D(r, Y, Y_0) \underbrace{\int d^2 \mathbf{b} T_p^2(\mathbf{b})}_{\stackrel{\text{def}}{=} \sigma_0^D}$$

- σ_0 : effective transverse area, from fit to HERA inclusive data
([Lappi and Mäntysaari, PRD 88, 114020 \(2013\)](#))
- σ_0^D : depends on proton shape
- $N(r, Y)$ and $N_D(r, Y, Y_0)$ obey BK and KL evolutions, resp.

- ▷ I.P. profile for proton:

$$T_p(\mathbf{b}) = \frac{\Gamma\left(\frac{1}{\omega}, \frac{2\pi b^2}{\sigma_0 \omega}\right)}{\Gamma\left(\frac{1}{\omega}\right)}$$

- ω : steepness parameter
- $\omega = 1$: gaussian; $\omega = 0$: hard disk (step function)

Initial condition for the evolution

- Initial condition for the first-step (BK) evolution at $Y=0$ ($x=x_{\text{init}} = 0,01$):

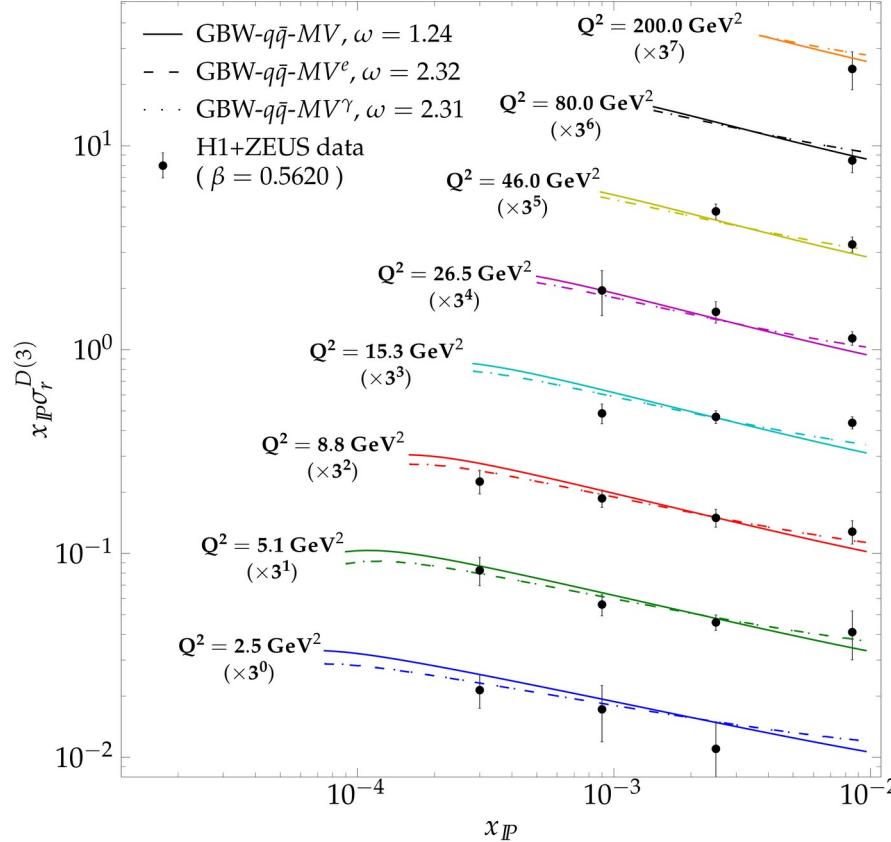
$$\mathcal{N}(r) = 1 - \exp \left[-\frac{(r^2 Q_{s0}^2)^\gamma}{4} \ln \left(e \cdot e_c + \frac{1}{r \Lambda_{\text{QCD}}} \right) \right]$$

- (original) **MV**: $\gamma = e_c = 1$, Q_{s0} is free param.
- MVe**: $\gamma = 1$, Q_{s0} and e_c are free params.
- MV $^\gamma$** : $e_c = 1$, Q_{s0} and are free params.

- Free parameters obtained from fits (light quarks only) to HERA inclusive data

Parametrization	$Q_{s0}^2(\text{GeV}^2)$	γ	e_c	$\sigma_0/2$ (mb)	C^2
MV	0.104	1	1	18.81	14.5
MV^γ	0.159	1.129	1	16.35	7.05
MVe	0.060	1	18.9	16.36	7.2

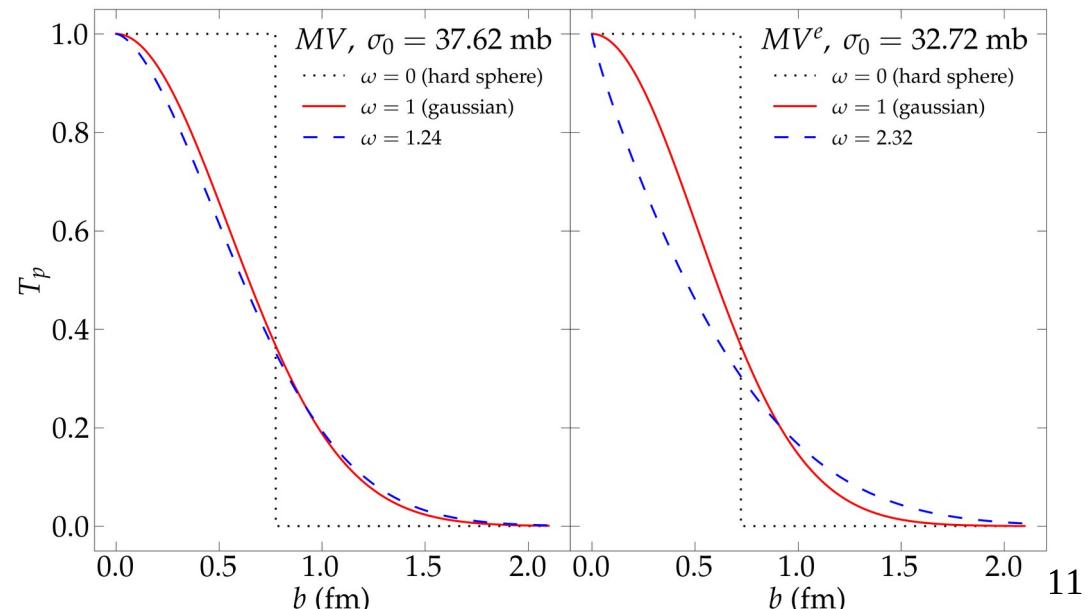
Proton shape: Optimal steepness



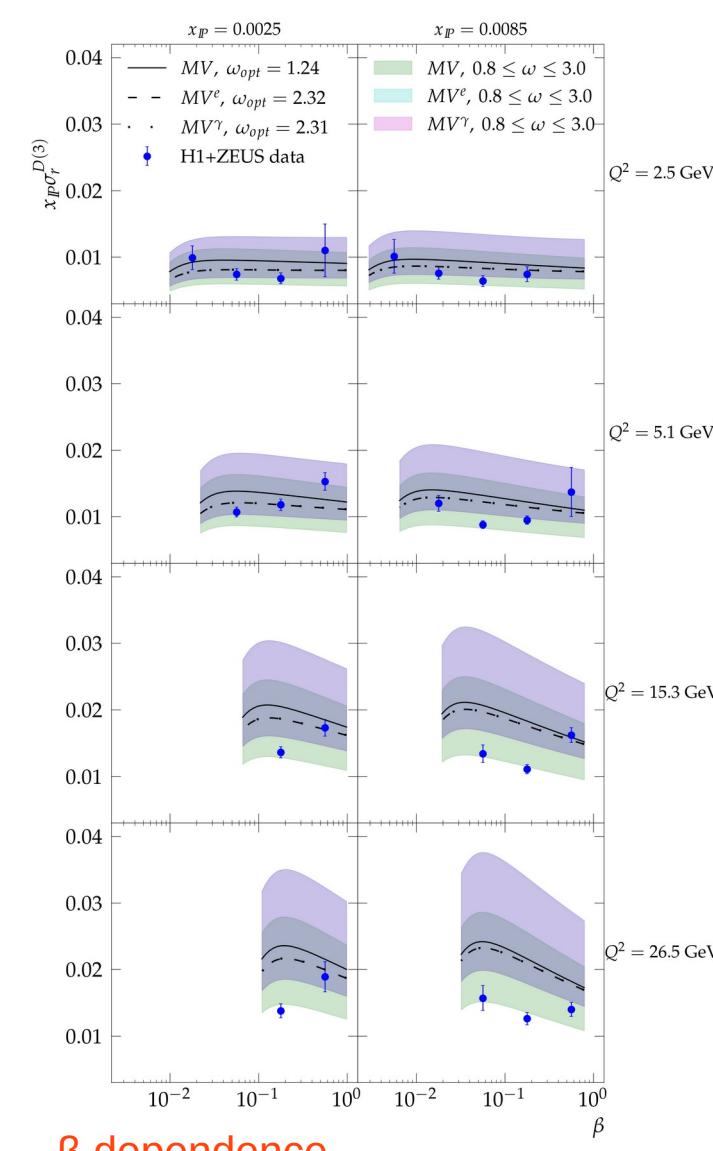
$$T_p(\mathbf{b}) = \frac{\Gamma\left(\frac{1}{\omega}, \frac{2\pi b^2}{\sigma_0 \omega}\right)}{\Gamma\left(\frac{1}{\omega}\right)}$$

- ▷ **GBW-qq** result is fitted to diffractive HERA combined data for $\beta > 0.5$ (24 data points) using the chosen proton shape.
- ▷ Optimal steepness:

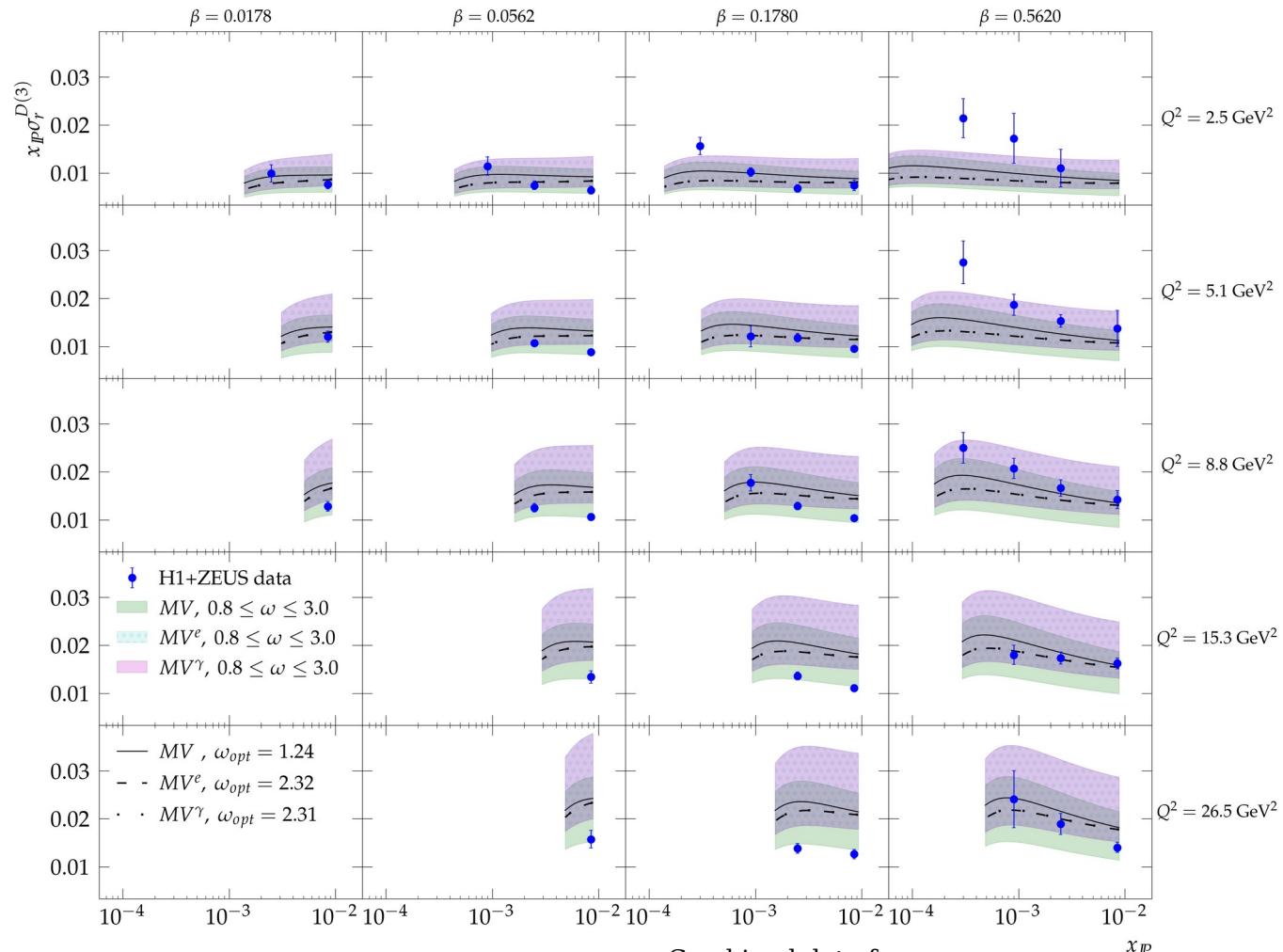
- $\omega = 1.24$ ($\chi^2/\text{dof} \approx 1.87$) for **MV**
- $\omega = 2.32$ ($\chi^2/\text{dof} \approx 1.08$) for **MV^e**
- $\omega = 2.31$ ($\chi^2/\text{dof} \approx 1.09$) for **MV^{\gamma}**



Comparison to HERA combined data

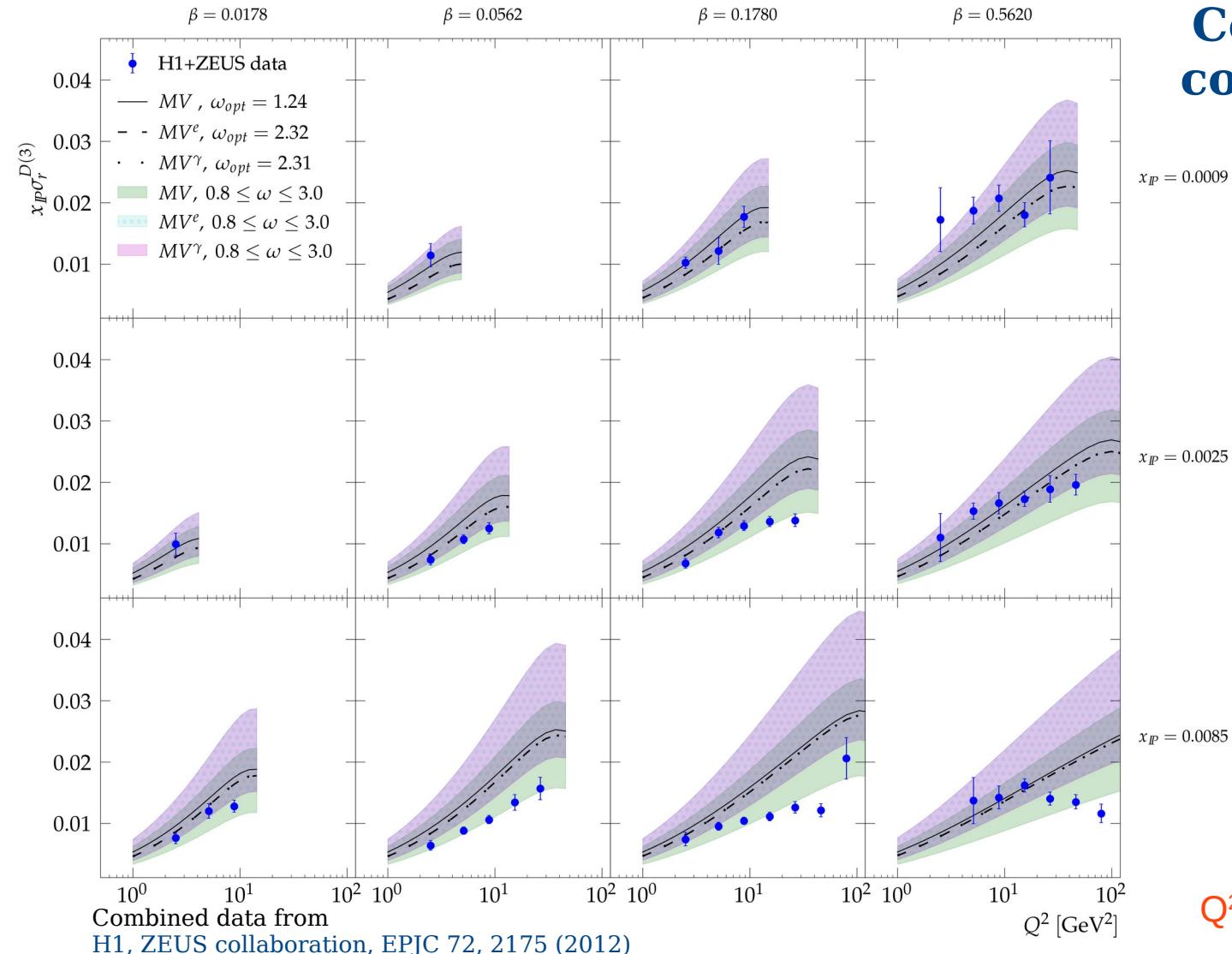


x_{IP} dependence



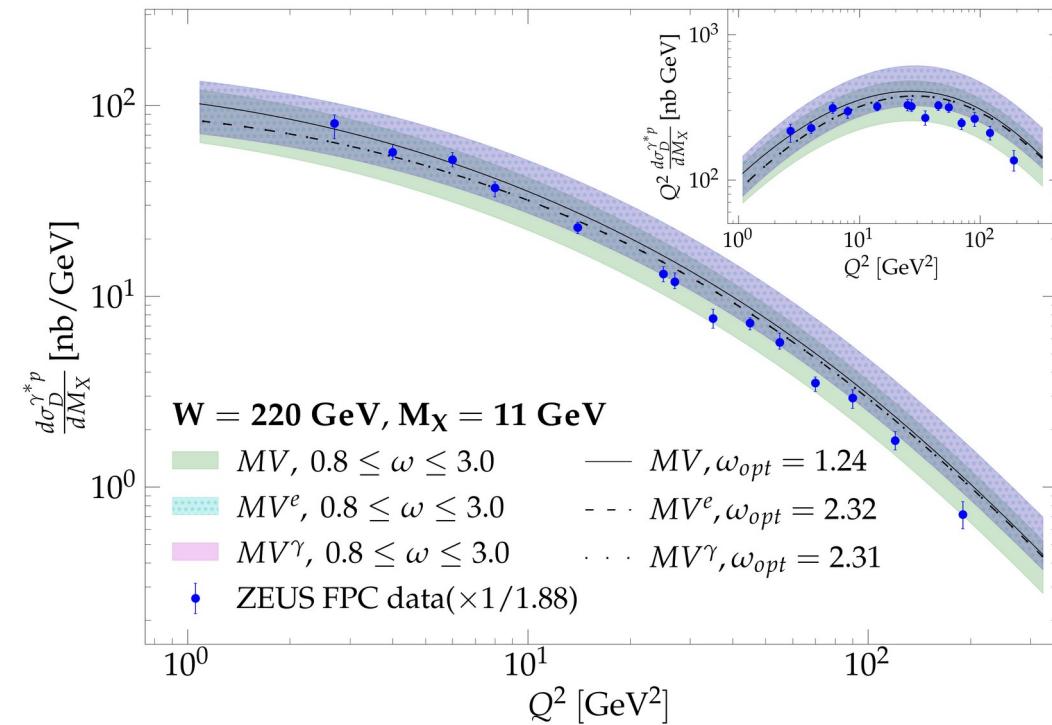
Combined data from
H1, ZEUS collaboration, EPJC 72, 2175 (2012)

Comparison to HERA combined data (cont.)



Q^2 dependence

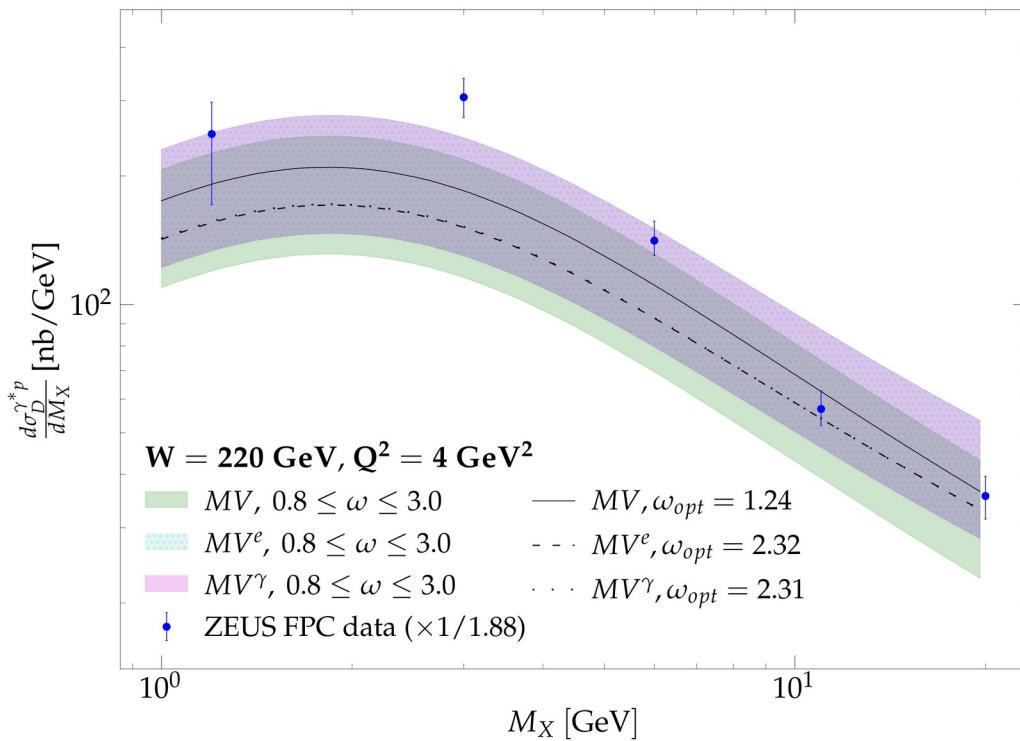
- Some remarks:
- Good description for data with $\beta \lesssim 0.1$.
 - The numerics appear to overestimate toward higher Q^2 .



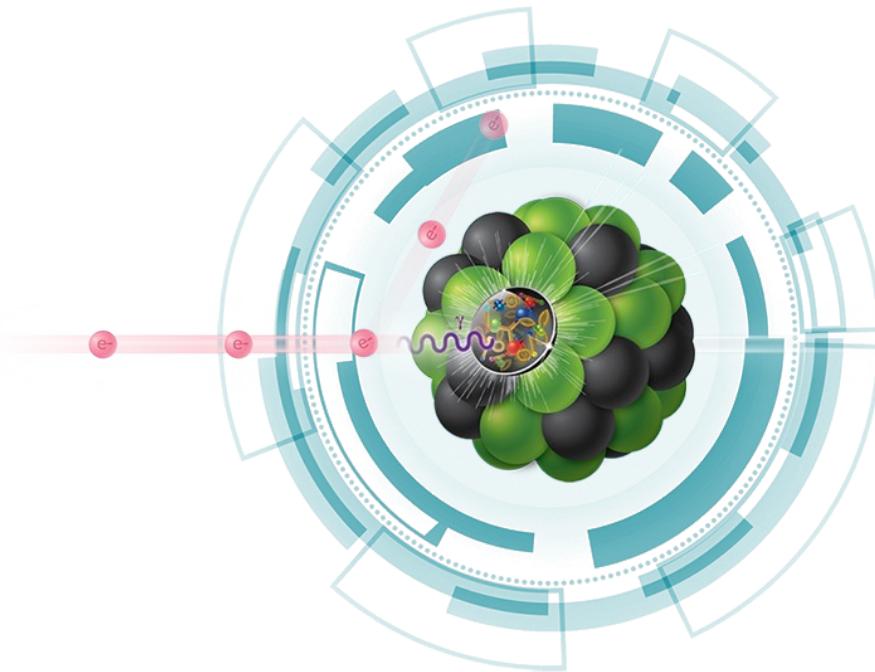
Comparison to ZEUS FPC data

$$\frac{d\sigma_D^{\gamma^* p}}{dM_X}$$

FPC data from
ZEUS collaboration, NPB 713, 3 (2005) & NPB 800, 1 (2008)



- FPC data is corrected for target dissociation by a scaling factor.
- Q^2 dependence reflects the leading-twist behavior.
- Good agreement at high mass.



Scattering off nucleus: EIC prediction

Impact parameter treatment and initial condition

- ▷ Neglect the dependence on the orientation of transverse vectors (**r** and **b**)
- ▷ BK & KL evolutions are solved for each **b** independently.
- ▷ Initial condition (optical Glauber):

$$N_A(r, b) = 1 - \exp \left[-AT_A(b) \frac{\sigma_0}{2} \frac{(r^2 Q_{s0}^2)^{\gamma}}{4} \ln \left(e \cdot e_c + \frac{1}{r \Lambda_{QCD}} \right) \right]$$

- $T_A(b)$: Wood-Saxon I.P. profile, $\int d^2\mathbf{b} T_A(b) = 1$.
- ▷ Unphysical increase of gluon density at $|\mathbf{b}| \gtrsim b_c \sim 6.3$ fm ([Lappi and Mäntysaari, PRD 88, 114020 \(2013\)](#)) → Assuming no nuclear (gluon) modification for total cross section at such large $|\mathbf{b}|$:

$$N_A(r, Y, b > b_c) = AT_A(b) \frac{\sigma_0 N(r, Y)}{2}$$

$$N_{D,A}(r, Y, Y_0, b > b_c) = A^2 T_A^2(b) \frac{\sigma_0^2 N_D(r, Y, Y_0)}{4}$$



impulse approximation (IA) treatment in this dilute regime

Quantifying the nuclear effect

- Usually, the following ratio is used: $\frac{F_{2,A}^{D(3)}}{AF_{2,p}^{D(3)}}$ (e.g., Kowalski et. al., PRC 78, 045201 (2008))
 - Analogous to $F_2(\text{eA})/\text{[AF}_2(\text{ep})]$
(recall: $\text{AF}_2(\text{ep})$ = the *impulse approximation (IA)* for F_2)
 - Nuclear form factor \otimes nuclear gluonic effect
- To disentangle the effects of nuclear form factor and of the nuclear gluonic content \rightarrow the full/IA ratio for diffraction:

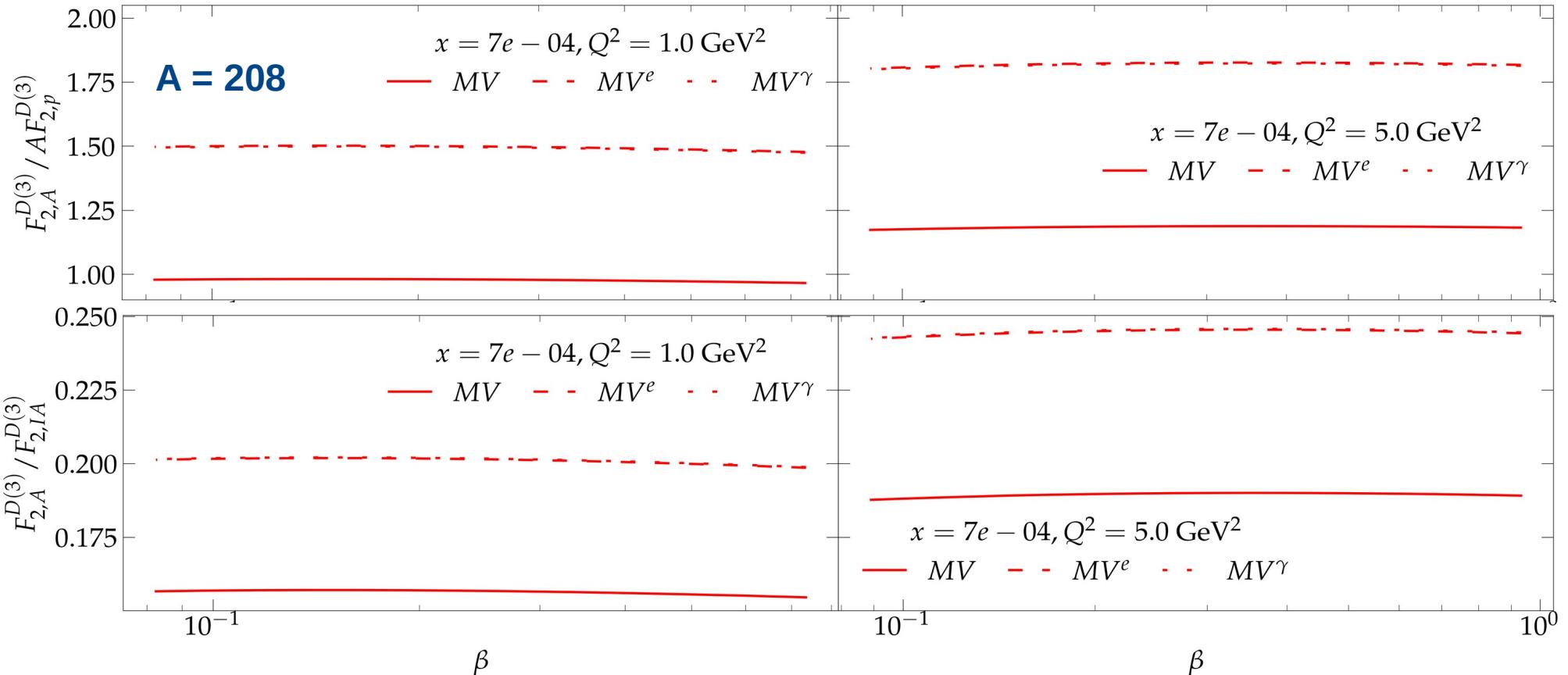
$$R_{IA} = \frac{F_{2,A}^{D(3)}}{F_{2,IA}^{D(3)}} = \frac{d\sigma_D^{y^* A}/dM_X^2}{d\sigma_{D,IA}^{y^* A}/dM_X^2}$$

IA approximation:

$$\sigma_{D,IA}^{q\bar{q}A} = \frac{d\sigma_D^{q\bar{q}p}}{d|\mathbf{t}|}(|\mathbf{t}|=0) \times \Phi_A = \frac{N(r, Y, Y_0) \sigma_0^2}{4} A^2 \int d^2 \mathbf{b} T_A^2(b)$$

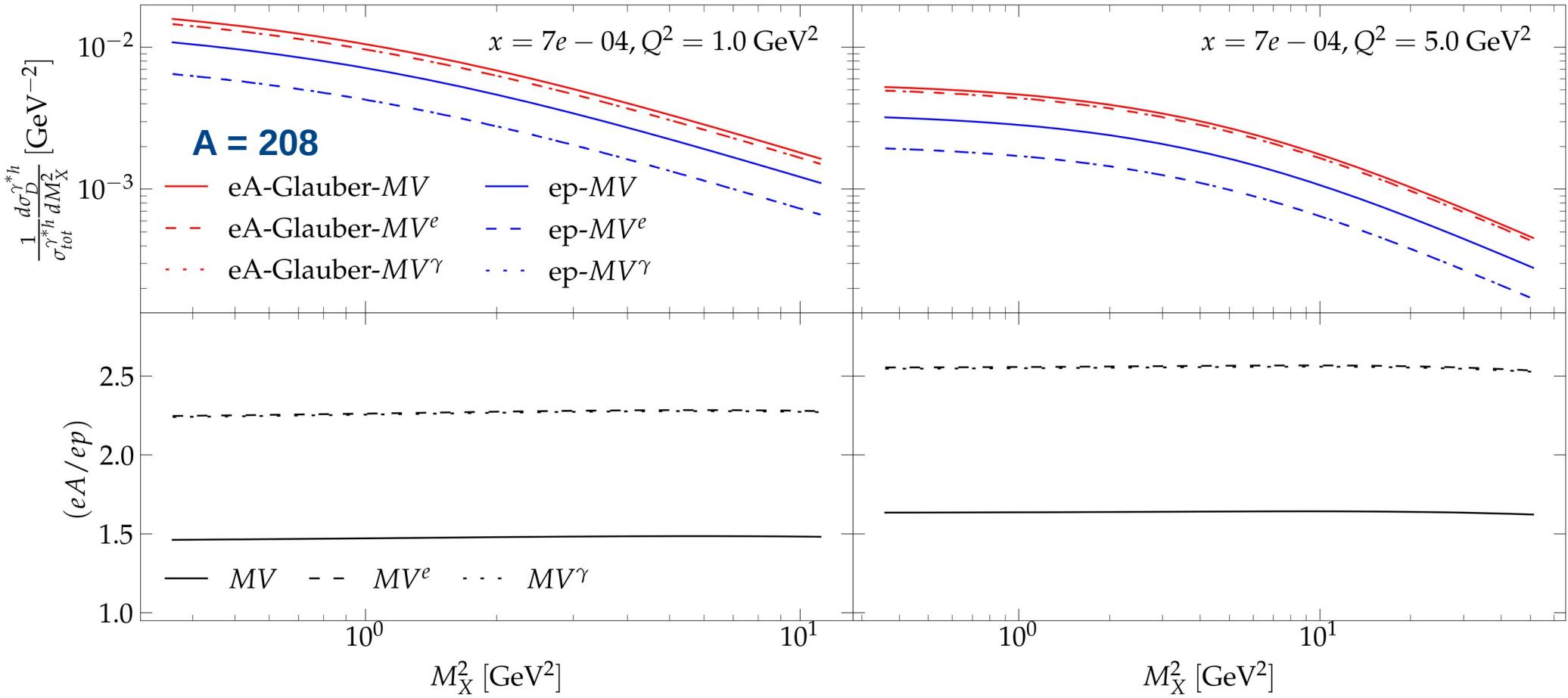
$= \frac{N_D(r, Y, Y_0)}{4\pi} \left| \int d^2 \mathbf{b} T_p(b) \right|^2$
 $\Phi_A \equiv A^2 \int d|\mathbf{t}| \left| \int d^2 \mathbf{b} e^{-i \mathbf{b} \cdot \Delta} T_A(b) \right|^2$
 $(|\mathbf{t}| = -\Delta^2)$

Diffractive structure function (prelim.)



- Very weak dependence on β .
- Gluon shadowing (saturation?).
- Nuclear enhancement (or small suppression) when including the nuclear form factor effect.

Diffractive mass spectra (prelim.)



→ Large nuclear enhancement compared to the result from the (qq + qqg) saturation model

(Kowalski et. al., PRC 78, 045201 (2008); EIC White Paper (2012))

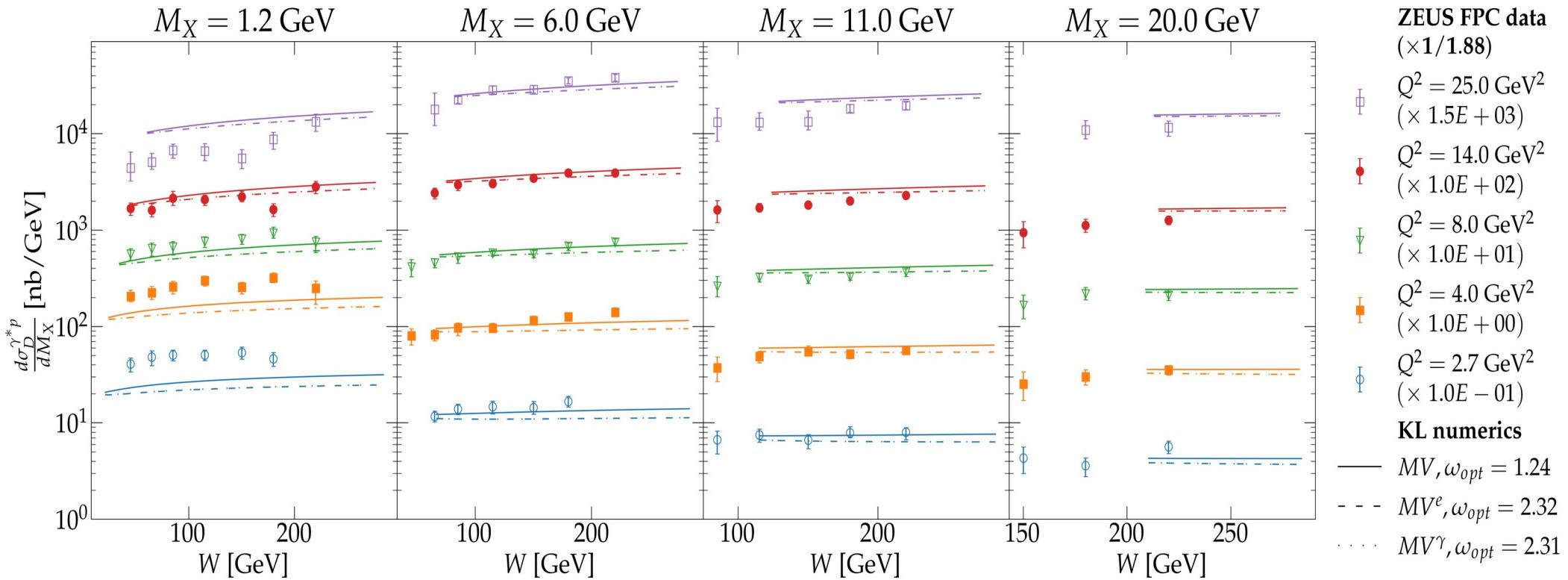
Final remarks

- ▷ Calculation of DIS diffractive observables with rcKL evolution, with free parameters obtained from fits to HERA inclusive data.
- ▷ KL evolution provides a reasonably good description to HERA data, up to the uncertainty in proton shape, especially for $\beta \lesssim 0.1$.
 - HERA diffractive data prefer the I.P. profile steeper than the gaussian shape.
- ▷ Some analyses for DDIS on nuclear target at the EIC accessible kinematics
 - Large nuclear modification effect (gluon shadowing + nuclear form factor)
- ▷ Work in progress!

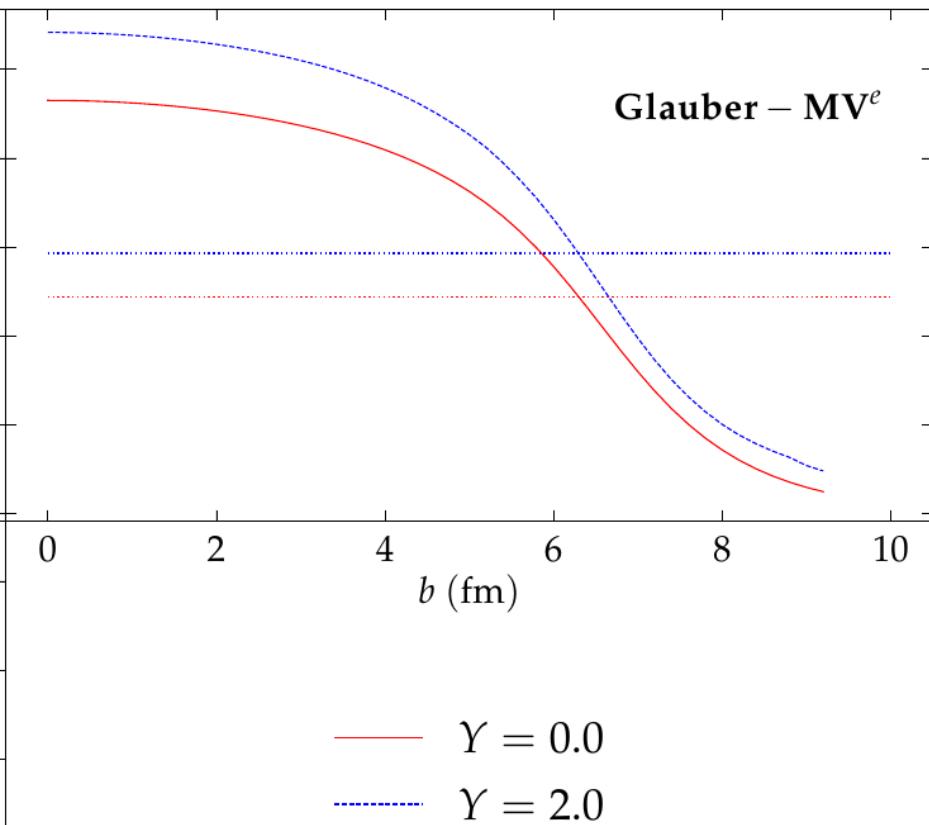
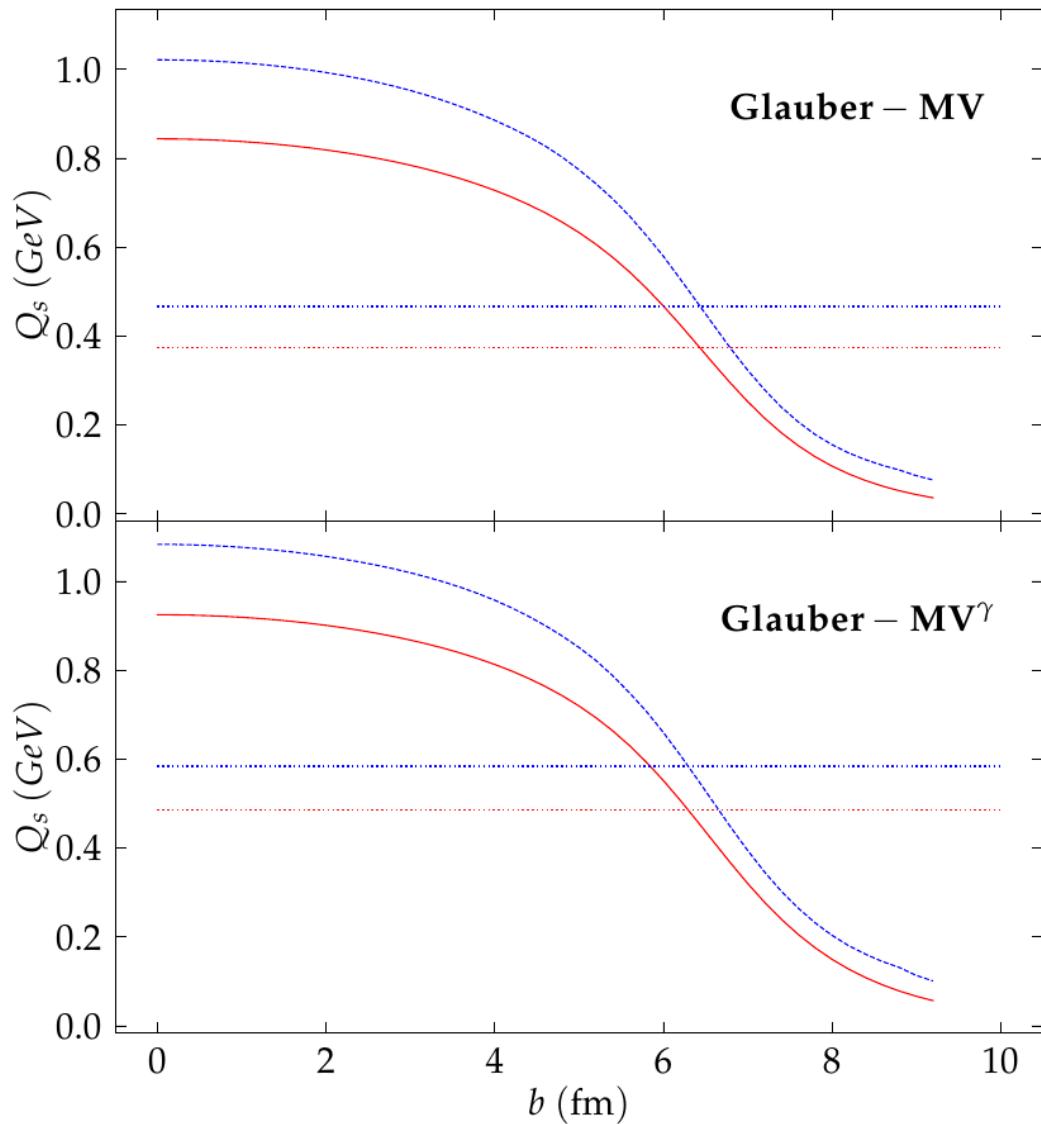
Back-up

Comparison to ZEUS FPC data (cont)

$$\frac{d\sigma_D^{\gamma^* p}}{dM_x}$$



FPC data from
ZEUS collaboration, NPB 713, 3 (2005) & NPB 800, 1 (2008)



b-dependence of saturation scale
(horizontal lines are of proton)

$$\tilde{T}^2(|\mathbf{t}|) = \left| \int d^2 b e^{-i\mathbf{b}\cdot\Delta} T(b) \right|^2$$

$\sigma_0 = 32.72 \text{ mb}$

