

## Back-to-back dijet production in DIS: small- $x$ TMD factorization at NLO

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International Workshop on Deep Inelastic Scattering  
and Related Subjects

March 29th, 2023

Based on

- (1) [2108.06347](#) [*JHEP 11 (2021) 222*]
- (2) [2208.13872](#) [*JHEP 11 (2022) 169*]
- (3) [2304.XXXX](#) [*in progress*]

In collaboration with

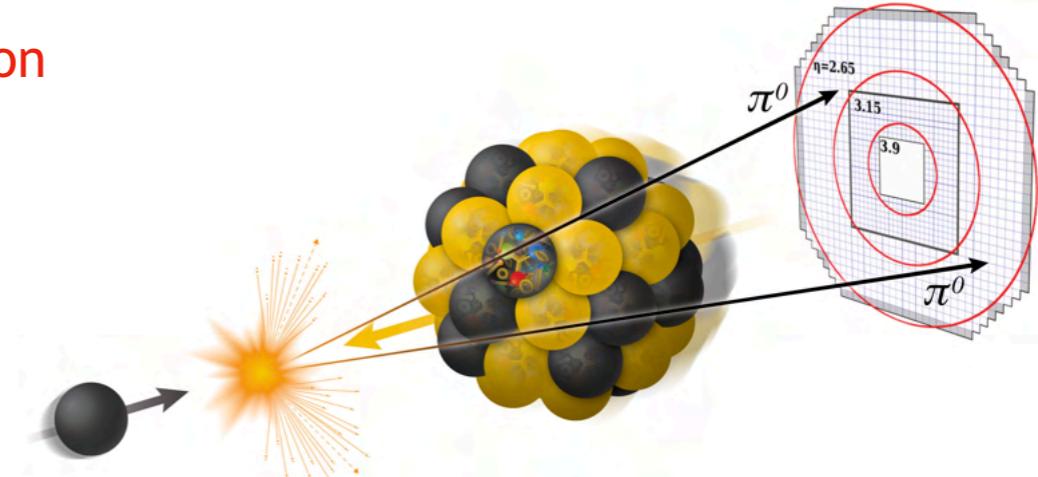
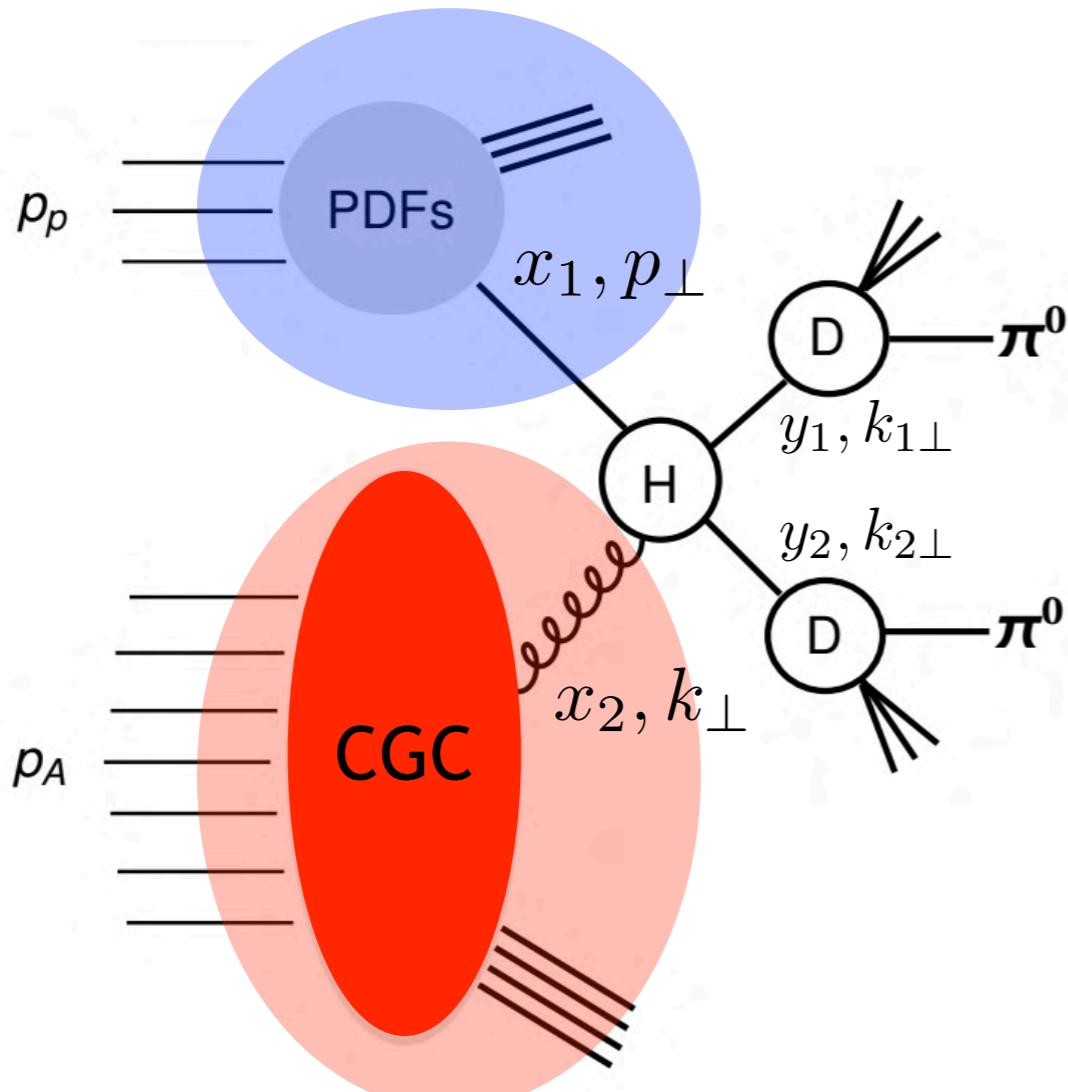
Paul Caucal (Nantes)  
Björn Schenke (BNL)  
Tomasz Stebel (Jagiellonian)  
Raju Venugopalan (BNL)

# Forward dihadrons in proton-nucleus collisions

Azimuthal correlations as a probe for gluon saturation

D. Kharzeev, E. Levin, L. McLerran (2005)

Hybrid dilute-dense formalism



$$x_1 = \frac{1}{\sqrt{s}}(k_{1\perp} e^{y_1} + k_{2\perp} e^{y_2}) \sim 1$$

$$x_2 = \frac{1}{\sqrt{s}}(k_{1\perp} e^{-y_1} + k_{2\perp} e^{-y_2}) \ll 1$$

$$p_{\perp} \sim \Lambda_{\text{QCD}}$$

$$k_{\perp} \sim Q_s(x_2)$$

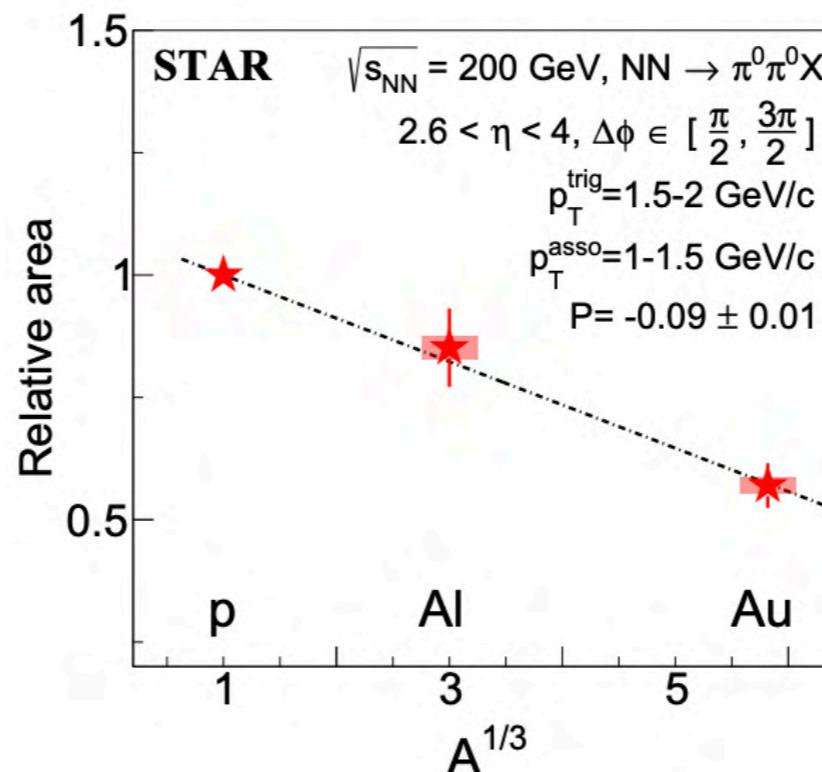
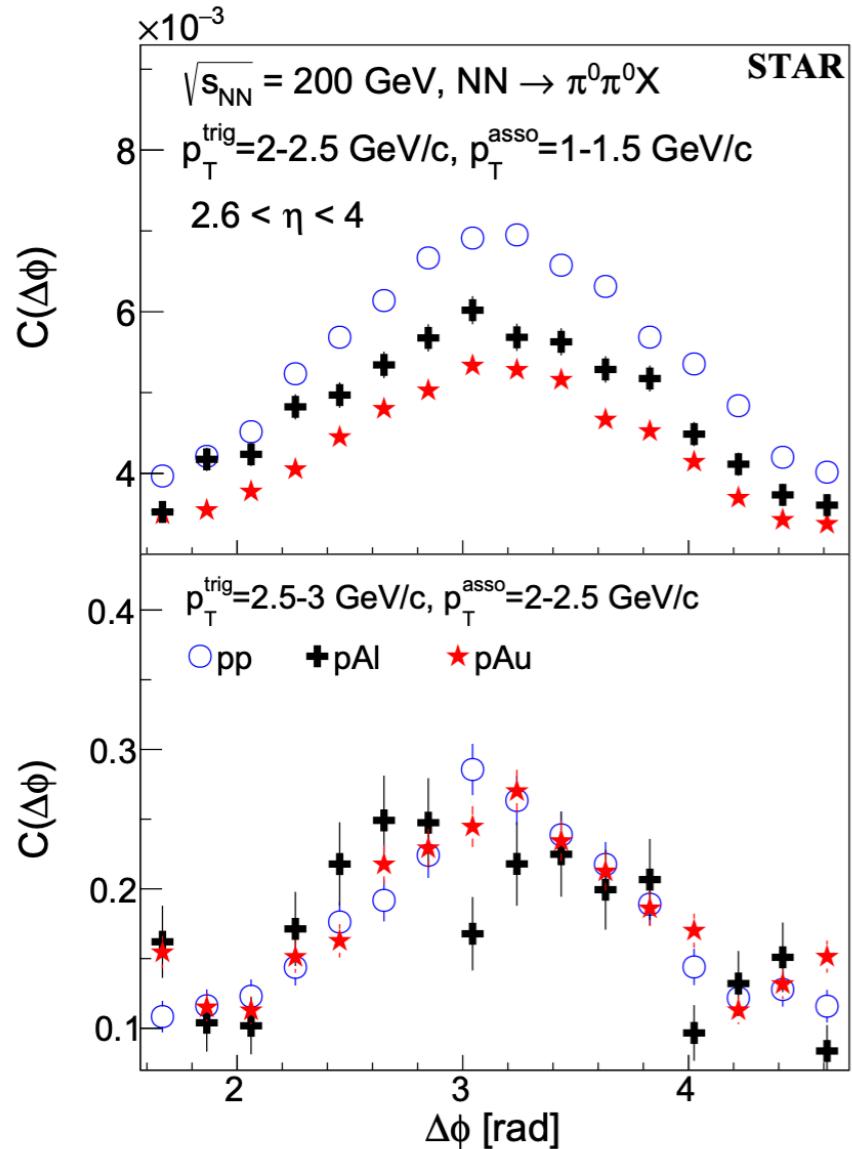
$$Q_s(x_2) \gg \Lambda_{\text{QCD}}$$

Dihadron momentum  
imbalance  $\sim Q_s$

# Forward dihadron at RHIC

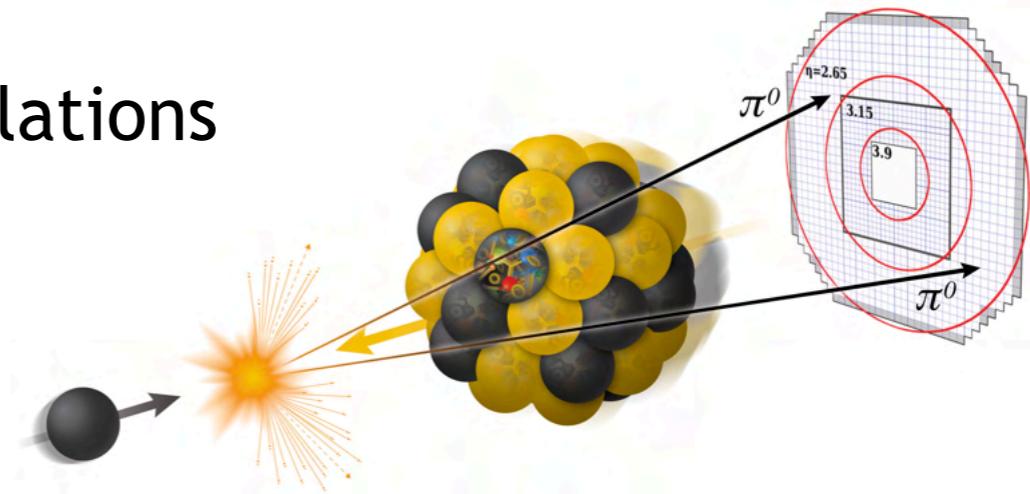
## Signatures of saturation in azimuthal correlations

STAR Collaboration. *Phys. Rev. Lett.* 129, 092501 (2022)



Suppression characteristic  
of saturation

$$Q_s^2 \propto A^{1/3}$$



Xiaoxuan Chu and Elke Aschenauer  
looking at the STAR detector

## Status:

Lots of recent progress in understanding saturation physics in the precision era (NLO) by many of you in this very audience

The more differential the process (e.g. two-particle correlations) the harder the calculations (both analytically and numerically)

## Our goal:

Promote two-particle observables in saturation to NLO

## Observable:

Inclusive Dijet production in DIS

## Why?

Will be measured at EIC and theoretically cleaner

# Outline

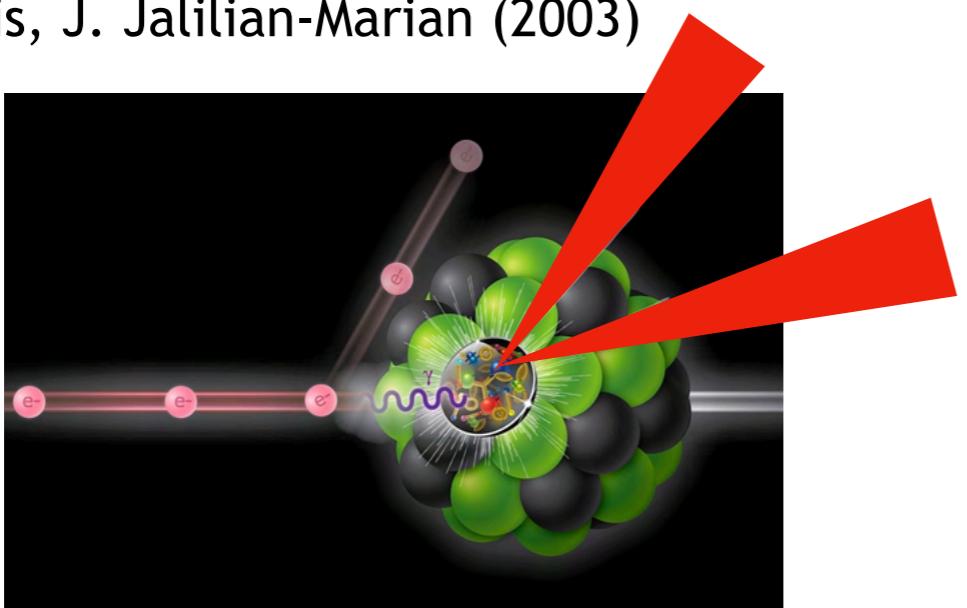
- Dijet production in DIS at small-x at NLO  
P. Caucal, FS, R. Venugopalan. [2108.06347 \[JHEP 11 \(2021\) 222\]](#)
- The back-to-back limit: Sudakov and the small-x gluon TMD  
P. Caucal, FS, B. Schenke ,R. Venugopalan. [2208.13872 \[JHEP 11 \(2022\) 169\]](#)
- Complete small-x TMD factorization at NLO  
P. Caucal, FS, B. Schenke, T. Stebel ,R. Venugopalan. 2304? XXXX
- Outlook



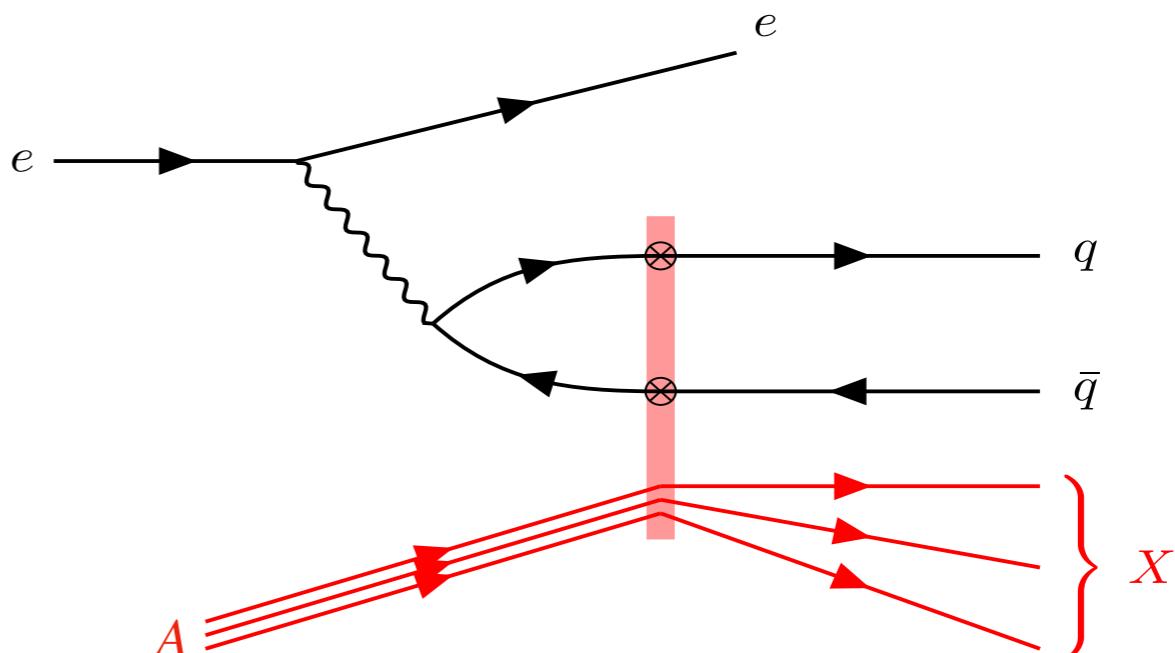
Paul Caucal Björn Schenke Tomasz Stebel Raju Venugopalan

# Dijet production in DIS at LO

F. Gelis, J. Jalilian-Marian (2003)



No need for hybrid factorization. Pin down photon kinematics from electron. Only one channel.



## Dijet differential cross-section:

$$\frac{d\sigma^{\gamma_\lambda^* + A \rightarrow q\bar{q} + X}}{d^2 k_{1\perp} d^2 k_{2\perp} d\eta_1 d\eta_2} \propto \int d^8 \mathbf{X}_\perp e^{-i \mathbf{k}_{1\perp} \cdot (\mathbf{x}_\perp - \mathbf{x}'_\perp)} e^{-i \mathbf{k}_{2\perp} \cdot (\mathbf{y}_\perp - \mathbf{y}'_\perp)} \\ \times \langle \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp \mathbf{x}'_\perp) \rangle_Y \mathcal{R}^\lambda(\mathbf{x}_\perp - \mathbf{y}_\perp, \mathbf{x}'_\perp - \mathbf{y}'_\perp)$$

$$\Xi_{\text{LO}}(x_\perp, y_\perp; y'_\perp x'_\perp) = 1 - S^{(2)}(x_\perp, y_\perp) - S^{(2)}(y'_\perp, x'_\perp) + S^{(4)}(x_\perp, y_\perp; y'_\perp, x'_\perp)$$

**dipoles**      **quadrupole**

# Implicitly contain saturation scale $Q_s$

## First numerical evaluation

H. Mäntysaari, N. Mueller, FS, B. Schenke (PRL 2019)

# Dijet production in DIS at NLO

Our calculation in a nutshell

P. Caucal, FS, and R. Venugopalan. *JHEP* 11 (2021) 222

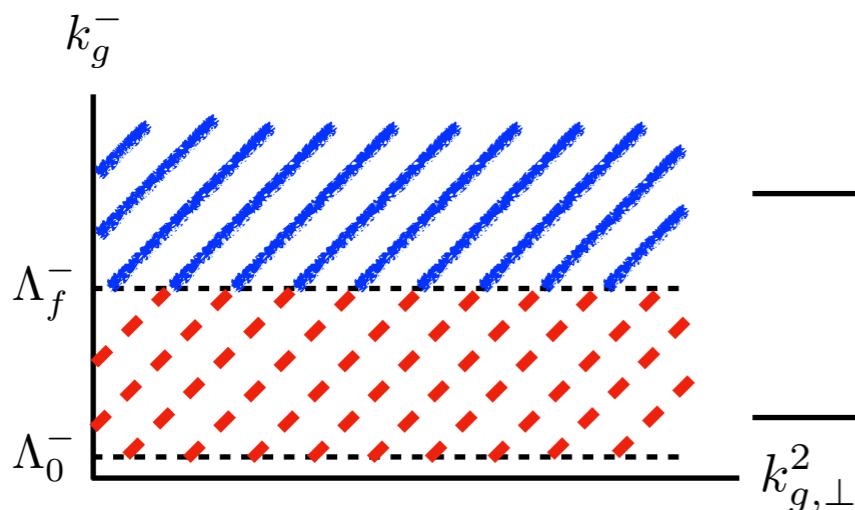
- Covariant perturbation theory Feynman rules in momentum space

Dimensional regularization +  
longitudinal momentum cut-off  
+ small-R cone algorithm

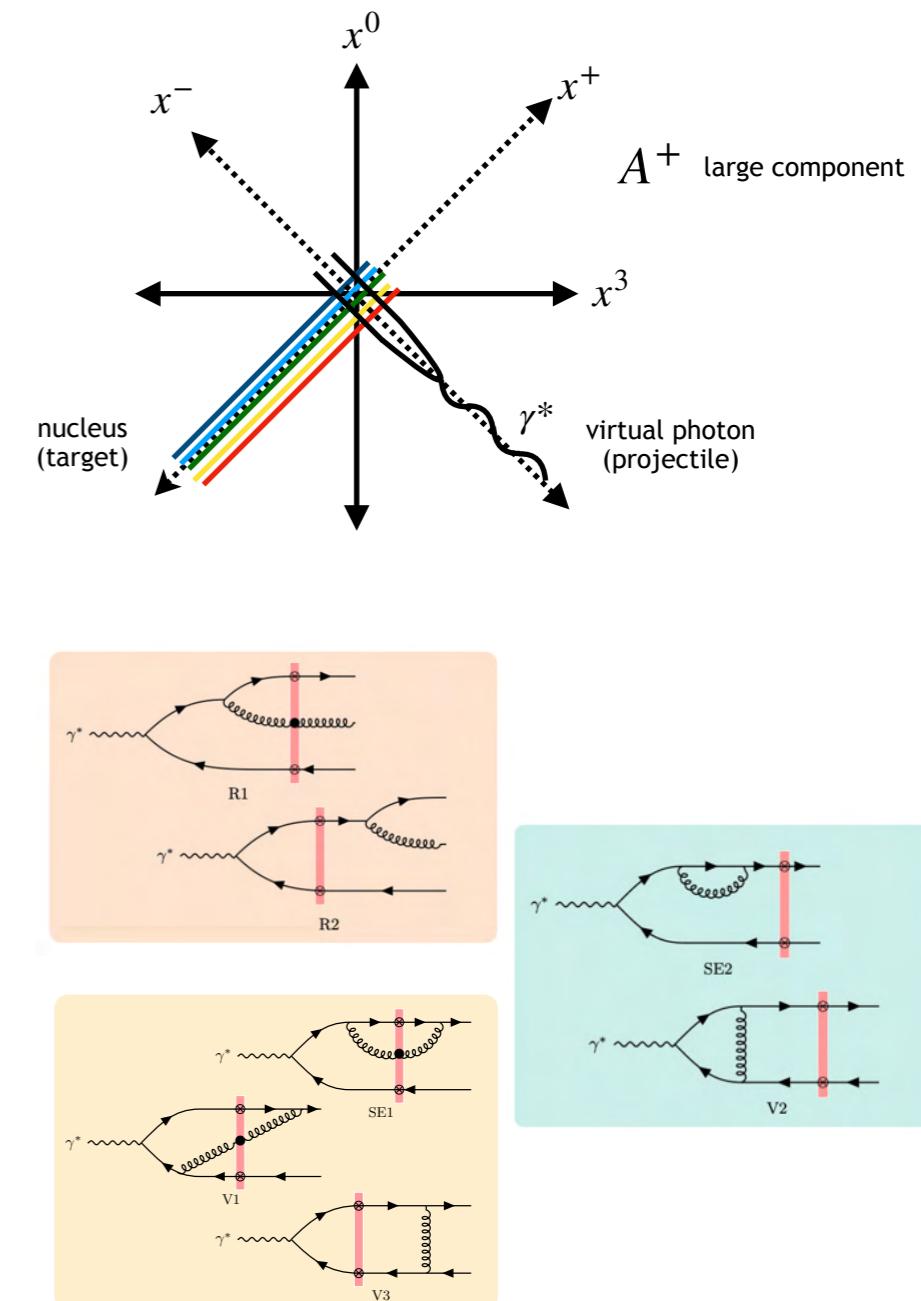
$$\int_{\Lambda_0^-} dk_g^- \frac{d}{k_g^-} \mu^\varepsilon \int \frac{d^{2-\varepsilon} k_{g\perp}}{(2\pi)^{2-\varepsilon}} f_{\Lambda^-}(k_g^-, k_{g\perp})$$

- We showed cancellation of UV, soft and collinear divergences
- Absorbed large energy/rapidity logs into JIMWLK resummation
- Isolated genuine  $\mathcal{O}(\alpha_s)$  contributions (aka NLO impact factor)

Gluon phase space (evolution vs impact factor)



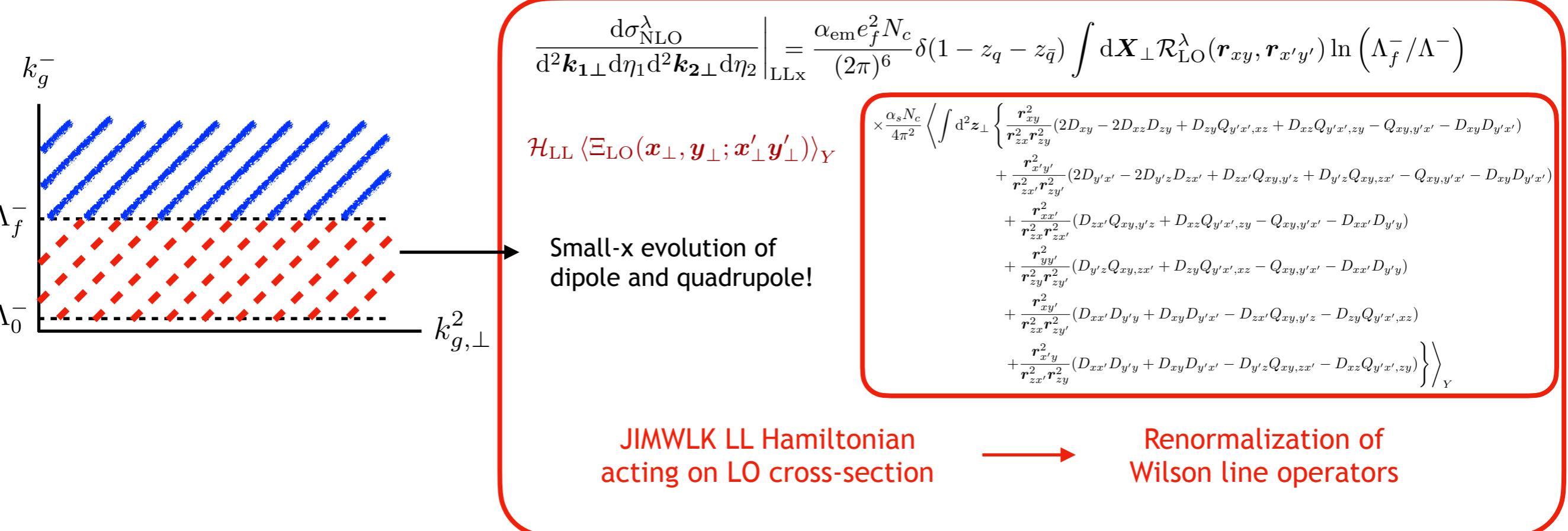
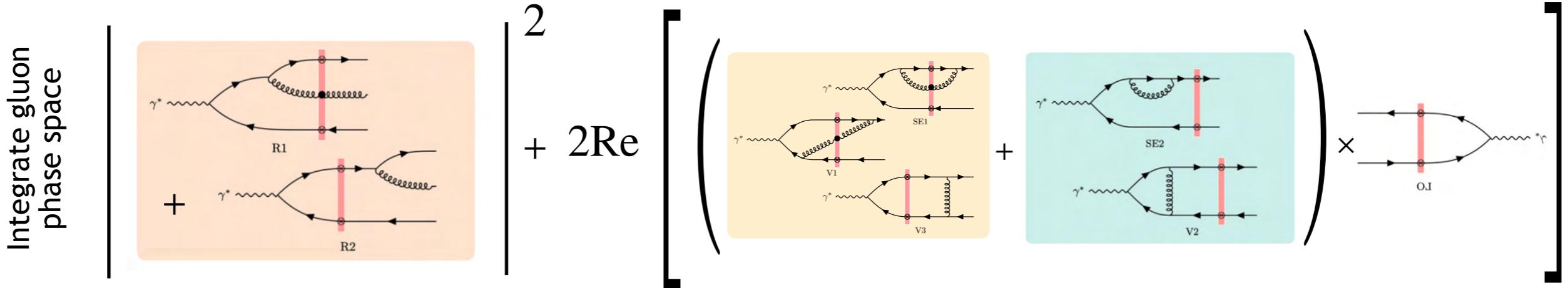
- Impact factor
- Finite piece (free of large rapidity logs)
- Large rapidity (high-energy) logs
- Resummed via JIMWLK renormalization



# Dijet production in DIS at NLO

Small-x evolution: JIMWLK factorization

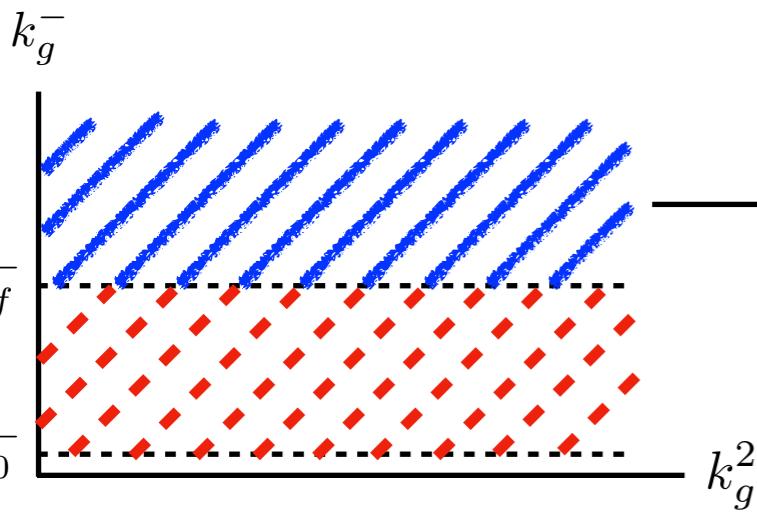
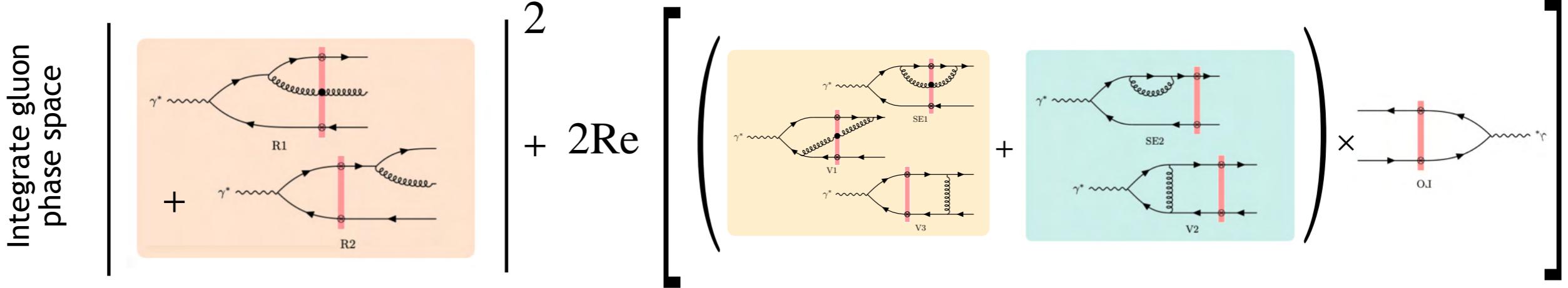
P. Caucal, FS, and R. Venugopalan. *JHEP* 11 (2021) 222



# Dijet production in DIS at NLO

## Impact factor

P. Caucal, FS, and R. Venugopalan. *JHEP* 11 (2021) 222



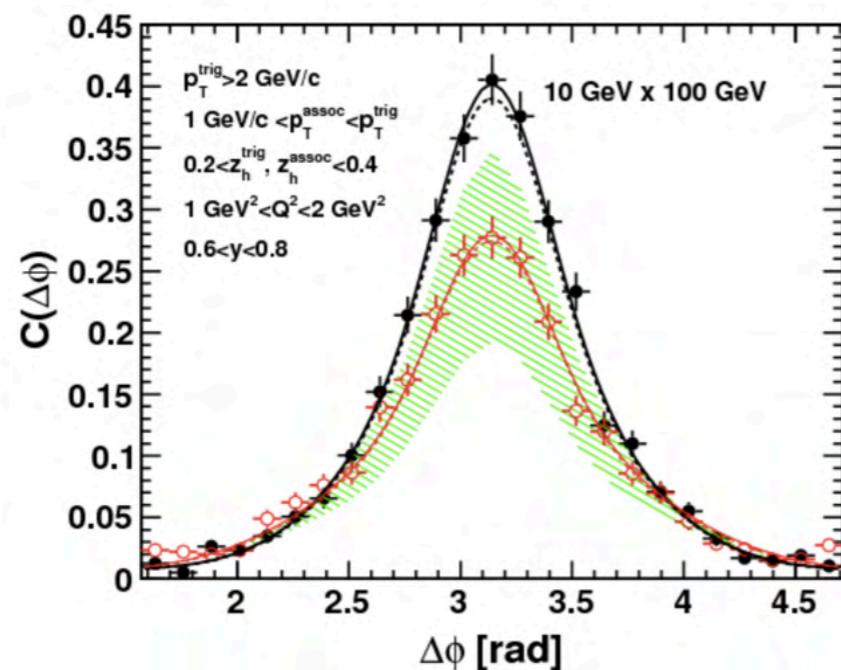
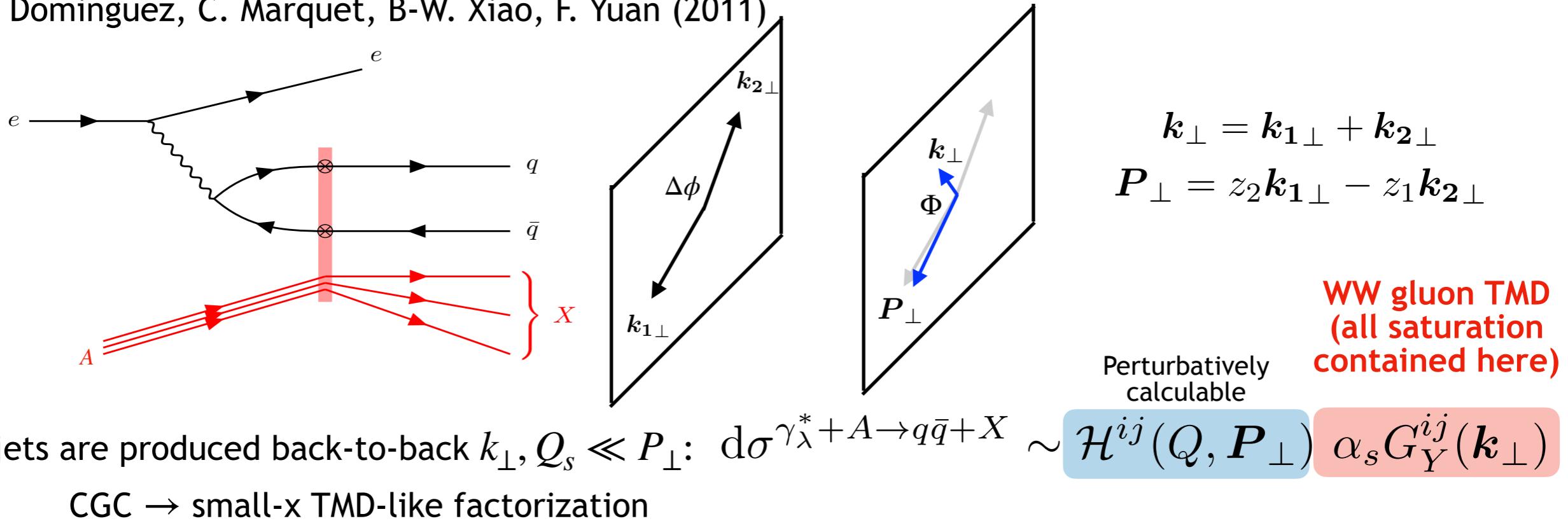
Only longitudinally polarized photon shown,  
lengthier expressions for transversely  
polarized photon

$$\begin{aligned}
 d\sigma_{R_2 \times R_2, \text{sud2}} &= \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \\
 &\quad \times C_F \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \times \frac{\alpha_s}{\pi} \int_0^1 \frac{d\xi}{\xi} [1 - e^{-i\xi \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'}}] \ln \left( \frac{\mathbf{k}_{1\perp}^2 \mathbf{r}_{xx'}^2 R^2 \xi^2}{c_0^2} \right) \\
 d\sigma_{R_2 \times R'_2, \text{sud2}} &= \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \\
 &\quad \times \Xi_{\text{NLO,3}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \times \frac{(-\alpha_s)}{\pi} \int_0^1 \frac{d\xi}{\xi} [1 - e^{-i\xi \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy'}}] \ln \left( \frac{\mathbf{P}_\perp^2 \mathbf{r}_{xy'}^2 \xi^2}{z_2^2 c_0^2} \right) \\
 d\sigma_{R, \text{no-sud, LO}}^{\gamma^* + A \rightarrow q\bar{q}g + X} &= \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^8} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} (4\alpha_s C_F) \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \\
 &\quad \times \frac{e^{-i\mathbf{k}_{g\perp} \cdot \mathbf{r}_{xx'}}}{(\mathbf{k}_{g\perp} - \frac{z_g}{z_1} \mathbf{k}_{1\perp})^2} \left\{ 8z_1 z_2^3 (1-z_2)^2 Q^2 \left( 1 + \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2} \right) K_0(\bar{Q}_R r_{xy}) K_0(\bar{Q}_R r_{x'y'}) \delta_z^{(3)} \right. \\
 &\quad \left. - \mathcal{R}_{\text{LO}}^L(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \Theta(z_1 - z_g) \delta_z^{(2)} \right\} + (1 \leftrightarrow 2) \\
 d\sigma_{R, \text{no-sud, NLO}_3}^{\gamma^* + A \rightarrow q\bar{q}g + X} &= \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^8} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} 8z_1^3 z_2^3 Q^2 K_0(\bar{Q} r_{x'y'}) \\
 &\quad \times \frac{\alpha_s}{\pi} \int_0^{z_1} \frac{dz_g}{z_g} \left[ K_0(\bar{Q}_{V3} r_{xy}) \left( \left( 1 - \frac{z_g}{z_1} \right)^2 \left( 1 + \frac{z_g}{z_2} \right) (1+z_g) e^{i(P_\perp + z_g \mathbf{q}_\perp) \cdot \mathbf{r}_{xy}} K_0(-i\Delta_{V3} r_{xy}) \right. \right. \\
 &\quad \left. \left. - \left( 1 - \frac{z_g}{2z_1} + \frac{z_g}{2z_2} - \frac{z_g^2}{2z_1 z_2} \right) e^{i\frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy}} \mathcal{J}_\odot(\mathbf{r}_{xy}, \left( 1 - \frac{z_g}{z_1} \right) \mathbf{P}_\perp, \Delta_{V3}) \right) \right. \\
 &\quad \left. + K_0(\bar{Q} r_{xy}) \ln \left( \frac{z_g P_\perp r_{xy}}{c_0 z_1 z_2} \right) + (1 \leftrightarrow 2) \right\} \Xi_{\text{NLO,3}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) + c.c. \\
 d\sigma_{V, \text{no-sud, LO}}^{\lambda=L} &= \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} 8z_1^3 z_2^3 Q^2 K_0(\bar{Q} r_{x'y'}) \\
 &\quad \times \frac{\alpha_s}{\pi} \int_0^{z_1} \frac{dz_g}{z_g} \left\{ \frac{1}{\mathbf{r}_{xx}^2} \left[ \left( 1 - \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2} \right) e^{-i\frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx}} K_0(Q X_V) - \Theta(z_f - z_g) K_0(\bar{Q} r_{xy}) \right] \Xi_{\text{NLO,1}} \right. \\
 &\quad \left. - \frac{1}{\mathbf{r}_{xx}^2} \left[ \left( 1 - \frac{z_g}{z_1} + \frac{z_g}{2z_2} - \frac{z_g^2}{2z_1 z_2} \right) e^{-i\frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy}} K_0(\bar{Q} r_{xy}) - \Theta(z_f - z_g) e^{-i\frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy}} K_0(\bar{Q} r_{xy}) \right] C_F \Xi_{\text{LO}} \right. \\
 &\quad \left. - \frac{\mathbf{r}_{xx} \cdot \mathbf{r}_{xy}}{\mathbf{r}_{xx}^2 \mathbf{r}_{xy}^2} \left[ \left( 1 - \frac{z_g}{z_1} \right) \left( 1 + \frac{z_g}{z_2} \right) \left( 1 - \frac{z_g}{2z_1} - \frac{z_g}{2(z_2 + z_g)} \right) e^{-i\frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx}} K_0(Q X_V) \right. \right. \\
 &\quad \left. \left. - \Theta(z_f - z_g) K_0(\bar{Q} r_{xy}) \right] \Xi_{\text{NLO,1}} + (1 \leftrightarrow 2) \right\} + c.c. \\
 d\sigma_{V, \text{no-sud, other}}^{\lambda=L} &= \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} 8z_1^3 z_2^3 Q^2 K_0(\bar{Q} r_{x'y'}) \int_0^{z_1} \frac{dz_g}{z_g} \\
 &\quad \times \frac{\alpha_s}{\pi} \int \frac{d^2 \mathbf{z}_\perp}{\pi} \left\{ \frac{1}{\mathbf{r}_{xx}^2} \left[ \left( 1 - \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2} \right) e^{-i\frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx}} K_0(Q X_V) - \Theta(z_f - z_g) K_0(\bar{Q} r_{xy}) \right] \Xi_{\text{NLO,1}} \right. \\
 &\quad \left. - \frac{1}{\mathbf{r}_{xx}^2} \left[ \left( 1 - \frac{z_g}{z_1} + \frac{z_g}{2z_2} - \frac{z_g^2}{2z_1 z_2} \right) e^{-i\frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy}} K_0(\bar{Q} r_{xy}) - \Theta(z_f - z_g) e^{-i\frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy}} K_0(\bar{Q} r_{xy}) \right] C_F \Xi_{\text{LO}} \right. \\
 &\quad \left. - \frac{\mathbf{r}_{xx} \cdot \mathbf{r}_{xy}}{\mathbf{r}_{xx}^2 \mathbf{r}_{xy}^2} \left[ \left( 1 - \frac{z_g}{z_1} \right) \left( 1 + \frac{z_g}{z_2} \right) \left( 1 - \frac{z_g}{2z_1} - \frac{z_g}{2(z_2 + z_g)} \right) e^{-i\frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx}} K_0(Q X_V) \right. \right. \\
 &\quad \left. \left. - \Theta(z_f - z_g) K_0(\bar{Q} r_{xy}) \right] \Xi_{\text{NLO,1}} + (1 \leftrightarrow 2) \right\} + c.c. \\
 d\sigma_{R, \text{no-sud, other}}^{\gamma^* + A \rightarrow q\bar{q}g + X} &= \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(3)}}{(2\pi)^8} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} 8z_1^3 z_2^3 Q^2 \int \frac{d^2 \mathbf{z}_\perp}{\pi} \frac{d^2 \mathbf{z}'_\perp}{\pi} e^{-i\mathbf{k}_{g\perp} \cdot \mathbf{r}_{zz'}} \\
 &\quad \alpha_s \left\{ - \frac{\mathbf{r}_{zx} \cdot \mathbf{r}_{z'x'}}{\mathbf{r}_{zx}^2 \mathbf{r}_{z'x'}^2} K_0(Q X_R) K_0(\bar{Q}_R r_{w'y'}) \left( 1 + \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2} \right) \Xi_{\text{NLO,1}}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{w}'_\perp, \mathbf{y}'_\perp) \right. \\
 &\quad + \frac{\mathbf{r}_{zy} \cdot \mathbf{r}_{z'y'}}{\mathbf{r}_{zy}^2 \mathbf{r}_{z'y'}^2} K_0(Q X_R) K_0(\bar{Q}_R r_{w'y'}) \left( 1 + \frac{z_g}{2z_1} + \frac{z_g}{2z_2} \right) \Xi_{\text{NLO,1}}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{w}'_\perp, \mathbf{y}'_\perp) \quad d\sigma_{\text{sud1}} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \times \frac{\alpha_s}{\pi} \\
 &\quad + \frac{1}{2} \frac{\mathbf{r}_{zx} \cdot \mathbf{r}_{z'y'}}{\mathbf{r}_{zx}^2 \mathbf{r}_{z'y'}^2} K_0(Q X_R) K_0(Q X'_R) \left( 1 + \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2} \right) \Xi_{\text{NLO,4}}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \\
 &\quad - \frac{1}{2} \frac{\mathbf{r}_{zy} \cdot \mathbf{r}_{z'y'}}{\mathbf{r}_{zy}^2 \mathbf{r}_{z'y'}^2} K_0(Q X_R) K_0(Q X'_R) \left( 1 + \frac{z_g}{2z_1} + \frac{z_g}{2z_2} \right) \Xi_{\text{NLO,4}}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \\
 &\quad + (1 \leftrightarrow 2) + c.c. \left. \right\} - \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^8} \alpha_s \Theta(z_f - z_g) \times \text{"slow"} \\
 &\quad \times \left\{ C_F \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \left[ \ln \left( \frac{z_f}{z_1} \right) \ln \left( \frac{\mathbf{r}_{xy}^2}{|\mathbf{r}_{xy}| |\mathbf{r}_{x'y'}|} \right) + \ln \left( \frac{z_f}{z_2} \right) \ln \left( \frac{\mathbf{r}_{yy'}^2}{|\mathbf{r}_{xy}| |\mathbf{r}_{x'y'}|} \right) \right] \right. \\
 &\quad \left. + \Xi_{\text{NLO,3}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \left[ \ln \left( \frac{z_1}{z_f} \right) \ln \left( \frac{\mathbf{r}_{xy}^2}{|\mathbf{r}_{xy}| |\mathbf{r}_{x'y'}|} \right) + \ln \left( \frac{z_2}{z_f} \right) \ln \left( \frac{\mathbf{r}_{yy'}^2}{|\mathbf{r}_{xy}| |\mathbf{r}_{x'y'}|} \right) \right] \right\}
 \end{aligned}$$

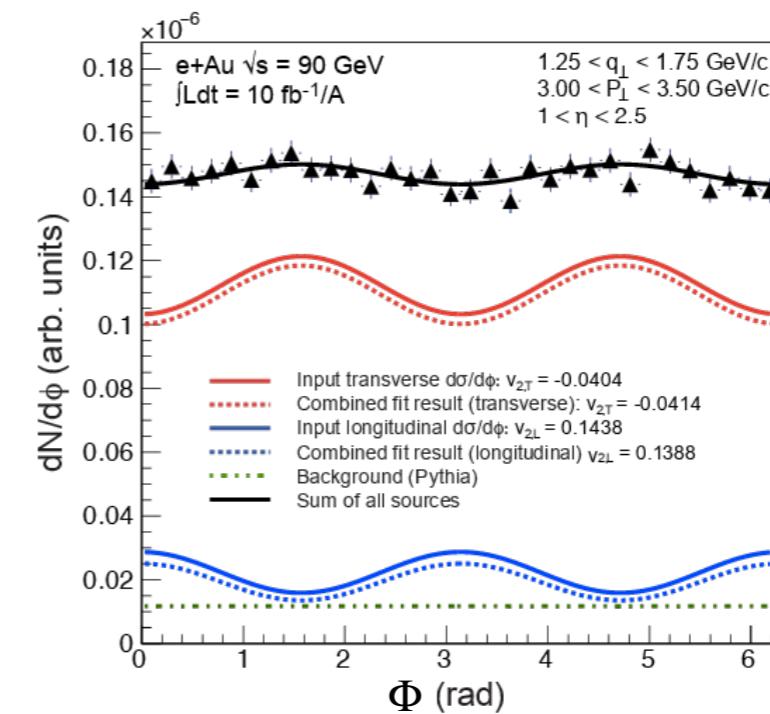
# Back-to-back dijets at LO

## Small-x Weizsäcker-Williams (WW) gluon distribution

F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan (2011)



Zheng, Aschenauer, Lee, Xiao (2014)



Dumitru, Skokov, Ullrich (2018)

# Back-to-back dijets at NLO

Does CGG-TMD correspondence hold at NLO?

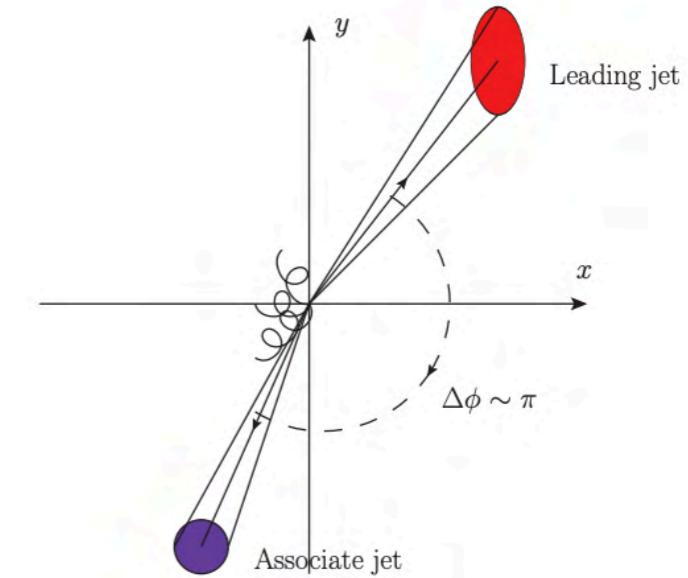
A.H. Mueller, B-W. Xiao, F. Yuan (2013)

$$q_\perp^2 \ll P_\perp^2 \ll s^2$$

$$\ln(s/P_\perp^2)$$

$$\ln^2(P_\perp^2/q_\perp^2)$$

Conjecture: joint (soft) small- $x$  + Sudakov resummation



$$d\sigma^{\gamma_\lambda^* + A \rightarrow q\bar{q} + X} \propto \mathcal{H}(Q, \mathbf{P}_\perp) \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \alpha_s \tilde{G}_Y^0(\mathbf{b}_\perp) e^{-S_{\text{Sud}}(\mathbf{b}_\perp, \mathbf{P}_\perp)}$$

Perturbative  
Sudakov factor:

$$S_{\text{Sud}}(b_\perp^2, P_\perp^2) = \int_{c_0^2/b_\perp^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \left[ A \ln \left( \frac{P_\perp^2}{\mu^2} \right) + B \right]$$

**Soft gluon emissions**



**Change profile of azimuthal correlations**

See also Y. Hatta, B-W. Xiao, F. Yuan, J. Zhou (2021)

**Can we derive these results from our NLO calculation? Can we obtain finite NLO pieces?**

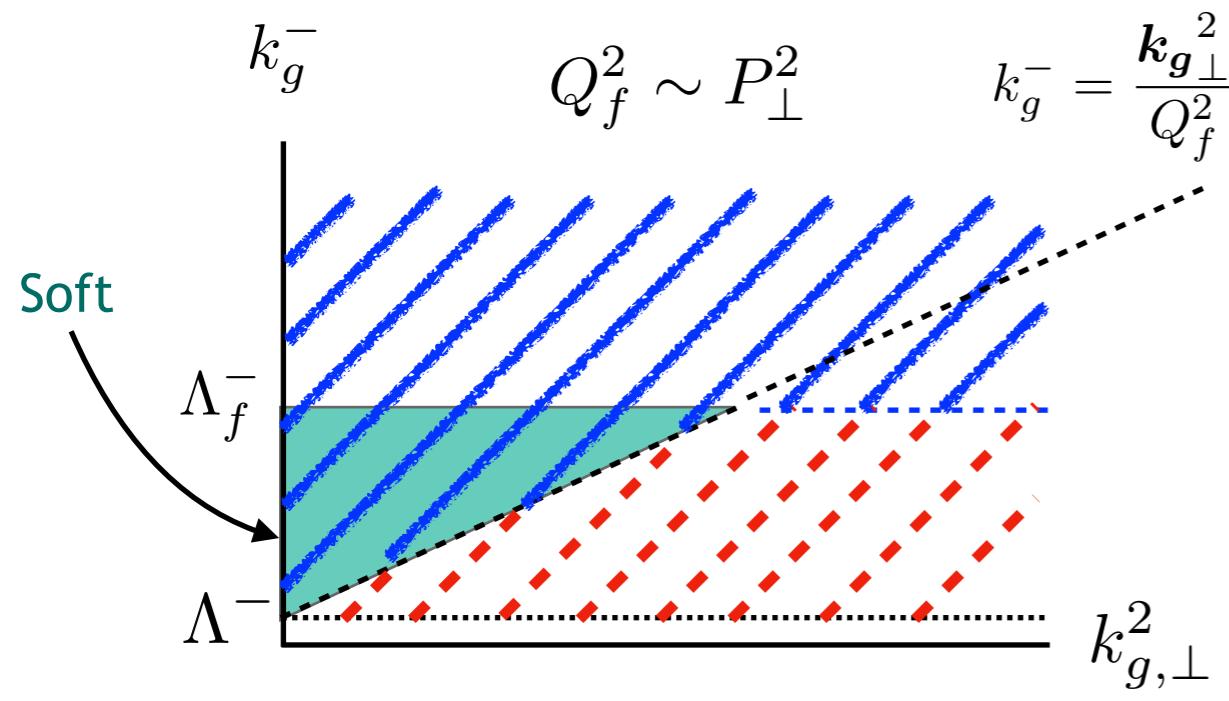
# Back-to-back dijets at NLO

Small-x and Sudakov interplay: The need for kinematic constraint

P. Caucal, FS, B. Schenke, and R. Venugopalan *JHEP 11 (2022) 169*

The back-to-back limit of our NLO dijet led to Sudakov double log with opposite sign

Solution: impose a kinematic constraint which also enforces ordering in  $k_g^+$  (target momenta)



$$k_g^- = \frac{k_{g\perp}^2}{Q_f^2} \Lambda_f^-$$

Proposed by P. Taels, T. Altinoluk, C. Marquet, G. Beuf (2022) for dijet photo-production

Small-x evolution for WW follows well-known BK-JIMWLK equations amended with a kinematic constraint to separate small-x and soft gluons

$$d\sigma^{\gamma_\lambda^* + A \rightarrow q\bar{q} + X} = \mathcal{H}^{\lambda,ij}(Q, \mathbf{P}_\perp) \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{-i \mathbf{q}_\perp \cdot \mathbf{b}_\perp} \alpha_s \tilde{G}_Y^{ij}(\mathbf{b}_\perp) e^{-S_{\text{Sud}}(\mathbf{b}_\perp, \mathbf{P}_\perp)} + \mathcal{O}(\alpha_s)$$

What about finite piece?

# Back-to-back dijets at NLO

Finite pieces: correlators beyond the WW

$$d\sigma^{\gamma_\lambda^* + A \rightarrow q\bar{q} + X} = \mathcal{H}^{ij}(Q, \mathbf{P}_\perp) \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \alpha_s \tilde{G}_Y^{ij}(\mathbf{b}_\perp) e^{-S_{\text{Sud}}(\mathbf{b}_\perp, \mathbf{P}_\perp)} + \mathcal{O}(\alpha_s)$$

Pure  $\mathcal{O}(\alpha_s)$

Subset involves complicated convolutions including **operators beyond WW**, but **needed for precision!**

$\Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp)$	$\langle 1 - D_{xy} - D_{y'x'} + Q_{xy,y'x'} \rangle$
$\Xi_{\text{NLO},1}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp)$	$\frac{N_c}{2} \langle 1 - D_{y'x'} + Q_{zy,y'x'} D_{xz} - D_{xz} D_{zy} \rangle - \frac{1}{2N_c} \Xi_{\text{LO}}$
$\Xi_{\text{NLO},2}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp)$	$\frac{N_c}{2} \langle 1 - D_{y'x'} + Q_{xz,y'x'} D_{zy} - D_{xz} D_{zy} \rangle - \frac{1}{2N_c} \Xi_{\text{LO}}$
$\Xi_{\text{NLO},3}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp)$	$\frac{N_c}{2} \langle 1 - D_{xy} - D_{y'x'} + D_{xy} D_{y'x'} \rangle - \frac{1}{2N_c} \Xi_{\text{LO}}$
$\Xi_{\text{NLO},4}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp)$	$\frac{N_c}{2} \langle 1 - D_{xz} D_{zy} - D_{y'z} D_{zx'} + Q_{xz,z'x'} Q_{y'z',zy} \rangle - \frac{1}{2N_c} \Xi_{\text{LO}}$

- Blue correlators collapse to the WW gluon TMD, red correlators result in other TMDs. e.g.

$$\Xi_{\text{LO}} \approx \mathbf{u}_\perp^i \mathbf{u}'_\perp^j \frac{1}{2N_c} \underbrace{(-2) \left\langle \text{Tr} \left[ (V(\mathbf{b}_\perp) \partial^i V^\dagger(\mathbf{b}_\perp)) (V(\mathbf{0}_\perp) \partial^j V^\dagger(\mathbf{0}_\perp)) \right] \right\rangle_Y}_{\alpha_s \tilde{G}_Y^{ij}(\mathbf{b}_\perp)}$$

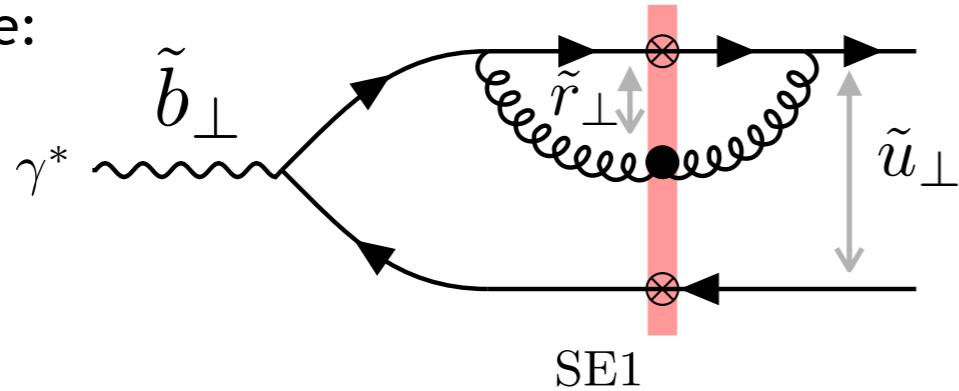
$$\Xi_{\text{NLO},1} \approx -\mathbf{u}'_\perp^j \frac{1}{2N_c} \underbrace{(-2) \left\langle \text{Tr} [V(\mathbf{b}_\perp) V^\dagger(\mathbf{z}_\perp)] \text{Tr} [V(\mathbf{z}_\perp) V^\dagger(\mathbf{b}_\perp) \partial^j V(\mathbf{0}_\perp) V^\dagger(\mathbf{0}_\perp)] \right\rangle_Y}_{\alpha_s \tilde{G}_{Y,\text{NLO},1}^j(\mathbf{b}_\perp, \mathbf{z}_\perp)}$$

breaks TMD factorization?

# Back-to-back dijets at NLO

Finite pieces: restoring TMD factorization

Example:



Correlation limit at NLO

$$q_\perp \ll P_\perp \rightarrow \tilde{u}_\perp \ll \tilde{b}_\perp$$

Still involves operator beyond WW

Key observation: Perturbative factor constraint  $\tilde{r}_\perp$ :

$$K_0 \left( \bar{Q} \sqrt{u_\perp^2 + \frac{z_g}{z_1 z_2} r_\perp^2} \right) \longrightarrow r_\perp^2 \lesssim \frac{z_1 z_2}{z_g} u_\perp^2 \sim \frac{1}{z_g P_\perp^2}$$

$z_g \sim \mathcal{O}(1)$  Expand around small  $r_\perp$   
Recover WW!

$$\begin{aligned} \tilde{\Xi}_{\text{NLO},1} = & \left[ C_F \tilde{\mathbf{u}}_\perp^i \mathbf{u}'_\perp^j + \left( \frac{N_c}{2} + \frac{1}{2N_c} \frac{z_g}{(z_1 - z_g)} \right) \tilde{\mathbf{r}}_\perp^i \mathbf{u}'_\perp^j \right] \\ & \times \frac{1}{N_c} \left\langle \text{Tr} V(\tilde{\mathbf{b}}_\perp) \partial^i V^\dagger(\tilde{\mathbf{b}}_\perp) \partial^j V(\mathbf{b}'_\perp) V^\dagger(\mathbf{b}'_\perp) \right\rangle_Y. \end{aligned}$$

$z_g \ll 1$  Can't expand in  $r_\perp$

Operator beyond WW non-linear evolution!

Operators can be found in the Evolution of WW by  
F. Dominguez, A. Mueller, S. Munier, B-W. Xiao (2011)

$r_\perp \sim 1/k_{g\perp}$  Kinematic constraint interpolates between  
non-linear (small  $k_{g\perp}^2$  gluons) and linear (large  $k_{g\perp}^2$  gluons) regimes

# Back-to-back dijets at NLO

Complete small- $x$  TMD factorization at NLO

$$d\sigma^{\gamma^* + A \rightarrow \text{dijet} + X} = \mathcal{H}_{\text{NLO}}^{ij, \lambda}(Q, \mathbf{P}_\perp; \mu_F; R) \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{-i \mathbf{q}_\perp \cdot \mathbf{b}_\perp} \alpha_s \tilde{G}_Y^{ij}(\mathbf{b}_\perp) e^{-S_{\text{Sud}}(\mathbf{b}_\perp, \mu_F)} + \mathcal{O}(\alpha_s^2)$$

fully analytic result

$$d\sigma^{(0), \lambda=L} = \alpha_{\text{em}} \alpha_s e_f^2 \delta_z^{(2)} \mathcal{H}_{\text{LO}}^{0, \lambda=L} \left\{ 1 + \frac{\alpha_s(\mu_h)}{\pi} \left[ \frac{N_c}{2} \tilde{f}_1^{\lambda=L}(\chi, z_f) + \frac{1}{2\pi N_c} \tilde{f}_2^{\lambda=L}(\chi) \right] \right\}$$

$$\times \int \frac{d^2 \mathbf{r}_{bb'}}{(2\pi)^4} e^{-i \mathbf{q}_\perp \cdot \mathbf{r}_{bb'}} \hat{G}_{Y_f}^0(\mathbf{r}_{bb'}) \tilde{\mathcal{S}}(\mu_h^2, \mathbf{r}_{bb'}^2)$$

$$+ \alpha_{\text{em}} \alpha_s e_f^2 \delta_z^{(2)} \mathcal{H}_{\text{LO}}^{0, \lambda=L} \frac{\alpha_s(\mu_h)}{\pi} \left\{ \frac{N_c}{2} [1 + \ln(R^2)] + \frac{1}{2N_c} [-\ln(z_1 z_2 R^2)] \right\}$$

$$\times \int \frac{d^2 \mathbf{r}_{bb'}}{(2\pi)^4} e^{-i \mathbf{q}_\perp \cdot \mathbf{r}_{bb'}} \hat{h}_{Y_f}^0(\mathbf{r}_{bb'}) \tilde{\mathcal{S}}(\mu_h^2, \mathbf{r}_{bb'}^2)$$

$$\begin{aligned} f_1^{\lambda=L}(\chi, z_f) = & 7 - \frac{3\pi^2}{2} - \frac{3}{2} \ln\left(\frac{z_1 z_2 R^2}{\chi^2}\right) - \ln(z_1) \ln(z_2) + 2 \ln\left(\frac{(1+\chi^2) z_f}{z_1 z_2}\right) \\ & - \ln(1+\chi^2) \ln\left(\frac{1+\chi^2}{z_1 z_2}\right) + \left\{ \text{Li}_2\left(\frac{z_2 - z_1 \chi^2}{z_2(1+\chi^2)}\right) - \frac{1}{4(z_2 - z_1 \chi^2)} \right. \\ & \left. + \frac{(1+\chi^2)(z_2(2z_2 - z_1) + z_1(2z_1 - z_2)\chi^2)}{4(z_2 - z_1 \chi^2)^2} \ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right) + (1 \leftrightarrow 2) \right\} \end{aligned}$$

Analogous expression  
for elliptic  
anisotropy  $d\sigma^{(2), \lambda=L}$

$$\chi = \frac{\bar{Q}}{P_\perp}$$

Similar expression  
for  $f_2$

The first proof of TMD factorization at NLO at small- $x$   
(modulo the non-linear evolution of the WW)

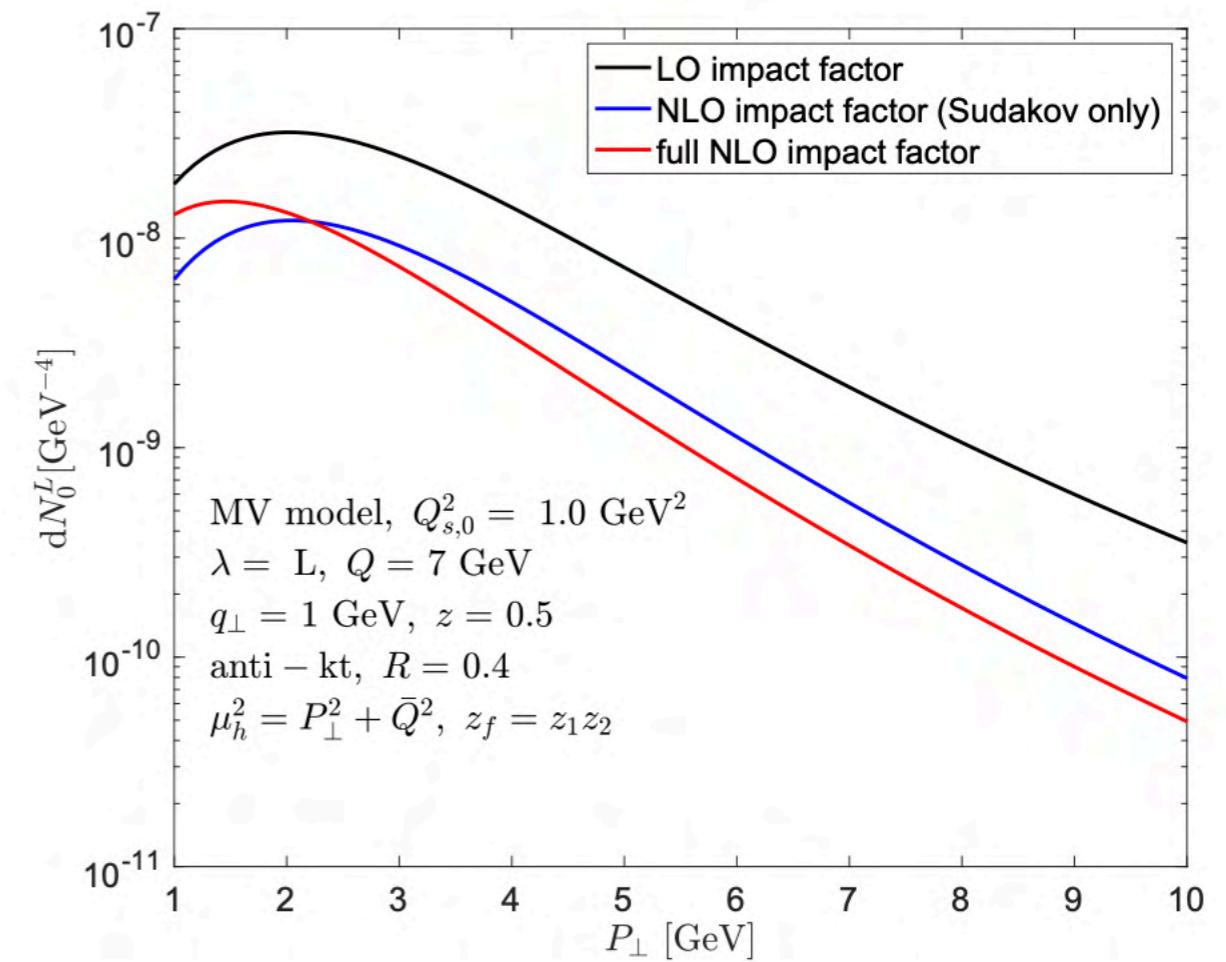
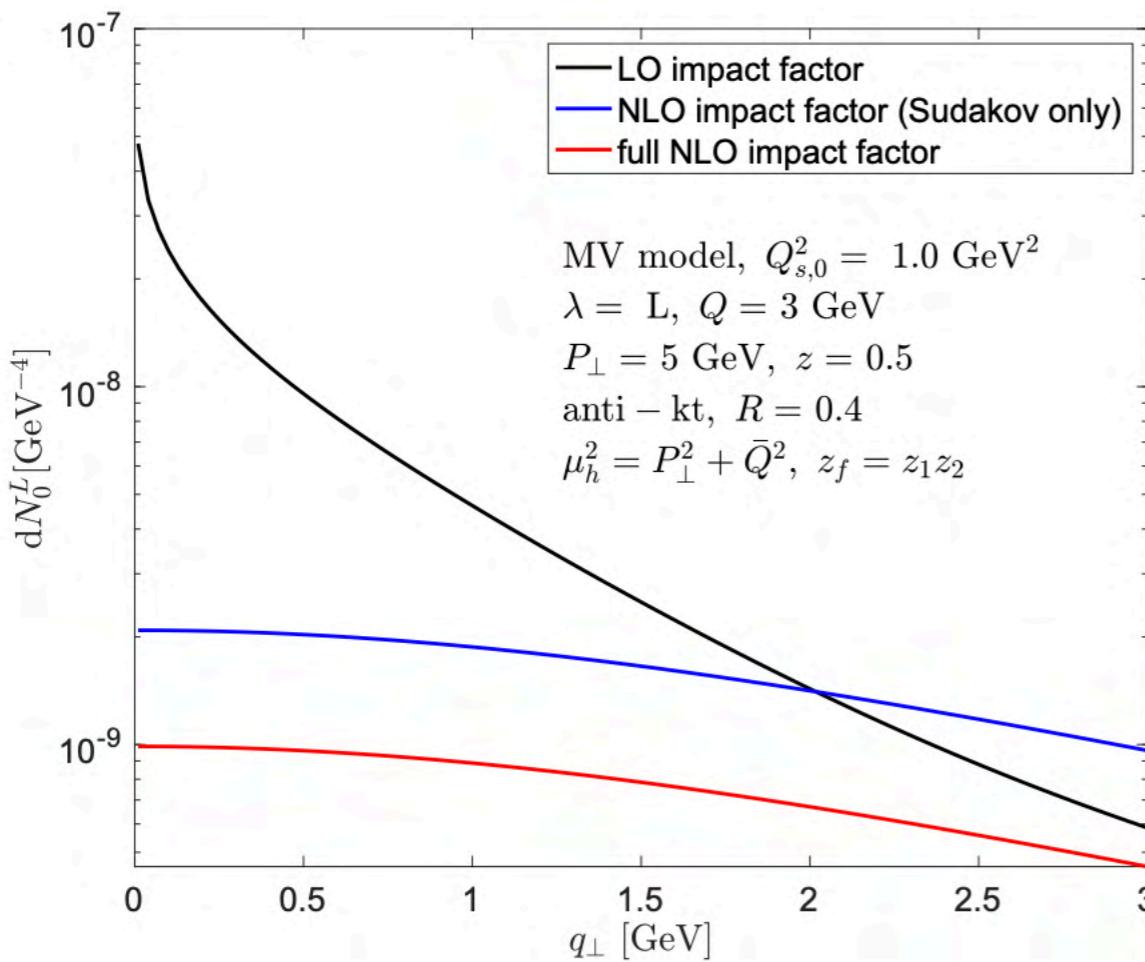
# Back-to-back inclusive dijets in DIS at NLO

Preliminary numerical results (only  $\gamma_L^* + A \rightarrow \text{dijet} + X$ )

- WW from MV model (Gaussian approximation) without rapidity evolution
- Sudakov form factor with running of the coupling at two-loops + non-perturbative contribution

$$\frac{dN^{\lambda=L}}{d^2\mathbf{P}_\perp d^2\mathbf{q}_\perp d\eta_1 d\eta_2} = dN^{(0),\lambda=L} \left[ 1 + 2 \sum_{n=1}^{\infty} v_{2n}^{\lambda=L} \cos(2n\phi) \right]$$

$\phi$  angle between  $\mathbf{P}_\perp$  and  $\mathbf{q}_\perp$



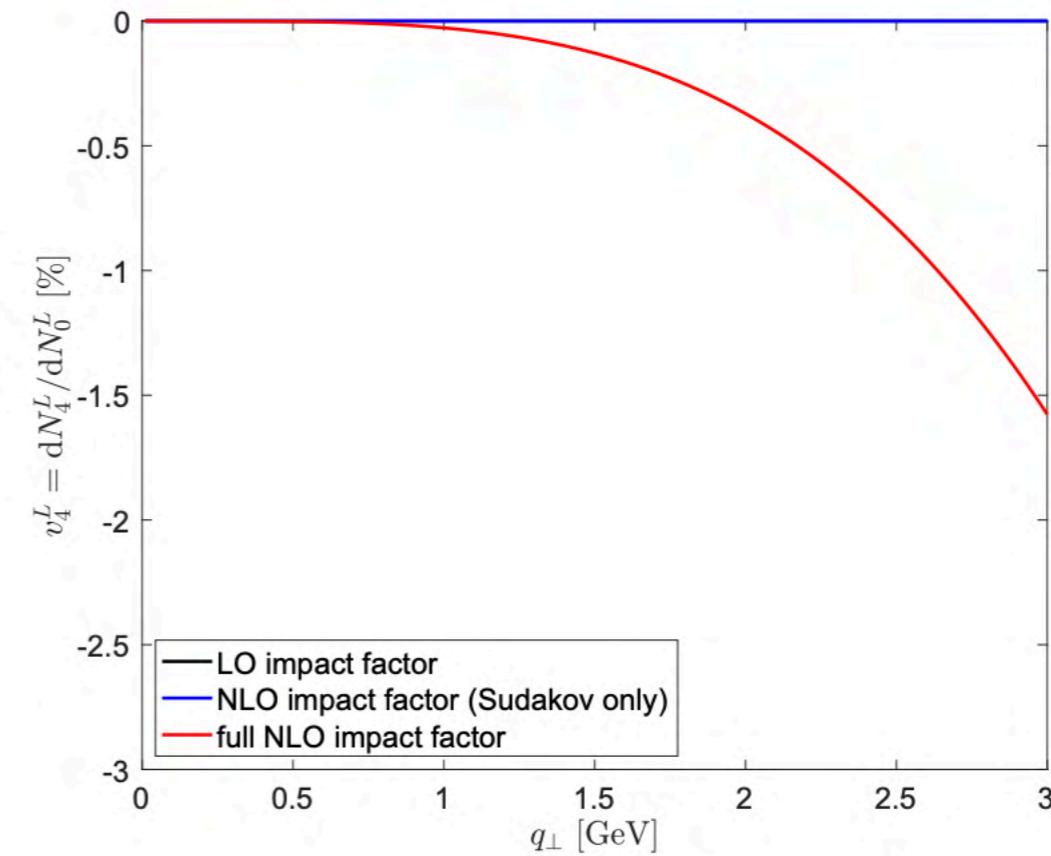
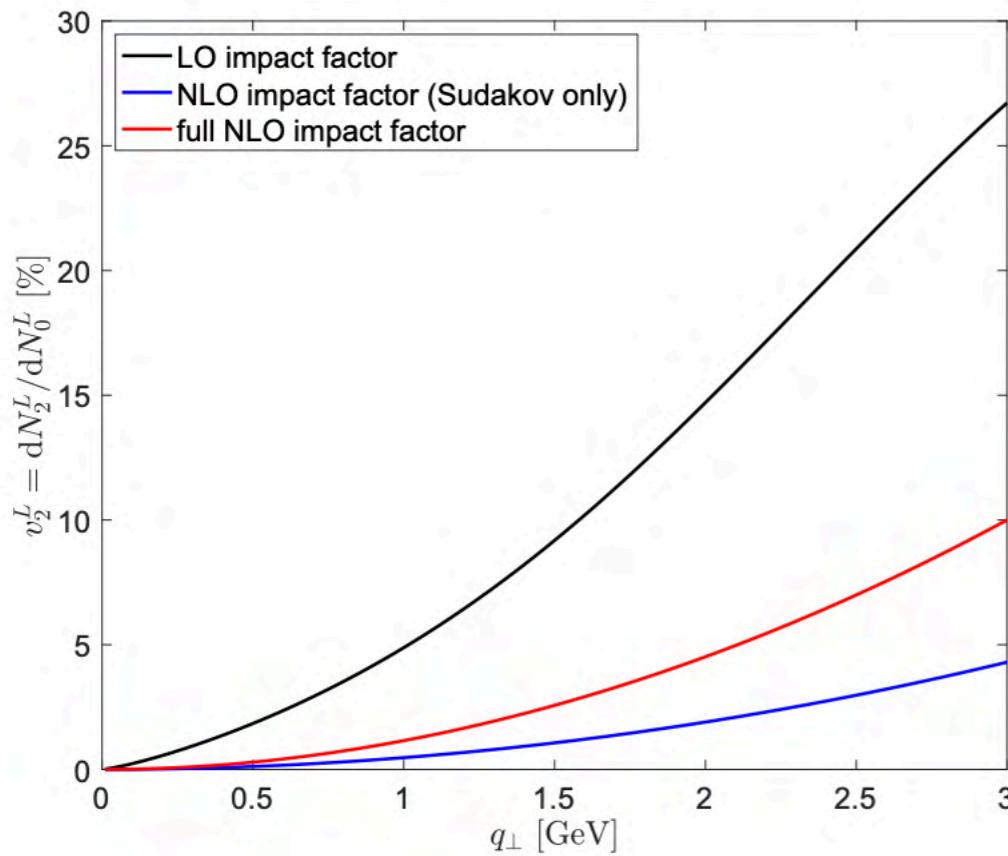
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$\phi$  angle between  $\mathbf{P}_\perp$  and  $\mathbf{q}_\perp$



# Summary

## Motivation:

2-particle azimuthal correlations



powerful observables to search for saturation

## Results:

full NLO calculation  
dijets in DIS

back-to-back limit



small-x and soft gluon resummation

NLO finite pieces

preliminary numerical results

Outlook: Include kinematically constrained small-x evolution in numerical result

Other observables: Dihadrons, UPCs, all two-particle observables

Could SCET-like techniques help us promote results beyond NLO/  
more complex observables?