Quantum structure of the 3D gluon distribution

Yacine Mehtar-Tani (BNL)
In collaboration with R. Boussarie (in preparation)

2112.01412 [hep-ph]

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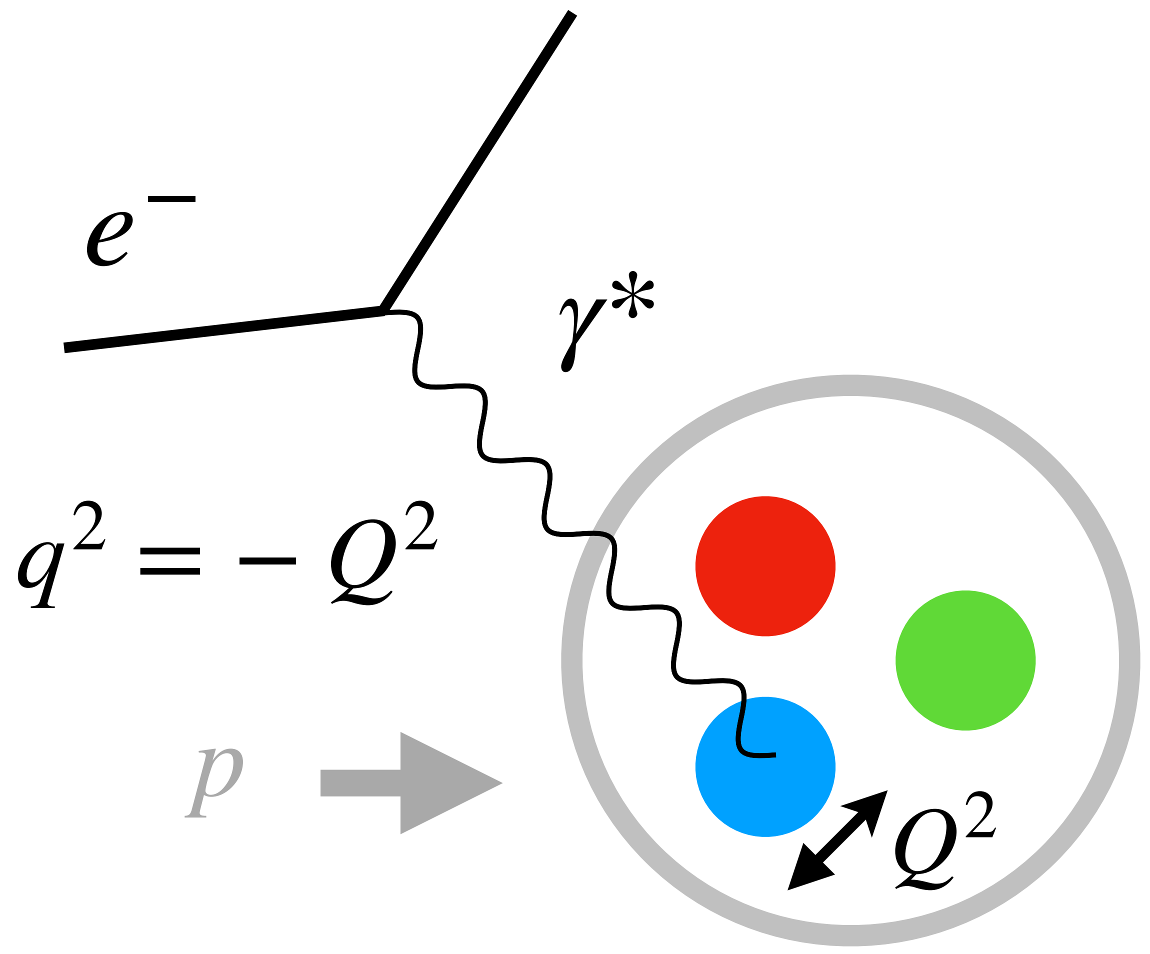
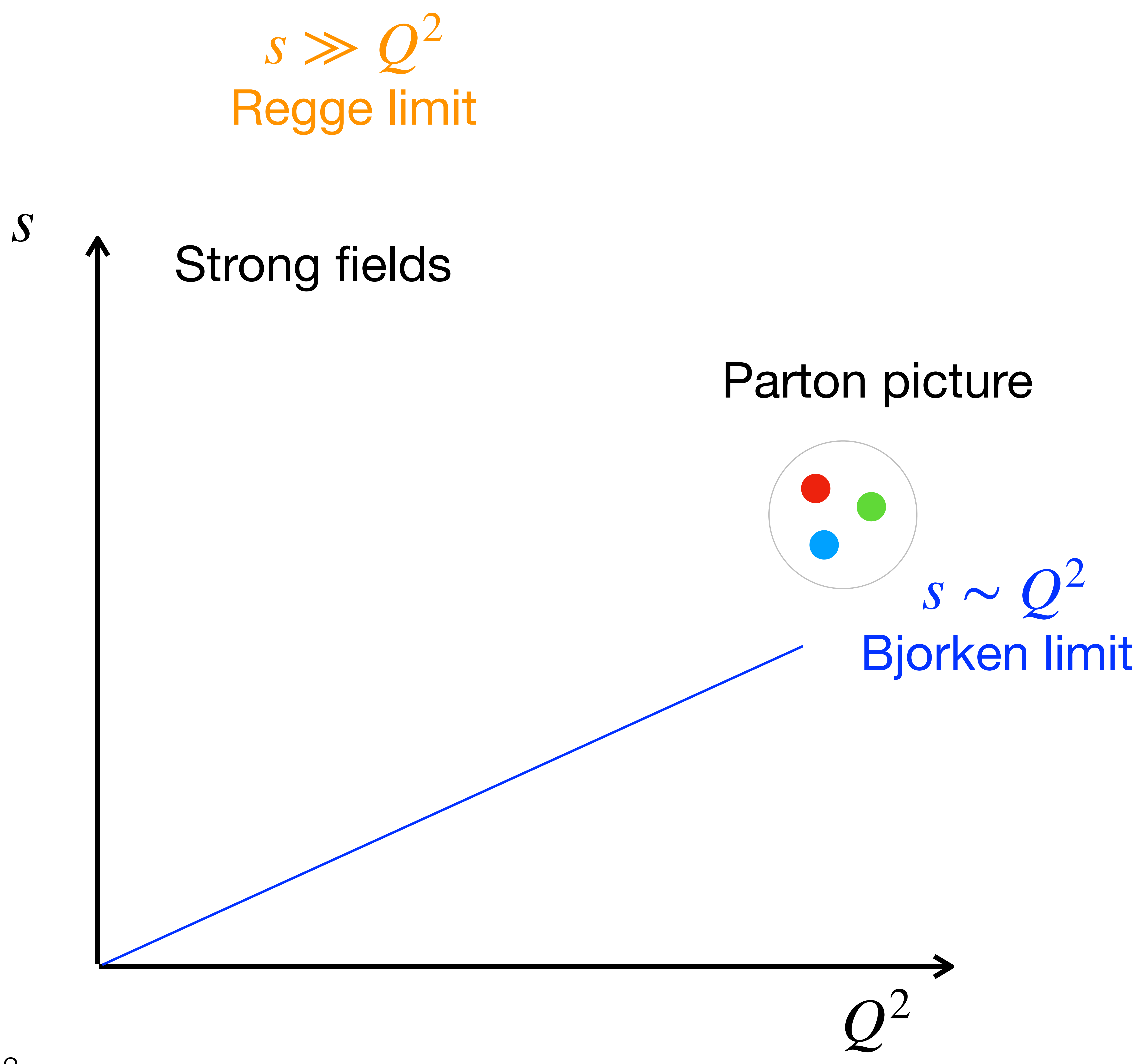
QCD landscape

Probing partonic content of hadrons
with an electromagnetic probe



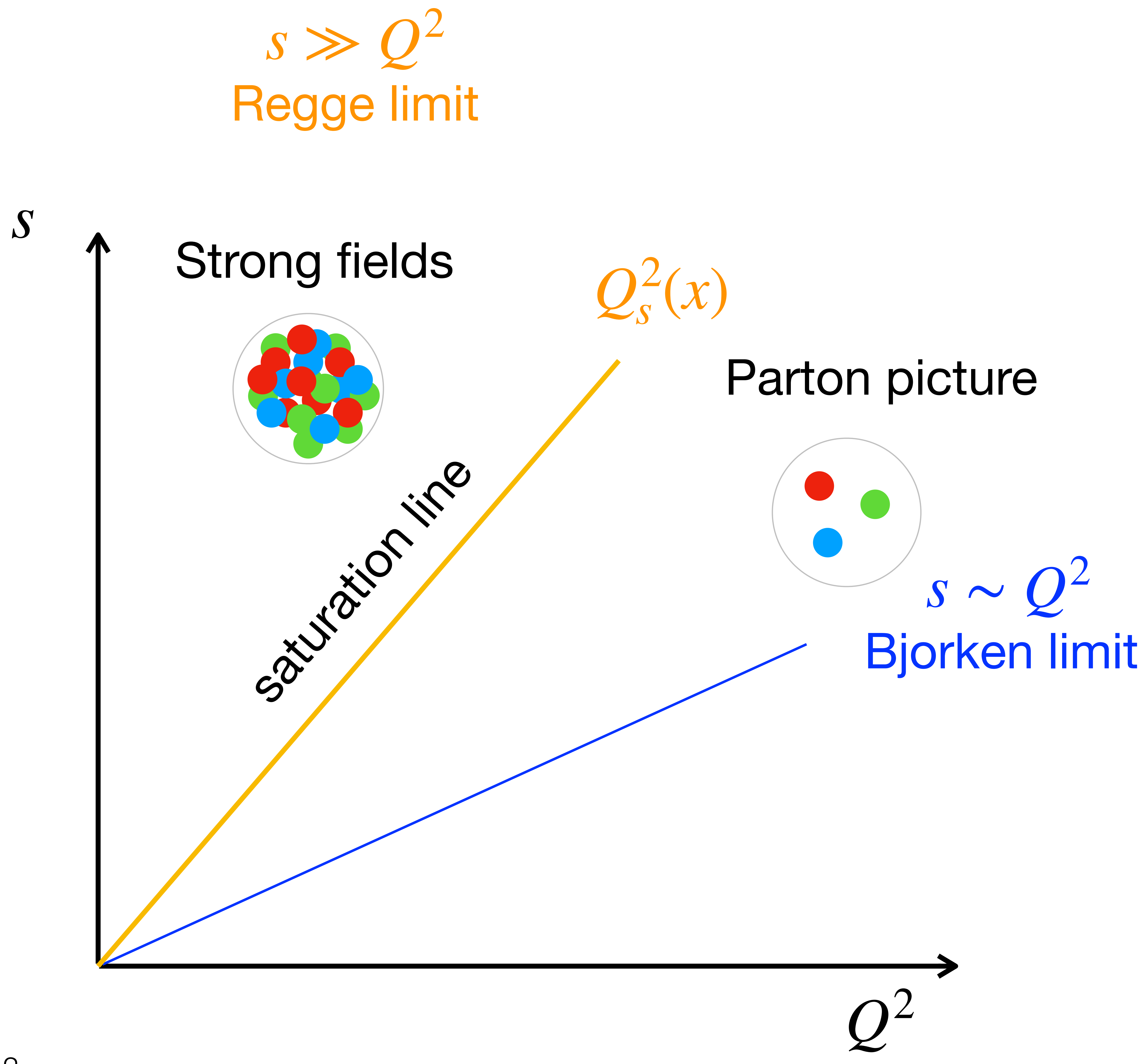
Q^2: resolution scale

$x = Q^2/s$: longitudinal mom. fraction



QCD landscape

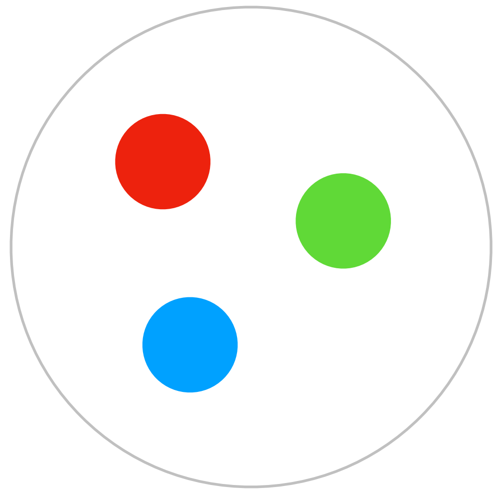
- Saturation regime: breakdown of the parton picture
- Relevant d.o.f.'s: strong classical fields $A^\mu \sim g^{-1} \gg 1$



Motivation

Bjorken limit

$$s \sim Q^2$$

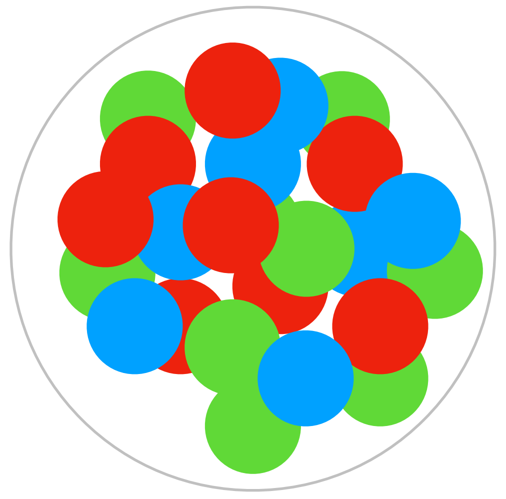


Partons

$$\longleftrightarrow$$

Regge limit

$$s \gg Q^2$$



Strong fields

How to connect the two regimes from first principles?

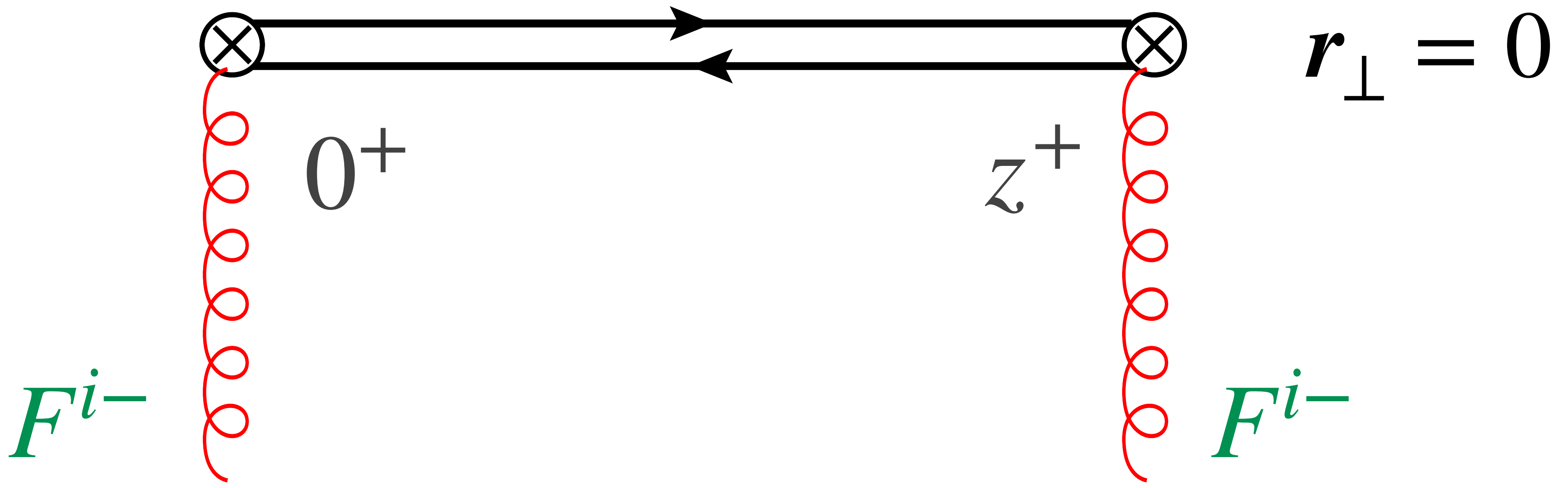
Two different kinds of gluon distributions

Bjorken limit

$$f(x) + O(Q^{-2})$$

Gluon PDF at moderate x

$$\langle P | F^{i-}(z^+)\, W F^{i-}(0^+)\, W^\dagger | P \rangle\, e^{iz^+ x P^-}$$



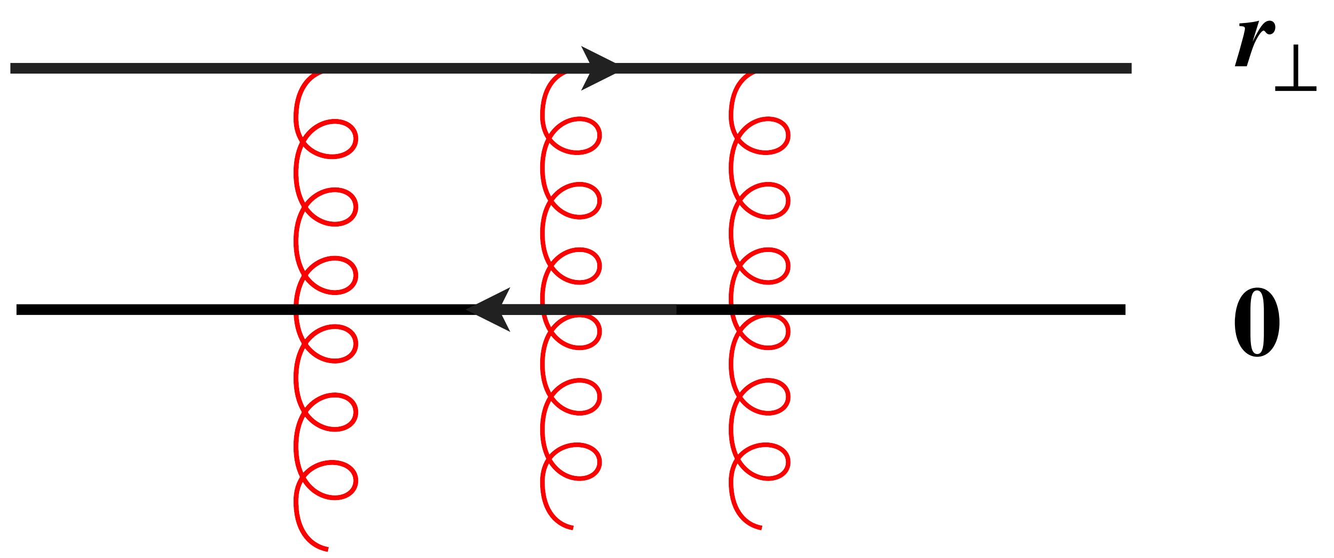
$$F^{i-} \equiv \partial^i A^- - \partial^- A^i - ig[A^i, A^-]$$

Regge limit

$$f(x = 0, k_\perp) + O(s^{-1})$$

Dipole gluon distribution at small x

$$\langle P | \mathrm{Tr}\, U_{\boldsymbol{r}}\, U^\dagger_{\boldsymbol{0}} | P \rangle$$



$$U_{\boldsymbol{r}} \equiv [+\infty, -\infty]_{\boldsymbol{r}} = P \exp\left(ig \int_{-\infty}^{+\infty} dz^+ A^- \right)$$

Diagnosing small x

- Dipole distribution evaluated in the strict x=0 limit
- QCD factorization: Hard part integrated over x

Bjorken limit

$$s \sim Q^2$$

$$\sigma \sim \int \mathrm{d}x\, H(x) f(x)$$

Regge limit

$$s \gg Q^2$$

$$\sigma \sim f(0) \int \mathrm{d}x\, H(x)$$

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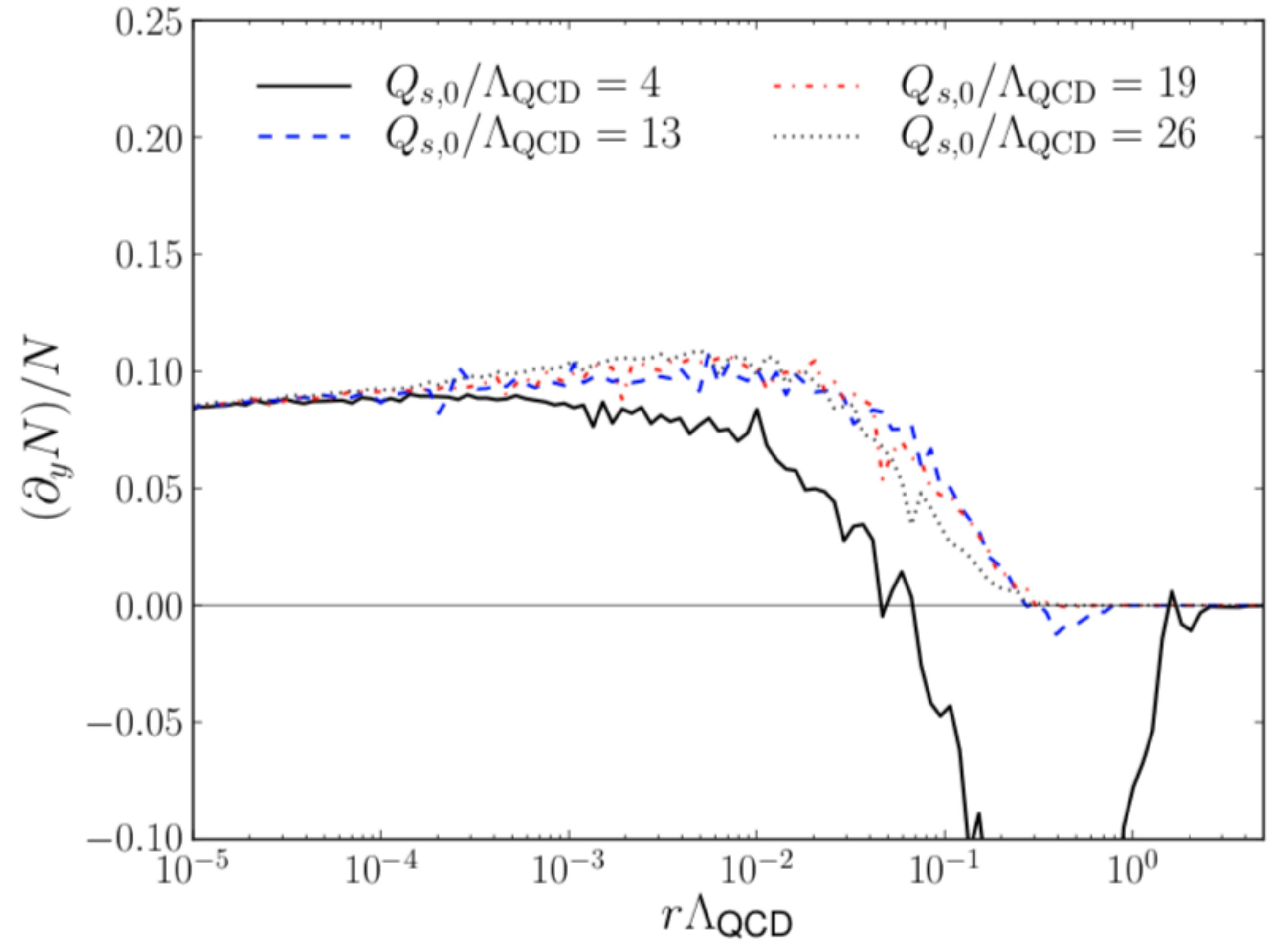
Diagnosing small x

- Instability of NLO BFKL/BK: rapidity $Y \equiv \log(q^+/\Lambda^+)$ evolves independently from $r_\perp$ violating $k^- = xP^-$ ordering (producing large collinear logs)

Beuf (2014) Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos (2015-2019)

Lappi and Mäntysaari (2015)



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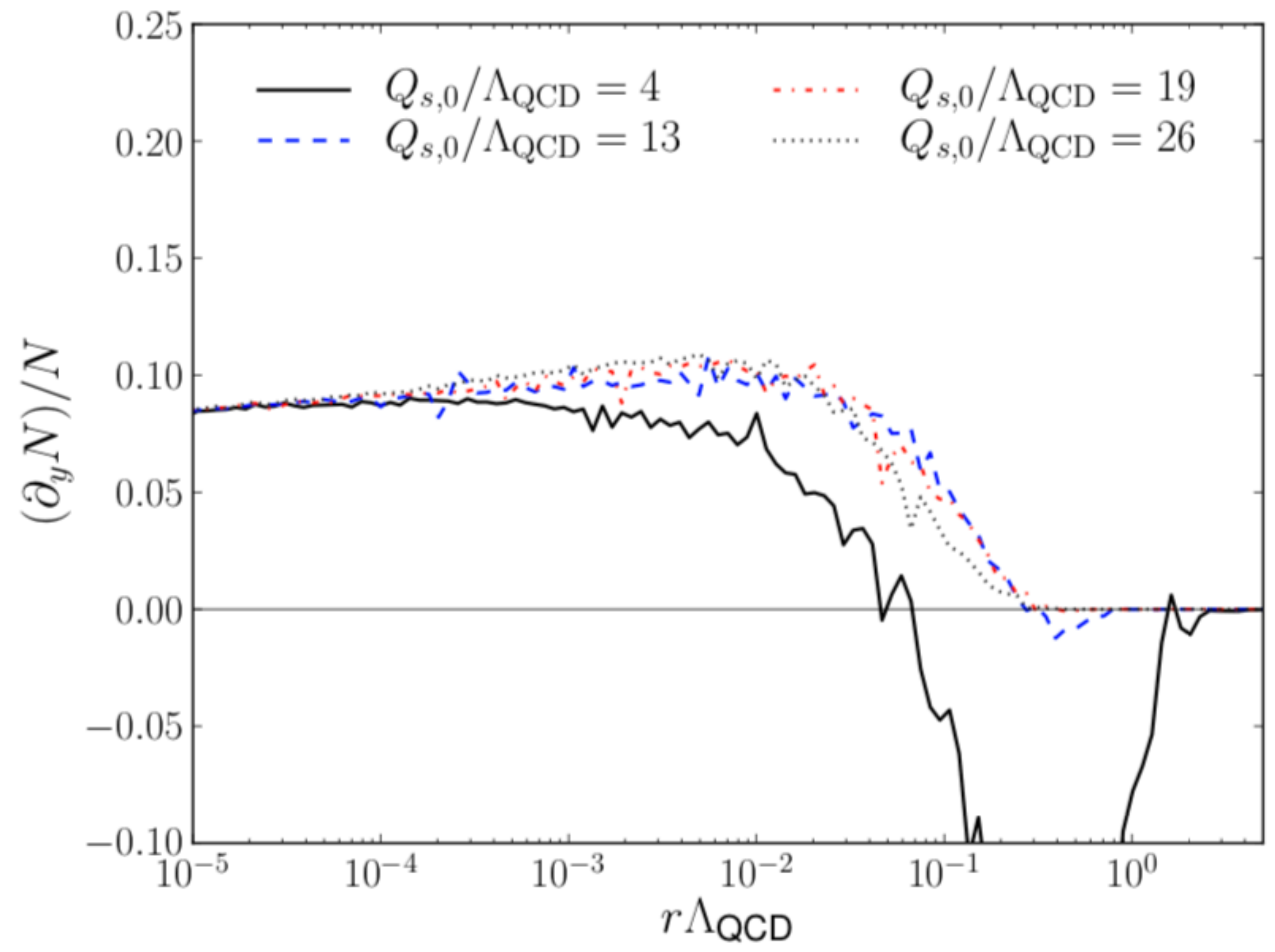

- Similar issues in forward hadron production in pA and forward dijet production in DIS

- Ad hoc solutions: restoring kinematic constraint in k^-, resummation, better choice of evolution variable $\eta \sim \log k^-$

Salam (1998), Shi, Wang, Wei, Xiao (2021) Liu, Xie, Kang, Liu, (2022) Caucal, Salazar, Schenke, Venugopalan, (2022) Taels, Altinoluk, Beuf, Marquet (2022)

Beyond shockwave approximation

- Sub-eikonal expansion around the shock wave $\delta(x^+)$ [Agostini, Altinoluk, Armesto, Beuf, Martinez, Moscoso, Salgado]
- Expansion in the boost parameter [Chirilli] ; [Altinoluk, Beuf, Czajka, Tymowska]
- Addition of a single hard scattering [Jalilian-Marian]

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Our approach:

- Revisit the shock wave factorization scheme to restore the x dependence of the gluon distribution - consistent with factorization in k^+ [Balitsky-Tarasov]
- Perform a partial twist expansion to connect Regge and Bjorken limits

$$f(k_\perp, x) + O\left(\frac{x_{\mathrm{Bj}}}{Q^2}\right)$$

Revisiting high energy factorization

- Restore x dependence: ordering of interactions in x^+ $\rightarrow$ built in k^- ordering

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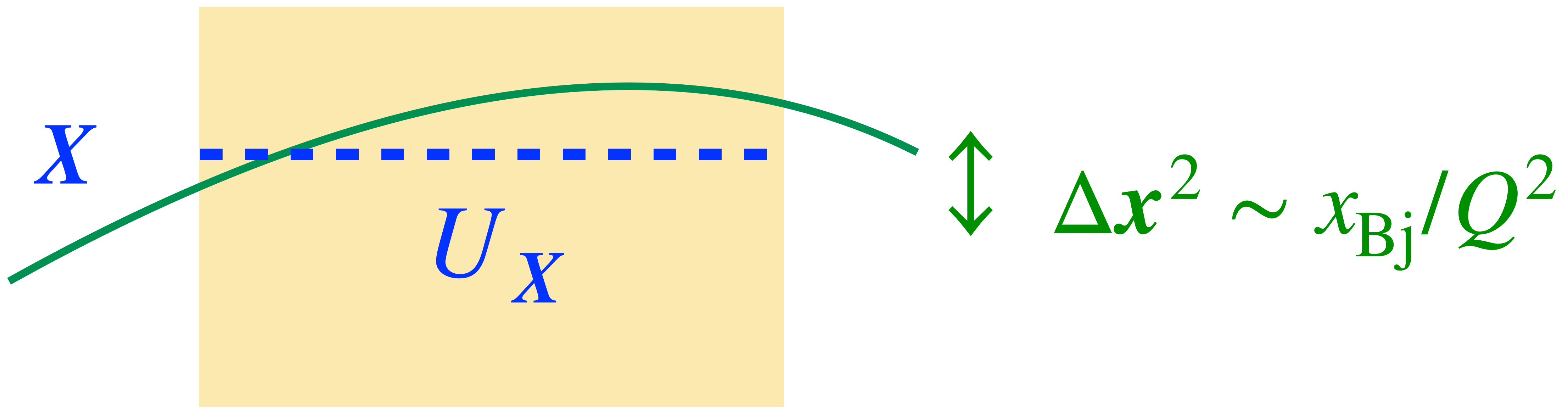

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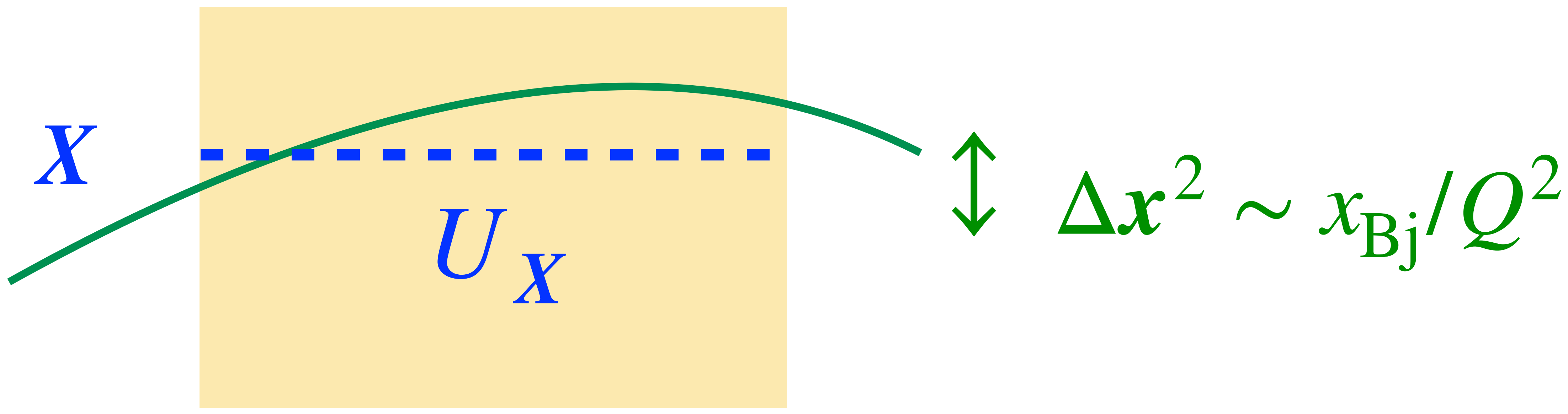

$$G \sim G_0(\Delta \boldsymbol{x}, \Delta x^+) \; U_{\boldsymbol{X}}$$

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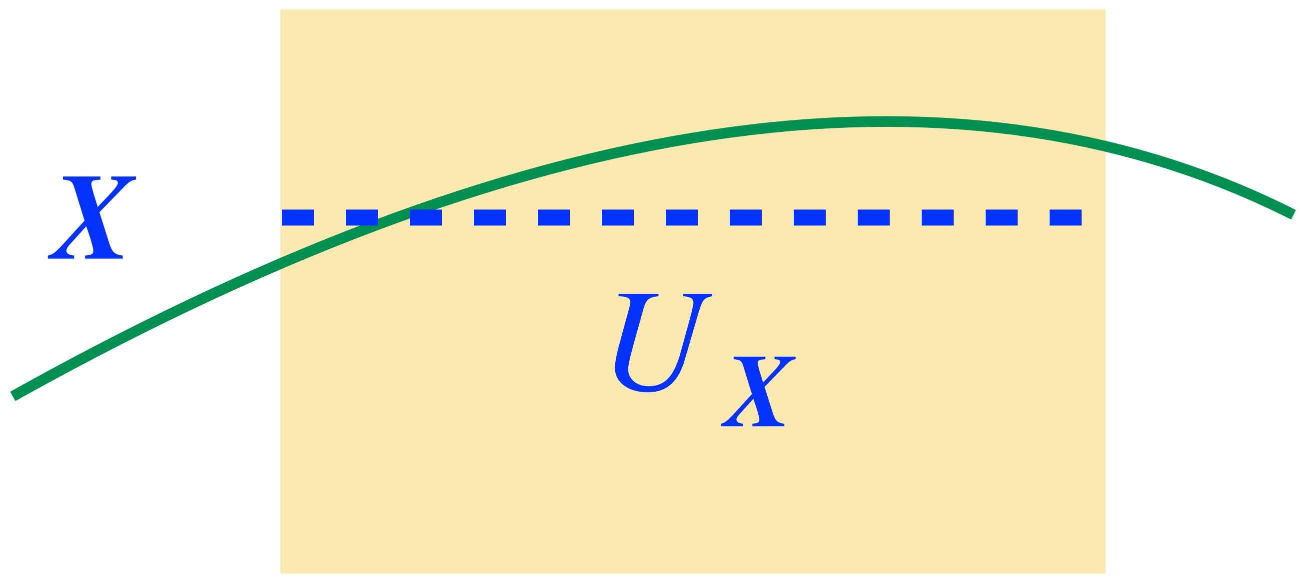

$\updownarrow \; \Delta \boldsymbol{x}^2 \sim x_{\mathrm{Bj}}/Q^2$

$$G \sim G_0(\Delta \boldsymbol{x}, \Delta x^+) \;\; U_{\boldsymbol{X}}$$

$$\mathrm{e}^{-ixP^-\Delta x^+}$$

quantum phase

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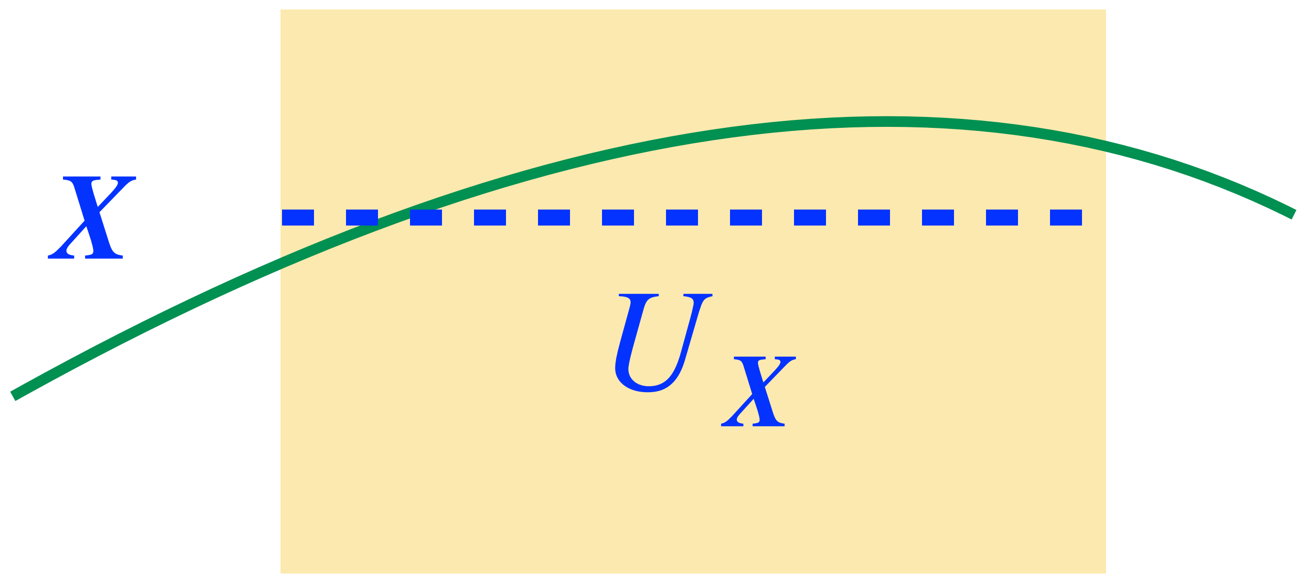

$\Delta \boldsymbol{x}^2 \sim x_{\mathrm{Bj}}/Q^2$

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$\mathrm{e}^{-ixP^- \Delta x^+}$

quantum phase

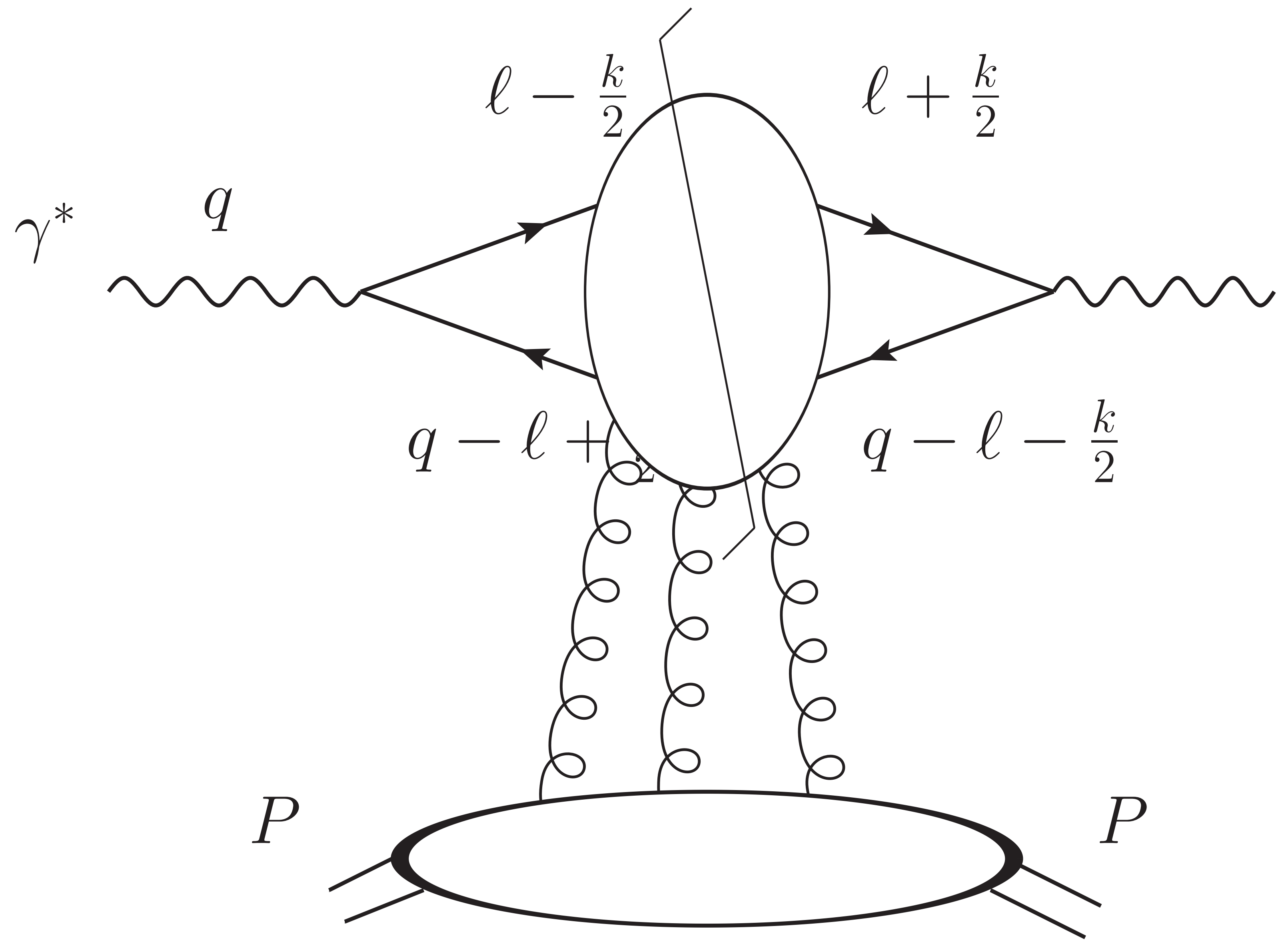
Wilson line

Revisiting rapidity factorization in DIS

- PTE to leading power yields the factorization formula (for the transverse photon cross-section), in momentum space,

Boussarie, MT (2020-2021)

$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 \mathrm{d}\,z \int_0^1 \mathrm{d}x \int_{\boldsymbol{\ell},\boldsymbol{k}} H\left(\boldsymbol{\ell}, \boldsymbol{k}, z, x_{Bj}\right) \delta\left(x - x_{Bj} - \frac{\boldsymbol{\ell}^2}{2z\bar{z}q^+}\right) xG^{ij}(x, \boldsymbol{k}) + O\left(k_\perp^2/s\right)$$



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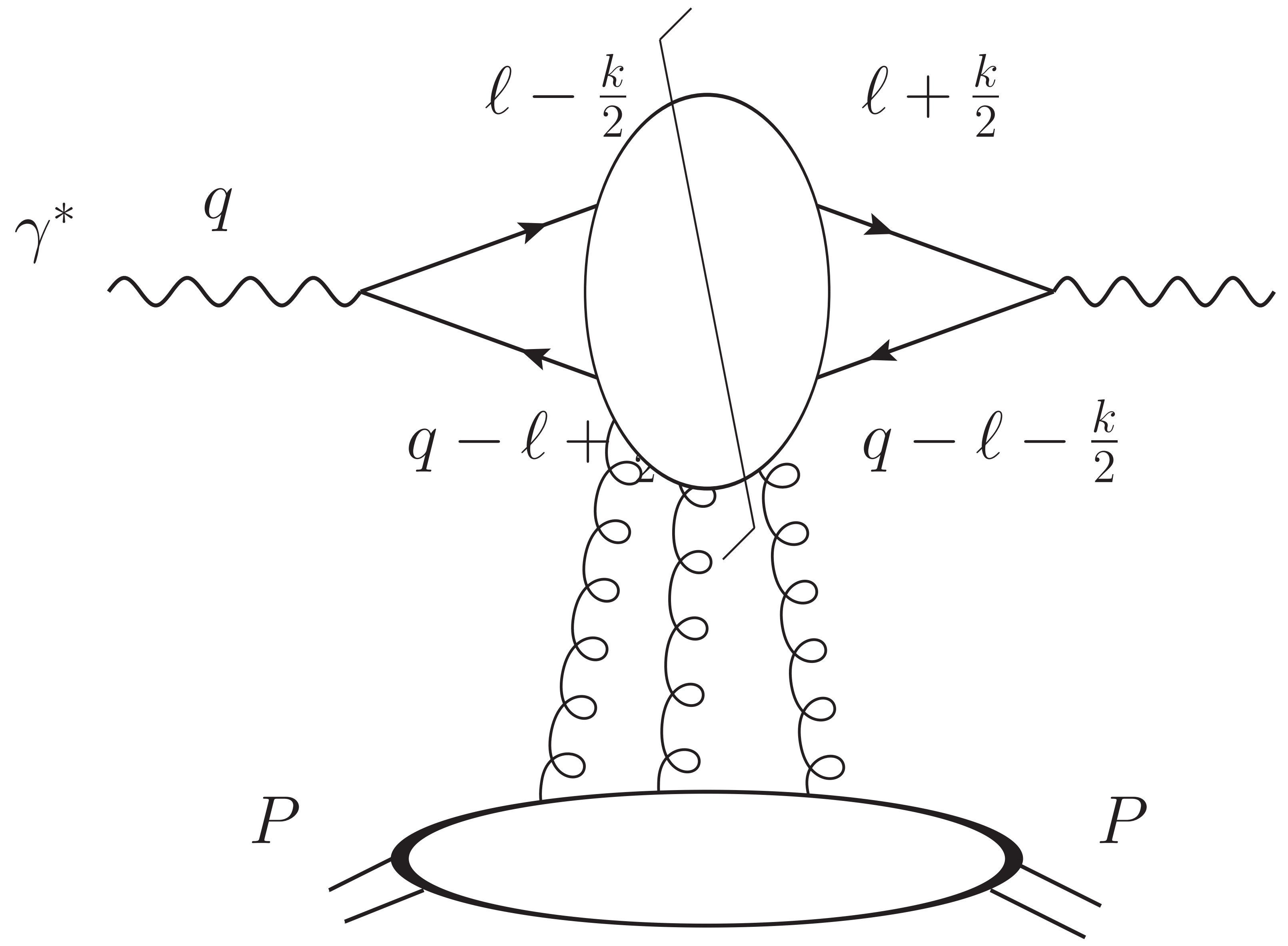
- Same wave functions as small x!



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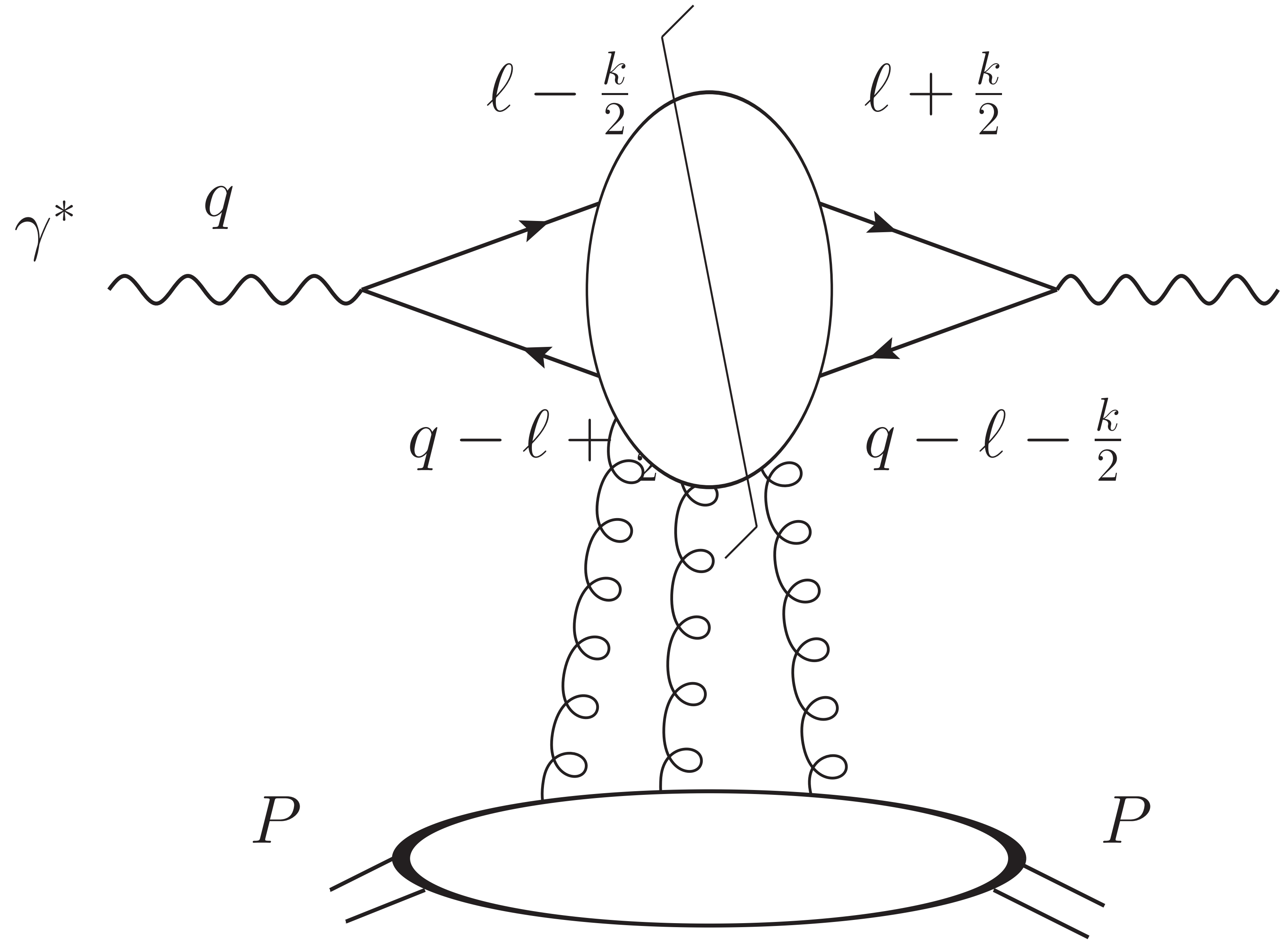
- The delta function relates x in the gluon distribution to x_{Bj} (kinematic constraint in momentum space)



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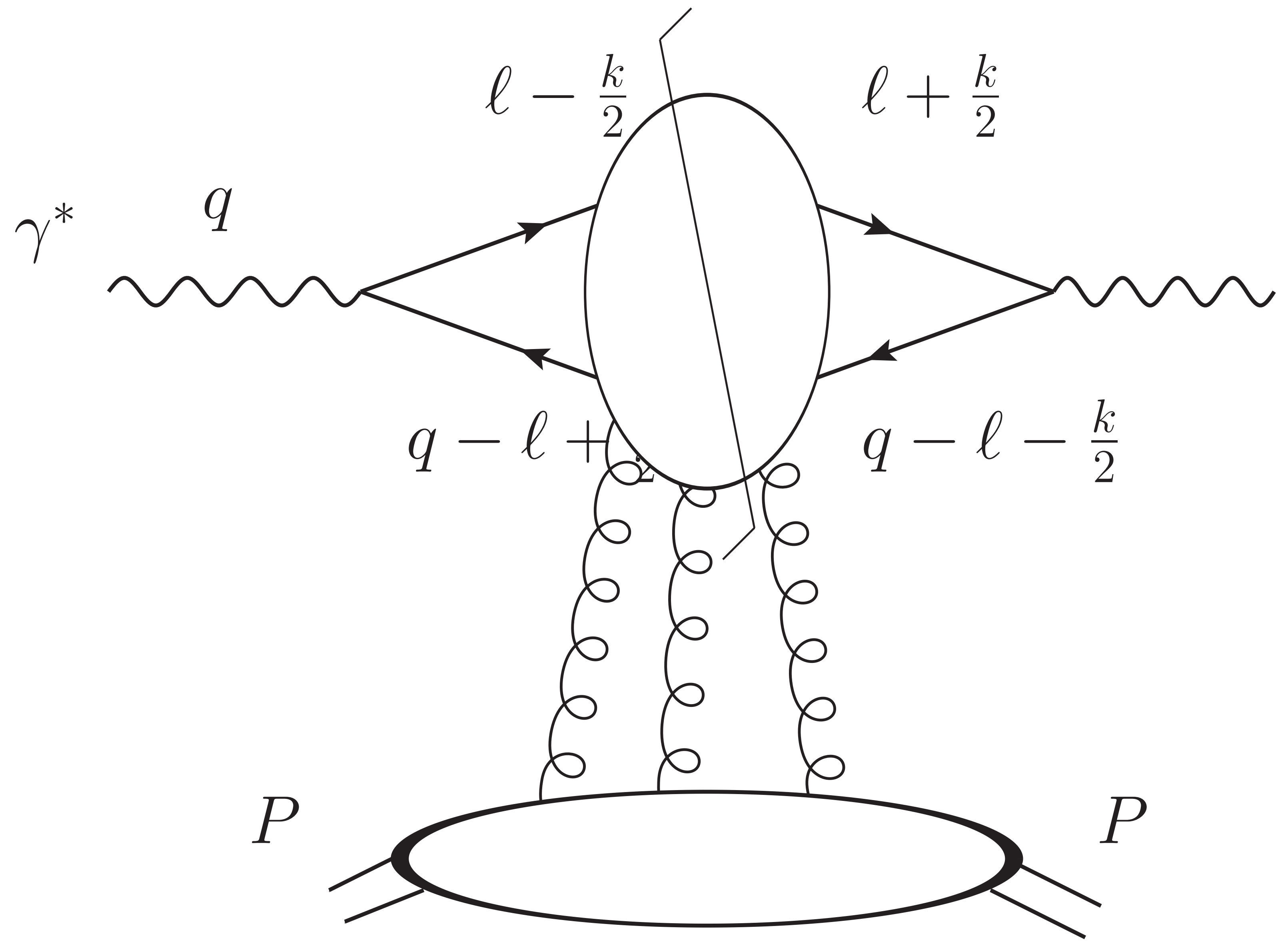
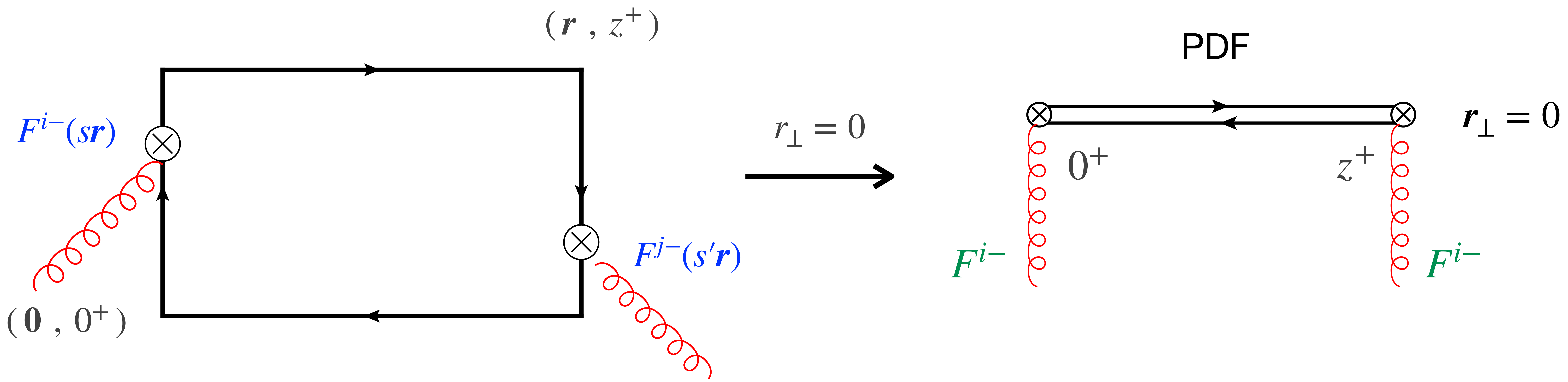
- Same wave functions as small x!
- The delta function relates x in the gluon distribution to x_{Bj} (kinematic constraint in momentum space)
- Gluon distribution different than small x



3D gluon distribution

Boussarie, MT (2020-2021)

$$xG^{ij}(x,k_\perp) \equiv 2\int_{s,s'}\int \frac{\mathrm{d}z^+\mathrm{d}\boldsymbol{r}}{(2\pi)^3P^-}\,\mathrm{e}^{\,ixP^-z^+-i\boldsymbol{k}\cdot\boldsymbol{r}}\,\left\langle P\,\middle|\,\mathrm{Tr}\,[\,0\,,z^+]_{\boldsymbol{r}}\,F^{j-}\,(z^+,s'\boldsymbol{r})\,[\,z^+\,,0]_{\boldsymbol{0}}\,F^{i-}\,(\,0\,,s\boldsymbol{r})\,\middle|\,P\right\rangle$$



- Note that this uPDF involves finite Wilson lines in contrast with gluon TMD's such as Weizsacker-Williams

Interpolating scheme for exclusive Compton scattering

Overarching scheme

$$\int \mathrm{d}x \int \mathrm{d}^d k\, G^{ij}(x,\xi,k,\Delta) H^{ij}(x,\xi,k,\Delta)$$

Bjorken limit

$$\int \mathrm{d}x H^{ij}(x,\xi,0,\Delta) \times \left[\int \mathrm{d}^d k\, G^{ij}(x,\xi,k,\Delta)\right]$$

Regge limit

$$\lim_{\xi\to 0} \int \mathrm{d}^d k\, G^{ij}(0,\xi,k,\Delta) \times \int \mathrm{d}x H^{ij}_{\mathrm{cut}}(x,\xi,k,\Delta)$$

We found an interpolating scheme

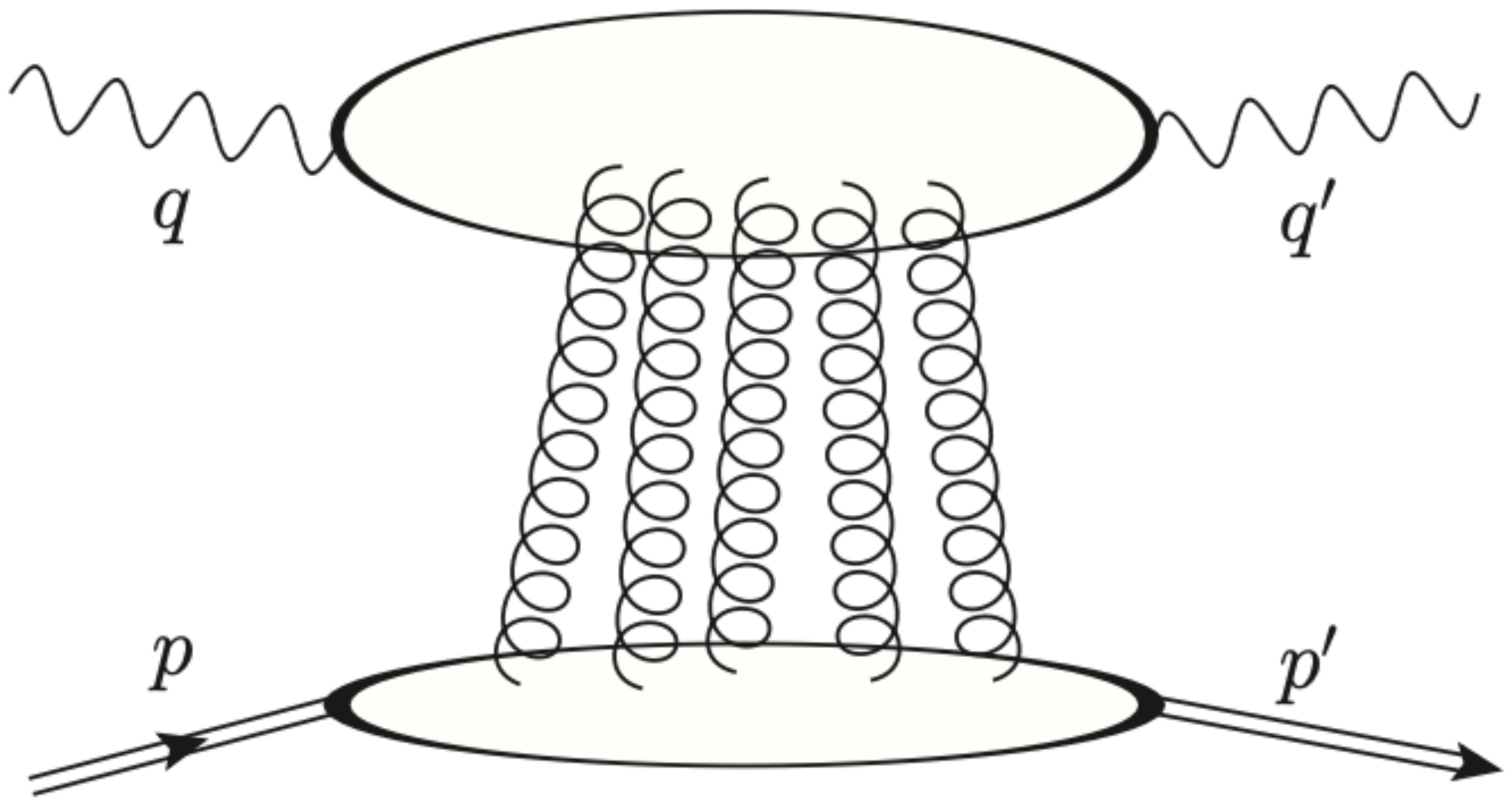
Cross-checks I: NLO DIS

- In the collinear limit $Q^2 \to \infty$, we recover the 1-loop contribution to the DIS structure function

$$F_T(x_{Bj}, Q^2) = \frac{\alpha_s}{\pi} \sum_f q_f^2 \int_{x_{Bj}}^1 dy \; xg(x_{Bj}/y, \mu^2)$$

$$\times \left[\frac{1}{\epsilon} \left(\frac{\mathrm{e}^{\gamma_E}}{4\pi} \right)^{\epsilon} P_{qg}(y) + \left[(1-y)^2 + y^2 \right] \log \left[\frac{Q^2(1-y)}{\mu^2 y} \right] - 1 + 4y(1-y) \right]$$

Double, Spacelike, and Timelike exclusive Compton Scattering



Longitudinal momentum variables:

$$x, \quad \xi \sim \frac{-q^2 + q'^2}{4q \cdot P}, \quad x_{\mathrm{Bj}} = \frac{-q^2 - q'^2}{4q \cdot P}$$

Can we restore the dependence on all 3 variables in a CGC-like scheme?

Cross-checks II: NLO DVCS, TCS and DDVCS

Boussarie, MT (in preparation)

$$\overline{Q}^2 \equiv \frac{Q^2 - Q'^2}{2}, \quad x_{\rm Bj} \equiv \frac{\overline{Q}^2}{2q^+P^-}$$

LL transition:

$$\mathcal{T}_{LL}^{\rm Bjorken} = i\alpha_{\rm em}\alpha_s \sum_f q_f^2 2q^+P^- \frac{16QQ'}{(Q^2+Q'^2)^2}\int {\rm d}x \frac{G^{ii}(x,\xi,\boldsymbol{\Delta})}{x-\xi} \times \left[\left(\frac{x_{\rm Bj}}{\xi} - \frac{x_{\rm Bj}+\xi}{x-\xi}\right)\ln\left(\frac{x_{\rm Bj}+x-i0}{x_{\rm Bj}+\xi-i0}\right) + \left(\frac{x_{\rm Bj}}{\xi} - \frac{x_{\rm Bj}-\xi}{x-\xi}\right)\ln\left(\frac{x_{\rm Bj}-x-i0}{x_{\rm Bj}-\xi-i0}\right)\right],$$

TT transition:

$$\begin{aligned}
\mathcal{T}_{hh';p}^{\rm Bjorken} &= 2\alpha_{\rm em}\alpha_s \sum_f q_f^2 \epsilon^{mn} e_h^m e_{h'}^{\prime n*} \frac{1}{\epsilon}\left(\frac{{\rm e}^{\gamma_E}}{4\pi}\frac{Q^2+Q'^2}{2\mu^2}\right)^\epsilon \\
&\times \int {\rm d}x \frac{\widetilde{G}(x,\xi,\boldsymbol{\Delta})}{(x+\xi-i0x_{\rm Bj})^2(x-\xi+i0x_{\rm Bj})^2} \\
&\times \{(x^2-2xx_{\rm Bj}+\xi^2)\ln(x_{\rm Bj}-x-i0) - (x^2+2xx_{\rm Bj}+\xi^2)\ln(x_{\rm Bj}+x-i0) \\
&+ 2x(x_{\rm Bj}+\xi)\ln(x_{\rm Bj}+\xi-i0) + 2x(x_{\rm Bj}-\xi)\ln(x_{\rm Bj}-\xi-i0) \\
&- 2\epsilon(2x^2-3xx_{\rm Bj}+\xi^2)\ln(x_{\rm Bj}-x-i0) + 2\epsilon(2x^2+3xx_{\rm Bj}+\xi^2)\ln(x_{\rm Bj}+x-i0) \\
&- 6\epsilon x(x_{\rm Bj}+\xi)\ln(x_{\rm Bj}+\xi-i0) - 6\epsilon x(x_{\rm Bj}-\xi)\ln(x_{\rm Bj}-\xi-i0) \\
&+ \frac{1}{2}\epsilon(x^2-2xx_{\rm Bj}+\xi^2)\ln^2\left(\frac{x_{\rm Bj}-x-i0}{\xi}\right) - \frac{1}{2}\epsilon(x^2+2xx_{\rm Bj}+\xi^2)\ln^2\left(\frac{x_{\rm Bj}+x-i0}{\xi}\right) \\
&+ \epsilon x(x_{\rm Bj}+\xi)\ln^2\left(\frac{x_{\rm Bj}+\xi-i0}{\xi}\right) + \epsilon x(x_{\rm Bj}-\xi)\ln^2\left(\frac{x_{\rm Bj}-\xi-i0}{\xi}\right)\Big\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{hh';u}^{\rm Bjorken} &= -2\alpha_{\rm em}\alpha_s \sum_f q_f^2 \delta^{mn} e_h^m e_{h'}^{\prime n*} \int {\rm d}x \frac{G(x,\xi,\boldsymbol{\Delta})}{(x+\xi-i0x_{\rm Bj})(x-\xi+i0x_{\rm Bj})} \\
&\times \frac{1}{\epsilon}(1-2\epsilon)\left(\frac{{\rm e}^{\gamma_E}}{4\pi}\frac{Q^2+Q'^2}{2\mu^2}\right)^\epsilon \\
&\times \Big\{\frac{x^2-2x_{\rm Bj}x+2x_{\rm Bj}^2-\xi^2}{(x+\xi-i0x_{\rm Bj})(x-\xi+i0x_{\rm Bj})}\ln(x_{\rm Bj}-x-i0) \\
&+ \frac{x^2+2x_{\rm Bj}x+2x_{\rm Bj}^2-\xi^2}{(x+\xi-i0x_{\rm Bj})(x-\xi+i0x_{\rm Bj})}\ln(x_{\rm Bj}+x-i0) \\
&+ \frac{x_{\rm Bj}-\xi}{\xi}\frac{x^2-2x_{\rm Bj}\xi-\xi^2}{(x+\xi-i0x_{\rm Bj})(x-\xi+i0x_{\rm Bj})}\ln(x_{\rm Bj}-\xi-i0) \\
&- \frac{x_{\rm Bj}+\xi}{\xi}\frac{x^2+2x_{\rm Bj}\xi-\xi^2}{(x+\xi-i0x_{\rm Bj})(x-\xi+i0x_{\rm Bj})}\ln(x_{\rm Bj}+\xi-i0) \\
&+ \epsilon\ln(x_{\rm Bj}-x-i0) + \epsilon\ln(x_{\rm Bj}+x-i0) \\
&- \epsilon\frac{x_{\rm Bj}+\xi}{\xi}\ln(x_{\rm Bj}+\xi-i0) + \epsilon\frac{x_{\rm Bj}-\xi}{\xi}\ln(x_{\rm Bj}-\xi-i0) \\
&+ \frac{\epsilon}{2}\frac{(x^2-2x_{\rm Bj}x+2x_{\rm Bj}^2-\xi^2)}{(x+\xi-i0x_{\rm Bj})(x-\xi+i0x_{\rm Bj})}\ln^2\left(\frac{x_{\rm Bj}-x-i0}{\xi}\right) \\
&+ \frac{\epsilon}{2}\frac{(x^2+2x_{\rm Bj}x+2x_{\rm Bj}^2-\xi^2)}{(x+\xi-i0x_{\rm Bj})(x-\xi+i0x_{\rm Bj})}\ln^2\left(\frac{x_{\rm Bj}+x-i0}{\xi}\right) \\
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&- \frac{\epsilon}{2}\frac{x_{\rm Bj}+\xi}{\xi}\frac{x^2+2x_{\rm Bj}\xi-\xi^2}{(x+\xi-i0x_{\rm Bj})(x-\xi+i0x_{\rm Bj})}\ln^2\left(\frac{x_{\rm Bj}+\xi-i0}{\xi}\right)\Big\},
\end{aligned}$$

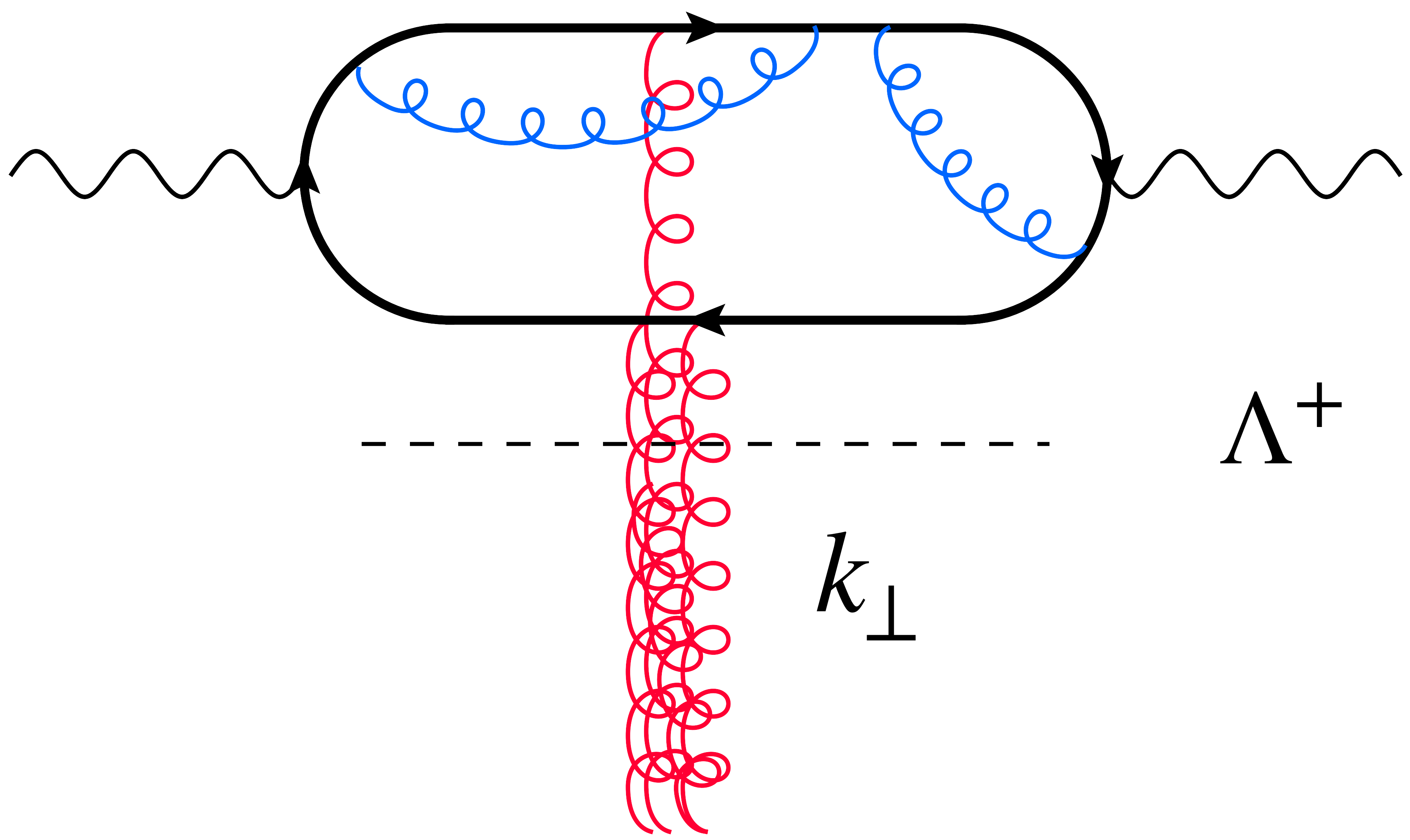
[Hoodboy, Ji (1998)]

[Pire, Szymanowski, Wagner (2011)]

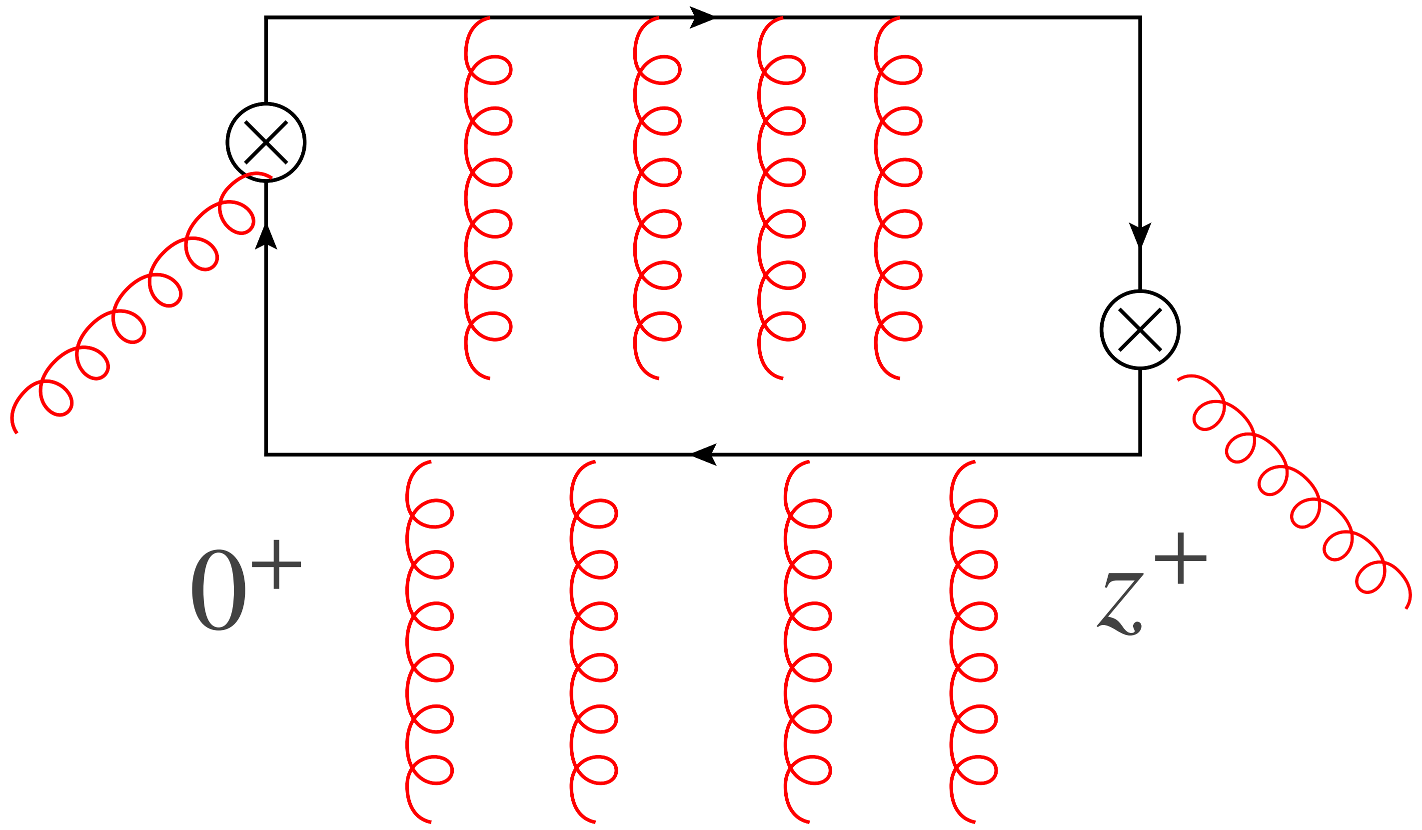
Structure of quantum corrections

- Rapidity evolution: increase the space of the gluon kt-distribution from 1 to 2 variables

$$\varphi_G(\boldsymbol{k};\Lambda^+) \equiv \frac{1}{N_c}\int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{r}\cdot\boldsymbol{k}}\,\langle \mathrm{tr}\, U_{\boldsymbol{0}} U^\dagger_{\boldsymbol{r}}\rangle_{\Lambda^+} \quad \to \quad G^{ij}_{\Lambda^+}(x,\boldsymbol{k})$$

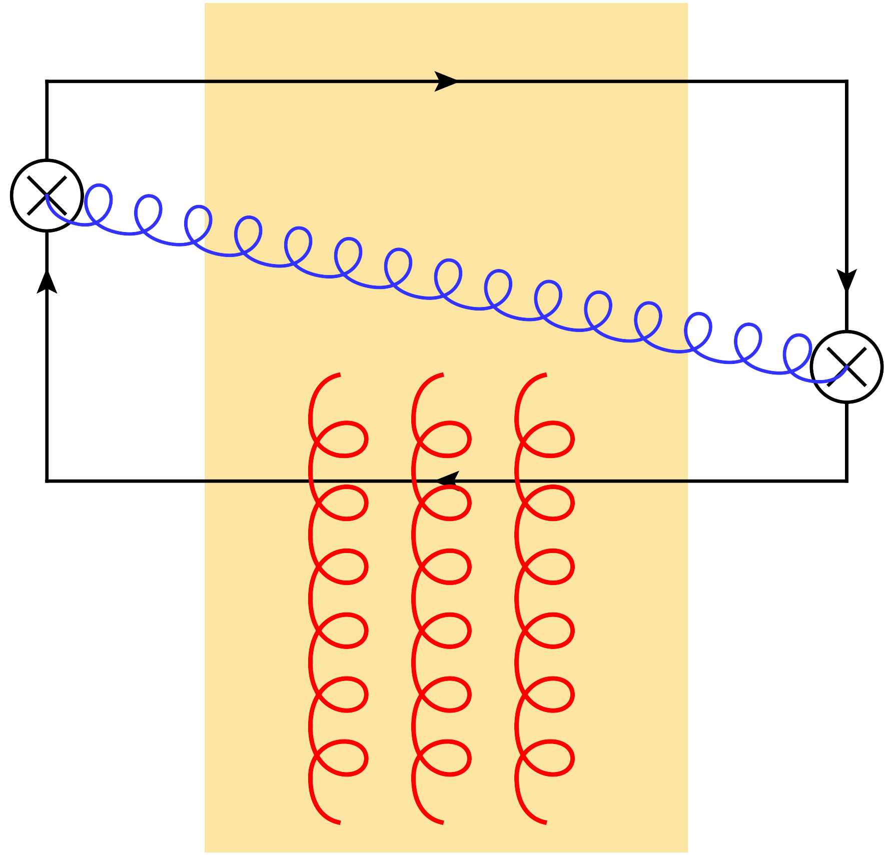


Structure of quantum corrections (in progress)



Background field method:

$$A^\mu \;\rightarrow\; a^\mu + A^\mu$$



Enhanced at small x

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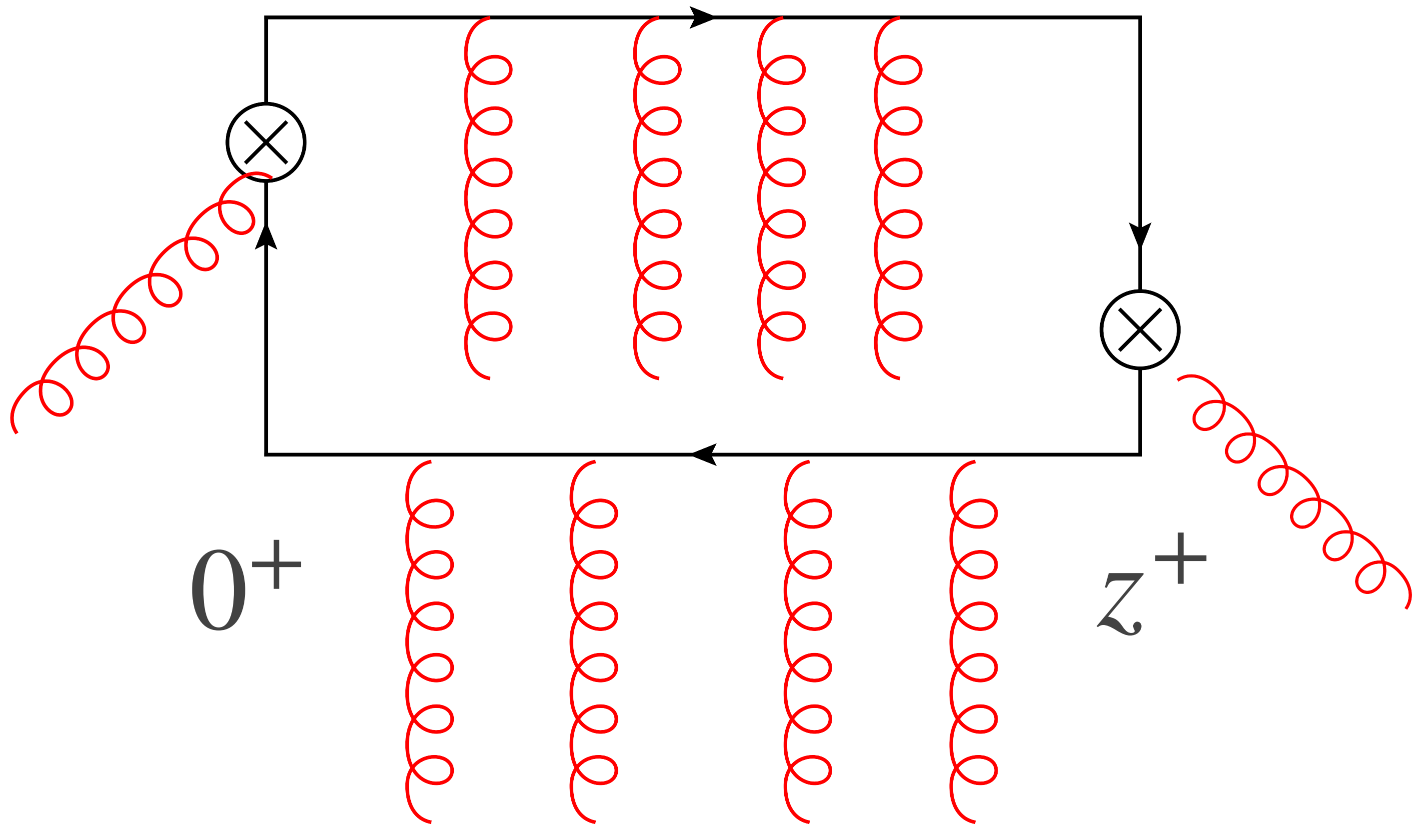

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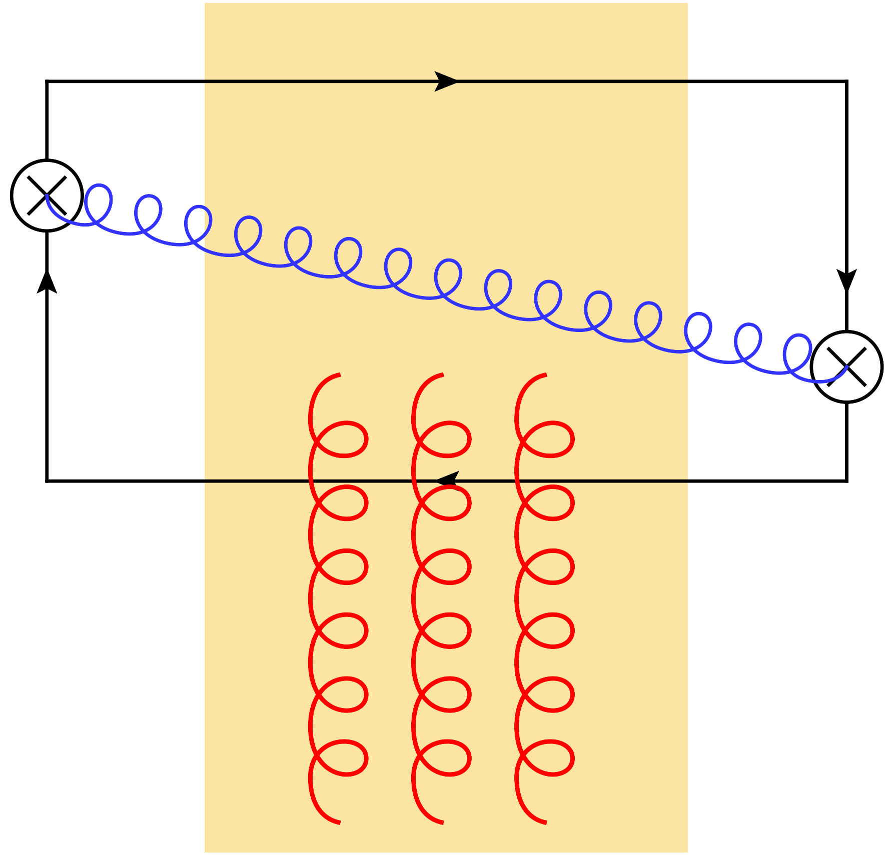

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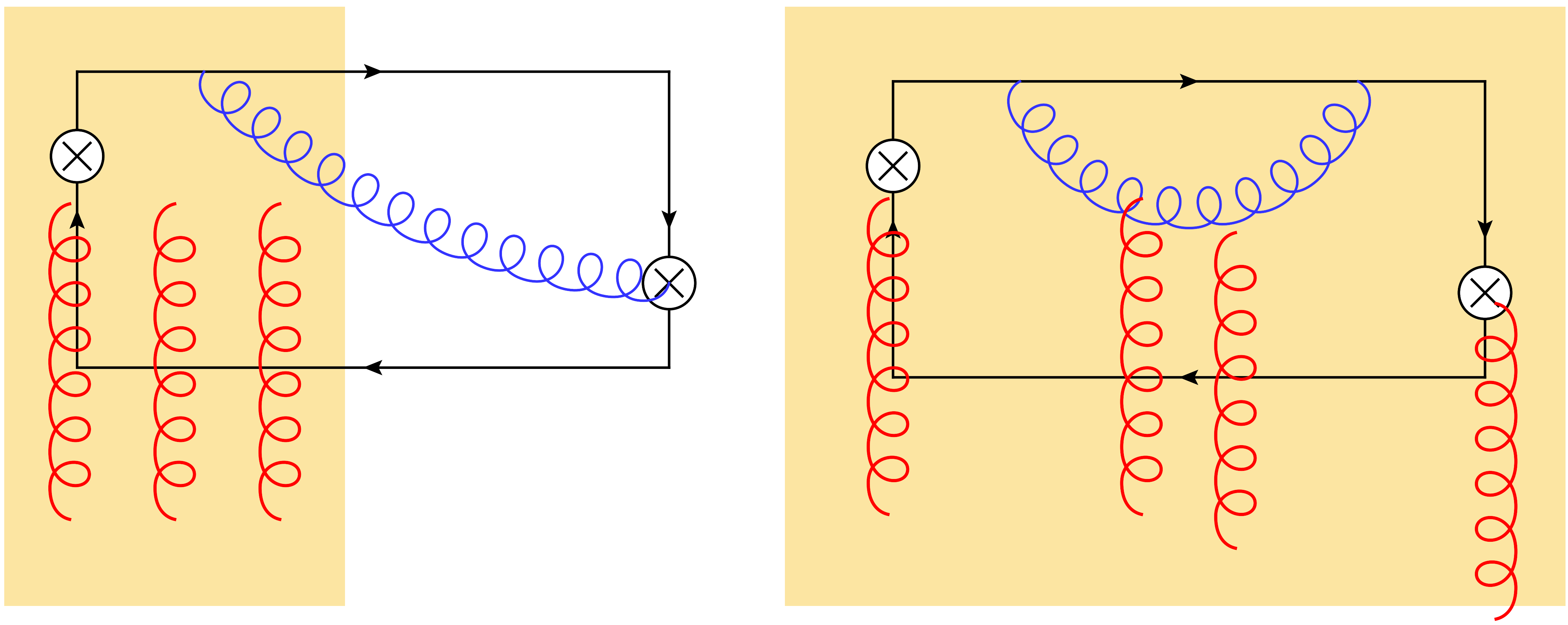
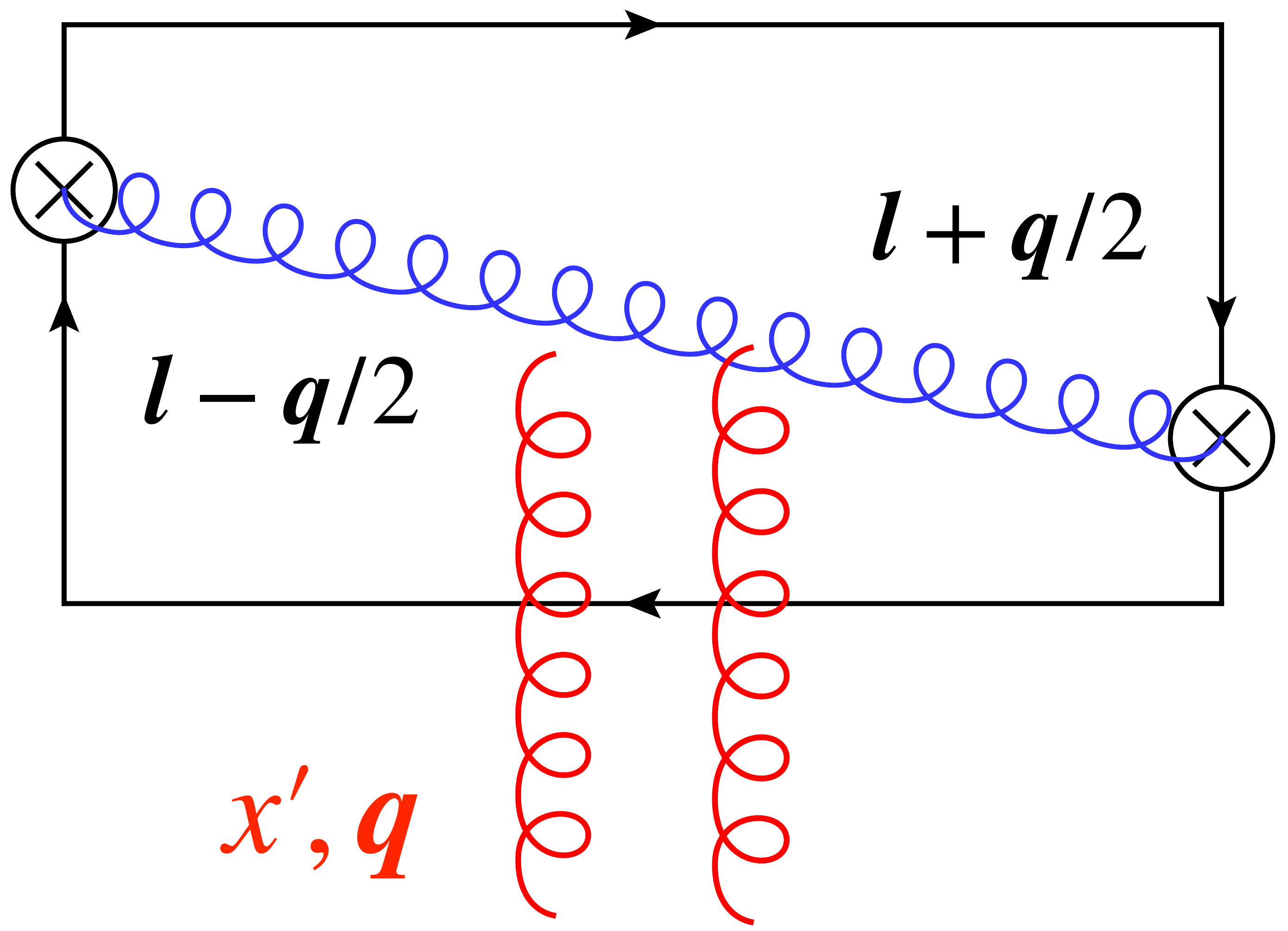


Suppressed at small x as O(1/s) (necessary to recover DGLAP at leading twist)

Dynamical kinematic constraint: linear regime (in progress)

$$\Delta G^{ij}(x,\boldsymbol{k}) \sim \bar{\alpha}\int \frac{\mathrm{dk}^+}{k^+}\int \mathrm{d}x' \int_{\boldsymbol{l},\boldsymbol{q}} K^{ij,lm}(x,\boldsymbol{l},\boldsymbol{k},\boldsymbol{q})\,\delta\left(x'-x-\frac{\boldsymbol{l}^2}{2k^+P^-}\right) xG^{lm}(x',\boldsymbol{q}) \;+\; \mathrm{non-lin}$$

- Similar to DIS factorization formula
- k^+ is the evolution variable
- The ordering in x is explicit $x' > x$
- Rapidity and collinear divergence recovered in the corresponding limits



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