

Longitudinal Double-Spin Asymmetry at Small x

Small- x Effective Hamiltonian and The Pure Glue Case

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Origin of Nucleon Spin

Jaffe-Manohar spin sum rule for proton

The RHIC Spin Collaboration (2015)

$$S_q + L_q + S_G + L_G = \frac{1}{2}$$

Quark Spin

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx \Delta \Sigma(x, Q^2)$$

$$S_q(Q^2 = 10\text{GeV}^2) \approx [0.15, 0.20]$$

$$x \in [0.001, 0.7]$$

Gluon Spin

$$S_G(Q^2) = \int_0^1 dx \Delta G(x, Q^2)$$

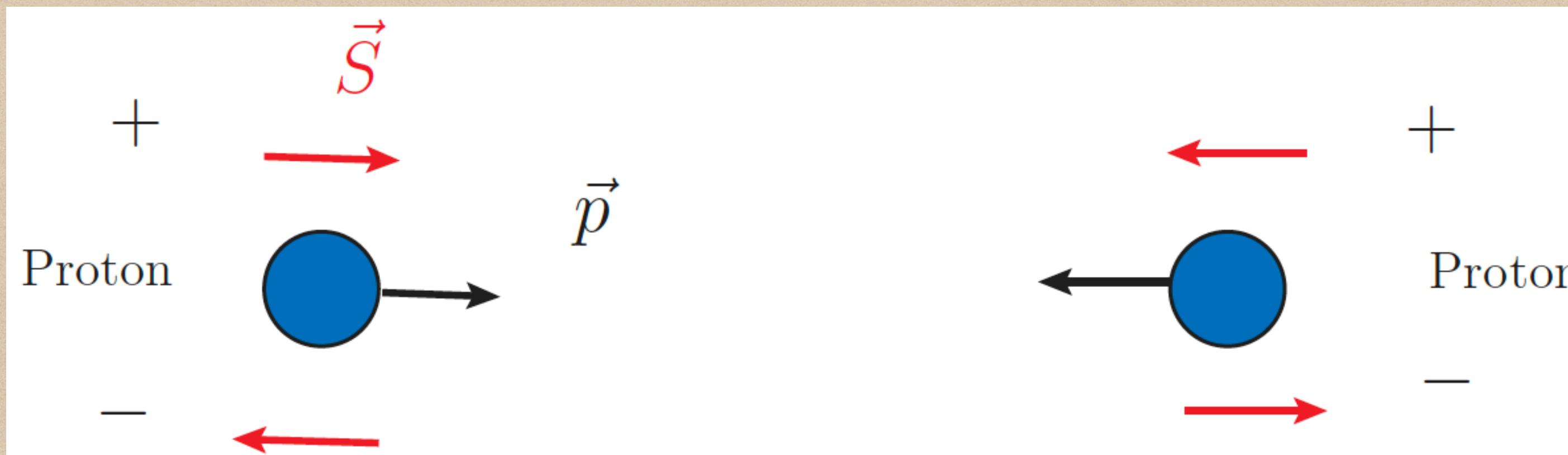
$$S_G(Q^2 = 10\text{GeV}^2) \approx [0.13, 0.26]$$

$$x \in [0.05, 0.7]$$

**Missing spin of the proton maybe in Quark and Gluon Orbital Angular Momentum L_q and L_G
and/or smaller values of x**

Longitudinal Double-Spin Asymmetry

How to measure quark and gluon intrinsic spin contribution inside proton?



$$A_{LL} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

RHIC has measured A_{LL} for the productions of jets, dijets, π^0 , π^\pm , direct photons...

RHIC Spin Collaboration, arXiv: 2302.00605

Longitudinal Double-Spin Asymmetry

Longitudinal double-spin asymmetry is related to parton helicity distribution.

$$A_{\text{LL}} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

Collinear Factorization (also parity invariance)

*Babcock, Monday and Sivers (1979),
de Florian, Sassot, Stratmann and Vogelsang (2008)(2014)*

$$d\Delta\sigma = \sum_{ab} \int dx_a \int dx_b \Delta f_a(x_a, Q^2) \Delta f_b(x_b, Q^2) d\Delta\hat{\sigma}_{ab}(x_a, x_b, p_T, \alpha_s(Q^2), p_T/Q)$$

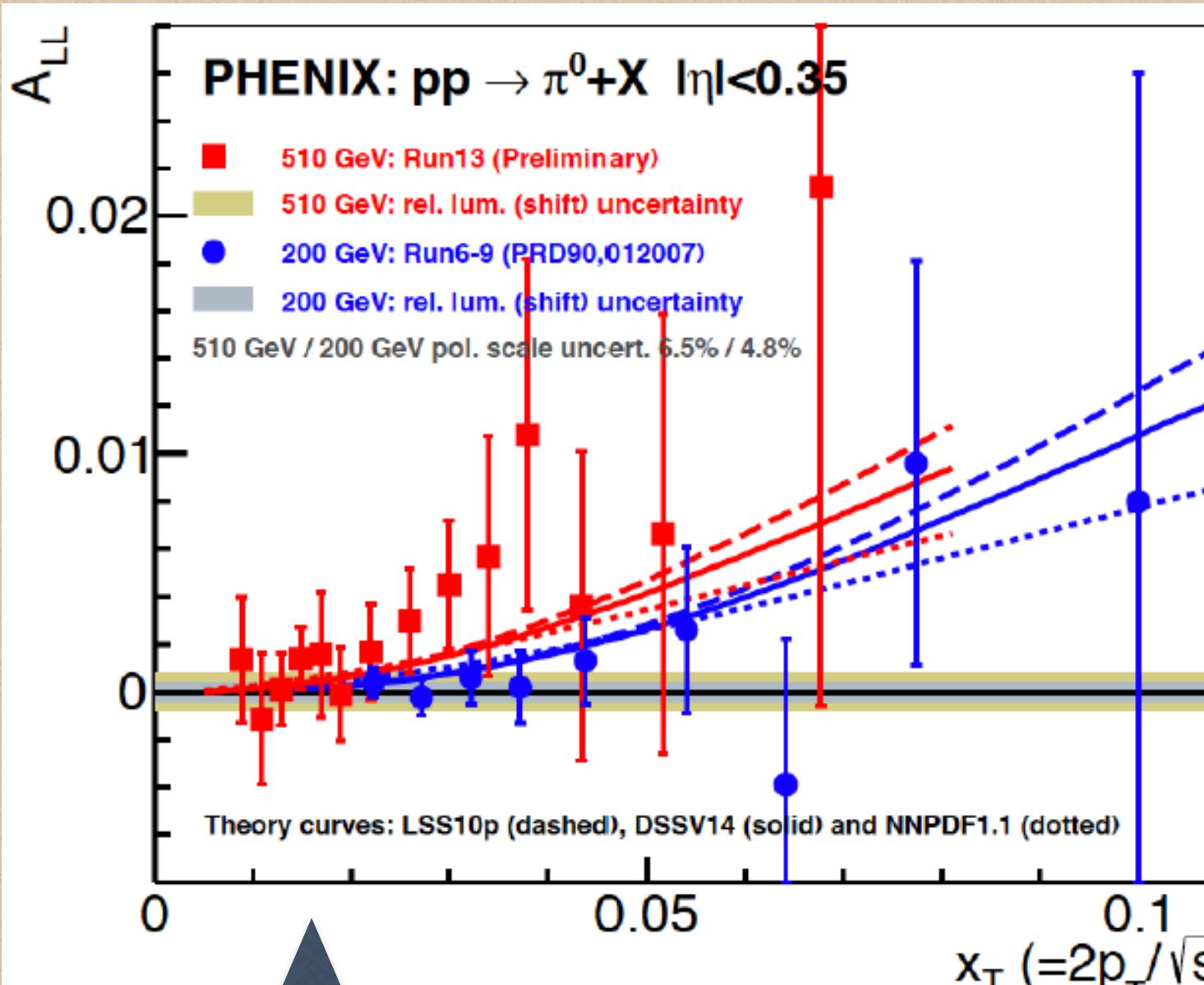
(Anti) quark and gluon helicity distribution $\Delta f_j(x, Q^2) \equiv f_j^+(x, Q^2) - f_j^-(x, Q^2)$

Partonic level double-spin asymmetry $d\Delta\hat{\sigma} = d\hat{\sigma}^{++} - d\hat{\sigma}^{+-}$

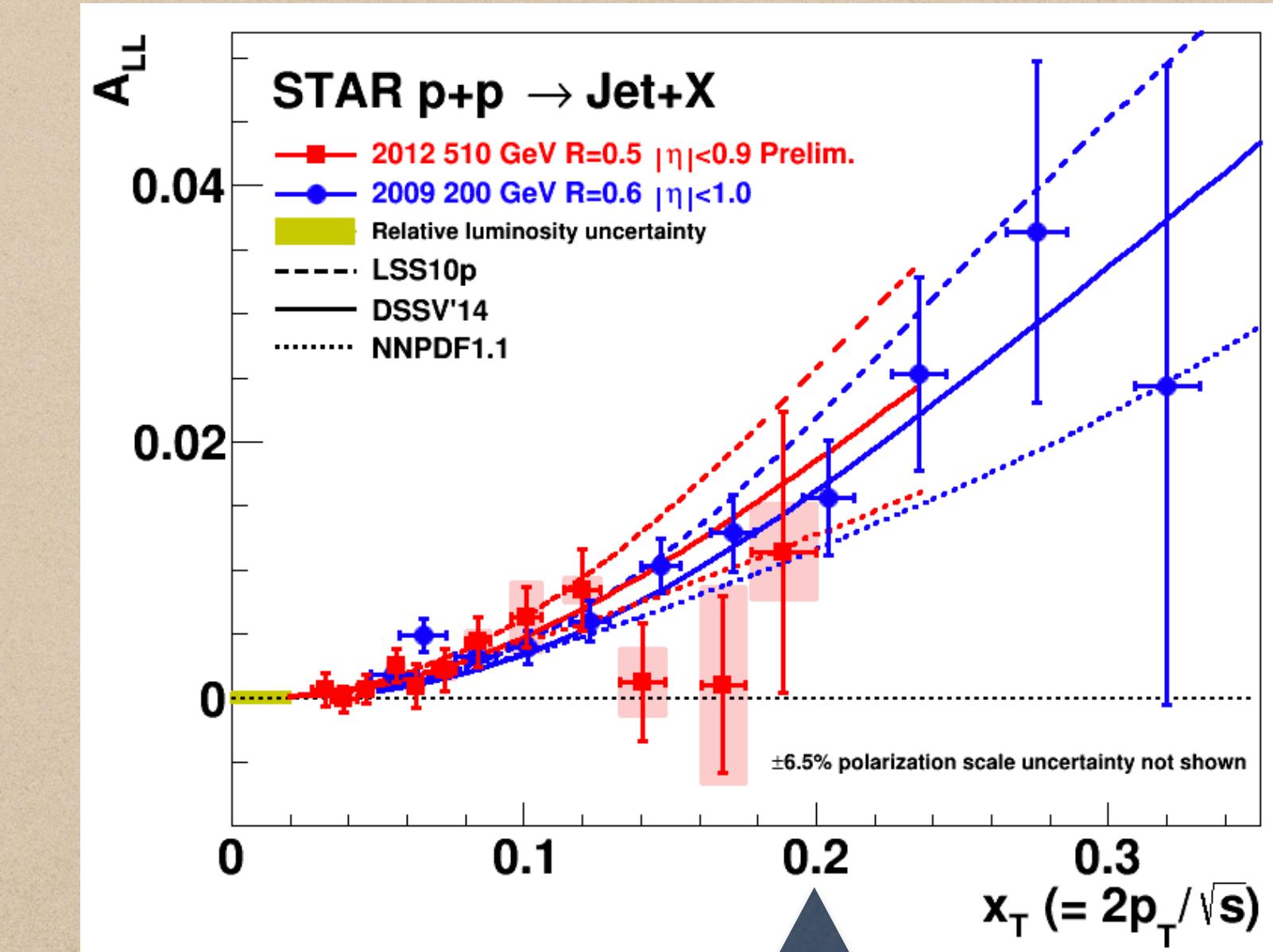
Longitudinal Double-Spin Asymmetry at small x

Collinear Factorization (applicable for large transverse momentum)

$$d\Delta\sigma = \sum_{ab} \int dx_a \int dx_b \Delta f_a(x_a, Q^2) \Delta f_b(x_b, Q^2) d\Delta\hat{\sigma}_{ab}(x_a, x_b, p_T, \alpha_s(Q^2), p_T/Q)$$



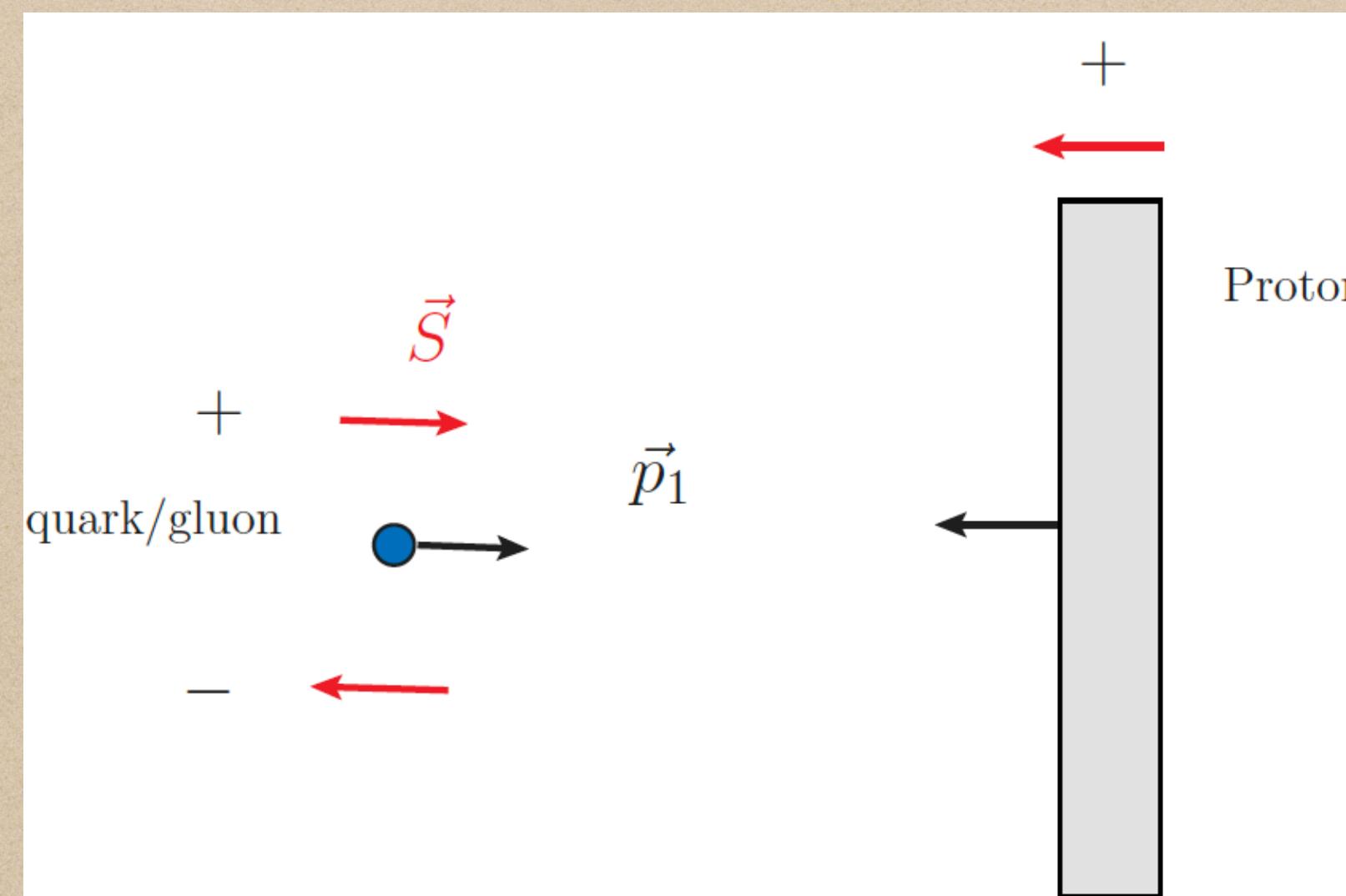
*Low transverse momentum region,
sensitive to small x gluons, collinear
factorization probably breaks down*



*Large transverse momentum region,
collinear factorization successful, but
not sensitive to small x gluons.*

RHIC Spin
Collaboration (2015)

Double-spin asymmetry at small x : Soft gluon production

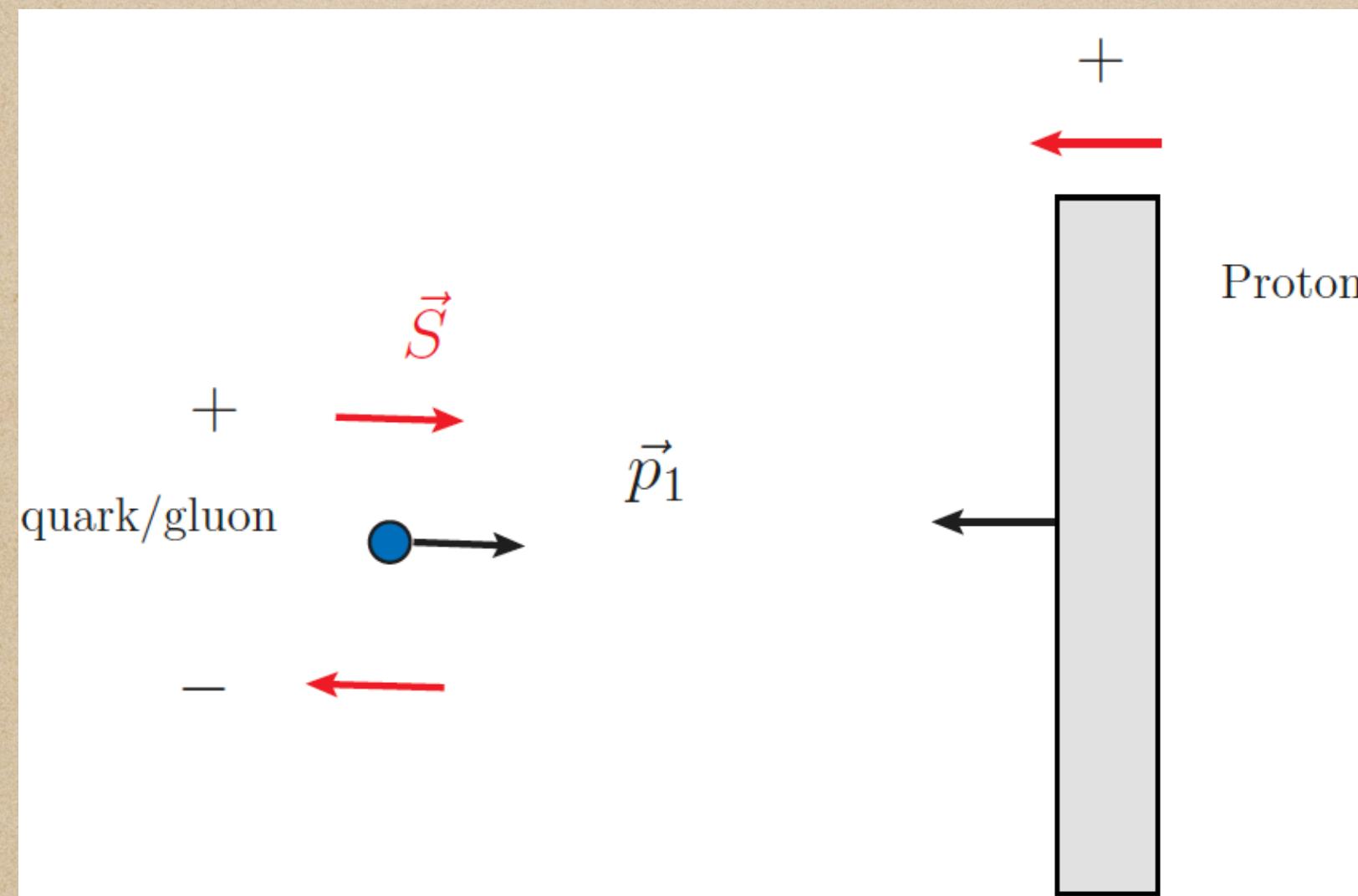


quark/gluon + proton \longrightarrow gluon + X

$$A_{\text{LL}}^g \equiv \frac{d\Delta\sigma}{d^2\mathbf{p}dy} = \frac{d\sigma^+}{d^2\mathbf{p}dy} - \frac{d\sigma^-}{d^2\mathbf{p}dy}$$

Goal: A_{LL}^g for Gluon production at midrapidity

The Formalism: Small- x Effective Hamiltonian



The Shockwave Picture of High energy Scatterings:
proton is treated as background gluon and quark fields

S-matrix element for highly boosted states

$$S_{\text{fi}} = \langle \phi_f | e^{i\omega \hat{K}^3} \mathcal{P}\exp \left\{ -i \int_{-\infty}^{+\infty} dz^+ V_I(z^+) \right\} e^{-i\omega \hat{K}^3} | \phi_i \rangle$$

$$= \langle \phi_f | \mathcal{P}\exp \left\{ -i \int_{-\infty}^{+\infty} dz^+ e^{i\omega \hat{K}^3} V_I(z^+) e^{-i\omega \hat{K}^3} \right\} | \phi_i \rangle.$$

S-matrix element of boosted interaction for unboosted states

Light-cone Hamiltonian
in the background fields

Boosting the background fields,
expanding in powers of $\xi = e^{-\omega}$

Small-x effective Hamiltonian
up to linear order in ξ

Bjorken, Kogut and Soper
(1971)

The Formalism: Small- x Effective Hamiltonian

Light-Cone Gauge $A^+ = 0$.

Expansion in ξ is equivalent to expansion in x : $\xi = e^{-|Y_P - Y_T|} \sim xe^{-m_N/Q}$

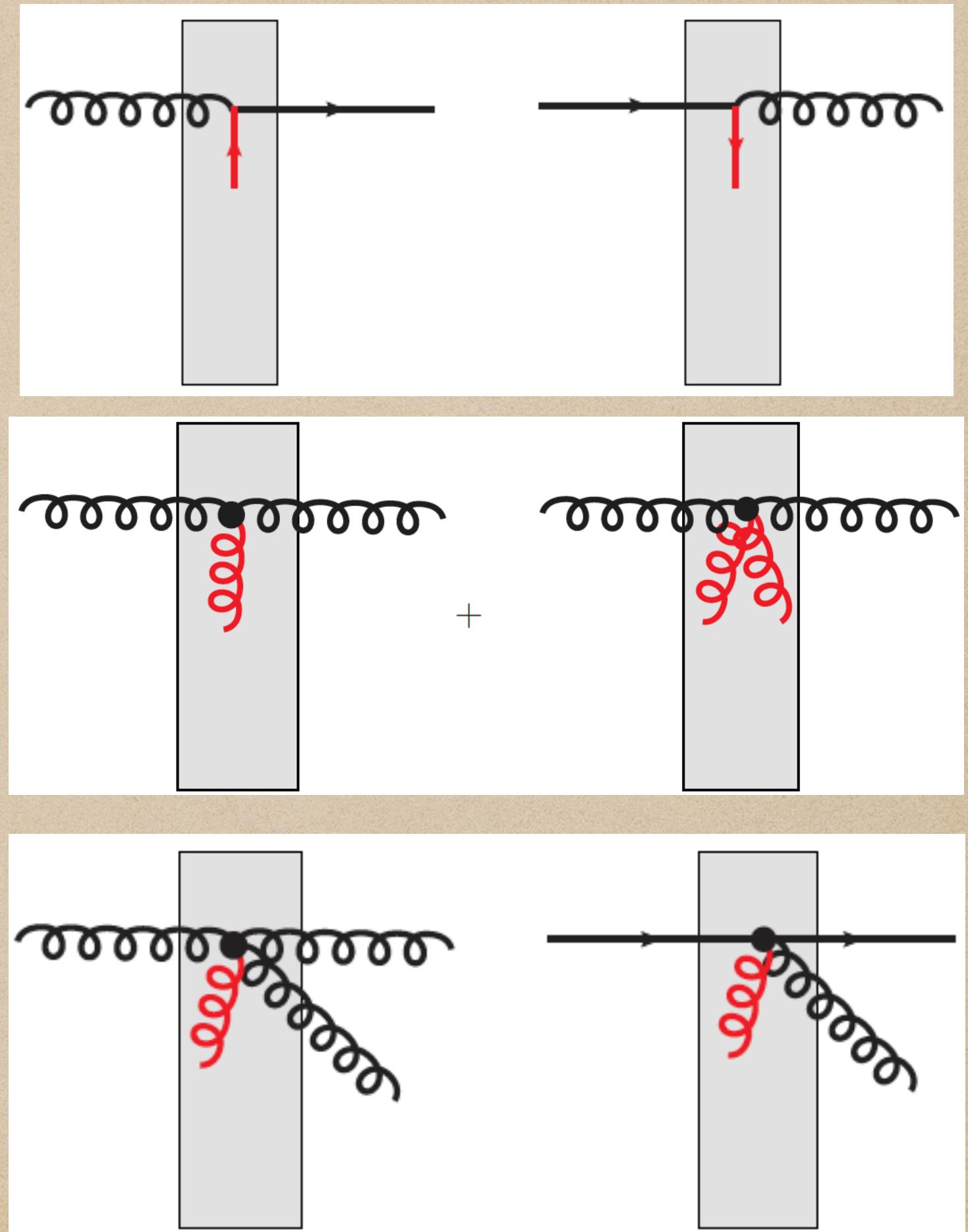
Order ξ^0 : $V_{(0)} = a_b^- J_b^+ = a_b^- \left(g \bar{\Psi} \gamma^+ t^b \Psi - ig [A^i, F^{+i}]^b \right)$

Order $\xi^{\frac{1}{2}}$: $V_{(\frac{1}{2})} = g \bar{\Psi}_G \gamma^i A_i \psi_B + g \bar{\Psi}_B \gamma^i A_i \Psi_G$

Order ξ^1 :
$$\begin{aligned} V_{(1)} = & -\frac{1}{2} A_a^i \left((\mathcal{D}_l \mathcal{D}^l)^{ab} g_{ij} + 2ig(f_{ij})^{ab} \right) A_b^j + \frac{i}{\sqrt{2}} \Psi_G^\dagger \left(g f_{ji} S^{ij} - \mathcal{D}_l \mathcal{D}^l \right) \frac{1}{\partial_-} \Psi_G \\ & + ig \left[A_i, A_j \right]_b (\mathcal{D}^i A^j)_b + (\mathcal{D}_i A^i)_b \frac{1}{\partial_-} \left(-ig \left[\partial_- A^j, A_j \right]^b + \sqrt{2} g \Psi_G^\dagger t^b \Psi_G \right) \\ & + \frac{1}{\sqrt{2}} g \Psi_G^\dagger A_j \gamma^j \gamma^i \mathcal{D}_i \frac{1}{\partial_-} \Psi_G + h.c. \end{aligned}$$

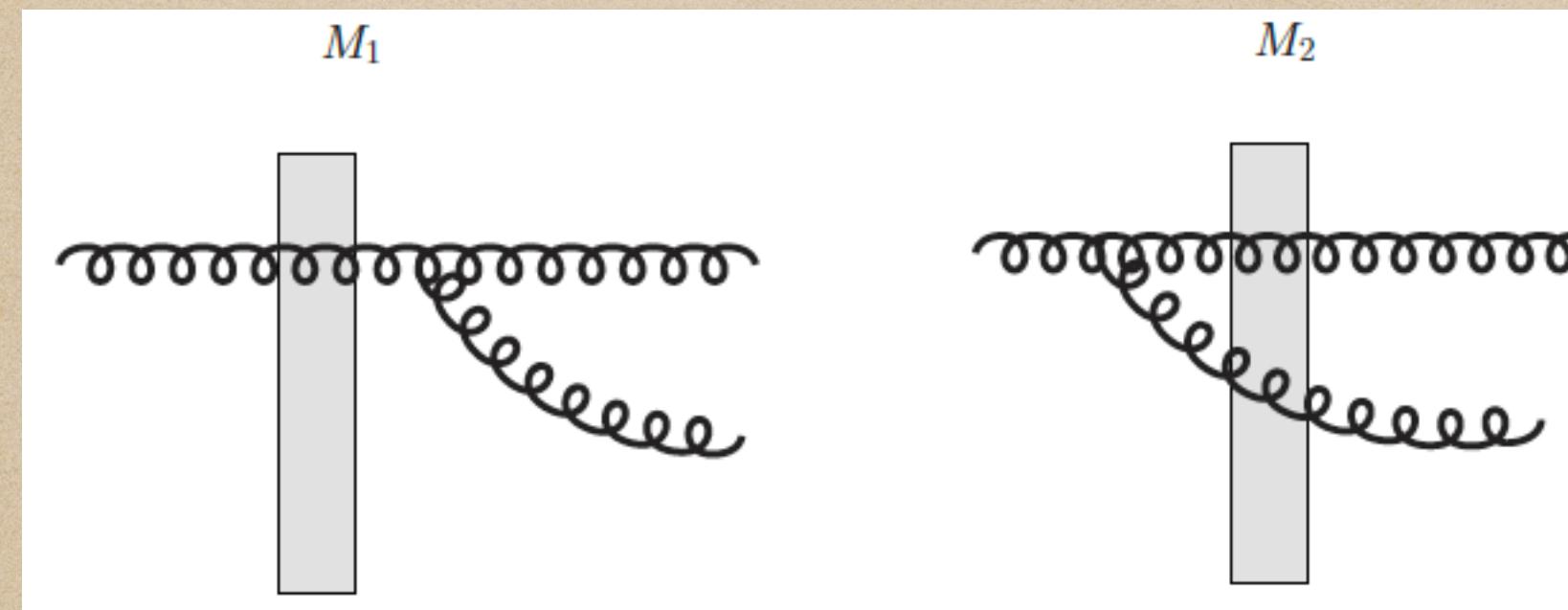
$$\Psi = \Psi_G + \Psi_B \quad \mathcal{D}^i = \partial^i + ig[a^i, .]$$

Background fields: a^-, a^i, ψ_B Quantum fields: Ψ_G, A^i



A_{LL}^g at small x : Pure Glue Case

Eikonal Order

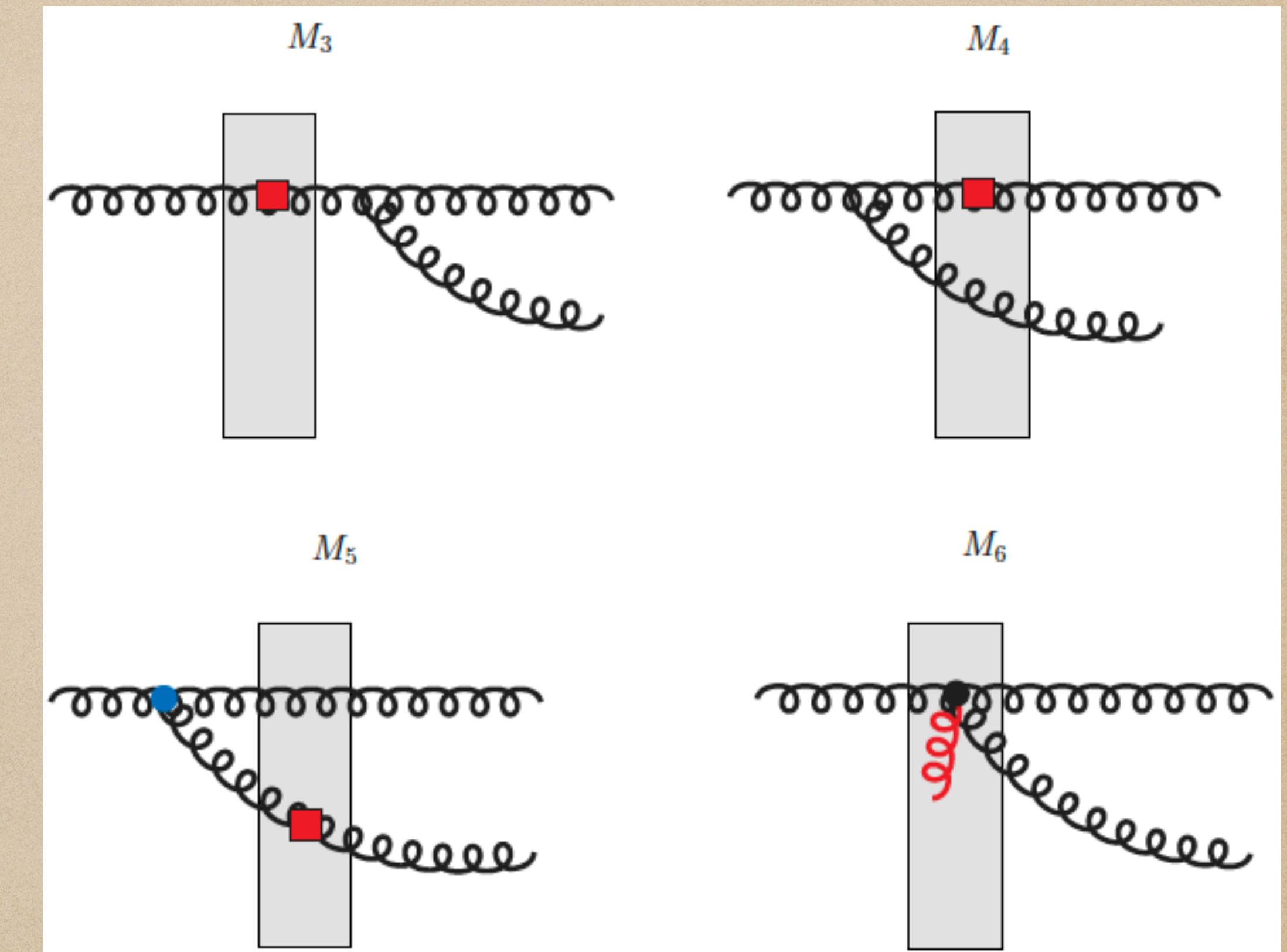


Red square: Polarized Wilson Lines

$$U_{\mathbf{x}}^{G[1]}(k^+) = -\frac{2ig}{2k^+} \int_{x_0^+}^{x^+} dw^+ U_{\mathbf{x}}(x^+, w^+) f_{12}(w^+, \mathbf{x}) U_{\mathbf{x}}(w^+, x_0^+),$$

$$U_{\mathbf{x}', \mathbf{x}}^{G[2]}(k^+) = \frac{i}{2k^+} \int_{x_0^+}^{x^+} dw^+ U_{\mathbf{x}'}(x^+, w^+) \int_{\mathbf{z}} \delta(\mathbf{x}' - \mathbf{z}) \left[\overleftarrow{\mathcal{D}}_l \overrightarrow{\mathcal{D}}^l(w^+, \mathbf{z}) \right] \delta(\mathbf{z} - \mathbf{x}) U_{\mathbf{x}}(w^+, x_0^+).$$

Sub-eikonal Order (spin dependent)



Gluon radiation inside the shockwave

Balitsky and Tarasov (2015)

Polarized Wilson Line Correlators

Final result depends on two polarized Wilson line correlators.

$$\langle\langle \text{Tr} [U_{\mathbf{x}_1} U_{\mathbf{x}_0}^{G[1]\dagger} + U_{\mathbf{x}_0}^\dagger U_{\mathbf{x}_1}^{G[1]}] \rangle\rangle$$

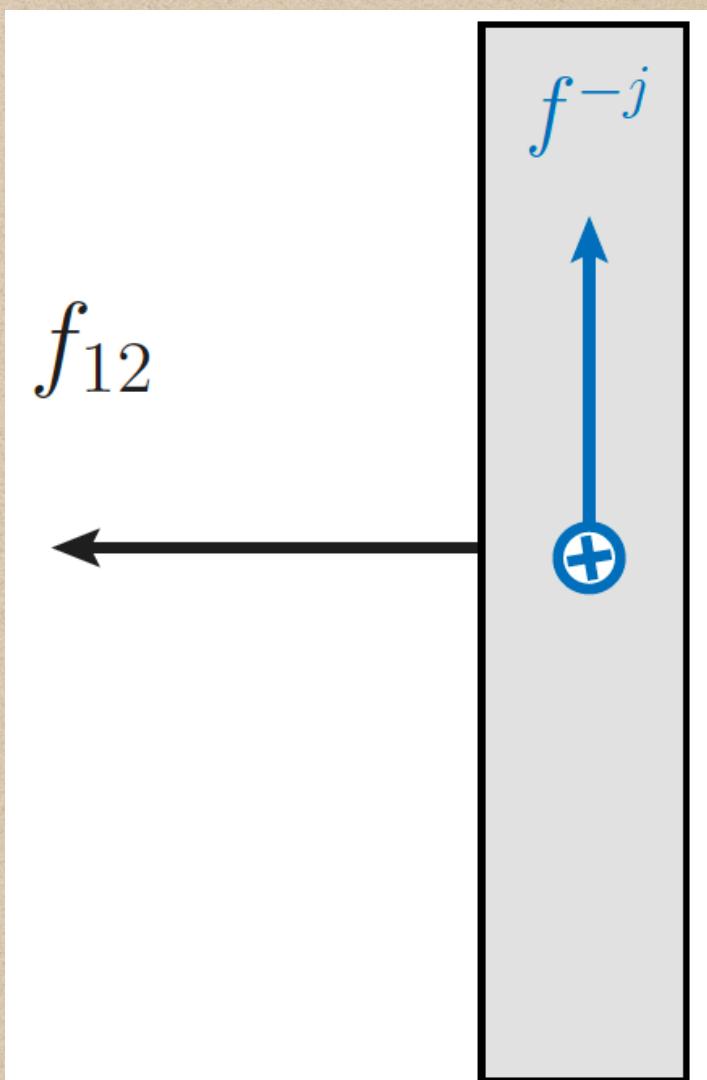
$$\langle\langle \text{Tr} [U_{\mathbf{x}_0}^\dagger U_{\mathbf{x}_1}^{j,G[2]}(k^+) - U_{\mathbf{x}_0}^{j,G[2]\dagger}(k^+) U_{\mathbf{x}_1}] \rangle\rangle$$

**Magnetically Polarized
Wilson Line**

$$U_{\mathbf{x}}^{G[1]}(k^+) = -\frac{2ig}{2k^+} \int_{-\infty}^{+\infty} dw^+ U_{\mathbf{x}}(+\infty, w^+) f_{12}(w^+, \mathbf{x}) U_{\mathbf{x}}(w^+, -\infty),$$

**Electrically Polarized
Wilson Line**

$$U_{\mathbf{x}}^{j,G[2]}(k^+) = \frac{ig}{2k^+} \int_{-\infty}^{+\infty} dw^+ w^+ U_{\mathbf{x}}(+\infty, w^+) f^{-j}(w^+, \mathbf{x}) U_{\mathbf{x}}(w^+, -\infty).$$



No contributions from transverse magnetic field f^{+j} and longitudinal electric field f^{+-} .

High order in eikonicity

Longitudinal momentum exchange

Two Gluon TMDs

In the small x limit:

Cougoelic, Kovchegov, Tarasov, and Tawabutr (2022)

Gluon Helicity TMD

$$\Delta G_L(x, \mathbf{k}^2) = \frac{4i}{g^2} \epsilon^{ij} \mathbf{k}^i \int_{\mathbf{x}_1, \mathbf{x}_0} e^{-i\mathbf{k} \cdot (\mathbf{x}_1 - \mathbf{x}_0)} \langle \langle \text{Tr} \left[U_{\mathbf{x}_0}^\dagger U_{\mathbf{x}_1}^{j, G[2]}(k^+) - U_{\mathbf{x}_0}^{j, G[2]\dagger}(k^+) U_{\mathbf{x}_1} \right] \rangle \rangle$$

Twist-3 Gluon TMD

$$\Delta H_{3L}^\perp(x, \mathbf{k}^2) = \frac{4}{g^2} \int_{\mathbf{x}_1, \mathbf{x}_0} e^{-i\mathbf{k} \cdot (\mathbf{x}_1 - \mathbf{x}_0)} \frac{1}{2} \langle \langle \text{Tr} \left[U_{\mathbf{x}_1} U_{\mathbf{x}_0}^{G[1]\dagger} + U_{\mathbf{x}_0}^\dagger U_{\mathbf{x}_1}^{G[1]} \right] \rangle \rangle$$

Gluon correlation functions:

Mulders and Rodrigues (2001)

$$\Gamma^{\mu\nu;\rho\sigma}(k, P, S) = \int d^4 \mathbf{x} e^{ik \cdot x} \left\langle P, S \mid \text{Tr} \left[F^{\mu\nu}(0) \mathcal{U}^{[+]}(0, x) F^{\rho\sigma}(x) \mathcal{U}^{[-]}(x, 0) \right] \mid P, S \right\rangle$$

→ $\Delta G_L(x, \mathbf{k}^2) \subset \Gamma^{-i;-j}(k, P, S), \quad \Delta H_{3L}^\perp(x, \mathbf{k}^2) \subset \Gamma^{ij;l-}(k, P, S)$

$$\int_{\mathbf{k}} \Delta H_{3L}^\perp(x, \mathbf{k}^2) = 0$$

Twist-3 gluon TMD does not contribute to gluon helicity PDF

A_{LL}^g at small x : Results

$$\frac{d\sigma_\lambda}{d^2\mathbf{p}_1 dy_1} = \lambda \frac{\alpha_s^2}{2\pi} \frac{1}{C_F} \int_{\mathbf{k}} \left[\frac{-6\mathbf{p}_1 \cdot (\mathbf{p}_1 - \mathbf{k})}{\mathbf{p}_1^2 |\mathbf{p}_1 - \mathbf{k}|^2} + \frac{4}{|\mathbf{p}_1 - \mathbf{k}|^2} \right] \Delta H_{3L}^\perp(x, \mathbf{k}^2) + \left[\frac{4(\mathbf{p}_1 \times \mathbf{k})^2}{\mathbf{p}_1^2 |\mathbf{p}_1 - \mathbf{k}|^2 \mathbf{k}^2} + \frac{(\mathbf{k} - \mathbf{p}_1) \cdot \mathbf{k}}{|\mathbf{k} - \mathbf{p}_1|^2 \mathbf{k}^2} \right] \Delta G_L(x, \mathbf{k}^2)$$

Large transverse momentum limit:

$$\frac{d\sigma_\lambda}{d^2\mathbf{p}_1 dy_1} \Big|_{\mathbf{p}_1 \rightarrow \infty} \simeq \lambda \frac{\alpha_s^2}{2\pi} \frac{1}{C_F} \frac{2}{\mathbf{p}_1^2} \Delta G_L(x, Q^2)$$

Reproducing collinear factorization

Low transverse momentum limit:

$$\frac{d\sigma_\lambda}{d^2\mathbf{p}_1 dy_1} \Big|_{\mathbf{p}_1 \rightarrow 0} \simeq \lambda \frac{\alpha_s^2}{2\pi} \frac{1}{C_F} \int_{\mathbf{k}} \frac{1}{\mathbf{k}^2} [4\Delta H_{3L}^\perp(x, \mathbf{k}^2) + 3\Delta G_L(x, \mathbf{k}^2)]$$

Collinear factorization breaks down

At low transverse momentum, both the Twist-3 gluon TMD and the gluon helicity TMD contribute comparably to double-spin asymmetry

Conclusion and Outlook

- We developed an effective Hamiltonian approach to study small-x physics beyond Eikonal approximation with a focus on spin-related processes.
- For a pure glue system, the A_{LL}^g for soft gluon production depends on a new twist-3 gluon TMD in addition to the gluon helicity TMD.
- The twist-3 gluon TMD does not contribute to gluon helicity PDF, it contributes comparably to the A_{LL}^g in the low transverse momentum regime,
- Suggesting that using double-spin asymmetry to extract gluon helicity distribution at small x within the collinear factorization formalism might overestimate the contribution.
- Future works: including quarks for a full calculation, examine double-spin asymmetry at small x for other observables in pp collisions and ep/eA collisions, including small-x helicity evolution.