

# Exclusive $\eta_c$ at the EIC from the small- $x$ evolved odderon

Sanjin Benić (University of Zagreb)

SB, Horvatić, Kaushik, Vivoda, in preparation

DIS2023, East Lansing, March 30, 2023



# Odderon

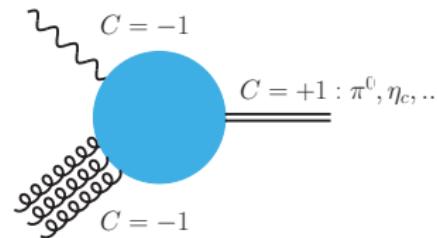
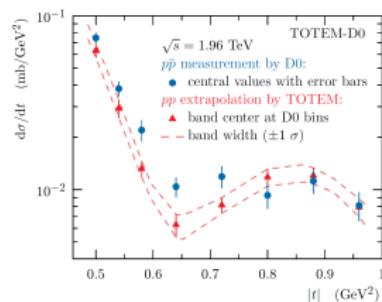
- 50 years ago Lukaszuk, Nicolescu: odderon is a  $C = -1$  exchange

Lukaszuk, Nicolescu, LNC 8 (1973) 405

Joynson, Leader, Nicolescu, Lopez, NCA 30 (1975) 345

Ewerz, 0306137

- QCD: three gluons  $\text{tr} [\{A^\mu, A^\nu\} A^\rho] \sim d_{abc}$



- elastic  $p p$  vs  $p \bar{p}$  cross section

TOTEM, D0, PRL 127 (2021) 6, 062003

Staszewski, DIS 2023 plenary, 03/27, 10:00

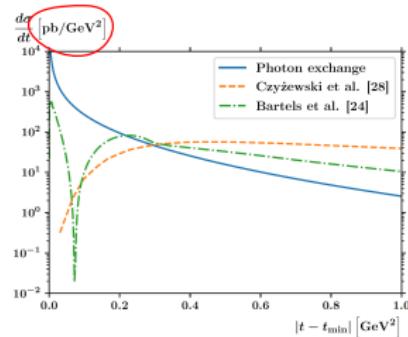
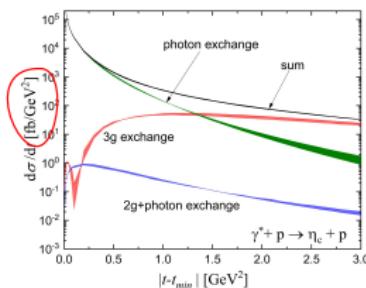
- exclusive productions to fix the  $C$  parity of the final state

Czyzowski, Kwiecinski, Motyka, Sadzikowski, PLB 398 400 (1997)

Bartels, Braun, Colferai, Vacca, EPJC 20 323 (2001)

# Odderon in $ep \rightarrow ep\eta_c$

- no experimental measurement so far  $\rightarrow$  EIC?  
(null result from H1 at HERA for exclusive  $\pi^0$ )  
H1, PLB 544 (2002) 35-43
- more recent computations at moderate  $x$  lead to even lower cross sections than the original estimates



Dumitru, Stebel, PRD 99 (2019) 9, 094038

Jia, Mo, Pan, Zhang, 2207.14171

- this work: consider nuclear targets and take into account evolution effects

# Odderon in the CGC

- high energy collisions → Wilson lines

$$V(\mathbf{z}_\perp) = \mathcal{P} \exp \left[ ig \int_{-\infty}^{\infty} dy^- A^+(y^-, \mathbf{z}_\perp) \right]$$

- Color Glass Condensate (CGC): emergence of a saturation scale  $Q_S^2 \sim A^{1/3}$   
→ better theoretical control for a large nuclei

$$Q_S^2 \gg \Lambda_{\text{QCD}}^2$$

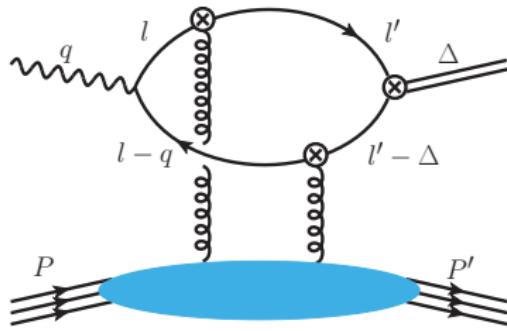
Dumitru, DIS 2023 plenary, 03/27, 11:30

- odderon as imaginary part of the dipole

$$\mathcal{O}(\mathbf{x}_\perp, \mathbf{y}_\perp) \equiv -\frac{1}{2iN_c} \text{tr} \left\langle V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) - V(\mathbf{y}_\perp) V^\dagger(\mathbf{x}_\perp) \right\rangle$$

Kovchegov, Szymanowski, Wallon, PLB 586, 267 (2004)  
Hatta, Itakura, McLerran, Nucl.Phys.A 760 (2005) 172-207

# Amplitude



$$\mathbf{r}_\perp = \mathbf{x}_\perp - \mathbf{y}_\perp$$

$$\mathbf{b}_\perp = \frac{\mathbf{x}_\perp + \mathbf{y}_\perp}{2}$$

- CGC vertex

$$\tau(p, p') = (2\pi)\delta(p^- - p'^-) \gamma^- \text{sgn}(p^-) \int_{\mathbf{z}_\perp} e^{-i(\mathbf{p}_\perp - \mathbf{p}'_\perp) \cdot \mathbf{z}_\perp} V^{\text{sgn}(p^-)}(\mathbf{z}_\perp)$$

$$\begin{aligned} \langle \mathcal{M}_\lambda \rangle = & \epsilon q_c \int_{\mathbf{r}_\perp} \int_{\mathbf{l}''} (2\pi)\delta(\mathbf{l}^- - \mathbf{l}'^-) \theta(\mathbf{l}^-) \theta(\mathbf{q}^- - \mathbf{l}^-) e^{-i(\mathbf{l}'_\perp - \mathbf{l}_\perp - \frac{1}{2}\Delta_\perp) \cdot \mathbf{r}_\perp} \\ & \times (-iN_c) \mathcal{O}(\mathbf{r}_\perp, \Delta_\perp) \text{tr} [S(\mathbf{l}) \not{e}(\lambda, \mathbf{q}) S(\mathbf{l} - \mathbf{q}) \gamma^- S(\mathbf{l}' - \Delta) (\mathbf{i} \gamma_5) S(\mathbf{l}') \gamma^-] \end{aligned}$$

# Amplitude

- only transverse photon polarizations  $\lambda = \pm 1$  survive in the eikonal approximation

$$\langle \mathcal{M}_\lambda \rangle = q^- \lambda e^{i\lambda\phi_\Delta} \langle \mathcal{M} \rangle \quad \frac{d\sigma}{d|t|} = \frac{1}{16\pi} |\langle \mathcal{M} \rangle|^2$$

- polarization independent part of the amplitude

$$\begin{aligned} \langle \mathcal{M} \rangle &= 8\pi i e q_c N_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^\infty r_\perp dr_\perp \mathcal{O}_{2k+1}(r_\perp, \Delta_\perp) \\ &\times \mathcal{A}(r_\perp) \left[ J_{2k}(r_\perp \delta_\perp) - \frac{2k+1}{r_\perp \delta_\perp} J_{2k+1}(r_\perp \delta_\perp) \right]. \end{aligned}$$

- result proportional to  $m_c$

$$\mathcal{A}(r_\perp) = (-1) \frac{\sqrt{2} m_c}{2\pi} \frac{1}{z\bar{z}} [K_0(\varepsilon r_\perp) \partial_{r_\perp} \phi_P(z, r_\perp) - \varepsilon K_1(\varepsilon r_\perp) \phi_P(z, r_\perp)]$$

Dumitru, Stebel, PRD 99 (2019) 9, 094038  
SB, Horvatić, Kaushik, Vivoda, in preparation

# BK equation

- odderon  $\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)$  is explicitly  $\mathbf{b}_\perp$ -dependent  
→ in principle need to solve the fully impact parameter dependent Balitsky-Kovchegov (BK) equation

$$\frac{\partial \mathcal{D}(\mathbf{r}_\perp, \mathbf{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int_{\mathbf{r}_{1\perp}} \frac{\mathbf{r}_\perp^2}{\mathbf{r}_{1\perp}^2 \mathbf{r}_{2\perp}^2} [\mathcal{D}(\mathbf{r}_{1\perp}, \mathbf{b}_{1\perp}) \mathcal{D}(\mathbf{r}_{2\perp}, \mathbf{b}_{2\perp}) - \mathcal{D}(\mathbf{r}_\perp, \mathbf{b}_\perp)]$$

$$\mathbf{r}_{2\perp} = \mathbf{r}_\perp - \mathbf{r}_{1\perp}$$

$$\begin{aligned}\mathcal{D}(\mathbf{r}_\perp, \mathbf{b}_\perp) &= \frac{1}{N_c} \text{tr} \left\langle V \left( \mathbf{b}_\perp + \frac{\mathbf{r}_\perp}{2} \right) V^\dagger \left( \mathbf{b}_\perp - \frac{\mathbf{r}_\perp}{2} \right) \right\rangle \\ &= 1 - \mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp) + i\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)\end{aligned}$$

- in general non-local in  $\mathbf{b}_\perp$
- this work: local approximation →  $\mathbf{b}_\perp$  becomes an external parameter

Kowalski, Lappi, Marquet, Venugopalan, PRC 78 (2008) 045201  
Lappi, Mäntysaari, PRD 88 (2013) 114020

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Kowalski, Lappi, Marquet, Venugopalan, PRC 78 (2008) 045201  
Lappi, Mäntysaari, PRD 88 (2013) 114020

# BK equation

- non-linear terms couple the Pomeron-Odderon system

$$\begin{aligned}\frac{\partial \mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp)}{\partial Y} &= \frac{\alpha_S N_c}{2\pi^2} \int_{\mathbf{r}_{1\perp}} \frac{\mathbf{r}_\perp^2}{\mathbf{r}_{1\perp}^2 \mathbf{r}_{2\perp}^2} \left[ \mathcal{N}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) + \mathcal{N}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp) \right. \\ &\quad \left. + \mathcal{N}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) \mathcal{N}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{O}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) \mathcal{O}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) \right] \\ \frac{\partial \mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)}{\partial Y} &= \frac{\alpha_S N_c}{2\pi^2} \int_{\mathbf{r}_{1\perp}} \frac{\mathbf{r}_\perp^2}{\mathbf{r}_{1\perp}^2 \mathbf{r}_{2\perp}^2} \left[ \mathcal{O}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) + \mathcal{O}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp) \right. \\ &\quad \left. - \mathcal{N}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) \mathcal{O}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{O}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) \mathcal{N}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) \right]\end{aligned}$$

Kovchegov, Szymanowski, Wallon, PLB 586, 267 (2004)

Hatta, Itakura, McLerran, Nucl.Phys.A 760 (2005) 172-207

Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)

Yao, Hagiwara, Hatta, PLB 790 (2019) 361-366

- small  $r_\perp$  limit:  $\mathcal{N} \rightarrow 0$  (linear)  $\rightarrow \mathcal{O} \sim e^{-\#Y}$
- large  $r_\perp$  limit:  $\mathcal{N} \rightarrow 1$  (saturation)  $\rightarrow \mathcal{O} \sim e^{-\#Y}$
- in numerical computations we are replacing  $\frac{\alpha_S N_c}{2\pi^2} \frac{\mathbf{r}_\perp^2}{\mathbf{r}_{1\perp}^2 \mathbf{r}_{2\perp}^2}$  by a running coupling kernel with Balitsky's prescription

# Initial conditions

- $\mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp)$  is fixed through a HERA fit

$$\mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp) = 1 - \exp \left[ -\frac{1}{4} \mathbf{r}_\perp^2 A T_A(\mathbf{b}_\perp) \frac{\sigma_0}{2} Q_{S,0}^2 \log \left( \frac{1}{r_\perp \Lambda_{\text{QCD}}} + e_c e \right) \right]$$

Lappi, Mäntysaari, PRD 88 (2013) 114020

- for  $\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)$  we use the Jeon-Venugopalan (JV) model

$$W[\rho] = \exp \left[ - \int_{\mathbf{x}_\perp} \left( \frac{\delta^{ab} \rho^a \rho^b}{2\mu^2} - \frac{d_{abc} \rho^a \rho^b \rho^c}{\kappa} \right) \right]$$

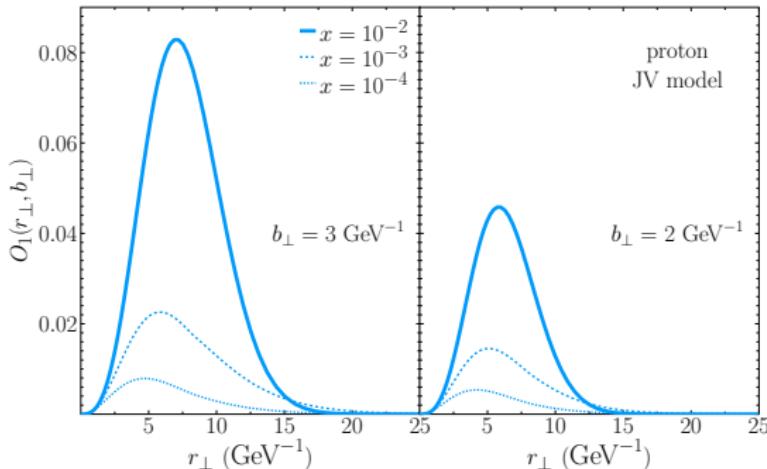
Jeon, Venugopalan, PRD 71 (2005) 125003

$$\begin{aligned} \mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp) &= \frac{\lambda}{8} \left[ A^{2/3} \frac{dT_A(\mathbf{b}_\perp)}{db_\perp} R_A \frac{\sigma_0}{2} \right] A^{1/2} (Q_{S,0} r_\perp)^3 (\hat{\mathbf{r}}_\perp \cdot \hat{\mathbf{b}}_\perp) \\ &\times \log \left( \frac{1}{r_\perp \Lambda_{\text{QCD}}} + e_c e \right) \exp \left[ -\frac{1}{4} \mathbf{r}_\perp^2 A T_A(\mathbf{b}_\perp) \frac{\sigma_0}{2} Q_{S,0}^2 \log \left( \frac{1}{r_\perp \Lambda_{\text{QCD}}} + e_c e \right) \right] \end{aligned}$$

- in the JV model  $\lambda_{\text{JV}} = -\frac{3}{16} \frac{N_c^2 - 4}{(N_c^2 - 1)^2} \frac{(Q_{S,0} R_A)^3}{\alpha_S^3 A^{3/2}}$   
SB, Horvatić, Kaushik, Vivoda, in preparation

# Numerical solutions

- rapid drop of the odderon with evolution
- does not obey geometric scaling

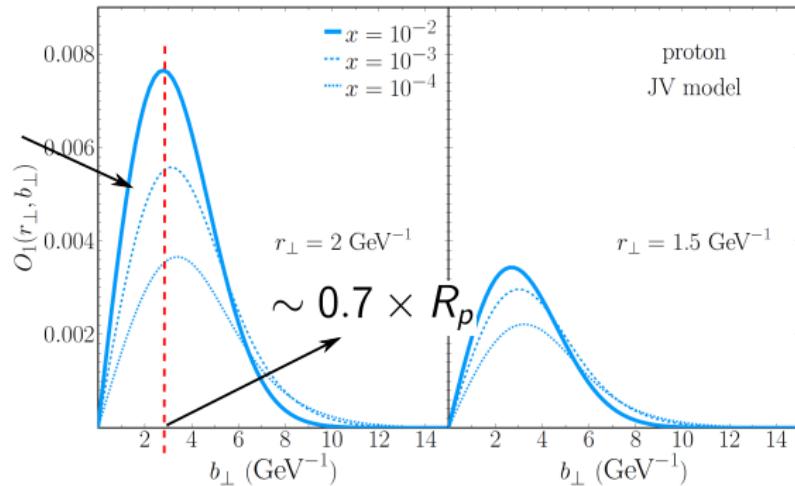


- the peak position ( $r_{\perp} \sim Q_S^{-1}$  initially) moves slowly with evolution  
Motyka, PLB 637, 185 (2006)  
Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)  
Yao, Hagiwara, Hatta, PLB 790 (2019) 361-366  
SB, Horvatić, Kaushik, Vivoda, in preparation

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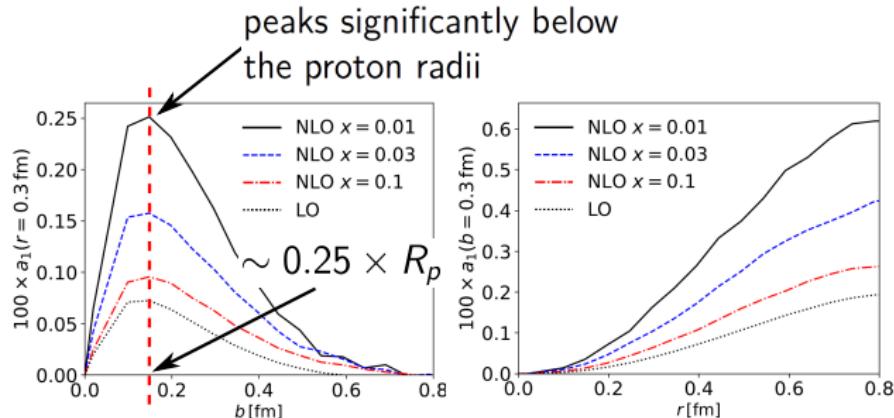
$$\sim \frac{dT_p(b_\perp)}{db_\perp}$$



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SB, Horvatić, Kaushik, Vivoda, in preparation

# Initial conditions

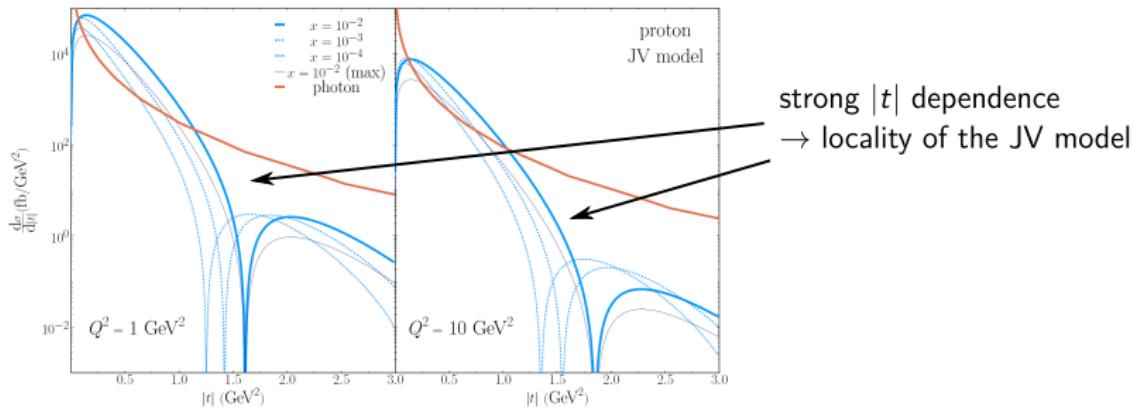
- recent computation at NLO using quark light-cone wavefunctions for the proton by Dumitru, Mäntysaari, Paatelainen (DMP)



Dumitru, Mäntysaari, Paatelainen, PRD 107 (2023) 1, L011501

- fitted the DMP model at  $x = 0.01$  to a functional form similar to the JV model
- assumption** in modeling the nuclei: *A* dependence inherited from JV model

# Results: proton target in the JV model

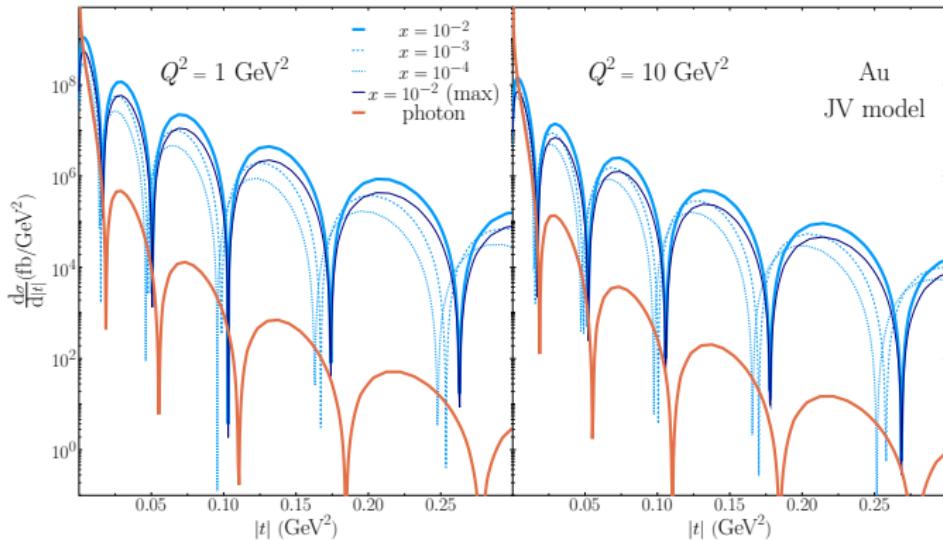


- generic feature: odderon contribution vanishes in the forward limit  $|t| \rightarrow 0$
- a significant background from photon  
→ depends on the model coupling  $\lambda$

SB, Horvatić, Kaushik, Vivoda, in preparation

# Results: Au target in the JV model

- JV model: odderon dominates already at low  $|t|$  and down to  $x = 10^{-4}$  ( $\sim A^2$  enhancement)

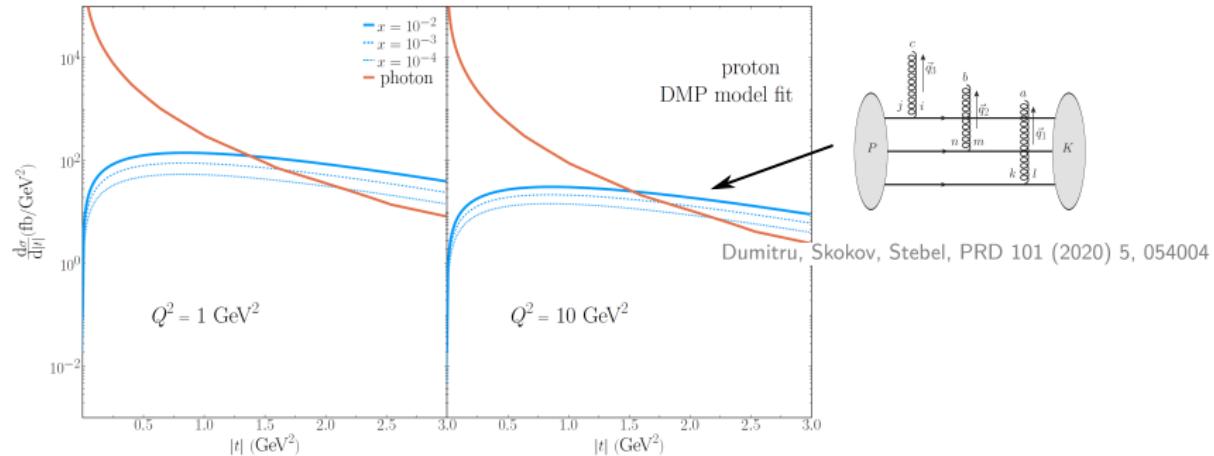


→ depends on the model coupling  $\lambda$

- diffractive dips controlled mostly by geometry even for the odderon

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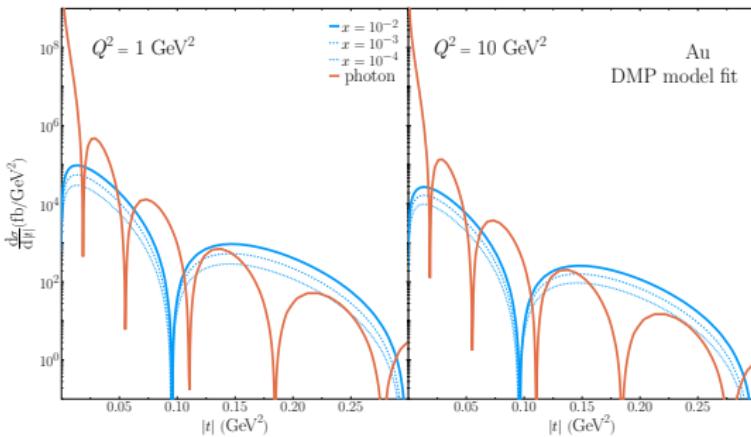
# Results: proton target in the DMP



- significantly smaller  $|t|$ -slope comes from a shift of the peak of the odderon in  $b_{\perp}$   
→ opportunity to isolate the odderon in the high  $|t| > 1.5 \text{ GeV}^2$  region
- roughly similar to the result by Dumitru and Stebel  
Dumitru, Stebel, PRD 99 (2019) 9, 094038  
SB, Horvatić, Kaushik, Vivoda, in preparation

# Results: Au target in the DMP

- DMP model: a shift in the diffractive dips
- based on the previous assumption of a smaller slope parameter in  $\gamma^* p \rightarrow \eta_c p$  collision and its nuclear scaling as  $\sim A^{2/3}$

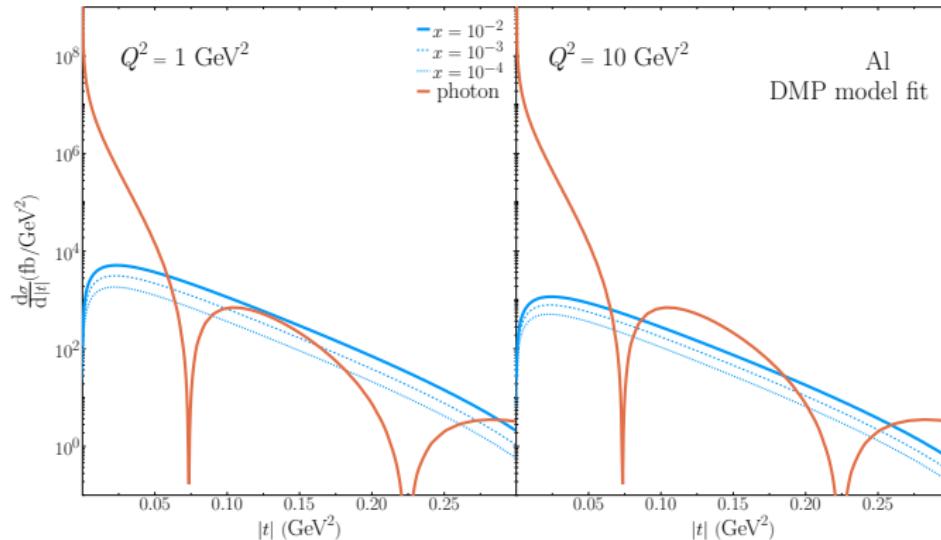


- the odderon fills up the diffractive dip controlled by the known QED contribution
  - the odderon coupling  $\lambda$  does not need to be large
  - a new way to probe the odderon?

SB, Horvatić, Kaushik, Vivoda, in preparation

# Results: Al target in the DMP

- with a lighter nuclei the QED diffractive dips are more separated  
→ use different nuclei species as a lever to extract the odderon contribution more favorably



SB, Horvatić, Kaushik, Vivoda, in preparation

# Conclusions

- collisions on a proton target require large  $|t| > 1.5 \text{ GeV}^2$  to extract the odderon
- a new idea for the nuclei:
  - odderon contribution fills up the diffractive dips of a known QED origin
  - a potential new application at the EIC!
- how to measure  $\eta_c$ ?
  - light quarks? → DVMP at small- $x$
- forward limit  $|t| \rightarrow 0$  cross section vanishes
  - consider the spin-flip contribution from the gluon Sivers function
- GPD limit?
  - Ma, NPA 727 (2003) 333-352

Boussarie, Hatta, Szymanowski, Wallon, PRL 124 (2020) 17, 172501

- GPD limit?

Ma, NPA 727 (2003) 333-352

# The photon contribution

- photon has  $C = -1 \rightarrow$  important background for extracting the odderon
- replace QCD with QED Wilson line

$$U(\mathbf{x}_\perp) = e^{ieq_c \Lambda(\mathbf{x}_\perp)} \quad e\Lambda(\mathbf{x}_\perp) = 4\pi Z\alpha \int_{\mathbf{k}_\perp} \frac{\mathcal{T}_A(\mathbf{k}_\perp)}{\mathbf{k}_\perp^2} e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}$$

- get the photon distribution in  $(\mathbf{r}_\perp, \mathbf{b}_\perp)$  space

$$\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp) \rightarrow -\frac{1}{2i} [U(\mathbf{x}_\perp)U^\dagger(\mathbf{y}_\perp) - U(\mathbf{y}_\perp)U^\dagger(\mathbf{x}_\perp)] \equiv \Omega(\mathbf{r}_\perp, \mathbf{b}_\perp)$$

- at all orders

$$\Omega(\mathbf{r}_\perp, \mathbf{b}_\perp) = \sin(\mathbf{r}_\perp \cdot \mathbf{b}_\perp \mathcal{F}(b_\perp)) \quad \mathcal{F}(b_\perp) \equiv \frac{4\pi Z q_c \alpha}{b_\perp^2} \int_0^{b_\perp} b'_\perp db'_\perp \mathcal{T}_A(b'_\perp)$$

- Fourier modes:  $\Omega_{2k+1}(r_\perp, b_\perp) = (-1)^k J_{2k+1}(r_\perp b_\perp \mathcal{F}(b_\perp))$

SB, Horvatić, Kaushik, Vivoda, in preparation

# Leading twist estimates

- $m_c \rightarrow \infty, |t| \rightarrow 0$  limit
- the odderon contribution to the cross section

$$\frac{d\sigma}{d|t|} \simeq \frac{9\pi q_c^2 \alpha \alpha_S^6 A^2 C_{3F}^2 \mathcal{R}_{\mathcal{P}}^2(0)}{N_c m_c^5} \frac{|t| T_A^2(\sqrt{|t|})}{m_c^4}$$

SB, Horvatić, Kaushik, Vivoda, in preparation

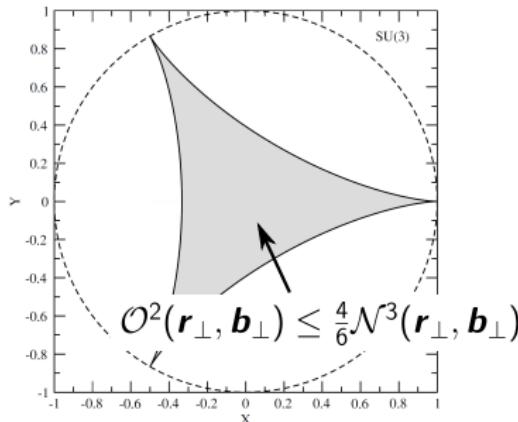
- contrast with the photon cross section

$$\frac{d\sigma}{d|t|} \simeq \frac{\pi q_c^4 \alpha^3 Z^2 N_c \mathcal{R}_{\mathcal{P}}^2(0)}{m_c^5} \frac{\mathcal{T}_A^2(\sqrt{|t|})}{|t|}$$

Jia, Mo, Pan, Zhang, 2207.14171

# Further computation details

- a bound on  $\lambda$  from group theory



Kaiser, JPA 39, 15287 (2006)

Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)

Lappi, Mäntysaari, PRD 88 (2013) 114020

- $\eta_c$  meson wave-function: Boosted Gaussian  
Dumitru, Stebel, PRD 99 (2019) 9, 094038
- electromagnetic profile  $T_p(\mathbf{b}_\perp)$ : Dirac form factor  $F_1(Q^2)$   
Ye, Arrington, Hill, Lee, PLB 777, 8-15 (2018)
- electromagnetic profile  $T_A(\mathbf{b}_\perp)$ : Woods-Saxon

