DIS dijet production at next-to-eikonal accuracy and its back-to-back limit

Guillaume Beuf

National Centre for Nuclear Research (NCBJ), Warsaw

with Tolga Altinoluk, Alina Czajka and Cyrille Marquet, (*to appear*).

DIS2023, Michigan State University, East Lansing, March 27-31, 2023

TMD vs CGC approaches

For a process with a hard $\mathbf{P}$ and a not so hard $\mathbf{k}$ transverse momenta:

- TMD factorization: leading power (twist 2) in the limit $|\mathbf{k}| \ll |\mathbf{P}| \sim \sqrt{s}$
- CGC result: leading power (eikonal) in the limit $|\mathbf{k}| \sim |\mathbf{P}| \ll \sqrt{s}$

Consistency of both approaches shown in the double limit $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$ (Dominguez, Marquet, Xiao, Yuan, 2011)

Power corrections in $|\mathbf{k}|/|\mathbf{P}|$ in the regime $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$ studied from the CGC approach (Altinoluk, Boussarie, Kotko, 2019)

$\Rightarrow$ What about power corrections in $\mathbf{P}^2/s$ or $|\mathbf{P}||\mathbf{k}|/s$ beyond the eikonal limit?

Eikonal approximation in the CGC

In the CGC approach to dense-dilute high-energy scattering:

Dense target represented by a strong semiclassical gluon field $\mathcal{A}^{\mu}(x)$

The Eikonal approx. can be understood as the limit of **infinite boost** of $\mathcal{A}^{\mu}(x)$:

- Under a boost of parameter γ_t along the "$-$" direction, $\mathcal{A}^-$ is enhanced and $\mathcal{A}^+$ is suppressed: $\mathcal{A}^- = O(\gamma_t) \gg \mathcal{A}_\perp = O(1) \gg \mathcal{A}^+ = O(1/\gamma_t)$
  $\Rightarrow$ Only $\mathcal{A}^-$ is kept
- Lorentz contraction of $\mathcal{A}^{\mu}(x)$ (shockwave limit)
  $\Rightarrow$ Partons from the projectile interact instantly in x^+ with the target, without transverse motion within the target
- $\mathcal{A}^{\mu}(x)$ independent on x^- (static limit) due to Lorentz time dilation
  $\Rightarrow$ No p^+ transfer from the target

Background field in the eikonal limit: $\mathcal{A}^{\mu}(x^+, x^-, \mathbf{x}) \approx \delta^{\mu -} \mathcal{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$

Next-to-Eikonal corrections to the CGC

Next-to-Eikonal (NEik) power corrections to the standard CGC formalism:

- Of order $1/\gamma_t$ at the level of the boosted background field
- Of order $1/s$ at the level of a cross section

$\rightarrow$ They arise from relaxing either of the 3 approximations:

1. Interactions with $\mathcal{A}_\perp$ field taken into account, not only $\mathcal{A}^-$
2. Target with finite width $\Rightarrow$ transverse motion of the parton within the medium
3. x^- dependence of $\mathcal{A}^\mu(x)$ beyond infinite Lorentz dilation
   $\rightarrow$ Treated as gradient expansion around a common x^- value:
   $\frac{\partial_- \mathcal{A}^-(x)}{\mathcal{A}^-(x)} = O(1/\gamma_t)$

   $\Rightarrow$ Possibility of (small) p^+ exchange with the target

DIS dijet at NEik accuracy

S-matrix element at NEik accuracy (longitudinal photon polarization)
(Altinoluk, G.B., Czajka, Tymowska, arXiv:2212.10484)

$$S_{q_1\bar{q}_2\leftarrow\gamma_L^*} = S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{Gen. Eik}} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dyn. target}} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dec. on }q} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dec. on }\bar{q}}$$

with

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{Gen. Eik}} = -2Q\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+v(2)\,\frac{(q^++k_1^+-k_2^+)(q^++k_2^+-k_1^+)}{4(q^+)^2}\,\theta(q^++k_1^+-k_2^+)\,\theta(q^++k_2^+-k_1^+)$$
$$\times\int_{\mathbf{v},\mathbf{w}} e^{-i\mathbf{v}\cdot\mathbf{k}_1}\,e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\hat{Q}\,|\mathbf{w}-\mathbf{v}|\right)\int db^-\,e^{ib^-(k_1^++k_2^+-q^+)}\left[\mathcal{U}_F\left(\mathbf{v},b^-\right)\mathcal{U}_F^\dagger\left(\mathbf{w},b^-\right)-1\right]$$

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dyn. target}} = 2\pi\delta(k_1^++k_2^+-q^+)\,iQ\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+v(2)\,\frac{(k_1^+-k_2^+)}{(q^+)^2}\int d^2\mathbf{v}\,e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\,e^{-i\mathbf{w}\cdot\mathbf{k}_2}$$
$$\times\left[\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)-\frac{(\bar{Q}^2-m^2)}{2\bar{Q}}\,|\mathbf{w}-\mathbf{v}|\,\mathrm{K}_1\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)\right]\left[\mathcal{U}_F\left(\mathbf{v},b^-\right)\overleftrightarrow{\partial_{b^-}}\mathcal{U}_F^\dagger\left(\mathbf{w},b^-\right)\right]\Big|_{b^-=0}$$

DIS dijet at NEik accuracy

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dec. on } q} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\,\frac{ee_f}{2\pi}\,(-1)Q\,\frac{k_2^+}{(q^+)^2}\int d^2\mathbf{v}\,e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\,e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)$$
$$\times\;\bar{u}(1)\gamma^+\left[\frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)}_{F;ij}(\mathbf{v}) - i\,\mathcal{U}^{(2)}_F(\mathbf{v}) + \mathcal{U}^{(1)}_{F;j}(\mathbf{v})\left(\frac{(\mathbf{k}_2^j-\mathbf{k}_1^j)}{2} + \frac{i}{2}\,\partial_{\mathbf{w}^j}\right)\right]\mathcal{U}_F^\dagger(\mathbf{w})\,v(2)$$

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dec. on } \bar{q}} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\,\frac{ee_f}{2\pi}\,(-1)Q\,\frac{k_1^+}{(q^+)^2}\int d^2\mathbf{v}\,e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\,e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)$$
$$\times\;\bar{u}(1)\gamma^+\left[\mathcal{U}_F(\mathbf{v})\left(\frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)\dagger}_{F;ij}(\mathbf{w}) - i\,\mathcal{U}^{(2)\dagger}_F(\mathbf{w}) + \left(\frac{i}{2}\,\overleftarrow{\partial}_{\mathbf{v}^j} - \frac{(\mathbf{k}_2^j-\mathbf{k}_1^j)}{2}\right)\mathcal{U}^{(1)\dagger}_{F;j}(\mathbf{w})\right)\right]v(2)$$

<u>decorated Wilson lines:</u>

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\,\mathcal{U}_F\left(\frac{L^+}{2}, v^+;\mathbf{v}\right)\overleftrightarrow{\mathcal{D}_{\mathbf{v}^j}}\,\mathcal{U}_F\left(v^+, -\frac{L^+}{2};\mathbf{v}\right)$$
$$\mathcal{U}^{(2)}_{F}(\mathbf{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\,\mathcal{U}_F\left(\frac{L^+}{2}, v^+;\mathbf{v}\right)\overleftarrow{\mathcal{D}_{\mathbf{v}^j}}\,\overrightarrow{\mathcal{D}_{\mathbf{v}^j}}\,\mathcal{U}_F\left(v^+, -\frac{L^+}{2};\mathbf{v}\right)$$
$$\mathcal{U}^{(3)}_{F;ij}(\mathbf{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\,\mathcal{U}_F\left(\frac{L^+}{2}, v^+;\mathbf{v}\right)gt\cdot\mathcal{F}_{ij}(\underline{v})\,\mathcal{U}_F\left(v^+, -\frac{L^+}{2};\mathbf{v}\right)$$

Rewriting NEik corrections as $\mathcal{F}^{\mu\nu}$ insertions

The relation between derivatives of the Wilson lines and field strength insertions:

$$\partial_\mu \mathcal{U}_F(x^+,y^+;\mathbf{v},v^-) + igt\cdot\mathcal{A}_\mu(x^+,\mathbf{v},v^-)\mathcal{U}_F(x^+,y^+;\mathbf{v},v^-) - ig\mathcal{U}_F(x^+,y^+;\mathbf{v},v^-)t\cdot\mathcal{A}_\mu(y^+,\mathbf{v},v^-)$$
$$= -ig\int_{y^+}^{x^+} dv^+ \mathcal{U}_F(x^+,v^+;\mathbf{v},v^-)t\cdot\mathcal{F}_\mu{}^-(v)\mathcal{U}_F(v^+,y^+;\mathbf{v},v^-) \qquad \text{for } \mu\neq +$$

DIS dijet production cross section at NEik accuracy written in terms of field strength insertions!

Expressions are lengthy before considering the back-to-back limit! e.g.

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\bigg|^{\text{dec. on } q}_{\text{NEik corr.}} = (2q^+)\,2\pi\delta(k_1^+ + k_2^+ - q^+)\;8k_1^+k_2^+Q^2\left(\frac{ee_f}{2\pi}\right)^2\frac{k_1^+k_2^+}{(q^+)^3}\,\frac{k_2^+}{2(q^+)^3}$$
$$\times\, 2\mathrm{Re}\int_{\mathbf{v},\mathbf{v}',\mathbf{w},\mathbf{w}'} e^{i\mathbf{k}_1\cdot(\mathbf{v}'-\mathbf{v})}\,e^{i\mathbf{k}_2\cdot(\mathbf{w}'-\mathbf{w})}\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}'-\mathbf{v}'|\right)\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)$$
$$\times\,\mathrm{Tr}\bigg\langle\Big[\mathcal{U}_F(\mathbf{w}')\mathcal{U}_F^\dagger(\mathbf{v}')-1\Big]\bigg[\bigg(-i\mathcal{U}_F^{(2)}(\mathbf{v}) + \frac{(\mathbf{k}_2^j-\mathbf{k}_1^j)}{2}\mathcal{U}_{F;j}^{(1)}(\mathbf{v})\bigg)\mathcal{U}_F^\dagger(\mathbf{w}) + \frac{i}{2}\mathcal{U}_{F;j}^{(1)}(\mathbf{v})\;\partial_{\mathbf{w}^j}\mathcal{U}_F^\dagger(\mathbf{w})\bigg]\bigg\rangle$$

with

$$\mathcal{U}_F^{(2)}(\mathbf{v}) = \int_{z^+,z'^+}(z^+-z'^+)\,\theta(z^+-z'^+)\mathcal{U}_F(+\infty,z^+,\mathbf{v})[-igt\cdot\mathcal{F}_j{}^-(z^+,\mathbf{v})]\mathcal{U}_F(z^+,z'^+;\mathbf{v})[-igt\cdot\mathcal{F}_j{}^-(z'^+,\mathbf{v})]\mathcal{U}_F(z'^+,-\infty;\mathbf{v})$$

$$\mathcal{U}_{F;j}^{(1)}(\mathbf{v}) = -2\int_{z^+} z^+\,\mathcal{U}_F(+\infty,z^+;\mathbf{v})[-igt\cdot\mathcal{F}_j{}^-(z^+,\mathbf{v})]\mathcal{U}_F(z'^+,-\infty;\mathbf{v})$$

$$\mathcal{U}_{F;j}^{(1)}(\mathbf{v})\;\partial_{\mathbf{w}^j}\mathcal{U}_F^\dagger(\mathbf{w}) = -2\int_{z^+,w^+} z^+\,\mathcal{U}_F(+\infty,z^+;\mathbf{v})[-igt\cdot\mathcal{F}_j{}^-(z^+,\mathbf{v})]\mathcal{U}_F(z'^+,-\infty;\mathbf{v})$$
$$\times\,\mathcal{U}_F^\dagger(w^+,-\infty;\mathbf{w})[igt\cdot\mathcal{F}_j{}^-(w^+,\mathbf{w})]\mathcal{U}_F^\dagger(+\infty,w^+;\mathbf{w})$$

Remark: Terms with $\mathcal{F}_{ij}$ insertions cancel at cross section level for γ_L^*, but survive for γ_T^*

Back-to-back limit

Back-to-back limit of dijets are conveniently expressed in terms of:

(dijet momentum imbalance) $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ and (relative momentum) $\mathbf{P} = (z_2\mathbf{k}_1 - z_1\mathbf{k}_2)$

$z_1 = k_1^+/(k_1^+ + k_2^+)$ and $z_2 = k_2^+/(k_1^+ + k_2^+) = 1 - z_1$ such that

$$\boxed{\mathbf{k}_1 = \mathbf{P} + z_1\mathbf{k}} \qquad \boxed{\mathbf{k}_2 = -\mathbf{P} + z_2\mathbf{k}}$$

back-to-back correlation limit: $|\mathbf{k}| \ll |\mathbf{P}|$

In coordinate space:

(conjugate to $\mathbf{k}$) $\mathbf{b} = (z_1\mathbf{v} + z_2\mathbf{w})$ and (conjugate to P) $\mathbf{r} = \mathbf{v} - \mathbf{w}$

such that

$$\boxed{\mathbf{v} = \mathbf{b} + z_2\,\mathbf{r}} \qquad \boxed{\mathbf{w} = \mathbf{b} - z_1\,\mathbf{r}}$$

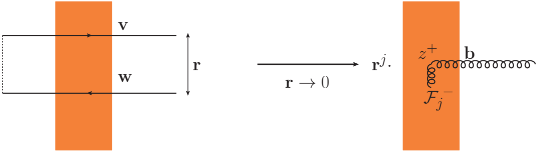
back-to-back correlation limit: $|\mathbf{r}| \ll |\mathbf{b}|$

Small $\mathbf{r}$ expansion for the eikonal contribution

Open dipole from the Generalized Eikonal term for $\mathbf{r} = \mathbf{v} - \mathbf{w} \to 0$:

$$\Big[\mathcal{U}_F\Big(\mathbf{v}, b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{w}, b^-\Big) - 1\Big] = -\frac{\mathbf{r}^j}{2}\Big[\mathcal{U}_F\Big(\mathbf{b}, b^-\Big)\overleftrightarrow{\partial_{\mathbf{b}^j}}\mathcal{U}_F^\dagger\Big(\mathbf{b}, b^-\Big)\Big] + O(\mathbf{r}^2)$$

$$= \mathbf{r}^j(-igt^a)\int_{z^+}\mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}, b^-\Big)_{ab}\mathcal{F}_j^{b\,-}(z^+, \mathbf{b}, b^-) + O(\mathbf{r}^2)$$

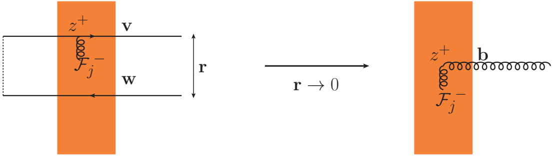


Note: 0th order in the $\mathbf{r}$ expansion trivial $\to$ first order needed

Small $\mathbf{r}$ limit for the NEik corrections

For the open decorated dipole with $\mathcal{U}^{(1)}_{F;j}$:

$$\begin{aligned}\mathcal{U}^{(1)}_{F;j}(\mathbf{v})\mathcal{U}^{\dagger}_{F}\Big(\mathbf{w}\Big) &= -2\int_{z^+} z^+\,\mathcal{U}_F\Big(+\infty, z^+;\mathbf{v}\Big)(-ig)t\cdot\mathcal{F}_j^{\;-}(z^+,\mathbf{v})\mathcal{U}_F\Big(z^+,-\infty;\mathbf{v}\Big)\mathcal{U}^{\dagger}_{F}(\mathbf{w})\\ &= 2igt^a\int_{z^+} z^+\,\mathcal{U}_A\Big(+\infty, z^+;\mathbf{b}\Big)_{ab}\mathcal{F}_j^{b\;-}(z^+,\mathbf{b}) + O(|\mathbf{r}|)\end{aligned}$$



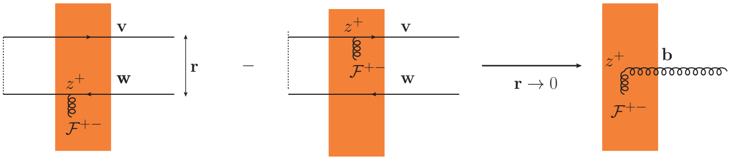
- Nontrivial result already at 0th order in the small $\mathbf{r}$ expansion due to the decoration
- Similar result as for the Generalized Eikonal contribution, except for the z^+ factor

Small $\mathbf{r}$ limit for the NEik corrections

For the open decorated dipole due to the dynamics of the target:

$$\begin{aligned}
&\left[\mathcal{U}_F\Big(\mathbf{v},b^-\Big)\overleftrightarrow{\partial_{b^-}}\mathcal{U}_F^\dagger\Big(\mathbf{w},b^-\Big)\right]\Big|_{b^-=0}\\
&=\int_{z^+}\Big\{\mathcal{U}_F(\mathbf{v})\mathcal{U}_F^\dagger\Big(z^+,-\infty;\mathbf{w}\Big)igt\cdot\mathcal{F}^{+-}(z^+,\mathbf{w})\mathcal{U}_F^\dagger\Big(+\infty,z^+;\mathbf{w}\Big)\\
&-\mathcal{U}_F\Big(+\infty,z^+;\mathbf{v}\Big)(-ig)t\cdot\mathcal{F}^{+-}(z^+,\mathbf{v})\mathcal{U}_F\Big(z^+,-\infty;\mathbf{v}\Big)\mathcal{U}_F^\dagger(\mathbf{w})\Big\}\\
&=2igt^a\int_{z^+}\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b}\Big)_{ab}\mathcal{F}_b^{+-}(z^+,\mathbf{b})+O(|\mathbf{r}|)
\end{aligned}$$

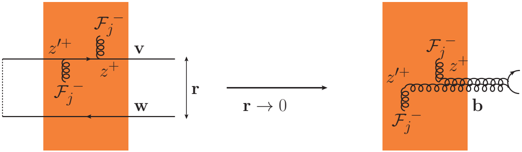
$\Rightarrow$ Involves the longitudinal chromoelectric field $\mathcal{F}^{+-}$ instead of the transverse field $\mathcal{F}_j^{\ -}$



Small $\mathbf{r}$ limit for the NEik corrections

For the open decorated dipole with $\mathcal{U}_F^{(2)}$:

$$\mathcal{U}_F^{(2)}(\mathbf{v})\mathcal{U}_F^{\dagger}(\mathbf{w}) = \int_{z^+,z'^+} (z^+ - z'^+)\,\theta(z^+ - z'^+)\mathcal{U}_F(+\infty, z^+, \mathbf{v})[-igt\cdot\mathcal{F}_j^{\;-}(z^+,\mathbf{v})]$$
$$\times\mathcal{U}_F(z^+, z'^+; \mathbf{v})[-igt\cdot\mathcal{F}_j^{\;-}(z'^+,\mathbf{v})]\mathcal{U}_F(z'^+, -\infty; \mathbf{v})\mathcal{U}_F^{\dagger}(\mathbf{w})$$
$$= -g^2(t^a t^b)\int_{z^+,z'^+} (z^+ - z'^+)\,\theta(z^+ - z'^+)\mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}\Big)_{aa'}\mathcal{F}_j^{a'\,-}(z^+,\mathbf{b})$$
$$\times\mathcal{U}_A\Big(+\infty, z'^+; \mathbf{b}\Big)_{bb'}\mathcal{F}_j^{b'\,-}(z'^+,\mathbf{b}) + O(|\mathbf{r}|)$$



2 Field strength insertions at amplitude level
⇒ At least 3 at cross section level (beyond TMDs)

Back-to-back cross section: (Generalized) Eikonal piece

The dijet cross section for the longitudinal photon in the back-to-back correlation limit:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}} = \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Gen. Eik}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{NEik corr.}}$$

with

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Gen.Eik}} = 2q^+ \int d(\Delta b^-) e^{i\Delta b^-(k_1^+ + k_2^+ - q^+)} (ee_f)^2 (q^+ + k_1^+ - k_2^+)^2 (q^+ - k_1^+ + k_2^+)^2 \frac{2k_1^+ k_2^+}{(q^+)^6} \frac{Q^2 \mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \hat{Q}^2)^4}$$
$$\times g^2 T_F \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \bigg\langle \mathcal{F}_i^{a\,-}\Big(z'^+, \mathbf{b}', -\frac{\Delta b^-}{2}\Big) \Big[\mathcal{U}_A^\dagger\Big(+\infty, z'^+; \mathbf{b}', -\frac{\Delta b^-}{2}\Big) \mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}, \frac{\Delta b^-}{2}\Big)\Big]_{ab}$$
$$\times \mathcal{F}_j^{b\,-}\Big(z^+, \mathbf{b}, \frac{\Delta b^-}{2}\Big)\bigg\rangle$$

On the other hand:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Gen. Eik}} = \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Strict Eik}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{NEik corr. from Gen. Eik}}$$

with

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Strict.Eik}} = 2q^+ 2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 T_F 32 z_1^3 z_2^3 Q^2 \frac{\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2 + \bar{Q}^2)^4}$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \Big\langle \mathcal{F}_i^{a\,-}(z'^+, \mathbf{b}') \Big[\mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}') \mathcal{U}_A(+\infty, z^+; \mathbf{b})\Big]_{ab} \mathcal{F}_j^{b\,-}(z^+, \mathbf{b})\Big\rangle$$

- **Strict eikonal contribution: Twist-2 gluon TMDs** (both linearly polarized and unpolarized)
- *NEik corr. from Gen. Eik: Either 2 or 3-body terms: twist 4?* (study is still in progress!)

Back-to-back cross section: NEik terms

Explicit NEik corrections to the dijet cross section for the longitudinal photon in the back-to-back correlation limit:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|^{\text{corr. lim.}}_{\text{NEik corr.}} = \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|^{\text{corr. lim.}}_{\text{dyn. target}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|^{\text{corr. lim.}}_{\text{dec. on } q+\bar{q}}$$

with

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|^{\text{corr. lim.}}_{\text{dyn. target}} = -(2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 T_F \frac{(k_1^+k_2^+)^2(k_1^+ - k_2^+)}{(q^+)^6}\frac{16Q^2\mathbf{P}^i}{(\mathbf{P}^2+\bar{Q}^2)^3}\left[1 - \frac{(\bar{Q}^2 - m^2)}{\mathbf{P}^2+\bar{Q}^2}\right]2\text{Re}$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_b^{+-}(z^+,\mathbf{b})\right\rangle$$

⇒ NEik. correction stemming from the dynamics of the target is a **twist-3 gluon TMD**. (Mulders, Rodrigues arXiv:0009343)

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|^{\text{corr. lim.}}_{\text{dec. on } q+\bar{q}} = 2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 T_F 16 z_1^2 z_2^2 Q^2 \frac{\mathbf{P}^i(2\mathbf{P}^j - (z_2 - z_1)\mathbf{k}^j)}{(\mathbf{P}^2+\bar{Q}^2)^3}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}$$
$$\times \int_{z^+,z'^+} i(z^+ - z'^+)\left\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\right\rangle$$
$$+ 3\,\text{body contributions}$$

- The term proportional to k^j is a kinematical twist 3 contribution.
- **The main contribution from this term is a contribution to twist-2 gluon TMDs.**

Twist 2 term from NEik corrections

Leading twist contributions from Strict Eik. and NEik (dec. on $q+\bar{q}$) terms can be combined:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{Strict Eik}}^{\text{corr. lim.}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{dec. on } q+\bar{q}}^{\text{corr. lim.}} \simeq 2\pi(2q^+)\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 T_F 32 z_1^3 z_2^3 Q^2 \frac{\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4}$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \left[1 + i(z^+ - z'^+)\frac{\mathbf{p}^2+\bar{Q}^2}{2q^+ z_1 z_2}\right] \Big\langle \mathcal{F}_i^{a\;-}(z'^+,\mathbf{b}') \Big[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\Big]_{ab} \mathcal{F}_j^{b\;-}(z^+,\mathbf{b})\Big\rangle$$

On the other hand, the " $-$ " momentum extracted from the target can be defined from the conservation relation:

$$xP^-_{tar.} \equiv \check{k}_1^- + \check{k}_2^- - q^- = \frac{\mathbf{k}_1^2+m^2}{2k_1^+} + \frac{\mathbf{k}_2^2+m^2}{2k_2^+} + \frac{Q^2}{2q^+} = \frac{\mathbf{P}^2+\bar{Q}^2}{2q^+z_1z_2} + \frac{\mathbf{k}^2}{2q^+}$$

- $\mathbf{k}^2$ term is a kinematical twist 4 contribution (can be neglected to our accuracy!)

The leading twist contribution can be summed into a phase! $\Rightarrow$ x dependence of the twist 2 gluon TMDs

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{Strict Eik}}^{\text{corr. lim.}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{dec. on } q+\bar{q}}^{\text{corr. lim.}} \simeq 2\pi(2q^+)\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 T_F 32 z_1^3 z_2^3 Q^2 \frac{\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4}$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} e^{i(z^+-z'^+)xP^-_{tar.}} \Big\langle \mathcal{F}_i^{a\;-}(z'^+,\mathbf{b}') \Big[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\Big]_{ab} \mathcal{F}_j^{b\;-}(z^+,\mathbf{b})\Big\rangle$$

Summary

To further understand the interplay between CGC and TMD, we studied the back-to-back limit of the NEik corrections to the DIS dijet cross-section. Various contributions are obtained:

- One twist 2 term, interpreted as the first order expansion of the x phase from the gluon TMD definition
- Kinematical twist 3 terms
- Twist 3 gluon TMDs, with one ${\mathcal{F}_i}^{-}$ replaced by $\mathcal{F}^{+-}$ or $\mathcal{F}^{ij}$
- Twist 3 correlators of 3 field strengths

Remark: Twist 3 terms typically proportional to $\mathbf{P}\cdot\mathbf{k}$
(for unpolarized target)
$\Rightarrow$ v_1-type azimuthal modulation in terms of $\mathbf{P}$ and $\mathbf{k}$