T-Odd Leading-Twist Quark TMDs at Small-x : Sub-Eikonal Evolution of the Sivers Function

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Quark TMDs



The leading twist quark TMDs give various correlations between the transverse momentum and polarizations of the quarks within a hadron with the polarization of the parent hadron

Their scale evolution in Q^2 is given by the CSS equations, but the small- x evolution is an ongoing effort

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The *T*-odd leading-twist quark TMD called the Sivers f_{1T}^{\perp} encodes spin-orbit coupling within the hadronic state

This function changes sign between SIDIS and the similar Drell-Yan (DY) process

$$f_{1T}^{\perp \text{ SIDIS}}(x, k_T^2) = -f_{1T}^{\perp \text{ DY}}(x, k_T^2)$$

Operator Definition of TMDs

Quark TMDs are defined by the non-local operator product in the hadron state

$$\Phi^{[\Gamma]} = \int \frac{\mathrm{d}r^{-} \,\mathrm{d}^{2} r_{\perp}}{2 \,(2\pi)^{3}} e^{ik \cdot r} \langle P, S | \bar{\psi}(r) \mathcal{U}[r,0] \Gamma \psi(0) | P, S \rangle$$

The unintegrated quark density f_1^q and the Sivers function $f_{1T}^{\perp q}$ are given by the taking the matrix to be $\gamma^+/2$

$$f_1^q(x,k_T^2) - \frac{\underline{k} \times \underline{S}_P}{M_P} f_{1T}^{\perp q}(x,k_T^2) = \int \frac{\mathrm{d}r^- \,\mathrm{d}^2 r_\perp}{2\,(2\pi)^3} e^{ik\cdot r} \langle P,S | \bar{\psi}(r) \mathcal{U}[r,0] \frac{\gamma^+}{2} \psi(0) | P,S \rangle$$

Small-*x* TMDs from polarized Wilson lines

Kovchegov, Sievert and Pitonyak (2015-2019) developed a Light Cone Operator Treatment (LCOT) for deriving small- *x* evolution of TMDs

Studied small- x evolution equations for the quark helicity TMD, gluon helicity TMD, quark transversity TMD, and quark Sivers function

Rewriting the TMD operator definitions at small- x yields modified dipole correlators



Small-*x* TMDs from polarized Wilson lines

Simplify

- \circ Rewrite operator definition in small- x limit using shockwave formalism
- Expand to a given order in eikonality
- Obtain expression for TMD in terms of 'polarized dipoles'

Evolve

- Calculate small- x gluon/quark emissions in dipole
- $\,\circ\,$ Take (for example) large- N_c limit to obtain closed equations

Solve

- Solve integral equations analytically (if possible) or numerically
- Plug evolved dipole back into TMD definition

Gauge Link to Dipole



Sub-eikonal corrections

The (anti)quark propagator through the shockwave can include sub-eikonal corrections to allow for spin-dependence

From the helicity and transversity TMDs, at least need sub-sub-eikonal corrections for general leading-twist quark TMD



$$\begin{split} \hat{V}_{\underline{w}}^{\dagger} &= \delta_{\chi,\chi'} V_{\underline{w}}^{\dagger} + \text{sub-eikonal corrections} \\ &= \delta_{\chi,\chi'} V_{\underline{w}}^{\dagger} + V_{\underline{w}}^{\text{pol}\dagger} \end{split}$$

Operator Product to Dipoles

The quark correlator can be rewritten in terms of Wilson lines

$$f_{1}^{q}(x,k_{T}^{2}) - \frac{\underline{k} \times \underline{S}_{P}}{M_{P}} f_{1T}^{\perp q}(x,k_{T}^{2}) = -\frac{2p_{1}^{+}}{2(2\pi)^{3}} \int d^{2}\zeta_{\perp} d^{2}w_{\perp} \frac{d^{2}k_{1\perp} dk_{1}^{-}}{(2\pi)^{3}} e^{i(\underline{k}_{1}+\underline{k})\cdot(\underline{w}-\underline{\zeta})}$$
$$\theta(k_{1}^{-}) \frac{1}{(xp_{1}^{+}k_{1}^{-}+\underline{k}_{1}^{2})(xp_{1}^{+}k_{1}^{-}+\underline{k}^{2})} \sum_{\chi_{1},\chi_{2}} \bar{v}_{\chi_{2}}(k_{2}) \frac{\gamma^{+}}{2} v_{\chi_{1}}(k_{1}) \left\langle \mathrm{T} V_{\underline{\zeta}}^{ij} \, \bar{v}_{\chi_{1}}(k_{1}) V_{\underline{w}}^{\dagger \, \mathrm{pol}, \, \mathrm{T}ji} v_{\chi_{2}}(k_{2}) \right\rangle$$

The polarized Wilson line $V_{\underline{w}}^{\dagger \text{ pol, T}}$ makes the correlator a transverse polarized dipole



Eikonal Flavor Non-Singlet Sivers Function

The flavor non-singlet Sivers function has a nonzero eikonal contribution

$$\begin{split} & \left[-\frac{\underline{k} \times \underline{S}_P}{M_P} f_{1\,T}^{\perp \,NS}(x, k_T^2) \right]_{\text{eikonal}} \subset \frac{4p_1^+}{(2\pi)^3} \int \mathrm{d}^2 \zeta_\perp \,\mathrm{d}^2 w_\perp \,\frac{\mathrm{d}^2 k_{1\perp} \,\mathrm{d} k_1^-}{(2\pi)^3} e^{i(\underline{k}_1 + \underline{k}) \cdot (\underline{w} - \underline{\zeta})} \theta(k_1^-) \\ & \times \left\{ \frac{\underline{k} \cdot \underline{k}_1}{(xp_1^+ k_1^- + \underline{k}_1^2)(xp_1^+ k_1^- + \underline{k}^2)} \left\langle \mathrm{T} \,\operatorname{tr} \left[V_{\underline{\zeta}} V_{\underline{w}}^{\dagger} \right] - \mathrm{T} \,\operatorname{tr} \left[V_{\underline{w}} V_{\underline{\zeta}}^{\dagger} \right] + \bar{\mathrm{T}} \,\operatorname{tr} \left[V_{\underline{\zeta}} V_{\underline{w}}^{\dagger} \right] - \bar{\mathrm{T}} \,\operatorname{tr} \left[V_{\underline{w}} V_{\underline{\zeta}}^{\dagger} \right] \right\} \\ & + \frac{\underline{k}_1^2}{(xp_1^+ k_1^- + \underline{k}_1^2)^2} \left\langle \mathrm{T} \,\operatorname{tr} \left[V_{\underline{\zeta}} V_{\underline{w}}^{\dagger} \right] - \mathrm{T} \,\operatorname{tr} \left[V_{\underline{w}} V_{\underline{\zeta}}^{\dagger} \right] \right\rangle \right\} \end{split}$$

The terms in angle brackets give us the imaginary part of eikonal dipoles

Dipole Odderon

In the color dipole picture, the odderon is the antisymmetric, imaginary piece of a dipole correlator



Eikonal Sivers Function = Spin-Dependent Odderon

The imaginary correlator in the Sivers function is exactly the odderon amplitude, so we have

$$-\frac{\underline{k} \times \underline{S}_{P}}{M_{P}} f_{1\,T}^{\perp\,q}(x,k_{T}^{2})\Big|_{\text{eikonal}} = \frac{4i\,N_{c}\,p_{1}^{+}}{(2\pi)^{3}} \int \mathrm{d}^{2}\zeta_{\perp}\,\mathrm{d}^{2}w_{\perp}\,\frac{\mathrm{d}^{2}k_{1\perp}\,\mathrm{d}k_{1}^{-}}{(2\pi)^{3}} e^{i(\underline{k}_{1}+\underline{k})\cdot(\underline{w}-\underline{\zeta})}\theta(k_{1}^{-}) \\ \times \left[\frac{2\,\underline{k}\cdot\underline{k}_{1}}{(xp_{1}^{+}k_{1}^{-}+\underline{k}_{1}^{2})(xp_{1}^{+}k_{1}^{-}+\underline{k}^{2})} + \frac{\underline{k}_{1}^{2}}{(xp_{1}^{+}k_{1}^{-}+\underline{k}_{1}^{2})^{2}}\right]\mathcal{O}_{\underline{\zeta}\underline{w}} \\ \left[f_{1T}^{\perp}(x,k_{T}^{2})\Big|_{\text{eikonal}} \sim \frac{1}{x}\right]$$

Agreement with the results of Boer et al (2016) and Zhou et al (2019)!

Eikonal spin-dependent effect even after evolution

Sub-Eikonal Flavor Non-Singlet Sivers Function

Previously we considered the known eikonal (spin-dependent odderon) contribution and the new sub-eikonal contribution

We neglected some sub-eikonal operators which mix and change the small- x asymptotics, which we have now restored

$$-\frac{\underline{k}\times\underline{S}_{P}}{M_{P}}f_{1T}^{\perp NS}(x,k_{T}^{2})\Big|_{\text{sub-eikonal}} = \frac{16N_{c}}{(2\pi)^{3}}\int d^{2}x_{10}\frac{d^{2}k_{1\perp}}{(2\pi)^{3}}\frac{e^{i(\underline{k}+\underline{k}_{1})\cdot\underline{x}_{10}}}{\underline{k}_{1}^{2}\underline{k}^{2}}\int_{\underline{\lambda}_{s}^{2}}^{1}\frac{dz}{z}$$

$$\times\left\{\underline{k}_{1}\cdot\underline{k}(k-k_{1})^{i}\left[\epsilon^{ij}S_{P}^{j}x_{10}^{2}F_{NS}^{A}(x_{10}^{2},z)+x_{10}^{i}\underline{x}_{10}\times\underline{S}_{P}F_{NS}^{B}(x_{10}^{2},z)+\epsilon^{ij}x_{10}^{j}\underline{x}_{10}\cdot\underline{S}_{P}F_{NS}^{C}(x_{10}^{2},z)\right]$$

$$+i\underline{k}_{1}\cdot\underline{k}\underline{x}_{10}\times\underline{S}_{P}F_{NS}^{[2]}(x_{10}^{2},z)-i\underline{k}\times\underline{k}_{1}\underline{x}_{10}\cdot\underline{S}_{P}F_{NS}^{\mathrm{mag}}(x_{10}^{2},z)\right\}$$

$$=\text{chromomagnetic interaction}+\text{covariant phase factor}$$

$$Previously missed!$$

Large- N_c Small-x Evolution for Sivers



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Large- N_c Linearized Small-x Evolution for NS Sivers in Double Logarithmic Approximation (DLA)

 $F_A^{NS}(x_{10}^2, z) = F_A^{NS\,(0)}(x_{10}^2, z)$ $+ \frac{\alpha_s N_c}{4\pi} \int^z \frac{dz'}{z'} \int^{\min\left[\frac{z}{z'}x_{10}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{21}^2}{x_{21}^2} \left[6 F_A^{NS}(x_{21}^2, z') - F_B^{NS}(x_{21}^2, z') + F_C^{NS}(x_{21}^2, z') \right]$ $F_B^{NS}(x_{10}^2, z) = F_B^{NS(0)}(x_{10}^2, z)$ $+ \frac{\alpha_s N_c}{4\pi} \int^z \frac{dz'}{z'} \int^{\min\left[\frac{z}{z'}x_{10}^2, \frac{1}{\Lambda^2}\right]} \int \left[-2 F_A^{NS}(x_{21}^2, z') + 5 F_B^{NS}(x_{21}^2, z') - F_C^{NS}(x_{21}^2, z')\right],$ $F_C^{NS}(x_{10}^2,z) = F_C^{NS\,(0)}(x_{10}^2,z)$ $+ \frac{\alpha_s N_c}{4\pi} \int_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int_{\max[x_{10}^2, \frac{1}{\lambda'^2}]}^{\min[\frac{z}{z'}, x_{10}^2, \frac{1}{\Lambda^2}]} \frac{dx_{21}^2}{x_{21}^2} \left[2 F^{NS \max}(x_{21}^2, z') + 6 F_C^{NS}(x_{21}^2, z') \right],$ $F^{NS\max}(x_{10}^2, z) = F^{NS\max(0)}(x_{10}^2, z)$ $+ \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma^{NS \max}(x_{10}^2, x_{21}^2, z') \right]$ $\left. + \, 2 \, \Gamma_A^{NS}(x_{10}^2,x_{21}^2,z') - \Gamma_B^{NS}(x_{10}^2,x_{21}^2,z') + 3 \, \Gamma_C^{NS}(x_{10}^2,x_{21}^2,z') \right]$

$$\begin{split} \Gamma_A^{NS}(x_{10}^2, x_{21}^2, z') &= F_A^{NS\,(0)}(x_{10}^2, z') + \frac{\alpha_s \, N_c}{4\pi} \int_{-\infty}^{z' \frac{x_{10}^2}{x_{10}^2}} \frac{\min\left[\frac{z''}{z''} \int_{\max[x_{10}^2, \frac{1}{z''_s}]}^{\min\left[\frac{z}{z''} x_{21}^2, \frac{1}{\Lambda^2}\right]} dx_{32}^2}{\max[x_{10}^2, \frac{1}{z''_s}]} \\ &\times \left[6 \, F_A^{NS}(x_{32}^2, z'') - F_B^{NS}(x_{32}^2, z'') + F_C^{NS}(x_{32}^2, z'') \right] \\ \Gamma_B^{NS}(x_{10}^2, x_{21}^2, z') &= F_B^{NS\,(0)}(x_{10}^2, z') + \frac{\alpha_s \, N_c}{4\pi} \int_{-\infty}^{z' \frac{x_{10}^2}{x_{10}^2}} \frac{dz''}{z''} \int_{\max[x_{10}^2, \frac{1}{z''_s}]}^{\min\left[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}\right]} dx_{32}^2 \\ &\times \left[-2 \, F_A^{NS}(x_{23}^2, z'') + 5 \, F_B^{NS}(x_{32}^2, z'') - F_C^{NS}(x_{32}^2, z'') \right], \\ \Gamma_C^{NS}(x_{10}^2, x_{21}^2, z') &= F_C^{NS\,(0)}(x_{10}^2, z') + \frac{\alpha_s \, N_c}{4\pi} \int_{-\frac{x_s}{x_0}}^{z' \frac{x_{11}}{x_0^2}} \int_{\max[x_{10}^2, \frac{1}{z''_s}]}^{\min\left[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}\right]} dx_{32}^2 \\ &\times \left[2 \, F^{NS\,\max}(x_{23}^2, z'') + 6 \, F_C^{NS}(x_{32}^2, z'') \right], \\ \Gamma^{NS\,\max}(x_{10}^2, x_{21}^2, z') &= F^{NS\,\max}(0)(x_{10}^2, z') \\ &+ \frac{\alpha_s \, N_c}{2\pi} \, \int_{\frac{z'}{x_{10}}}^{z'} \frac{dz''}{z''} \int_{\frac{z''}{x_{10}}}^{\min\left[x_{10}^2, x_{21}^2, \frac{z'}{x_0}\right]} \frac{dx_{32}^2}{x_{32}^2} \\ &\times \left[2 \, F^{NS\,\max}(0)(x_{10}^2, x_{21}^2, z') + 6 \, F_C^{NS}(x_{23}^2, z'') \right], \\ \Gamma^{NS\,\max}(x_{10}^2, x_{21}^2, z') &= F^{NS\,\max}(0)(x_{10}^2, x_{21}^2, z') \\ &+ \frac{\alpha_s \, N_c}{2\pi} \, \int_{\frac{z'}{x_{10}}}^{z'} \frac{dz''}{z''} \, \int_{\frac{z''}{x_{10}}}^{\min\left[x_{10}^2, x_{21}^2, \frac{z'}{x_{10}^2}\right]} \frac{dx_{32}^2}{x_{32}^2} \\ &\times \left[\Gamma^{NS\,\max}(x_{10}^2, x_{22}^2, z'') + 2 \, \Gamma_A^{NS}(x_{10}^2, x_{32}^2, z'') - \Gamma_B^{NS}(x_{10}^2, x_{32}^2, z'') + 3 \, \Gamma_C^{NS}(x_{10}^2, x_{32}^2, z'') \right] \right] \right\}$$

'Neighbor' dipole evolution equations

Dipole evolution equations

Numerical Solution for NS Sivers



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We find that a modified ansatz beyond the simple power law behavior describes the numerical solution well

$$F_{NS}=F_{NS}^{(0)}\,e^{lpha\eta+eta e^{-\gamma\eta}}$$
 'pre-asymptotic' correction usual intercept term

Interesting to find an exponential correction to the intercept, equivalently an infinite series of sub-leading power corrections

$$e^{\alpha\eta+\beta e^{-\gamma\eta}} \xrightarrow[x \to 0]{} \left(\frac{1}{x}\right)^{\alpha\sqrt{\frac{\alpha_s N_c}{4\pi}}} + \beta\left(\frac{1}{x}\right)^{(\alpha-\gamma)\sqrt{\frac{\alpha_s N_c}{4\pi}}} + \frac{\beta^2}{2}\left(\frac{1}{x}\right)^{(\alpha-2\gamma)\sqrt{\frac{\alpha_s N_c}{4\pi}}} + \dots$$

Numerical Results

The pre-asymptotic terms are very small compared to our numerical precision, so extrapolating to the continuum limit we find

$$F_A^{NS} \sim F_B^{NS} \sim F_C^{NS} \sim F_C^{NS} \sim \left(\frac{1}{x}\right)^{3.4\sqrt{\frac{\alpha_s N_c}{4\pi}}}$$

Plugging this into the definition of the Sivers function we find the sub-eikonal contribution as

$$\left. f_{1\,T}^{\perp NS}(x,k_T^2) \right|_{\text{sub-eikonal}} \sim \left(\frac{1}{x}\right)^{3.4\sqrt{\frac{\alpha_s N_c}{4\pi}}}$$

Conclusions

We obtained new sub-eikonal evolution after restoring the missing F_{mag} dipole

Small-*x* Sivers function is dominated by eikonal spin-dependent odderon with a sub-eikonal, energy dependent correction

$$\begin{aligned} f_{1\,T}^{\perp\,q}(x \ll 1, k_T^2) &= C_O(k_T^2, x) \, \frac{1}{x} + C_1(k_T^2) \, \left(\frac{1}{x}\right)^{3.4 \sqrt{\frac{\alpha_s N_c}{4\pi}}} + \dots \\ \uparrow & \uparrow \\ \text{Odderon} & \text{Sub-eikonal Correction} \end{aligned}$$

Odderon small-*x* evolution is known to have a zero intercept away from the saturation regime

Sub-eikonal correction provides a background which may be comparable with the odderon

Backup Slides

Small- $x \rightarrow$ Shockwave formalism

Fourier factor picks out long range correlations in the x^- direction

$$e^{ixP^+r^-} \to \text{large } r^- \text{ for small } x$$

Hadron is very Lorentz contracted, so interactions in gauge link happen over short x^- lifetime inside the shockwave



Eikonal power counting

We can expand in powers of x or equivalently inverse powers of CM energy s

Eikonal distributions $q(x, k_T) \sim \frac{1}{x}$, no COM energy suppression Sub-eikonal distributions $q(x, k_T) \sim x^0$, $\frac{1}{s}$ energy suppression Sub-sub-eikonal distributions $q(x, k_T) \sim x$, $\frac{1}{s^2}$ energy suppression

Small-*x* TMD diagrams



Fairly general analysis shows that only class B diagrams contribute to spin dependent TMDs at sub-eikonal order

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General polarized Wilson line



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General polarized Wilson line

Full **sub-sub-eikonal** polarized fundamental Wilson line for TMDs which depend on the proton's transverse spin

$$\begin{split} & V_{\underline{x},\underline{y};\chi',\chi} = V_{\underline{x}} \, \delta^2(\underline{x} - \underline{y}) \, \delta_{\chi,\chi'} + \int_{-\infty}^{\infty} \mathrm{d}z^- \, d^2z \, V_{\underline{x}}[\infty, z^-] \, \delta^2(\underline{x} - \underline{z}) \, \mathcal{O}_{\chi',\chi}^{\mathrm{pol}\,\mathrm{G}}(z^-, \underline{z}) \, V_{\underline{y}}[z^-, -\infty] \, \delta^2(\underline{y} - \underline{z}) \\ & + \int_{-\infty}^{\infty} \mathrm{d}z_1^- \, d^2z_1 \int_{z_1^-}^{\infty} \mathrm{d}z_2^- \, d^2z_2 \, \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z_2^-] \, \delta^2(\underline{x} - \underline{z}_2) \, \mathcal{O}_{\chi',\chi''}^{\mathrm{pol}\,\mathrm{G}}(z^-, \underline{z}_2) \, V_{\underline{z}_1}[z^-_2, z^-_1] \, \delta^2(\underline{z} - \underline{z}_1) \\ & \times \, \mathcal{O}_{\chi'',\chi}^{\mathrm{pol}\,\mathrm{G}}(z^-_1, \underline{z}_1) \, V_{\underline{y}}[z^-_1, -\infty] \, \delta^2(\underline{y} - \underline{z}_1) + \int_{-\infty}^{\infty} \mathrm{d}z_1^- \int_{z_1^-}^{\infty} \mathrm{d}z_2^- \, V_{\underline{x}}[\infty, z_2^-] \, \mathcal{O}_{\chi',\chi''}^{\mathrm{pol}\,\mathrm{G}}(z^-_2, z^-_1; \underline{x}, \underline{y}) \, V_{\underline{y}}[z^-_1, -\infty] \\ & + \int_{-\infty}^{\infty} \mathrm{d}z_1^- \int_{z_1^-}^{\infty} \mathrm{d}z_2^- \int_{z_2^-}^{\infty} \mathrm{d}z_3^- \int_{z_3^-}^{\infty} \mathrm{d}z_4^- \, d^2z \, \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z_4^-] \, \mathcal{O}_{\chi',\chi''}^{\mathrm{pol}\,\mathrm{G}}(\overline{z}_4^-, z^-_3; \underline{x}, \underline{z}) \, V_{\underline{z}}[z^-_3, z^-_2] \, \mathcal{O}_{\chi'',\chi}^{\mathrm{pol}\,\mathrm{G}}(\overline{z}_2^-, z^-_1; \underline{z}, \underline{y}) \, V_{\underline{y}}[z^-_1, -\infty] \\ & + \int_{-\infty}^{\infty} \mathrm{d}z_1^- \int_{z_1^-}^{\infty} \mathrm{d}z_2^- \int_{z_2^-}^{\infty} \mathrm{d}z_3^- \int_{z_3^-}^{\infty} \mathrm{d}z_4^- \, d^2z \, \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z^-_4] \, \mathcal{O}_{\chi',\chi''}^{\mathrm{pol}\,\mathrm{G}}(\overline{z}_4^-, z^-_3; \underline{x}, \underline{z}) \, V_{\underline{z}}[z^-_3, z^-_2] \, \mathcal{O}_{\chi'',\chi}^{\mathrm{pol}\,\mathrm{G}}(\overline{z}_2^-, z^-_1; \underline{z}, \underline{y}) \, V_{\underline{y}}[z^-_1, -\infty] \\ & + \int_{-\infty}^{\infty} \mathrm{d}z_1^- \int_{z_1^-}^{\infty} \mathrm{d}z_2^- \, d^2z_2 \, \int_{z_2^-}^{\infty} \mathrm{d}z_3^- \, V_{\underline{x}}[\infty, z^-_3] \, \delta^2(\underline{x}_2 - \underline{x}) \, \mathcal{O}_{\chi',\chi''}^{\mathrm{pol}\,\mathrm{G}}(\overline{q}}(\overline{z}_1, z^-_2, z^-_3; \underline{x}, \underline{z}_2) \delta^2(\underline{z}_2 - \underline{y}) V_{\underline{y}}[z^-_1, -\infty] \\ & + \int_{-\infty}^{\infty} \mathrm{d}z_1^- \, \int_{z_1^-}^{\infty} \mathrm{d}z_2^- \, \int_{z_2^-}^{\infty} \mathrm{d}z_3^- \, d^2z \, \chi_{\underline{x}'=\pm 1} \, V_{\underline{x}}[\infty, z^-_3] \, \delta^2(\underline{x} - \underline{z}) \, \mathcal{O}_{\chi',\chi'''}^{\mathrm{pol}\,\mathrm{G}}(\overline{z}_3; \underline{z}) \, V_{\underline{z}}[z^-_3, z^-_2] \, \mathcal{O}_{\chi'',\chi''}^{\mathrm{pol}\,\mathrm{G}}(\overline{z}_1, z^-_2, z^-_3; \underline{z}, \underline{z}) \, V_{\underline{y}}[z^-_1, -\infty] \\ & + \int_{-\infty}^{\infty} \mathrm{d}z_1^- \, \int_{z_1^-}^{\infty} \mathrm{d}z_2^- \, \int_{z_2^-}^{\infty} \mathrm{d}z_3^- \, \mathcal{U}_{\underline{x}}[\infty, z^-_3] \, \delta^2(\underline{x} - \underline{z}) \, \mathcal{O}_{\chi',\chi''}^{\mathrm{pol}\,\mathrm{G}}(\overline{z}_3; \underline{z}) \, V_{\underline{z}}[z^-_3, z^-_1] \, \mathcal{O}_{\chi'',\chi''}^{\mathrm{pol}\,\mathrm{G}}(\overline{z}_1, z^-_1; \underline{$$



cf. *Altinoluk et al* (2020), *Chirilli* (2021) subeikonal propagator

Sivers Flavor Non-Singlet Dipoles

$$\begin{split} F^{NS\,i}_{\underline{w},\underline{\zeta}}(z) &= \frac{1}{2N_c} \operatorname{Re} \left\langle \! \left\langle \operatorname{T} \operatorname{tr} \left[V_{\underline{\zeta}} V_{\underline{w}}^{i\dagger} \right] - \operatorname{T} \operatorname{tr} \left[V_{\underline{w}}^{i} V_{\underline{\zeta}}^{\dagger} \right] \right\rangle \! \right\rangle, \\ F^{NS\,[2]}_{\underline{w},\underline{\zeta}}(z) &= \frac{1}{2N_c} \operatorname{Im} \left\langle \! \left\langle \operatorname{T} \operatorname{tr} \left[V_{\underline{\zeta}} V_{\underline{w};\underline{k},\underline{k}_1}^{[2]\dagger} \right] - \operatorname{T} \operatorname{tr} \left[V_{\underline{w};\underline{k},\underline{k}_1} V_{\underline{\zeta}}^{\dagger} \right] \right\rangle \! \right\rangle, \\ F^{NS\,\max}_{\underline{w},\underline{\zeta}}(z) &= \frac{1}{2N_c} \operatorname{Re} \left\langle \! \left\langle \operatorname{T} \operatorname{tr} \left[V_{\underline{\zeta}} V_{\underline{w}}^{\max} \dagger \right] - \operatorname{T} \operatorname{tr} \left[V_{\underline{w}}^{\max} V_{\underline{\zeta}}^{\dagger} \right] \right\rangle \! \right\rangle \\ V^{i}_{\underline{x}} &= -\frac{p_1^{i}}{8s} \int_{-\infty}^{\infty} \mathrm{d} z^{-} \ V_{\underline{x}}[\infty, z^{-}] \left(\bar{D}_z^{i} - \bar{D}_z^{i} \right) V_{\underline{x}}[z^{-}, -\infty] \\ V^{(2)}_{\underline{x}} &= \frac{i p_1^{+}}{8s} \int_{-\infty}^{\infty} \mathrm{d} z^{-} \ V_{\underline{x}}[\infty, z^{-}] \left((\bar{D}_z^{i} - \bar{D}_z^{i})^2 - (\underline{k}_1 - \underline{k})^2 \right) V_{\underline{x}}[z^{-}, -\infty] \\ &- \frac{g^2 p_1^{+}}{4s} \int_{-\infty}^{\infty} \mathrm{d} z_1^{-} \int_{z_1^{-}}^{\infty} \mathrm{d} z_2^{-} V_{\underline{x}}[\infty, z_2^{-}] t^b \psi_{\beta}(z_2^{-}, \underline{x}) U_{\underline{x}}^{ba}[z_2^{-}, z_1^{-}] \left[\frac{\gamma^{+}}{2} \right]_{\alpha\beta} \ \bar{\psi}_{\alpha}(z_1^{-}, \underline{x}) t^a V_{\underline{x}}[z_1^{-}, -\infty] \\ &- \frac{g^2 p_1^{+}}{4s} \int_{-\infty}^{\infty} \mathrm{d} z_1^{-} \int_{z_1^{-}}^{\infty} \mathrm{d} z_2^{-} V_{\underline{x}}[\infty, z^{-}] F^{12}(z^{-}, \underline{x}) U_{\underline{x}}^{ba}[z_2^{-}, z_1^{-}] \left[\frac{\gamma^{+} \gamma^5}{2} \right]_{\alpha\beta} \ \bar{\psi}_{\alpha}(z_1^{-}, \underline{x}) t^a V_{\underline{x}}[z_1^{-}, -\infty] \end{split}$$

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