Small x resummation of photon impact factor and the $\gamma^*\gamma^*$ high energy scattering

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Outline

- Resummation
- Collinear Limit
- LO Transverse Renormalization Group Improved (RGI) Impact Factor
- NLO Transverse RGI Impact Factor
- Longitudinal Impact Factors
- Numerical Results
- Conclusions

Resummation: Gluon Green's Function (GGF)

- GGF is known to Next-to-leading order (NLO) in QCD.

- LL Balitsky-Fadin-Kuraev-Lipatov (BFKL) kernel

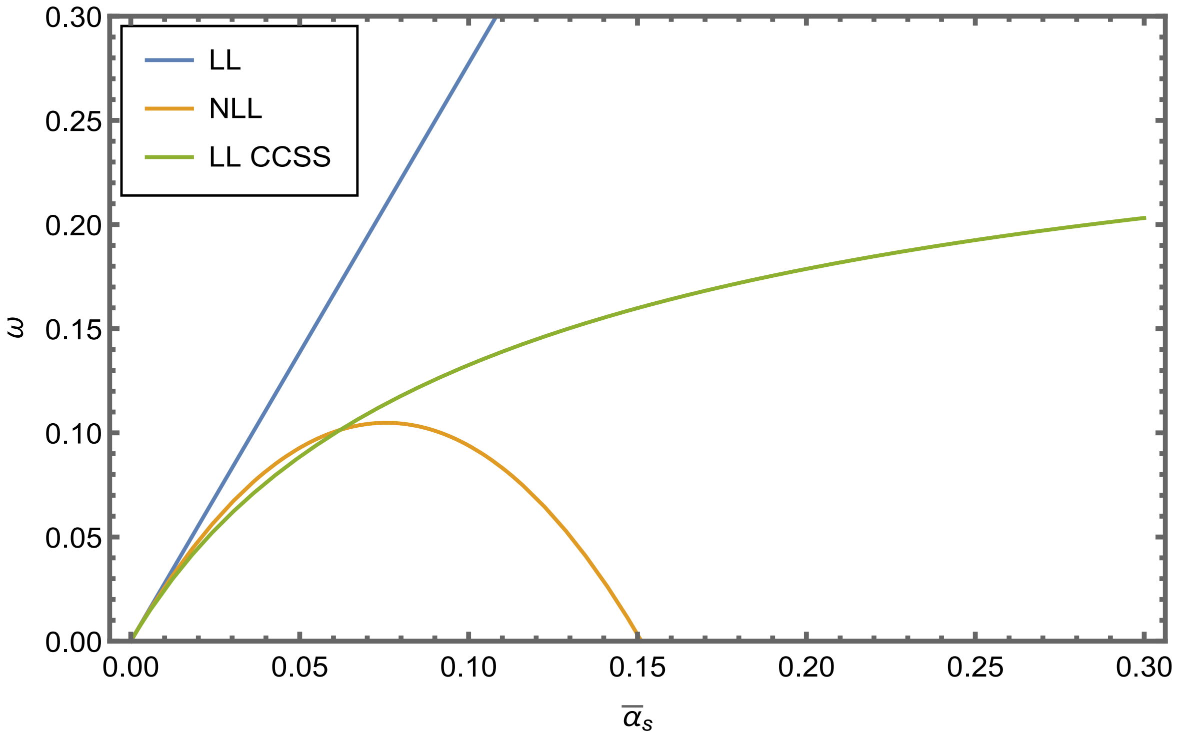
$$\omega = \bar{\alpha}_s \chi(\gamma) = \bar{\alpha}_s [\, 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)],$$

$\bar{\alpha}_s = \alpha_s(\mu)\frac{N_c}{\pi}$, with N_c the number of colors.

- NLL corrections to the BFKL equation are large and negative, and may lead to the instabilities.

- Resummation was developed which takes into account LL and NLL terms, the effects of kinematics, the DGLAP splitting function and the running coupling effects.

ω intercept



- LL Ciafaloni-Colferai-Salam-Staśto (CCSS) renormalization group improved resummation gives a kernel

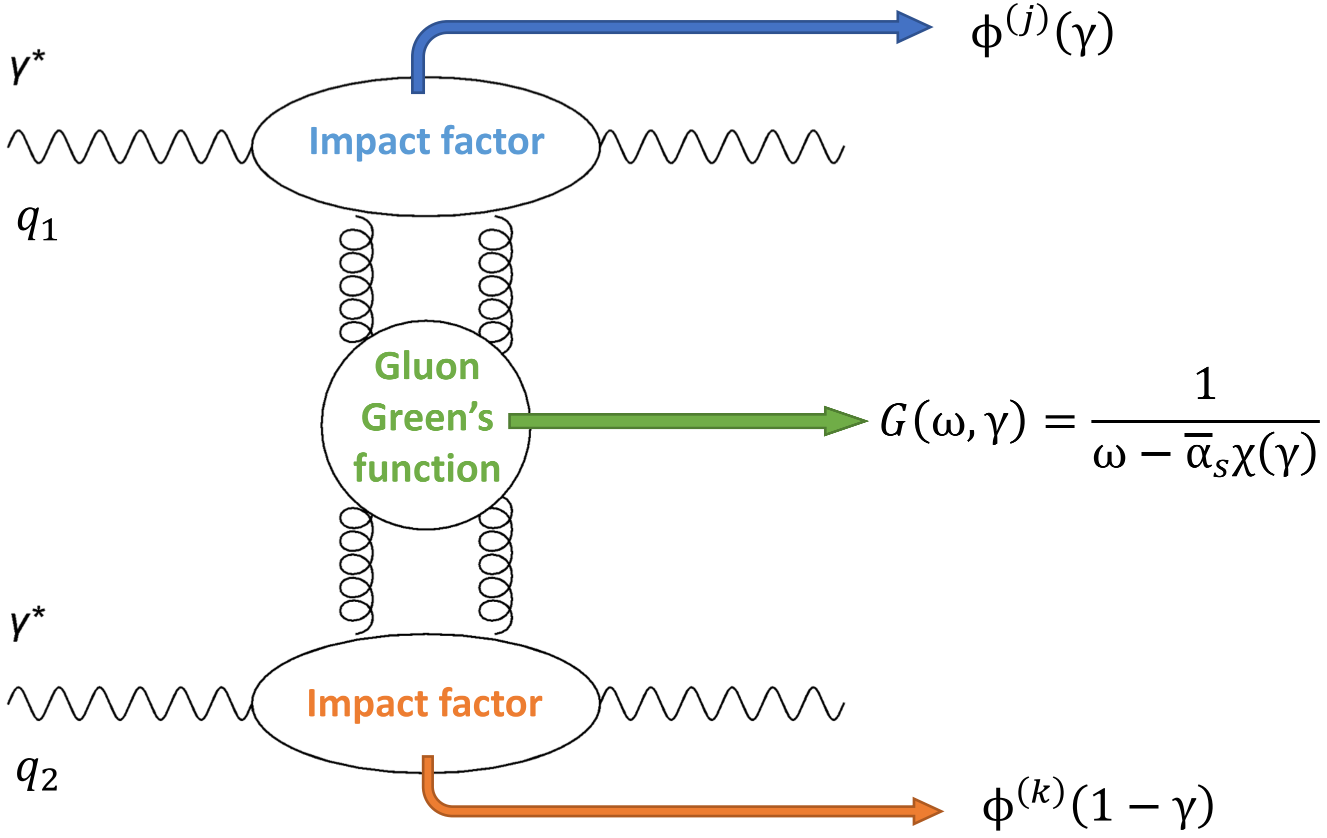
$$\chi^{\omega}(\gamma) = 2\psi(1) - \psi\left(\gamma + \frac{\omega}{2}\right) - \psi\left(1-\gamma+\frac{\omega}{2}\right) + \omega\left(A_{gg} - \frac{b}{2}\right)\left(\frac{1}{\gamma+\frac{\omega}{2}} + \frac{1}{1-\gamma+\frac{\omega}{2}}\right),$$

With A_{gg} the anomalous dimension without the $1/\omega$ term and $b_0 = \frac{11C_A - 4T_R N_f}{12\pi}$.

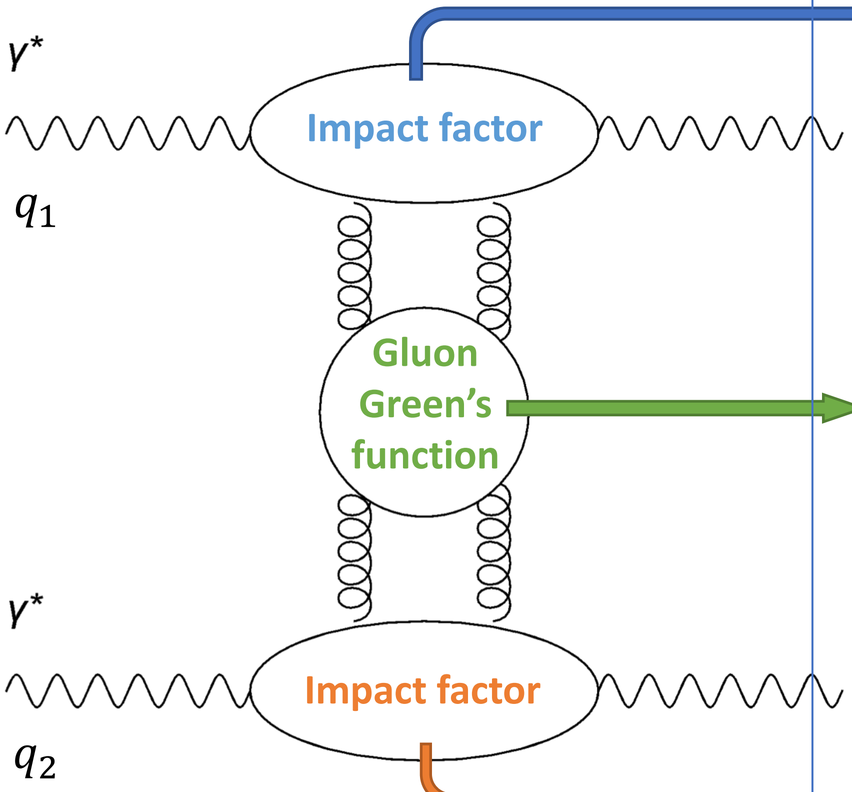
Resummation: $\gamma^*\gamma^*$scattering

$$\sigma^{(jk)}(s, Q_1, Q_2) = \frac{1}{2\pi Q_1 Q_2} \int \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{Q_1^2}{Q_2^2}\right)^{\gamma - \frac{1}{2}} \phi^{(j)}(\gamma)\, G(\omega, \gamma)\, \phi^{(k)}(1-\gamma)$$

- j, k denote the polarization of photons.
- Q_1^2, Q_2^2 are the negative photon virtualities.
$$Q_1^2 = -q_1^2, \qquad Q_2^2 = -q_2^2.$$
- s_0 is the Regge energy scale. The center-of-mass energies squared
$$s = (q_1 + q_2)^2$$
- The impact factors are known to NLO (By Balitsky, Chirilli & Kovchegov). Calculations in literature need heave tuning on parameters to agree the experiments.



Resummation



BFKL

$$\phi^{(j)}(\gamma)$$

$$G(\omega,\gamma) = \frac{1}{\omega - \bar{\alpha}_s \chi(\gamma)}$$

$$\phi^{(k)}(1-\gamma)$$

Renormalization group Improved (RGI)

$$\Phi^{(j)}(\omega,\gamma)$$

$$G(\omega,\gamma) = \frac{1}{\omega - \bar{\alpha}_s \mathrm{X}(\omega,\gamma)}$$

$$\Phi^{(k)}(\omega,1-\gamma)$$

Resummation

$$\sigma^{(jk)}(s,Q_1,Q_2)=\frac{1}{2\pi Q_1Q_2}\int\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega}\int\frac{d\gamma}{2\pi i}\left(\frac{Q_1^2}{Q_2^2}\right)^{\gamma-\frac{1}{2}}\Phi^{(j)}(\omega,\gamma)\,G(\omega,\gamma)\,\Phi^{(k)}(\omega,1-\gamma)$$

- $\int\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega}(\ldots)$ can be solved by deriving residue on ω.
- $G(\omega,\gamma)=\frac{1}{\omega-\bar{\alpha}_s X(\omega,\gamma)}$ implies the residue at $\omega=\bar{\alpha}_s X(\omega,\gamma)$, its solution
$$\omega^{\mathrm{eff}}(\gamma,\bar{\alpha}_s)=\bar{\alpha}_s X^{\mathrm{eff}}(\gamma,\bar{\alpha}_s)$$
- ω-integral singles out the residue at such pole,
$$\operatorname*{Res}_{\omega=\omega^{\mathrm{eff}}}\left[\frac{1}{\omega-\bar{\alpha}_s X(\omega,\gamma)}\right]=\frac{1}{1-\bar{\alpha}_s\partial_\omega X(\omega^{\mathrm{eff}},\gamma)}$$

BFKL		RGI
$\chi(\gamma)$	$=$	$X\left(\omega^{\mathrm{eff}},\gamma\right)$
$\phi^{(j)}(\gamma)\phi^{(k)}(1-\gamma)$	$=$	$\frac{\Phi^{(j)}\left(\omega^{\mathrm{eff}},\gamma\right)\Phi^{(k)}\left(\omega^{\mathrm{eff}},1-\gamma\right)}{1-\bar{\alpha}_s\partial_\omega X\left(\omega^{\mathrm{eff}},\gamma\right)}$

- LO: $1-\bar{\alpha}_s\partial_\omega X\left(\omega^{\mathrm{eff}},\gamma\right)$ does not contribute.
- NLO: Expanding in $\bar{\alpha}_s$ can give the correspondence.

Collinear Limit

- Take photon virtualities

$$Q_1^2 \gg Q_2^2,$$

and assume Strong ordering on the transverse momentum.

- LO BFKL kernel just by collinear and anti-collinear limits,

$$K^{coll}(Q^2, k^2) = \bar{\alpha}_s \left[\frac{\Theta(Q^2 - k^2)}{Q^2} + \frac{\Theta(k^2 - Q^2)}{k^2} \right].$$

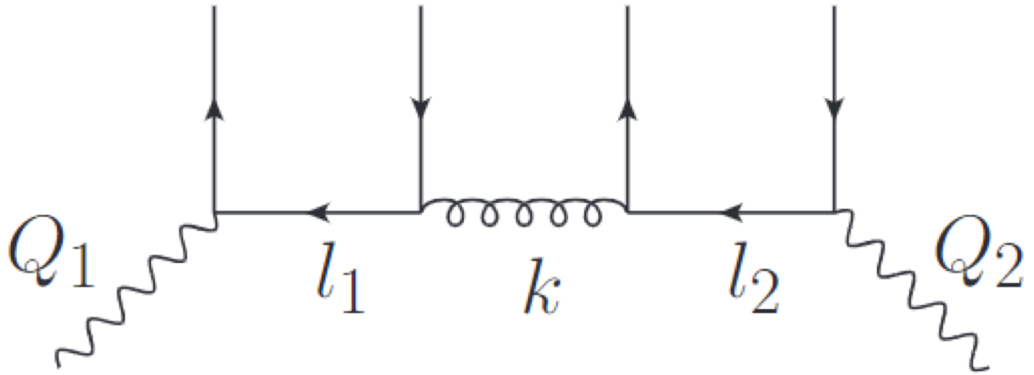
- Mellin transform in (ω, γ) space,

$$\chi^{\text{coll}}(\gamma) = \frac{1}{\gamma} + \frac{1}{1-\gamma}.$$

which agrees the leading poles of

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma).$$

LO $\gamma^*\gamma^*$scattering

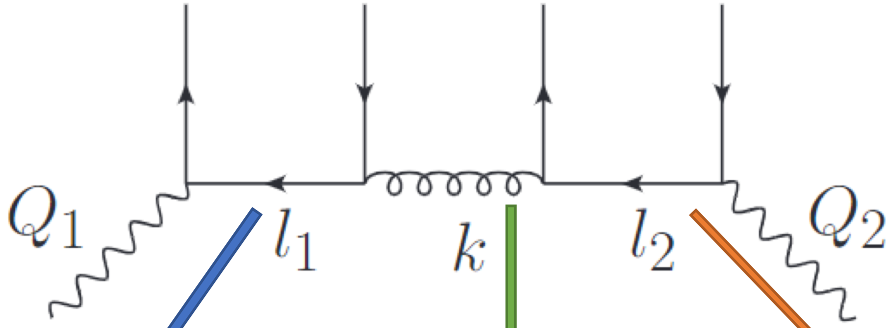


Strong ordering in LO $\gamma^*\gamma^*$scattering

$$Q_1^2 \gg l_1^2 \gg k^2 \gg l_2^2 \gg Q_2^2$$

LO transverse RGI impact factor

$$\sigma^{(jk)} = \frac{1}{2\pi Q_1 Q_2} \int \frac{d\omega}{2\pi i} \left(\frac{s}{Q_1^2}\right)^{\omega} I^{(jk)}(\omega, Q_1, Q_2)$$



$$I^{(TT)}(\omega, Q_1, Q_2) = (2\pi)^3 \alpha \left(\sum\nolimits_{q\in A} e_q^2\right) \times \boxed{\int_{Q_2^2}^{Q_1^2} \frac{dl_1^2}{l_1^2} \frac{\alpha_s(l_1^2)}{2\pi} P_{qg}(\omega)} \boxed{\int_{Q_2^2}^{l_1^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} P_{gq}(\omega)} \boxed{\int_{Q_2^2}^{k^2} \frac{dl_2^2}{l_2^2} \frac{\alpha}{2\pi} \left(\sum\nolimits_{q\in B} e_q^2\right) P_{q\gamma}(\omega)}$$

- $\sum_{q\in A(or\ B)} e_q^2$ sum the electric charges squared over different flavors of quarks.
- α is the electromagnetic coupling constant.
- $P_{ba}(\omega)$ is the splitting function that describe the fragmentation of parent parton a into parton b.

LO transverse RGI impact factor

- With running coupling features $$\alpha_s(k^2) = \frac{\alpha_s(\mu^2)}{1+\alpha_s(\mu^2) b_0 \ln\frac{k^2}{\mu^2}} \simeq \alpha_s(\mu^2)\left(1-\alpha_s(\mu^2) b_0 \ln\frac{k^2}{\mu^2}+\cdots\right),$$

$$b_0 = \frac{11C_A - 4T_R N_f}{12\pi}.$$

One can complete the integral on momentums and transfer the running coupling scale from $\mu^2 \to Q_1 Q_2$,

$$\alpha_s^2(\mu^2)\iiint(\ldots) \simeq \alpha_s^2(Q_1Q_2)\left[\frac{1}{3!}\log^3\frac{Q_1^2}{Q_2^2} - \alpha_s b_0 \frac{1}{4!}\log^4\frac{Q_1^2}{Q_2^2} + O(\alpha_s^4)\right].$$

- In Mellin space, this is

$$\alpha_s^2(Q_1Q_2)\,\frac{1}{\gamma^4}\left[1-\alpha_s b_0\frac{1}{\gamma}\right].$$

$\frac{1}{\gamma^4}$ is in agreement to the well know LO impact factor.

$\alpha_s b_0 \frac{1}{\gamma}$ will contribute to the Next-to-Leading order.

LO transverse RGI impact factor

- RGI integrand in the Mellin space

$$\Phi_0^{(T)}\, G_0\, \Phi_0^{(T)}$$
$$= (2\pi)^3\alpha\left(2\sum_q e_q^2\right)\frac{1}{\gamma}\cdot\frac{\alpha_s}{2\pi}\frac{P_{qg}(\omega)}{\gamma}\cdot\frac{\alpha_s}{2\pi}\frac{P_{gq}(\omega)}{\gamma}\cdot\frac{\alpha}{2\pi}\left(2\sum_q e_q^2\right)\frac{P_{q\gamma}(\omega)}{\gamma}$$
$$+(\gamma\to 1+\omega-\gamma)$$

- Both collinear ($\frac{1}{\gamma}$ terms) and anti-collinear terms $(\gamma \to 1+\omega-\gamma)$.
- This is asymmetric energy scale.
- For the symmetric energy scale, collinear $\frac{1}{\gamma+\frac{\omega}{2}}$ vs anti-collinear $\frac{1}{1-\gamma+\frac{\omega}{2}}$.

LO transverse RGI impact factor

- Splitting functions in the ω space

$$P_{qq}(\omega) = C_F\left(\frac{5}{4} - \frac{\pi^2}{3}\right)\omega + \mathcal{O}(\omega^2)$$

$$P_{gq}(\omega) = \frac{2C_F}{\omega}\left[1 + \omega A_{gq}(\omega)\right] \qquad A_{gq}(0) = -\frac{3}{4}$$

$$P_{qg}(\omega) = \frac{2}{3}T_R\left[1 + \omega A_{qg}(\omega)\right] \qquad A_{qg}(0) = -\frac{13}{12}$$

$$P_{gg}(\omega) = \frac{2C_A}{\omega}\left[1 + \omega A_{gg}(\omega)\right] \qquad A_{gg}(0) = -\frac{11}{6} + \bar{b}\,, \quad \bar{b} = \frac{11}{12} - \frac{T_R N_f}{3C_A}$$

$$P_{q\gamma}(\omega) = \frac{N_c}{T_R}P_{qg}(\omega)\,.$$

LO transverse RGI impact factor

- RGI integrand

$$\Phi_0^{(T)}\, G_0\, \Phi_0^{(T)}$$

$$= \boxed{\left[\alpha\alpha_s\left(\sum\nolimits_q e_q^2\right)2P_{qg}(\omega)\sqrt{2(N_c^2-1)}\left(\frac{1}{\gamma^2}+\frac{1}{(1+\omega-\gamma)^2}\right)\right]}^2 \times \frac{1}{\omega}\left(1+\omega A_{gq}(\omega)\right)$$

This agrees the LO ω-dependent impact factor by Bialas, Navelet and Peschanski.

- One possible choice,

$$\Phi_0^{(T)}(\omega,\gamma) = \Phi_0^{(T,\mathrm{BNP})}(\omega,\gamma)\left[1+\frac{\omega}{2}A_{gq}(\omega)\right]$$

NLO transverse RGI impact factor

BFKL | RGI

- General correspondence:

$$\chi(\gamma) = \mathrm{X}\left(\omega^{\text{eff}}, \gamma\right)$$

$$\phi^{(j)}(\gamma)\phi^{(k)}(1-\gamma) = \frac{\Phi^{(j)}\left(\omega^{\text{eff}}, \gamma\right)\Phi^{(k)}\left(\omega^{\text{eff}}, 1-\gamma\right)}{1-\bar{\alpha}_s\partial_\omega \mathrm{X}\left(\omega^{\text{eff}}, \gamma\right)}$$

- Now, we identify the NLO impact factors

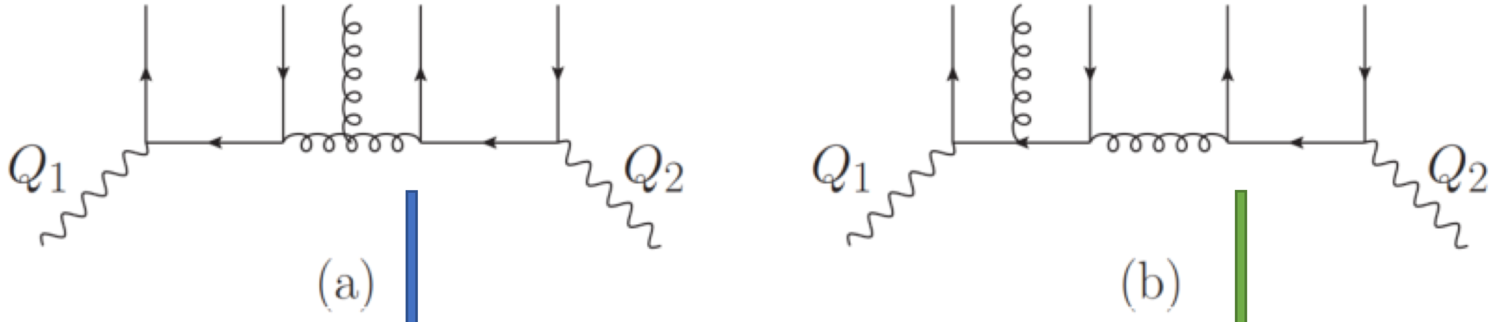
$$\phi_1(\gamma) + \phi_1(1-\gamma) = \Phi_1(\gamma) + \Phi_1(1-\gamma) + \chi_0(\gamma)[\partial_\omega \Phi_0(\omega,\gamma) + \partial_\omega \Phi_0(\omega, 1-\gamma)]_{\omega\to 0} + \phi_0(\gamma)\partial_\omega \mathrm{X}_0(\omega,\gamma)\Big|_{\omega\to 0}$$

By Ivanov, Murdaca, Papa

- Require the NLO impact factors to be symmetric in $\gamma \leftrightarrow 1-\gamma$,

$$\Phi_1(\gamma) = \frac{1}{2}[\Phi_1(\gamma) + \Phi_1(1-\gamma)]$$
$$= \frac{1}{2}\left[\phi_1(\gamma) + \phi_1(1-\gamma) - \phi_0(\gamma)\partial_\omega \mathrm{X}_0(\omega,\gamma)\Big|_{\omega\to 0} - \chi_0(\gamma)\left(\partial_\omega \Phi_0(\omega,\gamma) + \partial_\omega \Phi_0(\omega, 1-\gamma)\right)\Big|_{\omega\to 0}\right]$$

NLO transverse RGI impact factor



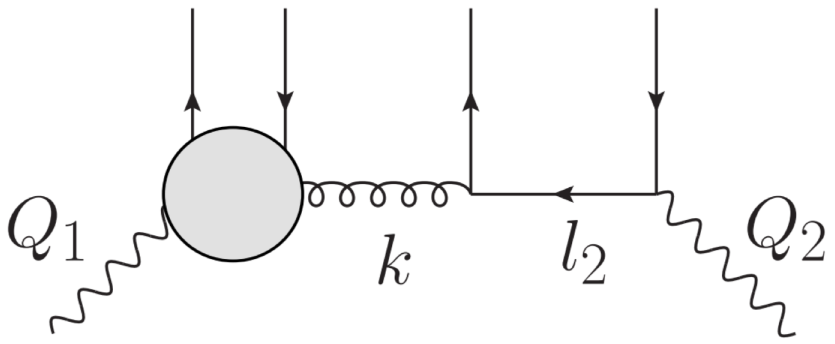
$$\tilde{I}_1^{(TT)}(\omega,\gamma) = \tilde{I}_0^{(TT)}(\omega,\gamma)\left[\boxed{\frac{\alpha_s}{2\pi}\frac{P_{gg}}{\gamma}} + \boxed{2\,\frac{\alpha_s}{2\pi}\frac{P_{qq}}{\gamma}} - \boxed{\frac{\alpha_s b_0}{\gamma}}\right]$$

From the LO collinear analysis

- One choice to attribute the NLO terms is GGF | Impact factors | Both
- There are other choices as collinear analysis is on the level of the cross section.

Longitudinal impact factors

LO LT cross section

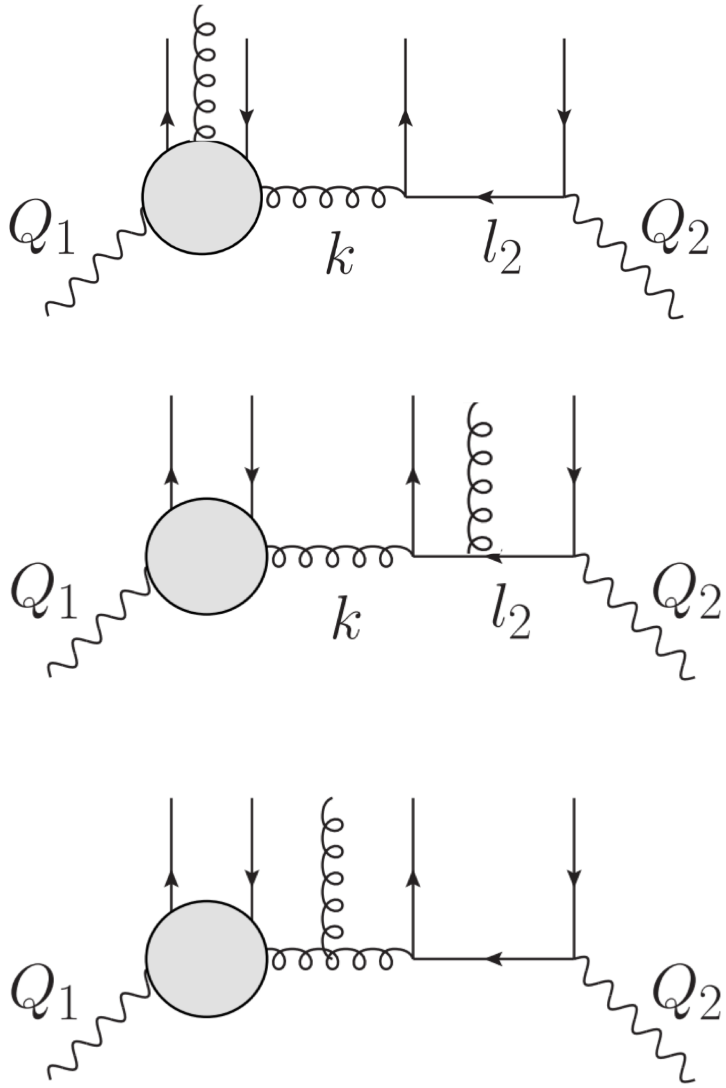


At LO,

$$\Phi_0^{(L)}(\omega,\gamma) = \Phi_0^{(L,\mathrm{BNP})}(\omega,\gamma)\left[1 + \frac{\omega}{2} A_{gq}(\omega)\right].$$

The construction is similar to the LO and NLO transverse impact factors.

NLO LT cross section



Numerical Results

- Possible schemes A, B on NLO calculation. For example, they feature different LO impact factors,

$$\Phi_0^{(A)}(\omega,\gamma) = \Phi_0^{(A,\mathrm{BNP})}(\omega,\gamma)\left[1+\frac{\omega}{2}A_{gq}\right],$$

$$\Phi_0^{(B)}(\omega,\gamma) = \Phi_0^{(B,\mathrm{BNP})}(\omega,\gamma)\left[1+\frac{\omega}{2}\left(A_{gq}-\Delta A-\frac{b_0}{2}\right)\right].$$

With $\Delta A = A_1 - A_{gg}$ and A_1 the non-singular part of the gluon anomalous dimension.

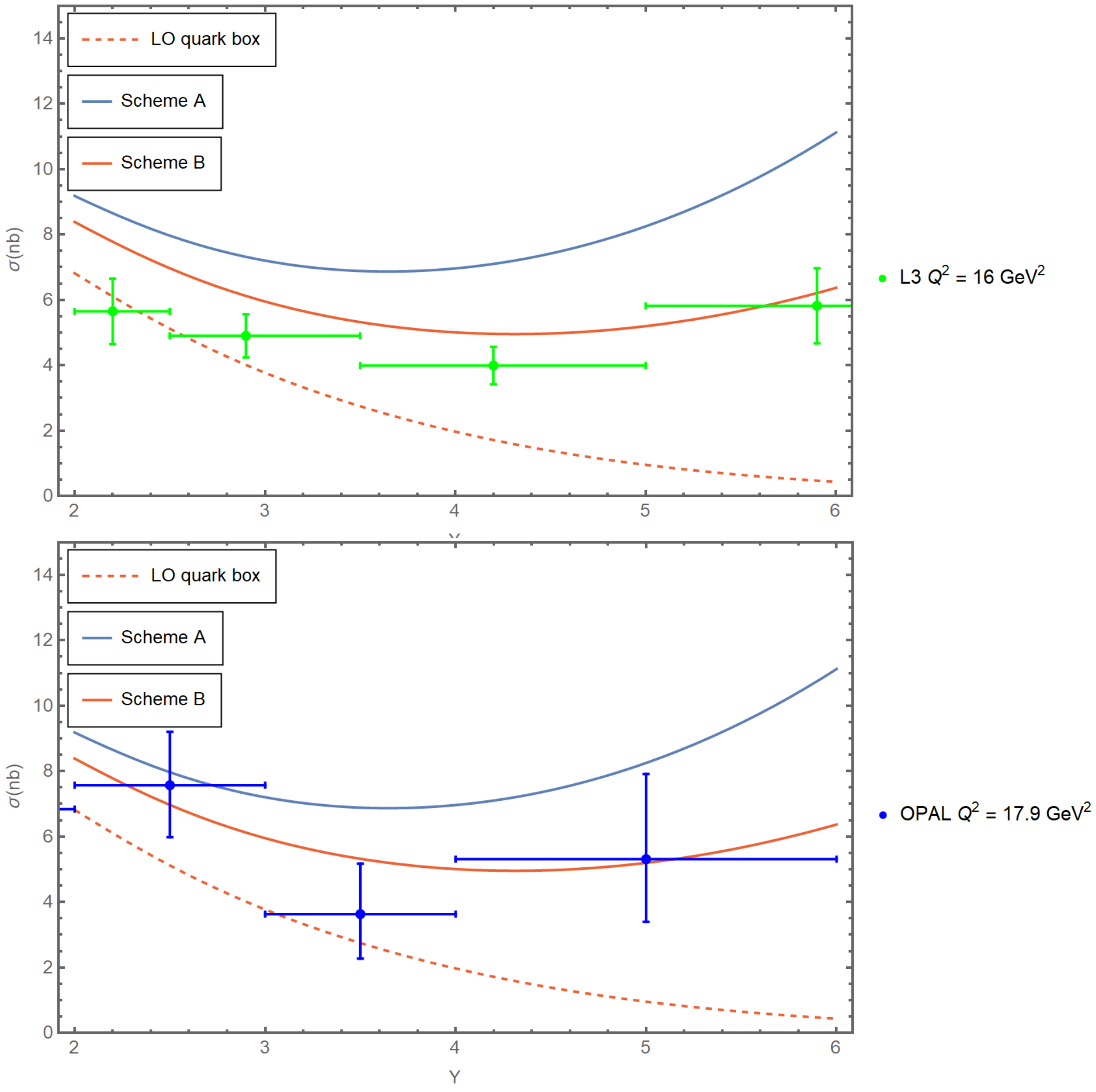
- Settings:

$$\mu^2 = s_0 = Q_1 Q_2 = 16 \text{ or } 17.9\ \mathrm{GeV}^2$$

- LO Quark box is calculated with charm mass

$$m_c = 1.4\ \mathrm{GeV}^2.$$

- We do not include the charm contribution in the factorization calculation. The factorization diagrams have more quark production, thus the charm threshold affects more.



Conclusions

- Resummation helps stabilizing the calculations.

- We calculate the RGI cross section by collinear analysis and construct the LO and NLO impact factors. RGI impact factors are matched to the fixed order impact factors calculations.

- There are flexibilities on moving different contributions to either BFKL eigenfunctions or impact factors, as we are loyal to the poles structure of the cross section of $\gamma^*\gamma^*$scattering.

- We do have some detailed constraints on the flexibilities.

Thanks.

Wanchen Li