

Continuum-Physical Nucleon Gluon PDF

Based on: arXiv:2210.09985v1 [hep-lat] 18 Oct 2022

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30 MARCH 2023



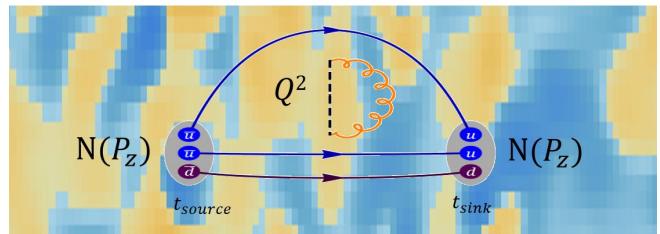
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Background and Introduction

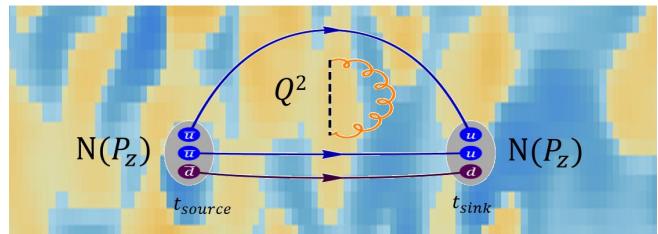
- The gluon PDF is an important input for high energy scattering experiments
- Phenomenological studies of the gluon PDF still see issues in the large- x range
- Gluon PDFs are difficult on the lattice due to large noise
- We present the first physical-continuum limit results of the unpolarized nucleon gluon PDF

Methodology Overview



Correlator
measurements
on the lattice

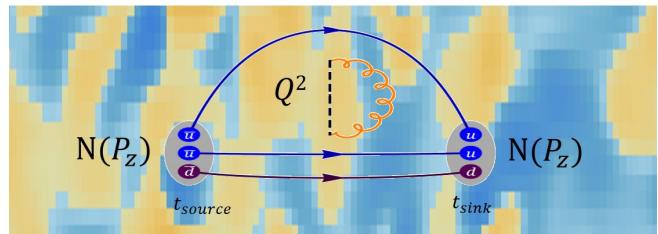
Methodology Overview



Correlator
measurements
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Find MEs from 2pt
and 3pt correlator
analysis

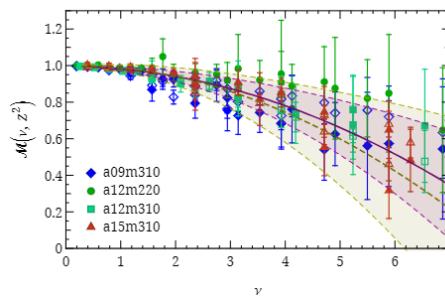
Methodology Overview



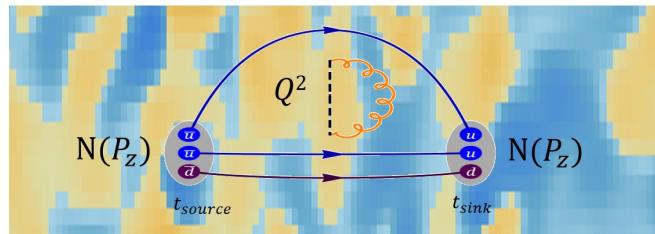
Correlator measurements on the lattice

Get Reduced pseudo-loffe Time Distribution (RpITD)

Find MEs from 2pt and 3pt correlator analysis



Methodology Overview

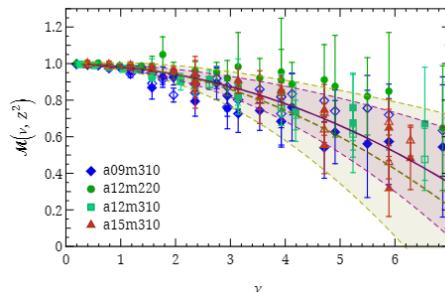


Correlator measurements on the lattice

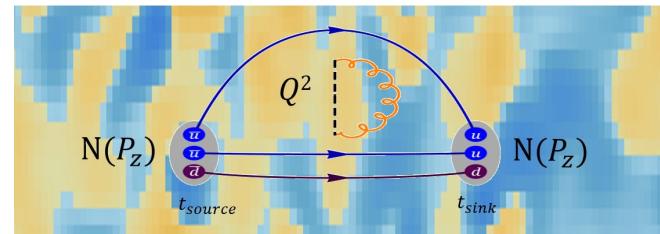
Get Reduced pseudo-loffe Time Distribution (RpITD)

Extrapolate to the physical-continuum limit RpITD

Find MEs from 2pt and 3pt correlator analysis



Methodology Overview



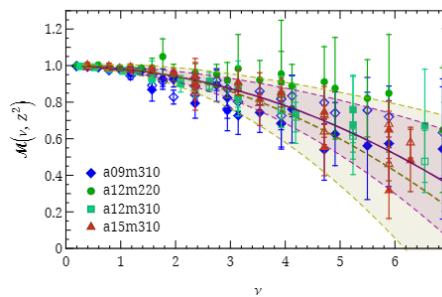
Correlator measurements on the lattice

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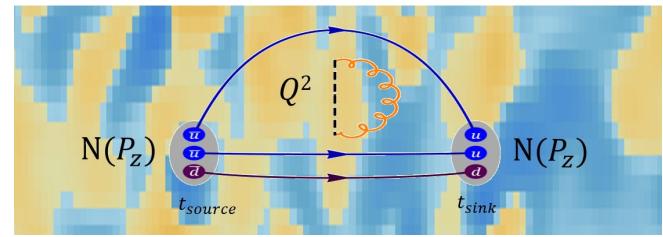
Extrapolate to the physical-continuum limit RpITD

Find MEs from 2pt and 3pt correlator analysis

Fit $xg(x)/\langle x \rangle_g$

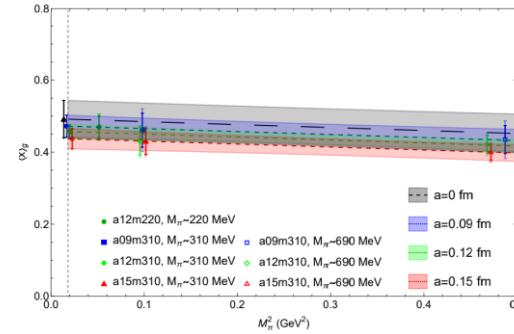


Methodology Overview

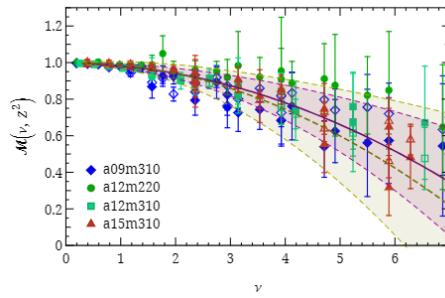


Correlator measurements on the lattice

Get Reduced pseudo-loffe Time Distribution (RpITD)

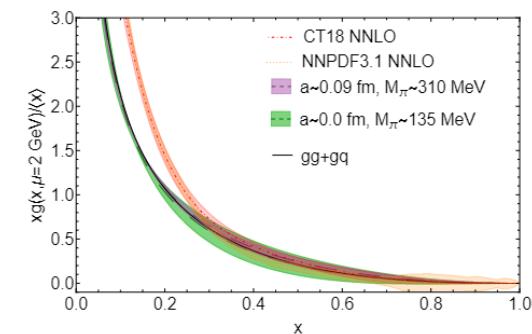


Find MEs from 2pt and 3pt correlator analysis



Extrapolate to the physical-continuum limit RpITD

Fit $xg(x)/\langle x \rangle_g$



$\langle x \rangle_g$ obtained from other work

General Lattice Setup

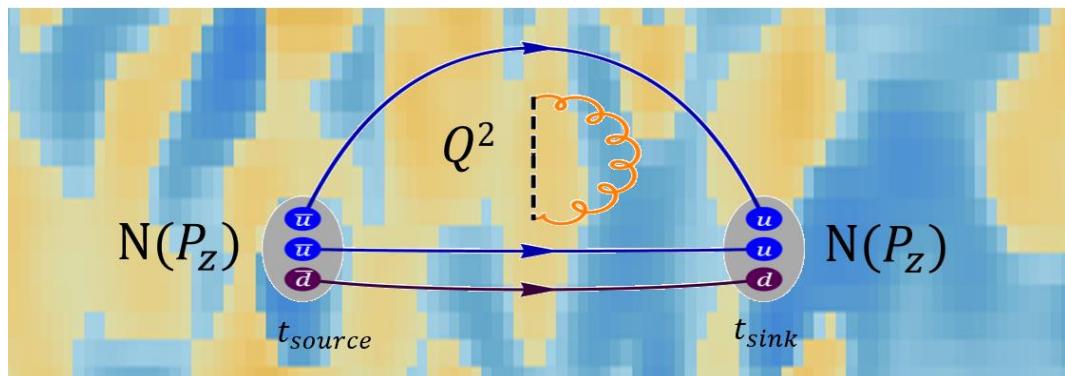
Calculation carried out with $N_f = 2 + 1 + 1$ highly improved staggered quarks (HISQ) generated by MILC collaboration

Five steps of hypercubic (HYP) smearing on gauge links

Wilson-clover fermions used in valence sector

Used lattice spacings $a \approx 0.15, 0.12, 0.09$ fm and pion masses $M_\pi \approx 220, 310$ MeV

Valence quark masses tuned to reproduce light and strange pion masses $M_l \approx 220, 310$ and $M_s \approx 700$ MeV



- Follana et al. PRD 75:054502, 2007.
A. Bazavov, et al. [MILC], PRD 82:074501 2010.
A. Bazavov, et al. [MILC], PRD 87:054505 2013.
A. Bazavov, et al. [F Lattice and MILC] PRD:074512 2018

More Ensemble Details

Ensemble	a09m310	a12m220	a12m310	a15m310
a (fm)	0.0888(8)	0.1184(10)	0.1207(11)	0.1510(20)
$L^3 \times T$	$32^3 \times 96$	$32^3 \times 64$	$24^3 \times 64$	$16^3 \times 48$
M_π^{val} (GeV)	0.313(1)	0.2266(3)	0.309(1)	0.319(3)
$M_{\eta_s}^{\text{val}}$ (GeV)	0.698(7)	N/A	0.6841(6)	0.687(1)
P_z (GeV)	[0, 3.05]	[0, 2.29]	[0, 2.14]	[0, 2.56]
N_{cfg}	1009	957	1013	900
$N_{\text{meas}}^{\text{2pt}}$	387,456	1,466,944	324,160	259,200
t_{sep}	[6, 10]	[6, 10]	[5, 9]	[4, 8]

Correlator Definitions

$$C_N^{2\text{pt}}(P_z; t) = \langle 0 | \Gamma \int d^3y e^{-iyP_z} \chi(\vec{y}, t) \chi(\vec{0}, 0) | 0 \rangle$$

$$\Gamma = \frac{1}{2}(1 + \gamma_4) \quad \chi(\vec{y}, t) = \epsilon^{lmn} [u(y)^l{}^T i\gamma_4 \gamma_2 \gamma_5 d^m(y)] u^n(y)$$

$$C_N^{3\text{pt}}(z, P_z; t_{\text{sep}}, t) =$$

$$\langle 0 | \Gamma \int d^3y e^{-iyP_z} \chi(\vec{y}, t_{\text{sep}}) \mathcal{O}_g(z, t) \chi(\vec{0}, 0) | 0 \rangle$$

$$\mathcal{O}(z) \equiv \sum_{i \neq z, t} \mathcal{O}(F^{ti}, F^{ti}; z) - \frac{1}{4} \sum_{i, j \neq z, t} \mathcal{O}(F^{ij}, F^{ij}; z) \quad \text{Balitsky et al, PLB 808:135621, 2020.}$$

$$\mathcal{O}(F^{\mu\nu}, F^{\alpha\beta}; z) = F_\nu^\mu(z) U(z, 0) F_\beta^\alpha(0)$$

2pt and 3pt Analysis

Applied Gaussian momentum smearing on the quark field to improve signal up to 3.0 GeV

For fitting, take correlators to be of form:

$$C_N^{2pt}(P_z, t) = |A_{N,0}|^2 e^{-E_{N,0}t} + |A_{N,1}|^2 e^{-E_{N,1}t} + \dots$$

and

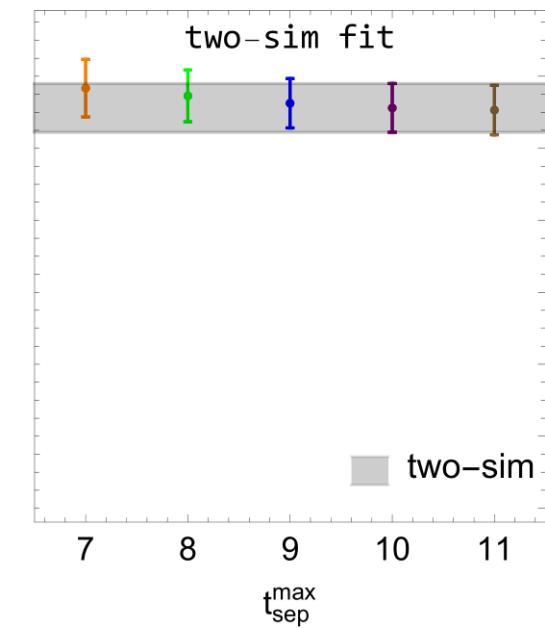
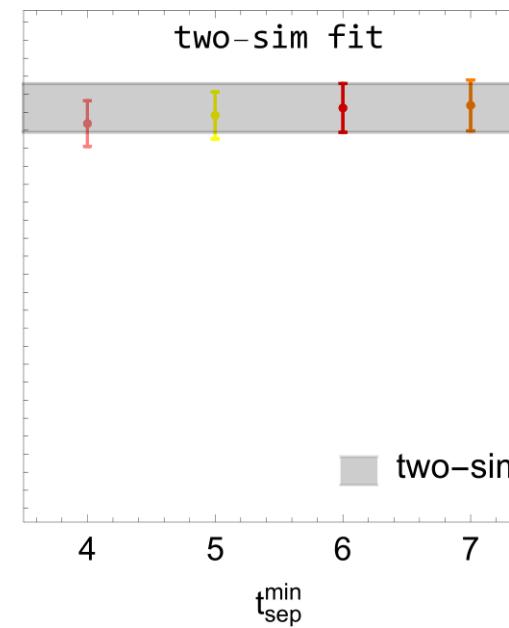
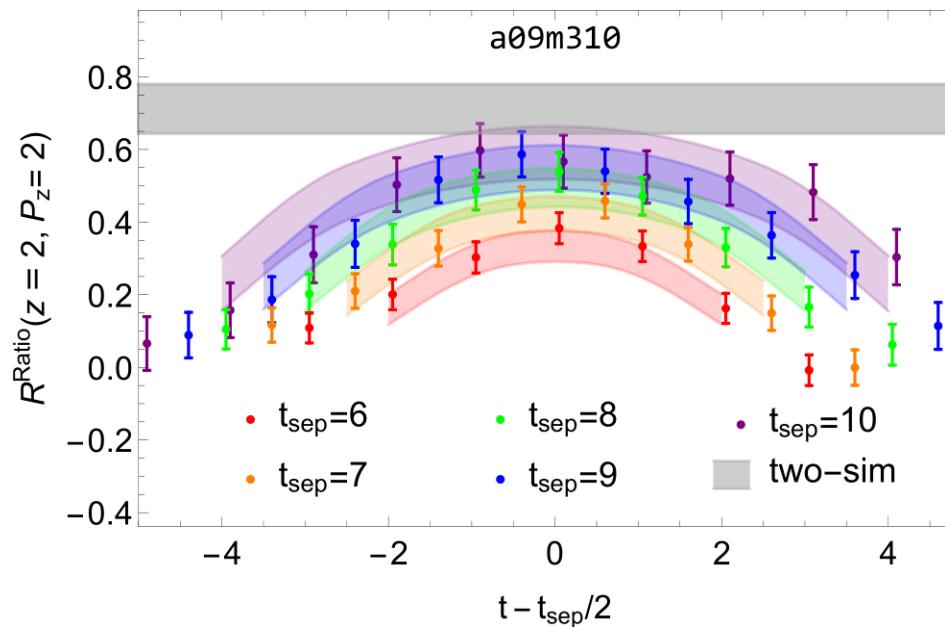
$$\begin{aligned} C_N^{3pt}(z, P_z, t, t_{sep}) = & \\ & |A_{N,0}|^2 \langle 0 | O_g | 0 \rangle e^{-E_{N,0}t_{sep}} + |A_{N,0}| |A_{N,1}| \langle 0 | O_g | 1 \rangle e^{-E_{N,1}(t_{sep}-t)} e^{-E_{N,0}t} + \\ & |A_{N,0}| |A_{N,1}| \langle 1 | O_g | 0 \rangle e^{-E_{N,0}(t_{sep}-t)} e^{-E_{N,1}t} + |A_{N,1}|^2 \langle 1 | O_g | 1 \rangle e^{-E_{N,1}t_{sep}} + \dots \end{aligned}$$

Correlator Ratio Plots and Fits

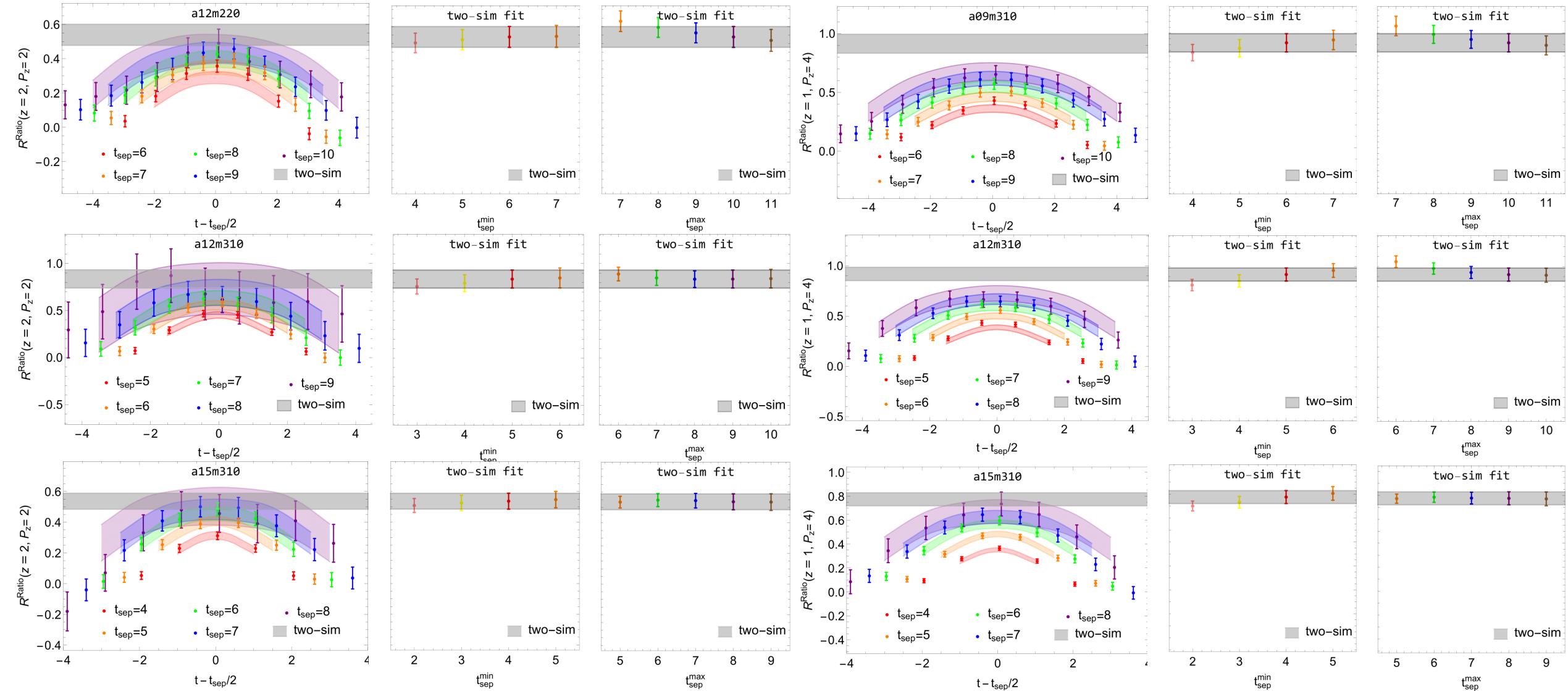
Plot the ratio:

$$R_N(z, P_z, t, t_{sep}) = \frac{C_N^{3pt}(z, P_z, t, t_{sep})}{C_N^{2pt}(P_z, t_{sep})}$$

Simultaneously fit 2pt and 3pt to obtain $\langle 0 | O_g | 0 \rangle$ matrix elements
(MEs)



Other Ratio Plots



Light

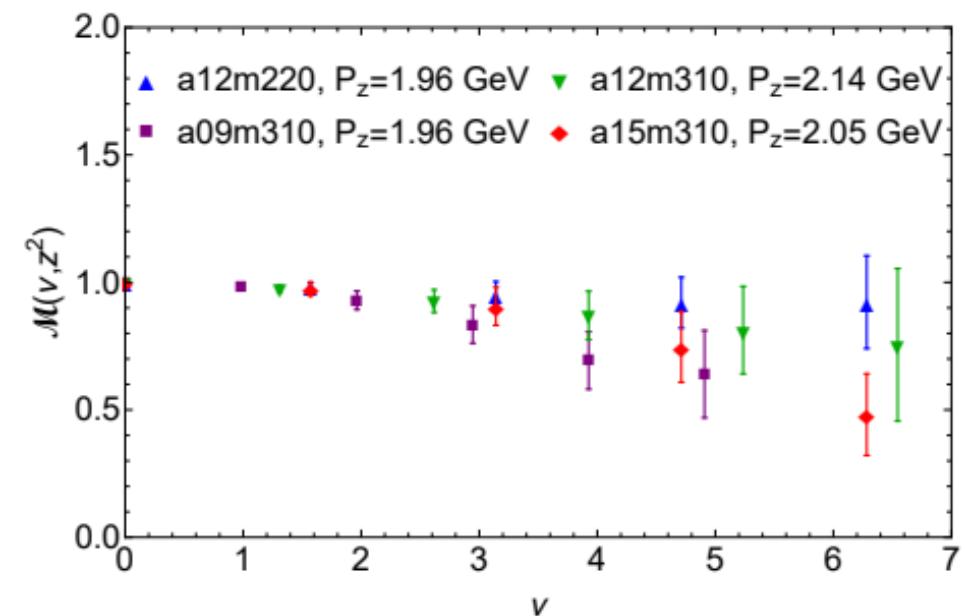
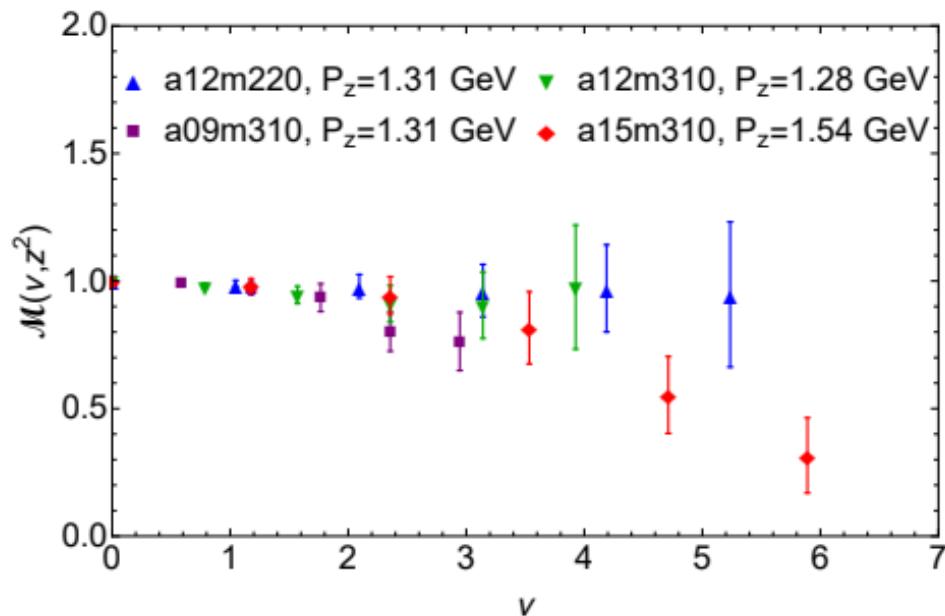
Strange

Reduced Pseudo Ioffe Time Distribution (RpITD)

Take the double ratio of the MEs

$$\mathcal{M}(\nu, z^2) = \frac{\mathcal{M}(zP_z, z^2)/\mathcal{M}(0 \cdot P_z, 0)}{\mathcal{M}(z \cdot 0, z^2)/\mathcal{M}(0 \cdot 0, 0)}$$

$$(\mathcal{M}(zP_z, z^2) = \langle 0(P_z) | O_g(z) | 0(P_z) \rangle) \quad (\nu = zP_z)$$

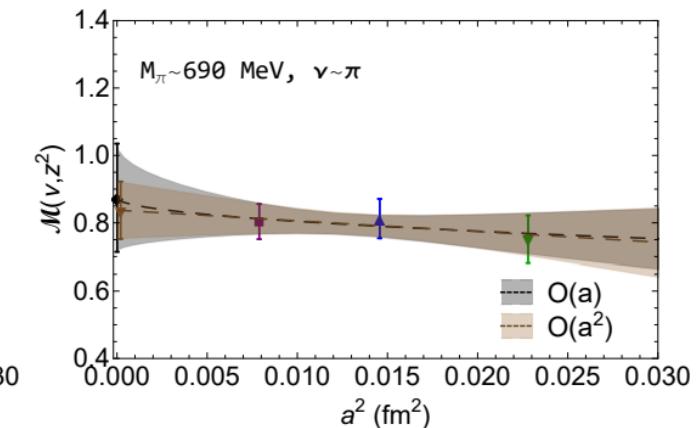
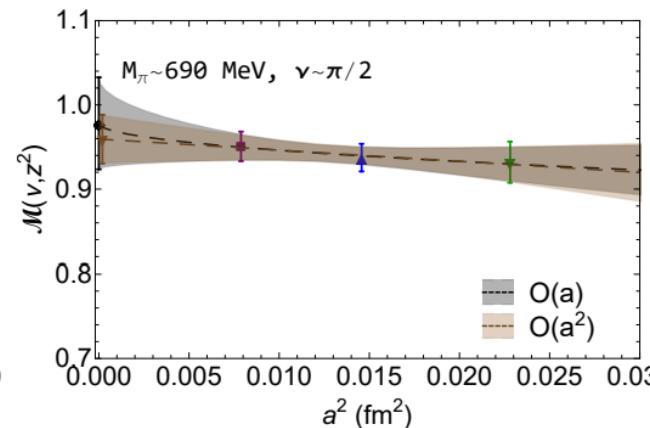
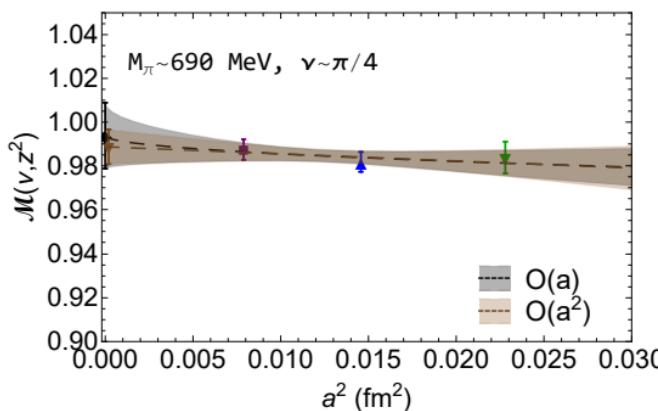
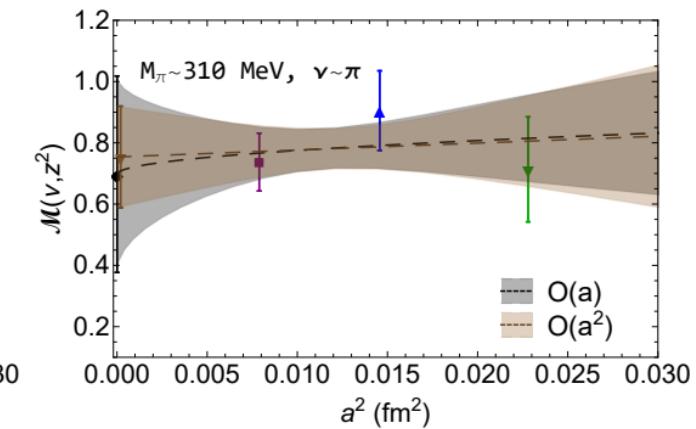
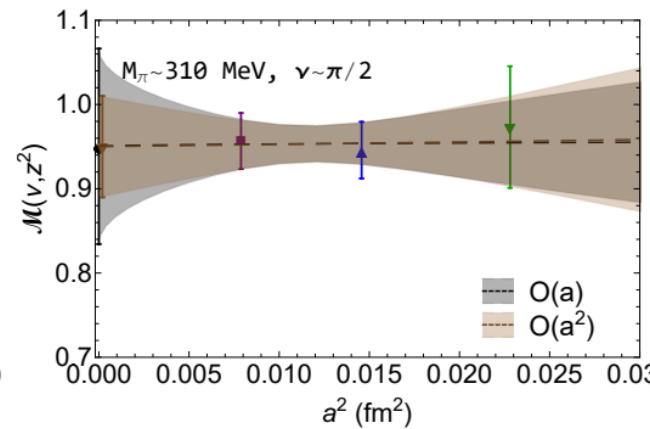
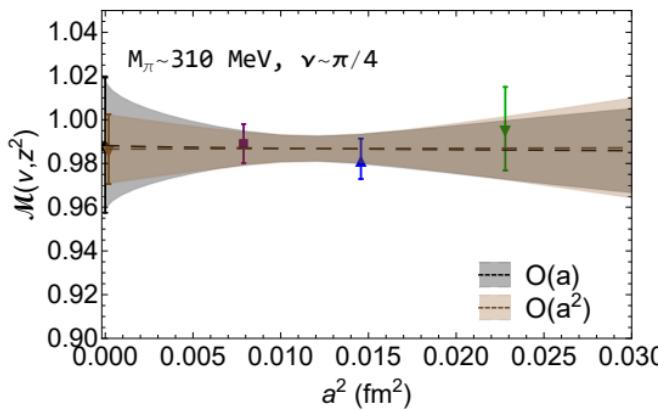


Continuum RplTD Data

Assumed fit form $n = 1, 2$

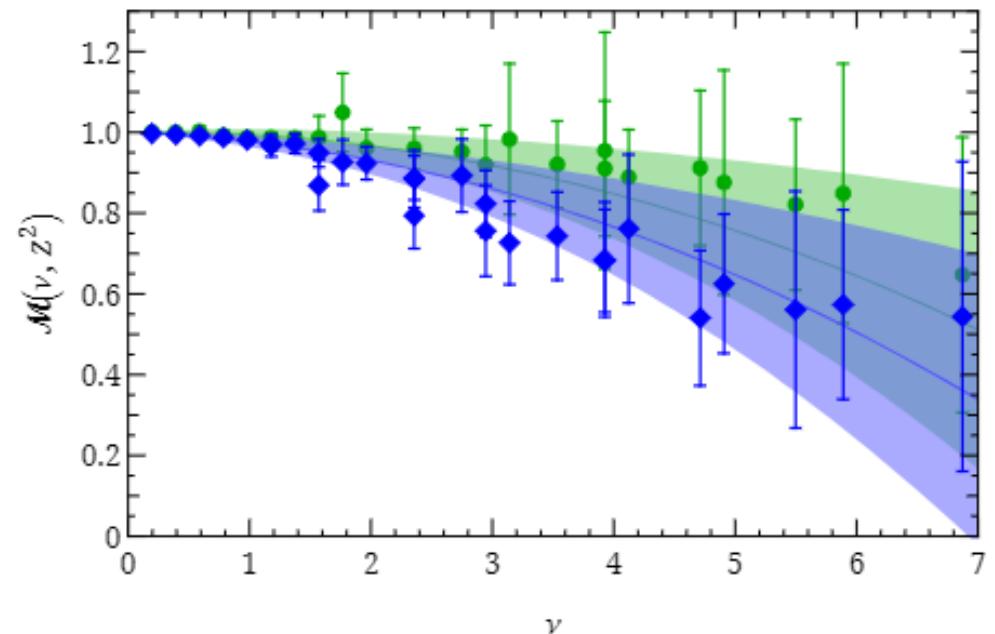
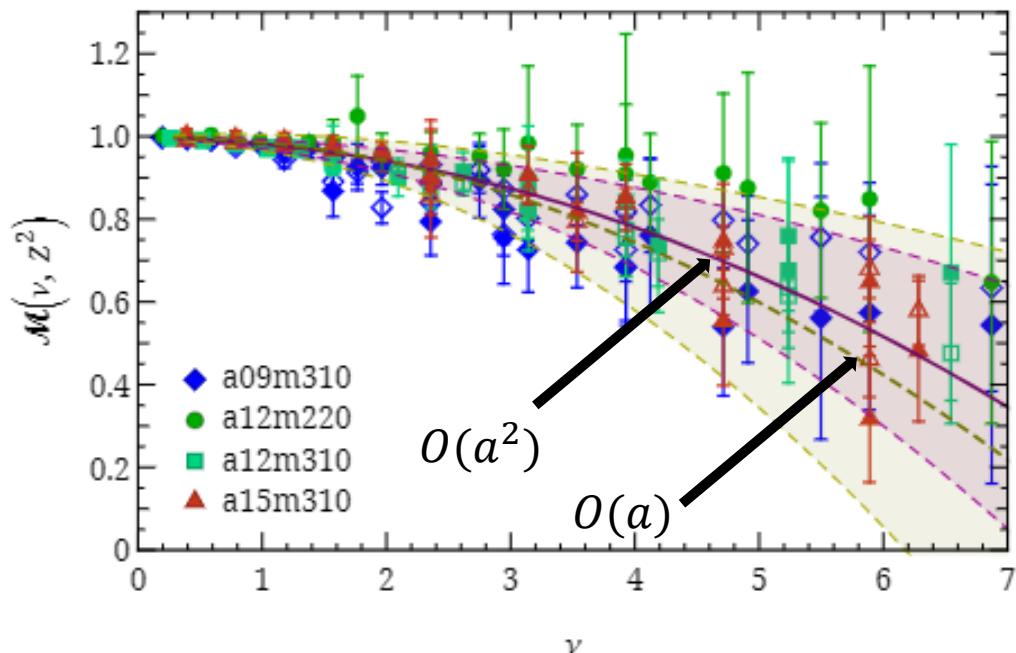
$$\mathcal{M}(\nu, z^2, a, M_\pi) = \mathcal{M}^{\text{cont}} + c_a a^n$$

Choose constant P_z and change z to get $\nu = \frac{\pi}{4}, \frac{\pi}{2}, \pi$



Extrapolate to Physical-Continuum limit

$$\mathcal{M}(\nu, z^2, a, M_\pi) = \left(\sum_{k=0}^{k_{\max}} \lambda_k(a, M_\pi) \nu^k + c_z(a, M_\pi) z^2 \right) \times (1 + c_a a^2 + c_M (M_\pi^2 - (M_\pi^{\text{phys}})^2))$$



Phenomenological Fit Form

Gluon matching kernel R_{gg} connects the RplTD to the PDF as shown

Balitsky et al, PLB 808:135621, 2020.

$$\mathcal{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x, \mu^2)}{\langle x \rangle_g} R_{gg}(x\nu, z^2 \mu^2)$$

(μ is the renormalization scale in the MS-bar scheme)

Use a typical global analysis fit form

$B(A + 1, C + 1)$ is beta function (integral of numerator)

$$f_g(x, \mu) = \frac{xg(x, \mu)}{\langle x \rangle_g(\mu)} = \frac{x^A(1-x)^C}{B(A+1, C+1)}$$

Minimize

$$\chi^2(\mu, a, M_\pi) =$$

$$\sum_{\nu, z} \frac{(\mathcal{M}^{\text{fit}}(\nu, \mu, z^2, a, M_\pi) - \mathcal{M}^{\text{lat}}(\nu, z^2, a, M_\pi))^2}{\sigma_{\mathcal{M}}^2(\nu, z^2, a, M_\pi)}$$

Results

Ball et al. EPJ 77(10):663, 2017.

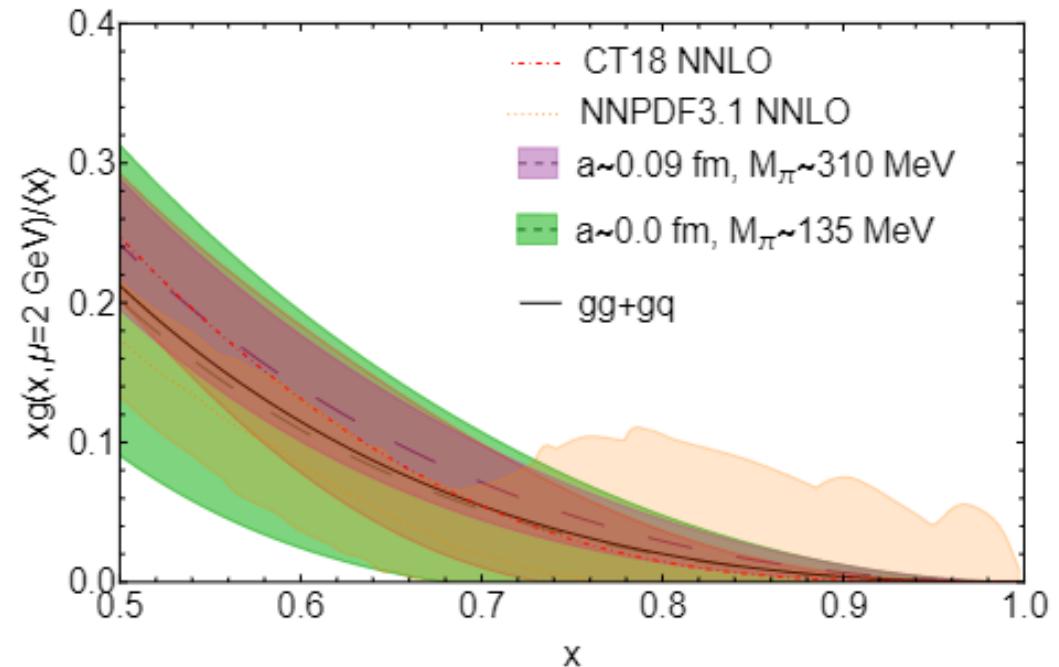
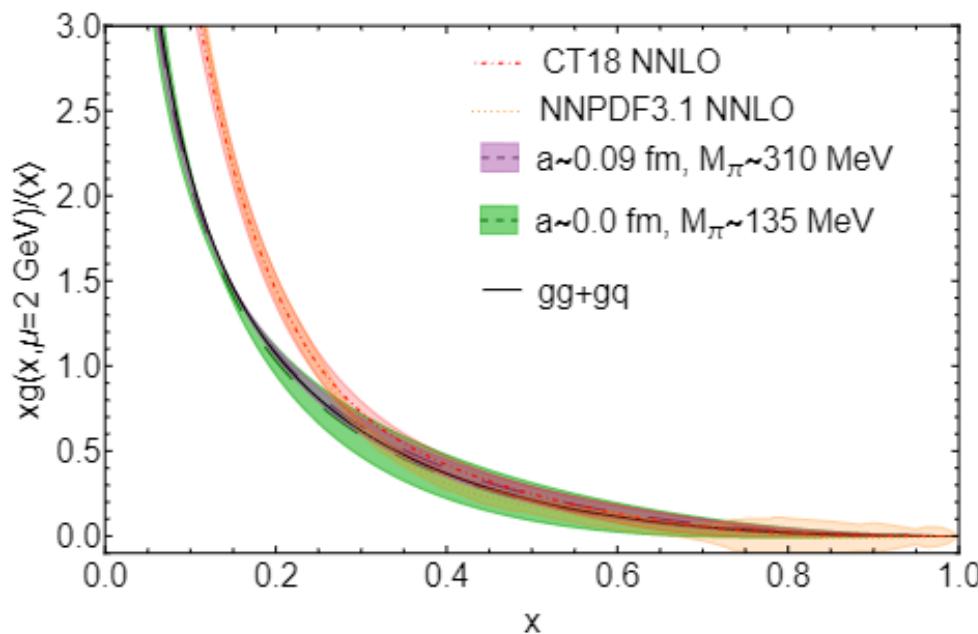
Hou et al. PRD 103(1):014013, 2021.

Fan et al. IJMPC 36(13):2150080, 2021.

Compare to global analysis PDFs

Add quark term from CT18 global fit

$$\frac{P_z}{P_0} \int_0^1 dx \frac{x q_S(x, \mu^2)}{\langle x \rangle_g} R_{gq}(x\nu, z^2 \mu^2)$$

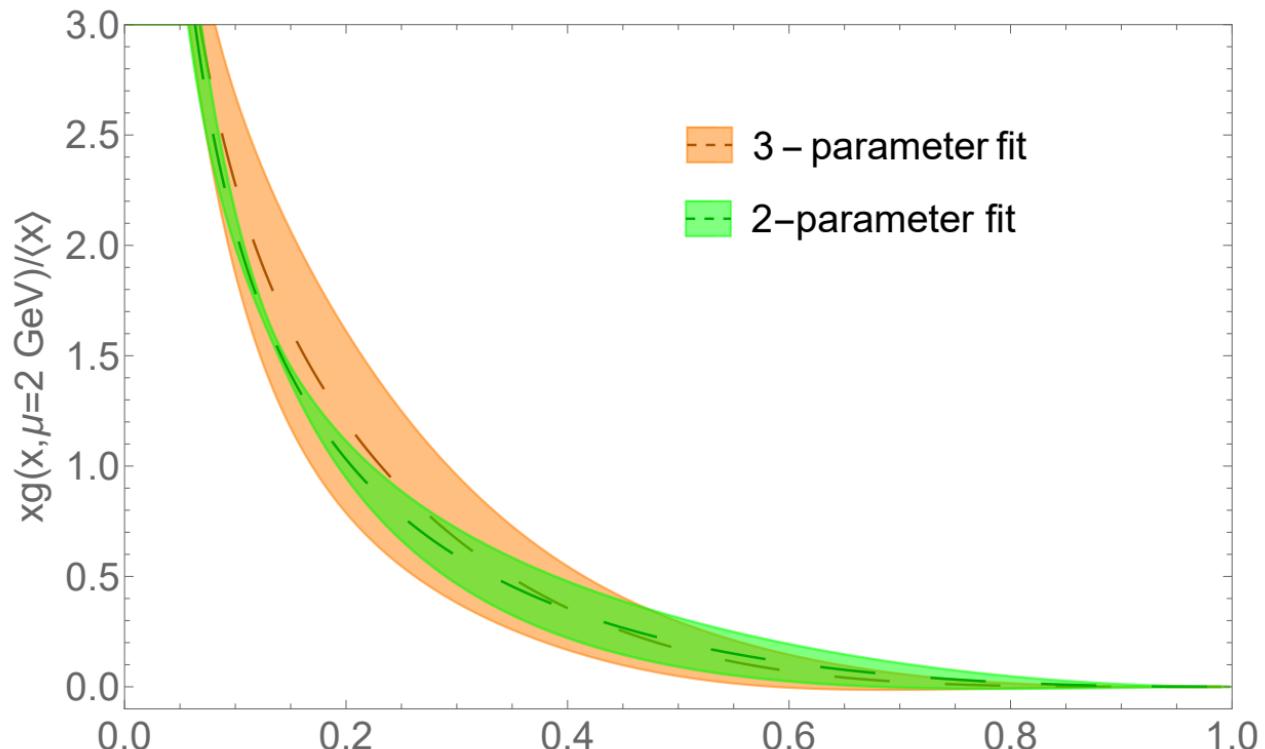


One Last Consideration

Try 3 parameter fit

$$f_{g,3}(x, \mu) = \frac{x^A(1-x)^C(1+Dx)}{B(A+1, C+1) + DB(A+2, C+1)}$$

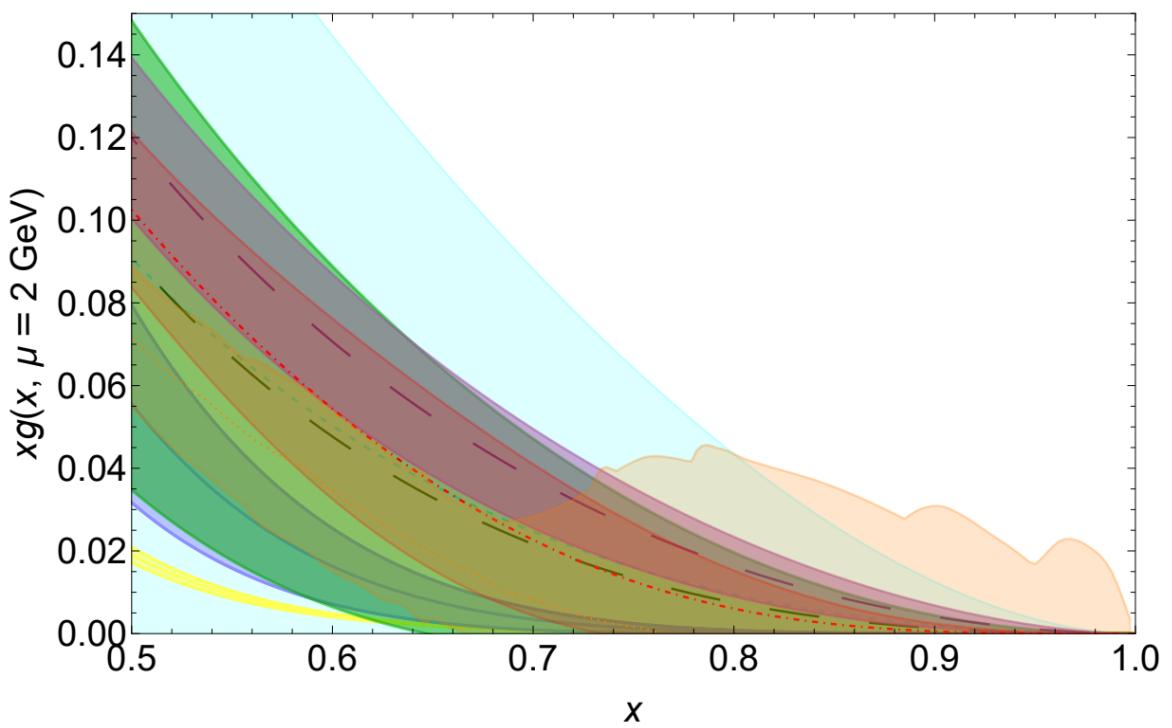
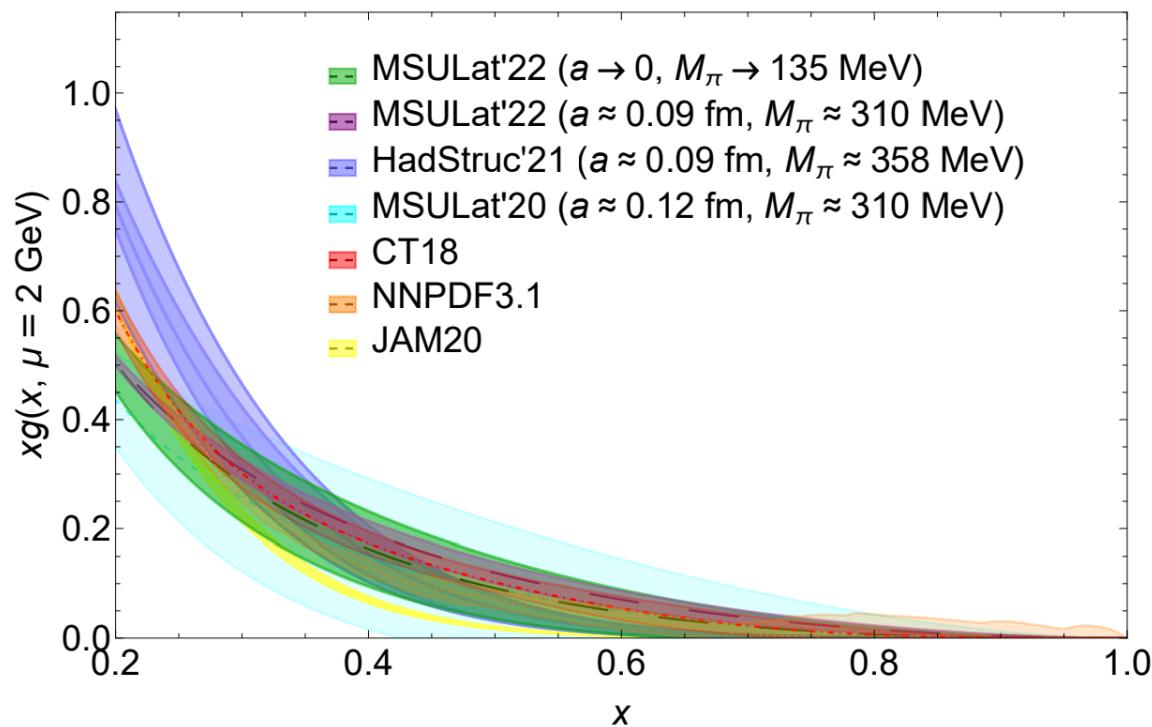
Similar results, more error



Final PDF Results

Multiply through by $\langle x \rangle$ from Fan et al. arXiv:2208.00980v1
[hep-lat] 2022

Ball et al. EPJ 77(10):663, 2017.
Hou et al. PRD 103(1):014013, 2021.
Fan et al. IJMPC 36(13):2150080, 2021.
Kahn et al. PRD 104(9):094516, 2021.
Moffat et al. PRD 104(1):016015, 2021.



Summary and Conclusion

Explained the analysis of the correlator data to get the RpITDs

Explored continuum and continuum-physical extrapolations for the RpITD

Matched the RpITD to gluon PDF, showing some differences from global analysis and HadStruc'21 lattice calculations

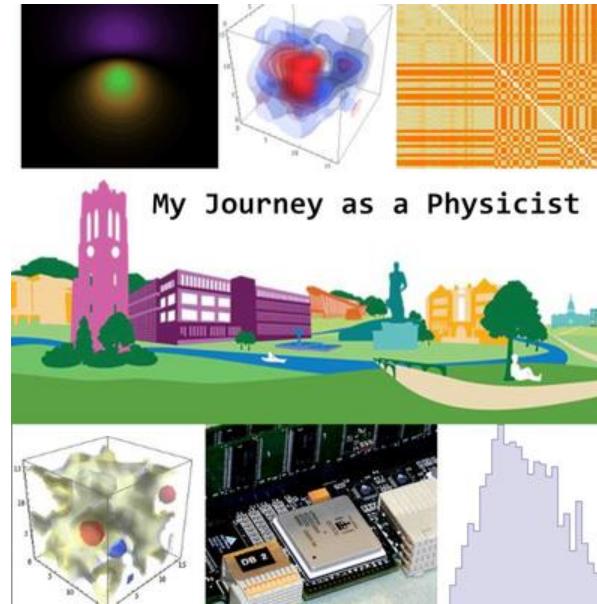
Noise in the calculation implies more data is needed

Need to better understand systematics

- i.e., differences in lattice calculations

Thank You!

Podcast plug:



Season 3 of H.W. Lin and B. Stanley's My Journey as a Physicist is hosted by Bill Good and features physicists working on the Long-Range Plan for Nuclear Science



Backup: Gluon Momentum Fractions

Ours: $\langle x \rangle_g = 0.492(52)_{stat+NPR} + (49)_{mixing}$

NNPDF3.1

HadStruc: $\langle x \rangle_g = 0.427(92)$

Alexandrou et al. 101.094513, 2020.

