Features of factorization studied in a Yukawa field theory

Tommaso Rainaldi
Old Dominion University

Based on

- **Basics of factorization in a Scalar Yukawa field theory** (2212.00757)
  (accepted for publication to **PRD**)
  (F. Aslan, L. Gamberg, J. O. Gonzalez-Hernandez, T.Rainaldi, T. C. Rogers)

- **The resolution to the problem of consistent large transverse momentum in TMDs** (2303.04921)
  (J. O. Gonzalez-Hernandez, T.Rainaldi, T. C. Rogers)

Motivation

➢ **Explicitly confirm factorization**

➢ **What is the difference between different treatments of TMD factorization?**

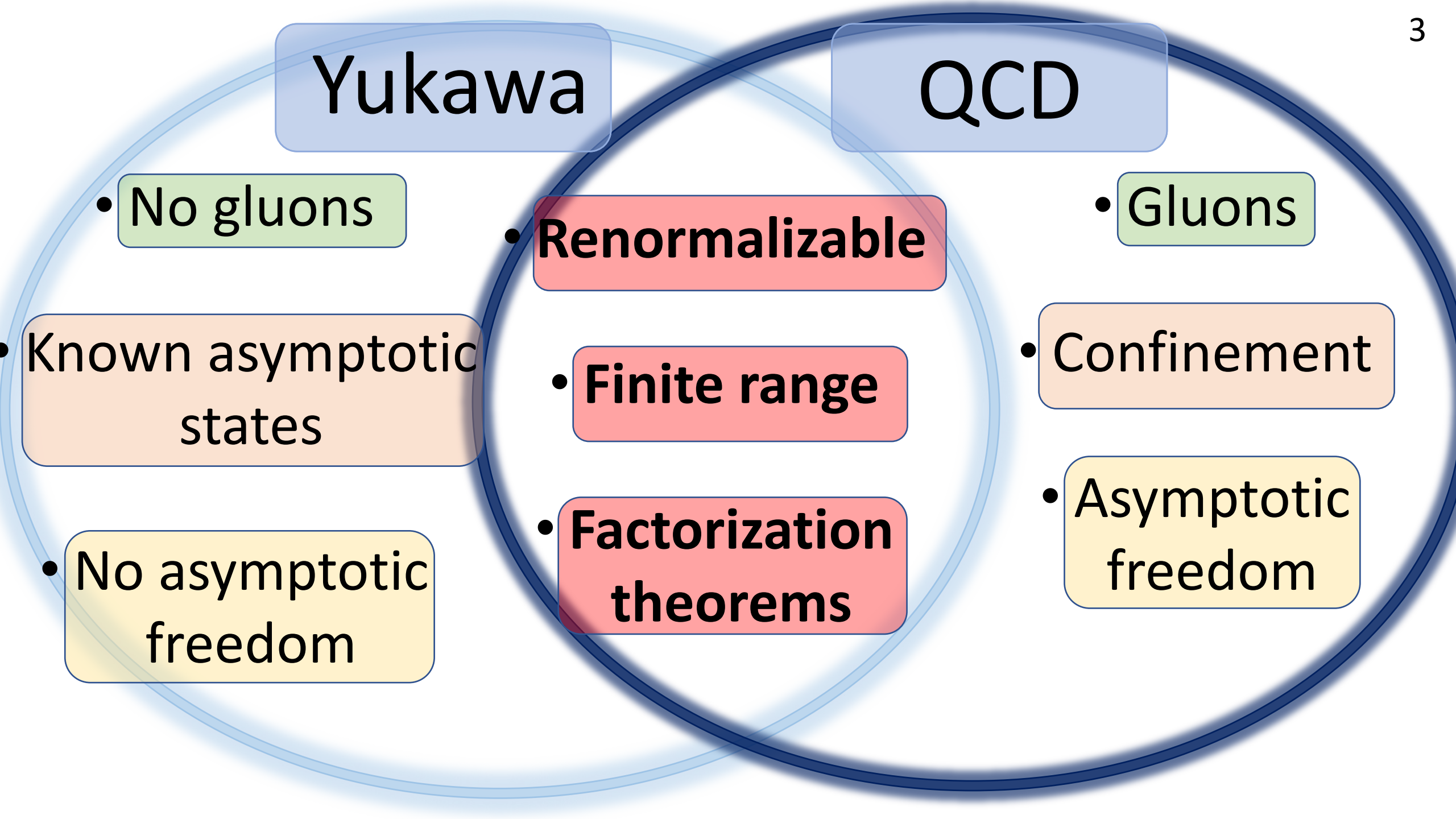
- W + Y : leading power accurate point-by-point in q_T
- Power law correction to TMD factorization ($1/Q^2$ corrections)

➢ **Different pdf definitions?**

- Renormalization or cutoff? ("On the normalization of integrated parton densities" (Guiot), "The complete tree-level result up to order 1/Q for polarized deep-inelastic leptoproduction" (Mulders, Tangerman))
- Positivity of pdfs? ("Positivity and renormalization of parton densities" (Collins, Rogers, Sato) , "Can MS bar parton distributions be negative"(Candido, Forte, Hekhorn))
- Parton model interpretation?

➢ **Test transverse coordinate TMD PDF techniques**

- b_* prescription
- Unconstrained or constrained g functions?
- Hadron Structure Oriented (HSO) approach ("Combining nonperturbative transverse momentum dependence with TMD evolution" (Gonzalez-Hernandez, Rogers, Sato))


Yukawa
QCD
No gluons
Known asymptotic states
No asymptotic freedom
Renormalizable
Finite range
Factorization theorems
Gluons
Confinement
Asymptotic freedom

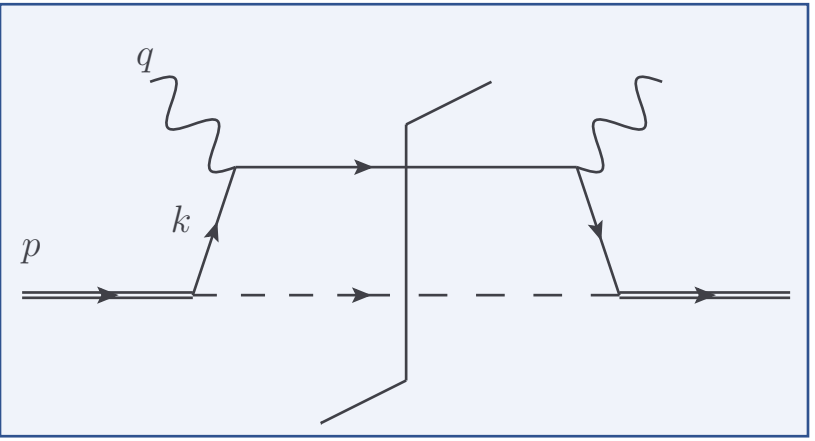
Scalar Yukawa theory

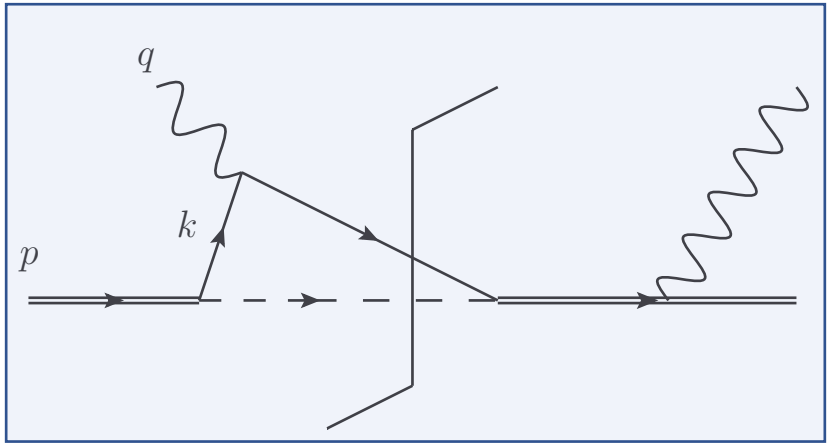
$$\mathcal{L}_I = -\lambda \overline{\Psi} \psi \phi + \text{h.c.}$$

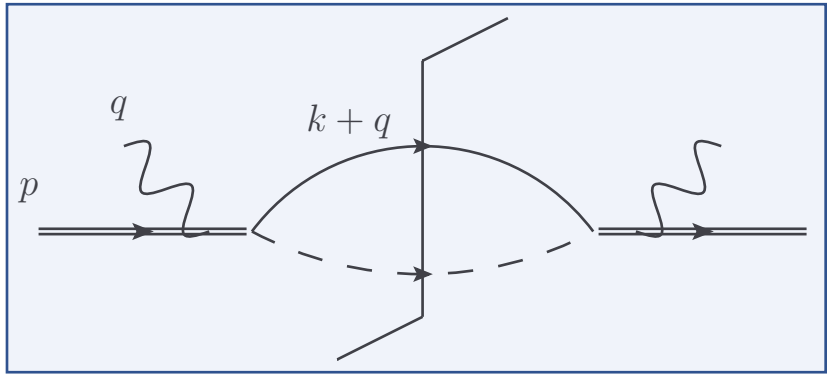
Ψ ⟷ "Hadron"

ψ ⟷ "Quark"

ϕ ⟷ "Meson"







How Factorization works

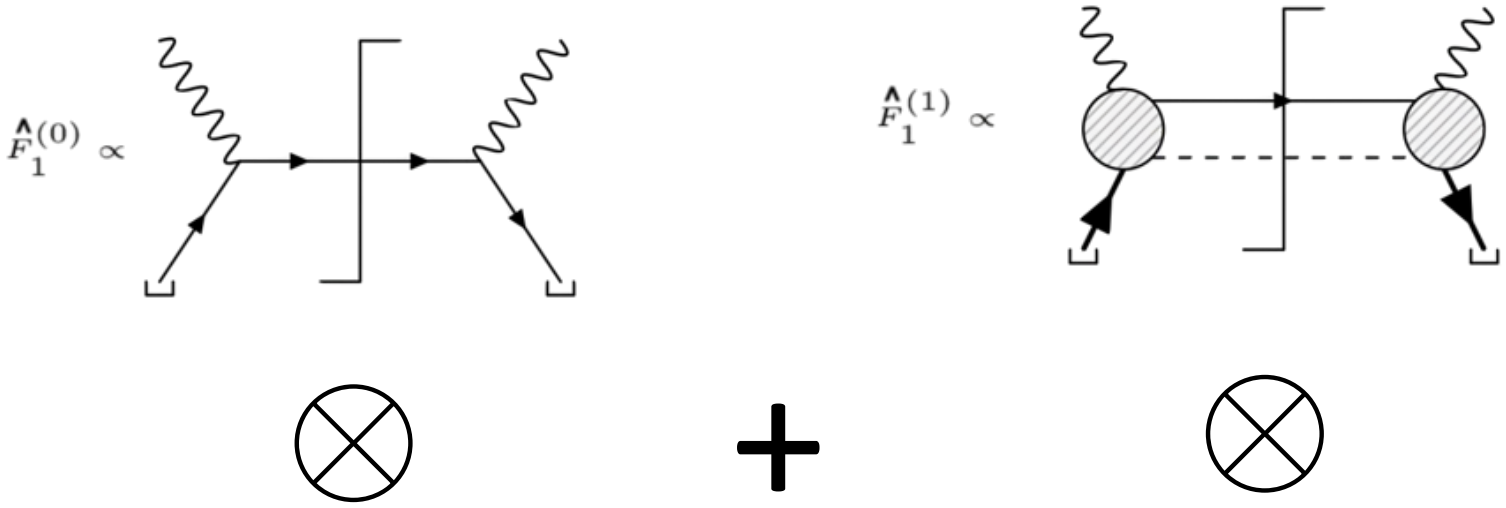
$$\sigma = \sum_{\text{partons}} \hat{\sigma} \otimes f + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

pQCD

Non Perturbative

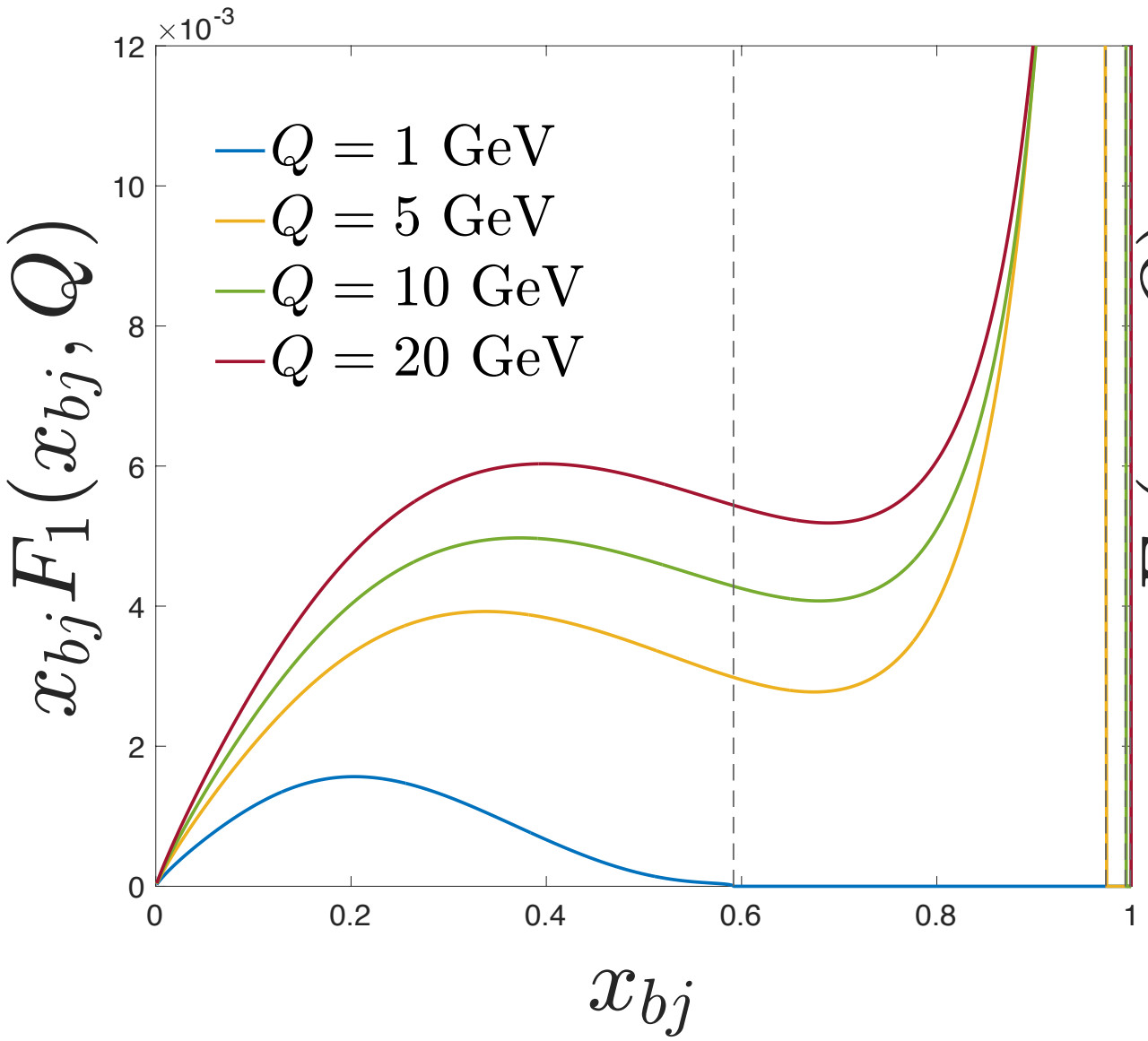
$$F_1 = \left(\widehat{F}_1^{(0)} + \widehat{F}_1^{(1)} + \ldots\right) \otimes \left(f^{(0)} + f^{(1)} + \ldots\right)$$

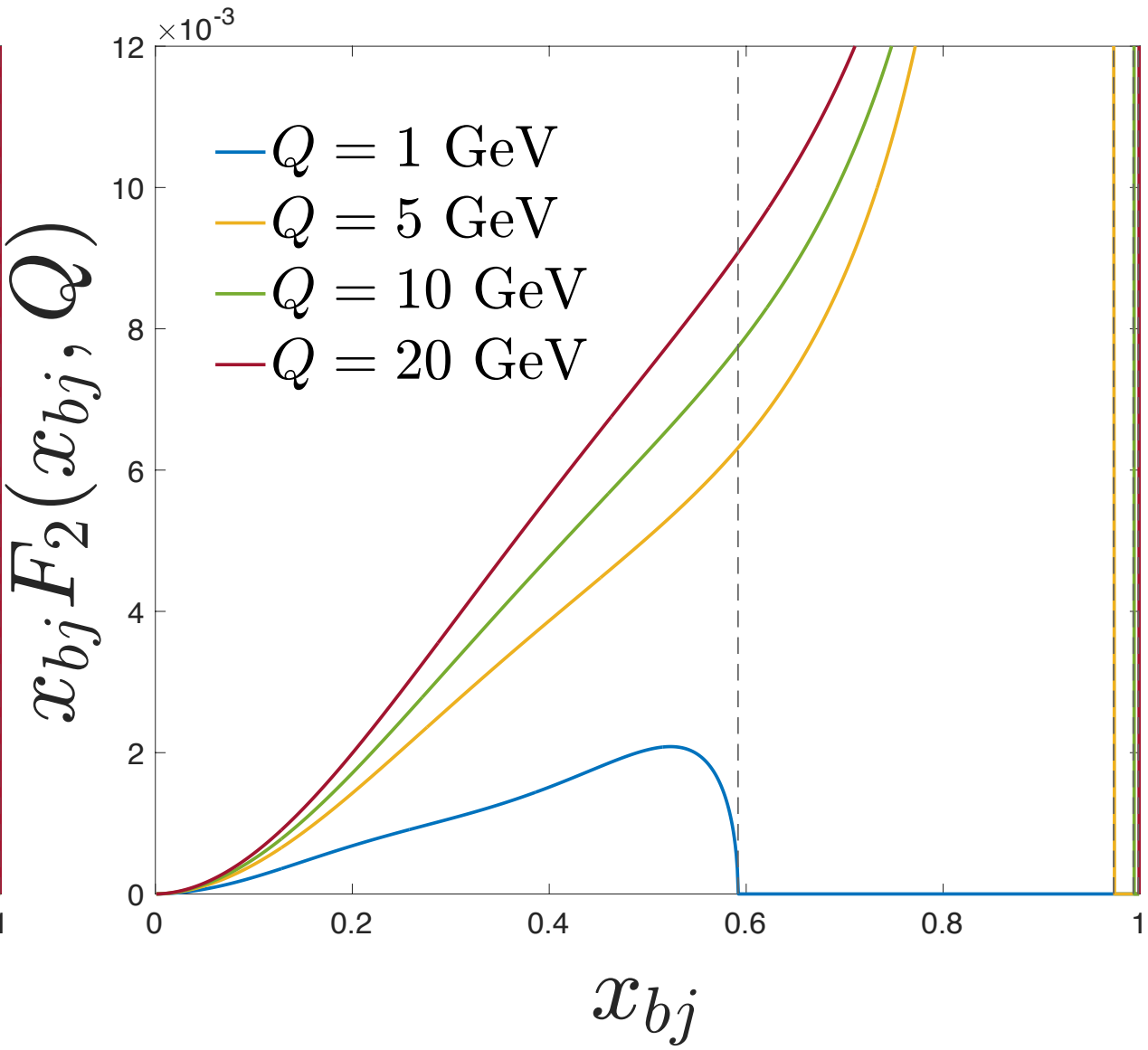
Massless partonic physics



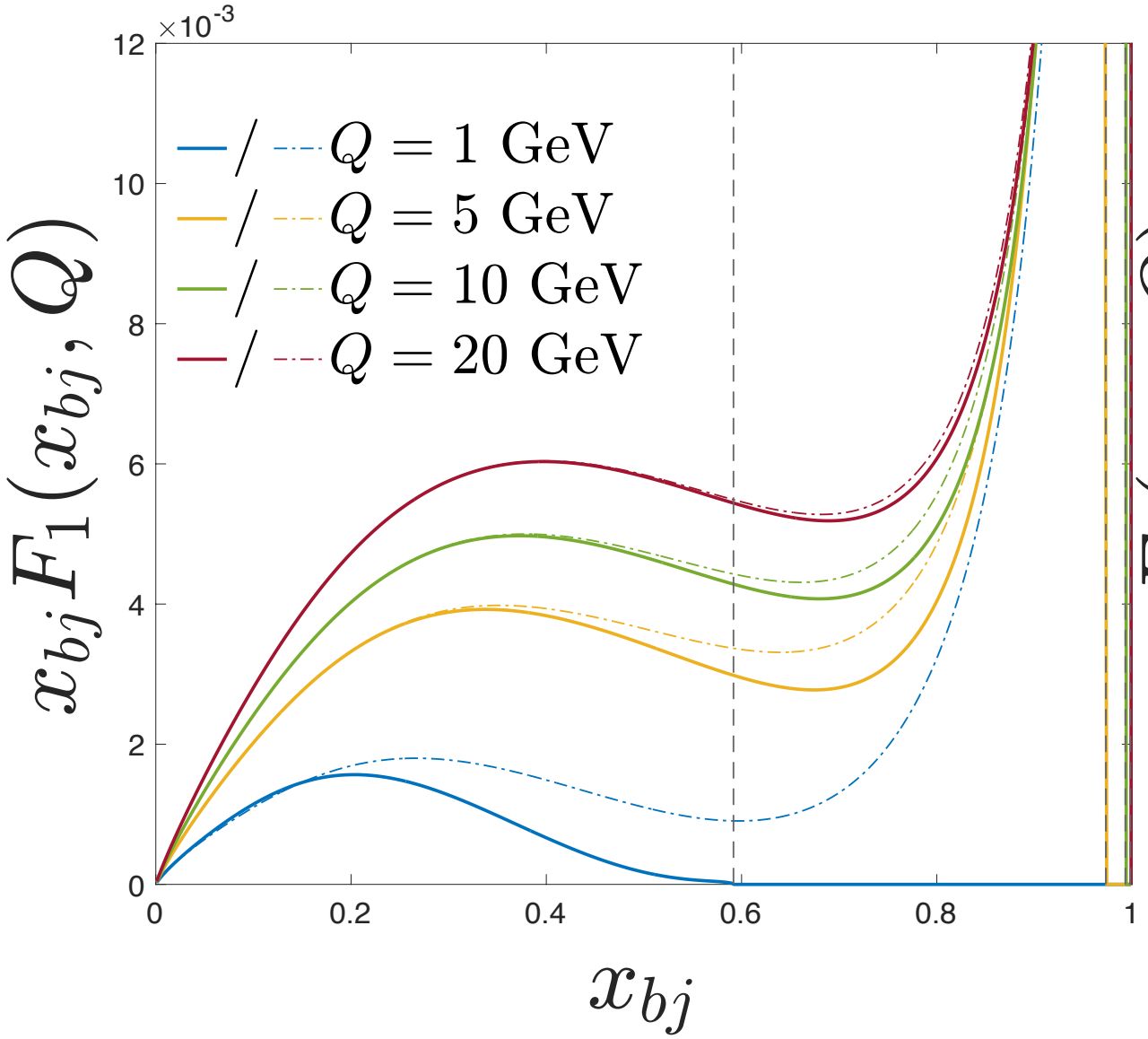
Intrinsic hadronic physics

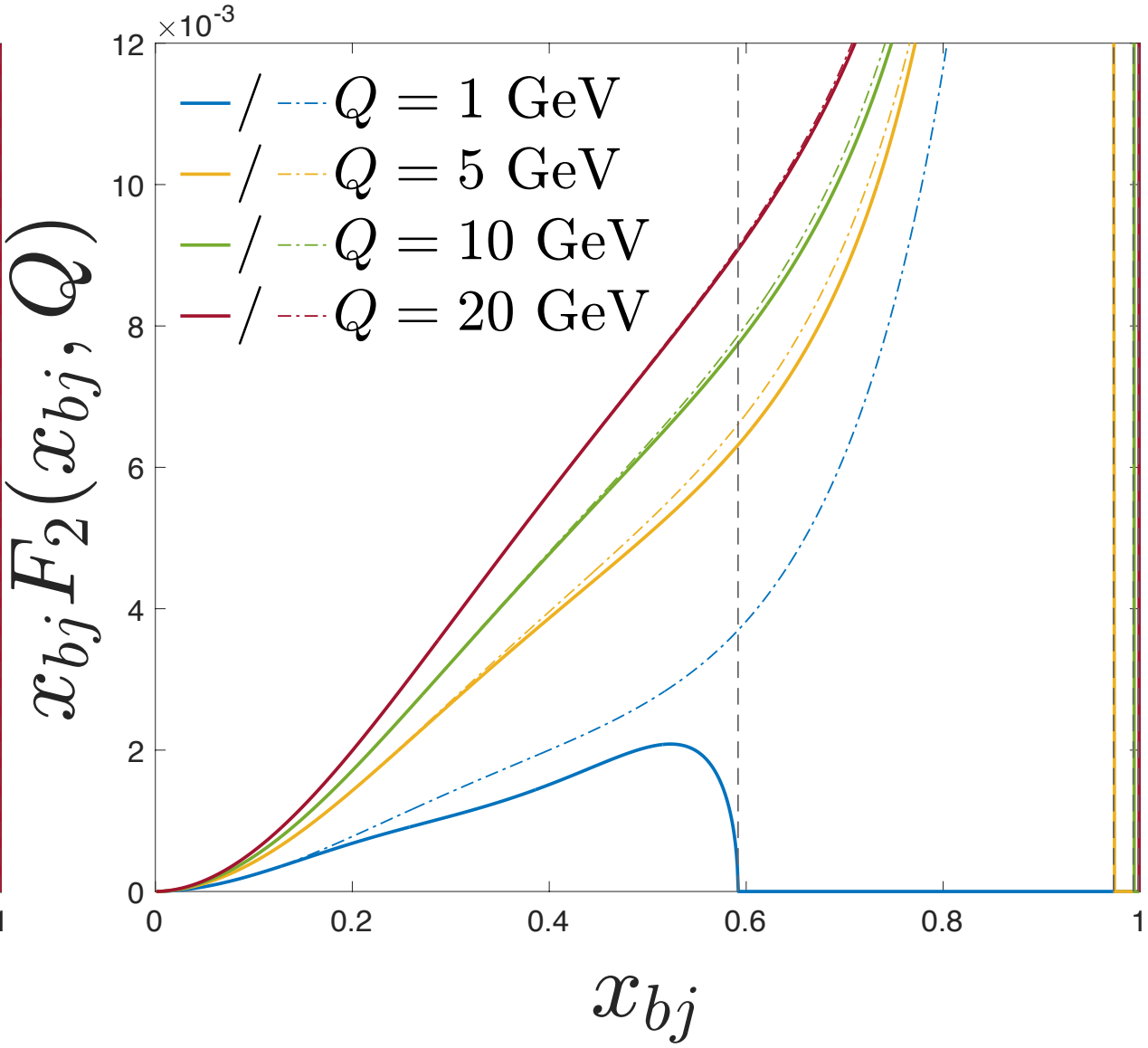
Unfactorized structure functions



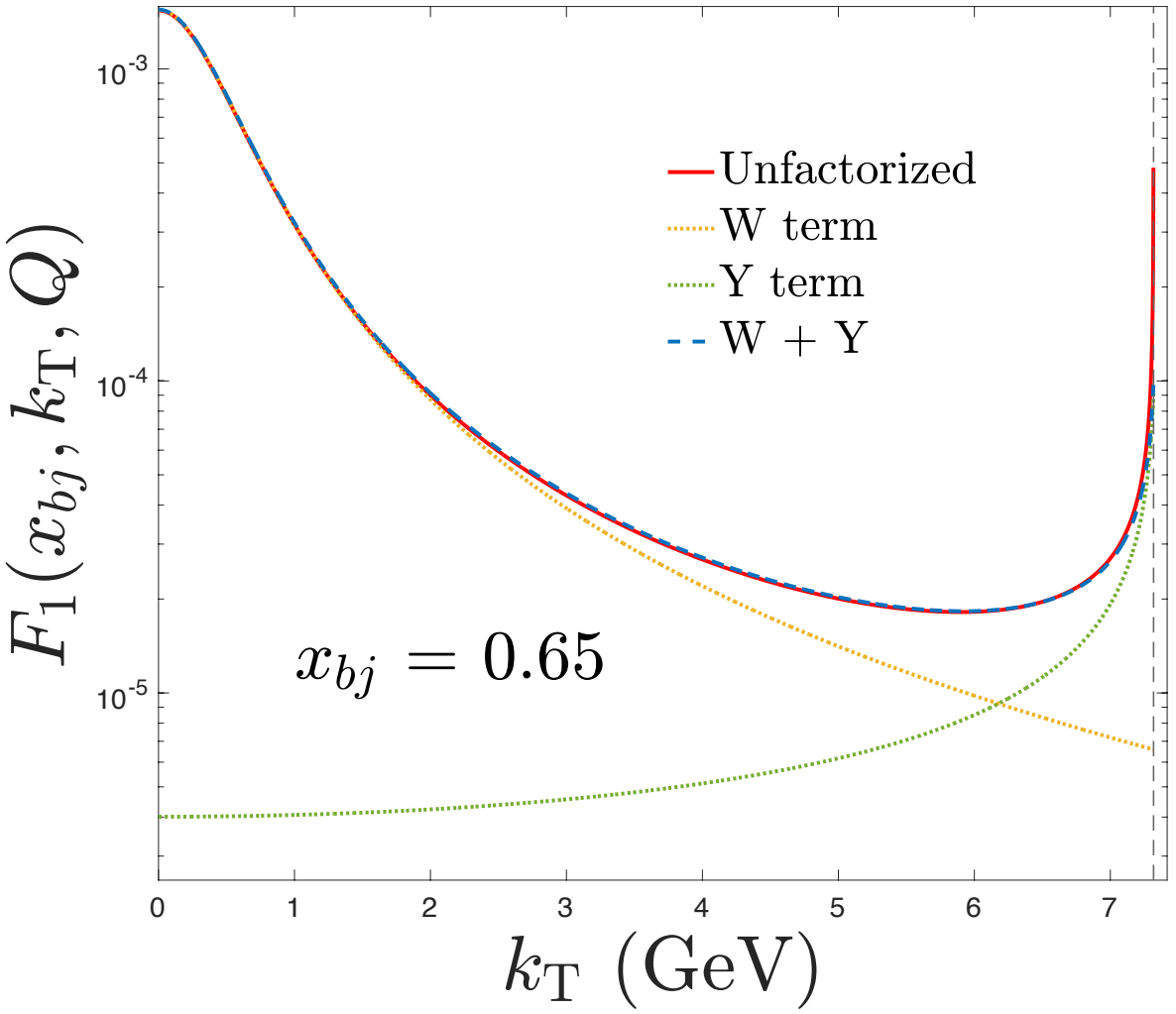


With factorized results





TMD W and Y terms contributions



TMD Observable = small kT + large kT

W + Y

TMD distributions

FO Collinear factorization

Collinear from TMD

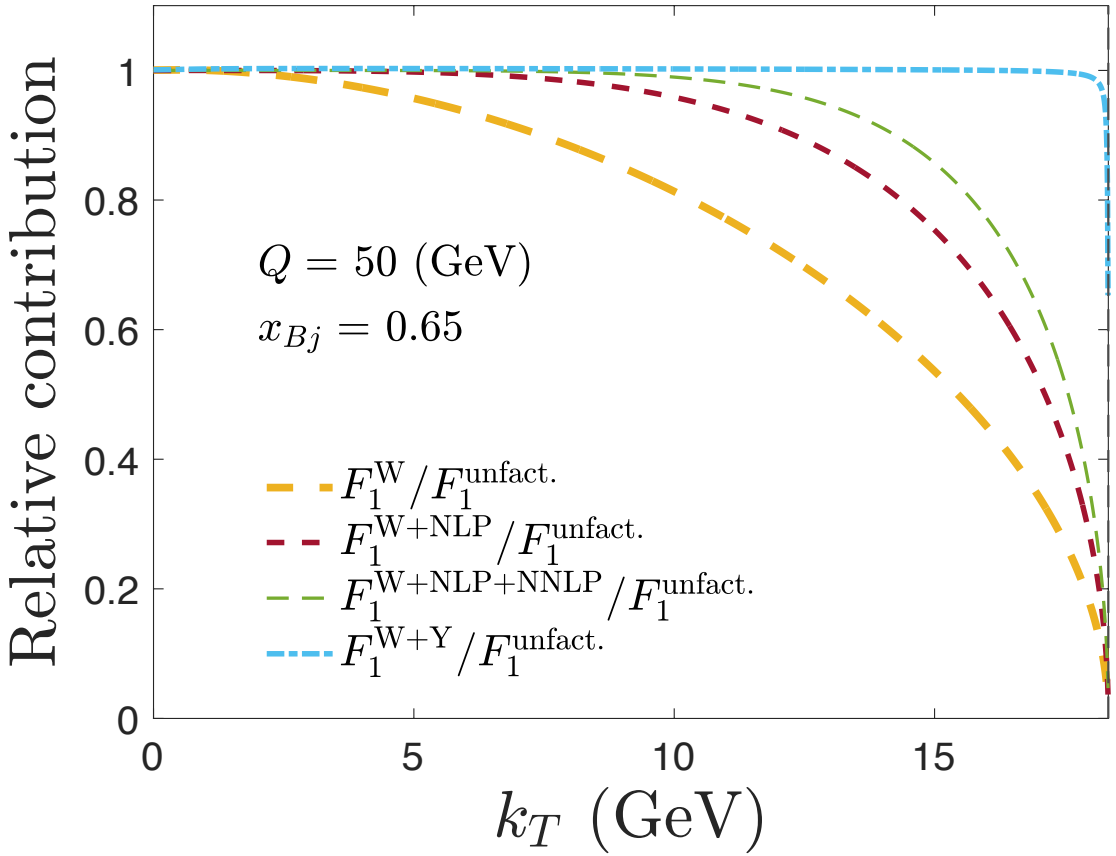
$$f(x;\mu) \overset{??}{=} \int \mathrm{d}^2\mathbf{k}_\mathrm{T}\, \mathrm{f}(\mathrm{x}, \mathbf{k}_\mathrm{T}; \mu)$$

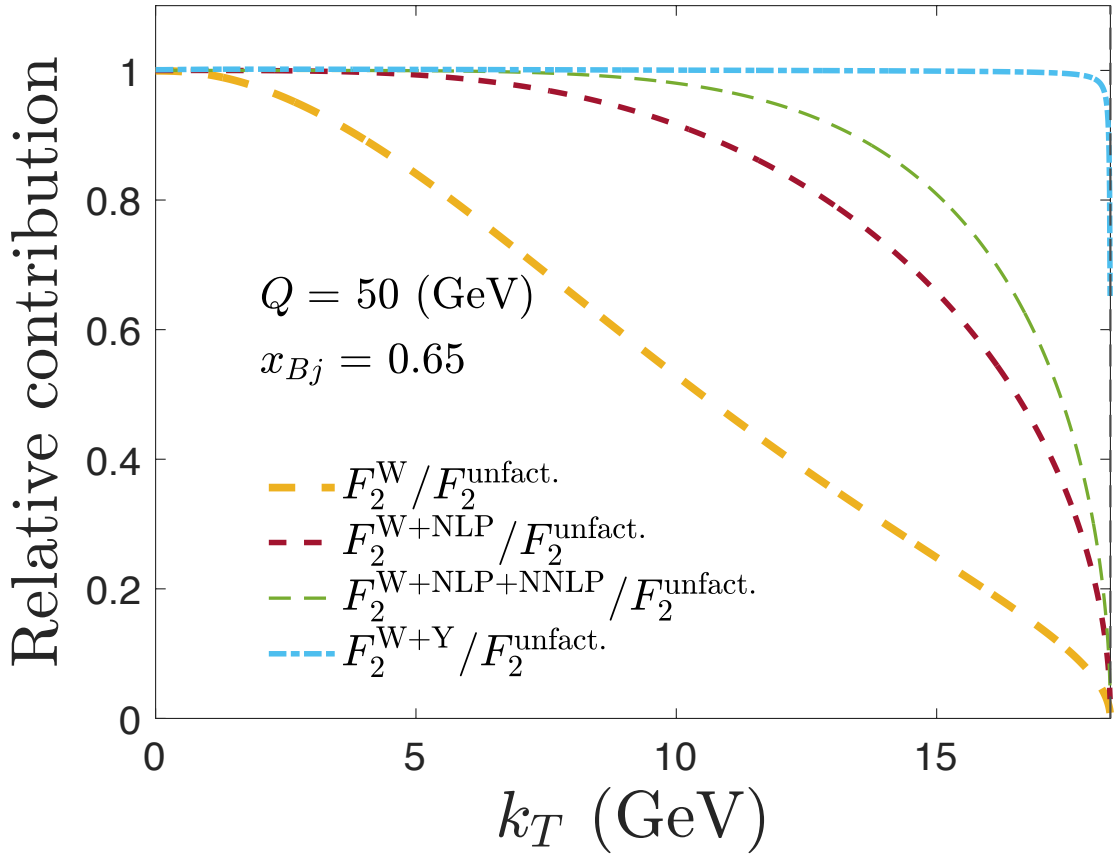
Both are necessary !!

W+Y and subleading power suppressed terms

small k_T $\leftrightarrow$ large k_T interpolation

$\frac{1}{Q^2}$ expansion





PDFs can be negative

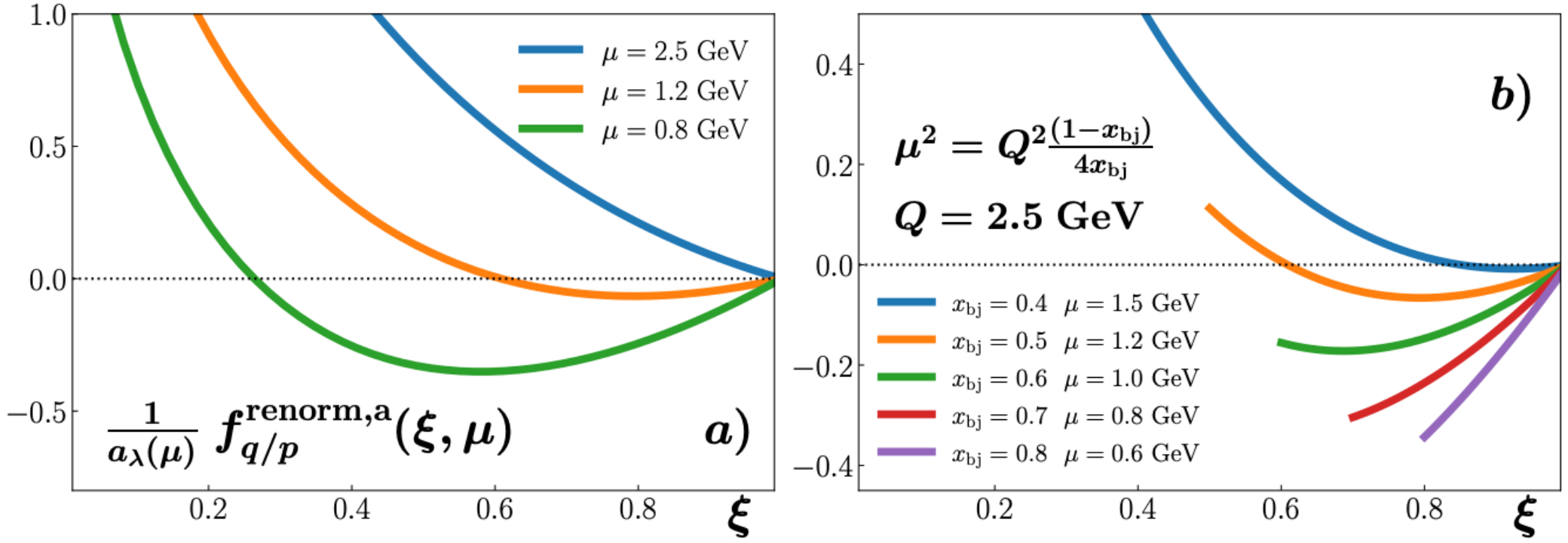
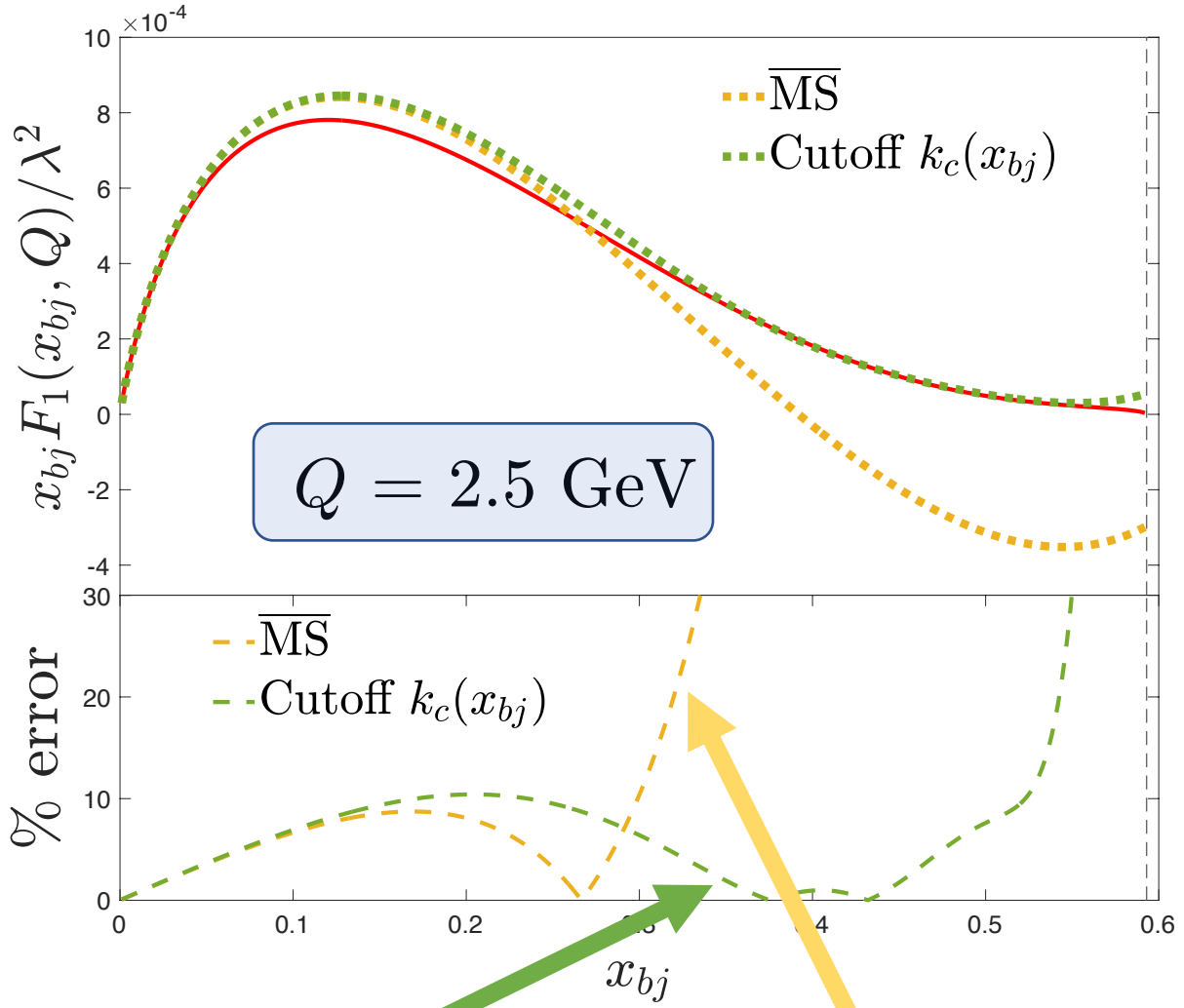


FIG. 2. (a) An example of the $\overline{\text{MS}}$ quark-in-proton pdf in Eq. (55) for several values of μ, with $m_q = 0.3$ GeV, $m_p = 1.0$ GeV, and $m_s = 1.5$ GeV. (b) Similar curves but in a form applicable to use in factorization for DIS with $\mu^2 = \hat{s}/4$, given $Q = 2.5$ GeV and several values for x_{bj}. The pdfs are only used in the range $x_{\text{bj}} \leq \xi \leq 1$, so the curves are restricted to this region.

From "Positivity and renormalization of parton densities" (Collins, Rogers, Sato)



$$\delta F^c(k_c(x_{bj})) \overset{?}{<} \delta F^{\overline{\rm MS}}$$

Different pdf definitions ?

Factorization at low Q

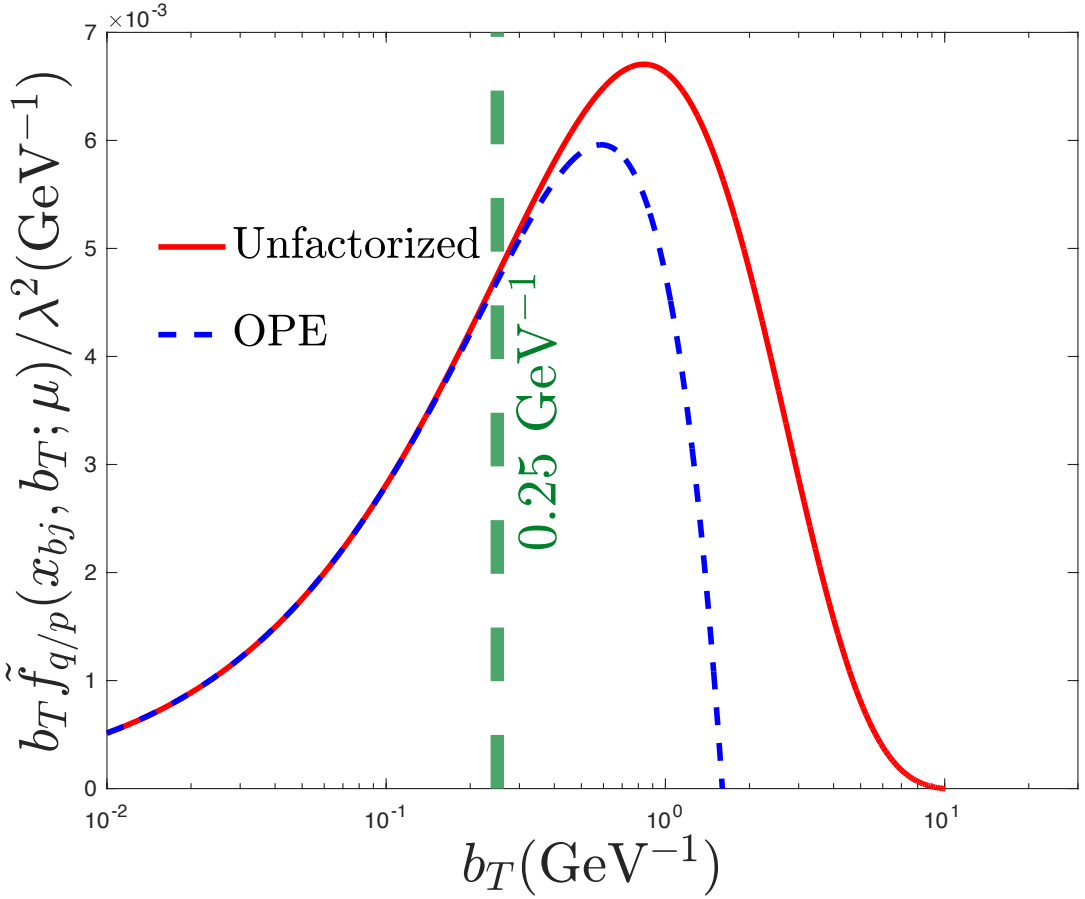
$$f^c(\xi;\mu) \equiv 2\pi \int_0^{k_c} \mathrm{dk_T k_T f}(\xi, \mathrm{k_T};\mu)$$

recover $\overline{\rm MS}$

$$\frac{dF}{d\log k_c} = \mathcal{O}\left(\frac{m^2}{k_c^2}, \alpha^2(\mu)\right)$$

Coordinate space pdf

$$\tilde{f}(x_{bj}, \boldsymbol{b}_T; \mu) = \int \mathrm{d}^2 \boldsymbol{k}_T e^{i\boldsymbol{k}_T \cdot \boldsymbol{b}_T} f(x_{bj}, \boldsymbol{k}_T; \mu)$$



Collinear perturbation theory (OPE) fails at large b_T

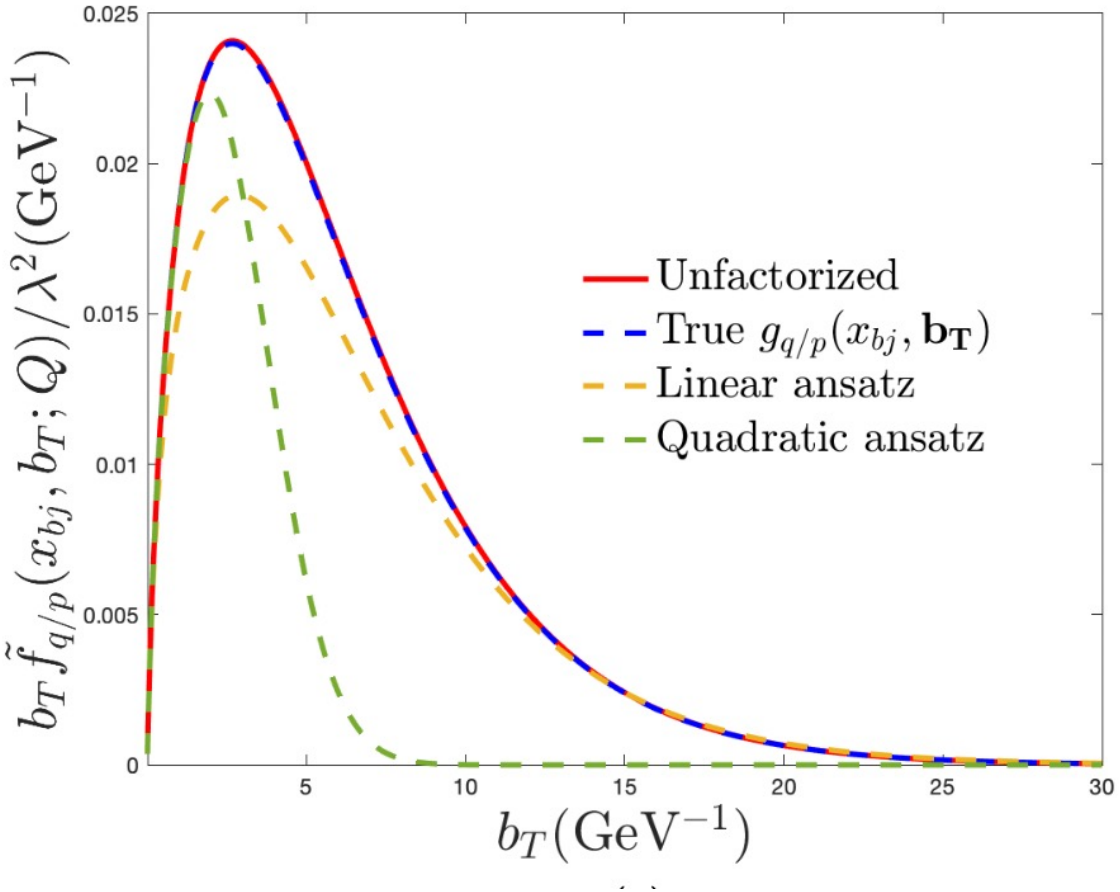
$$f(x_{bj}, \mathrm{b_T}) = \begin{cases} f^{\mathrm{OPE}}(x_{bj}, b_*) & b_T \ll b_{max} \\ f^{\mathrm{OPE}}(x_{bj}, b_*) e^{-g(x_{bj}, b_T)} & b_T > b_{max} \end{cases}$$

Non Perturbative

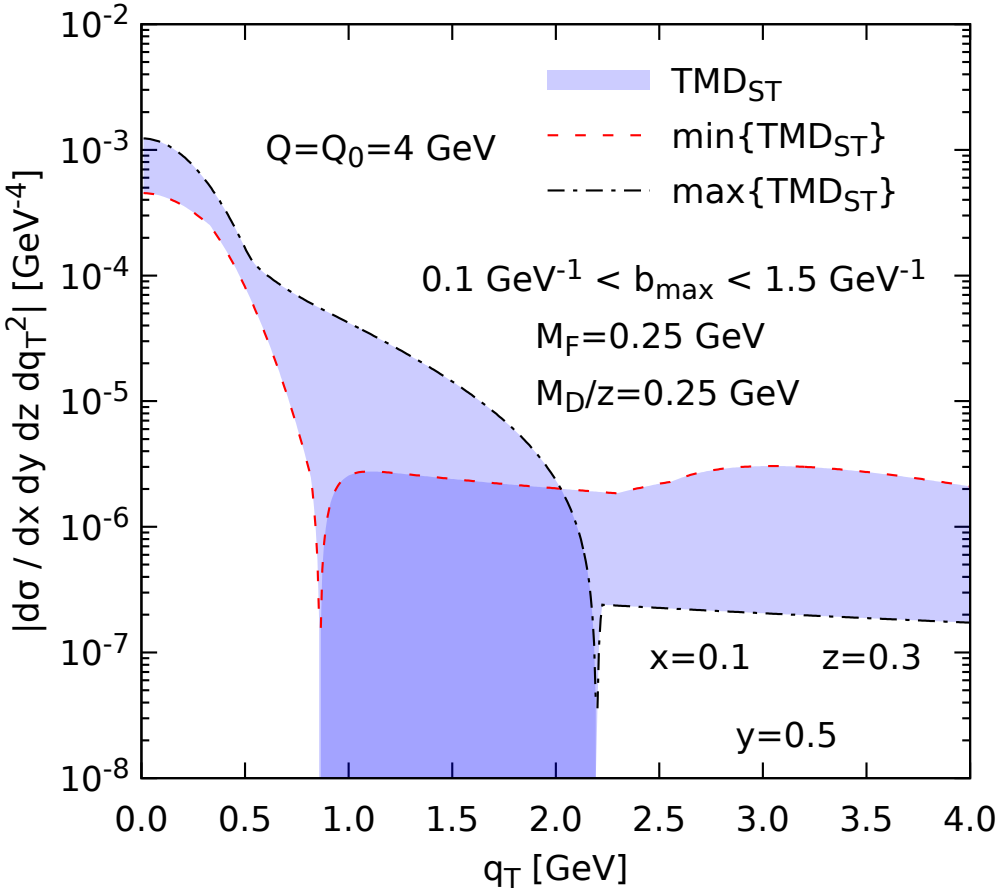
$$\frac{\mathrm{d}\tilde{f}(x_{bj}, \mathrm{b_T})}{\mathrm{db_{max}}} = \mathcal{O}(m \mathrm{b_{max}})$$

Common issues

Unconstrained g functions



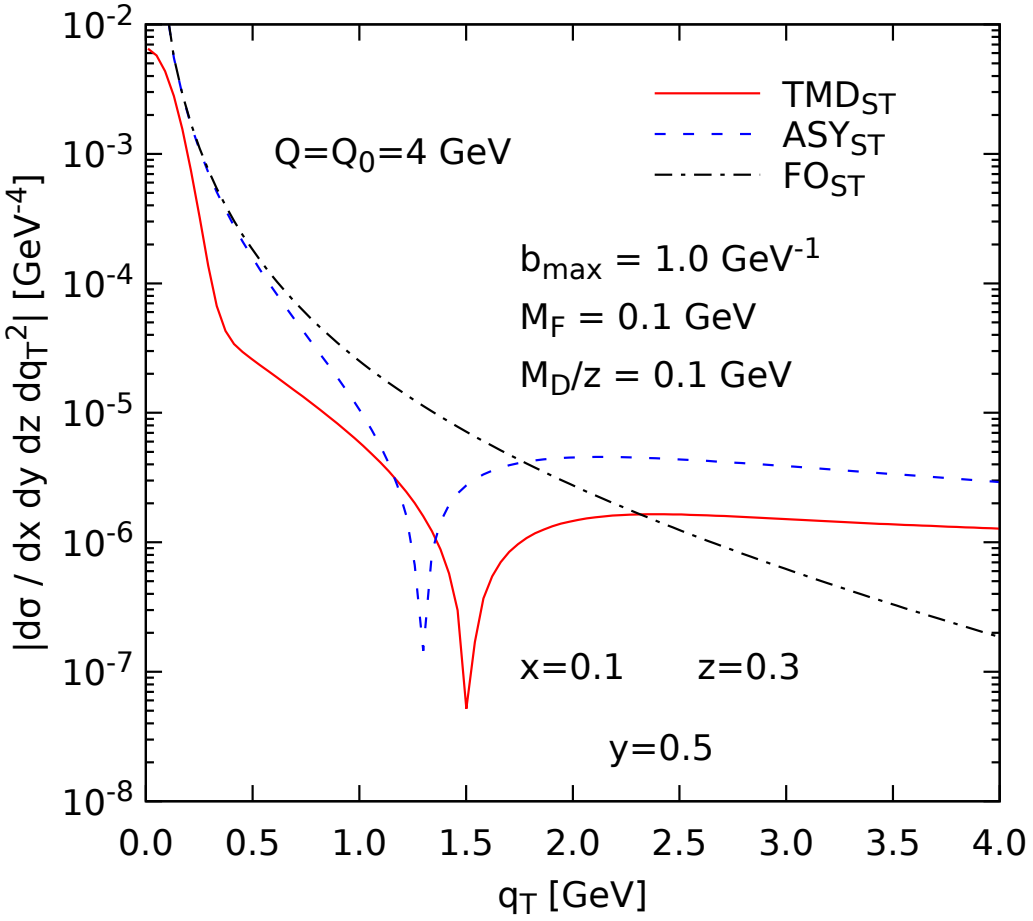
Large b_{max} dependence (QCD)



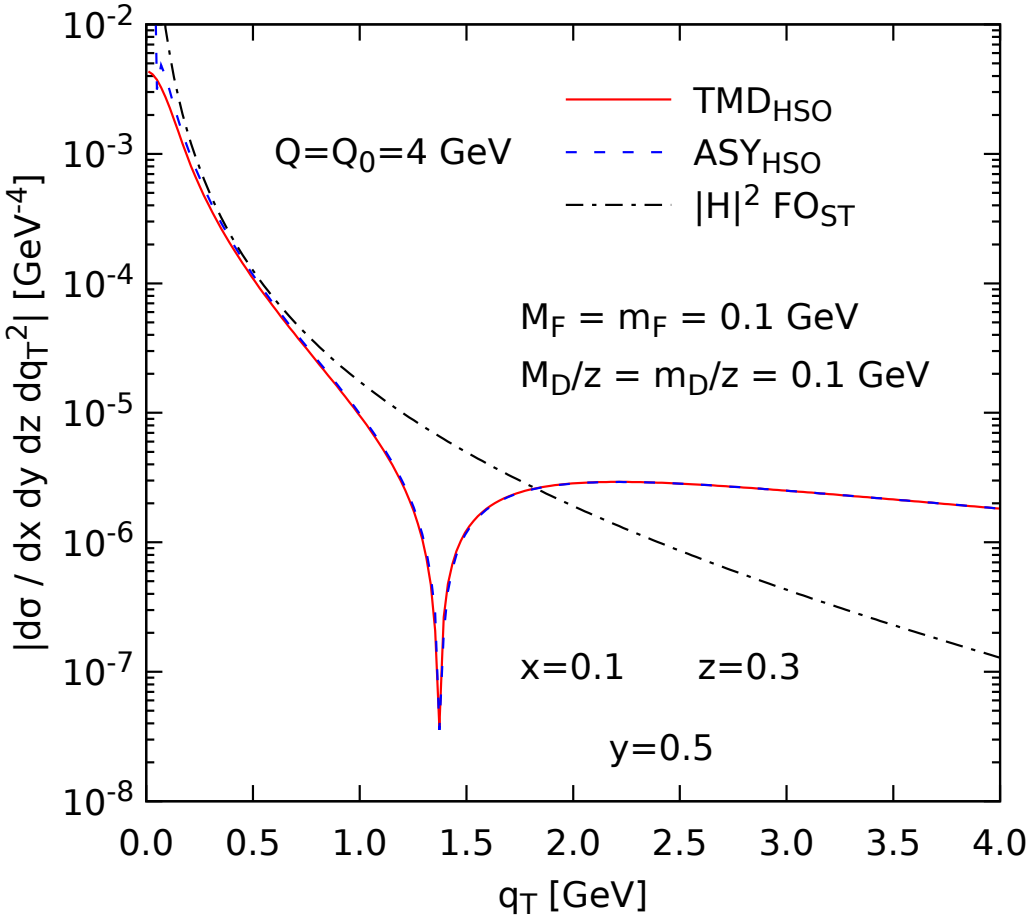
(from "2303.04921 " J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers)

Cutoff defined pdfs and constrained g functions (QCD SIDIS cross section)

Conventional



HSO



(from "2303.04921 " J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers)

(See Ted C. Rogers talk WG2)

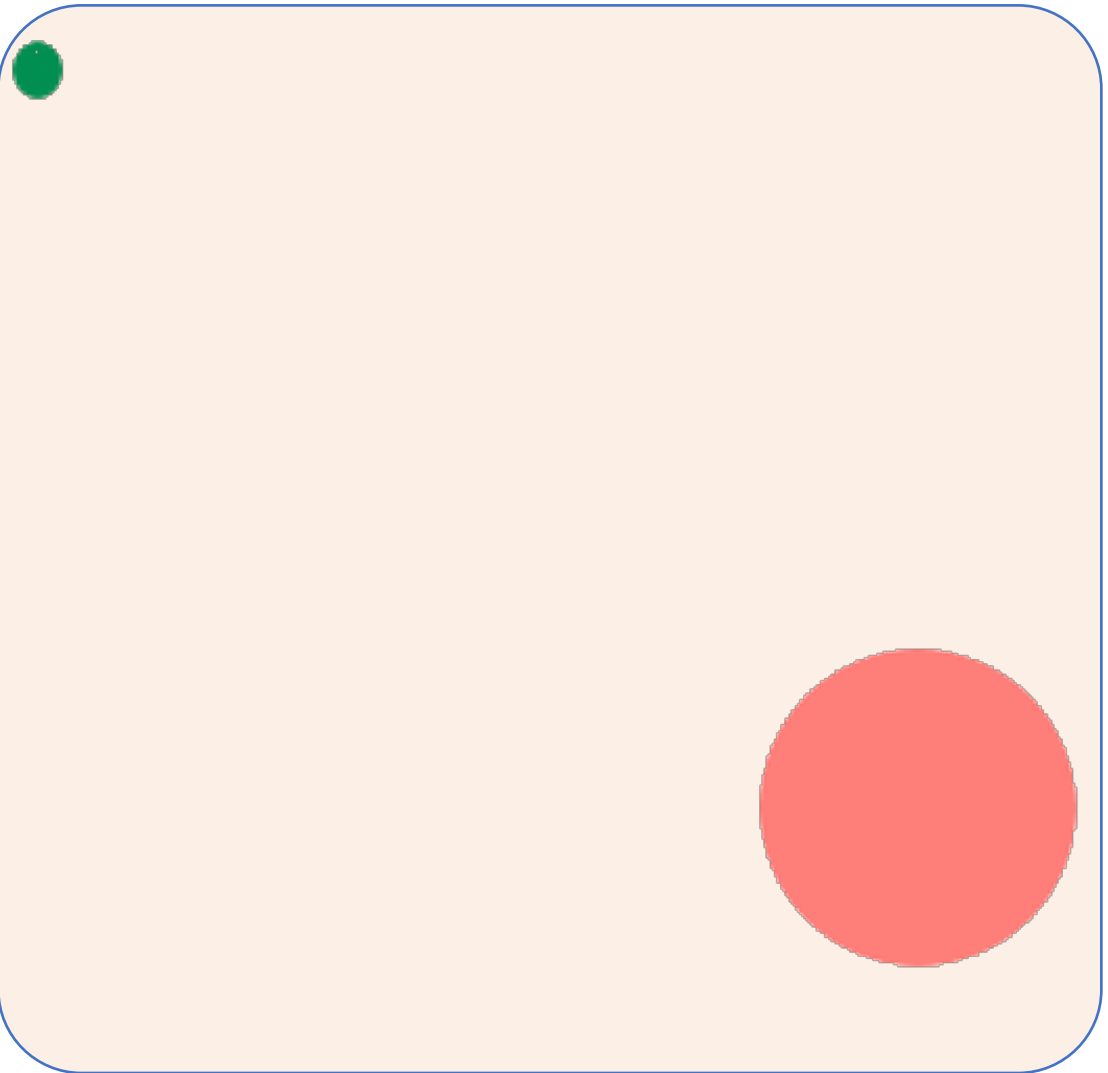
Summary

- Factorization theorem statement verified
- Differences between W+Y and power suppressed terms in TMDs
- Different pdf definitions might push factorization to lower Q and provide a consistent TMD parametrization
- Test b_T space techniques and avoid sources of large errors

Thank you !

Additional slides

Hadronic structure: scattering experiments



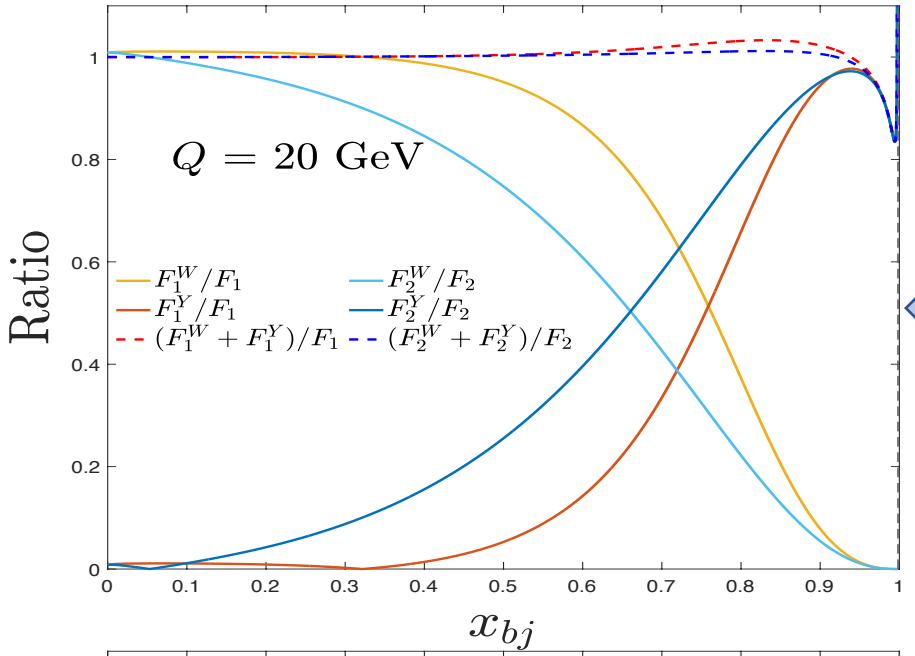
Statement of factorization :

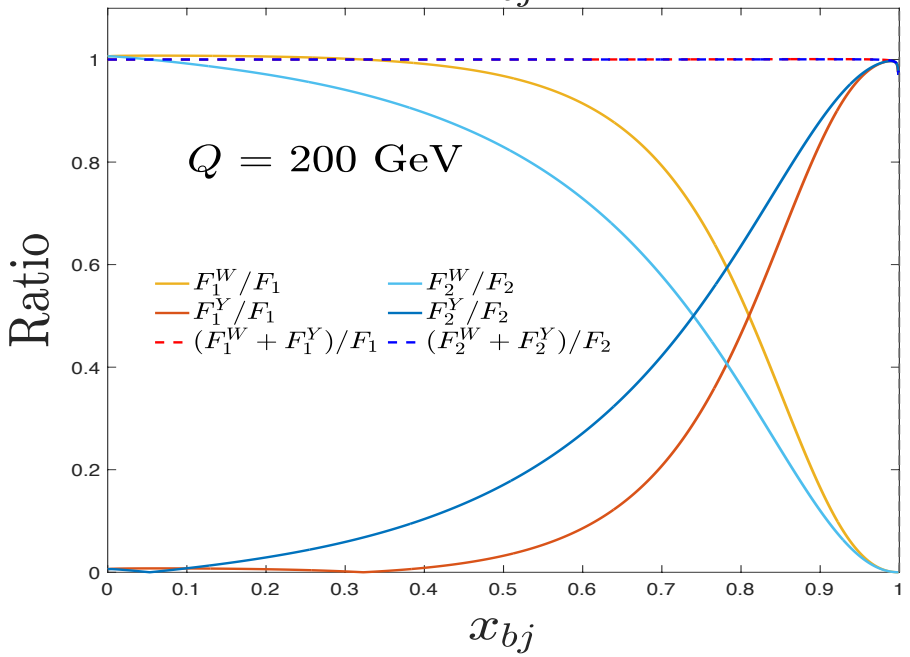
$$\sigma = \sum_{\text{partons}} \hat{\sigma} \otimes f + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

pQCD

Non Perturbative

The role of the integrated W and Y terms





Fixed Q

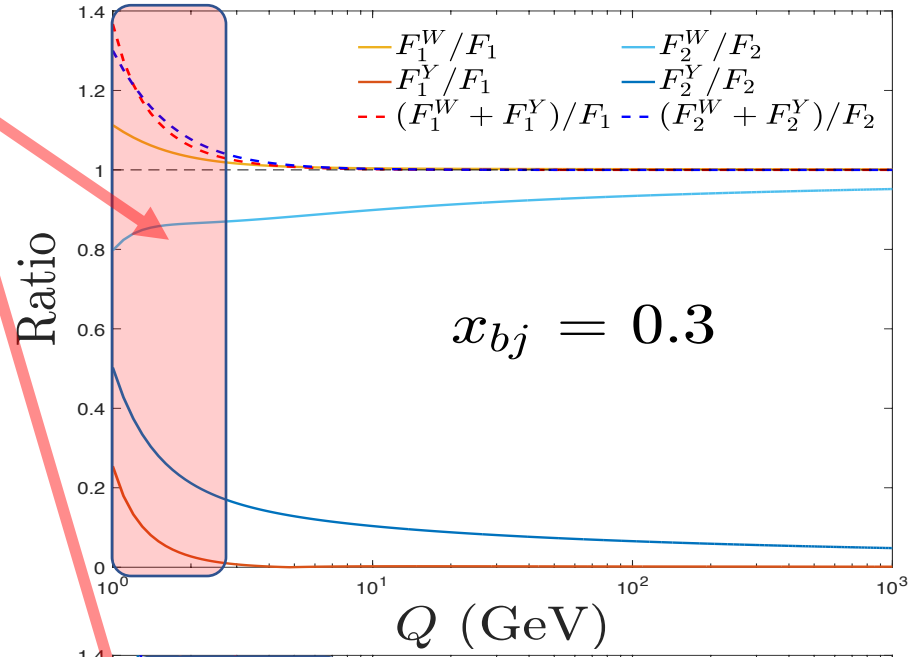
No "good" factorization

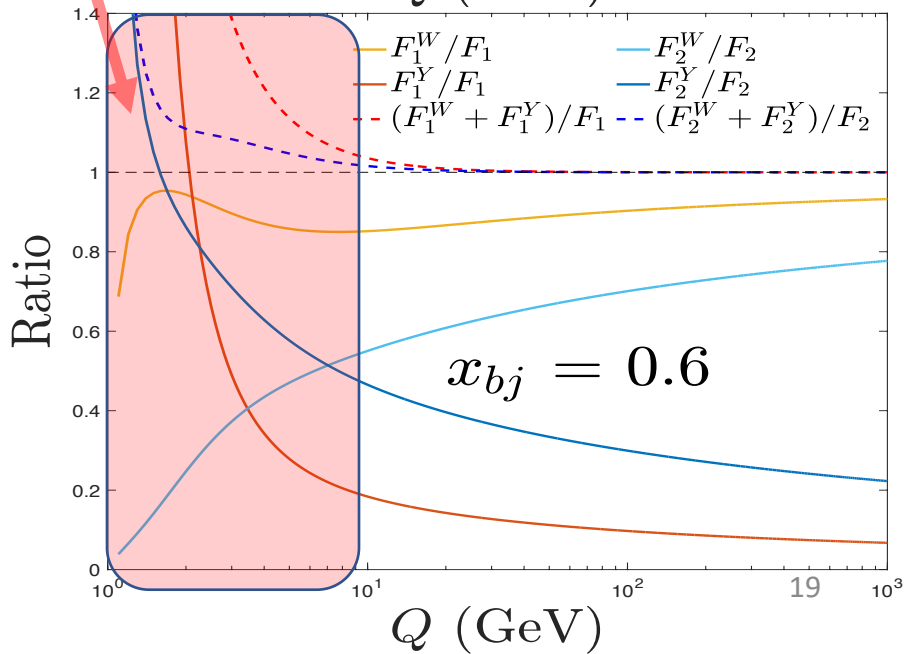
$$F_1(x_{Bj}, Q) =$$

$$\int \mathrm{d}^2\mathbf{k}_T F_1(x_{Bj}, k_T, Q) =$$

$$\int \mathrm{d}^2\mathbf{k}_T \left(\mathrm{W} + \mathrm{Y}\right) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

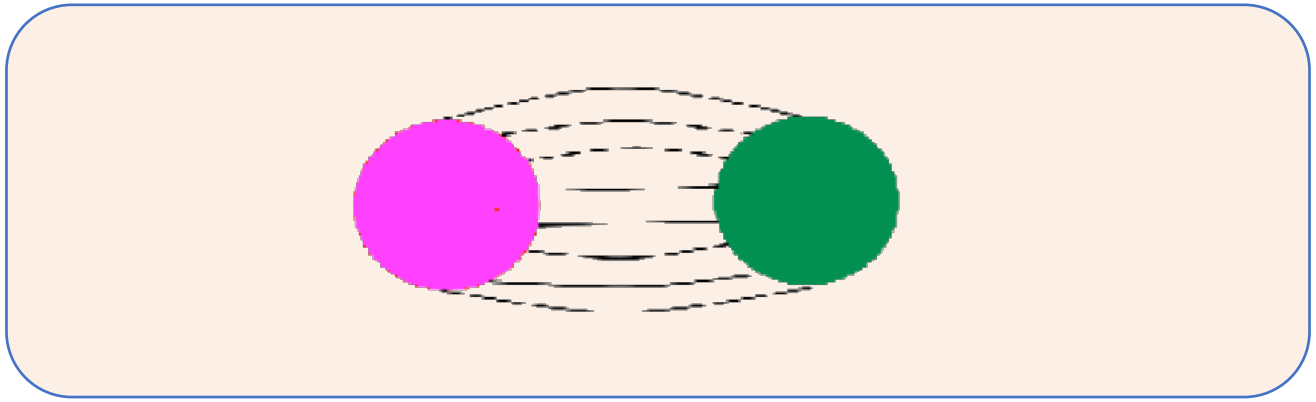
Fixed x_{Bj}



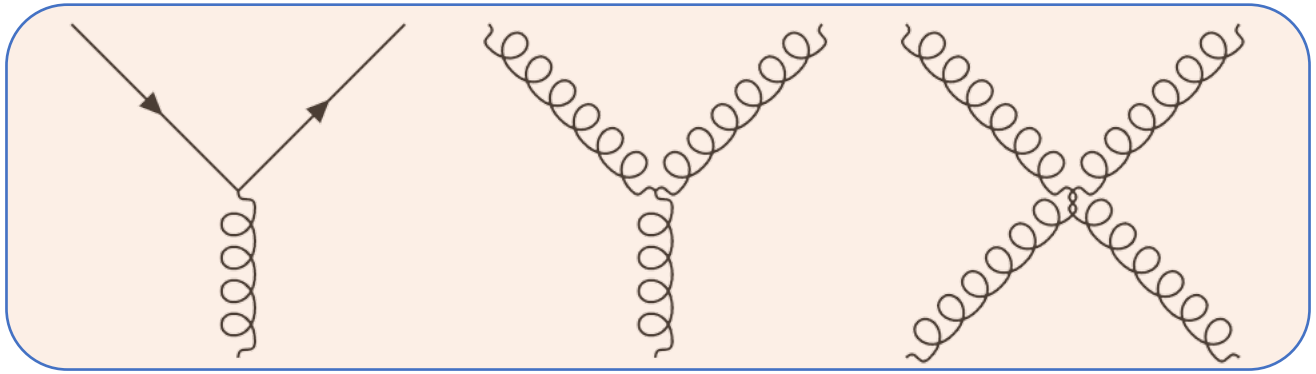


The difficulties with QCD

- Quark and gluons are never observed (**color confinement**)



- The interaction is **strong** (of order 1)
- Unlike photons, **gluons interact with themselves**



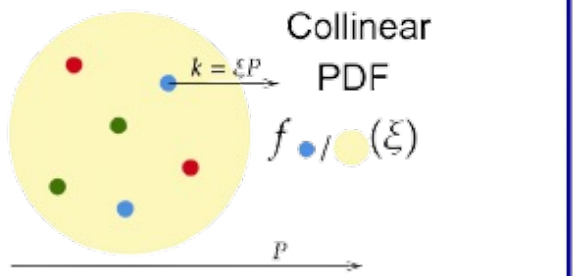
DIS Cross Section

$$\frac{d\sigma_{\mathrm{DIS}}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\psi} = \frac{y\alpha_{\mathrm{em}}^2}{Q^4} L_{\mu\nu} W_{\mathrm{DIS}}^{\mu\nu}$$

$$L^{\mu\nu} = 2(l_1^\mu l_2^\nu + l_1^\nu l_2^\mu - (l_1 \cdot l_2) g^{\mu\nu})$$

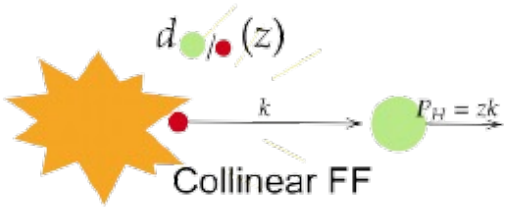
$$\begin{aligned} W^{\mu\nu} = & \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1 \\ & + \left(P^\mu - \frac{P\cdot q}{q^2} q^\mu \right) \left(P^\nu - \frac{P\cdot q}{q^2} q^\nu \right) \frac{F_2}{P\cdot q} \\ & + i\epsilon^{\mu\nu\alpha\beta} q^\alpha S^\beta \frac{g_1}{P\cdot q} \\ & + i\epsilon^{\mu\nu\alpha\beta} q^\alpha [(P\cdot q) S^\beta - (S\cdot q) P^\beta] \frac{g_2}{(P\cdot q)^2} \end{aligned}$$

Collinear Momentum



Parton Distribution Function: Probability distribution to find parton j in hadron H carrying momentum fraction ξ

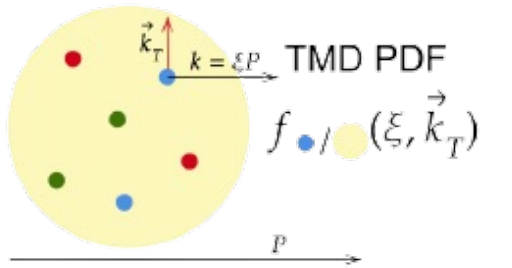
$$f_{j/H}(\xi)$$

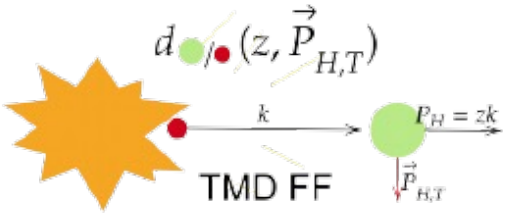


Fragmentation Function: Probability distribution to produce hadron H from parton j carrying momentum fraction z

$$d_{H/j}(z)$$

Transverse Momentum





Transverse Momentum Dependence (TMD)

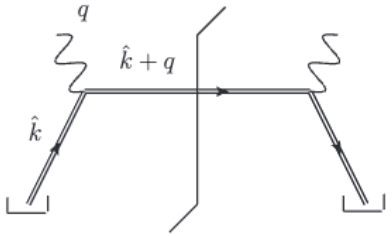
The partons will generally also move in the transverse direction:

$$f_{j/H}(\xi, \mathbf{k}_T)$$

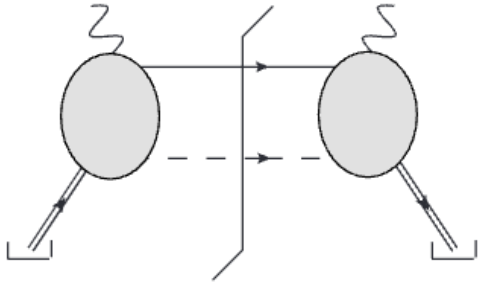
Similarly for the produced hadron

$$d_{H/j}(z, \mathbf{P}_{H,T})$$

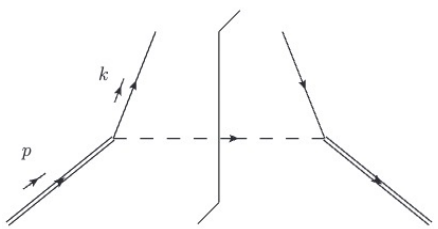
$$\widehat{F}^{(0)}_{1,p/p}(\xi/x_{\rm bj},Q)=\frac{1}{2}\delta(1-x_{\rm bj}/\xi)=$$



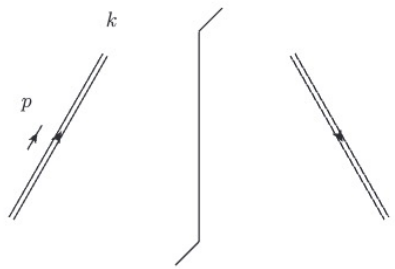
$$\widehat{F}^{(1)}_{1,q/p}(\xi,Q;\mu)_{\rm unsub}=$$



$$f^{(1)}_{q/p}(\xi;\mu)\overset{\xi\neq 1}{=}\int\frac{{\rm d}k^-\ {\rm d}^{2-2\epsilon}\boldsymbol{k}_{\rm T}}{(2\pi)^{4-2\epsilon}}{\rm Tr}\left[\frac{\gamma^+}{2}\right.$$

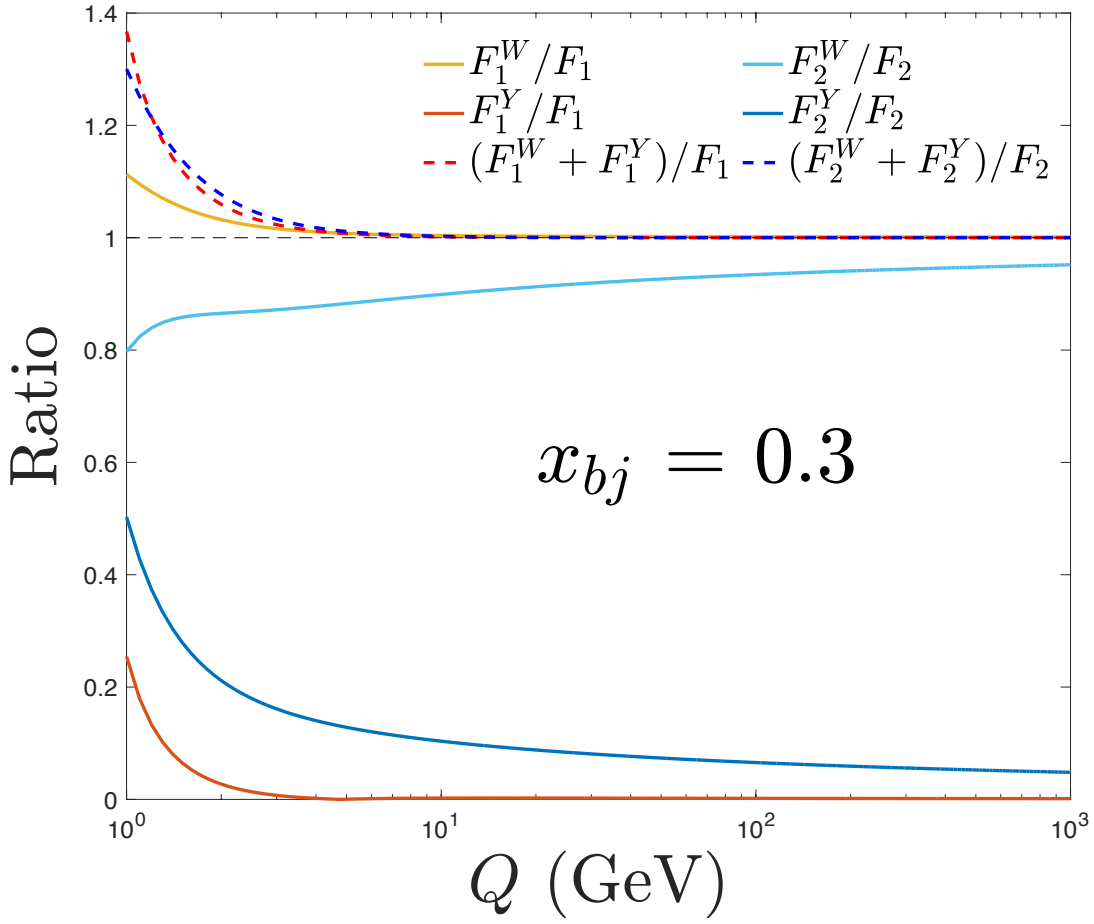


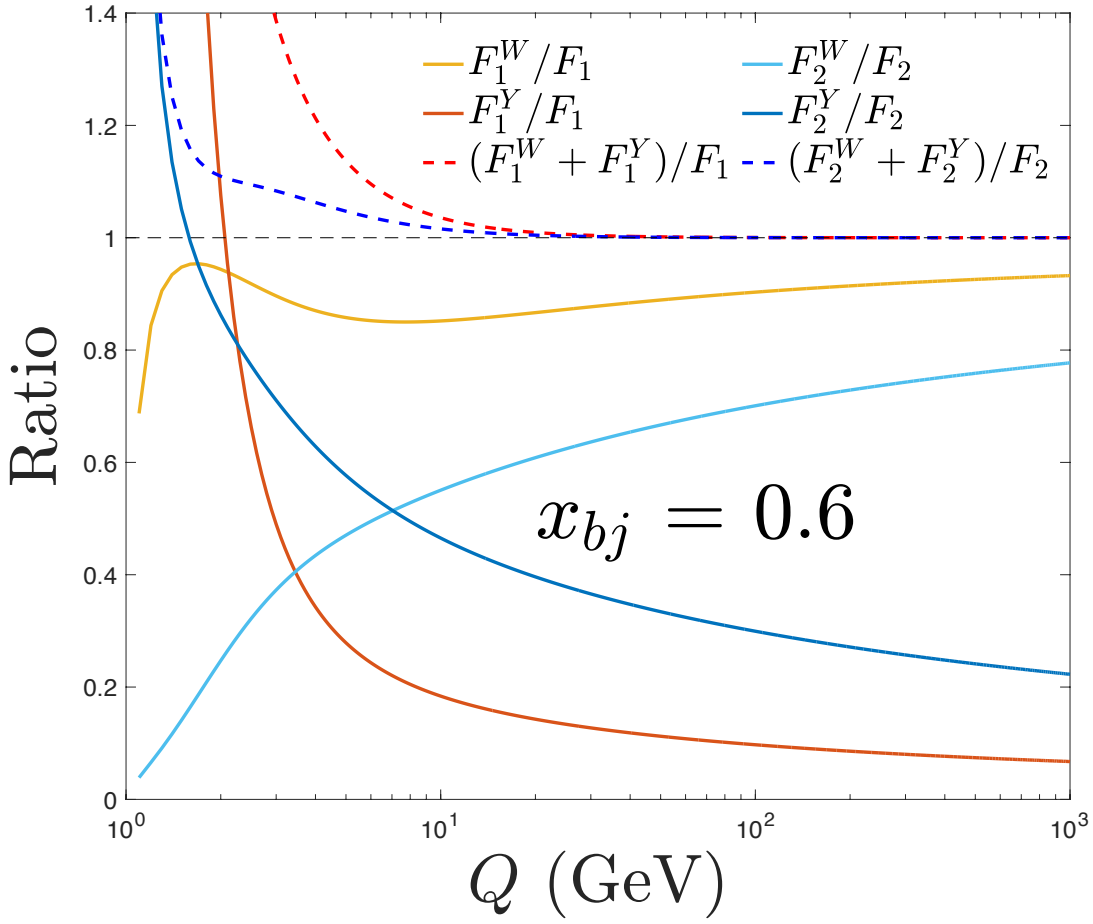
$$\left.\right]+\overline{\rm MS}\ \ {\rm C.T.}\quad f^{(0)}_{p/p}(\xi;\mu)=\int\frac{{\rm d}k^-\ {\rm d}^{2-2\epsilon}\boldsymbol{k}_{\rm T}}{(2\pi)^{4-2\epsilon}}{\rm Tr}\left[\frac{\gamma^+}{2}\right.$$



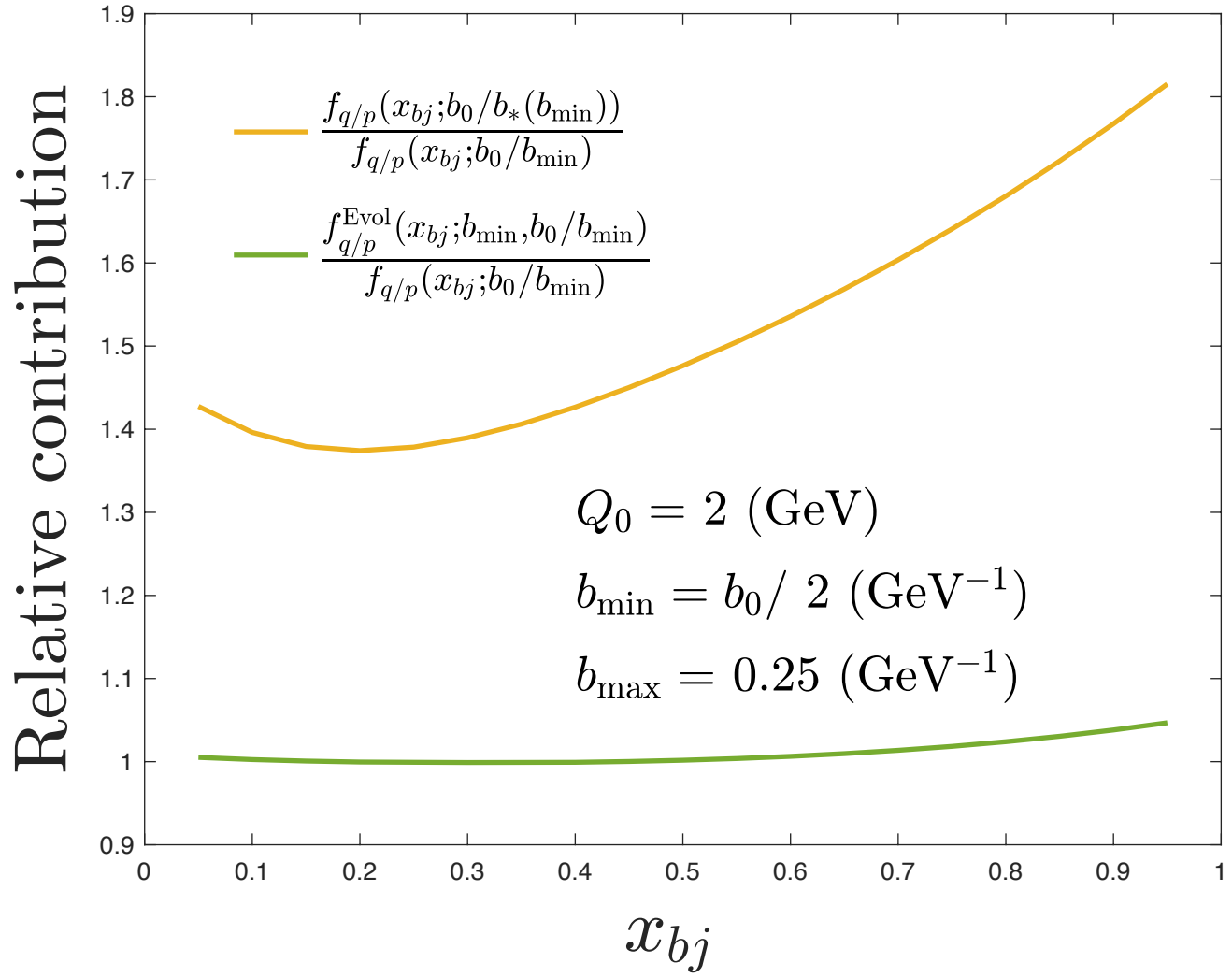
$$\left.\right]$$

For fixed x Bjorken

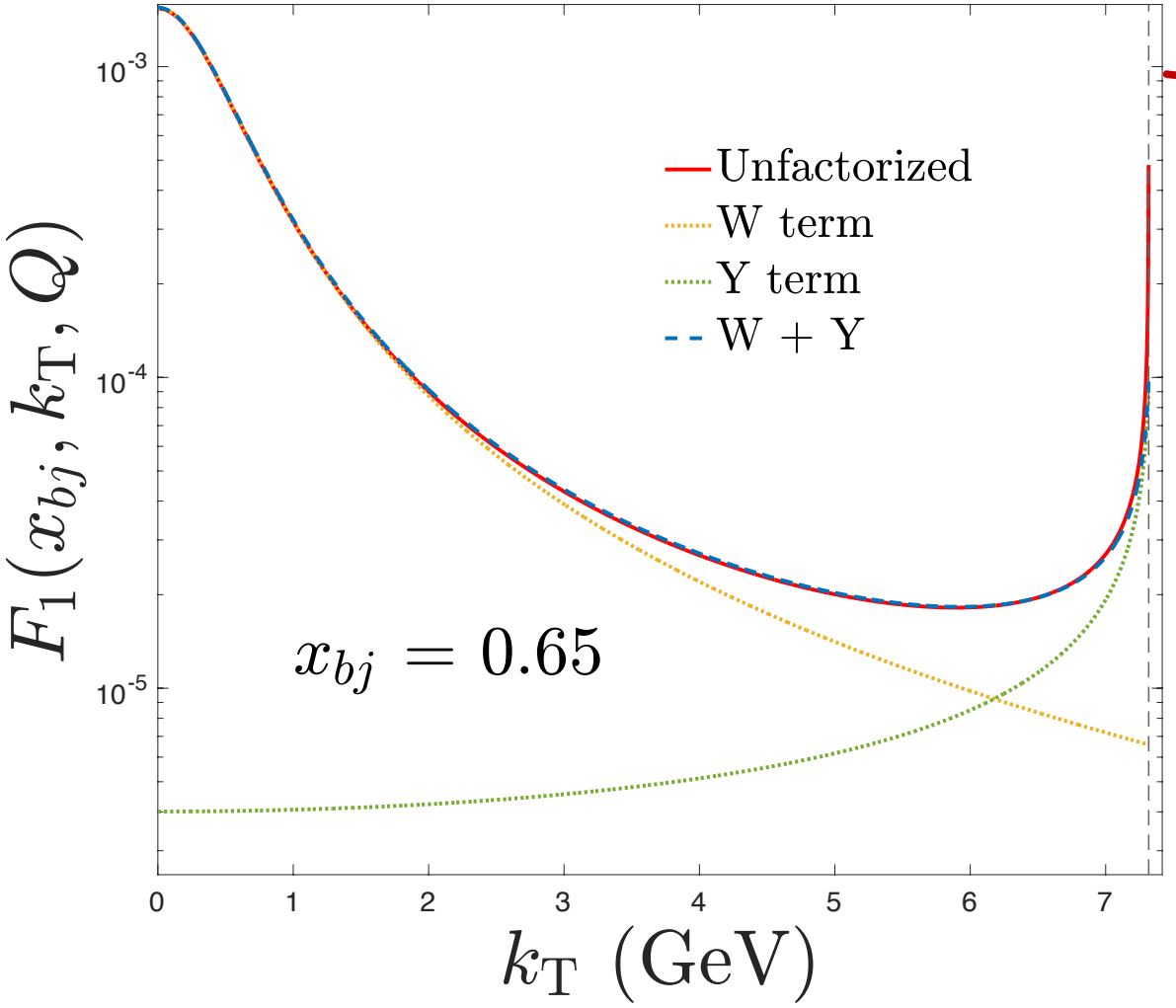




Recovering collinear pdfs from TMD pdfs?



TMD W and Y terms contributions



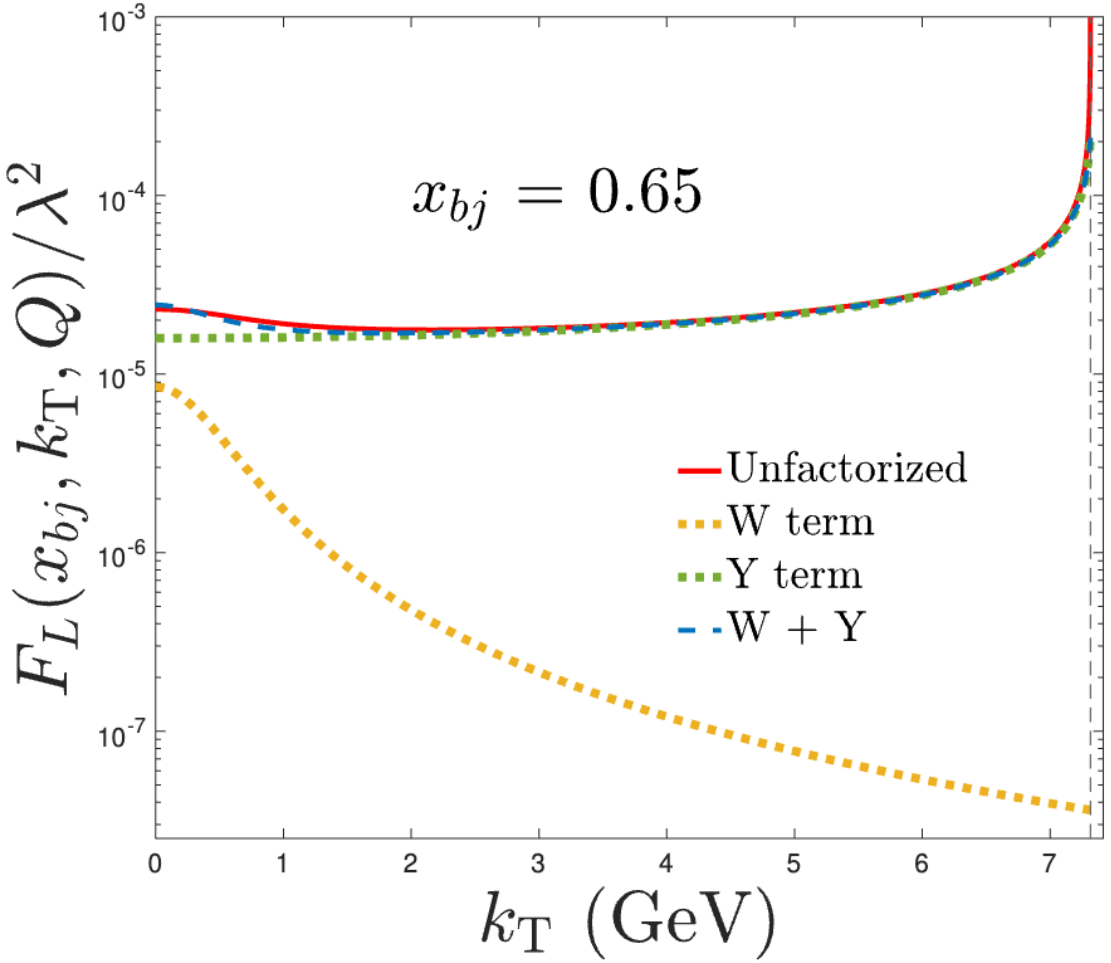
TMD Observable = small kT + large kT

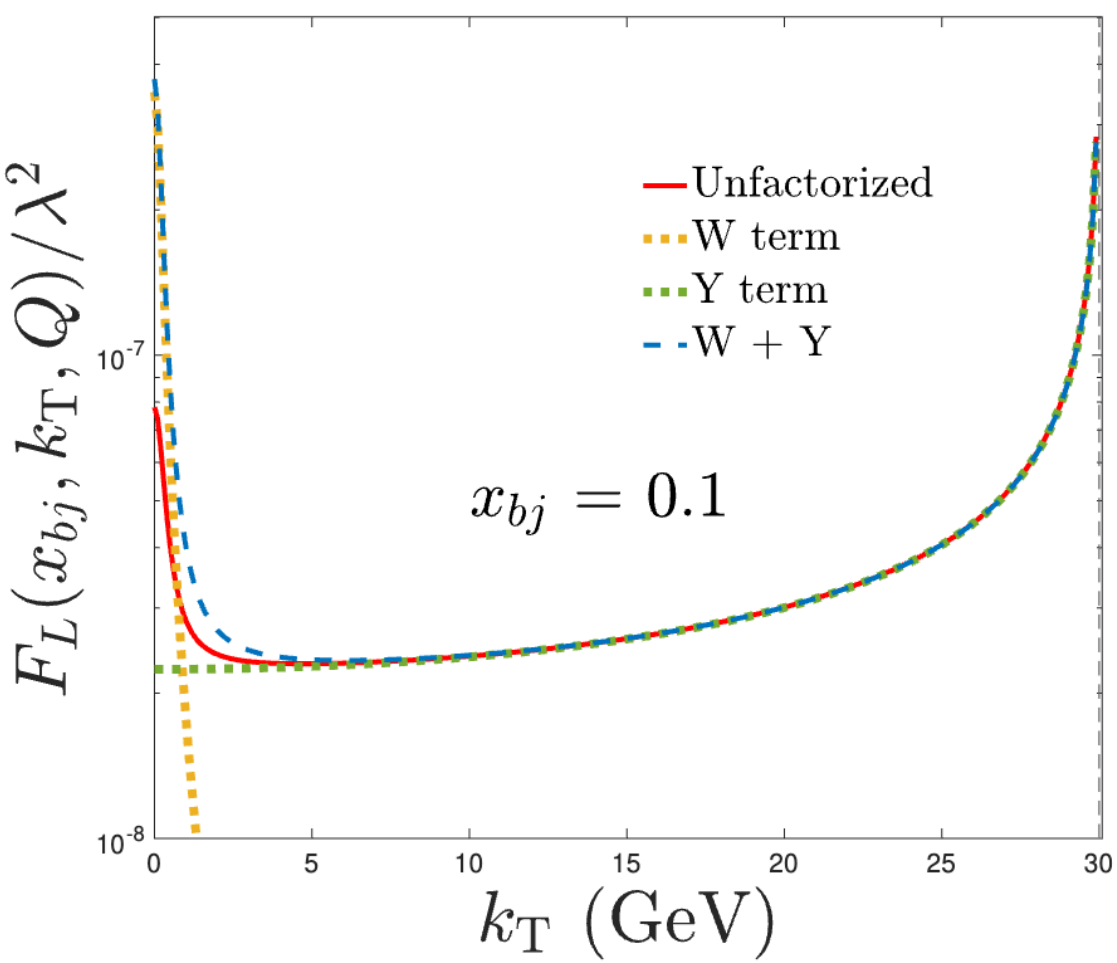
W + Y

Sometimes is ignored

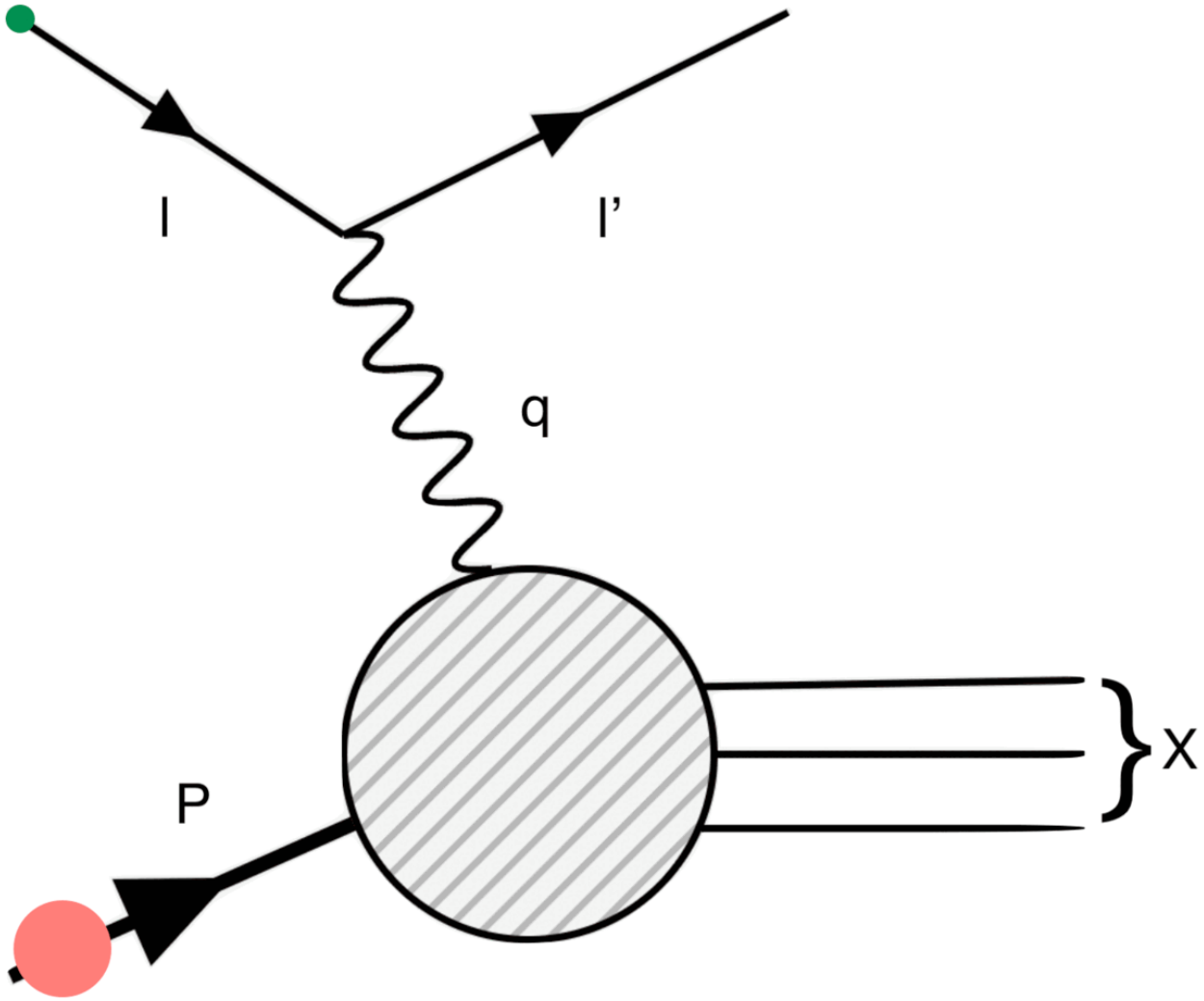
Both are necessary !!

W + Y for F_L


$x_{bj} = 0.65$
Unfactorized
W term
Y term
W + Y
$F_L(x_{bj}, k_T, Q)/\lambda^2$
k_T (GeV)


$x_{bj} = 0.1$
Unfactorized
W term
Y term
W + Y
$F_L(x_{bj}, k_T, Q)/\lambda^2$
k_T (GeV)

Typical scattering experiment

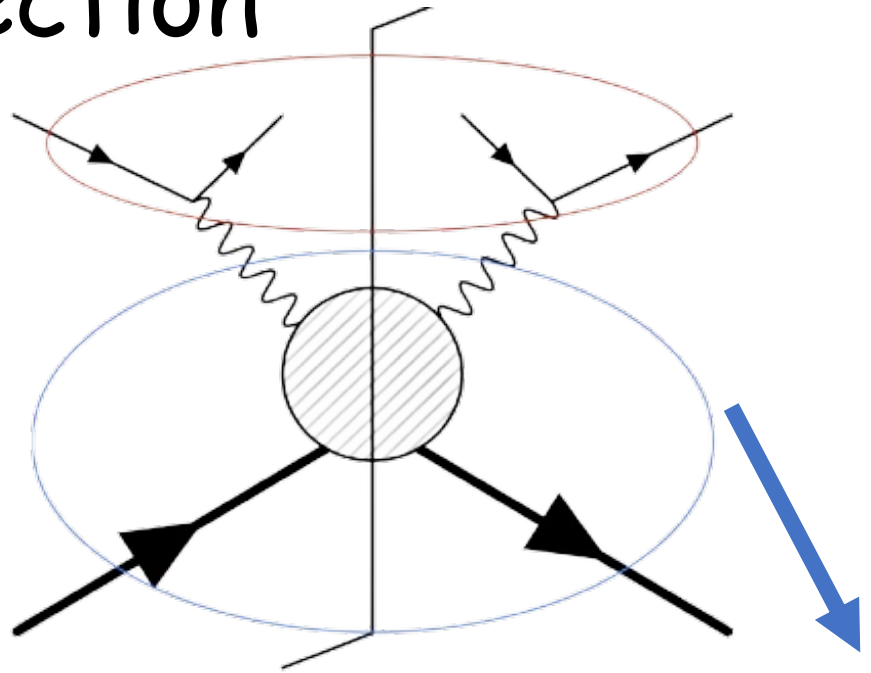


DIS

(Deep Inelastic Scattering)

DIS Cross Section

$$\frac{d\sigma_{\mathrm{DIS}}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\psi} \propto$$

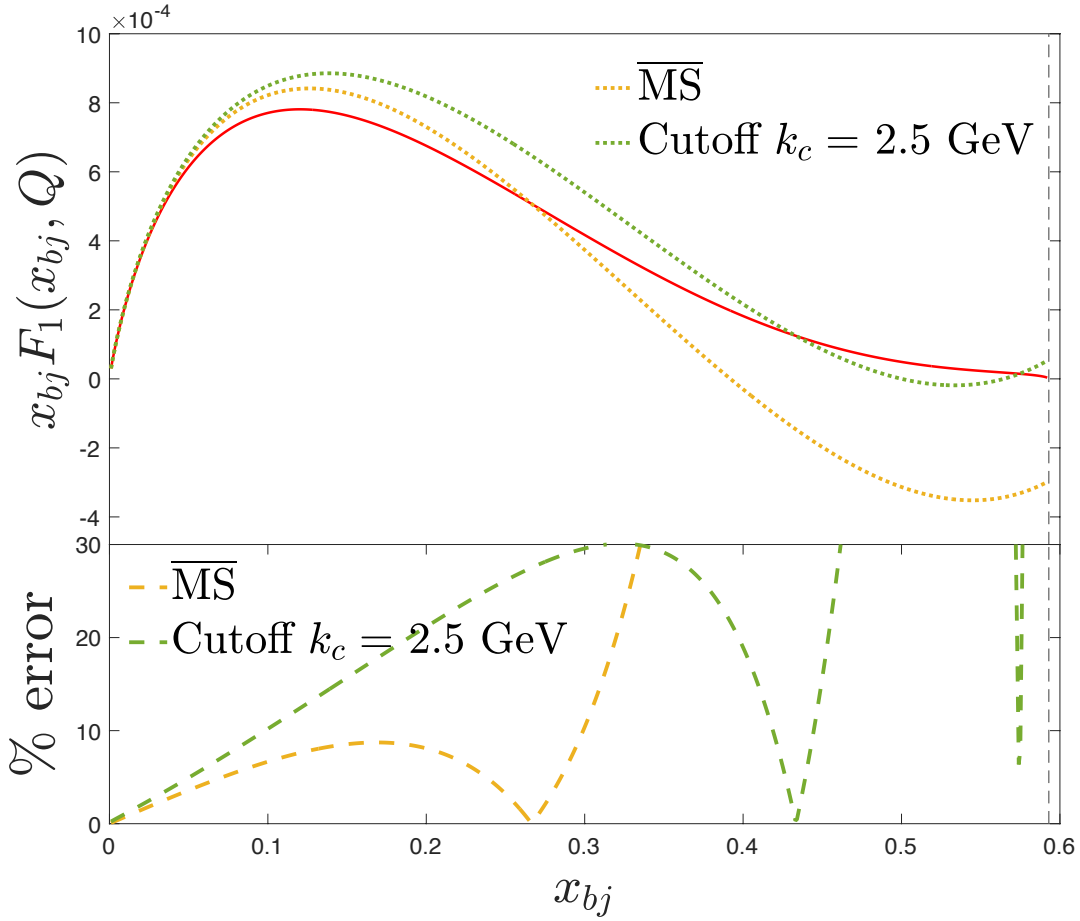


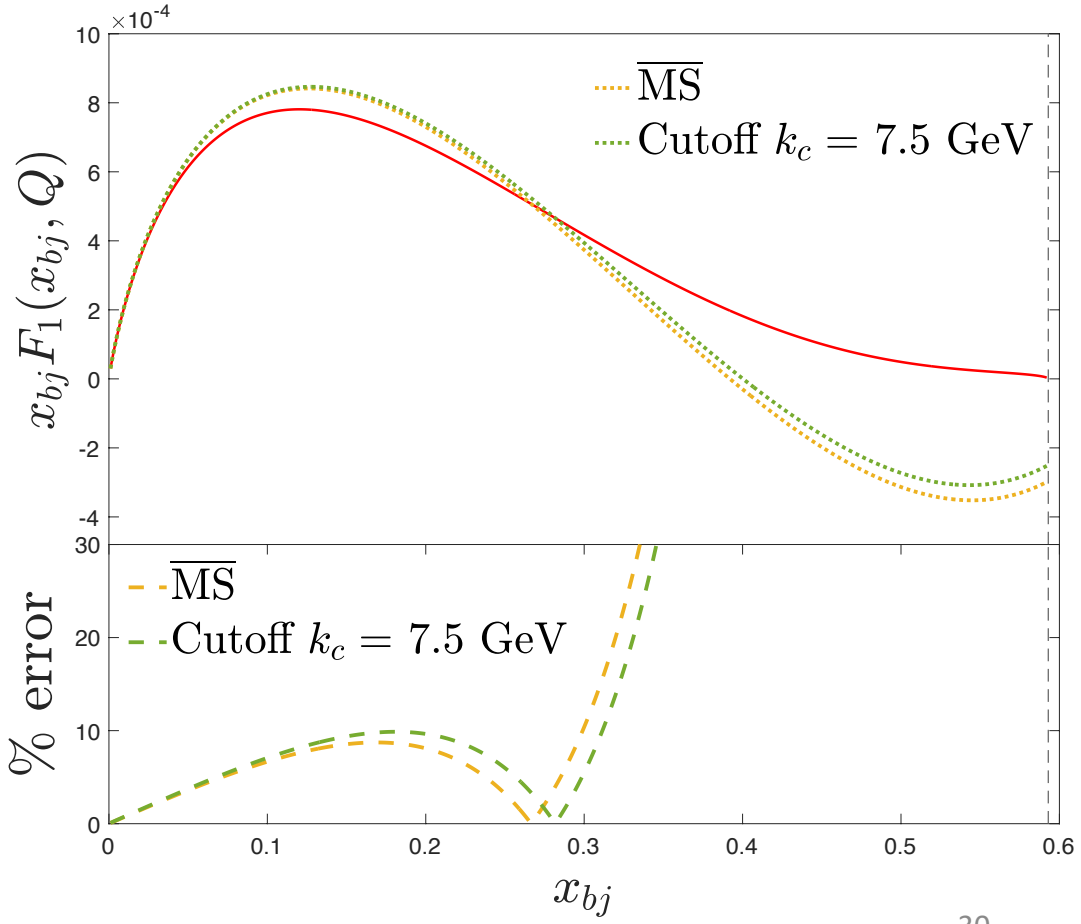
$$L^{\mu\nu} = 2(l_1^\mu l_2^\nu + l_1^\nu l_2^\mu - (l_1 \cdot l_2) g^{\mu\nu})$$

$$\begin{aligned} W^{\mu\nu} = & \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1 \\ & + \left(P^\mu - \frac{P\cdot q}{q^2} q^\mu \right) \left(P^\nu - \frac{P\cdot q}{q^2} q^\nu \right) \frac{F_2}{P\cdot q} \\ & + i\epsilon^{\mu\nu\alpha\beta} q^\alpha S^\beta \frac{g_1}{P\cdot q} \\ & + i\epsilon^{\mu\nu\alpha\beta} q^\alpha [(P\cdot q) S^\beta - (S\cdot q) P^\beta] \frac{g_2}{(P\cdot q)^2} \end{aligned}$$

The role of different definitions

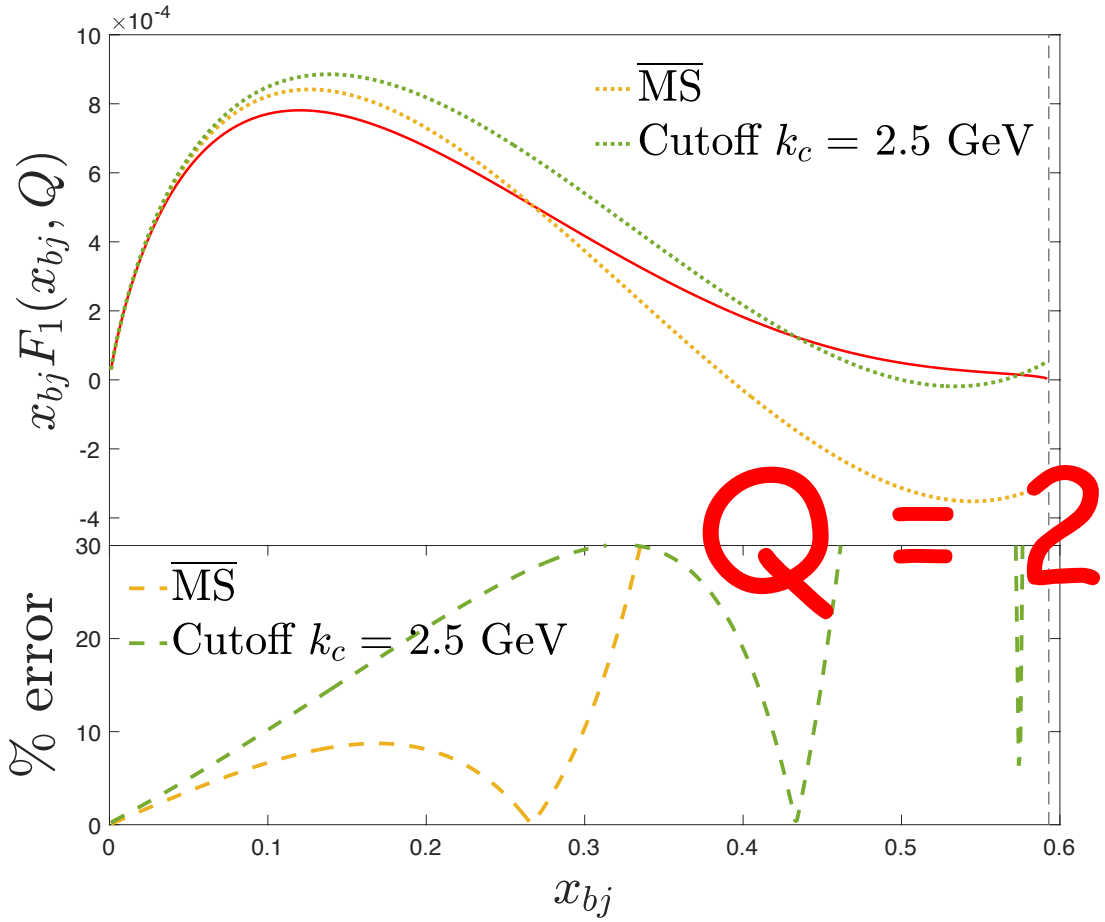
$$\delta F^{\text{Cutoff}}(k_c) < \delta F^{\overline{\text{MS}}}$$ Can happen !!



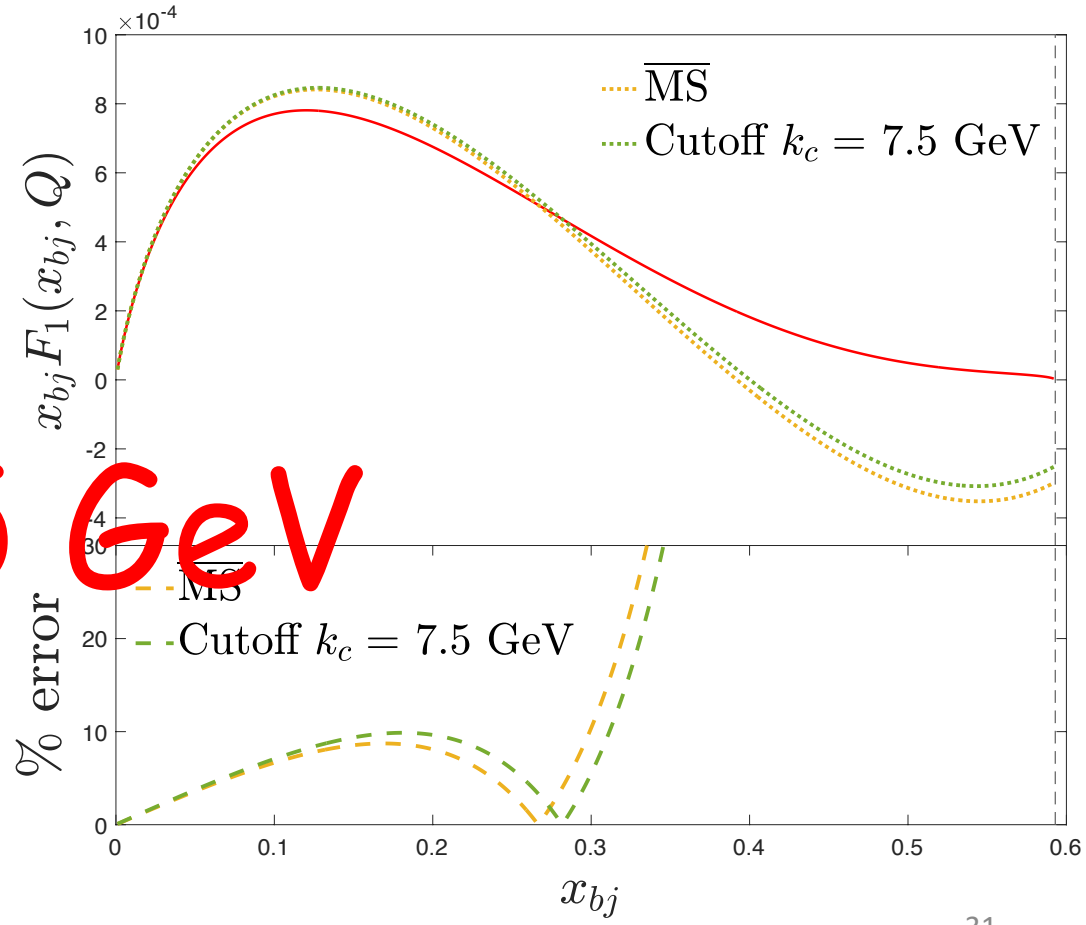


The role of different definitions

We can push factorization to smaller input scales



Q = 2.5 GeV



How do we get it?

For small b_T we can use collinear factorization because:

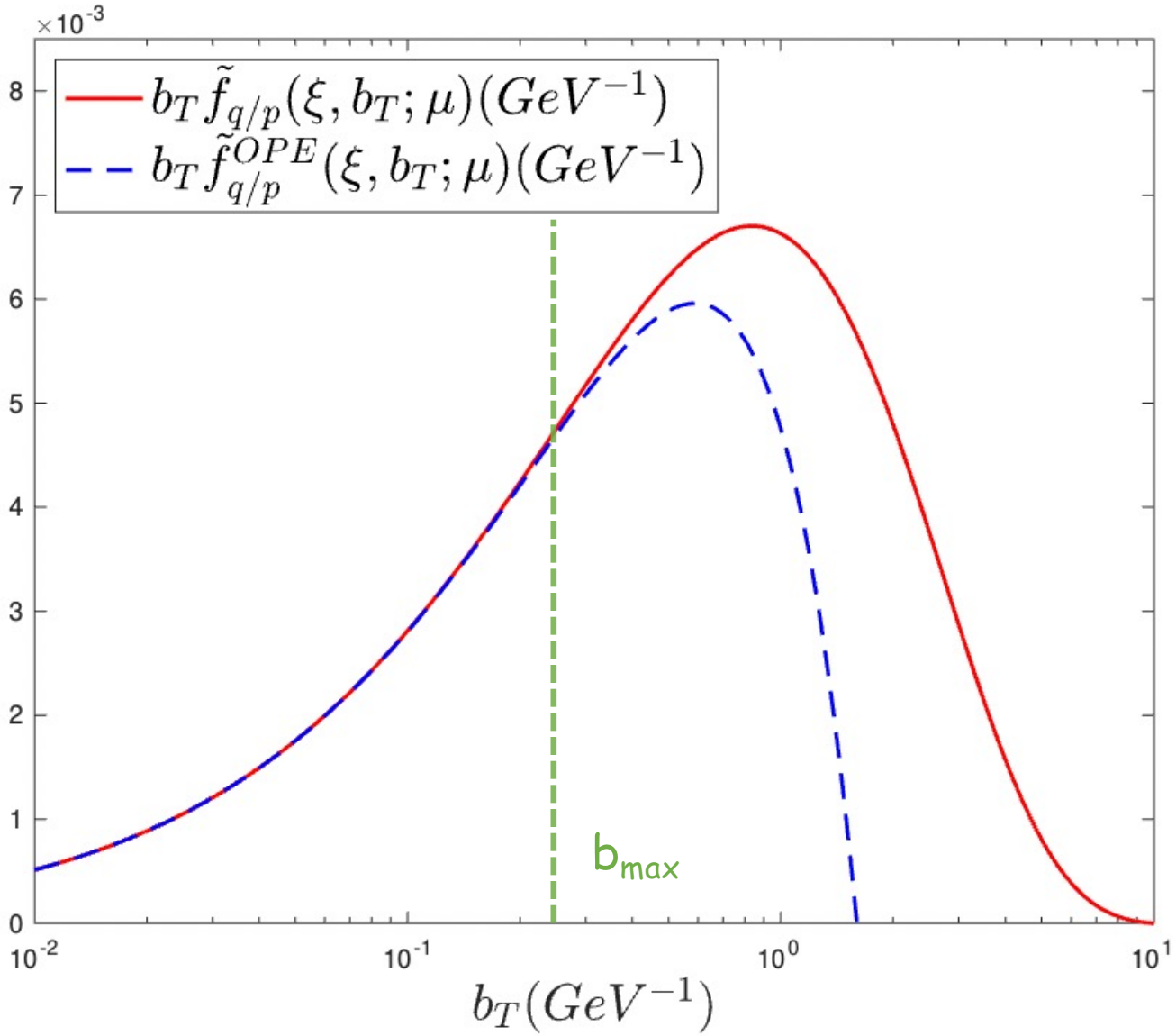
pQCD Experiments/pheno

$$\tilde{f}(x_{bj}, \boldsymbol{b}_T; \mu) = \tilde{C}(\xi, \boldsymbol{b}_T) \otimes \tilde{f}(\xi; \mu) + \mathcal{O}(m b_T)$$

$$\tilde{f}(x_{bj}, \boldsymbol{b}_T; \mu) = f(x_{bj}, \mu) - a_\lambda(\mu)(1 - x_{bj}) \ln\left(\frac{\mu^2 b_T^2 e^{2\gamma_E}}{4}\right) + \cdots + \mathcal{O}(m b_T)$$

How is it extended to all b_T ?
With the power of RG equations

OPE vs Unfactorized



Trick:

$$\tilde{f}(x_{bj}, \boldsymbol{b}_T; \mu) = \tilde{f}(x_{bj}, \boldsymbol{b}_*; \mu) \frac{\tilde{f}(x_{bj}, \boldsymbol{b}_T; \mu)}{\tilde{f}(x_{bj}, \boldsymbol{b}_*; \mu)}$$

$$= \tilde{f}(x_{bj}, \boldsymbol{b}_*; \mu) e^{-g(x_{bj}, \boldsymbol{b}_T)}$$

Non perturbative

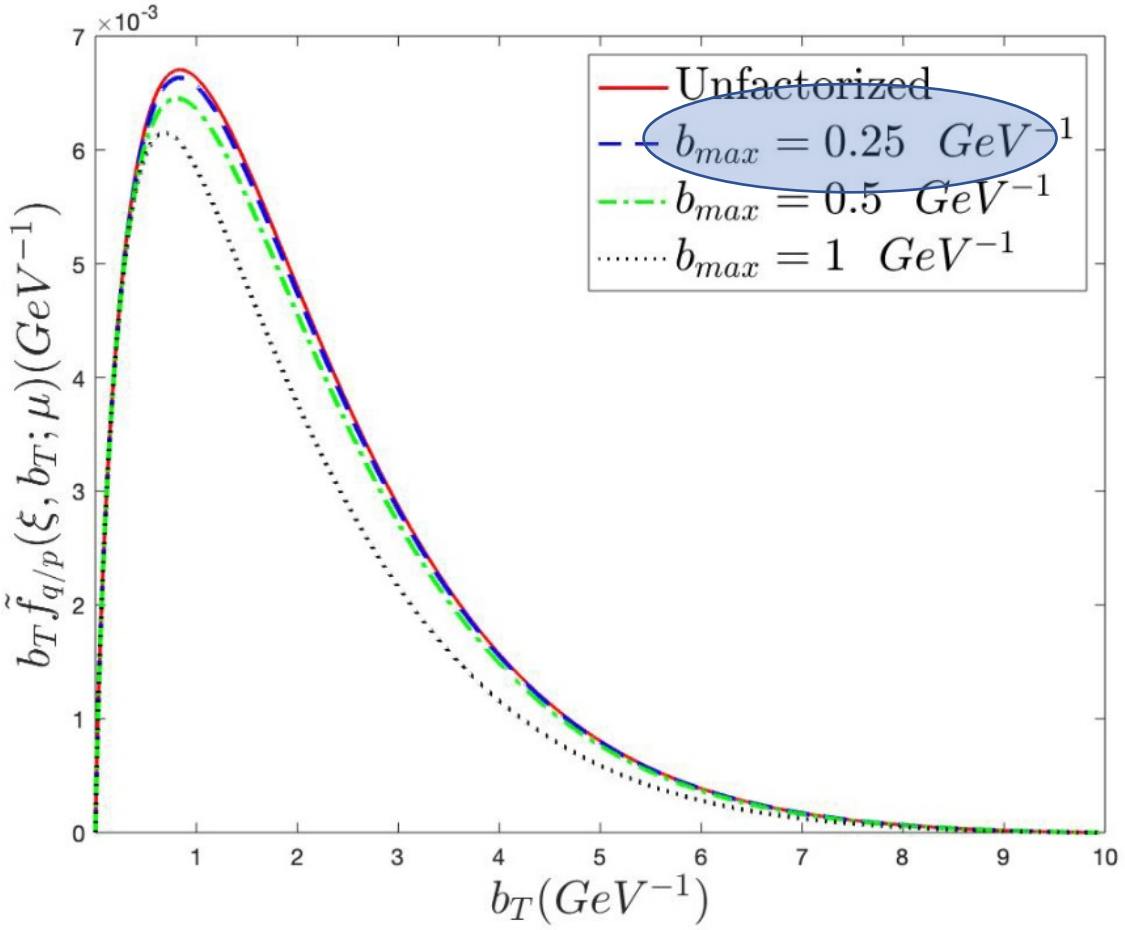
no μ dependence

OPE is valid

$$\boldsymbol{b}_*(\boldsymbol{b}_T) = \begin{cases} \boldsymbol{b}_T & b_T \ll b_{max} \\ b_{max} & b_T \gg b_{max} \end{cases}$$

Optimization of f and calibration of b_{max}

$$\tilde{f}^{Evol}(x_{bj}, \boldsymbol{b}_T; \mu = Q) = f(x_{bj}; \mu_{b_*}) \exp -2 \int_{\mu_{b_*}}^{Q} \frac{\mathrm{d}\mu}{\mu} \gamma_2(a_\lambda(\mu)) - g(x_{bj}, \boldsymbol{b}_T)$$



How large should it be?

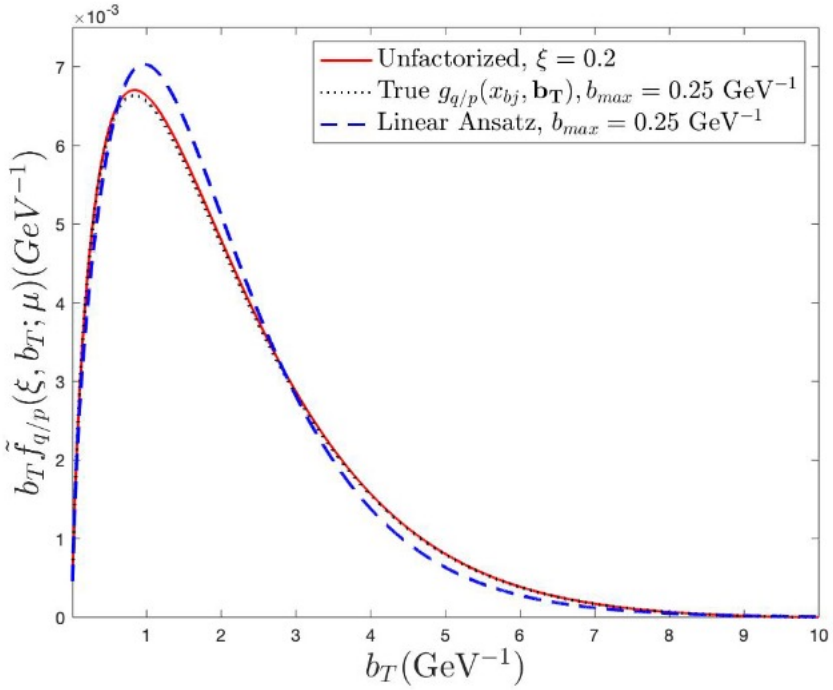
For sufficiently small b_{max}

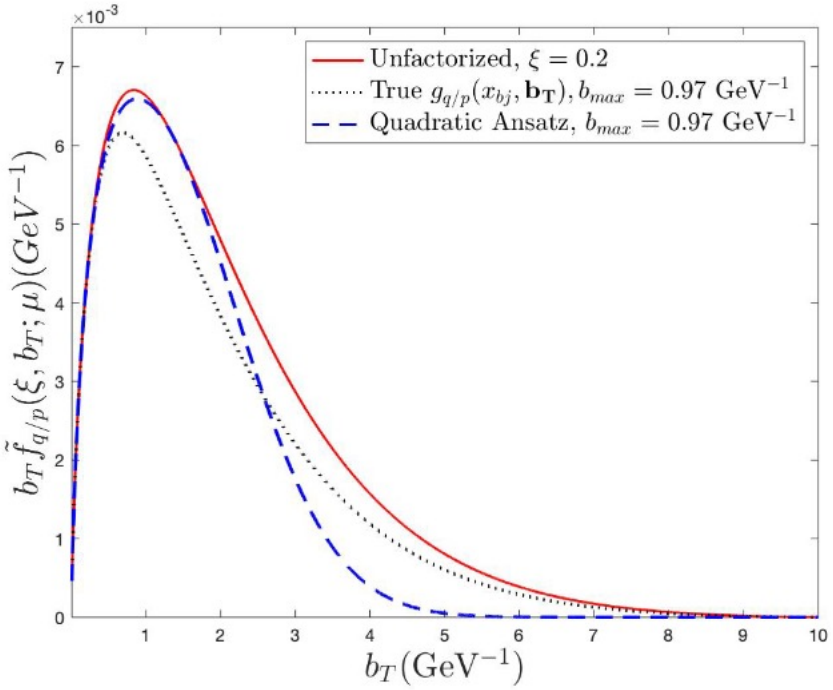
$$\frac{\mathrm{d}}{\mathrm{d}b_{max}} \tilde{f}^{\mathrm{Evol}}(x_{bj}, \boldsymbol{b}_T; \mu = Q) \to 0$$

It is often chosen too large

We use RG equation to optimize OPE and fit g functions

$$\tilde{f}^{Evol}(x_{bj}, \boldsymbol{b}_T; \mu = Q) = f(x_{bj}; \mu_{b_*}) \exp -2 \int_{\mu_{b_*}}^{Q} \frac{\mathrm{d}\mu}{\mu} \gamma_2(a_\lambda(\mu)) - g(x_{bj}, \boldsymbol{b}_T)$$





Parton model:

$$\pi \int_0^\infty \mathrm{d}k_T^2 f(x_{bj}, k_T) = f(x_{bj})$$

QCD and Yukawa:

$$\pi \int_0^\infty \mathrm{d}k_T^2 f(x_{bj}, k_T) = \infty$$

Same for b_T case

Recovering the parton model

$$\int_0^\infty \mathrm{d}^2\boldsymbol{k}_T \int_0^\infty \frac{\mathrm{d}^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{k}_T\cdot\boldsymbol{b}_T}\, \tilde{f}^{Evol}(x_{bj}, \boldsymbol{b}_c(\boldsymbol{b}_T, b_{min}); Q) = \tilde{f}^{Evol}(x_{bj}, b_{min}; Q)$$

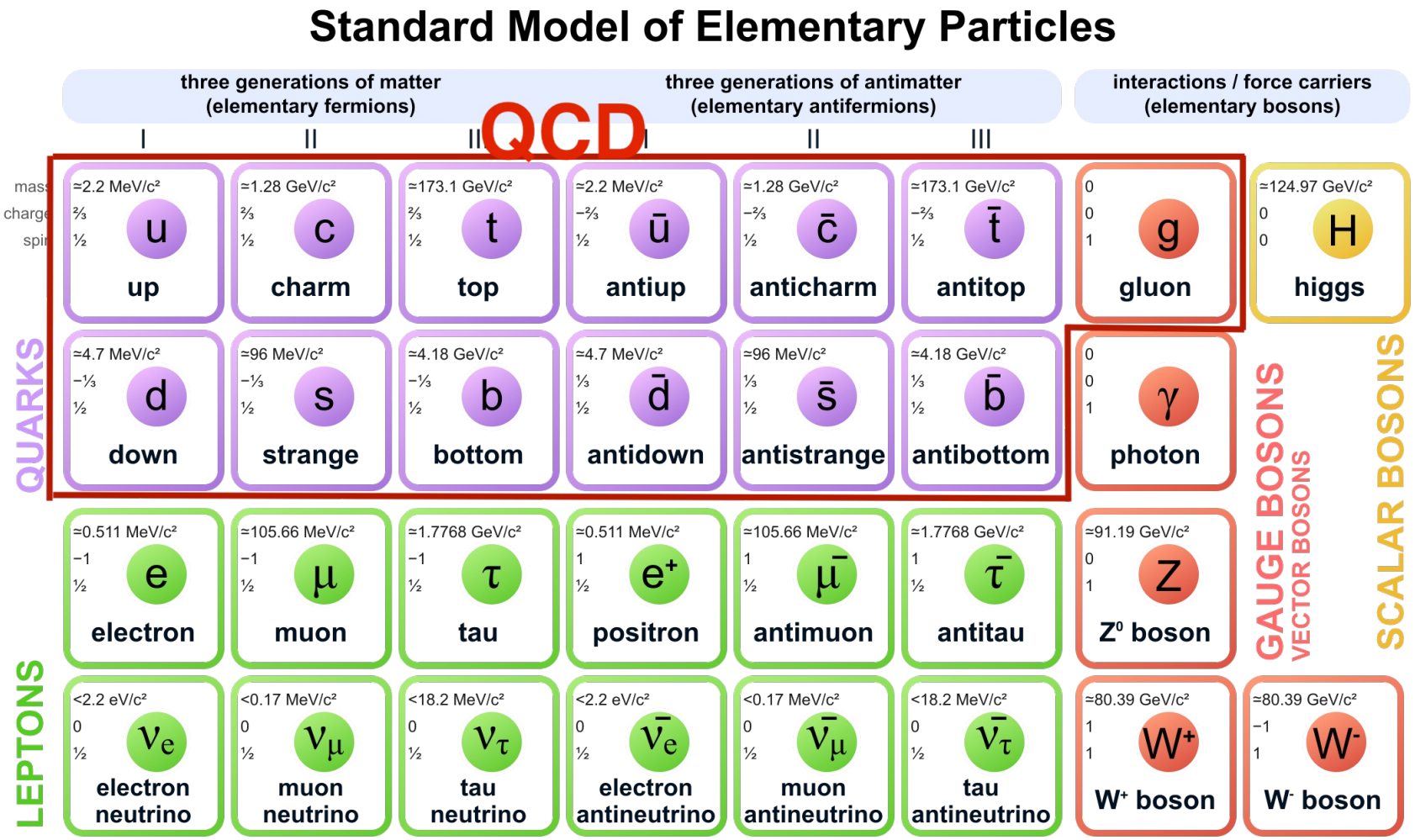
$$f(x_{bj}, b_0/b_*(b_{min})) \exp\left(-2\int_{b_0/b_*(b_{min})}^{Q} \frac{\mathrm{d}\mu}{\mu}\gamma_2(a_\lambda(\mu)) - g(x_{bj}, b_{min})\right)$$

$$f(x_{bj}, b_0/b_{min}) + \mathcal{O}\left(\frac{b_{min}}{b_{max}}, mb_{min}, mb_{max}, a_\lambda^2\right)$$

Collinear PDF

ONLY if
$b_{min}/b_{max} \ll 1$
or equivalently
$Q_0/Q \ll 1$

The constituents of the hadrons



$$\frac{d\sigma_{\mathrm{DIS}}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\psi} = \frac{2y\alpha_{\mathrm{em}}^2}{Q^2}\left[\left(F_1 + (1-y-\frac{\gamma^2 y^2}{4})\frac{F_2}{x_{bj}y^2}\right)\right.$$
$$\left. - \frac{2s_e x_{bj}}{yQ^2}\left[\frac{(y-2)}{(1+\gamma^2)}(q\cdot S)g_L - 2yg_T\left((S\cdot l') - \frac{(1-y)-\frac{\gamma^2}{2}y}{y(1+\gamma^2)}(q\cdot S)\right)\right]\right]$$