Collider physics with no PDFs

DIS2023 WG1

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  - More straightforward to compare to experimental data
- Physical basis previously discussed for example in Harland-Lang and Thorne 1811.08434, Hentschinski and Stratmann 1311.2825
- The novelty of our work:
  - Momentum space
  - Full three-flavor basis

Straightforward example with only two observables

Singlet and gluon approximation

$$(x, Q^2) = \sum_{q}^{n_f} \left[q(x, Q) + \overline{q}(x, Q^2) \right], \quad \text{where } n_f = 3$$

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$$(x, Q^2) = \sum_{q}^{n_\mathrm{f}} \left[q(x, Q) + \overline{q}(x, Q^2) \right], \quad \text{where } n_\mathrm{f} = 3$$

Express F_2 and F_L in terms of and gluon PDF:

$$F_2(x, Q^2) = x h e^2 i \quad (x, Q^2), \quad \text{where } h e^2 i \quad \frac{1}{n_\mathrm{f}} \sum_{q}^{n_\mathrm{f}} e_q^2$$

$$\frac{F_\mathrm{L}(x, Q^2)}{\frac{\alpha_\mathrm{s}(Q^2)}{2\pi}} = x h e^2 i \left[C_{F_\mathrm{L}} \qquad + n_\mathrm{f} C_{F_\mathrm{L} g} \quad g \right]$$

First non-zero order in α_s $\longrightarrow$ $F_2, \dfrac{F_\mathrm{L}}{\alpha_\mathrm{s}} \; / \; 1$

Straightforward example with only two observables

We need to invert this linear mapping

$$\begin{bmatrix} F_2 \\ F_\mathrm{L} / \frac{\alpha_\mathrm{s}}{2\pi} \end{bmatrix} = \begin{bmatrix} x h e^2 i & 0 \\ x h e^2 i C_{F_\mathrm{L}} & x h e^2 i n_\mathrm{f} C_{F_\mathrm{L} g} \end{bmatrix} \begin{bmatrix} \\ g \end{bmatrix}$$

Straightforward example with only two observables

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$$\begin{bmatrix} F_2 \\ F_L / \frac{\alpha_s}{2\pi} \end{bmatrix} = \begin{bmatrix} x he^2 i & 0 \\ x he^2 i C_{F_L} & x he^2 i n_f C_{F_L g} \end{bmatrix} \quad \begin{bmatrix} \\ g \end{bmatrix}$$

Singlet and gluon PDF in physical basis

$$(x, Q^2) = \frac{1}{he^2 i} 1 \quad \widetilde{F}_2$$

$$g(x, Q^2) = \int_x^1 \frac{dz}{z} \delta(1 \quad z) \left\{ \frac{C_F}{4 T_R n_f he^2 i} \left[\frac{x}{z} \frac{d}{d\frac{x}{z}} \quad 2 \right] \frac{F_2\left(\frac{x}{z}, Q^2\right)}{\frac{x}{z}} \right.$$

$$\left. + \frac{2\pi}{\alpha_s(Q^2)} \frac{1}{8 T_R n_f he^2 i} \left[\frac{x^2}{z^2} \frac{d^2}{d\left(\frac{x}{z}\right)^2} \quad 2\frac{x}{z} \frac{d}{d\frac{x}{z}} + 2 \right] \frac{F_L\left(\frac{x}{z}, Q^2\right)}{\frac{x}{z}} \right\}$$

$$\frac{1}{n_f he^2 i} \left\{ C_{g\widetilde{F}'_2} \quad \widetilde{F}'_2 + C_{g\widetilde{F}_2} \quad \widetilde{F}_2 + C_{g\widetilde{F}''_L} \quad \widetilde{F}''_L + C_{g\widetilde{F}'_L} \quad \widetilde{F}'_L + C_{g\widetilde{F}_L} \quad \widetilde{F}_L \right\}$$

Notation:

$$\widetilde{F}_2(x, Q^2) \quad \frac{F_2(x, Q^2)}{x}, \quad \widetilde{F}_L(x, Q^2) \quad \frac{2\pi}{\alpha_s(Q^2)} \frac{F_L(x, Q^2)}{x},$$

$$\widetilde{F}'_{2,L}(x, Q^2) \quad x \frac{d}{dx} \widetilde{F}_{2,L}(x, Q^2), \quad \widetilde{F}''_L(x, Q^2) \quad x^2 \frac{d^2}{dx^2} \widetilde{F}_L(x, Q^2)$$

Results for two observable basis

DGLAP equations of F_2 and F_L/α_s in physical basis:

$$\frac{\mathrm{d}F_2(x,Q^2)}{\mathrm{d}\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi}x\Bigg\{2P_{qg} \quad \Big[C_{g\widetilde{F^{00}}_L} \quad \widetilde{F^{00}}_L + C_{g\widetilde{F^{0}}_L} \quad \widetilde{F^{0}}_L + C_{g\widetilde{F}_L} \quad \widetilde{F}_L\Big]$$

$$+ P_{qq} \quad 1 \quad \widetilde{F}_2 + 2P_{qg} \quad \Big[C_{g\widetilde{F^{0}}_2} \quad \widetilde{F^{0}}_2 + C_{g\widetilde{F}_2} \quad \widetilde{F}_2\Big]\Bigg\}$$

$$\frac{\mathrm{d}}{\mathrm{d}\log(Q^2)}\left(\frac{F_L(x,Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}}\right) =$$

$$\frac{\alpha_s(Q^2)}{2\pi}x\Bigg\{\big[2C_{F_L} \quad P_{qg} + C_{F_L g} \quad P_{gg}\big] \quad \Big[C_{g\widetilde{F^{00}}_L} \quad \widetilde{F^{00}}_L + C_{g\widetilde{F^{0}}_L} \quad \widetilde{F^{0}}_L + C_{g\widetilde{F}_L} \quad \widetilde{F}_L\Big]$$

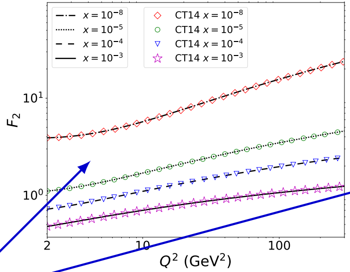
$$+ \Big[C_{F_L} \quad \Big(P_{qq} \quad 1 + 2P_{qg} \quad C_{g\widetilde{F}_2}\Big) + C_{F_L g} \quad \Big(n_f P_{gq} \quad 1 + P_{gg} \quad C_{g\widetilde{F}_2}\Big)\Big] \quad \widetilde{F}_2$$

$$+ \big[2C_{F_L} \quad P_{qg} + C_{F_L g} \quad P_{gg}\big] \quad C_{g\widetilde{F^{0}}_2} \quad \widetilde{F^{0}}_2\Bigg\}$$

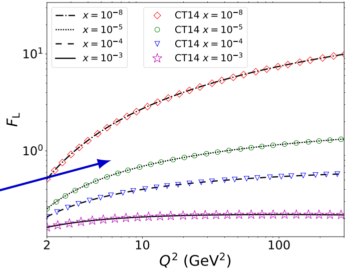
Scheme dependence of PDFs only at next order in α_s

! Should agree with evolution in terms of PDFs

Comparison with conventional DGLAP evolution

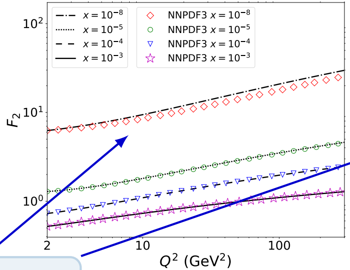


F_2 CT14 LO

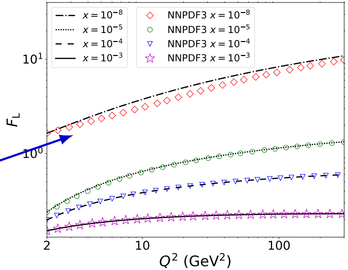


F_L CT14 LO

Nice match



F_2 NNPDF3.0 LO



F_L NNPDF3.0 LO

Small discrepancy after $x \lesssim 10^{-6}$

Six observable basis

- Full three-flavor basis: $u, u, d, d, s = s$, and g
  - *!* Need six linearly independent DIS structure functions

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- Full three-flavor basis: $u, u, d, d, s = s$, and g
  - ! Need six linearly independent DIS structure functions
- We choose structure functions: (again in first non-zero order in α_s)

Neutral current

$$F_2 = x \sum_q^{n_f} e_q^2 (q + \overline{q})$$

$$F_3 = 2 \sum_q^{n_f} (L_q^2 \quad R_q^2)(q \quad \overline{q}),$$

$$\text{where } L_q = T_q^3 \quad 2e_q \sin^2\theta_W$$

$$\text{and } R_q = \quad 2e_q \sin^2\theta_W$$

$$\frac{F_L(x, Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}} = x\Big[C_{F_L \widetilde{F}_2} \quad \sum_q^{n_f} e_q^2 (q + \overline{q})$$

$$+ \sum_q^{n_f} e_q^2 C_{F_L g} \quad g\Big]$$

Charged current

$$F_2^W = 2x(u + \overline{d} + \overline{s})$$

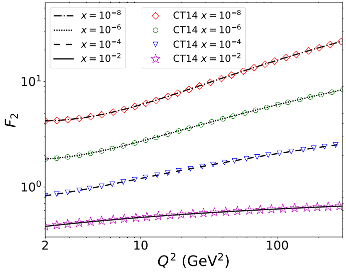
$$F_3^W = 2(u \quad \overline{d} \quad \overline{s})$$

$$F_{2c}^W = 2x\overline{s}$$

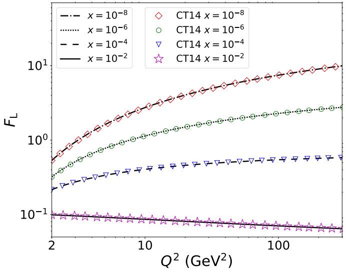
Derive DGLAP equations for these

Again, should agree with conventional DGLAP evolution

Comparison with conventional DGLAP evolution

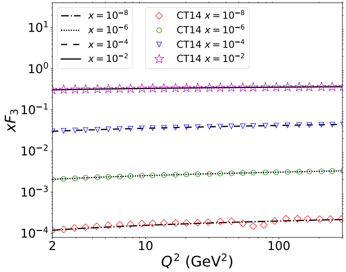


F_2 CT14 LO

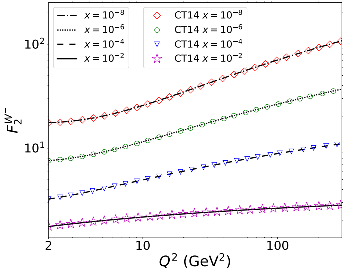


F_L CT14 LO

Match in full grid



xF_3 CT14 LO



F_2^W CT14 LO

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- We have implemented these equations numerically – agree with DGLAP-evolved PDFs
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- We have established physical basis in first non-zero order in α_s for six observables; F_2, F_L, F_3, F_2^W , F_3^W , and F_{2c}^W
- We have implemented these equations numerically – agree with DGLAP-evolved PDFs
- Scheme dependence of PDFs starts to play part at NLO in α_s
  ! By using a physical basis in second non-zero order in α_s, we will be able to avoid scheme dependence
- What next:
  - Establish physical basis at NLO
  - Study how LHC cross sections, e.g. Drell-Yan, are expressed in physical basis
  - Obtain physical basis including also heavy quarks

Backup: Inverting the gluon PDF

Gluon PDF in mellin space

$$g(n) = \frac{1}{n_f C_{F_L g}(n)} \left[\frac{1}{h e^2 i} \widetilde{F}_L(n) \quad C_{F_L} \quad (n) \quad (n) \right]$$

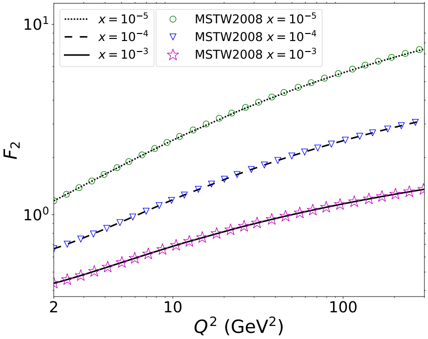
$$\frac{1}{C_{F_L g}(n)} = \frac{1}{8 T_R z_0^n} \int_0^1 \mathrm{d}z z^{n+2} \delta''(z \quad z_0),$$

where z_0 $2]0,1[$ is an arbitrary constant that cancels in final result.

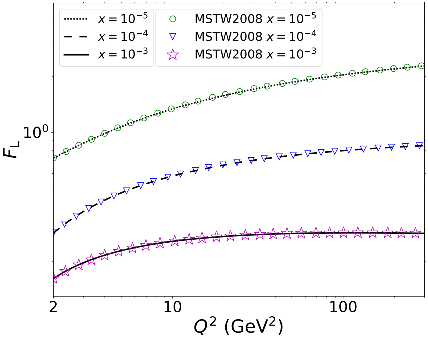
$$g(x, Q^2) = \int_x^1 \frac{\mathrm{d}z}{z} \delta(1 \quad z) \left\{ \frac{C_F}{4 T_R n_f h e^2 i} \left[\frac{x}{z} \frac{\mathrm{d}}{\mathrm{d}\frac{x}{z}} \quad 2 \right] \frac{F_2\left(\frac{x}{z}, Q^2\right)}{\frac{x}{z}} \right.$$
$$\left. + \frac{2\pi}{\alpha_s(Q^2)} \frac{1}{8 T_R n_f h e^2 i} \left[\frac{x^2}{z^2} \frac{\mathrm{d}^2}{\mathrm{d}\left(\frac{x}{z}\right)^2} \quad 2 \frac{x}{z} \frac{\mathrm{d}}{\mathrm{d}\frac{x}{z}} + 2 \right] \frac{F_L\left(\frac{x}{z}, Q^2\right)}{\frac{x}{z}} \right\}$$
$$\frac{1}{n_f h e^2 i} \left\{ C_{g\widetilde{F^0}_2} \quad \widetilde{F^0}_2 + C_{g\widetilde{F}_2} \quad \widetilde{F}_2 + C_{g\widetilde{F^{00}}_L} \quad \widetilde{F^{00}}_L + C_{g\widetilde{F^0}_L} \quad \widetilde{F^0}_L + C_{g\widetilde{F}_L} \quad \widetilde{F}_L \right\}$$

Backup: Comparison with MSTW2008 PDF set

DGLAP evolution for F_2 and F_L in two observable physical basis



F_2 MSTW LO



F_L MSTW LO

Backup: PDFs in six observable physical basis

$$\begin{bmatrix} F_2 \\ F_3 \\ F_2^{\mathrm{W}} \\ F_3^{\mathrm{W}} \\ F_{2c}^{\mathrm{W}} \end{bmatrix} = \begin{bmatrix} xe_d^2 & xe_d^2 & xe_u^2 & xe_u^2 & 2xe_s^2 \\ 2A_d & 2A_d & 2A_u & 2A_u & 0 \\ 0 & 2x & 2x & 0 & 2x \\ 0 & 2 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2x \end{bmatrix} \begin{bmatrix} d \\ \overline{d} \\ u \\ \overline{u} \\ \overline{s} \end{bmatrix},$$

where we have defined $A_q \quad L_q^2 \quad R_q^2$ in order to simplify equations.

$$xd(x,Q^2) = \frac{1}{A_u e_d^2 + A_d e_u^2}\Bigg[A_u F_2(x,Q^2) + \frac{e_u^2}{2} xF_3(x,Q^2) \quad \frac{A_u(2e_u^2 + e_d^2) \quad A_d e_u^2}{4} F_2^{\mathrm{W}} \quad (x,Q^2)$$

$$\frac{A_u(2e_u^2 \quad e_d^2) + A_d e_u^2}{4} xF_3^{\mathrm{W}} \quad (x,Q^2) + \frac{A_u(e_d^2 \quad 2e_s^2) \quad A_d e_u^2}{2} F_{2c}^{\mathrm{W}} \quad (x,Q^2)\Bigg]$$

$$x\overline{d}(x,Q^2) = \frac{1}{4} F_2^{\mathrm{W}} \quad (x,Q^2) \quad \frac{1}{4} xF_3^{\mathrm{W}} \quad (x,Q^2) \quad \frac{1}{2} F_{2c}^{\mathrm{W}}$$

$$xu(x,Q^2) = \frac{1}{4} F_2^{\mathrm{W}} \quad (x,Q^2) + \frac{1}{4} xF_3^{\mathrm{W}} \quad (x,Q^2)$$

$$x\overline{u}(x,Q^2) = \frac{1}{A_u e_d^2 + A_d e_u^2}\Bigg[A_d F_2(x,Q^2) \quad \frac{e_d^2}{2} xF_3(x,Q^2) \quad \frac{A_d(2e_d^2 + e_u^2) \quad A_u e_d^2}{4} F_2^{\mathrm{W}} \quad (x,Q^2)$$

$$+ \frac{A_d(2e_d^2 \quad e_u^2) + A_u e_d^2}{4} xF_3^{\mathrm{W}} \quad (x,Q^2) + A_d(e_d^2 \quad e_s^2) F_{2c}^{\mathrm{W}} \quad (x,Q^2)\Bigg]$$

$$x\overline{s}(x,Q^2) = xs(x,Q^2) = \frac{1}{2} F_{2c}^{\mathrm{W}} \quad (x,Q^2)$$

Backup: Results in six observable basis

$$\frac{\mathrm{d}F_2(x,Q^2)}{\mathrm{d}\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi}\left[P_{qq} \quad \widetilde{F}_2 + 2\sum_q^{n_f} e_q^2 P_{qg} \quad g\right], \quad \text{where } \widetilde{F}_2(x,Q^2) \quad \frac{F_2(x,Q^2)}{x}$$

$$\frac{\mathrm{d}xF_3(x,Q^2)}{\mathrm{d}\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x P_{qq} \quad F_3$$

$$\frac{\mathrm{d}F_2^{W} \quad (x,Q^2)}{\mathrm{d}\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x\left[P_{qq} \quad \widetilde{F}_2^{W} \quad + 6P_{qg} \quad g\right], \quad \text{where } \widetilde{F}_2^{W} \quad (x,Q^2) \quad \frac{F_2^{W} \quad (x,Q^2)}{x}$$

$$\frac{\mathrm{d}xF_3^{W} \quad (x,Q^2)}{\mathrm{d}\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x\left[P_{qq} \quad F_3^{W} \quad\quad 2P_{qg} \quad g\right]$$

$$\frac{\mathrm{d}F_{2c}^{W} \quad (x,Q^2)}{\mathrm{d}\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x\left[P_{qq} \quad \widetilde{F}_{2c}^{W} \quad + 2P_{qg} \quad g\right], \quad \text{where } \widetilde{F}_{2c}^{W} \quad (x,Q^2) \quad \frac{F_{2c}^{W} \quad (x,Q^2)}{x}$$

$$P_{qg} \quad g = \frac{C_F}{4\sum_q^{n_f} e_q^2}\left[\quad \widetilde{F}_2 + 2(z \quad 1) \quad \widetilde{F}_2\right] + \frac{1}{4\sum_q^{n_f} e_q^2}\left[\left(\frac{1}{x} \quad \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}\right) F_L(x,Q^2) + 1 \quad \widetilde{F}_L\right]$$

Backup: Results in six observable basis

$$
\frac{d}{d\log(Q^2)}\left(\frac{F_L(x,Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}}\right) =
$$

$$
\frac{\alpha_s(Q^2)}{2\pi} x \Bigg\{ \left[2C_{F_L\widetilde{F}_2} \quad P_{qg} + C_{F_L g} \quad P_{gg}\right] \quad \left[C_{g\widetilde{F^{00}}_L} \quad \widetilde{F^{00}}_L + C_{g\widetilde{F^{0}}_L} \quad \widetilde{F^{0}}_L + C_{g\widetilde{F}_L} \quad \widetilde{F}_L\right]
$$

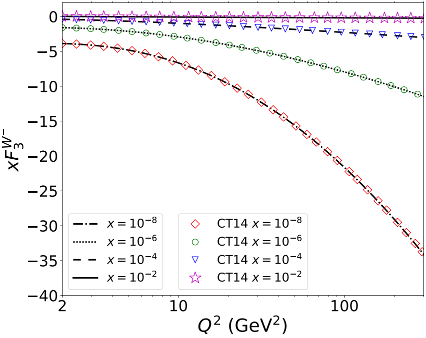
$$
+ \left[C_{F_L\widetilde{F}_2} \quad \left(P_{qq} + 2P_{qg} \quad C_{g\widetilde{F}_2}\right) + C_{F_L g} \quad P_{gg} \quad C_{g\widetilde{F}_2}\right] \quad \widetilde{F}_2
$$

$$
+ \left[2C_{F_L\widetilde{F}_2} \quad P_{qg} + C_{F_L g} \quad P_{gg}\right] \quad C_{g\widetilde{F^{0}}_2} \quad \widetilde{F^{0}}_2 + \sum_{q}^{n_f} e_q^2 C_{F_L g} \quad P_{gq} \quad \sum_q (q + \overline{q}) \Bigg\}
$$

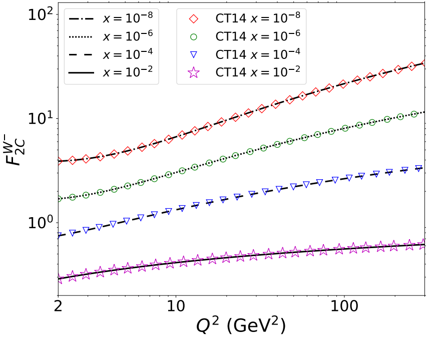
where $\sum_q (q + \overline{q})$ can be expressed in physical basis (see slide 11/9)

Backup: Comparison with conventional DGLAP evolution

Comparison for structure functions $F_3^{\rm W}$ and $F_{2c}^{\rm W}$ in six observable physical basis



$F_3^{\rm W}$ CT14 LO



$F_{2c}^{\rm W}$ CT14LO