

Towards NNPDFs @ aN3LO with MHOU

Giacomo Magni,
Nikhef Theory Group and VU Amsterdam

DIS2023,
Michigan State University, East Lansing
28 March 2022

In collaboration with:

R.D.Ball, A.Barontini, A.Candido, S.Carrazza, J.Cruz-Martinez, L.D.Debbio, S.Forte,
T.Giani, F.Hekhorn, Z.Kassabov, N.Laurenti, E.R.Nocera, J.Rojo, T.Rabemananjara, C.Schwan, R.Stegeman, M.Ubiali.

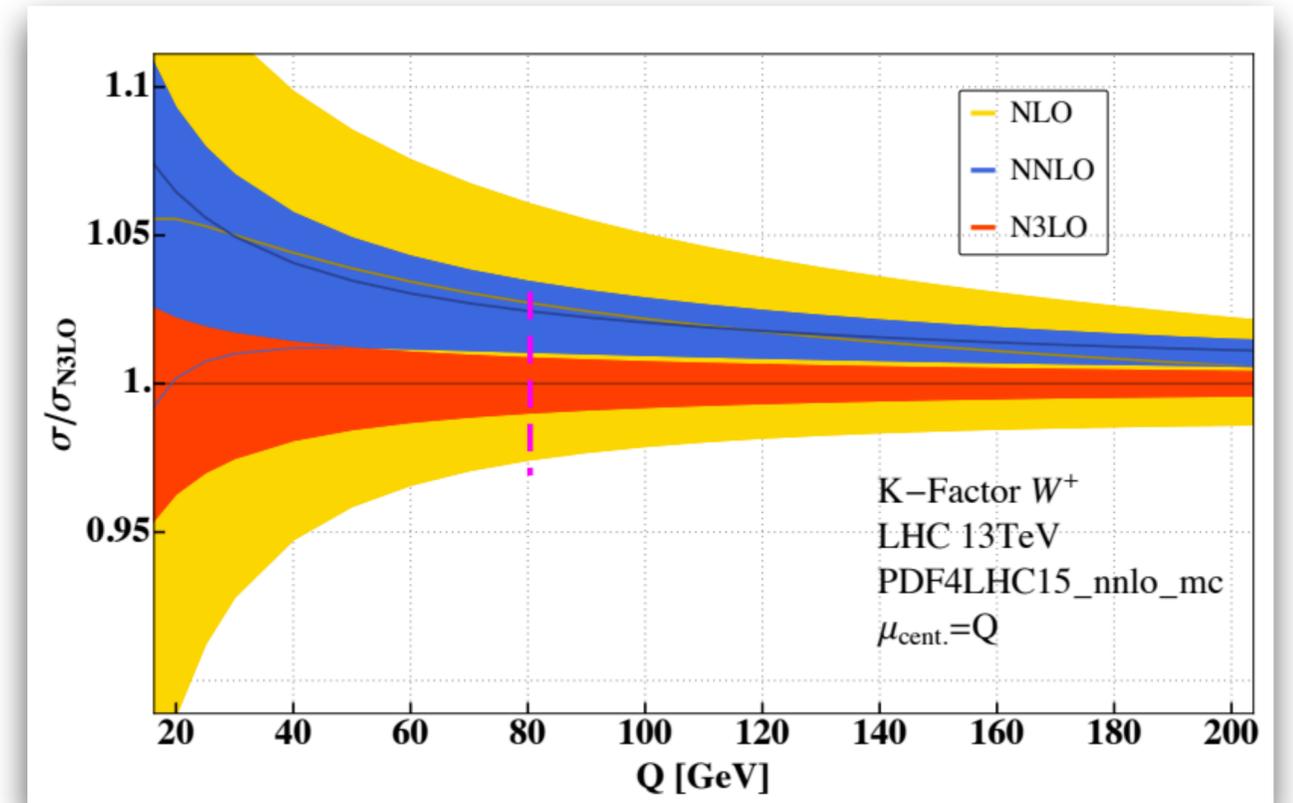


Introduction & Motivations

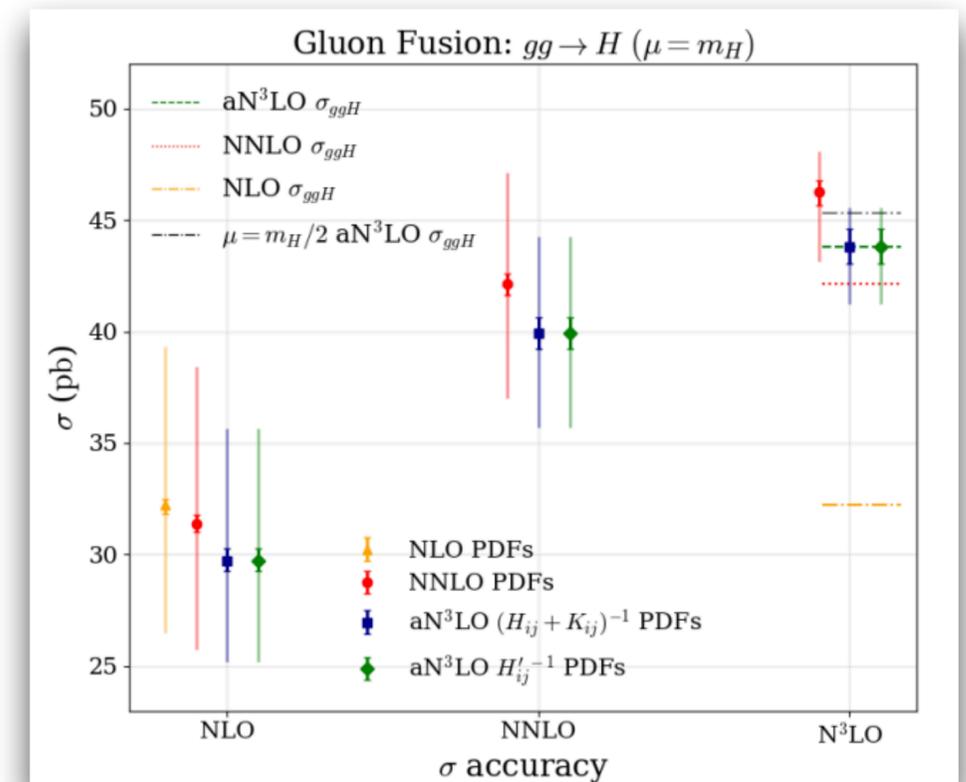
$$\sigma(x, Q^2) = \sum_i \int_x^1 \frac{dz}{z} f_i(z, \mu^2) \hat{\sigma}\left(\frac{x}{z}, \frac{Q^2}{\mu^2}, \alpha_s\right) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

- ▶ Predictions for LHC observables relies on two main ingredients: PDFs and partonic Matrix Elements.
- ▶ To reach higher accuracy we need to compute radiative corrections, of which QCD ones are generally the largest contribution.
- ▶ In the last years many LHC processes have been calculated up to QCD at N3LO:
 - $gg \rightarrow H$ Anastasiou, Duhr, Dulat, Herzog, Mistlberger [\[arxiv:1503.06056\]](#)
 - $qq \rightarrow H$ (VBF) Dreyer, Karlberg [\[arxiv:1606.00840\]](#); Duhr, Dulat, Mistlberger [\[arxiv:1904.09990\]](#); Duhr, Dulat, Hirschi, Mistlberger [\[arxiv:2004.04752\]](#)
 - $pp \rightarrow W^\pm$ Duhr, Dulat, Mistlberger [\[arxiv:2007.13313\]](#); Chen, Gehrmann, Glover, Huss, Yang, Xing Zhu [\[arxiv:2205.11426\]](#)
 - $pp \rightarrow Z/\gamma, pp \rightarrow VH$ Baglio, Duhr, Mistlberger, Szafrond [\[arxiv:2209.06138\]](#); Chen, Gehrmann, Glover, Huss, Yang, Xing Zhu [\[arxiv:2107.09085\]](#) Neumann, Campbell [\[arxiv:2207.07056\]](#)

From Duhr, Dulat, Mistlberger [\[arxiv:2007.13313\]](#)

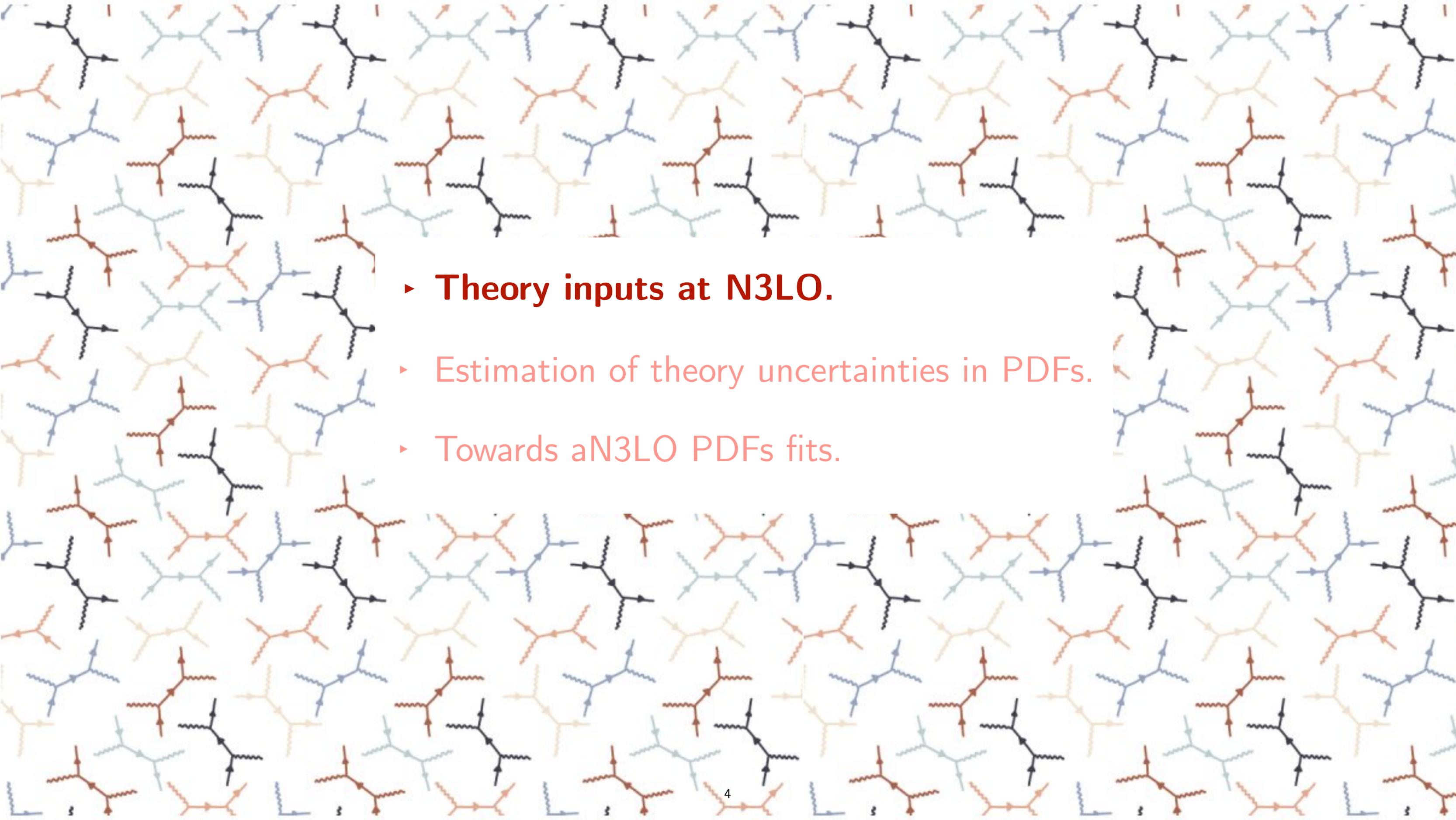


From Mc Gowan, Cridge, Harland-Lang, Thorne [\[arxiv:2207.04739\]](#)



How can we improve theory predictions in NNPDF4.0?

- ➔ NNPDF4.0 PDFs are still at NNLO accuracy in QCD, need to go **aN3LO**.
- ➔ Inclusion of **theory uncertainties** while determining PDFs is relevant at this level of accuracy.

- 
- The background of the slide is a repeating pattern of various Feynman diagrams. These diagrams are rendered in several colors: black, blue, orange, and light green. They represent different types of particle interactions, including vertices, propagators, and external lines, typical of quantum field theory calculations.
- ▶ **Theory inputs at N3LO.**
 - ▶ Estimation of theory uncertainties in PDFs.
 - ▶ Towards aN3LO PDFs fits.

PDFs determination @ aN3LO

Several theoretical inputs are needed in a PDF fit:

- ▶ The main ingredient are the QCD **splitting functions** which controls the DGLAP evolution.
- ▶ **VFNS matching conditions** for each running component.
- ▶ **DIS partonic coefficients** functions, accounting for massive corrections when possible.
- ▶ **Hadronic coefficients**: which can be included mainly through *k-factors*.

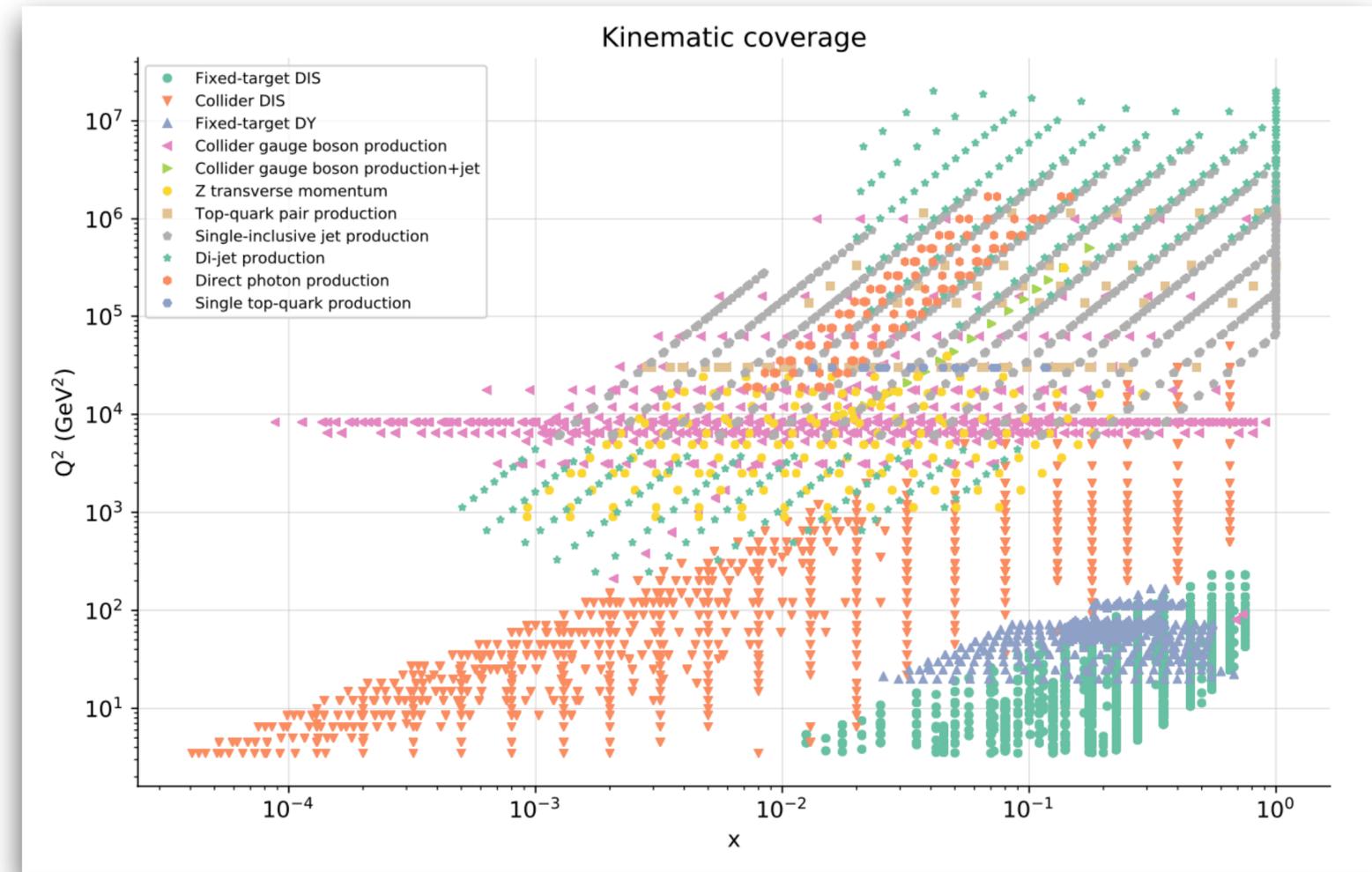
Not all of them are yet available at N3LO



- ➔ Construct reliable approximations from existing calculations.
- ➔ Determine theory uncertainties both from:

***Incomplete Higher Order corrections
IHOU***

***Missing Higher Order corrections
MHOU***



PDF evolution @ aN3LO

Splitting functions

Analytical calculations of the complete N3LO splitting functions are not available yet.

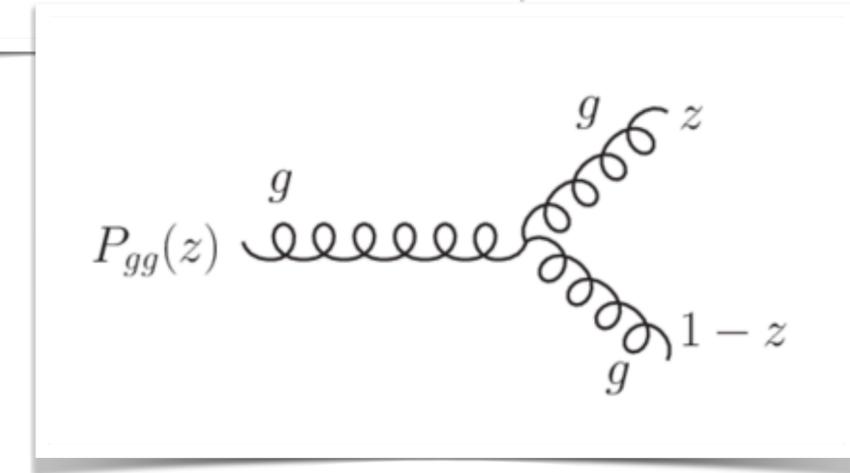
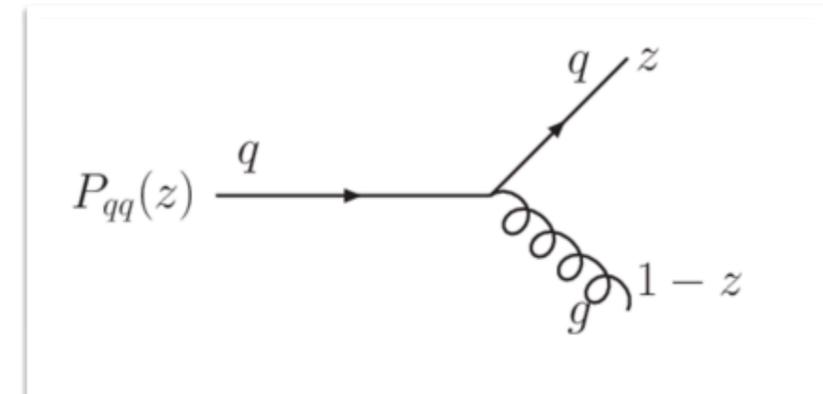
- ▶ The **Non Singlet** splitting functions can be estimated with quite precise accuracy for phenomenological studies:

Moch, Ruijl, Ueda, Vermaseren, Vogt [\[arXiv:1707.08315\]](#); Davies, Vogt, Ruijl, Ueda, Vermaseren [\[arXiv:1610.07477\]](#); Davies, Kom, Moch, Vogt [\[arXiv:2202.10362\]](#).

- ▶ The **Singlet** splitting functions are way more challenging and can be determined only with a finite accuracy.

Main ingredients used during the approximations are:

- ▶ **Large- n_f limit:** Davies, Vogt, Ruijl, Ueda, and Vermaseren. [\[arXiv:1610.07477\]](#)
- ▶ **Small- x limit:** Bonvini and Marzani [\[arXiv:1805.06460\]](#); Davies, Kom, Moch, Vogt. [\[arXiv:2202.10362\]](#)
- ▶ **Large- x limit:** Duhr, Mistlberger, Vita [\[arXiv:2205.04493\]](#); Henn, Korchemsky, Mistlberger [\[arXiv:1911.10174\]](#); Soar, Moch, Vermaseren, Vogt [\[arXiv:0912.0369\]](#).
- ▶ **Mellin Moments:** Moch, Ruijl, Ueda, Vermaseren, and Vogt [\[arXiv:2111.15561\]](#); Falcioni, Herzog, Loch, Vogt [\[arXiv:2302.07593\]](#)



N3LO Singlet splitting functions dependency on active flavors

	n_f^0	n_f^1	n_f^2	n_f^3
$\gamma_{gg}^{(3)}$	✓	✓	✓	✓
$\gamma_{gq}^{(3)}$	✓	✓	✓	✓
$\gamma_{qq}^{(3)}$		✓	✓	✓
$\gamma_{qq,ps}^{(3)}$		✓	✓	✓

PDF evolution @ aN3LO

Singlet splitting functions

$$\tilde{f}(N) = \int_0^1 x^{N-1} f(x) dx$$

Rule of thumb:
small- $N \rightarrow$ small- x ,
large- $N \rightarrow$ large- x

For more details see
[EKO N3LO documentation](#)

The approximation procedure is performed in Mellin space for each n_f part independently:

1. Parametrise the difference between the 4 (10) known moments and known limits with 4 functions $f_i(N)$.
2. Varying the sub-leading unknown $f_i(N)$ to produce a large set of parameterisation candidates (≈ 70).
3. Reduce the number of samples discarding too wiggly parameterisations and looking at the most representative cases.

- The spread among different linear combinations estimate IHO.
- Only theoretical inputs are considered.
- All the implemented approximations respect momentum sum rules.
- MHO can be added using Scale Variations

For example in $P_{gg}(x)$:

1. Theoretical constraint include:

- large- N :

$$\gamma_{gg}^{(3)}(N \rightarrow \infty) \approx A_{gg} S_1(N) + B_{gg} + \mathcal{O}\left(\frac{\ln(N)}{N}\right)$$

- small- N pole at $N = 0$, and $N = 1$ (leading contribution):

$$\gamma_{gg}^{(3)}(N \rightarrow 1) \approx C_4 \frac{1}{(N-1)^4} + C_3 \frac{1}{(N-1)^3} + \mathcal{O}\left(\frac{1}{(N-1)^2}\right)$$

2. Solve the constraint given by the 4 known Mellin

moments with many different candidates $\{f_1, f_2, f_3, f_4\}$:

$$f_1 = \frac{S_1(N)}{N} \quad f_3 = \left\{ \frac{1}{(N-1)}, \frac{1}{N} \right\}$$

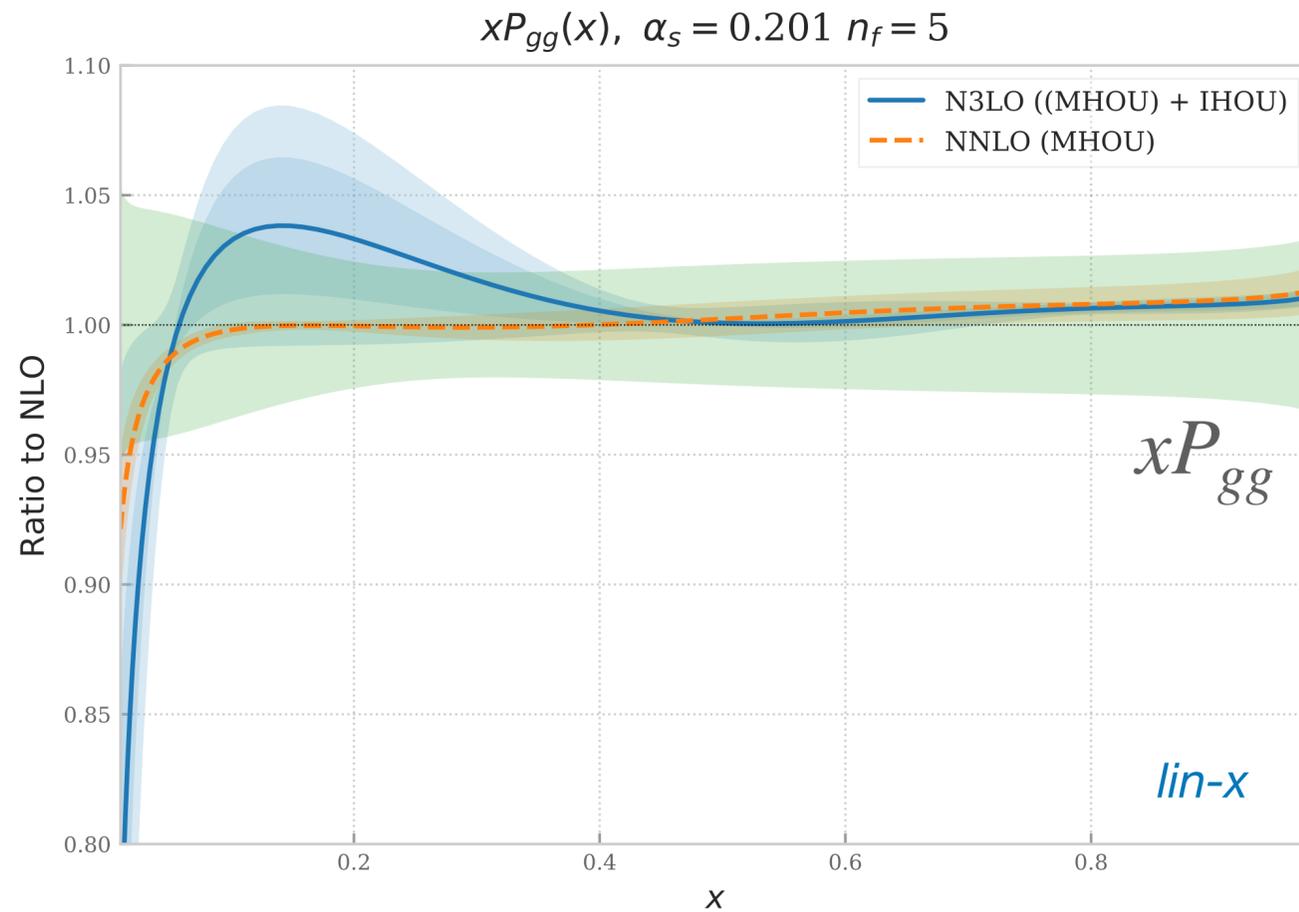
$$f_4 = \left\{ \frac{1}{(N-1)}, \frac{1}{N^4}, \frac{1}{N^3}, \frac{1}{N^2}, \frac{1}{N}, \frac{1}{(N+1)^3}, \frac{1}{(N+1)^2}, \right.$$

$$f_2 = \frac{1}{(N-1)^2}$$

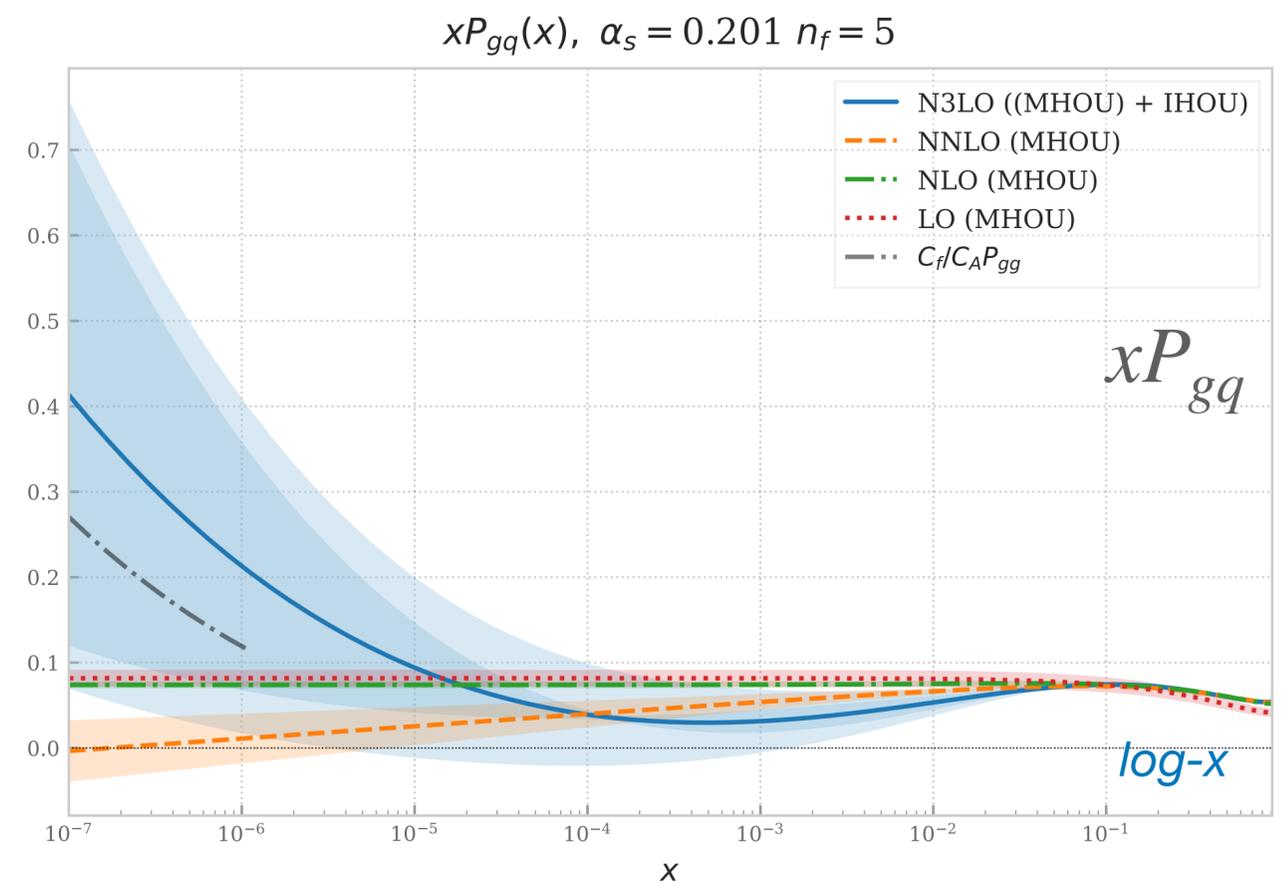
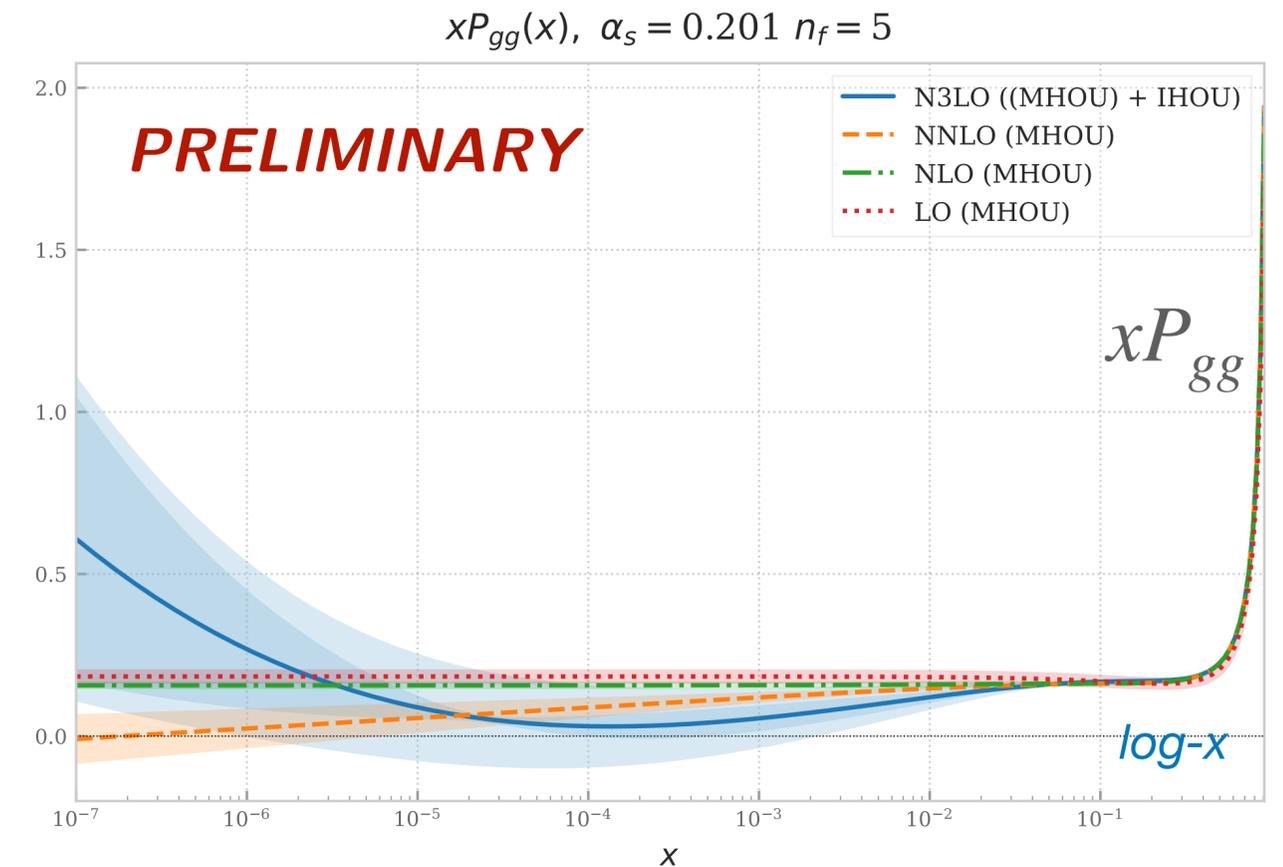
$$\left. \frac{1}{N+1}, \frac{1}{N+2}, \mathcal{M}[\ln(1-x)], \mathcal{M}[(1-x)\ln(1-x)], \frac{S_1(N)}{N^2} \right\}$$

PDF evolution @ aN3LO

Singlet splitting functions



- ▶ Large logs $1/x \ln^2(x), 1/x \ln(x)$ arise at N3LO.
- ▶ NNLO MHOU are not enough in small- x region.
- ▶ IHOU are not negligible. Having 10 moments available would be enough to reduce IHOU.
- ▶ Off diagonal terms P_{qg}, P_{gq} are more difficult to estimate (large- N goes to 0).

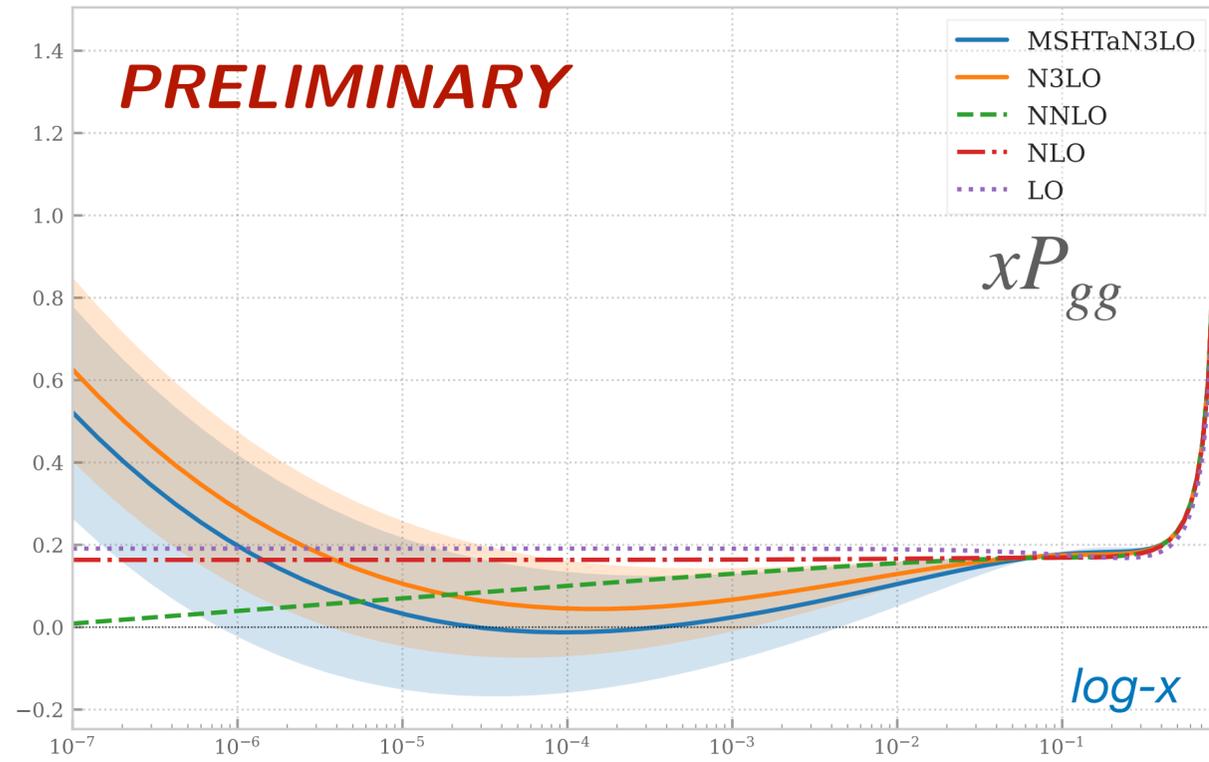


Comparison with MSHT

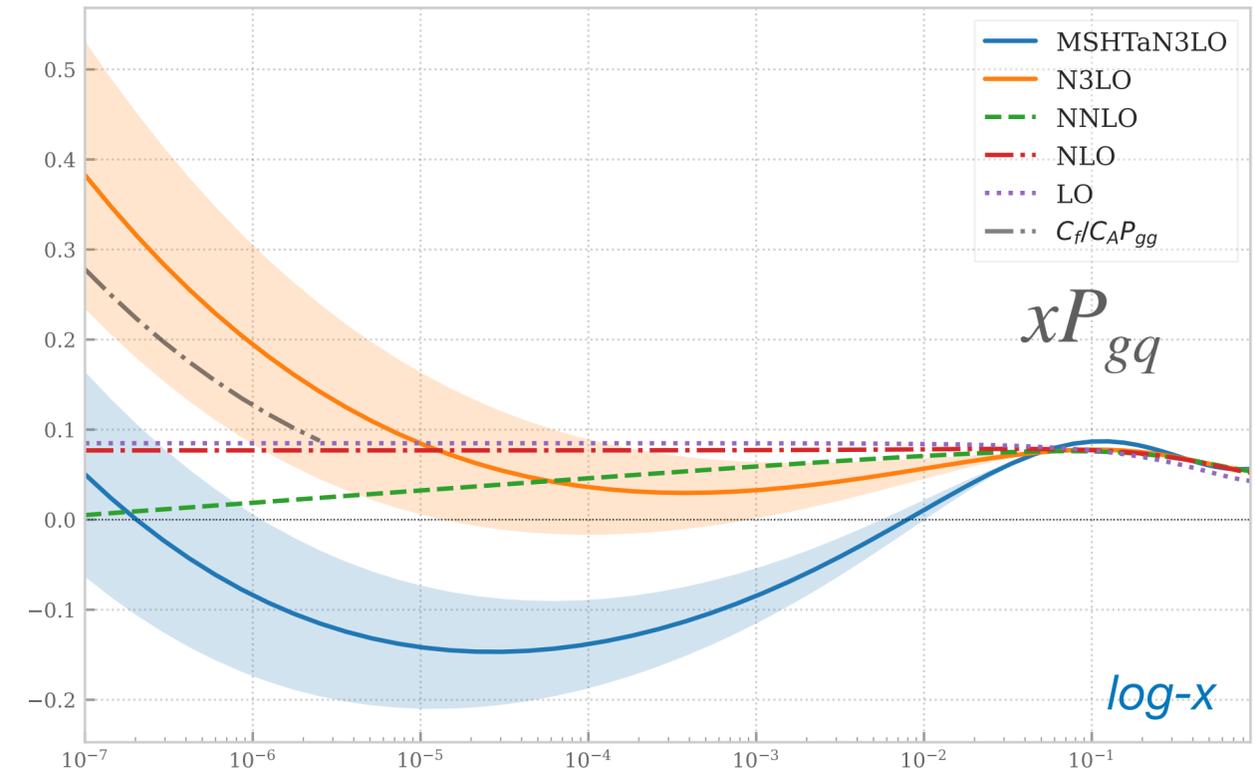
[MSHTaN3LO: \[arxiv:2207.04739\]](https://arxiv.org/abs/2207.04739)

See also [MSHT talk DIS2023](#)

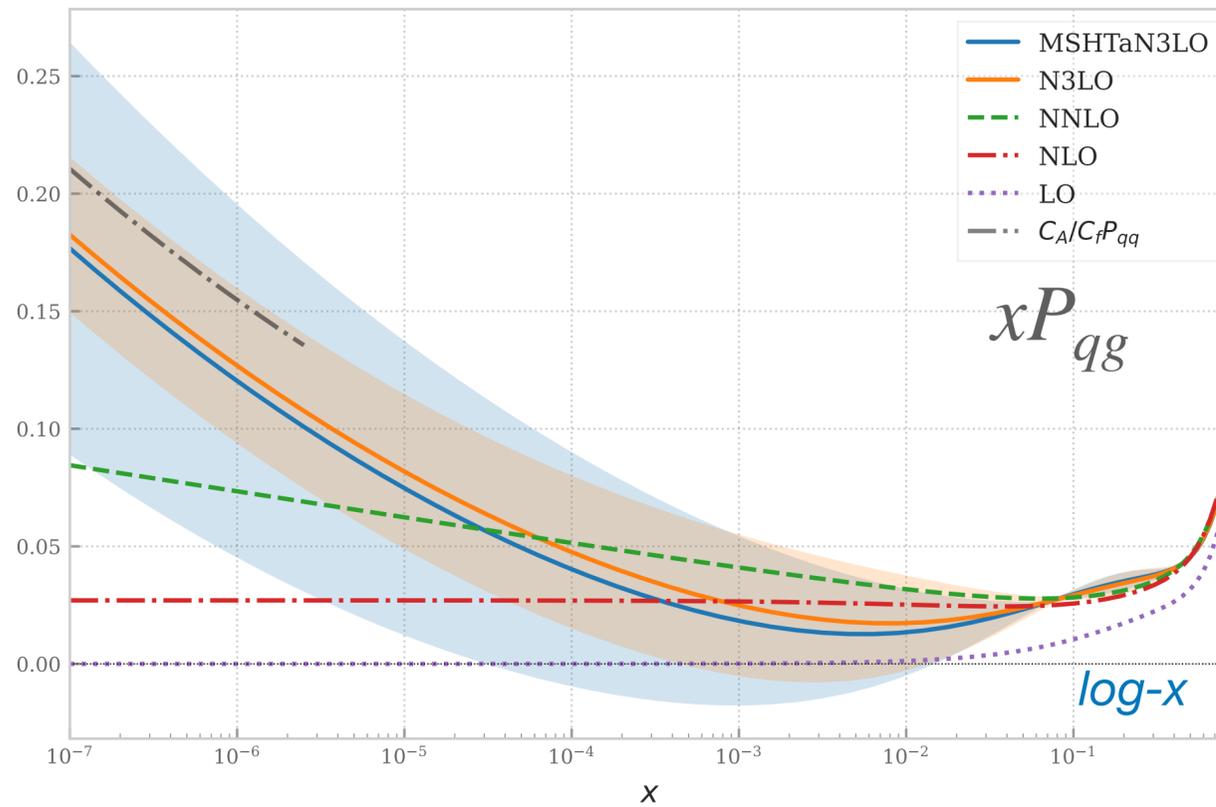
$xP_{gg}(x), \alpha_s = 0.2, n_f = 4$



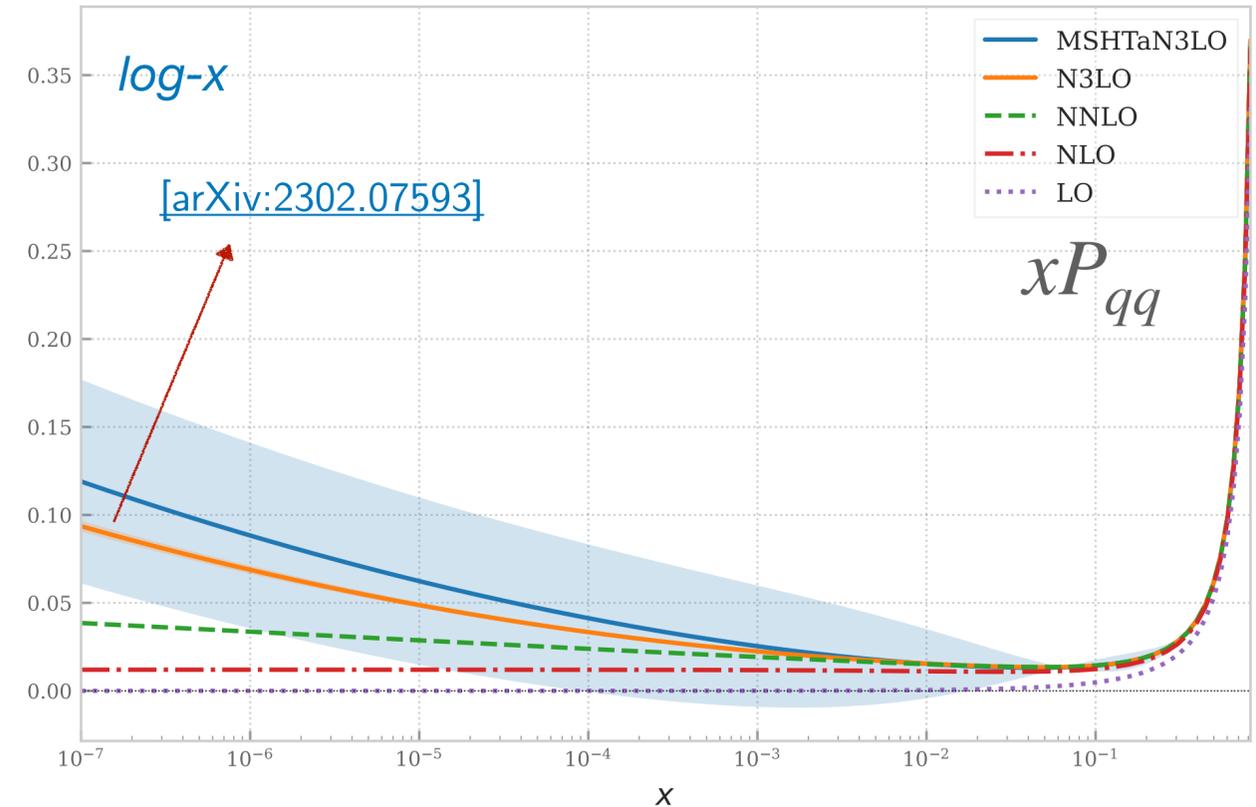
$xP_{gq}(x), \alpha_s = 0.2, n_f = 4$



$xP_{qg}(x), \alpha_s = 0.2, n_f = 4$



$xP_{qq}(x), \alpha_s = 0.2, n_f = 4$



DIS @ aN3LO

Structure Functions

DIS structure functions are known at N3LO in the **massless limit** for F_2, F_L, F_3 :

- DIS NC: Larin, Nogueira, Van Ritbergen, Vermaseren [\[arxiv:9605317\]](#) Moch, Vermaseren, Vogt [\[arxiv:0411112\]](#), [\[arxiv:0504242\]](#)
- DIS CC: Davies, Moch, Vermaseren, Vogt [\[arxiv:0812.4168\]](#) [\[arxiv:1606.08907\]](#)

DIS **Heavy structure functions** can be parametrised joining the known limits ($Q \rightarrow m_h^2$ and $Q \gg m_h^2$) with some damping functions.

$$C_{g,h}^3 = C_{g,h}^{(3,0)} + C_{g,h}^{(3,1)} \log\left(\frac{\mu}{m}\right) + C_{g,h}^{(3,2)} \log^2\left(\frac{\mu}{m}\right)$$

$$C_{g,h}^{(3,0)} = C_{g,h}^{thr}\left(z, \frac{m_h}{Q}\right) f_1(z) + C_{g,h}^{asy}\left(z, \frac{m_h}{Q}\right) f_2(z)$$

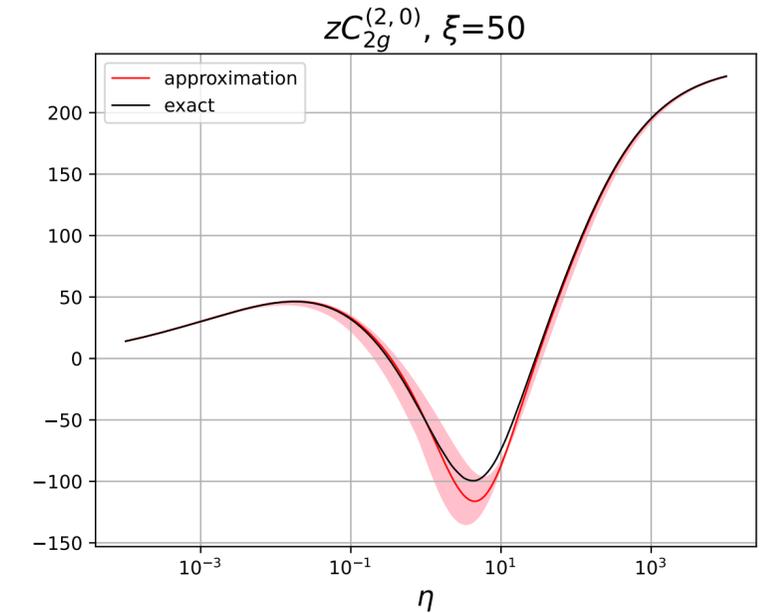
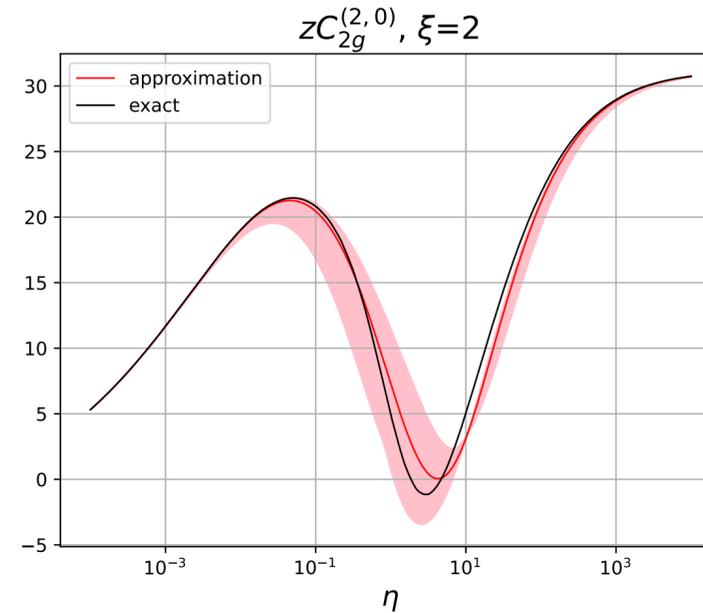
Kawamura, Lo Presti, Moch, Vogt [\[arxiv:1205.5727\]](#)

See also A. Pelloni talk at DIS2023 on 4-loop results Basdev-Sharma, Pelloni, Herzog, Vogt [\[arxiv:2211.16485\]](#)

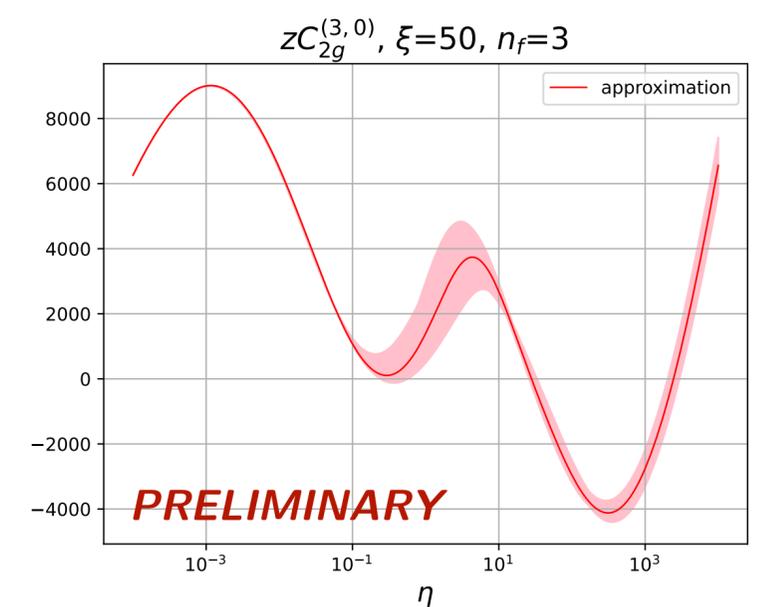
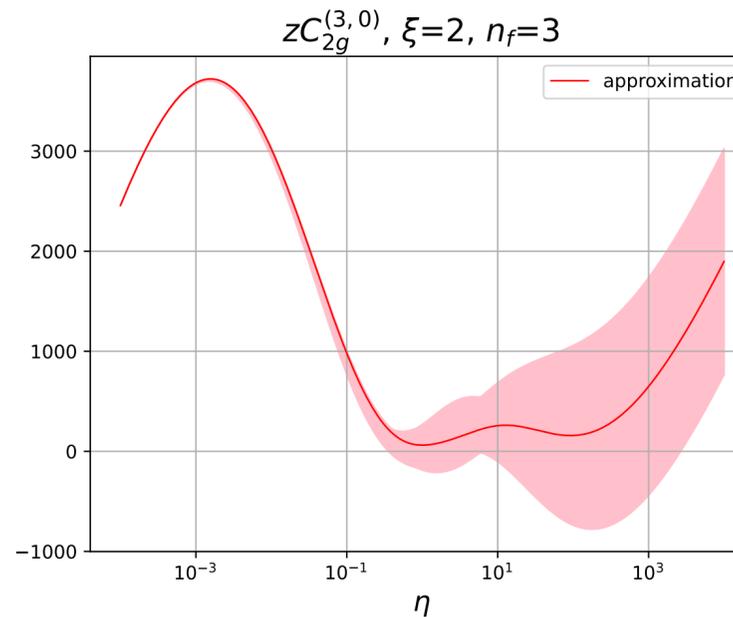
$$\eta = \frac{s}{4m_h^2} - 1 \quad \xi = \frac{Q^2}{m_h^2}$$

NNLO check

From N. Laurenti



aN3LO



DIS @ aN3LO

Variable Flavor Number Scheme

During a PDF fit all these contributions needs to be joined together using a proper **Variable Flavor Number** Scheme

PDFs matching conditions are now available at

N3LO almost completely, with the exception of $a_{H,g}^{(3)}$: Bierenbaum,

Blümlein, Klein [arXiv:0904.3563] Ablinger, Behring, Blümlein, De Freitas, Hasselhuhn, von Manteuffel, Round, Schneider, Wißbrock. [arXiv:1406.4654]; Ablinger, Behring, Blümlein, De Freitas, Goedicke, von Manteuffel, Schneider Schonwald [arXiv:2211.0546].

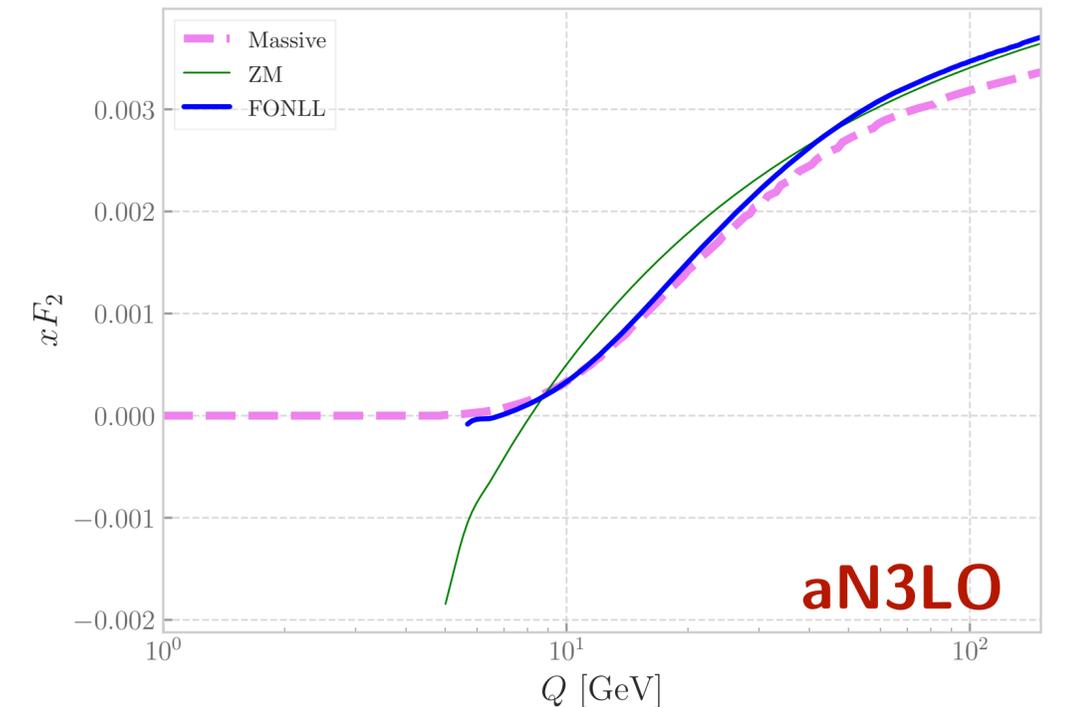
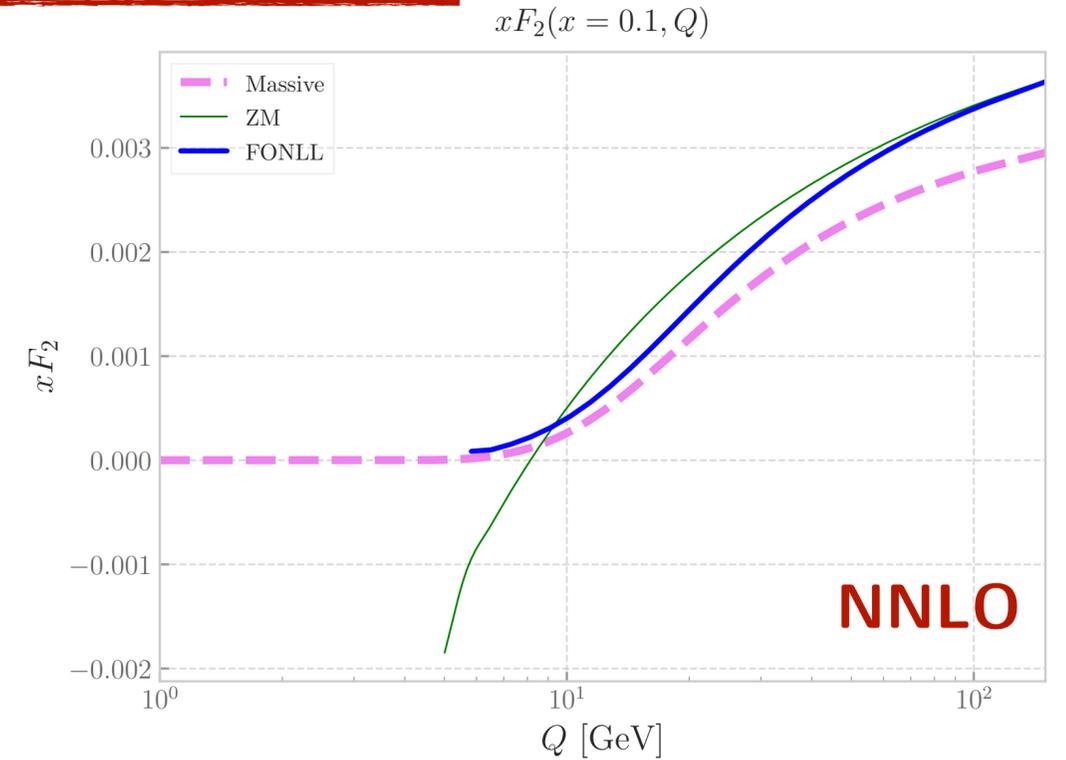
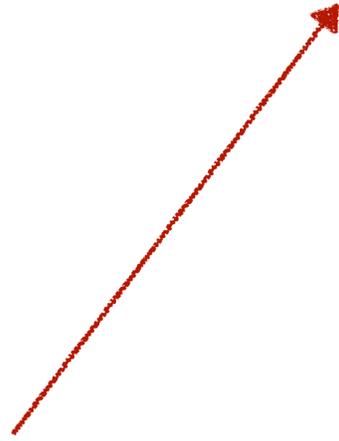
(Other works see slide 24)

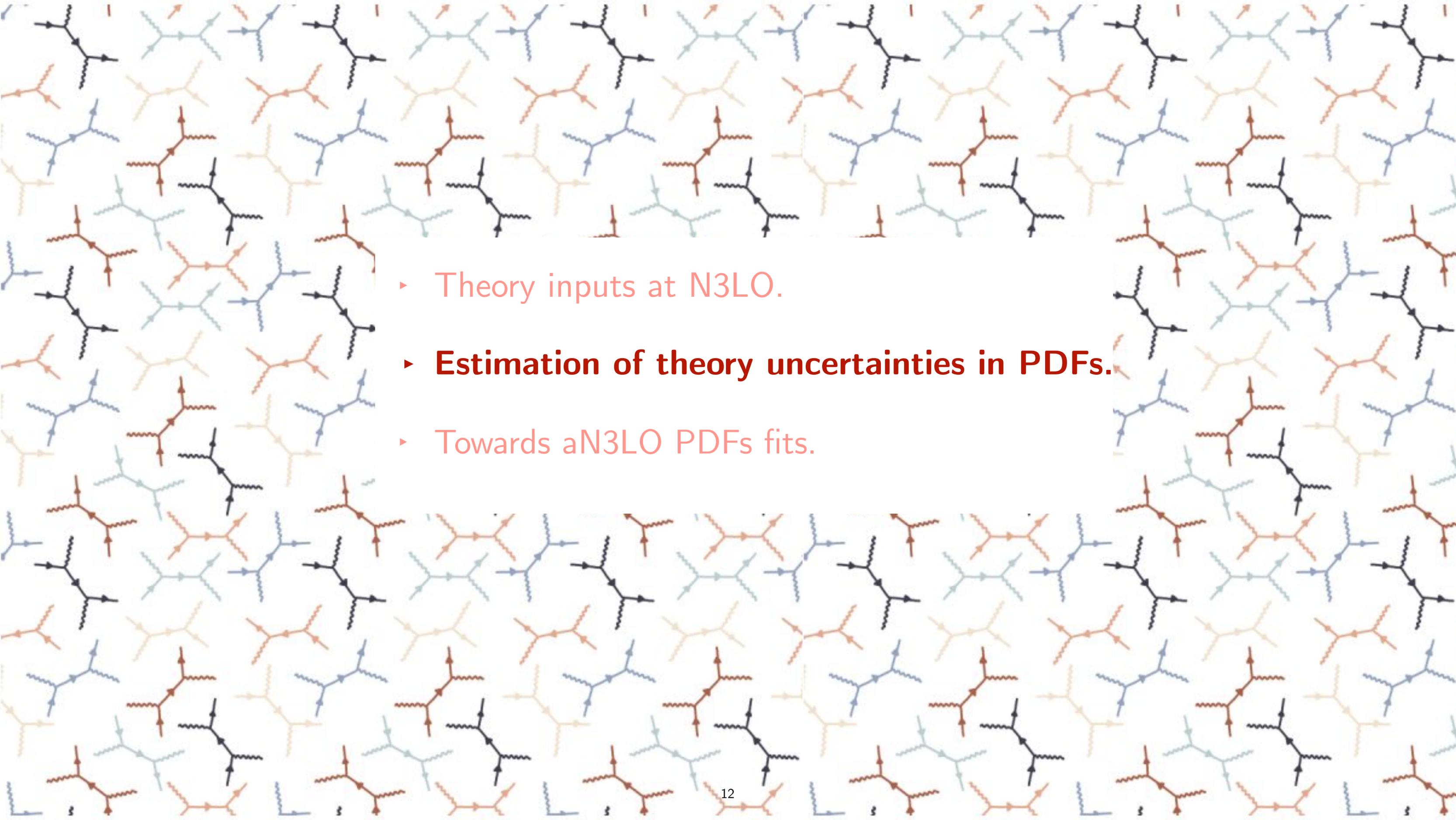
DIS structure functions are computed in the **FONLL** procedure

[arxiv:1001.2312]:

- ▶ Extended up to N3LO for the Heavy structure functions F_{heavy}
- ▶ Extended up to NNLO for light F_{light} + Massless N3LO contributions.

$$\begin{pmatrix} g \\ \Sigma^{(n_f)} \\ h^+ \end{pmatrix}^{n_f+1}(\mu_h^2) = \mathbf{A}_{S,h^+}^{(n_f)}(\mu_h^2) \begin{pmatrix} g \\ \Sigma^{(n_f)} \\ h^+ \end{pmatrix}^{n_f}(\mu_h^2)$$



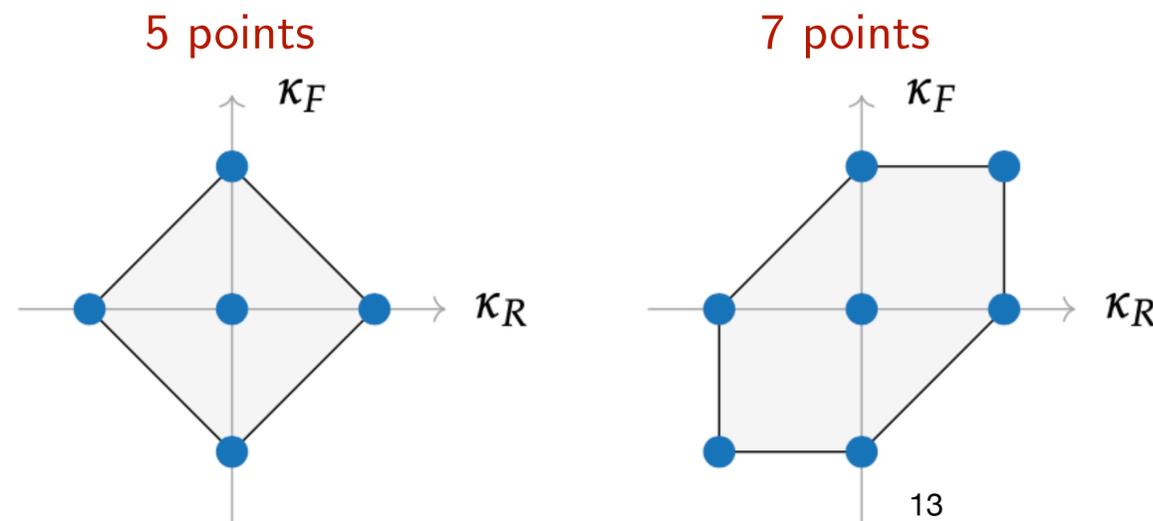
- 
- The background of the slide is a repeating pattern of Feynman diagrams. These diagrams are three-loop diagrams, specifically the sunset and sunrise topologies, rendered in various colors including black, blue, orange, and light green. Each diagram consists of vertices connected by lines representing particles, with some lines being wavy (representing gluons) and others straight (representing quarks).
- ▶ Theory inputs at N3LO.
 - ▶ **Estimation of theory uncertainties in PDFs.**
 - ▶ Towards aN3LO PDFs fits.

MHOU from scale variations

General formalism how to introduce theory uncertainties in PDFs have been addressed in various studies:

MSTH [[arxiv:1811.08434](https://arxiv.org/abs/1811.08434)], NNPDF [[arxiv:1906.10698](https://arxiv.org/abs/1906.10698)], [[arxiv:2105.05114](https://arxiv.org/abs/2105.05114)]

- ▶ Not a unique procedure. There are at least 3 different schemes that can be used to compute MHOU. Differences are always higher orders.
- ▶ **Factorization scale variations** are introduced during the DGLAP evolution.
- ▶ **Renormalization scale variations** are retained inside the coefficient functions and varied differently for different kind of processes.
- ▶ The way in which μ_f, μ_r are varied simultaneously define a so called point prescription.



Scale variation advantages:

- ▶ Justified by RGE invariance.
- ▶ Valid for every process.

PDF Evolution

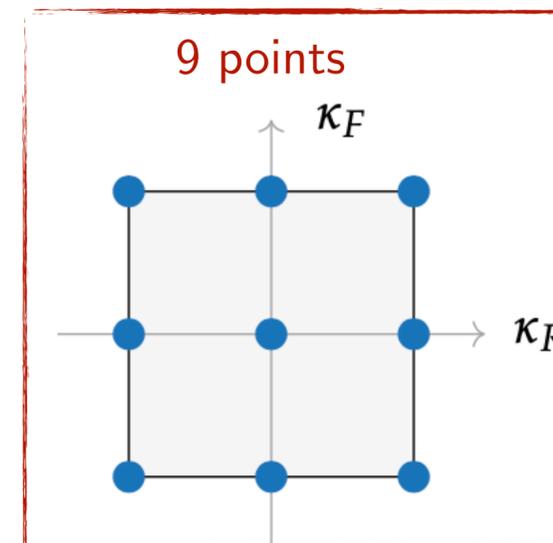


$$FK_{i,n}(Q, \mu_f, \mu_r) = E_{ij,n}(Q, \alpha_s, \mu_f) \otimes C_{j,n}(Q/\mu_r, \alpha_s)$$

$$n = \{1 \dots N_{dat}\}$$



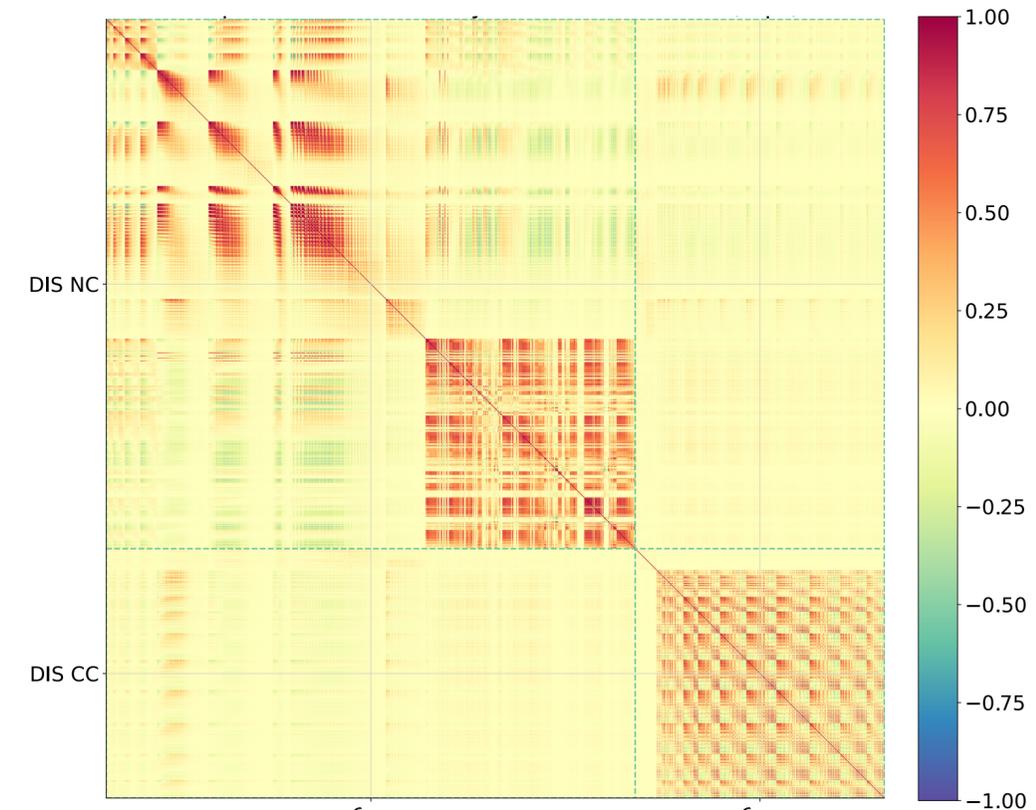
Partonic coefficients



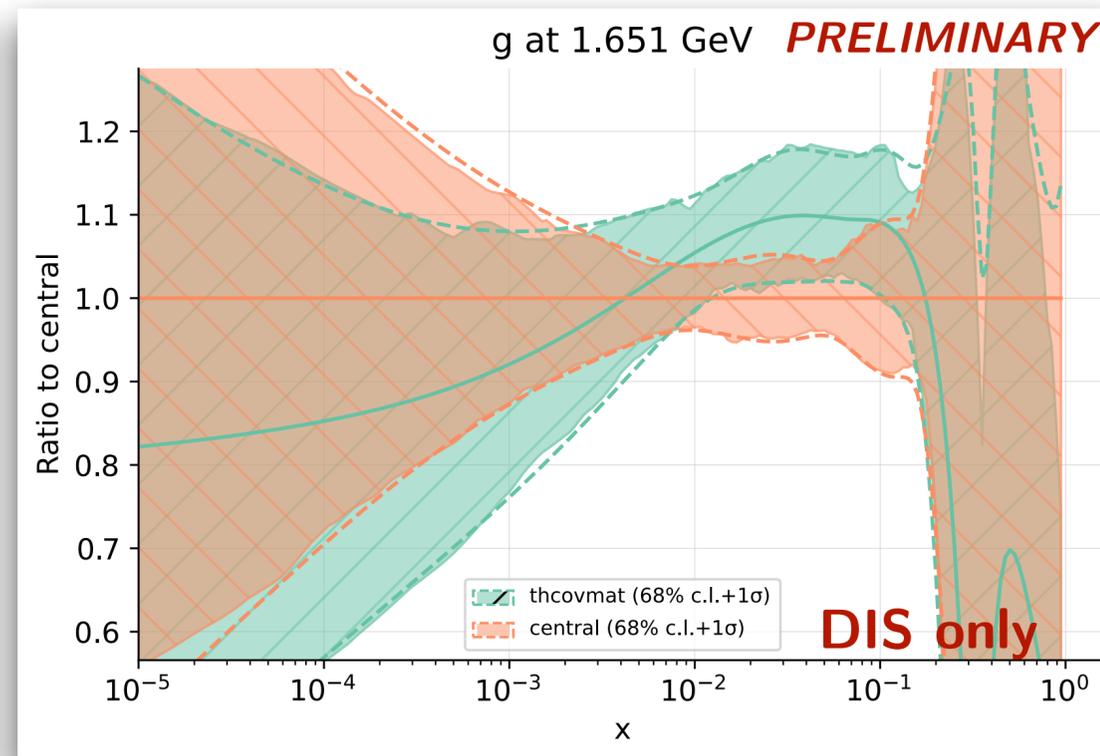
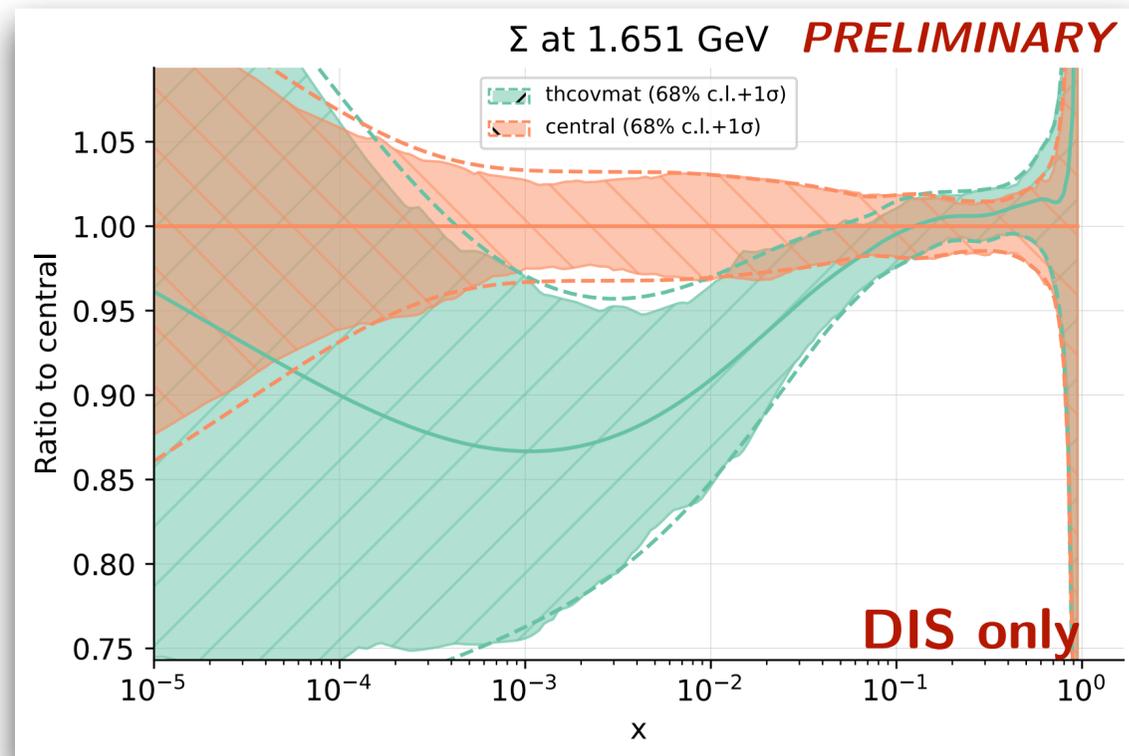
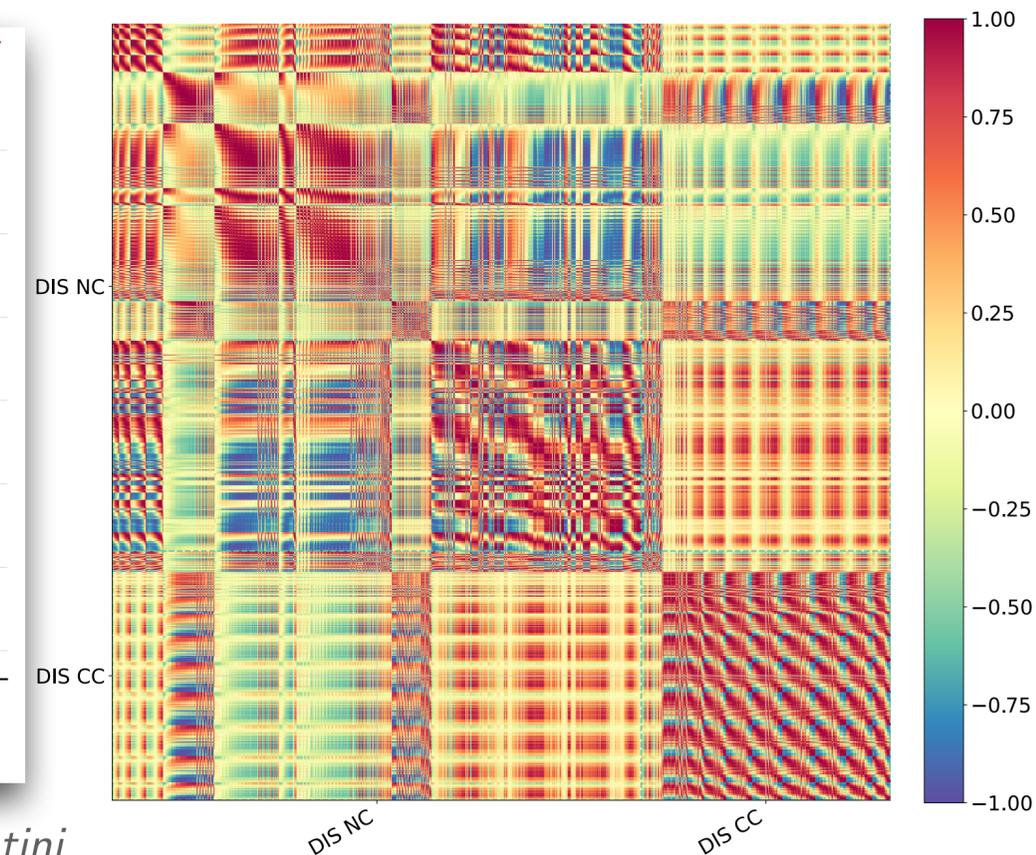
Impact of MHOU theory uncertainties

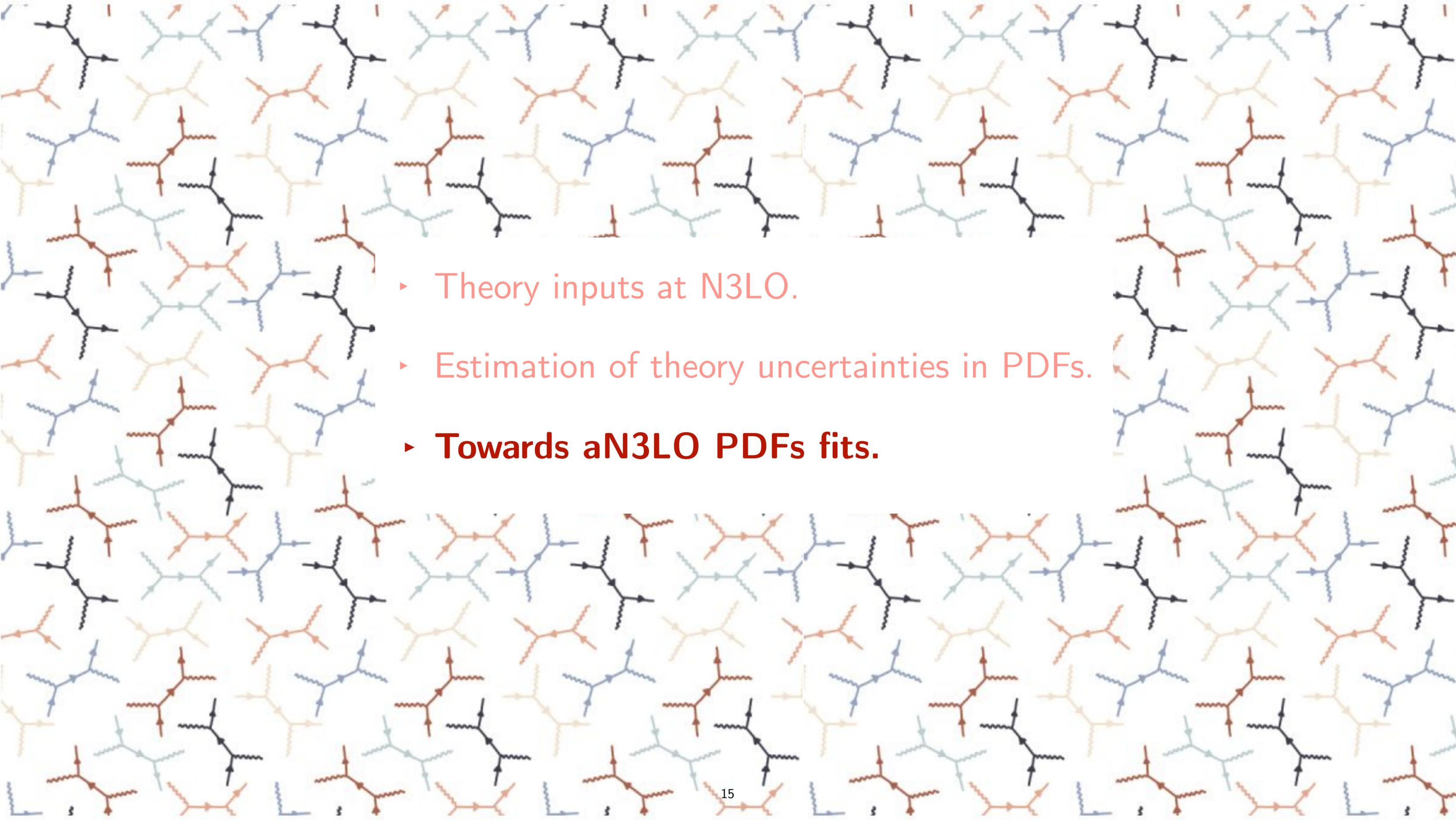
- ▶ $\mu_f/Q, \mu_r/Q$ are varied in the range $[0.5, 1, 2]$
- ▶ Theory uncertainties add correlations between datasets, which are not taken into account in the experimental covariance mat.
- ▶ Effects on the PDF fit are non-trivial.
- ▶ **DIS-only fits at NNLO** are ready, calculations for hadronic processes are ongoing.

Exp + MHOU correlations NNLO



MHOU correlations NNLO



- 
- ▶ Theory inputs at N3LO.
 - ▶ Estimation of theory uncertainties in PDFs.
 - ▶ **Towards aN3LO PDFs fits.**

Impact of IHOU theory uncertainties

What can be the effect of the uncertainties coming from the N3LO splitting functions?

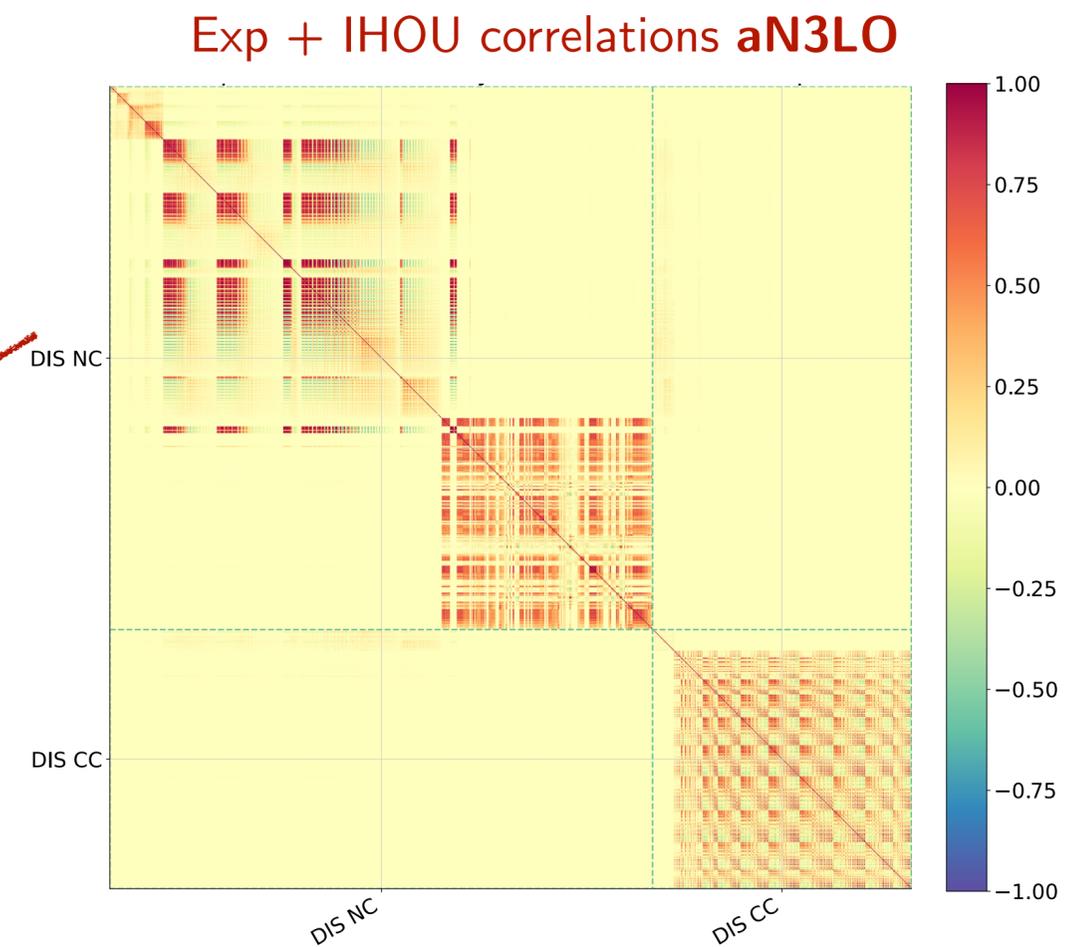
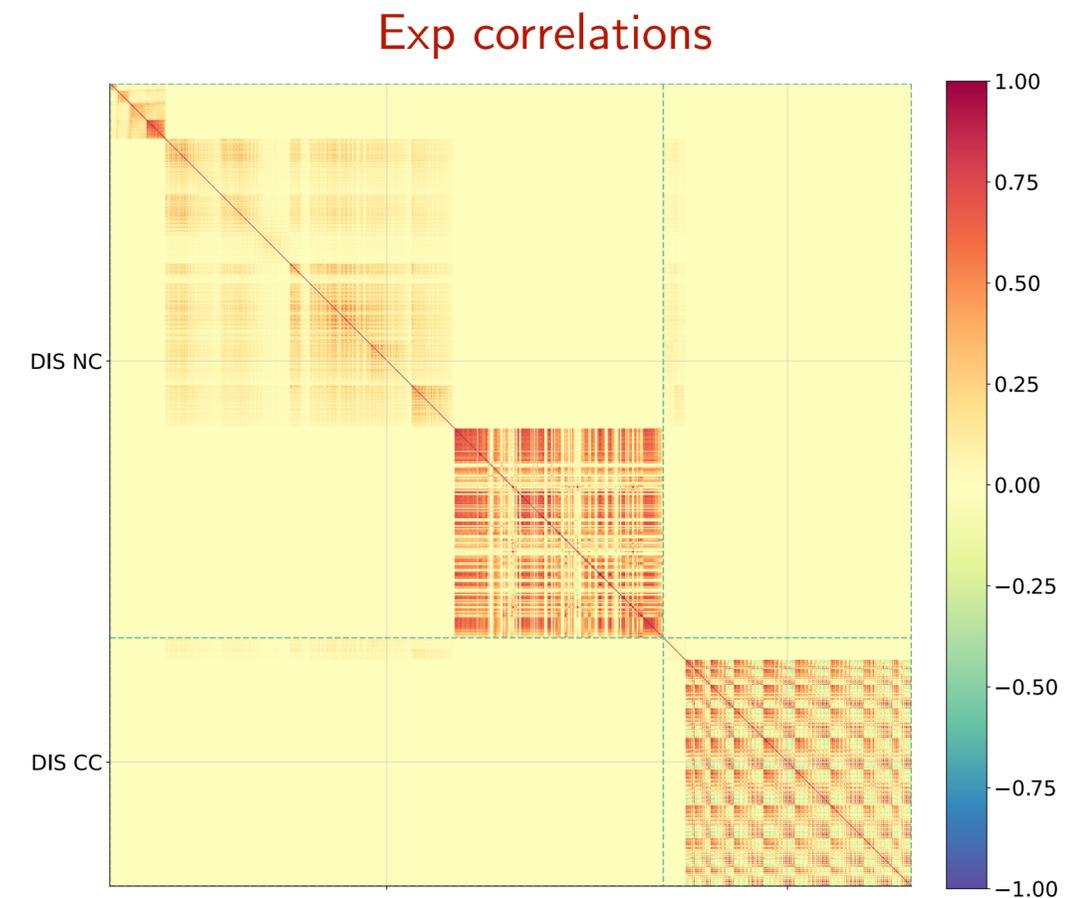
- ▶ Construct a theory covariance matrix by varying one single splitting function (during the DGLAP evolution) at the time:

$$\delta_{aN3LO_{mn}}^2 = \frac{1}{N_{dat}}(\sigma_{i,n} - \sigma_{0,n})(\sigma_{i,m} - \sigma_{0,m}), \quad n = \{0, \dots, N_{dat}\} \quad i = \{0, \dots, N_{var}\}$$

- ▶ This will produce an ≈ 70 point prescription theory covariance assuming that each variation is not correlated to the others.
- ▶ This source of uncertainty can be added to the MHOu theory covariance matrix obtained with scale variations:

$$\delta_{th}^2 = \delta_{MHOu}^2 + \delta_{aN3LO}^2$$

Larger effect on the small-x HERA data

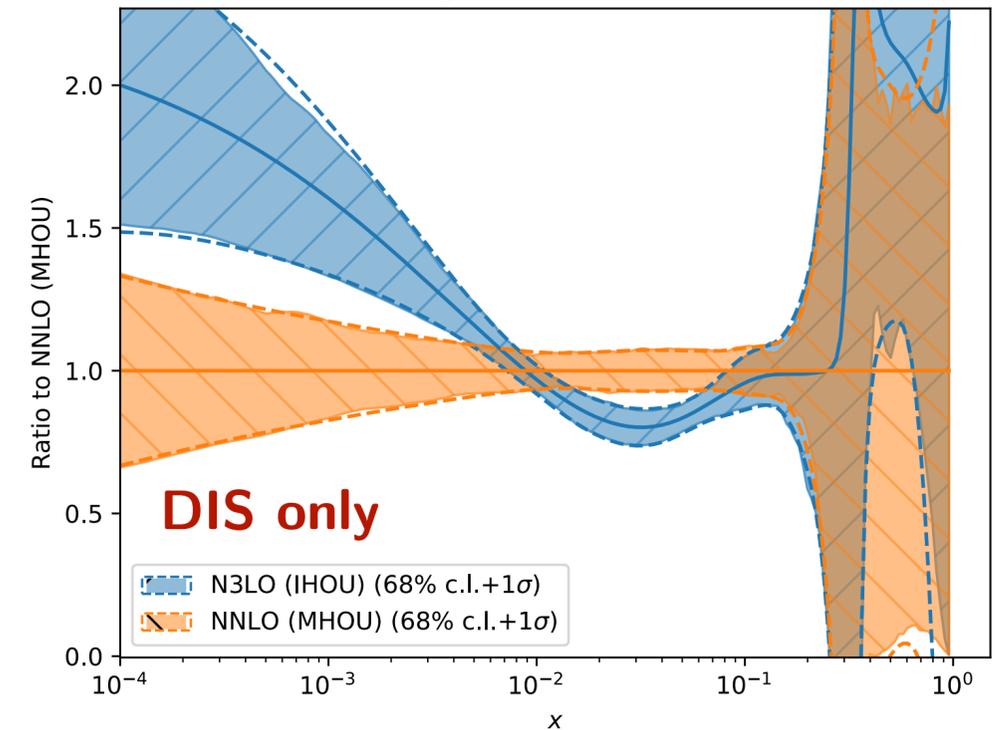


Towards aN3LO PDFs fits

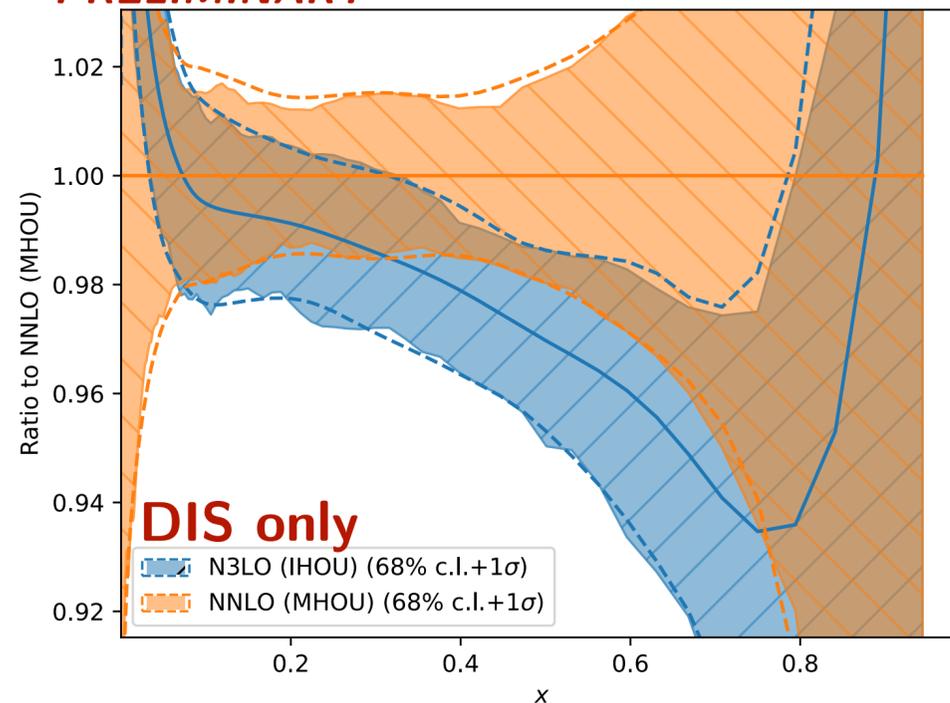
On the way of finalising computations for complete NNPDF4.0 datasets with MHOU and aN3LO.

- ▶ First runs of **aN3LO DIS only** fits show a quite visible impact of N3LO corrections in the **small- x** region for gluon g and Singlet Σ .
- ▶ At large- x PDFs are compatible within one sigma with NNLO.

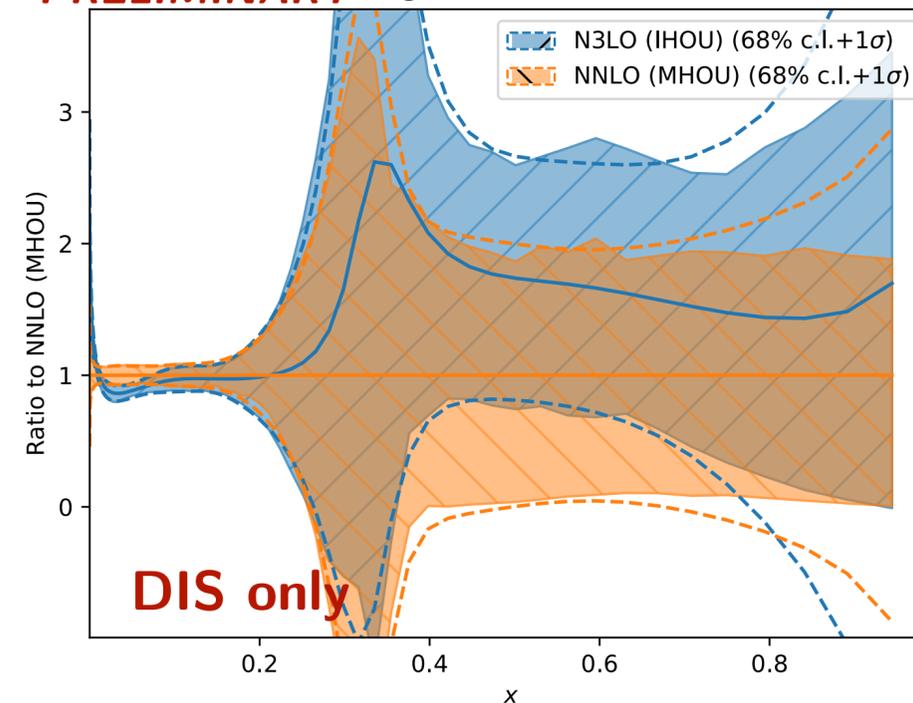
g at 1.651 GeV **PRELIMINARY**



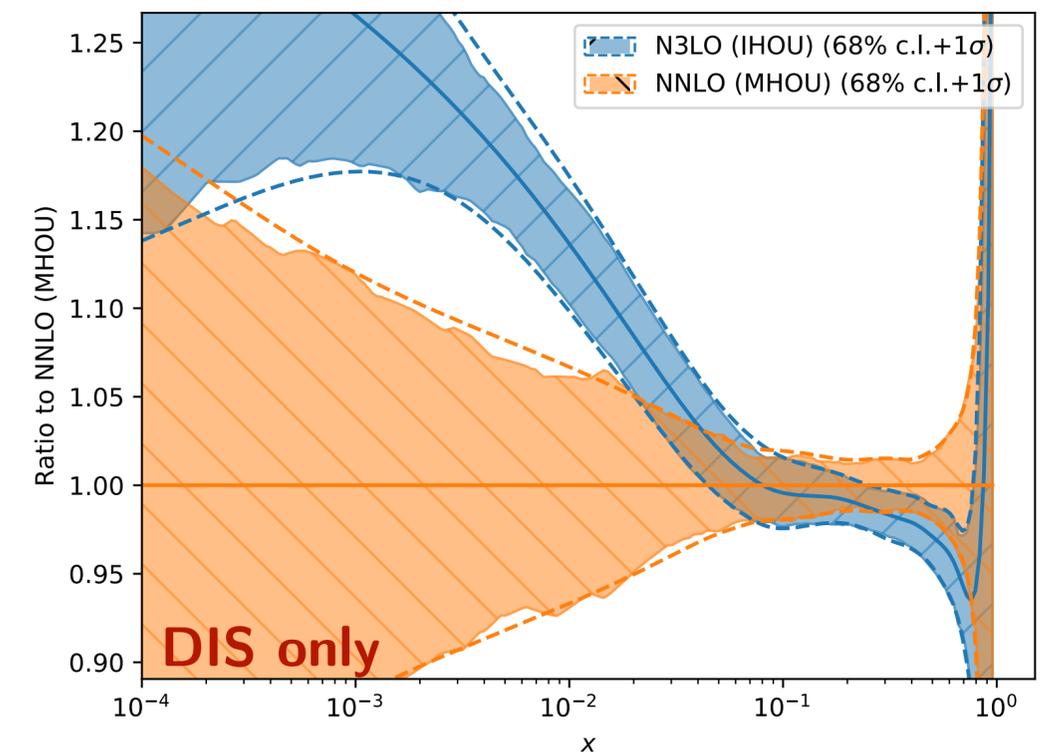
PRELIMINARY Σ at 1.651 GeV



PRELIMINARY g at 1.651 GeV



Σ at 1.651 GeV **PRELIMINARY**



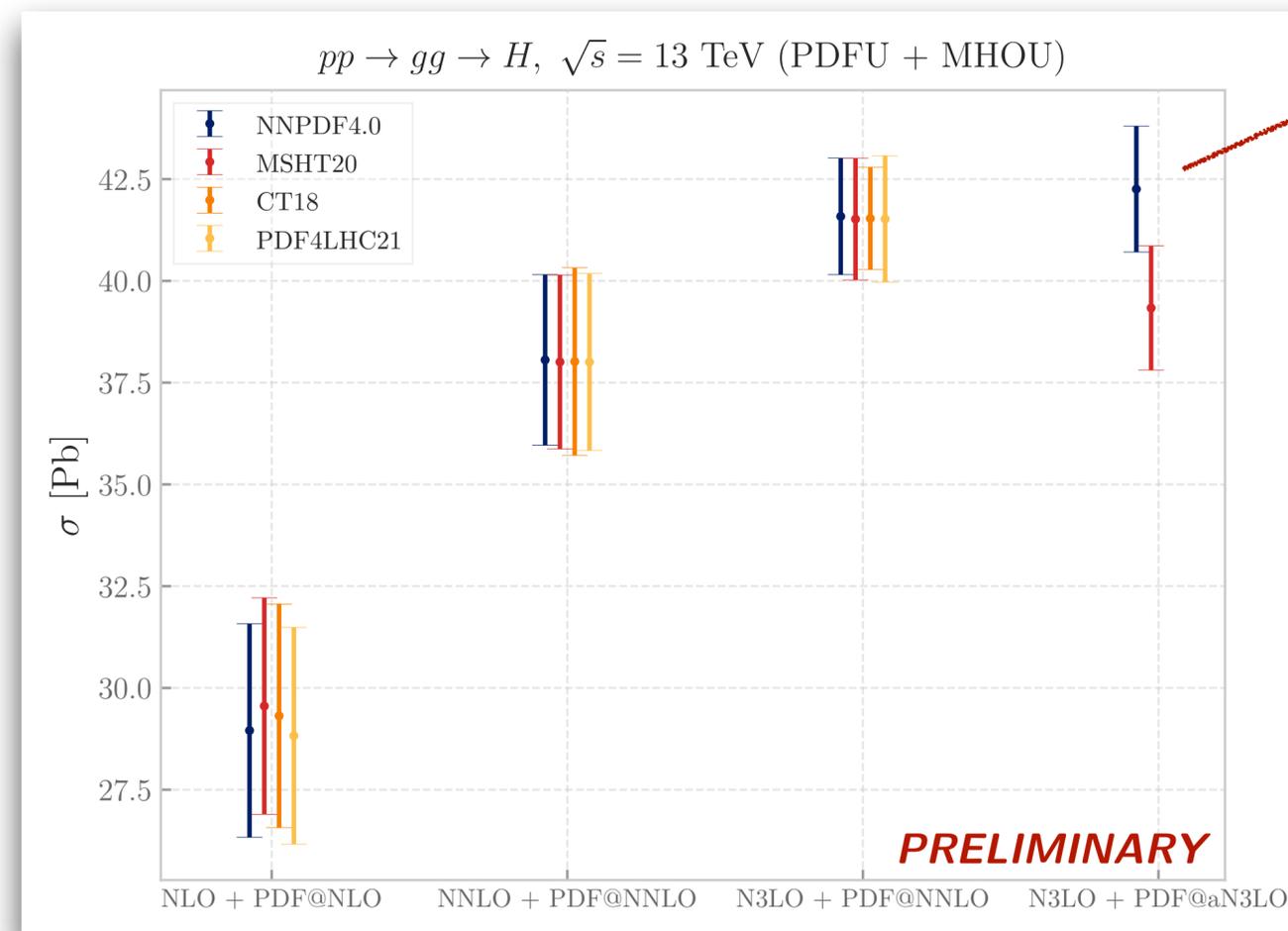
Outlook and future releases

Coming up NNPDF4.0 extensions:

- ▶ **4.0 MHO**: Faithful estimation of MHO from scale variations @ NNLO.
- ▶ **4.0 QED**: updated the determination of the photon PDF.
- ▶ **4.0 aN3LO**: inclusion of aN3LO QCD corrections [DGLAP, DIS, DY]

<https://github.com/NNPDF>

<https://nnpdf.mi.infn.it/>



DIS only

ggHiggs Bonvini et al. [\[arxiv:1306.6633\]](https://arxiv.org/abs/1306.6633)

Backup slides

Pipeline

A new tool chain for PDFs theory predictions

The code infrastructure needed to compute theory predictions has been completely rewritten and is now fully **open source**

One program, one job.

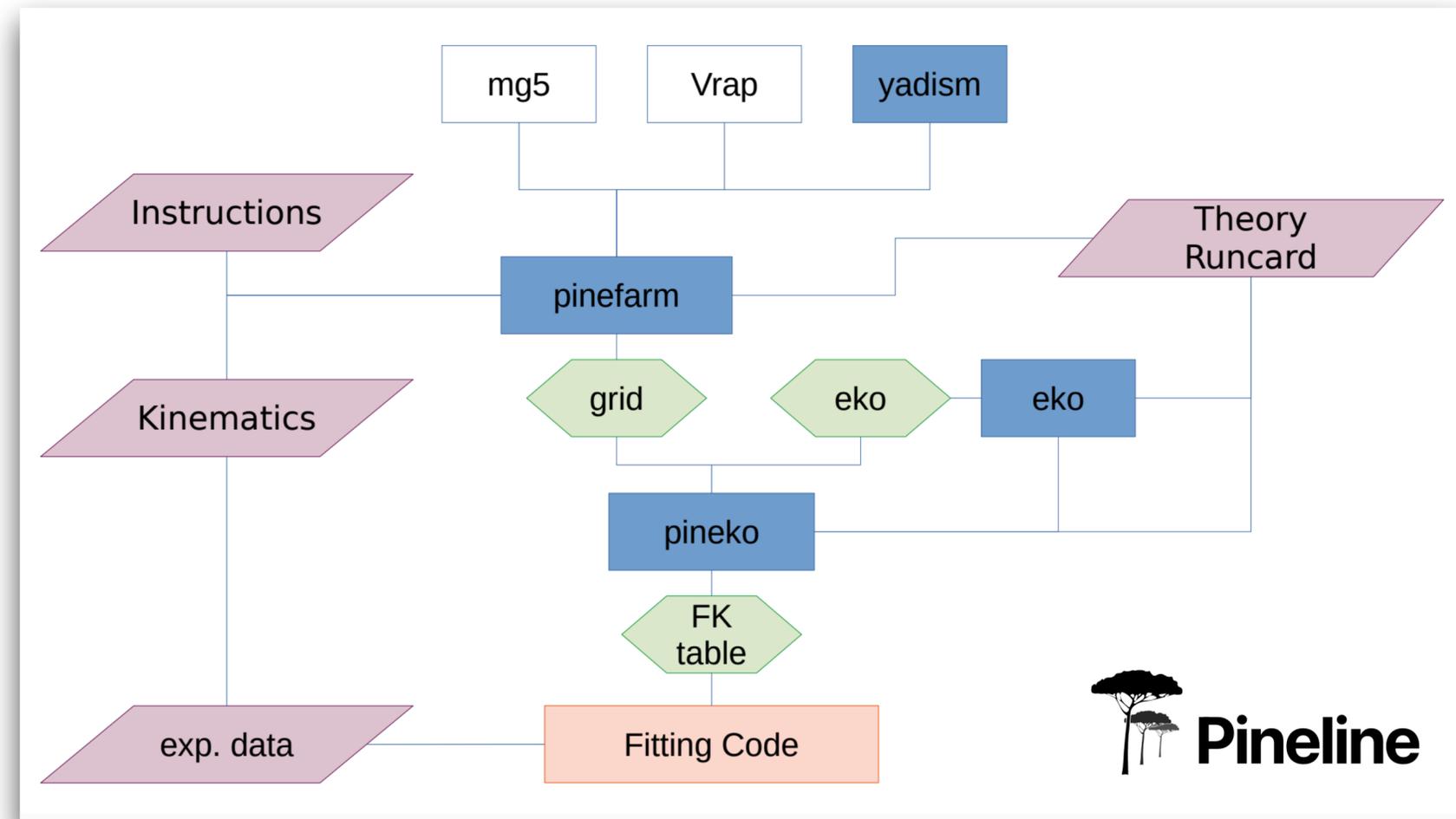
Easier to maintain. Mainly python written. Open-source

<https://nnpdf.github.io/pipeline/>

<https://github.com/NNPDF>

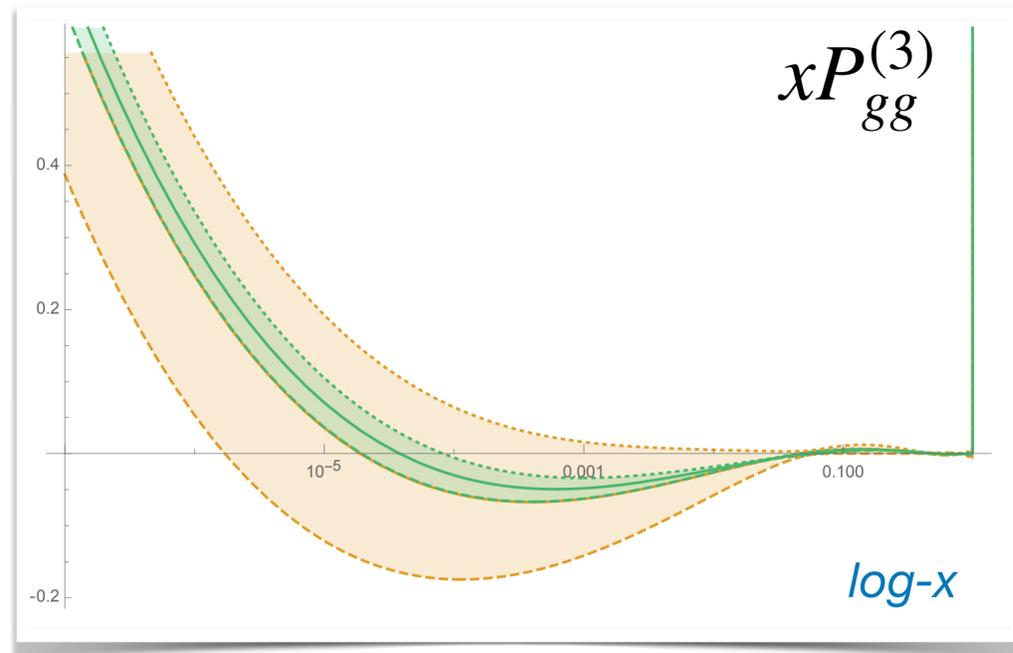
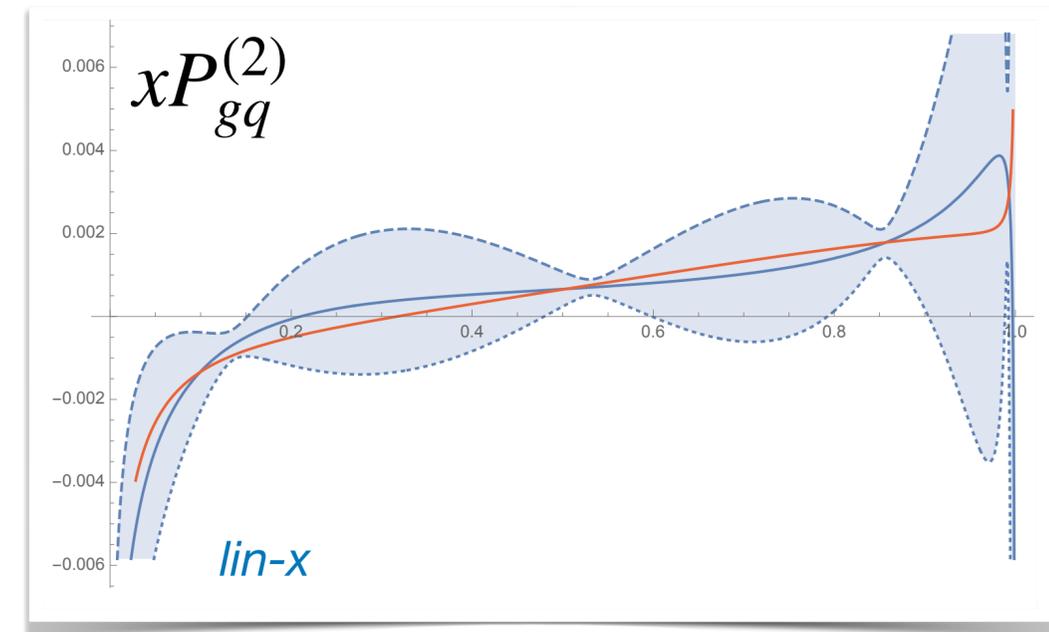
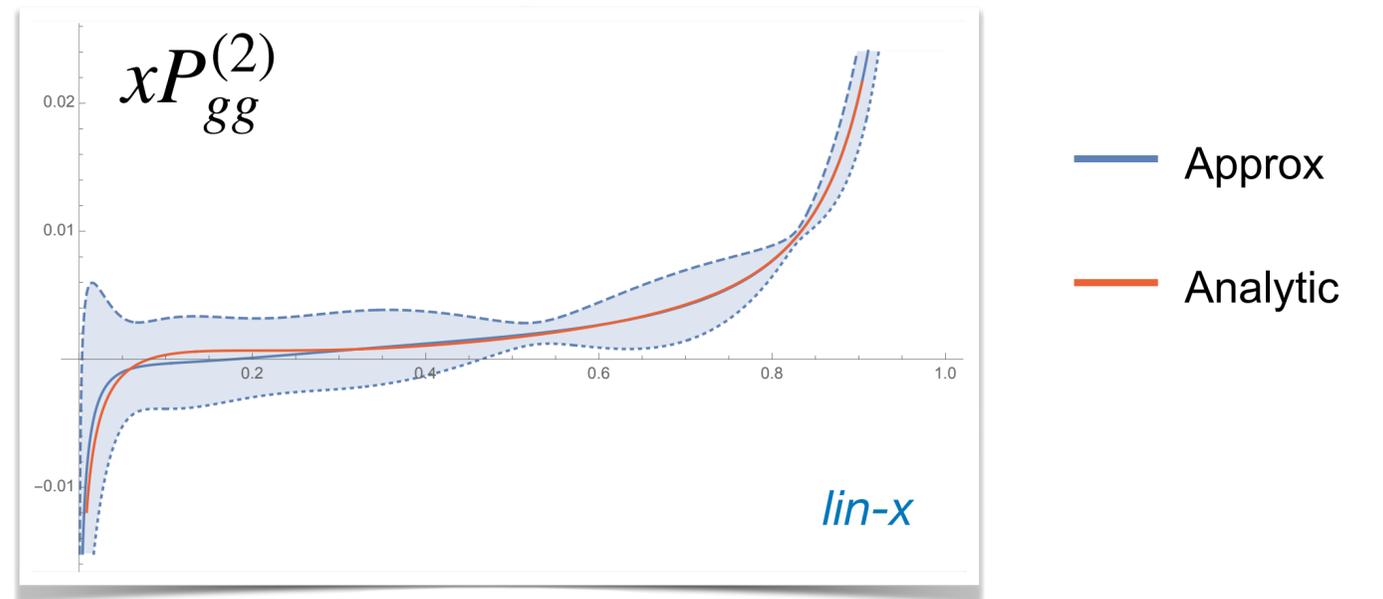
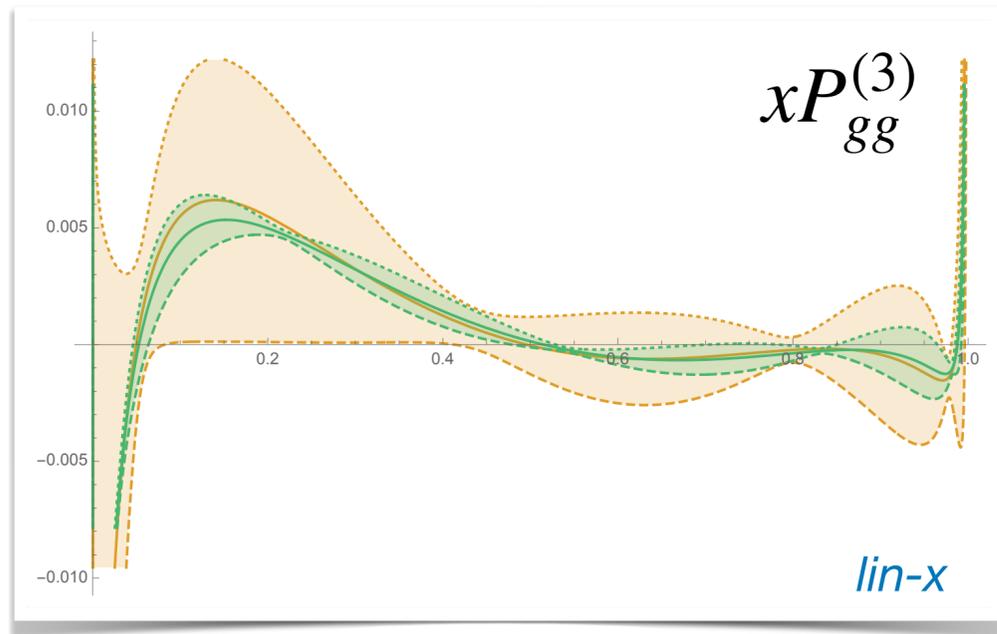


Barontini, Candido, Cruz-Martinez, Hekhorn, Schwan
[\[arxiv:2302.12124\]](https://arxiv.org/abs/2302.12124)



Splitting functions approximations tests

1. A possible way to validate the procedure is to **reproduce the known NNLO** singlet splitting functions using the very similar constrain that we have right now on the N3LO ones.

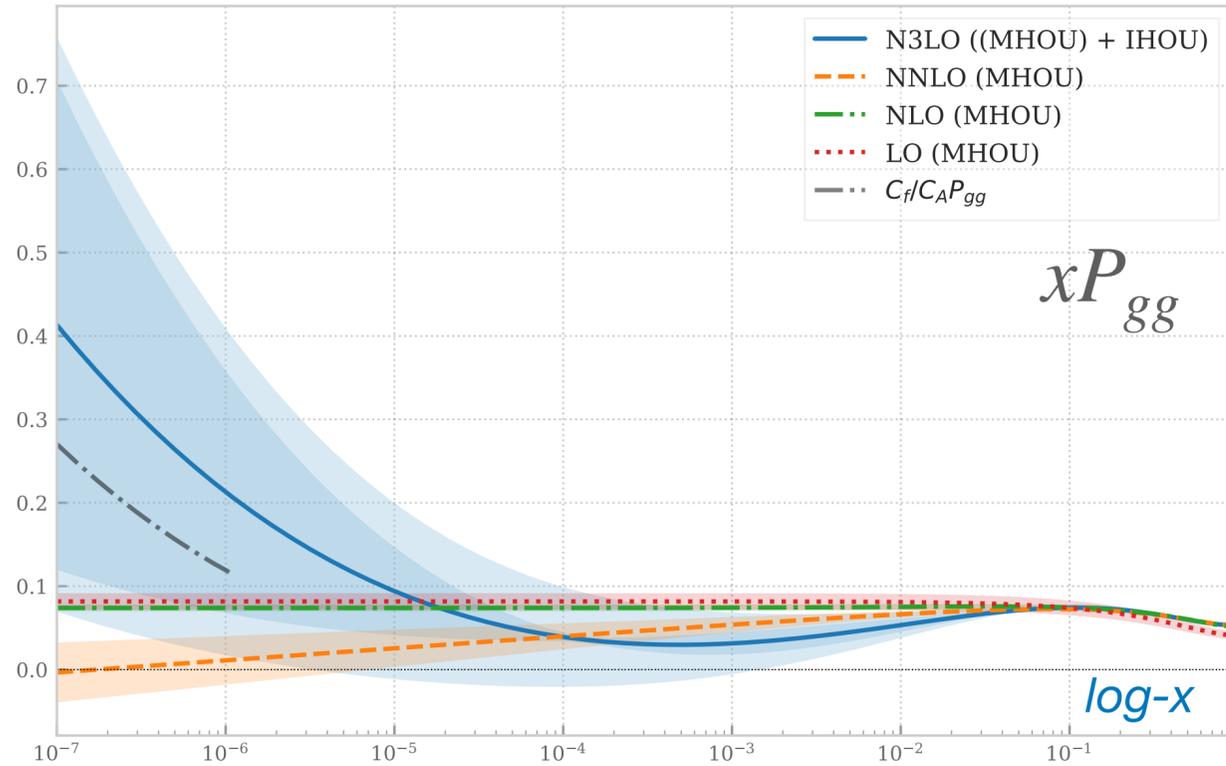


— 4 Moments fixed
— + N=3 fixed

2. Another way to validate the results is to **interpolate the known moments**, and construct a more constrained parametrisation now including 5/6 moments. If the procedure is working (the samples are varied enough) the uncertainty band obtained in this way should be small than the default one.

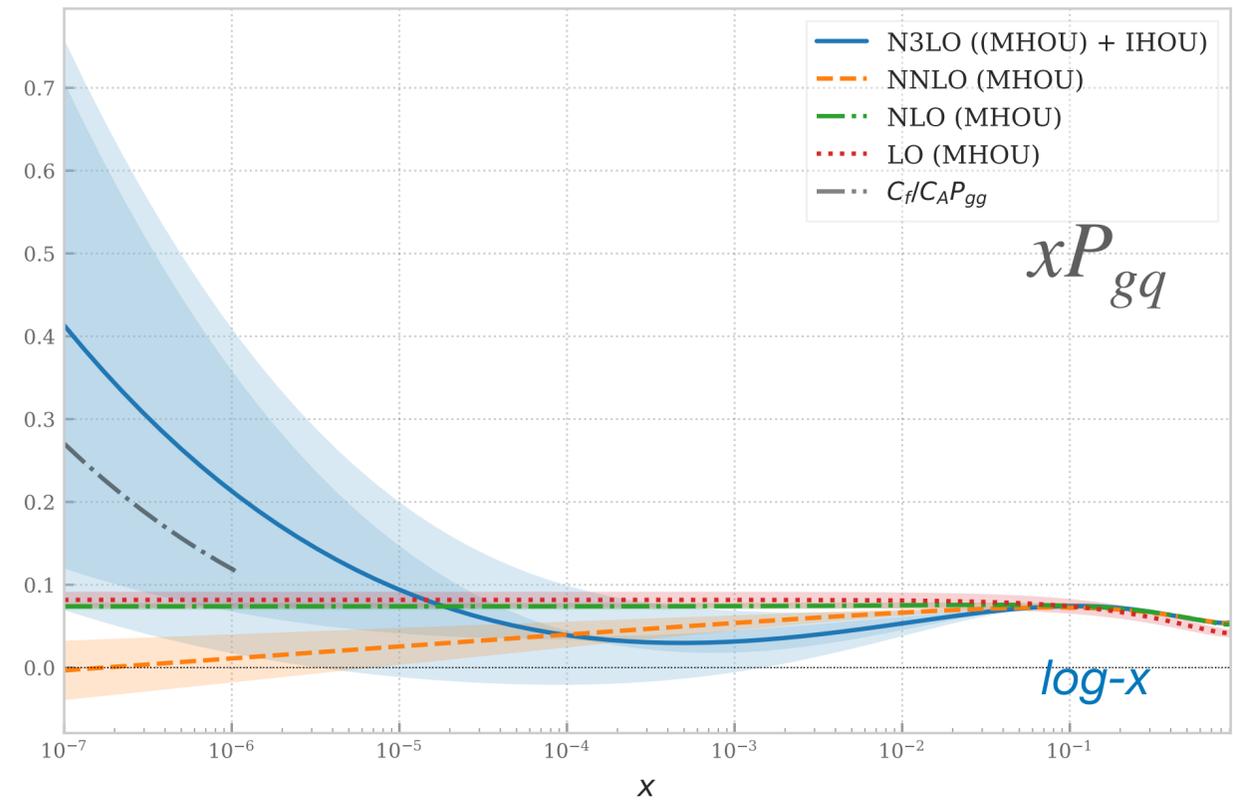
Splitting functions small-x

$xP_{gg}(x)$, $\alpha_s = 0.201$ $n_f = 5$

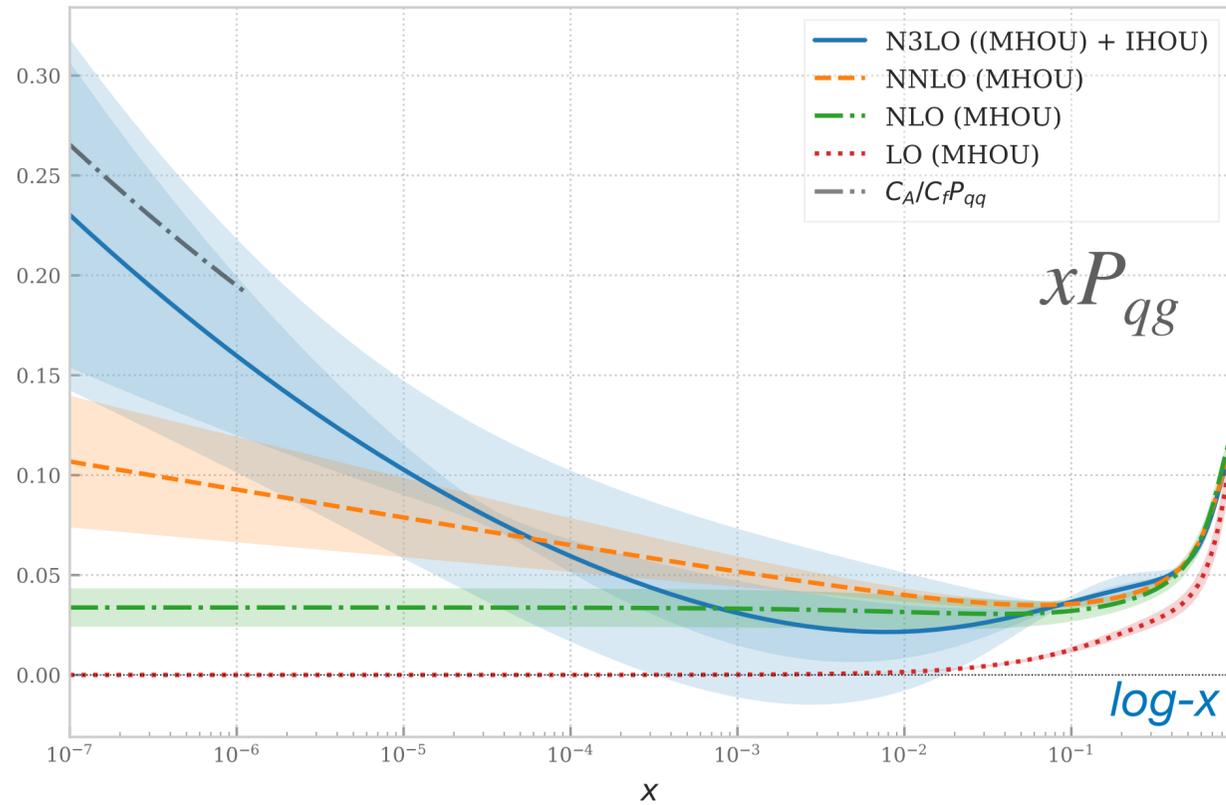


PRELIMINARY RESULTS

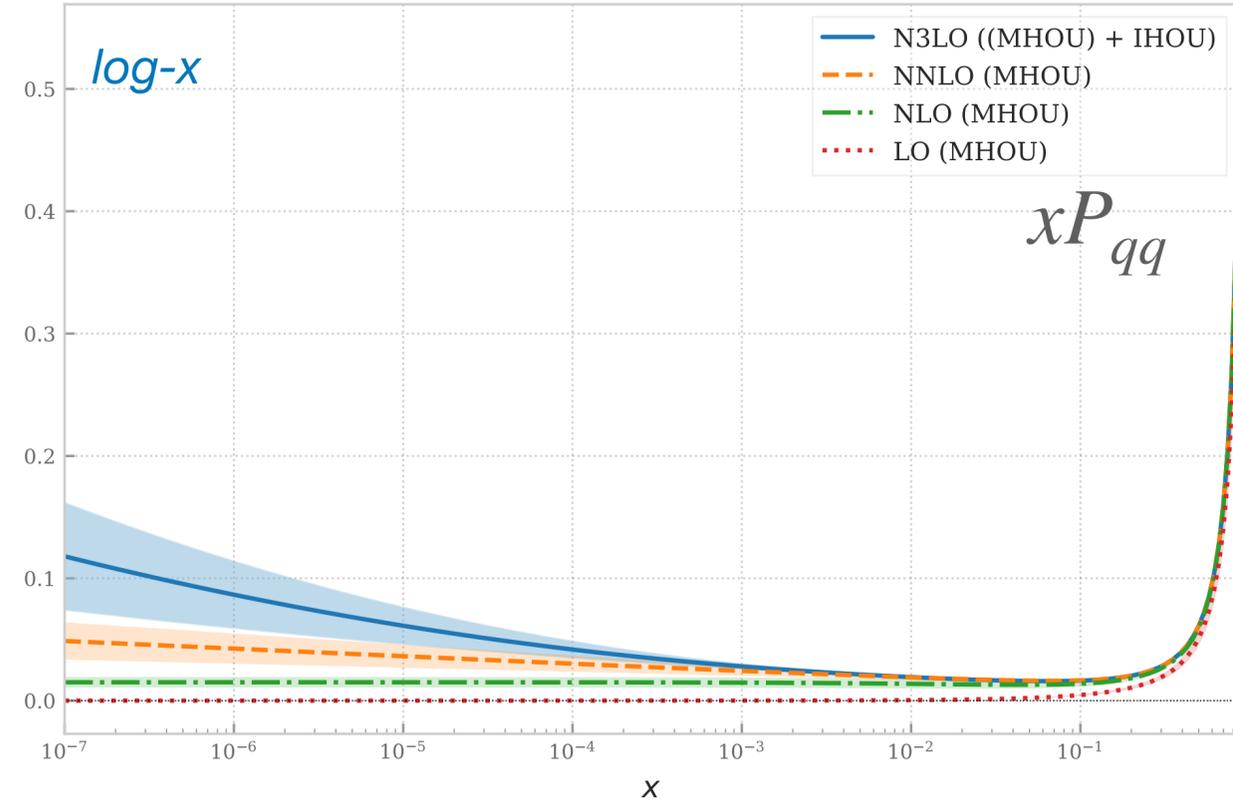
$xP_{gq}(x)$, $\alpha_s = 0.201$ $n_f = 5$



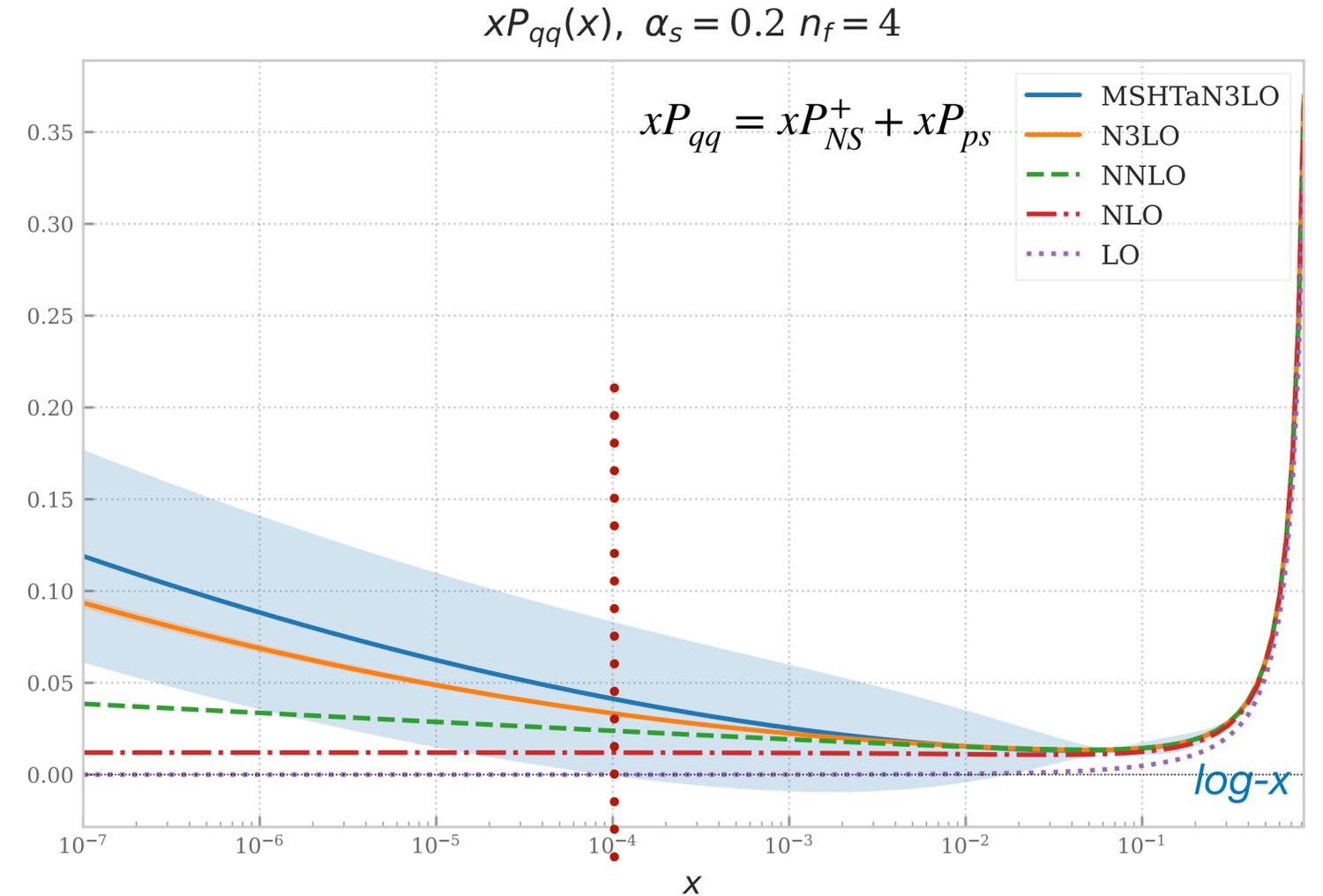
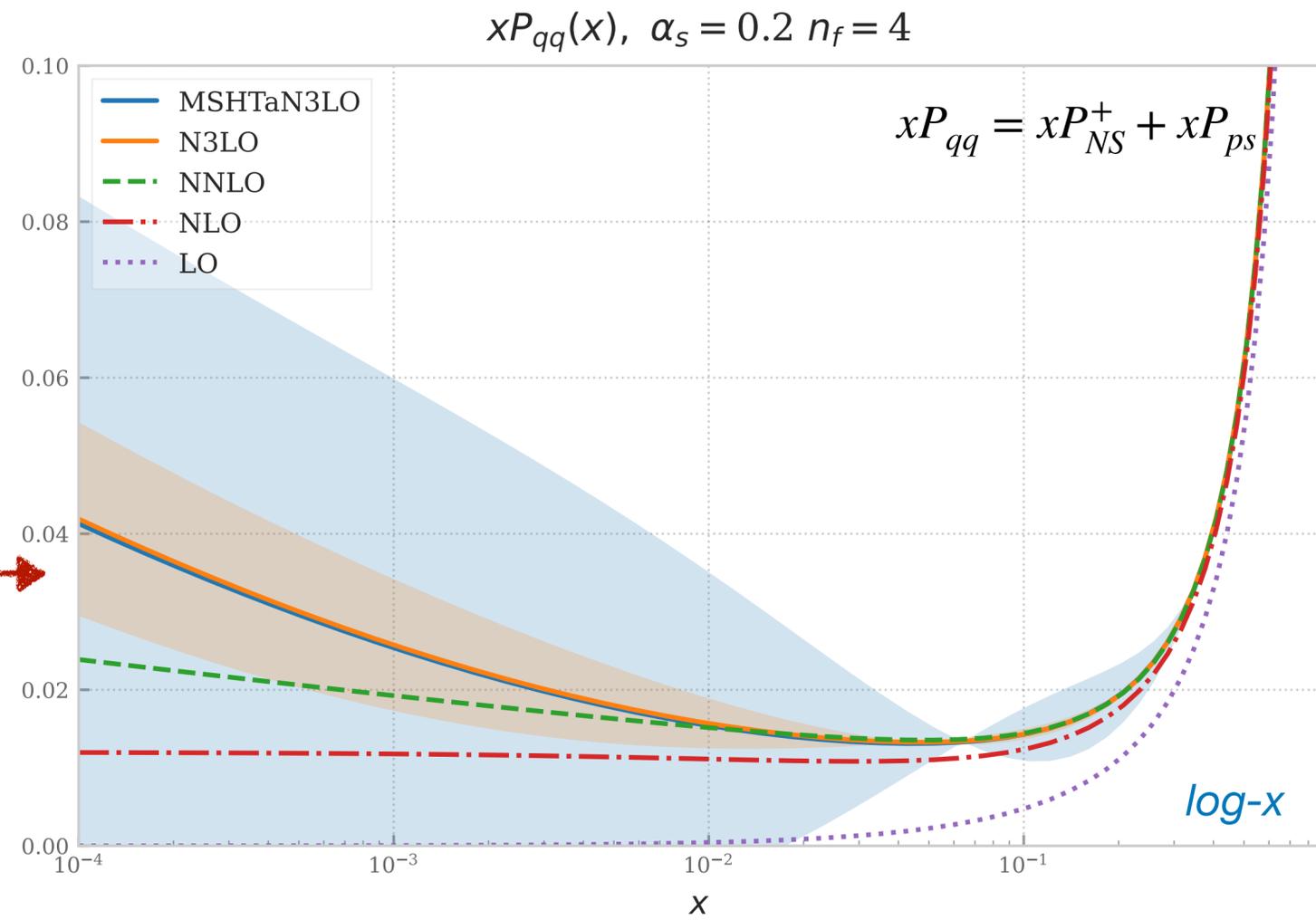
$xP_{qg}(x)$, $\alpha_s = 0.201$ $n_f = 5$



$xP_{qq}(x)$, $\alpha_s = 0.201$ $n_f = 5$

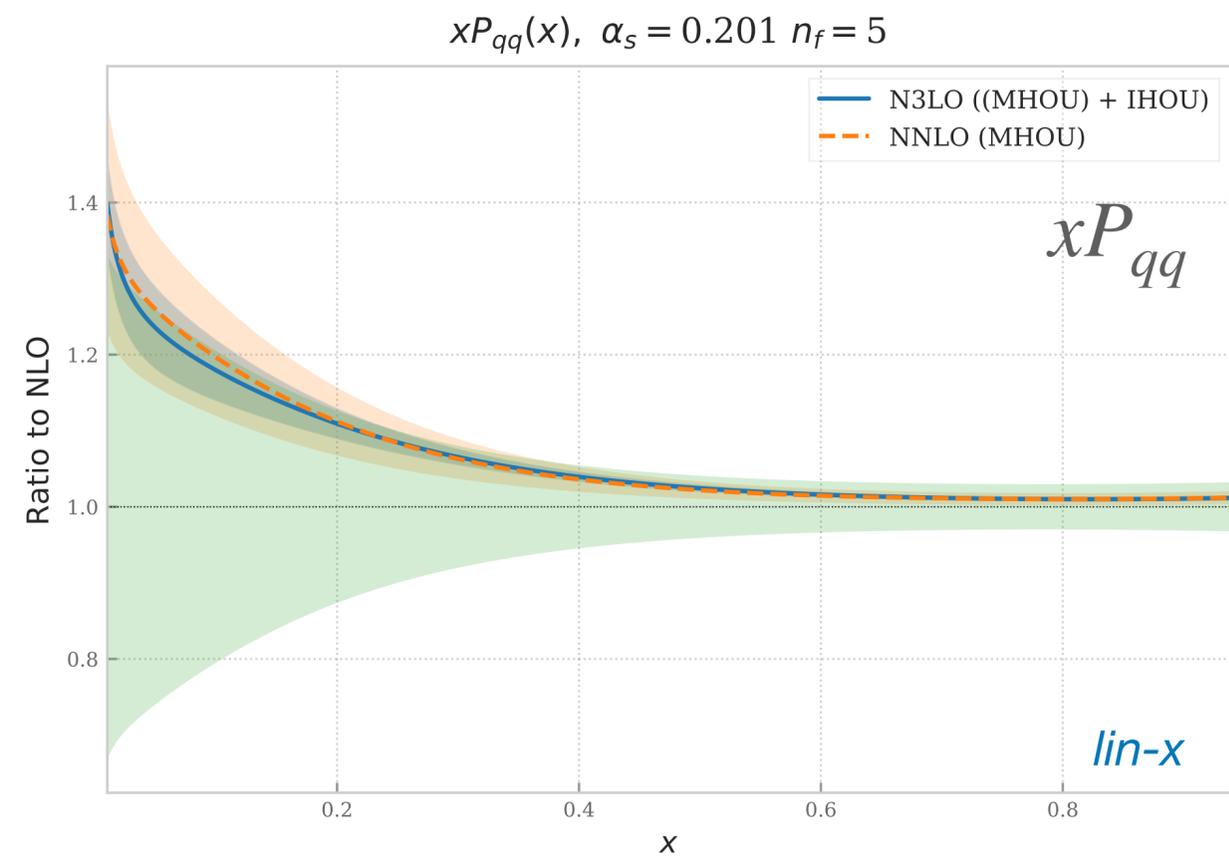
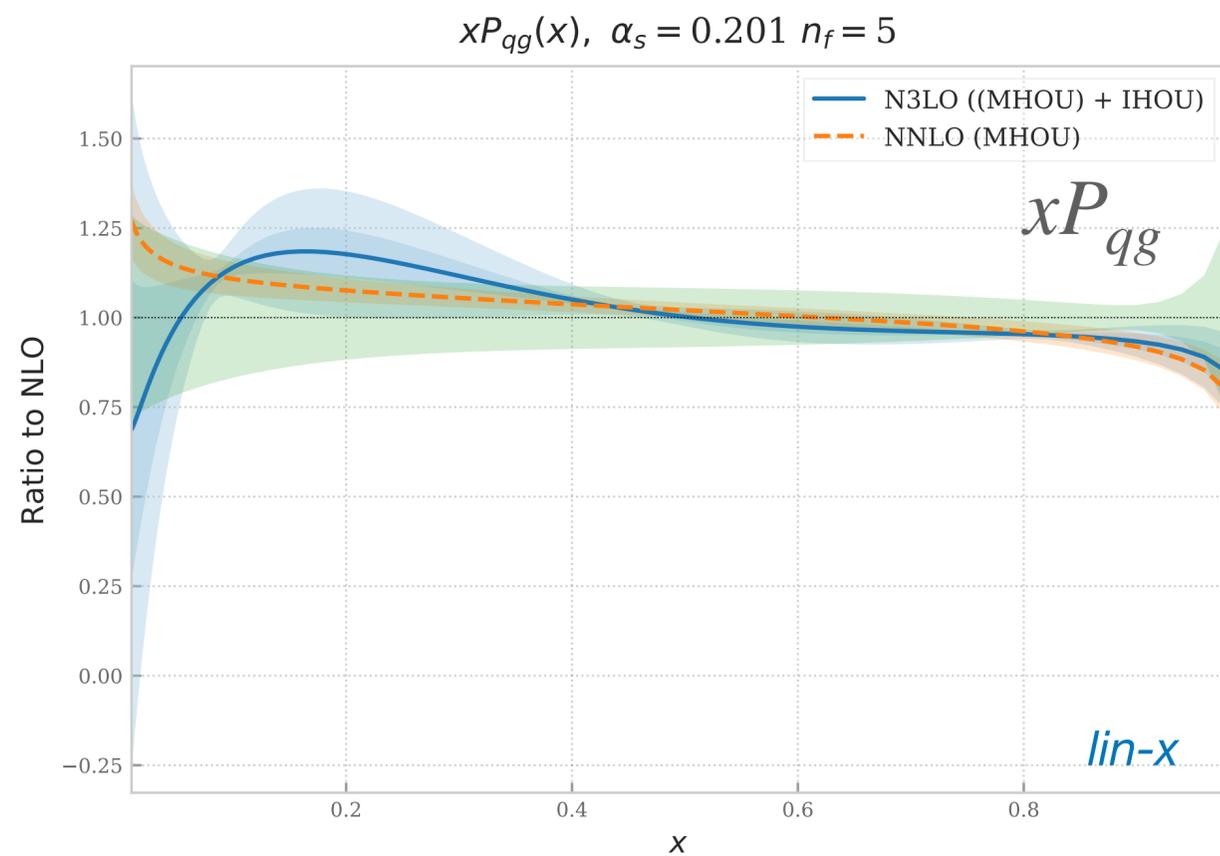
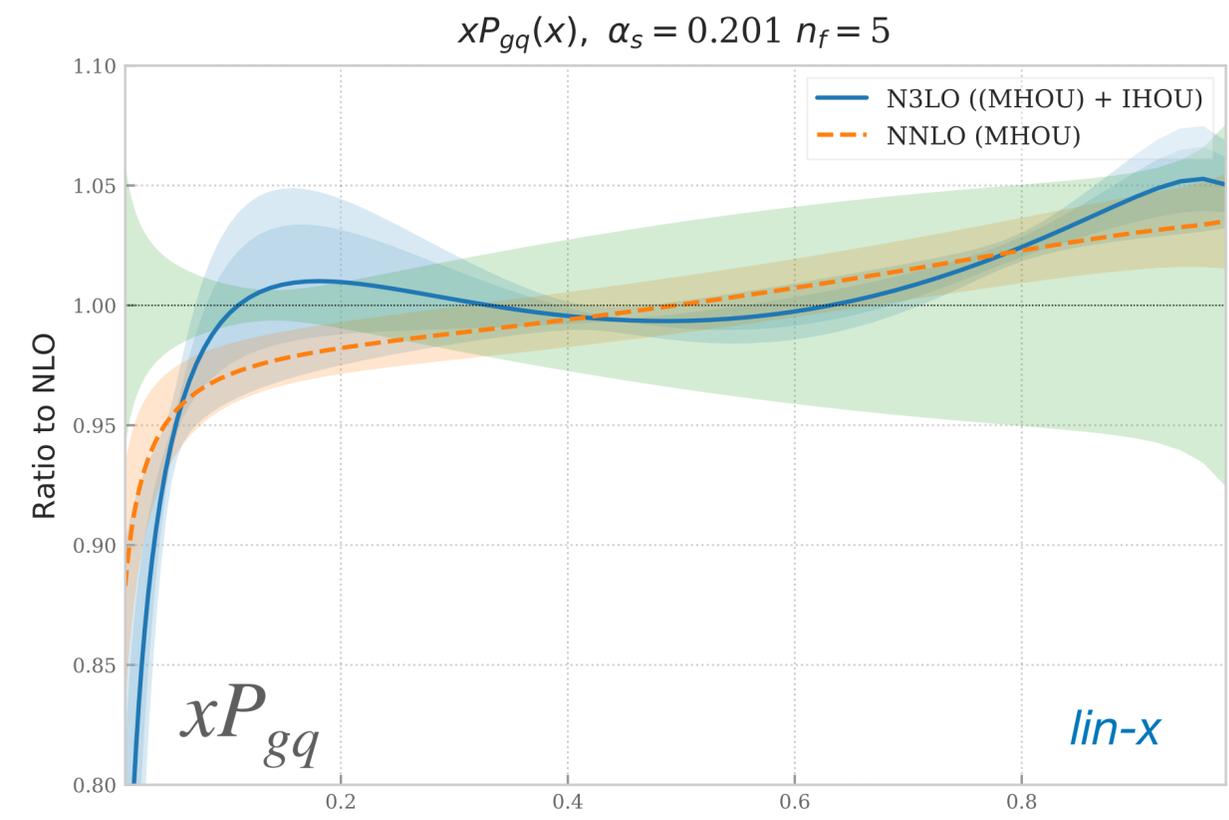
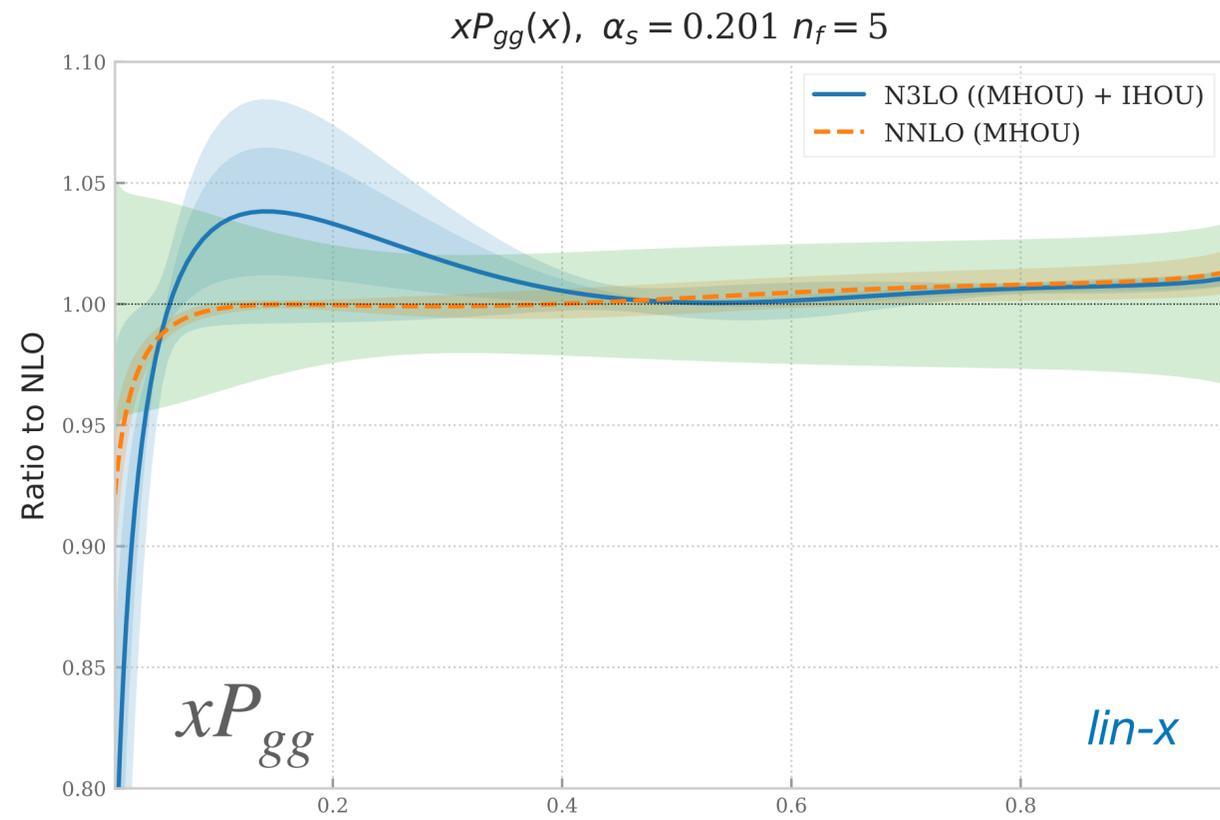


Splitting functions small-x



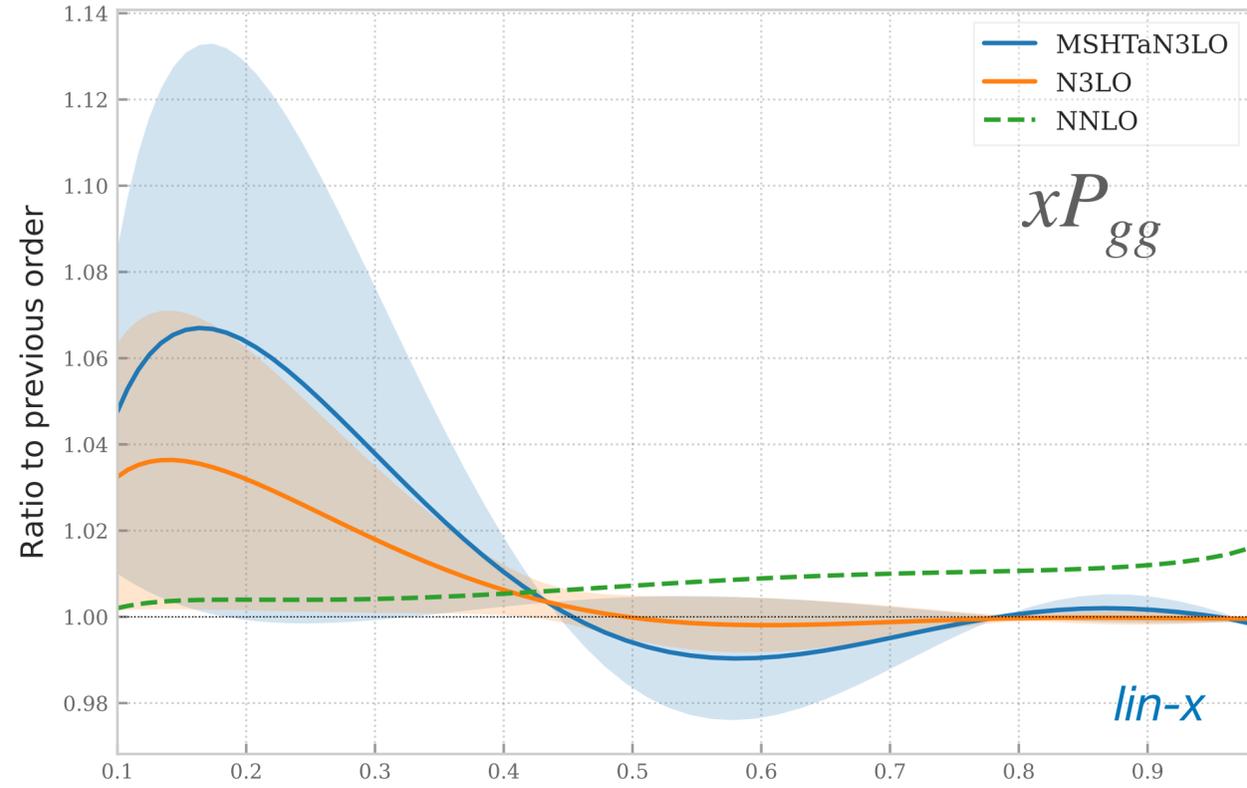
Splitting functions large- x

PRELIMINARY RESULTS



Comparison with MSHT large- x

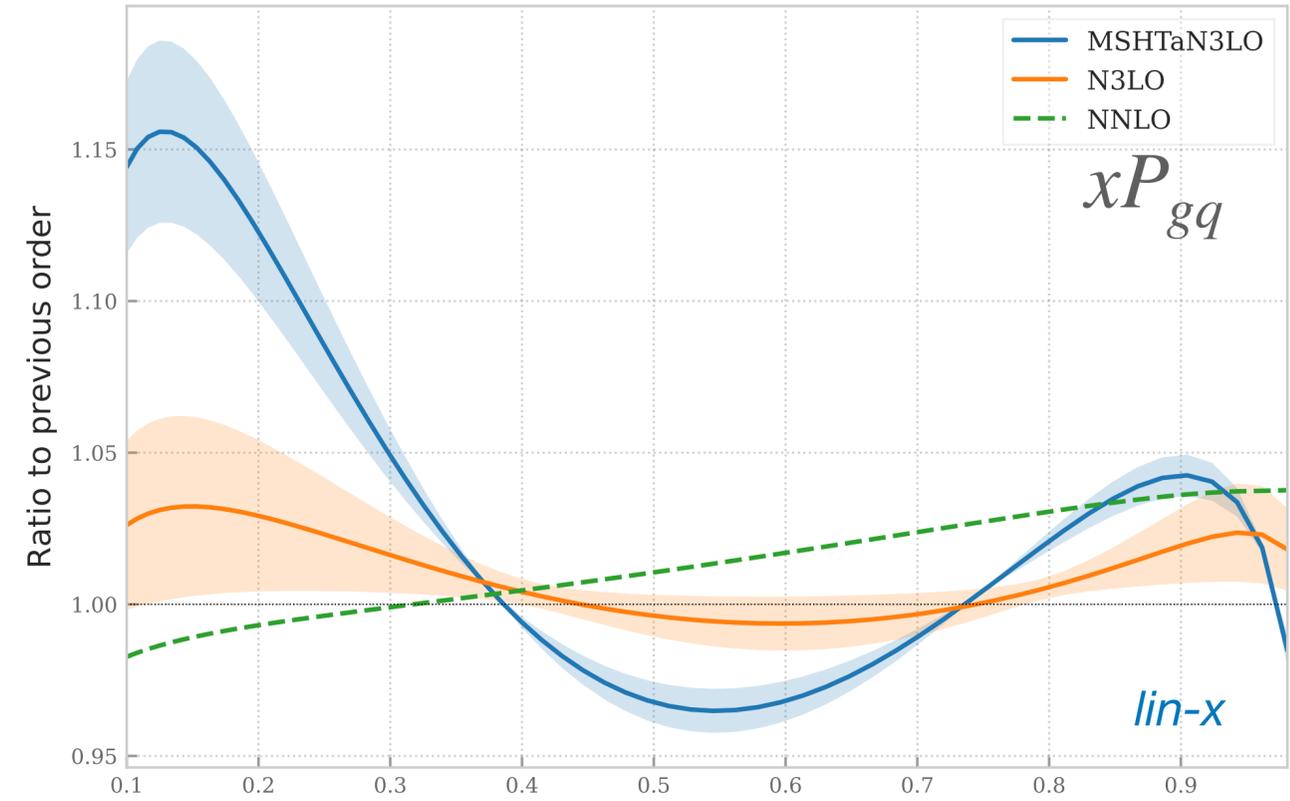
$xP_{gg}(x)$, $\alpha_s = 0.2$ $n_f = 4$



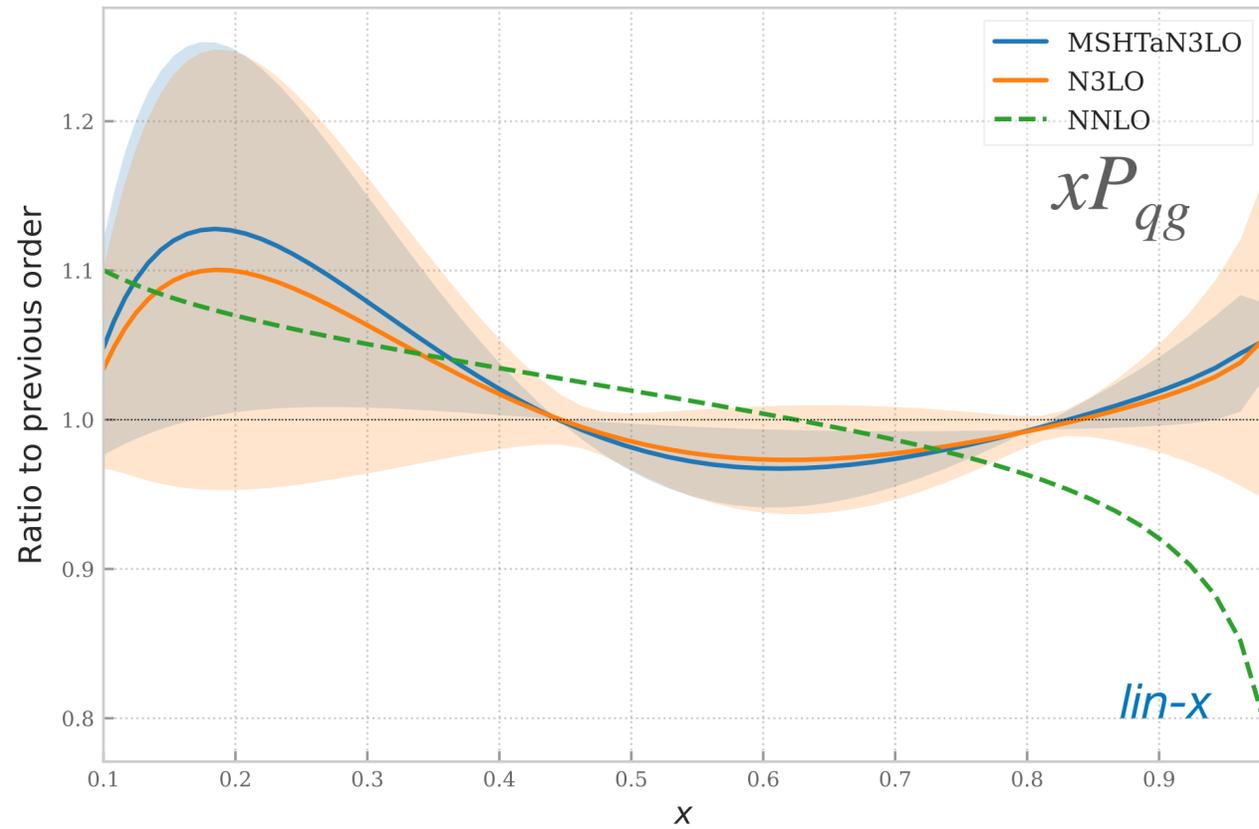
MSHTaN3LO: [arxiv:2207.04739]

PRELIMINARY RESULTS

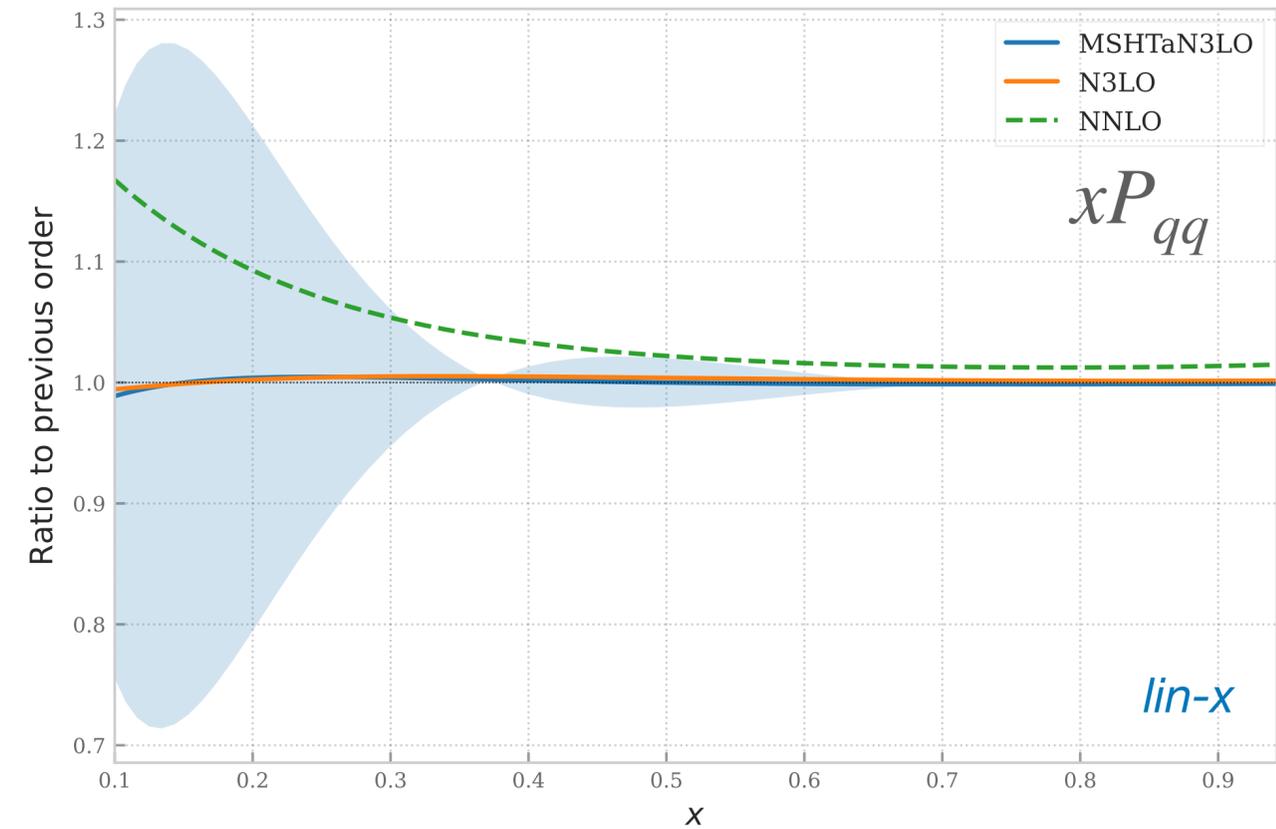
$xP_{gq}(x)$, $\alpha_s = 0.2$ $n_f = 4$



$xP_{qg}(x)$, $\alpha_s = 0.2$ $n_f = 4$



$xP_{qq}(x)$, $\alpha_s = 0.2$ $n_f = 4$

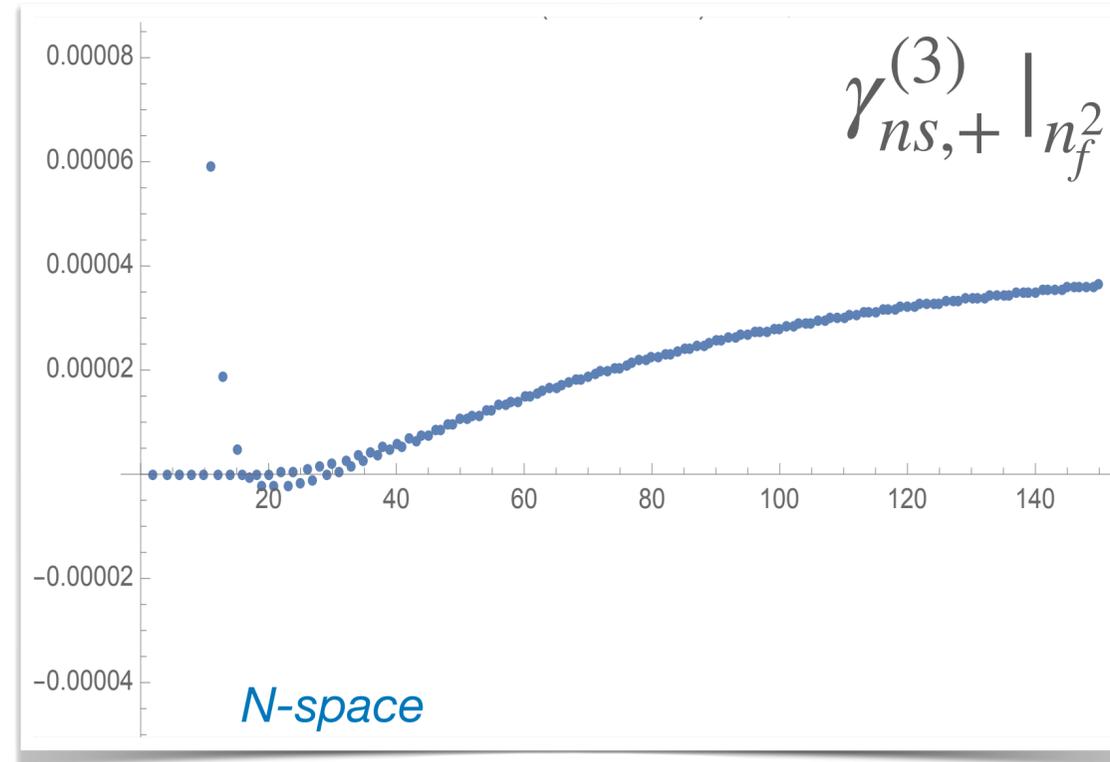


N3LO Splitting function [Non Singlet]

N3LO Non Singlet splitting functions
dependency on active flavors

	n_f^0	n_f^1	n_f^2	n_f^3
$\gamma_{ns,-}^{(3)}$	✓	✓	✓	✓
$\gamma_{ns,+}^{(3)}$	✓	✓	✓	✓
$\gamma_{ns,s}^{(3)}$		✓	✓	

Comparison w.r.t. known analytical part (%)



Main references:

- ▶ Moch, Ruijl, Ueda, Vermaseren, Vogt [arXiv:1707.08315].
- ▶ Davies, Vogt, Ruijl, Ueda, Vermaseren. [arXiv:1610.07477]
- ▶ Davies, Kom, Moch, Vogt . [arXiv:2202.10362].

Rule of thumb:
small-N → small-x,
large-N → large-x

- ▶ Estimation of the N3LO anomalous dimensions is based on the best available theoretical constraints:

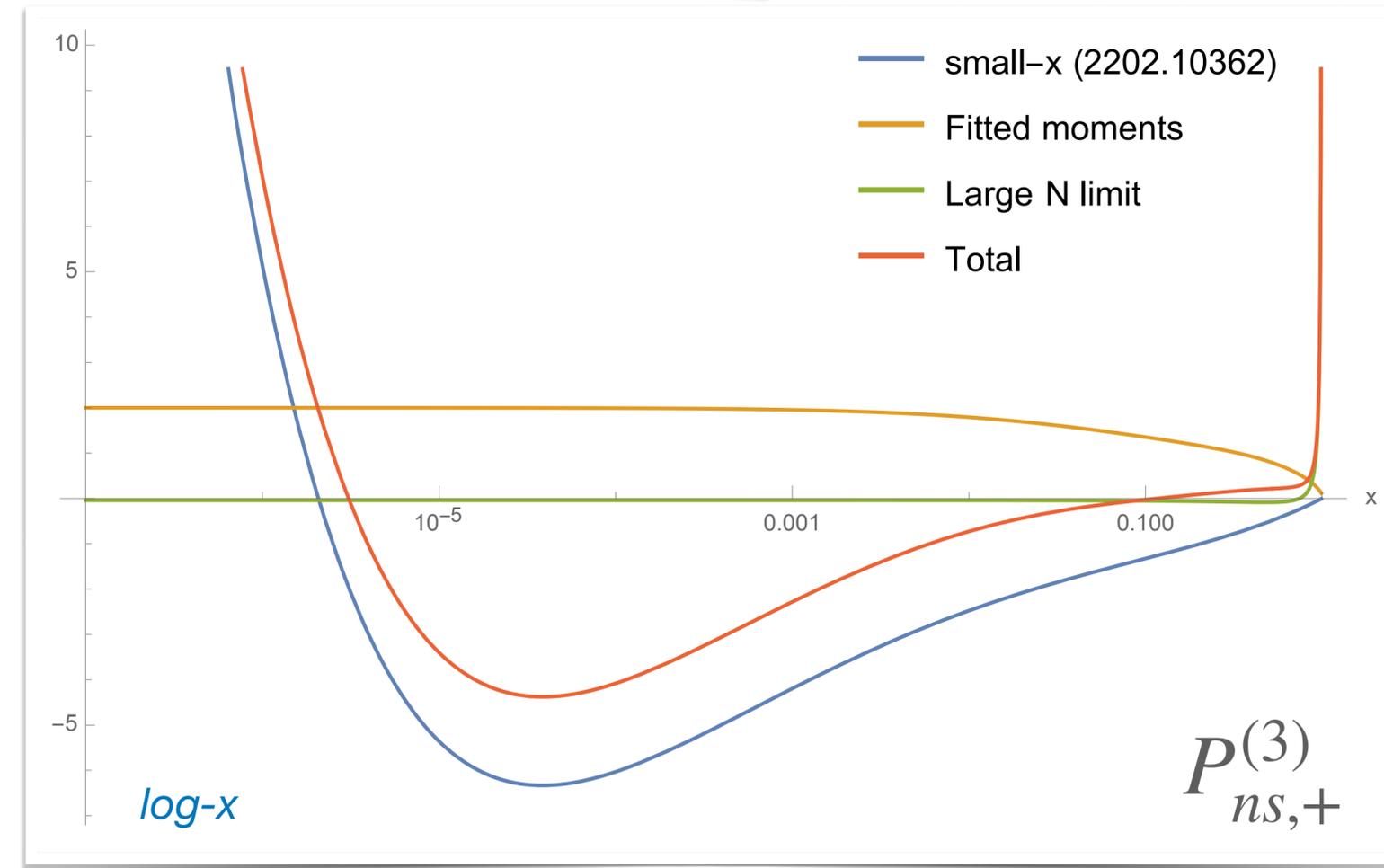
- large-N:
$$\gamma_{ns}^{(3)}(N \rightarrow \infty) \approx \Gamma_f S_1(N) + B + C \frac{S_1(N)}{N} + D \frac{1}{N} + \mathcal{O}\left(\frac{\ln(N)}{N^2}\right)$$

- small-N:
$$\gamma_{ns}^{(3)}(N \rightarrow 0) \approx \sum_{i=1}^7 C_i \frac{1}{N^i}$$

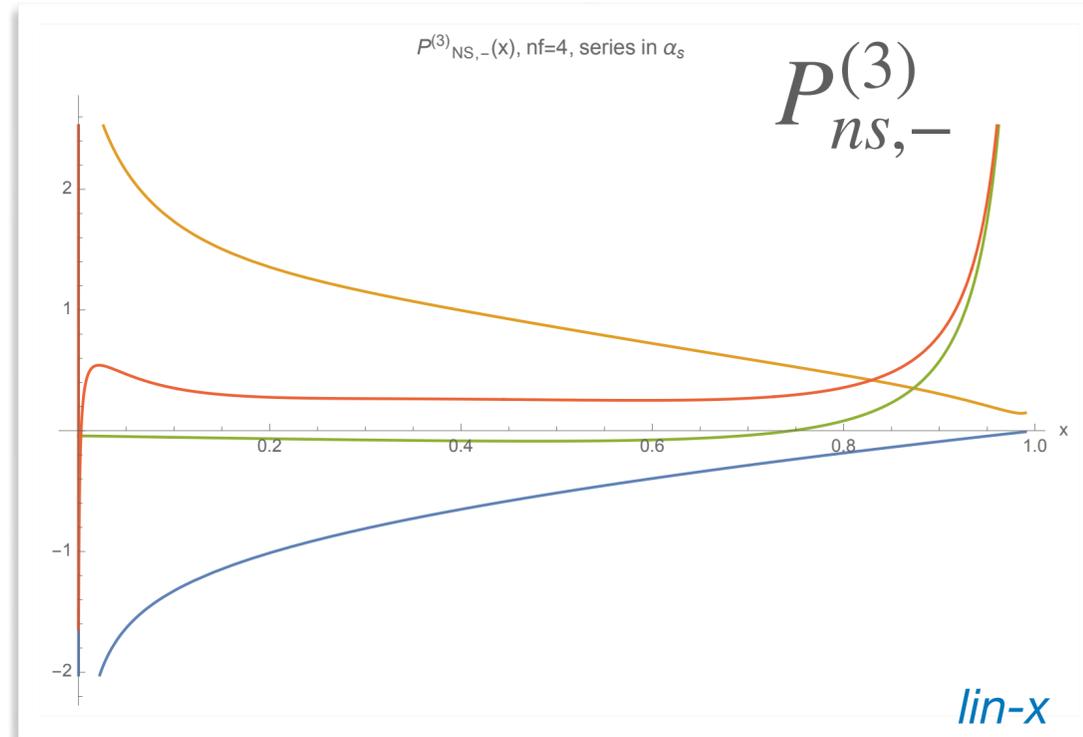
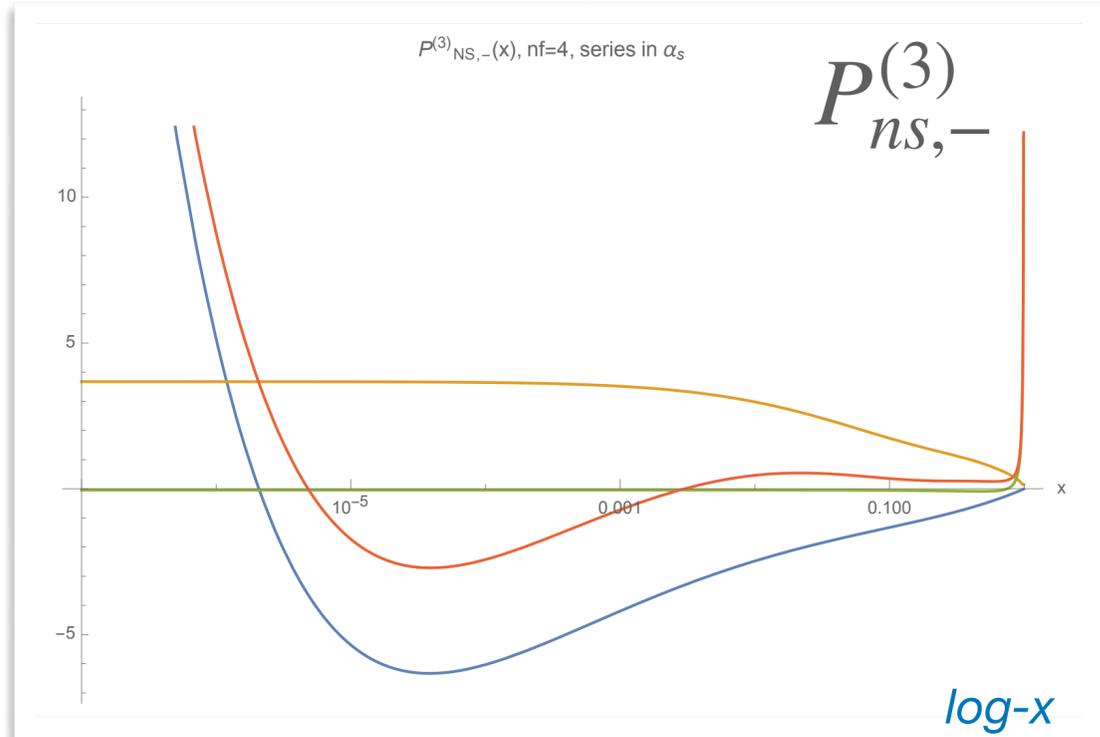
- 8 lowest Mellin moments

- ▶ For more details on the procedure used see [EKO N3LO ad documentation](#)

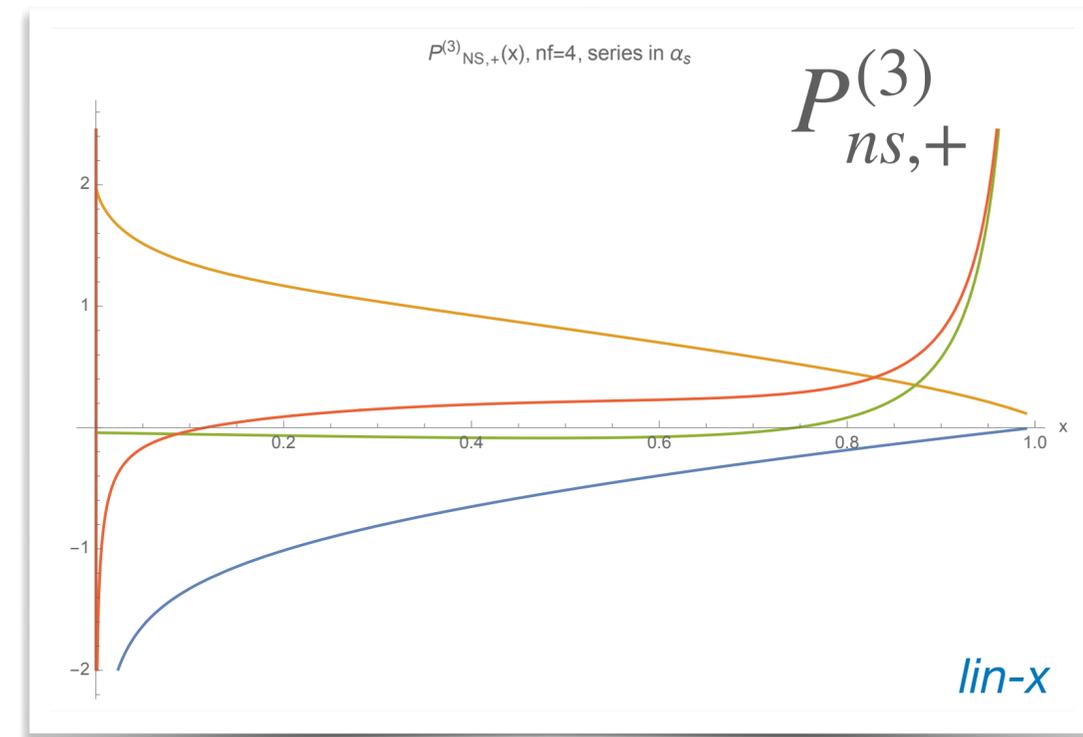
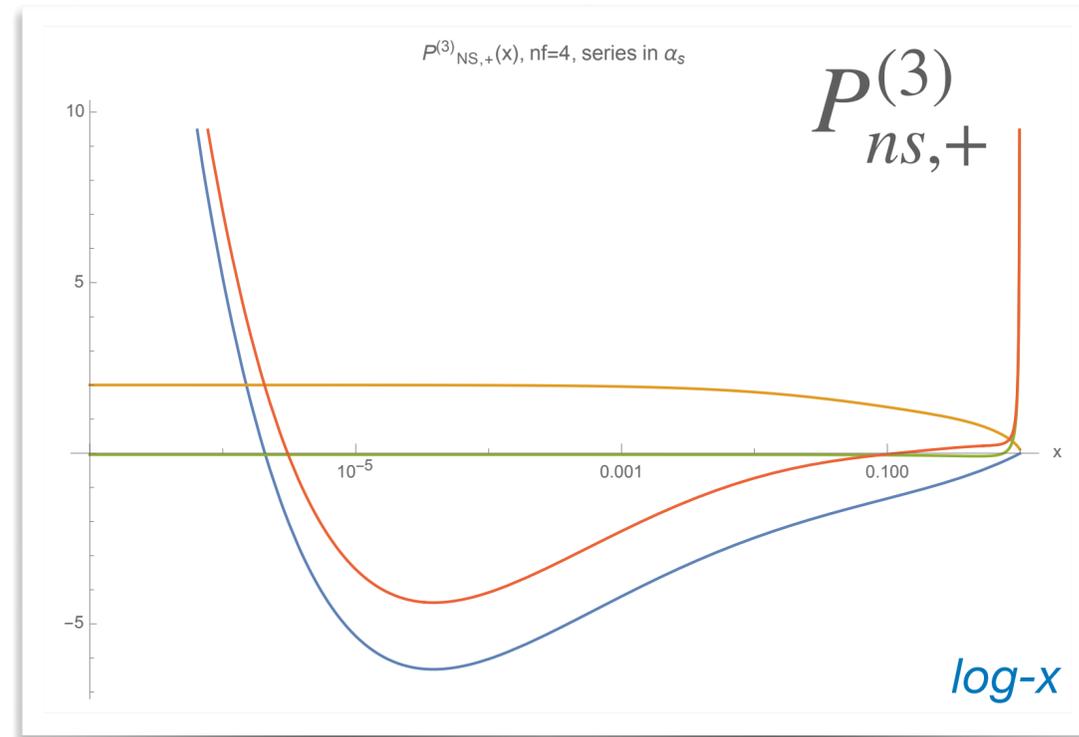
- ▶ Non singlet approximated spitting functions are compatible with the known analytical (and much more complex) parts within numerical accuracy.



N3LO Splitting function [Non Singlet]



- small-x (2202.10362)
- Fitted moments
- Large N limit
- Total

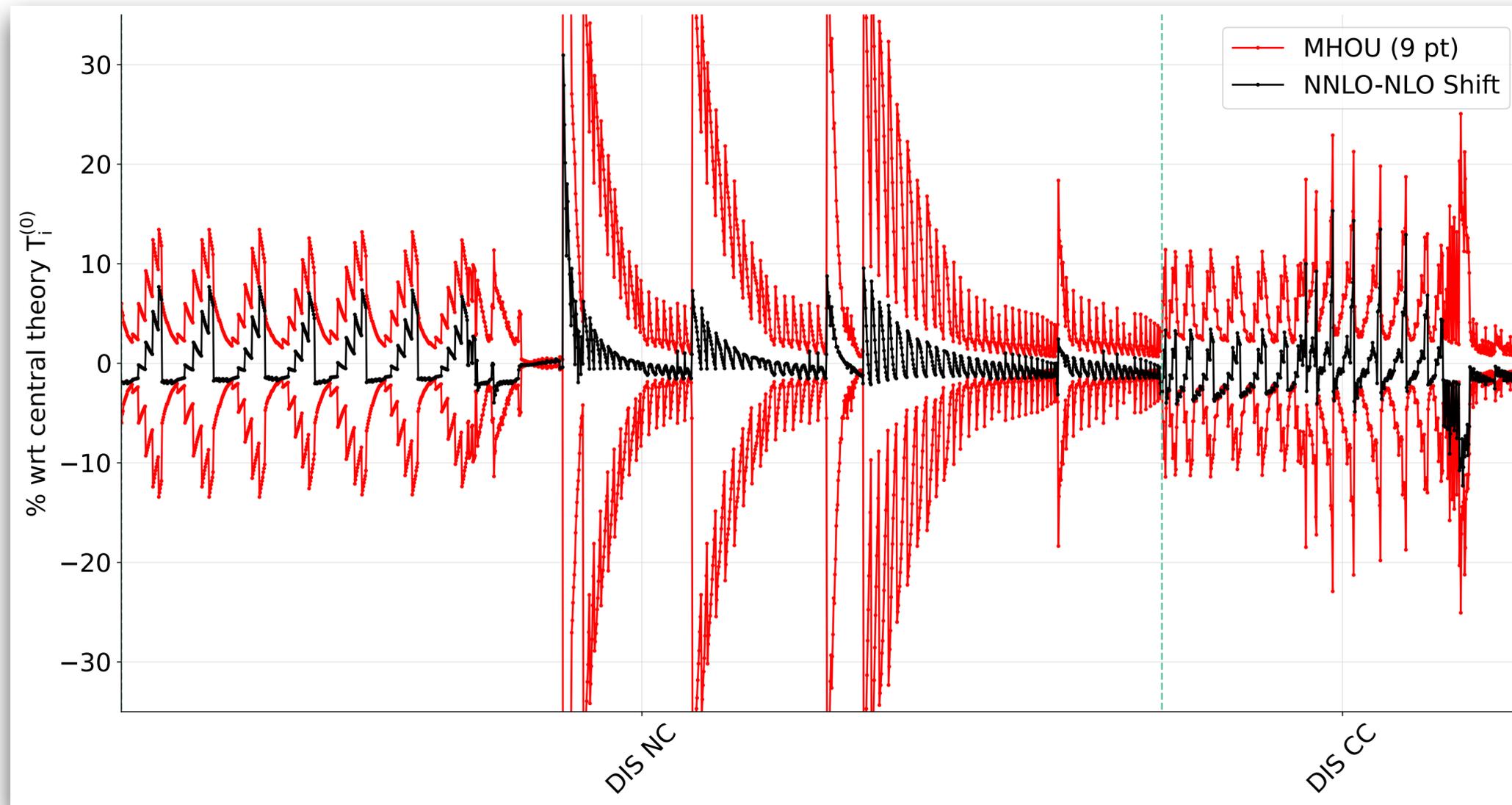


N3LO VFNS matching

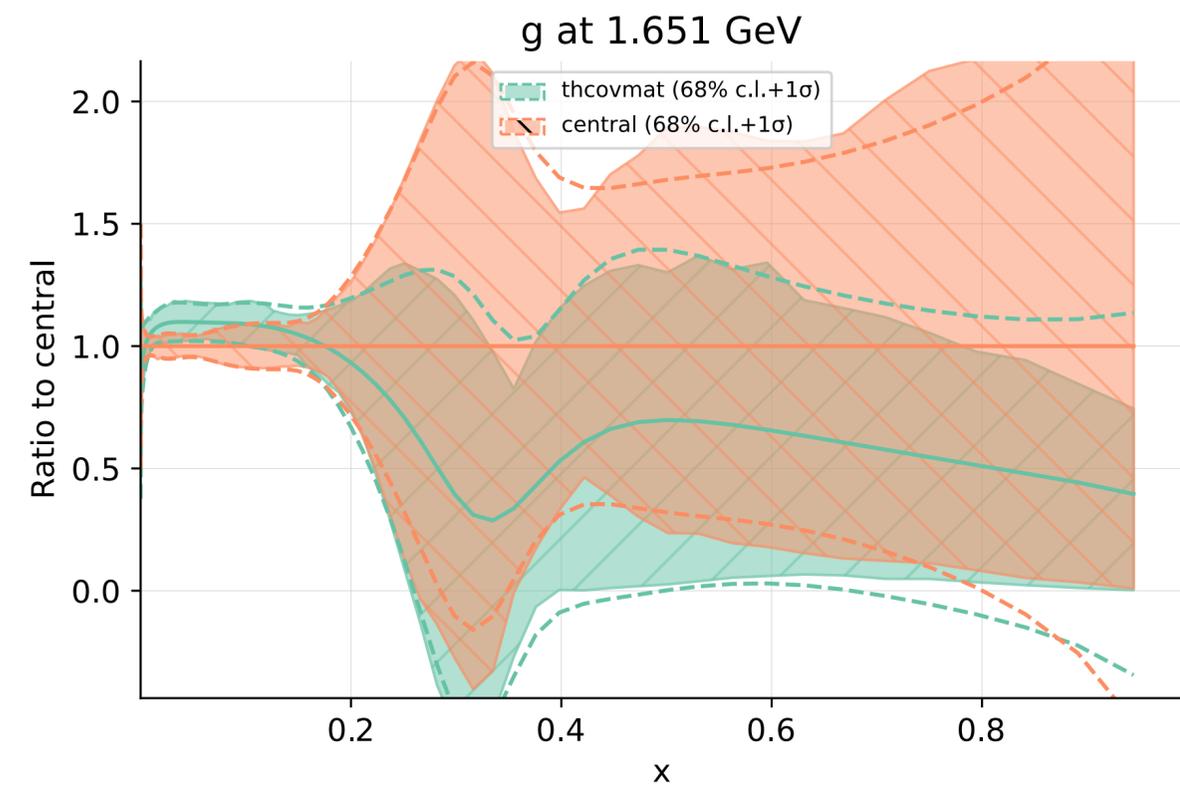
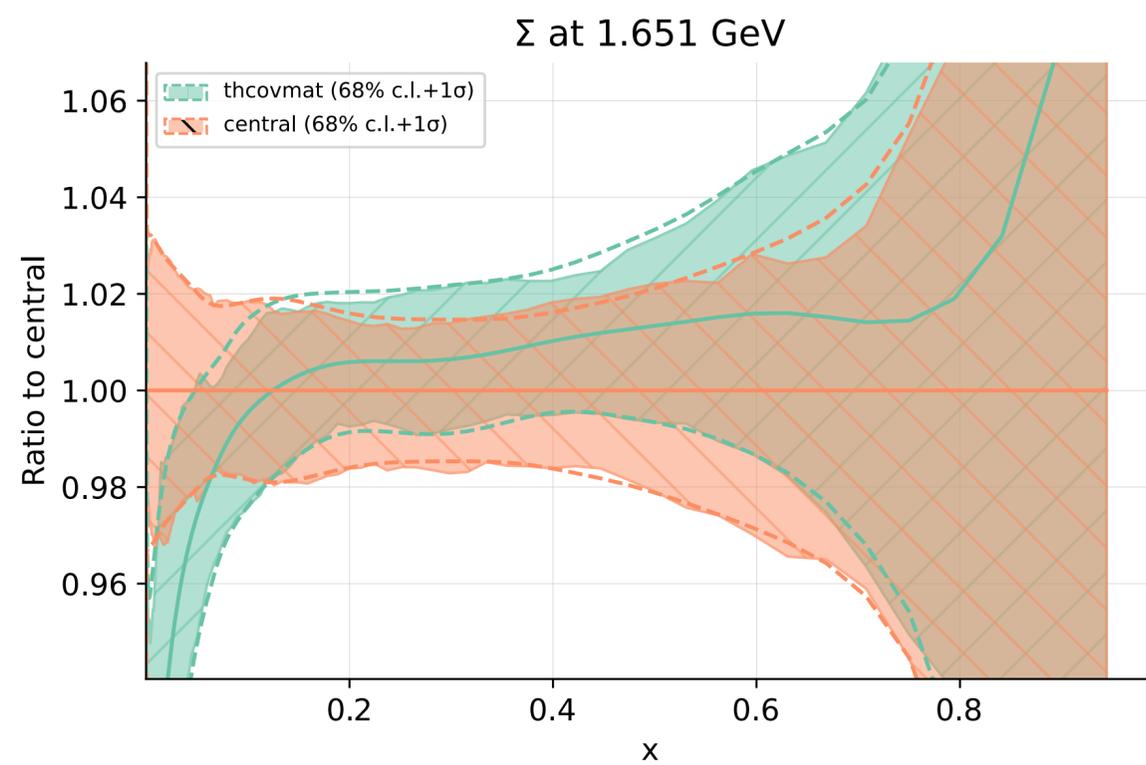
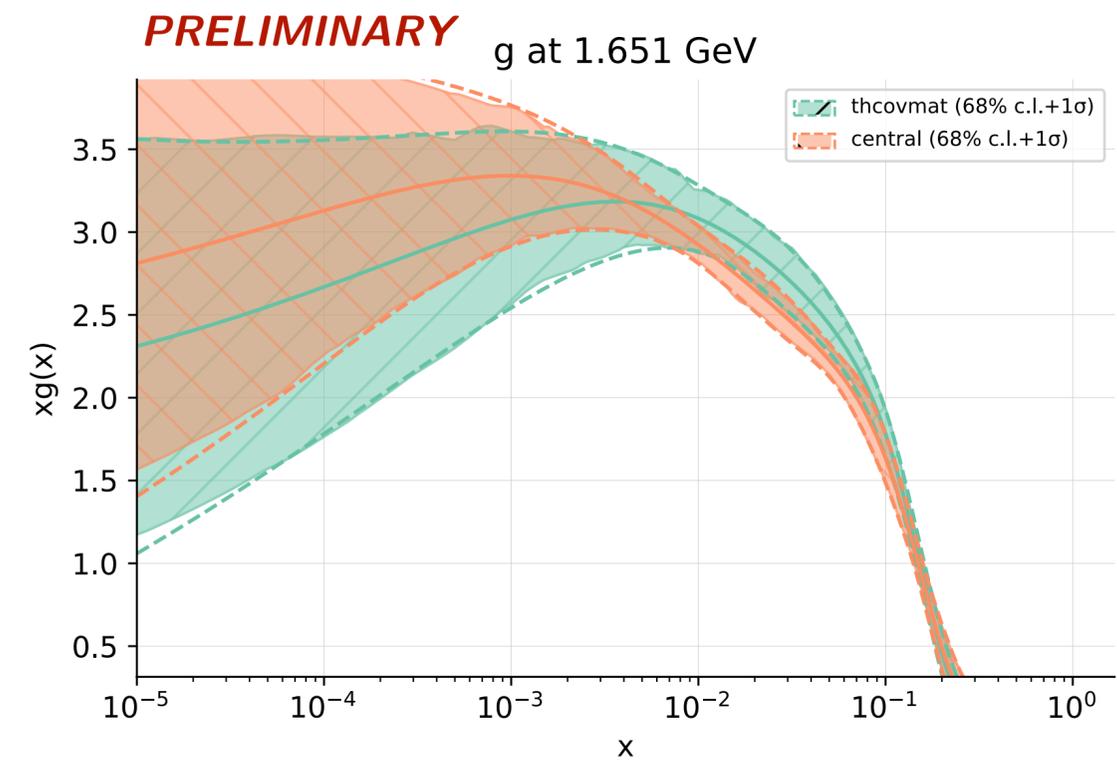
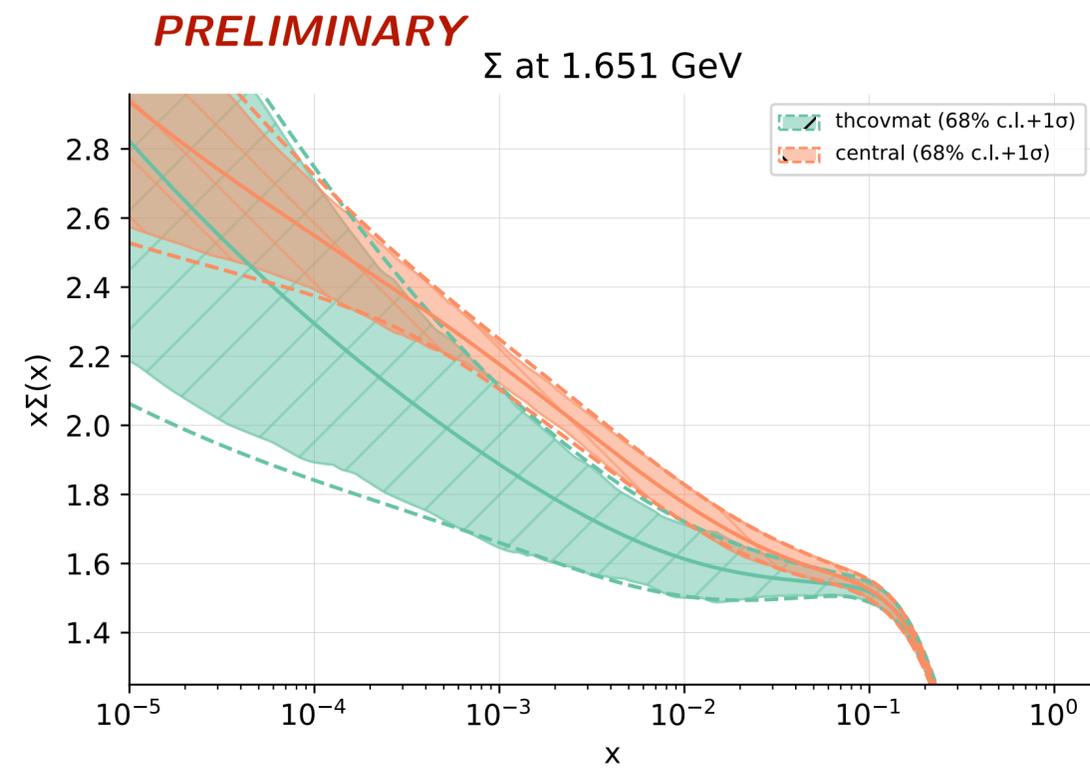
- ▶ I. Bierenbaum, J. Blümlein, and S. Klein. Mellin Moments of the $\mathcal{O}(\alpha_s^3)$ Heavy Flavor Contributions to unpolarized Deep-Inelastic Scattering at $Q^2 \gg m^2$ and Anomalous Dimensions. *Nucl. Phys. B*, 820:417–482, 2009. [arXiv:0904.3563](https://arxiv.org/abs/0904.3563), [doi:10.1016/j.nuclphysb.2009.06.005](https://doi.org/10.1016/j.nuclphysb.2009.06.005).
- ▶ J. Blümlein. Analytic continuation of mellin transforms up to two-loop order. *Computer Physics Communications*, 133(1):76–104, Dec 2000. URL: [http://dx.doi.org/10.1016/S0010-4655\(00\)00156-9](http://dx.doi.org/10.1016/S0010-4655(00)00156-9), [doi:10.1016/S0010-4655\(00\)00156-9](https://doi.org/10.1016/S0010-4655(00)00156-9).
- ▶ J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, C. Schneider, and F. Wißbrock. The 3-Loop Non-Singlet Heavy Flavor Contributions and Anomalous Dimensions for the Structure Function $F_2(x, Q^2)$ and Transversity. *Nucl. Phys. B*, 886:733–823, 2014. [arXiv:1406.4654](https://arxiv.org/abs/1406.4654), [doi:10.1016/j.nuclphysb.2014.07.010](https://doi.org/10.1016/j.nuclphysb.2014.07.010).
- ▶ J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. von Manteuffel, and C. Schneider. The 3-loop pure singlet heavy flavor contributions to the structure function $f_2(x, q^2)$ and the anomalous dimension. *Nuclear Physics B*, 890:48–151, Jan 2015. URL: <http://dx.doi.org/10.1016/j.nuclphysb.2014.10.008>, [doi:10.1016/j.nuclphysb.2014.10.008](https://doi.org/10.1016/j.nuclphysb.2014.10.008).
- ▶ J. Ablinger, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, and C. Schneider. The $\mathcal{O}(\alpha_s^3 T_f^2)$ contributions to the Gluonic Operator Matrix Element. *Nucl. Phys. B*, 885:280–317, 2014. [arXiv:1405.4259](https://arxiv.org/abs/1405.4259), [doi:10.1016/j.nuclphysb.2014.05.028](https://doi.org/10.1016/j.nuclphysb.2014.05.028).
- ▶ J. Ablinger, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, C. Schneider, and F. Wißbrock. The transition matrix element $a_{gq}(n)$ of the variable flavor number scheme at $\mathcal{O}(\alpha_s^3)$. *Nuclear Physics B*, 882:263–288, May 2014. URL: <http://dx.doi.org/10.1016/j.nuclphysb.2014.02.007>, [doi:10.1016/j.nuclphysb.2014.02.007](https://doi.org/10.1016/j.nuclphysb.2014.02.007).
- ▶ A. Behring, I. Bierenbaum, J. Blümlein, A. De Freitas, S. Klein, and F. Wißbrock. The logarithmic contributions to the $\mathcal{O}(\alpha_s^3)$ asymptotic massive Wilson coefficients and operator matrix elements in deeply inelastic scattering. *Eur. Phys. J. C*, 74(9):3033, 2014. [arXiv:1403.6356](https://arxiv.org/abs/1403.6356), [doi:10.1140/epjc/s10052-014-3033-x](https://doi.org/10.1140/epjc/s10052-014-3033-x).
- ▶ J. Ablinger, J. Blümlein, S. Klein, C. Schneider, and F. Wissbrock. The $\mathcal{O}(\alpha_s^3)$ Massive Operator Matrix Elements of $\mathcal{O}(n_f)$ for the Structure Function $F_2(x, Q^2)$ and Transversity. *Nucl. Phys. B*, 844:26–54, 2011. [arXiv:1008.3347](https://arxiv.org/abs/1008.3347), [doi:10.1016/j.nuclphysb.2010.10.021](https://doi.org/10.1016/j.nuclphysb.2010.10.021).
- ▶ J. Blümlein, J. Ablinger, A. Behring, A. De Freitas, A. von Manteuffel, and C. Schneider. Heavy Flavor Wilson Coefficients in Deep-Inelastic Scattering: Recent Results. *PoS, QCDEV2017:031*, 2017. [arXiv:1711.07957](https://arxiv.org/abs/1711.07957), [doi:10.22323/1.308.0031](https://doi.org/10.22323/1.308.0031).
- ▶ J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. Goedicke, A. von Manteuffel, C. Schneider and K. Schonwald. The Unpolarized and Polarized Single-Mass Three-Loop Heavy Flavor Operator Matrix Elements $A_{gg,Q}$ and $\Delta A_{gg,Q}$ [arXiv:2211.0546](https://arxiv.org/abs/2211.0546)

Impact of MHOU theory uncertainties

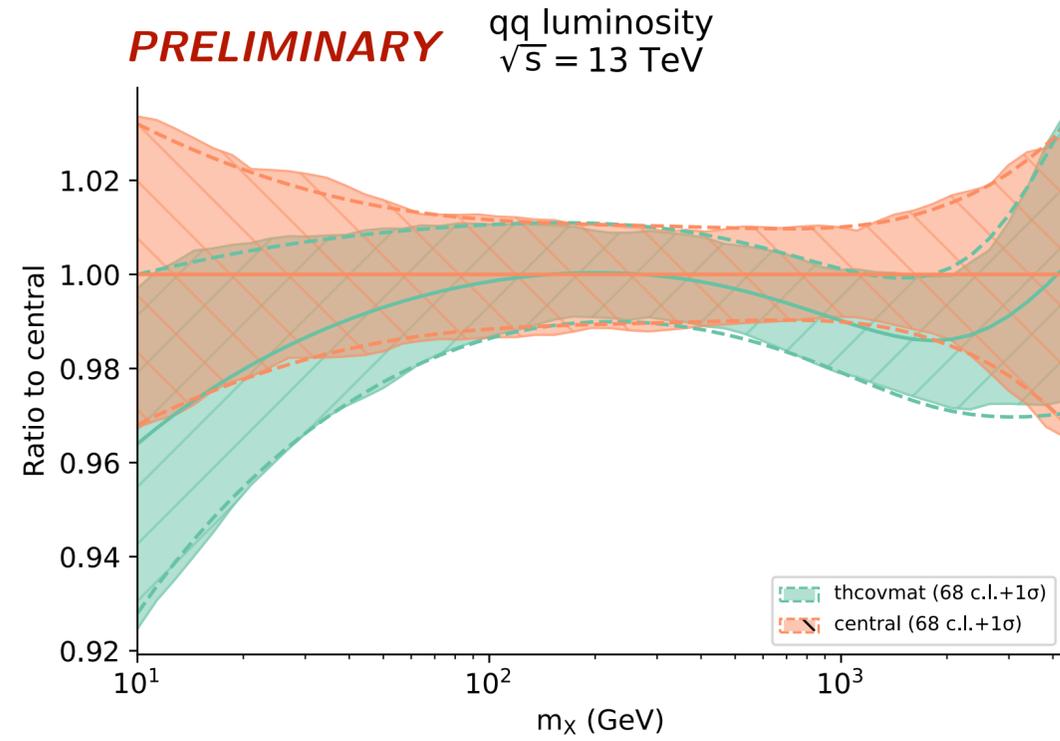
Are MHOU able to compatible with the exact shift from higher order QCD corrections?



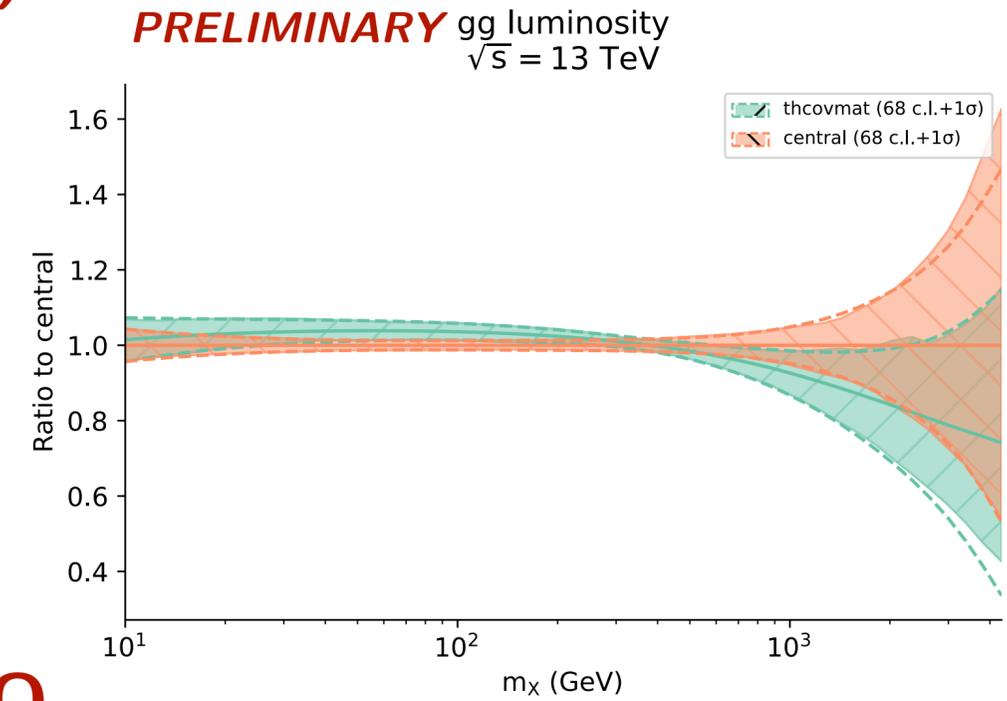
Impact of MHOU



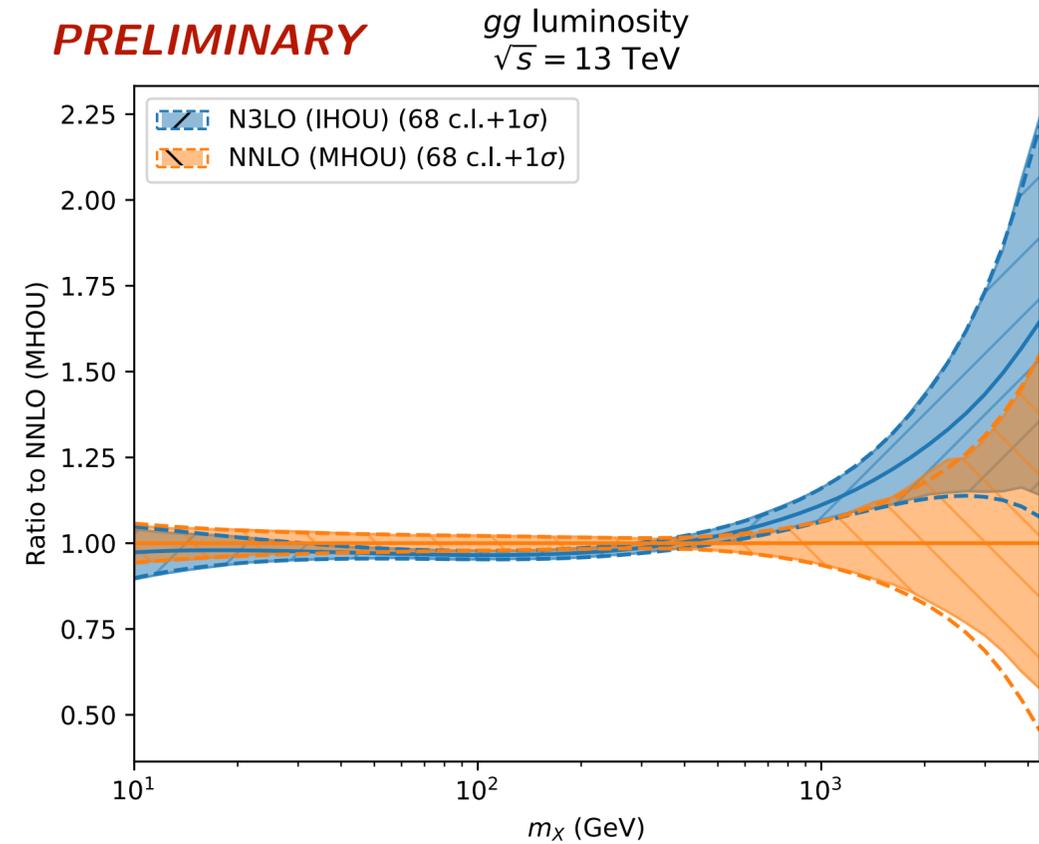
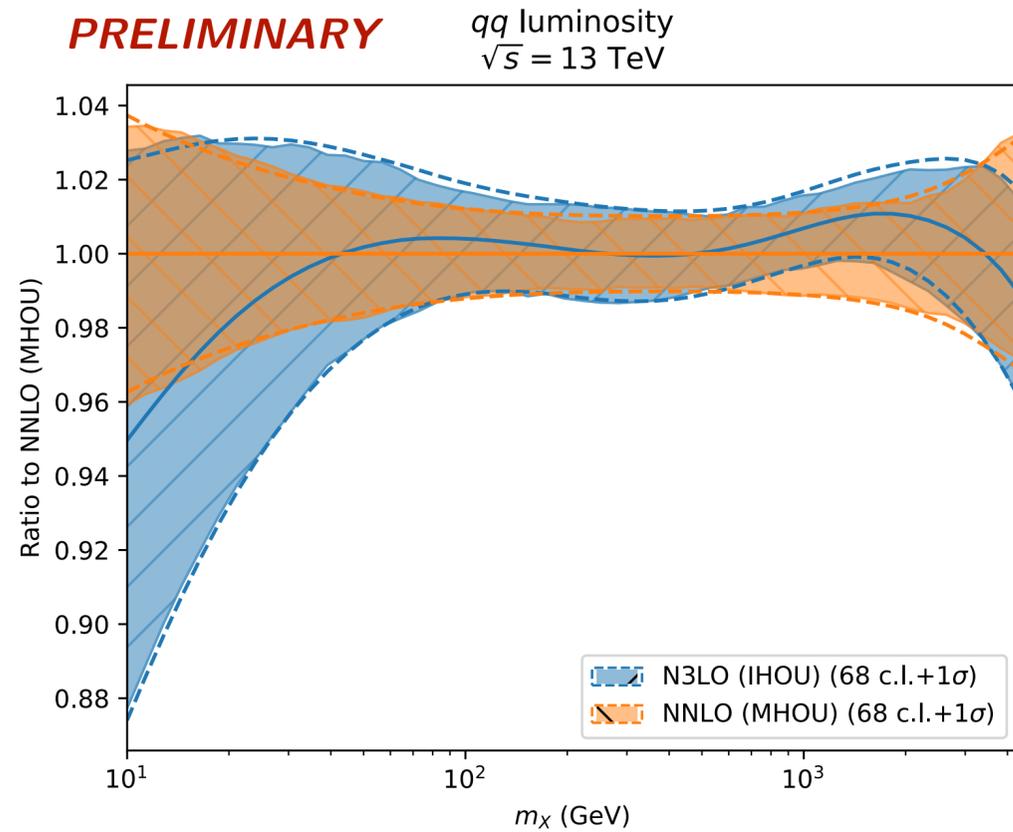
Luminosities DIS only



NNLO (MHOU)



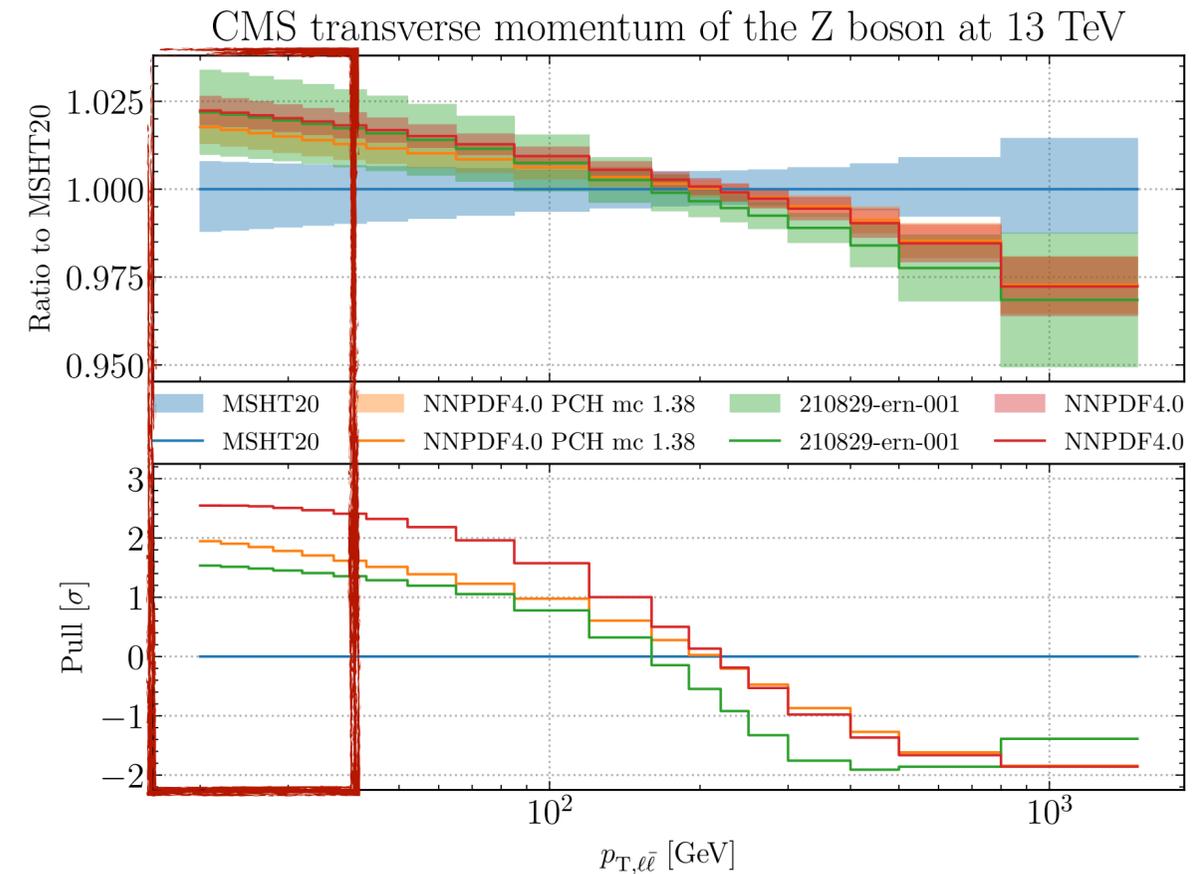
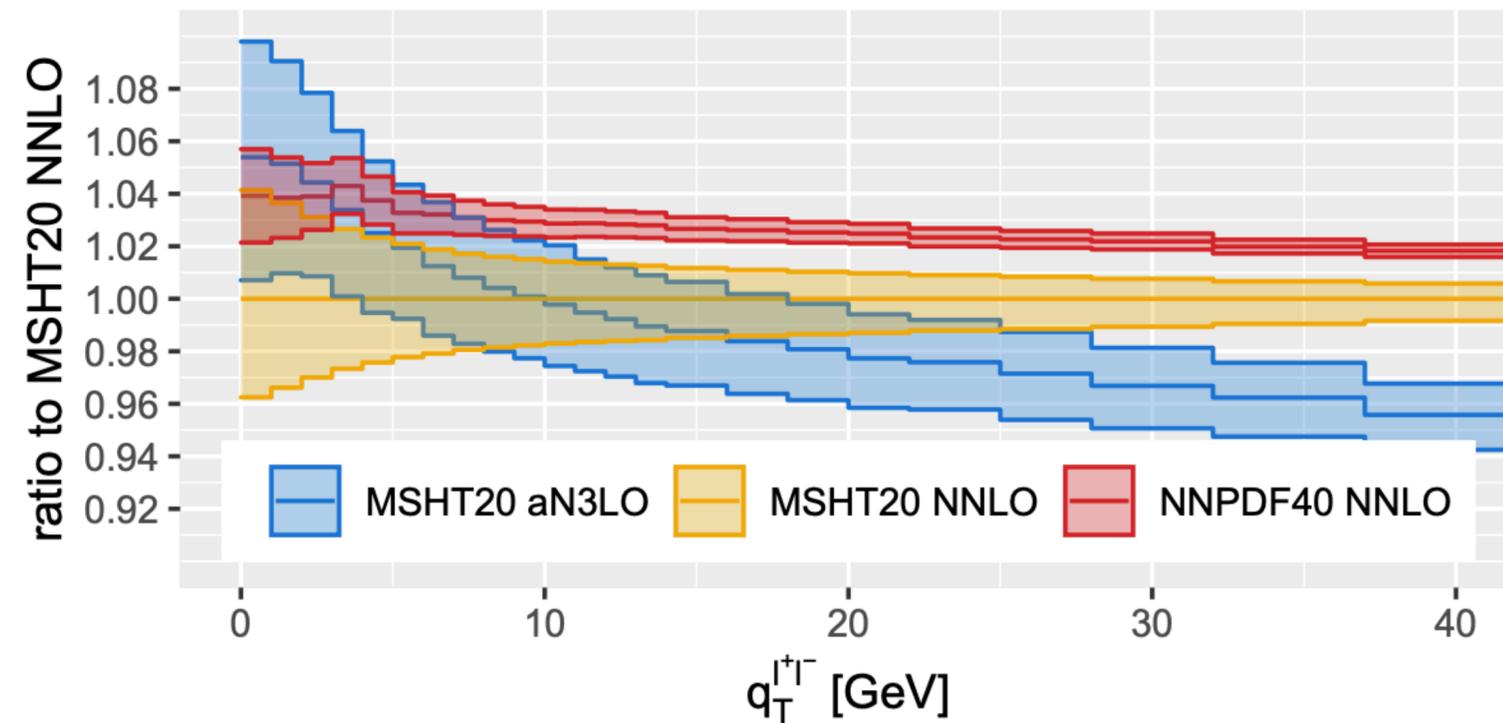
NNLO vs aN3LO



Drell Yan Z transverse momentum

Impact of PDFs uncertainties in the $p_T^{\ell\ell}$ distribution at $N3LO + N^4LL$
with CMS fiducial phase space

From Neumann, Campbell [\[arxiv:2207.07056\]](https://arxiv.org/abs/2207.07056)



Differences can originate by:

1. Fitting methodology
2. Charm mass value and treatment.
3. Different dataset variations.