Factorization of Lepton Radiation in SIDIS

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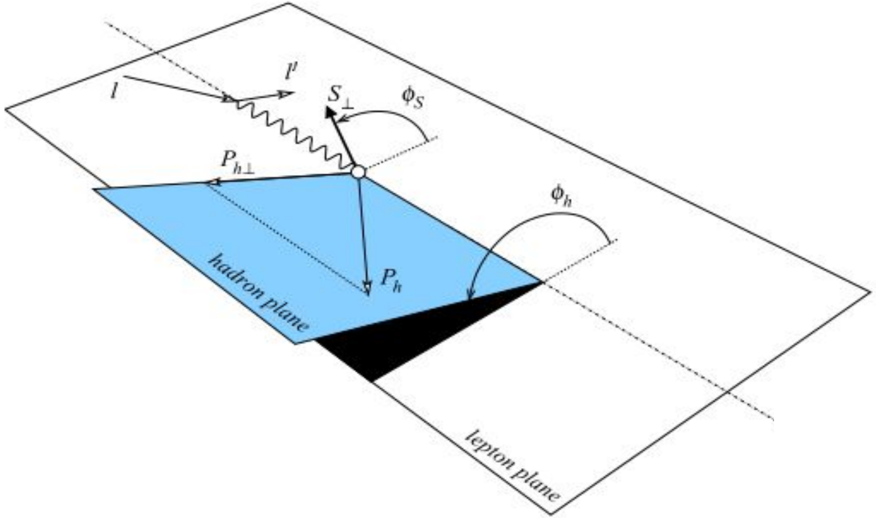
Semi-Inclusive DIS: Frontier of Hadron Structure

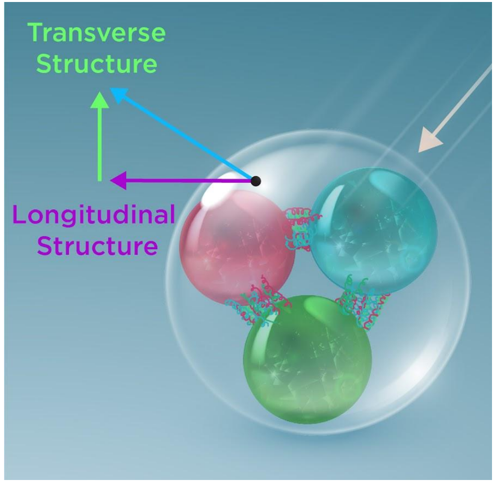
EIC²

Jefferson Lab

COMPASS

hermes

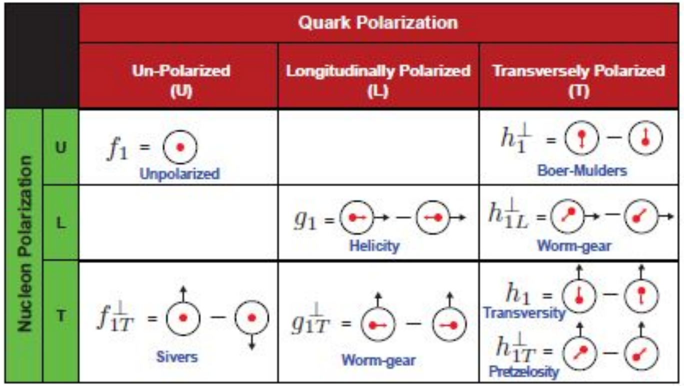


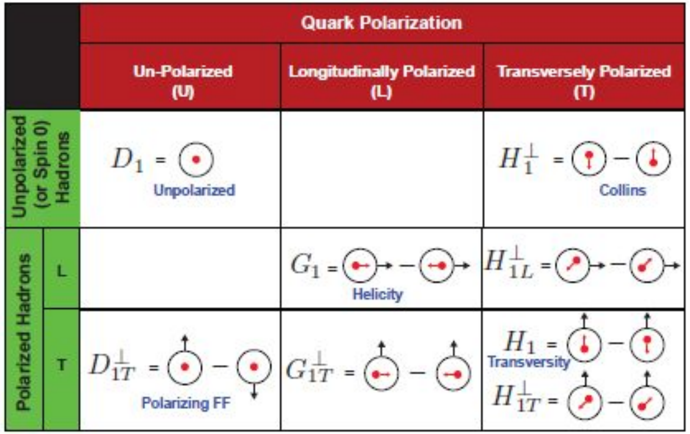


SIDIS Cross Section

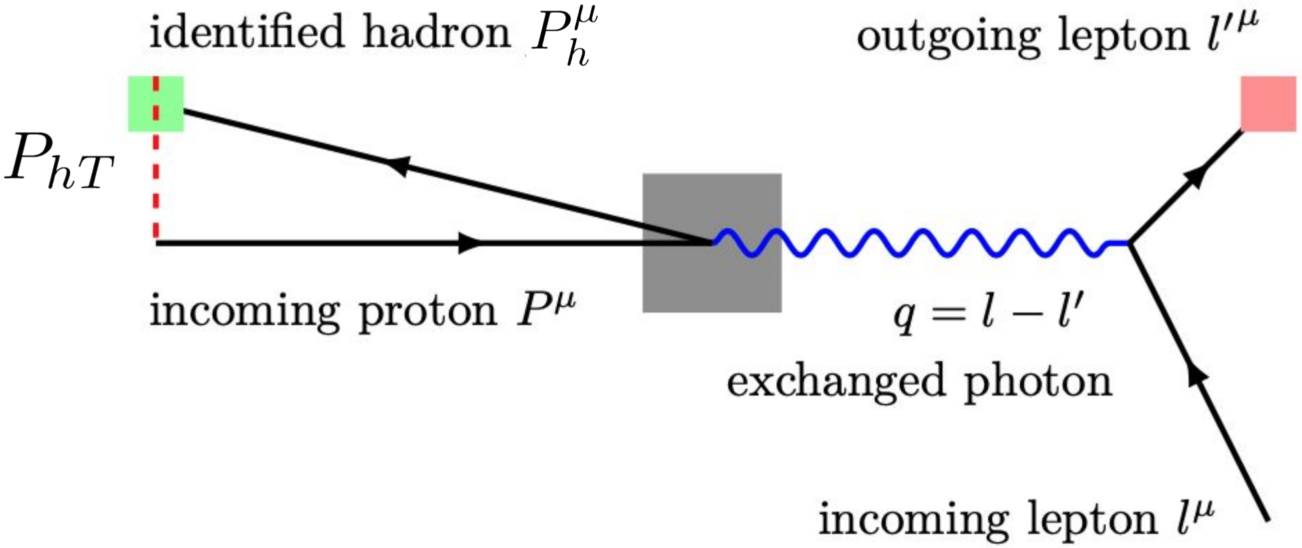
$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{\boxed{F_{UU,T}} + \varepsilon\,\boxed{F_{UU,L}} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,\boxed{F_{UU}^{\cos\phi_h}} \\
&\quad + \varepsilon\cos(2\phi_h)\,\boxed{F_{UU}^{\cos 2\phi_h}} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,\boxed{F_{LU}^{\sin\phi_h}} \\
&\quad + S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,\boxed{F_{UL}^{\sin\phi_h}} + \varepsilon\sin(2\phi_h)\,\boxed{F_{UL}^{\sin 2\phi_h}}\right] \\
&\quad + S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,\boxed{F_{LL}} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,\boxed{F_{LL}^{\cos\phi_h}}\right] \\
&\quad + |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(\boxed{F_{UT,T}^{\sin(\phi_h-\phi_S)}} + \varepsilon\,\boxed{F_{UT,L}^{\sin(\phi_h-\phi_S)}}\right) \\
&\qquad + \varepsilon\,\sin(\phi_h+\phi_S)\,\boxed{F_{UT}^{\sin(\phi_h+\phi_S)}} + \varepsilon\,\sin(3\phi_h-\phi_S)\,\boxed{F_{UT}^{\sin(3\phi_h-\phi_S)}} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,\boxed{F_{UT}^{\sin\phi_S}} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,\boxed{F_{UT}^{\sin(2\phi_h-\phi_S)}}\Big] \\
&\quad + |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,\boxed{F_{LT}^{\cos(\phi_h-\phi_S)}} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,\boxed{F_{LT}^{\cos\phi_S}} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,\boxed{F_{LT}^{\cos(2\phi_h-\phi_S)}}\Big]\Big\},
\end{aligned}
$$

Bacchetta et al., 2004



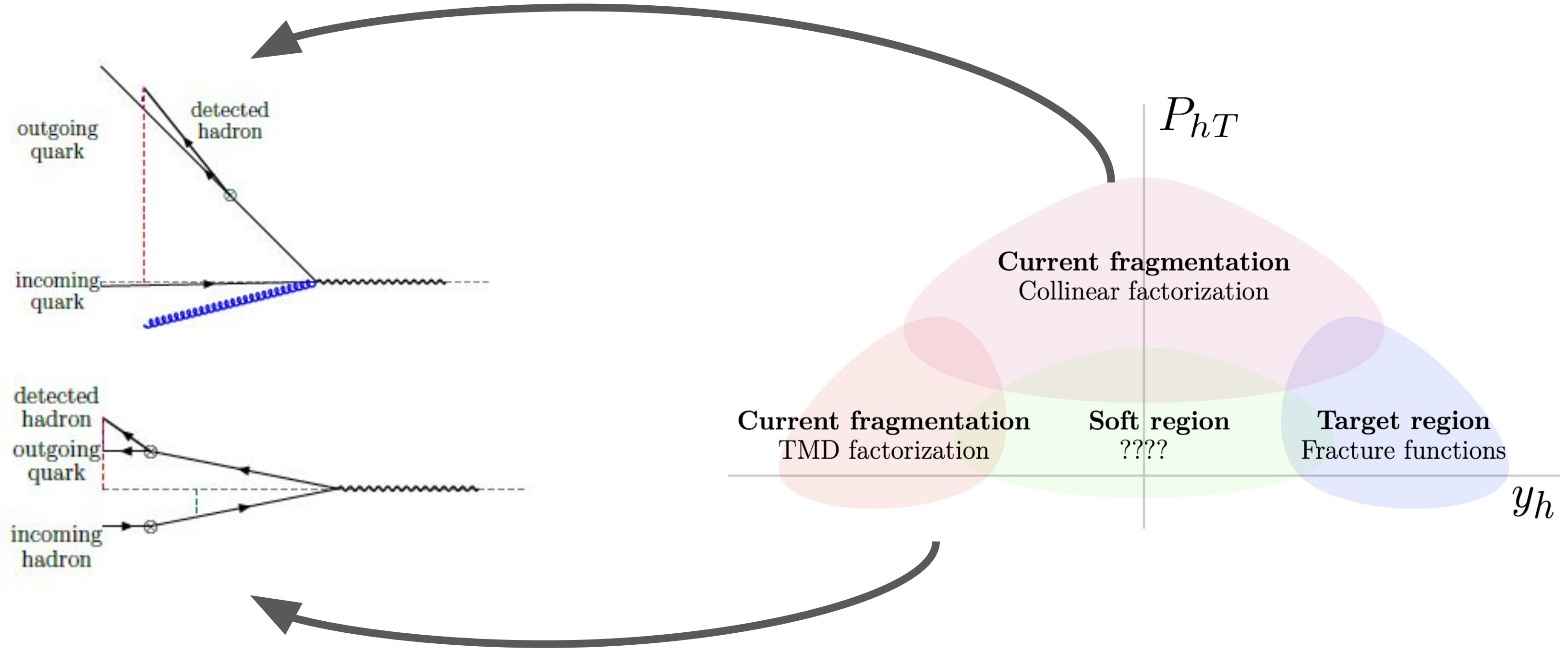


Breit Frame



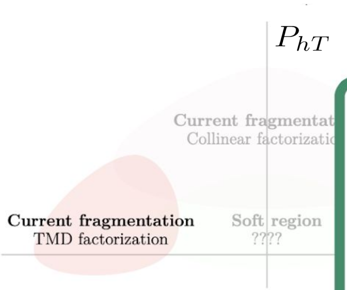
identified hadron P_h^μ
outgoing lepton l'^μ
P_{hT}
incoming proton P^μ
$q = l - l'$
exchanged photon
incoming lepton l^μ

Separation of Fragmentation Regions

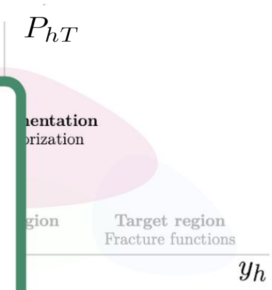


Standard Approach

small transverse momentum W



large transverse momentum FO



matching region

aka ASY **(=asymptotic)**

Scale Separation of Cross Section

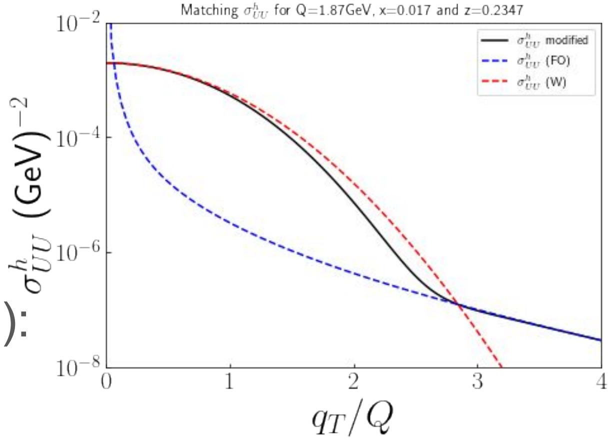
$$\frac{d\sigma}{dx\,dQ^2\,dz\,dP_{hT}} = \mathrm{W} + \mathrm{FO} - \mathrm{ASY} + \mathcal{O}(\Lambda^2_{QCD}/Q^2)$$

where

$$P_{hT} = q_T z_h$$

$$\sim \mathrm{W} \quad \text{for } q_{\mathrm{T}} \ll Q$$

$$\sim \mathrm{FO} \quad \text{for } q_{\mathrm{T}} \sim Q$$



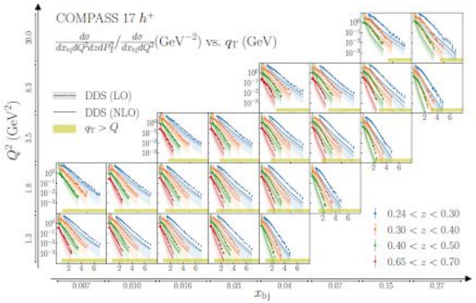
- Toy scheme used in this work (and in Liu et al., 2021):

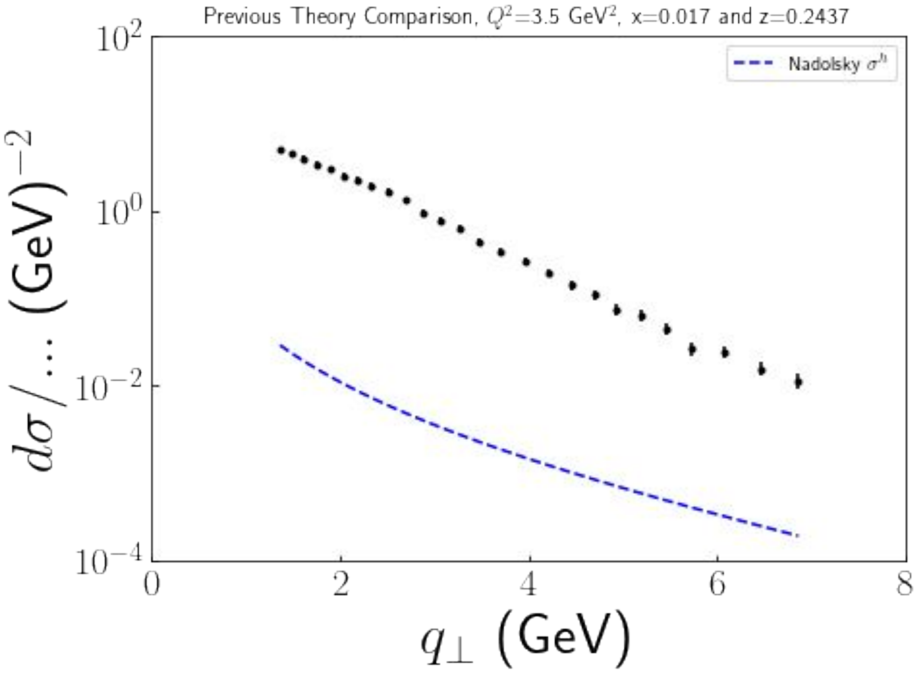
$$\sigma^h_{UU,\mathrm{Modified}} \approx \sigma^h_{UU,\mathrm{W}} R(q_T/Q) + (1 - R(q_T/Q))\sigma^h_{UU,\mathrm{FO}}$$

$$R = e^{-A(q_T/Q)^B}$$

Issues with Standard Approach

- Large discrepancy with data (example from COMPASS 2017 run, Aghasyan et al., 2018)
- Several attempted solutions
  - Next-to-leading order corrections (Wang et al., 2019)
  - Power Corrections (Liu and Qiu, 2020)

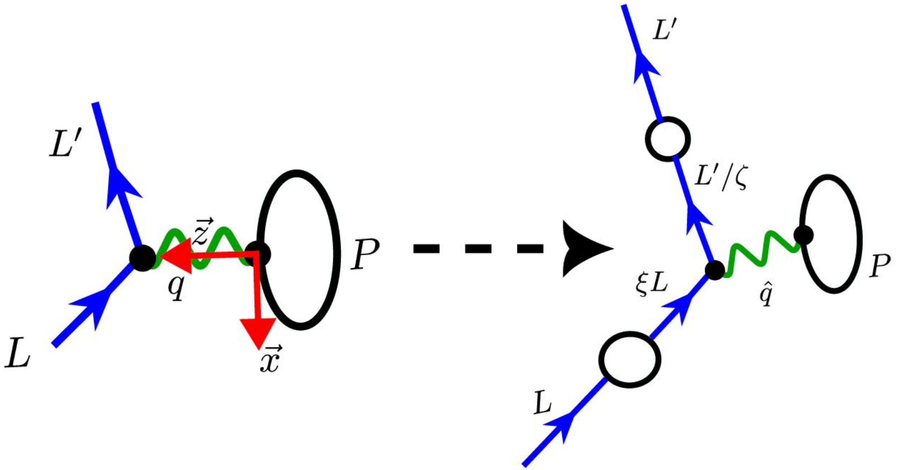


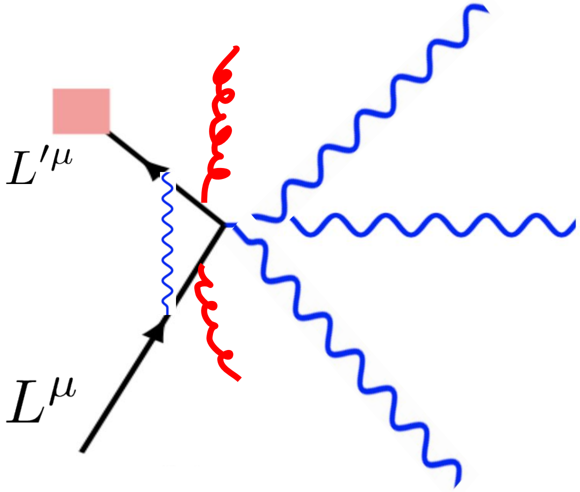


Gonzalez-Hernandez et al., 2018

QED Effects

- Radiation of photon from initial state changes the plane modulation
- Unobserved lepton radiation introduces ambiguity, as the radiation changes the unobserved photon’s momentum (Liu et al., 2021)
- Unified factorization method to address these issues





Kinematic Variables Appearing in Full SIDIS Cross Section

$$x_B = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

$$\cos(\phi_h) = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \cos(\phi_S) = -\frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}},$$
$$\sin(\phi_h) = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \sin(\phi_S) = -\frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P\cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \quad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$S_\parallel = \frac{S\cdot q}{P\cdot q}\frac{M}{\sqrt{1+\gamma^2}}, \quad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P\cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{P\cdot q\sqrt{1+\gamma^2}}$$

Kinematic Variables with QED effects Highlighted

$$x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

$$\cos(\phi_h) = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \qquad \cos(\phi_S) = -\frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}},$$

$$\sin(\phi_h) = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \qquad \sin(\phi_S) = -\frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P \cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \qquad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$S_\parallel = \frac{S \cdot q}{P \cdot q} \frac{M}{\sqrt{1+\gamma^2}}, \qquad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

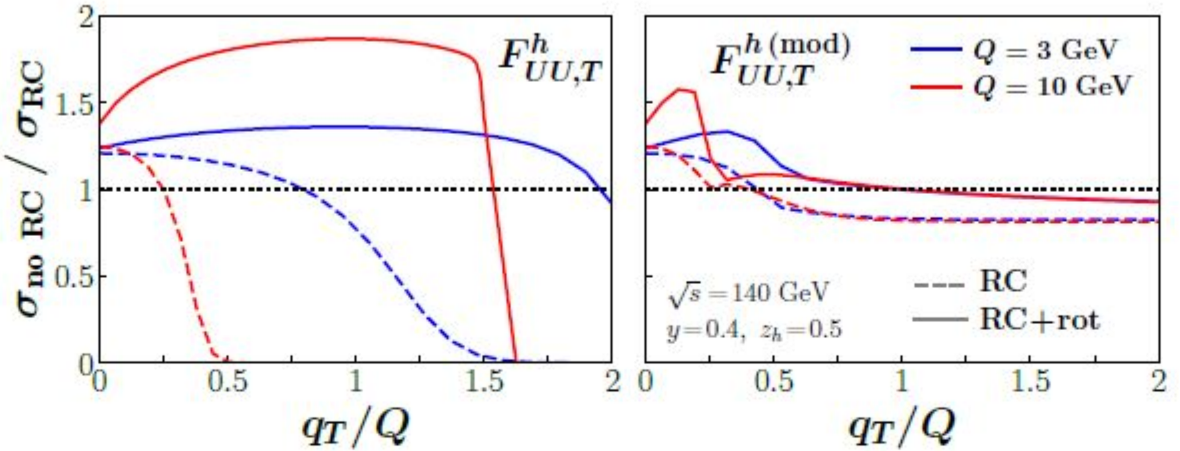
$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P \cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} \frac{P_\rho q_\sigma}{P \cdot q\sqrt{1+\gamma^2}}$$

Previous Work on QED Effects

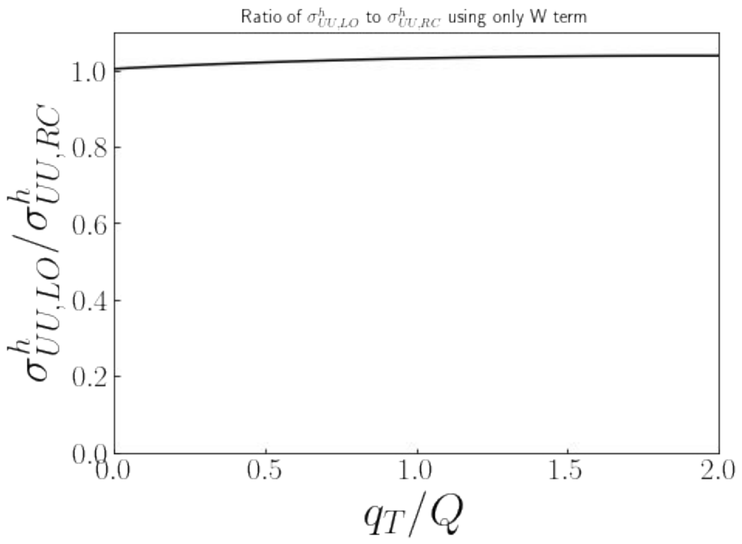
- Liu et al., 2020 and 2021 established new factorization approach
- Showed effects of radiative corrections to Gaussian approximation of W term with artificial fixed order tail

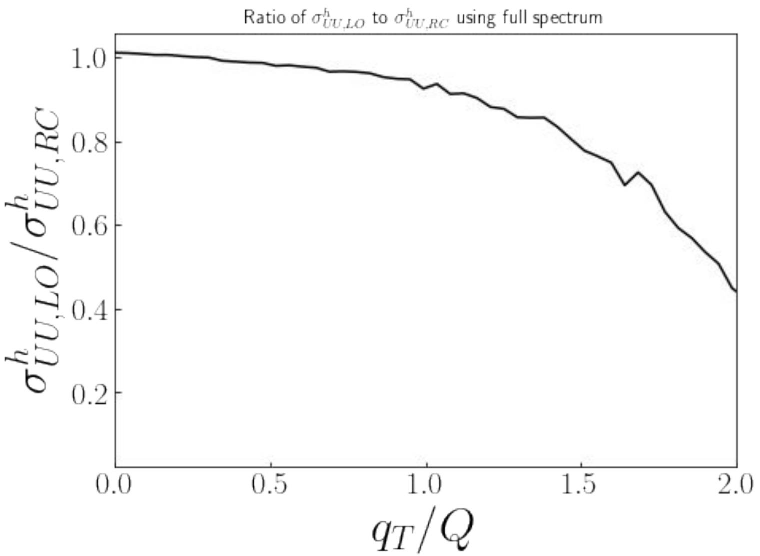
$$F^h_{UU,\text{Gaussian}}(x_B, y, z_h, Q^2, q_\perp) = \sum_q e_q^2 f_{q/h}(x_B, Q^2) D_{h'/q}(z_h, Q^2) \frac{e^{-(z_h q_\perp)^2/\langle (z_h q_\perp)^2\rangle}}{\pi \langle (z_h q_\perp)^2 \rangle}$$



Improved Fixed Order Implementation

- Fixed order calculation comes from Nadolsky et al., 1999



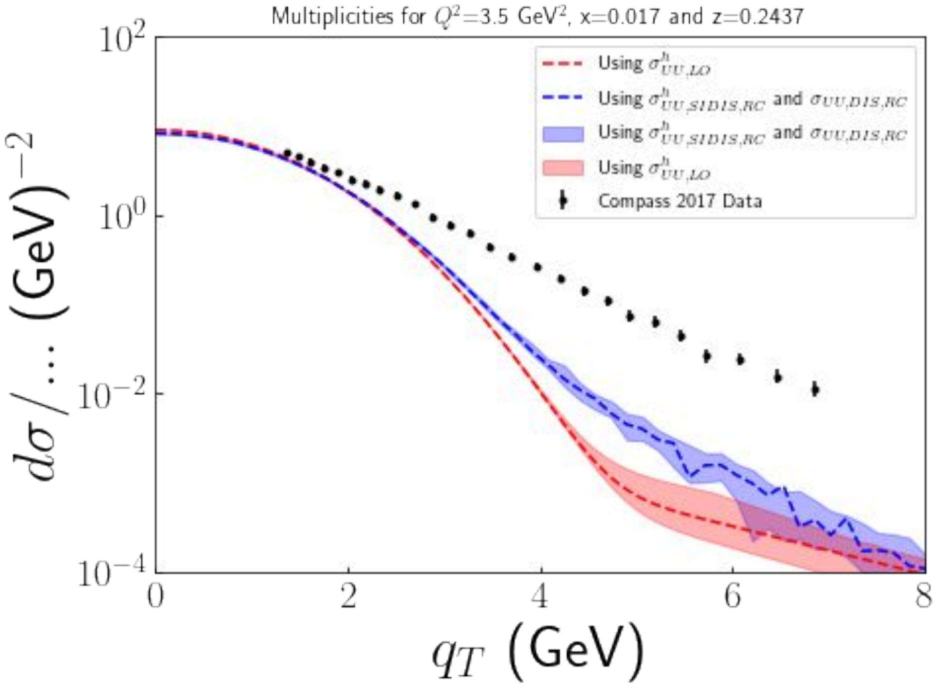


Results

- Take ratio of SIDIS cross section (integrated over ϕ_h) to IDIS cross section

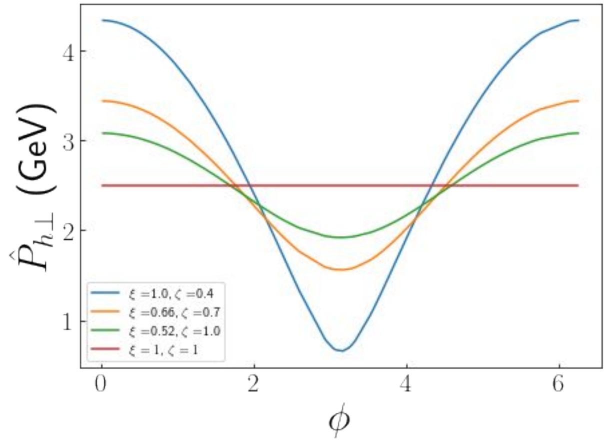
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B \mathrm{d}Q^2 \mathrm{d}z_h \mathrm{d}P_{hT}^2} \Big/ \frac{\mathrm{d}\sigma}{\mathrm{d}x_B \mathrm{d}Q^2}$$

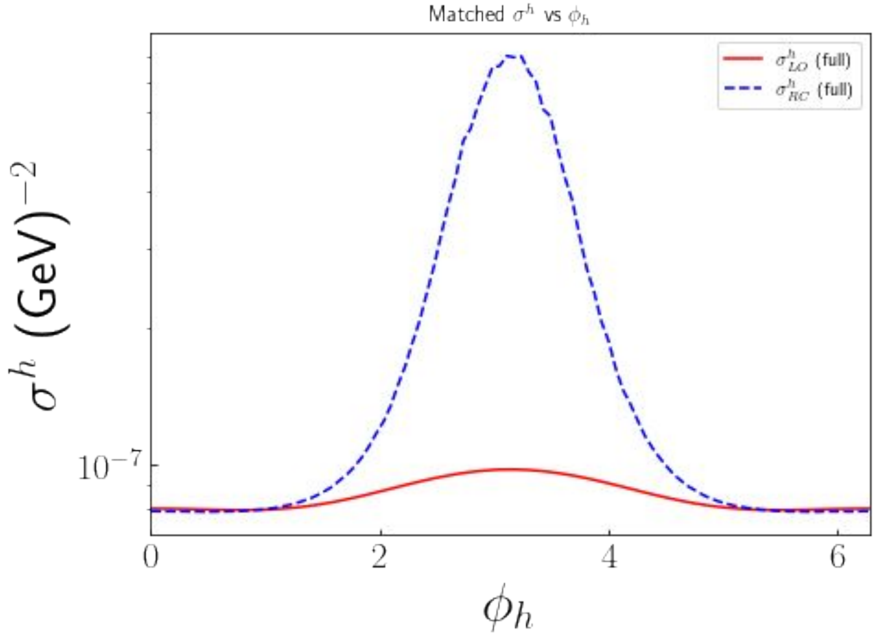
- Monte Carlo simulation used to integrate over radiative correction parameters (ξ,ζ) and ϕ_h
- $P_{hT} = q_T z_h$



Importance of Angular Effects

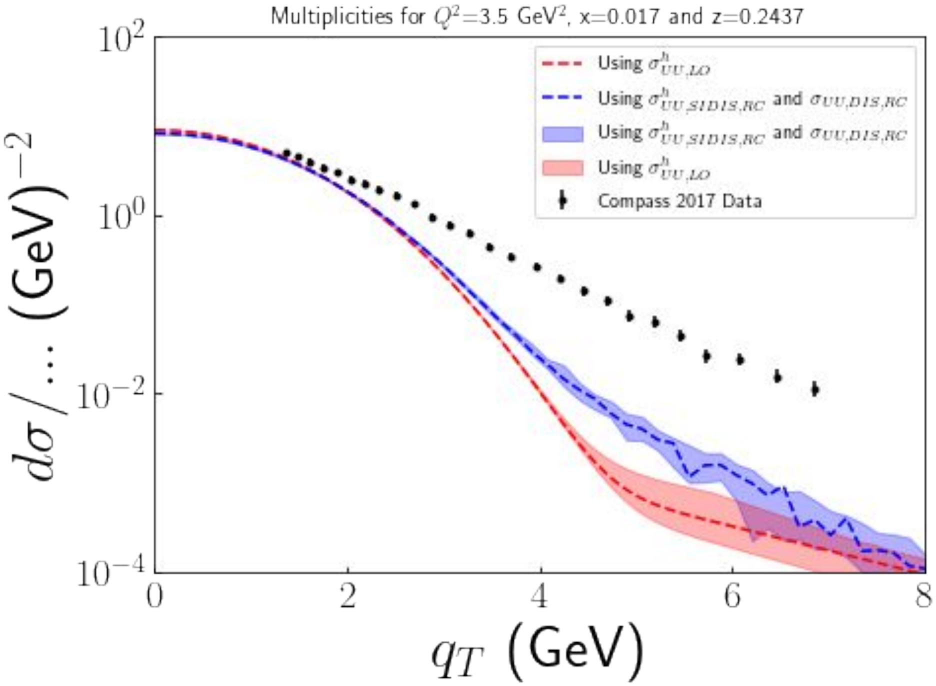
- After considering QED radiation effects, angular modulation appears, as $\hat{P}_{hT}$ depends on the angle





Conclusion

- Applied joint factorization scheme for SIDIS process
- Showed effects of QED radiation on cross section through W+Y formalism with improved fixed order tail
- Next Steps:
  - Expand unpolarized calculations to polarized cross section
  - Expand collinear factorization to TMD approach
  - Develop code to make prediction for PDFs and TMDs from SIDIS, eventually testing sensitivities of ΔG for future JLab 12 and EIC experiments



Acknowledgements

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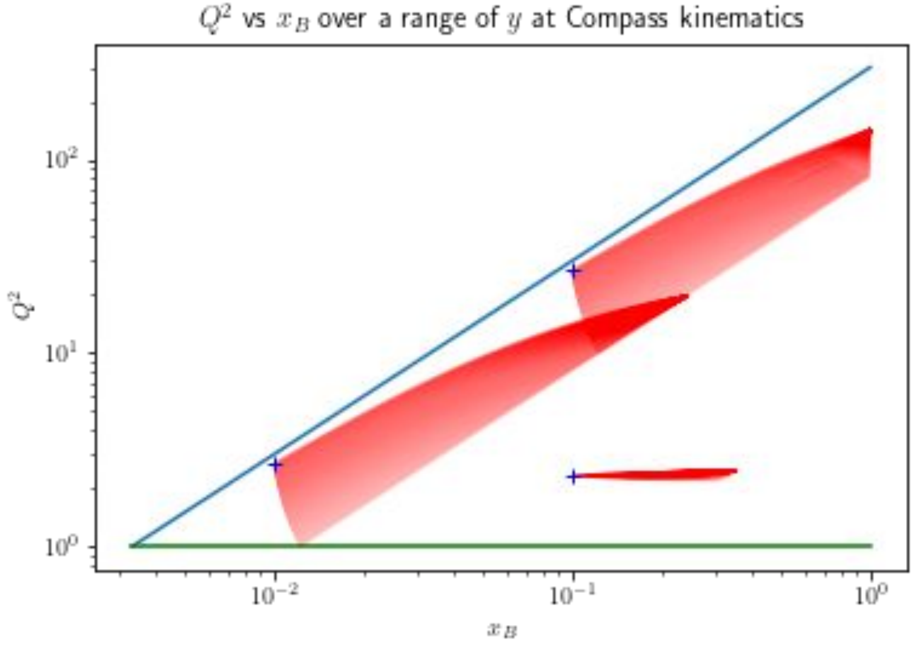
Backup Slides

Phase Space

- Primary kinematic variables are dependent on radiative correction parameters (ξ,ζ)

$$\hat{x}_B = \frac{\xi y x_B}{\xi\zeta - 1 + y} \qquad \hat{z}_h = \frac{\zeta y z_h}{\xi\zeta - 1 + y}$$

$$\hat{Q}^2 = \frac{\xi}{\zeta} Q^2 \qquad \hat{y} = \frac{\hat{Q}^2}{\xi S \hat{x}_B}$$



Transformation Between Lab Frame and Virtual Breit Frame

- Traditionally Breit frame is photon-hadron frame
- Lepton radiation makes frame determination ambiguous
- All historical factorization formula defined in photon hadron frame
- Introduce **virtual** photon-hadron frame which is determined by a given pair of ξ,ζ under one-photon exchange approximation

$$x^\mu = (x^+, x^-, \vec{x}_\perp = (x^1, x^2)) \qquad \tilde{x}^\sigma = R^\sigma_\nu \Lambda^\nu_\mu x^\mu$$

$$y \Rightarrow \tilde{y} = y$$

$$\begin{pmatrix} x \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{x} \\ \tilde{z} \end{pmatrix} = R(\theta(\xi, \zeta)) \begin{pmatrix} x \\ z \end{pmatrix}$$