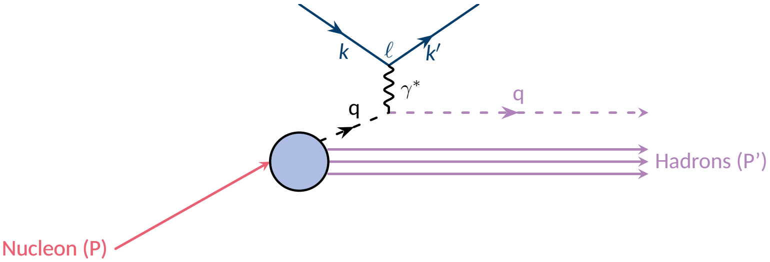
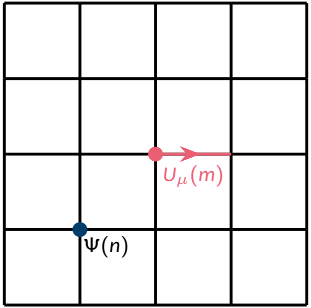


MOMENTS OF PDFS FROM LATTICE QCD AT THE PHYSICAL POINT

30. March 2023 | Marcel Rodekamp | Jülich Supercomputing Center, Forschungszentrum Jülich



Fermion Field $\Psi(n)$

Gauge Field $U_\mu(m) \approx e^{iaA_\mu(am)}$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U]\, e^{-S[U]} \mathcal{O}[U] = \frac{1}{\mathrm{N_{conf}}} \sum_{n=0}^{\mathrm{N_{conf}}-1} \mathcal{O}[U_n] \pm \sigma \tag{1}$$

Ensemble	Size	β	$a[\mathrm{fm}]$	$m_\pi[\mathrm{MeV}]$	$m_\pi L$	$\mathrm{N}_{\mathrm{conf}}$
Coarse	48^4	3.31	0.0804(13)	136(2)	3.9	212
Fine	64^4	3.5	0.0613(6)	133(1)	4.0	442

Ensemble	T/a	$p_x[2\pi/L]$
Coarse	3, 4, 5, 6, 7, 8, 10, 12	$0, -2$
Fine	10, 13, 16	$0, -1$

- Tree-level Symanzik-improved gauge action
- 2+1 flavour tree-level improved Wilson Clover fermions
- 2-level HEX smearing

[Dürr et al., 2011] [Hasan et al., 2019]

- Forward matrix element of local leading twist operator

$$\mathcal{O}^X = \bar{q}\Gamma^X_{\{\alpha}\overleftrightarrow{D}_{\mu\}}q \tag{2}$$

- Moments of parton distribution function

$$\langle N(P)|\mathcal{O}^X|N(P)\rangle = \langle x\rangle \bar{q}\Gamma^X_{\{\alpha}\mathrm{i}p_{\mu\}}q \tag{3}$$

- Renormalization

$$\langle x\rangle^{ren}_{\mathcal{O}} = Z_{\mathcal{O},\mathcal{O}'}\,\langle x\rangle_{\mathcal{O}'} \tag{4}$$

[Martinelli and Sachrajda, 1989] [Göckeler et al., 1996]

- Unpolarized $\langle x \rangle_{q+\bar{q}}$

$$\Gamma^{V}_{\alpha=\nu} = \gamma_{\nu} \tag{5}$$

- Polarized $\langle x \rangle_{\Delta q - \Delta \bar{q}}$

$$\Gamma^{A}_{\alpha=\nu} = \gamma_{\nu}\gamma_{5} \tag{6}$$

- Transversity $\langle x \rangle_{\delta q + \delta \bar{q}}$

$$\Gamma^{T}_{\alpha=\nu\rho} = \frac{1}{2}\left[\gamma_{\nu},\, \gamma_{\rho}\right] \tag{7}$$

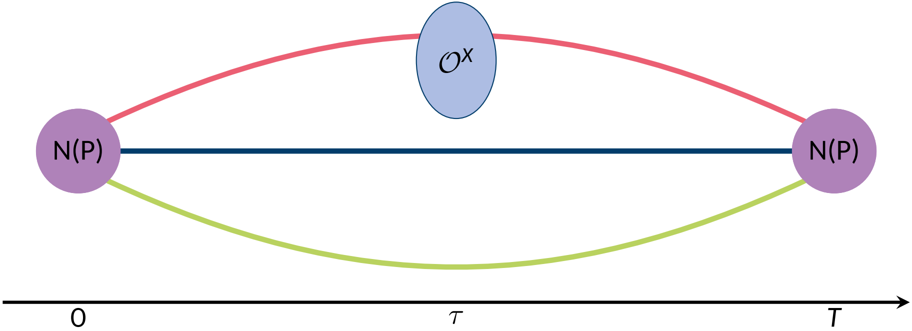
- (Finite) momentum

$$\vec{p} = (p_x, 0, 0) \tag{8}$$

- Irreducible representations $\tau_a^{(b)}$ of $H(4)$

[Göckeler et al., 1996] [Bali et al., 2019] [Alexandrou et al., 2020] [Mondal et al., 2021]

$$\langle N(P)|\mathcal{O}^X|N(P)\rangle = \lim_{\tau - T, \tau \to \infty} R(T,\tau) = \lim_{\tau - T, \tau \to \infty} \frac{C_{3pt}(T,\tau)}{C_{2pt}(T)} \quad (9)$$

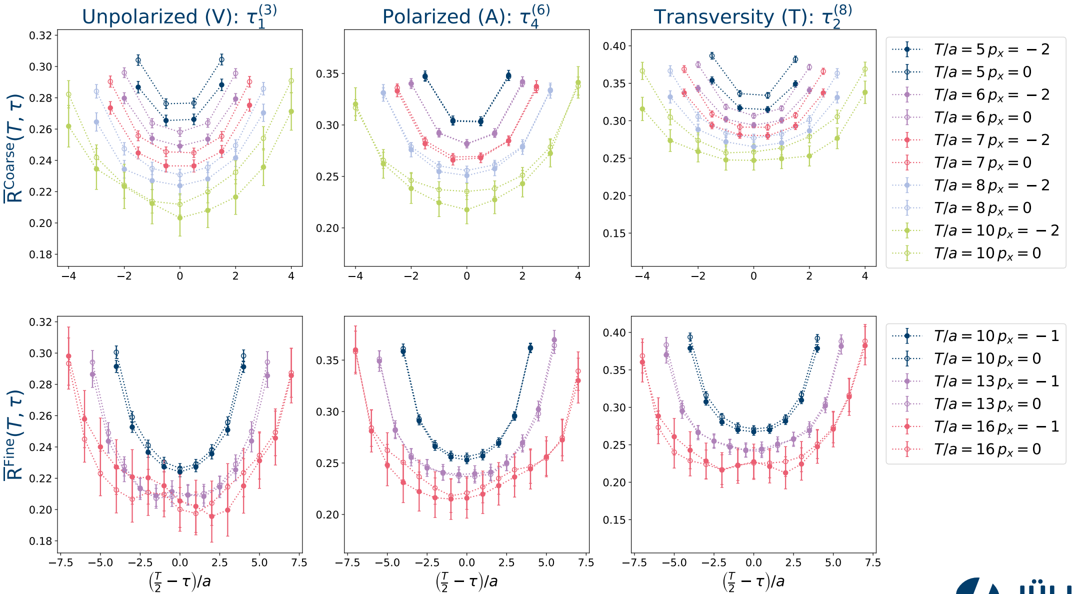


$$R(T,\tau) = \langle N(P)|\mathcal{O}^X|N(P)\rangle + \mathcal{O}\left(e^{-E_1}\right) \tag{10}$$

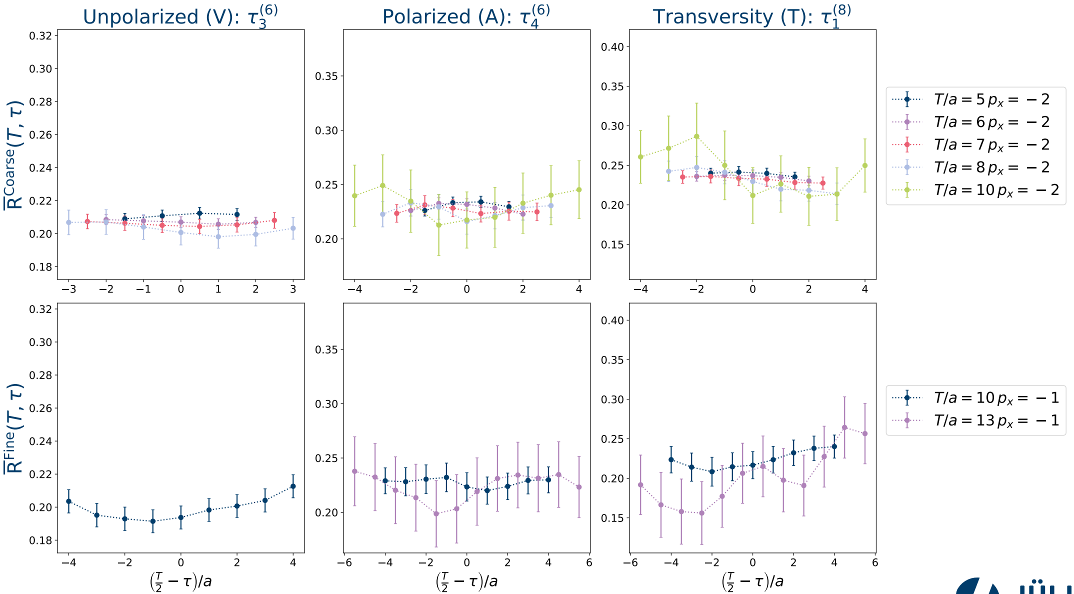
$$R(T,\tau) = \langle N(P)|\mathcal{O}^X|N(P)\rangle + \mathcal{O}\left(e^{-E_1}\right) \tag{10}$$

$$= \langle N(P)|\mathcal{O}|N(P)\rangle \left.\frac{1 + c_1 e^{-\frac{T}{2}\Delta E}\cosh\left[\left(T/2 - \tau\right)\Delta E\right] + c_2 e^{-T\Delta E}}{1 + c_3 e^{-T\Delta E}}\right|_{\Delta E = E_1 - E_0} + \mathcal{O}\left(e^{-\Delta E'}\right) \tag{11}$$

$$\vdots$$



Unpolarized (V): $\tau_1^{(3)}$
Polarized (A): $\tau_4^{(6)}$
Transversity (T): $\tau_2^{(8)}$
$\overline{R}^{\mathrm{Coarse}}(T,\tau)$
$\overline{R}^{\mathrm{Fine}}(T,\tau)$
$\left(\frac{T}{2}-\tau\right)/a$
$T/a = 5\, p_x = -2$
$T/a = 5\, p_x = 0$
$T/a = 6\, p_x = -2$
$T/a = 6\, p_x = 0$
$T/a = 7\, p_x = -2$
$T/a = 7\, p_x = 0$
$T/a = 8\, p_x = -2$
$T/a = 8\, p_x = 0$
$T/a = 10\, p_x = -2$
$T/a = 10\, p_x = 0$
$T/a = 10\, p_x = -1$
$T/a = 10\, p_x = 0$
$T/a = 13\, p_x = -1$
$T/a = 13\, p_x = 0$
$T/a = 16\, p_x = -1$
$T/a = 16\, p_x = 0$



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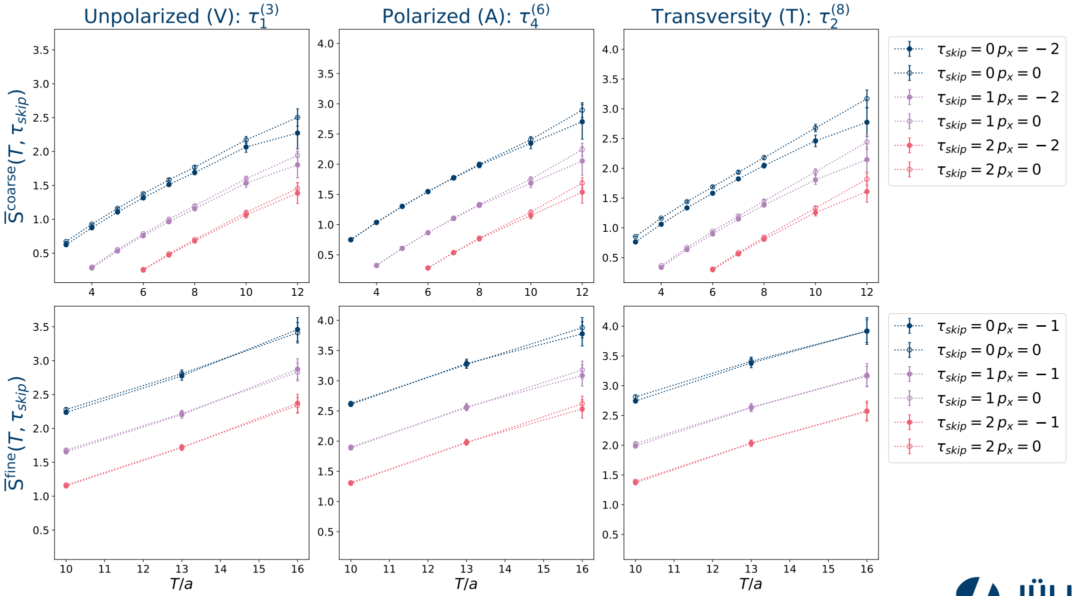
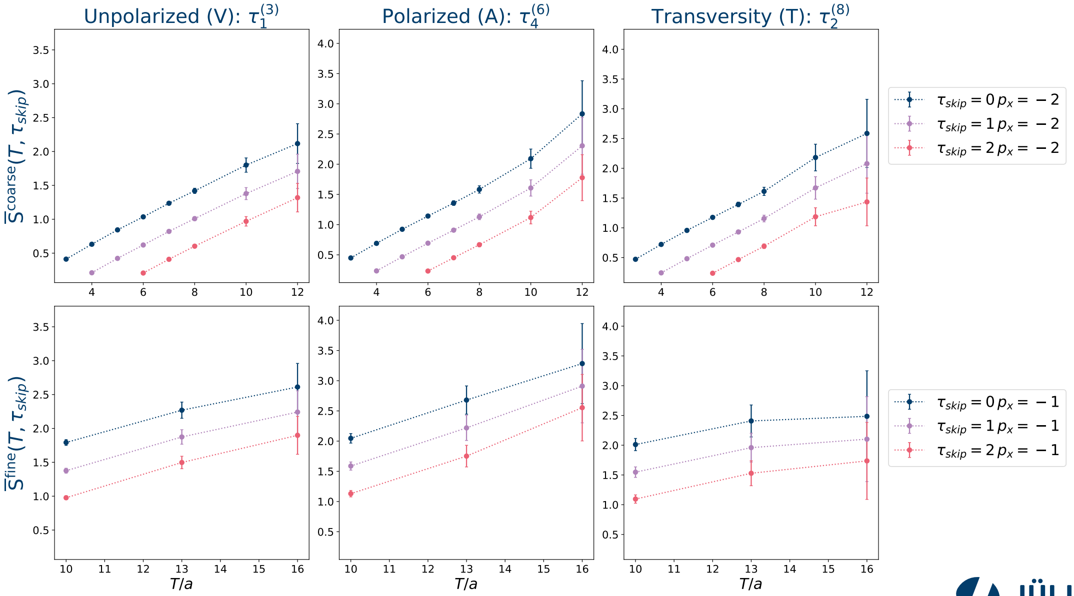
$$S(T,\tau_{skip}) = \sum_{\tau=\tau_{skip}}^{T-\tau_{skip}} R(T,\tau) = \langle N(P)|\mathcal{O}^X|N(P)\rangle\,(T-\tau_{skip}) + c \tag{12}$$

$$\boxed{\langle N(P)|\mathcal{O}^X|N(P)\rangle = \frac{S(T+a,\tau_{skip}) - S(T,\tau_{skip})}{a}} \tag{13}$$


Unpolarized (V): $\tau_1^{(3)}$
Polarized (A): $\tau_4^{(6)}$
Transversity (T): $\tau_2^{(8)}$
$\overline{S}^{\text{coarse}}(T, \tau_{skip})$
$\overline{S}^{\text{fine}}(T, \tau_{skip})$
T/a
$\tau_{skip} = 0\, p_x = -2$
$\tau_{skip} = 0\, p_x = 0$
$\tau_{skip} = 1\, p_x = -2$
$\tau_{skip} = 1\, p_x = 0$
$\tau_{skip} = 2\, p_x = -2$
$\tau_{skip} = 2\, p_x = 0$
$\tau_{skip} = 0\, p_x = -1$
$\tau_{skip} = 0\, p_x = 0$
$\tau_{skip} = 1\, p_x = -1$
$\tau_{skip} = 1\, p_x = 0$
$\tau_{skip} = 2\, p_x = -1$
$\tau_{skip} = 2\, p_x = 0$



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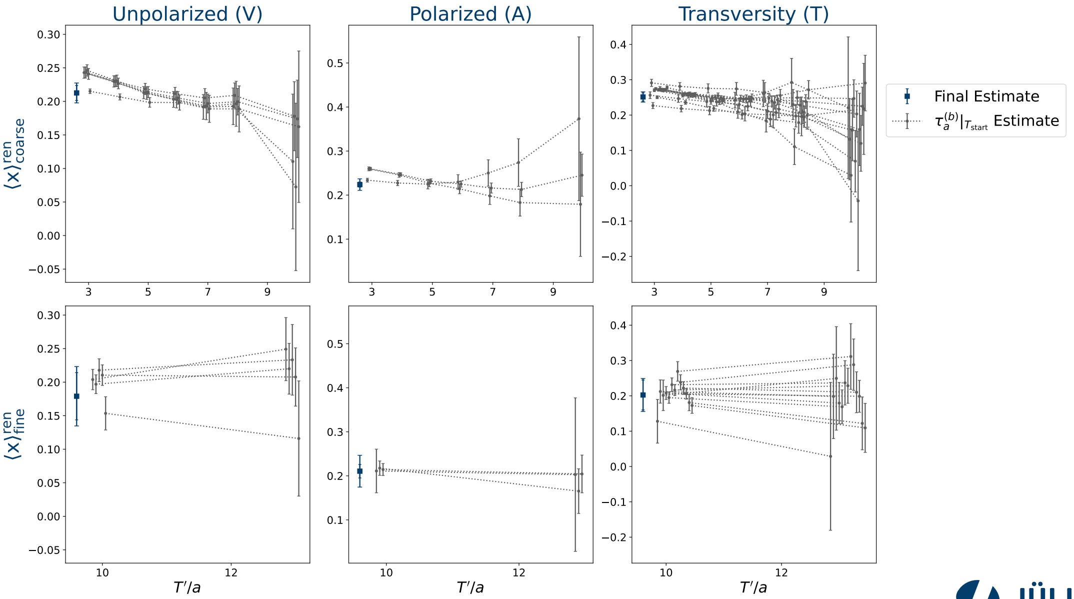
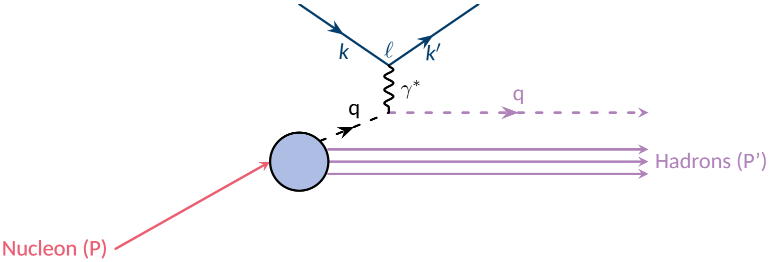
$$\langle x \rangle^{ren} = Z \cdot \frac{\langle N(P) | \mathcal{O}^{X} | N(P) \rangle}{\bar{q} \Gamma^{X}_{\{\alpha} p_{\mu\}} q} \tag{14}$$



Unpolarized (V)
Polarized (A)
Transversity (T)
$\langle x \rangle^{\mathrm{ren}}_{\mathrm{coarse}}$
$\langle x \rangle^{\mathrm{ren}}_{\mathrm{fine}}$
T'/a
Final Estimate
$\tau_a^{(b)}|_{T_{\mathrm{start}}}$ Estimate



MOMENTS OF PDFS FROM LATTICE QCD AT THE PHYSICAL POINT

Marcel Rodekamp, Michael Engelhardt, Jeremy Green, Stefan Krieg, Stefan Meinel, John Negele, Andrew Pochinsky, Sergey Syritsyn

Correlation Functions

$$C_{3pt}(T,\tau) = \int \mathrm{d}^3x\, \mathrm{d}^3y\, e^{-\mathrm{i}\vec{p}'\vec{x} - \mathrm{i}(\vec{p}'-\vec{p})\vec{y}}\, \mathrm{Tr}\left\{\Gamma_{pol}\left\langle \chi(\vec{x},T)\mathcal{O}^X\bar{\chi}(\vec{0},0)\right\rangle\right\} \tag{15}$$

$$C_{2pt}(T,\tau) = \int \mathrm{d}^3x\, e^{-\mathrm{i}\vec{p}\vec{x}}\, \mathrm{Tr}\left\{\Gamma_{pol}\left\langle \chi(\vec{x},T)\bar{\chi}(\vec{0},0)\right\rangle\right\} \tag{16}$$

$$\chi_\alpha = \varepsilon^{abc}\tilde{u}^T_{a\beta}C\gamma_5 P_+ \tilde{d}^\beta_b \tilde{u}_{c\alpha} \tag{17}$$

- $P_+ = 1/2\,(1+\gamma_4)$
- Smeared quark fields $\tilde{q}$

Analysis Algorithm

- Estimate ratios $R(T, \tau)$
- Estimate sum ratios $S(T, \tau_{skip})$
- Extract matrix elements $\langle N(P)|\mathcal{O}^X|N(P)\rangle$
  - Fit $S(T, \tau_{skip}) = M|_{T>T'} \cdot (T - \tau_{skip}) + c$
  - Finite difference $\frac{S(T+a_1,\tau_{skip})-S(T-a_2,\tau_{skip})}{a_1-a_2} = M|_{T=T'}$
- Obtain moments

$$\langle x\rangle^{ren} = \frac{1}{\#} \sum_{\tau_a^{(b)},P} Z_{\tau_a^{(b)}} \frac{\sum_{T'>T_{plateau}} M^{FFD,\tau_a^{(b)}}|_{T'}/\sigma_M^2}{\sum_{T'>T_{plateau}} 1/\sigma_M^2}$$

  - $T_{plateau} = \min_{T'} \left\{ \sqrt{\sigma_{syst}^2 + \sum_{\tau_a^{(b)}} \sigma_{M,\, stat}^2} \right\}$
  - $\sigma_{syst}^2 = \frac{1}{N_{bst}} \sum_{k=0}^{N_{bst}} \mathrm{RMS}_{methods,\tau_a^{(b)},P} \left\{ Z_{\tau_a^{(b)}} \langle x\rangle_k^{methods,\tau_a^{(b)},P} |_{T'} \right\}$

Operators

Unpolarized (V)	Polarized (A)	Transversity (T)
$\tau_1^{(3)}$ - $\frac{1}{2}\left[\mathcal{O}_{44} - \frac{1}{3}\sum_{\mu=1}^{3}\mathcal{O}_{\mu\mu}\right]$ - $\frac{1}{\sqrt{2}}[\mathcal{O}_{33} - \mathcal{O}_{44}]$ $\tau_3^{(6)}$ - $\frac{1}{\sqrt{2}}[\mathcal{O}_{14} - \mathcal{O}_{41}]$	$\tau_4^{(6)}$ - $\frac{1}{\sqrt{2}}[\mathcal{O}_{13} + \mathcal{O}_{31}]$ - $\frac{1}{\sqrt{2}}[\mathcal{O}_{34} + \mathcal{O}_{43}]$	$\tau_1^{(8)}$ - $\mathcal{O}_{211} - \mathcal{O}_{233}$ - $\mathcal{O}_{211} - \mathcal{O}_{244}$ - $\mathcal{O}_{233} - \mathcal{O}_{244}$ $\tau_2^{(8)}$ - $\mathcal{O}_{124} - \mathcal{O}_{241}$ - $\mathcal{O}_{142} - \mathcal{O}_{421}$ - $\mathcal{O}_{214} - \mathcal{O}_{142}$ - $\mathcal{O}_{124} + \mathcal{O}_{241} - \mathcal{O}_{412}$ - $\mathcal{O}_{142} + \mathcal{O}_{421} - \mathcal{O}_{214}$ - $\mathcal{O}_{214} + \mathcal{O}_{142} - \mathcal{O}_{421}$

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