

GLOBAL FITS OF PDFS WITH NON-LINEAR CORRECTIONS

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in collaboration with V. Guzey, I. Helenius, H. Paukkunen

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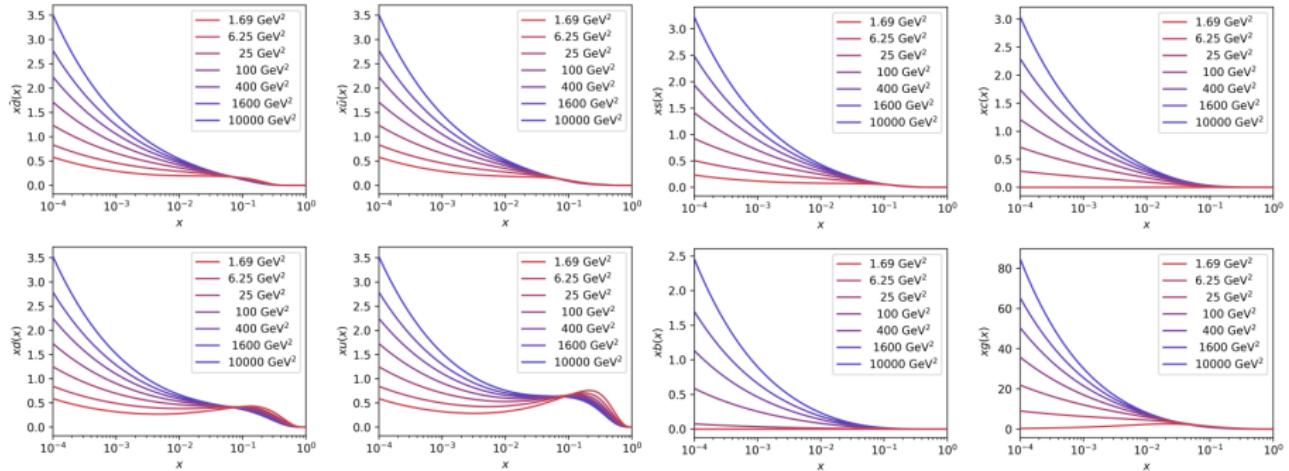


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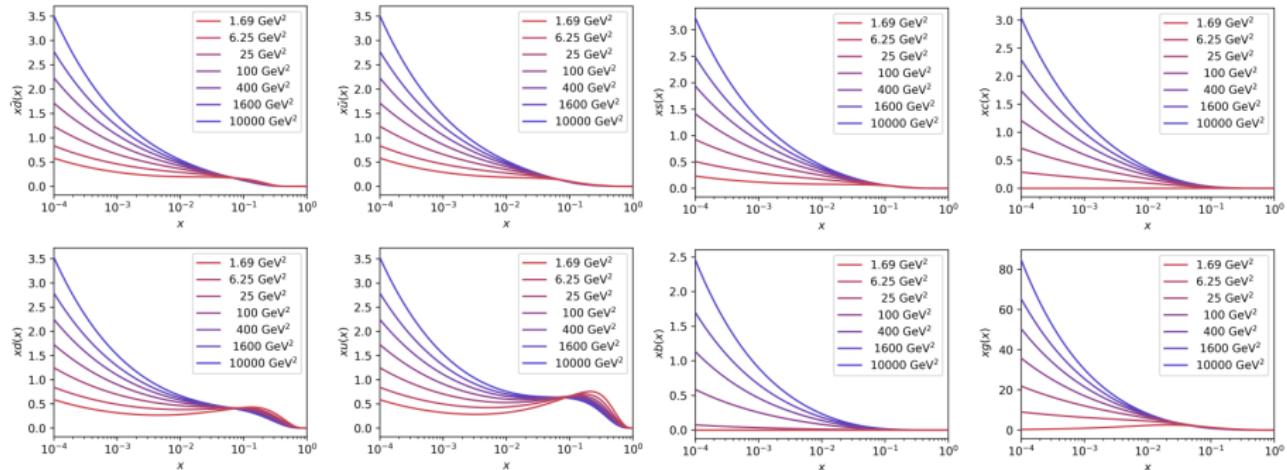
REMINDER - DGLAP EVOLUTION

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} f_i(x, \mu^2) \\ f_g(x, \mu^2) \end{pmatrix} = \sum_j \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{q_i q_j} \left(\frac{x}{\xi} \right) & P_{q_i g} \left(\frac{x}{\xi} \right) \\ P_{g q_j} \left(\frac{x}{\xi} \right) & P_{g g} \left(\frac{x}{\xi} \right) \end{pmatrix} \begin{pmatrix} f_j(\xi, \mu^2) \\ f_g(\xi, \mu^2) \end{pmatrix}$$



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PROBLEM:

Rapidly rising gluon at small x violates unitarity

GLUON RECOMBINATION - GLR EQUATION

[PHYS. REP. 100 (1983) 1, NUCL. PHYS. B268 (1986) 427]

Based on double-leading-logarithmic approximation in Q^2 and $\frac{1}{x}$

$$\begin{aligned}\frac{dx_B G(x_B, Q^2)}{d \ln Q^2} &= \text{linear terms} - 5.05 \left(\frac{\alpha_s}{RQ} \right)^2 \int_{x_B}^{x_0} \frac{dx_1}{x_1} [x_1 G^2(x_1, Q^2)]^2 \\ \frac{dx_B S(x_B, Q^2)}{d \ln Q^2} &= \text{linear terms} - 0.0010625 \left(\frac{\alpha_s}{RQ} \right)^2 [x_1 G^2(x_1, Q^2)]^2 \\ &\quad - 0.32 \frac{\alpha_s}{Q^2} \int_{x_B}^{x_0} \frac{dx_1}{x_1} \frac{x_B}{x_1} P_{MQ}^{GG \rightarrow q\bar{q}} x_1 H(x_1, Q^2)\end{aligned}$$

with

$$\frac{dx_1 H(x_1, Q^2)}{d \ln Q^2} = - 5.05 \left(\frac{\alpha_s}{RQ} \right)^2 \int_{x_B}^{x_0} \frac{dz}{z} [z G^2(z, Q^2)]^2$$

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- ▶ $\frac{dx_B S(x_B, Q^2)}{d \ln Q^2}$ does not appear naturally; requires special treatments,
e.g. mixing in NLL contributions
- ▶ violates unitarity at large x
- ▶ **Violates momentum sum rules**

ZHU + RUAN APPROACH

[NUCL. PHYS. B 559 (1999), 378-392]

Based on leading logarithmic approximation in Q^2

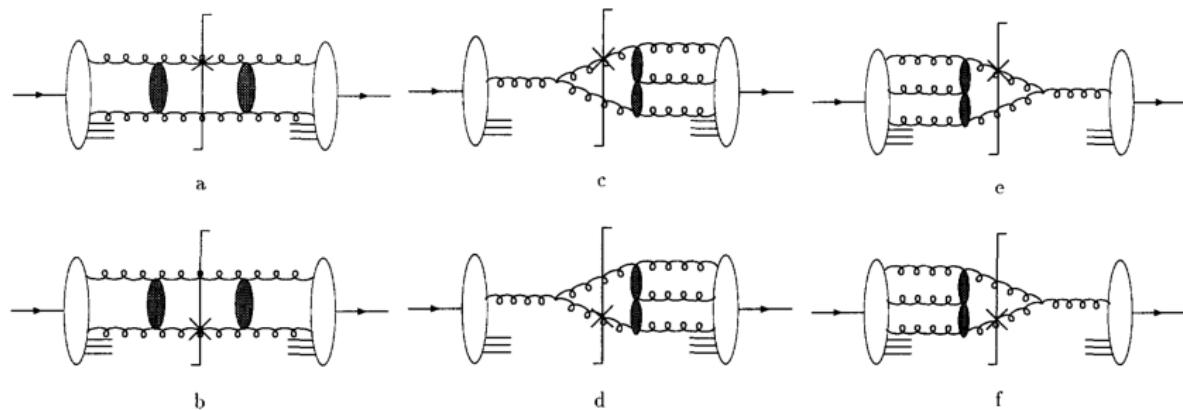
- ▶ Valid over the entire x range
- ▶ Includes transitions to quarks (and can be extended to $q\bar{q} \rightarrow G$, etc.)

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- ▶ $2 \rightarrow 2$ diagrams lead to antiscreening (a,b)
- ▶ $3 \rightarrow 2$ diagrams lead to screening (c,d,e,f)
- ▶ Same recombination function, but different kinematic regimes

ZHU + RUAN APPROACH

$$\frac{dx_B G(x_B, Q^2)}{d \ln Q^2} = \text{linear terms} + \frac{9}{32\pi^2} \left(\frac{1}{RQ} \right)^2 \int_{x_B/2}^{1/2} dx_1 x_B x_1 G^2(x_1, Q^2) \sum_i P_i^{GG \rightarrow G}(x_1, x_B)$$

$$- \frac{9}{16\pi^2} \left(\frac{1}{RQ} \right)^2 \int_{x_B}^{1/2} dx_1 x_B x_1 G^2(x_1, Q^2) \sum_i P_i^{GG \rightarrow G}(x_1, x_B)$$

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- Parameter R can be interpreted as the mean distance between gluons

$$\left. \begin{aligned} \frac{d \int_0^1 dx_B x_B G(x_B, Q^2)}{d \ln Q^2} &= 0 \\ \frac{d \int_0^1 dx_B x_B q(x_B, Q^2)}{d \ln Q^2} &= 0 \end{aligned} \right\} \Rightarrow \text{Momentum is conserved.}$$

NONLINEAR EVOLUTION IN PRACTICE - HOPPET

[COMPUT.PHYS.COMMUN. 180 (2009) 120-156]

Problem: PDF fitting requires DGLAP evolution to be performed thousands of times as fast as possible

- ▶ Convolution codes like HOPPET are highly optimized to solve the linear DGLAP evolutions quickly

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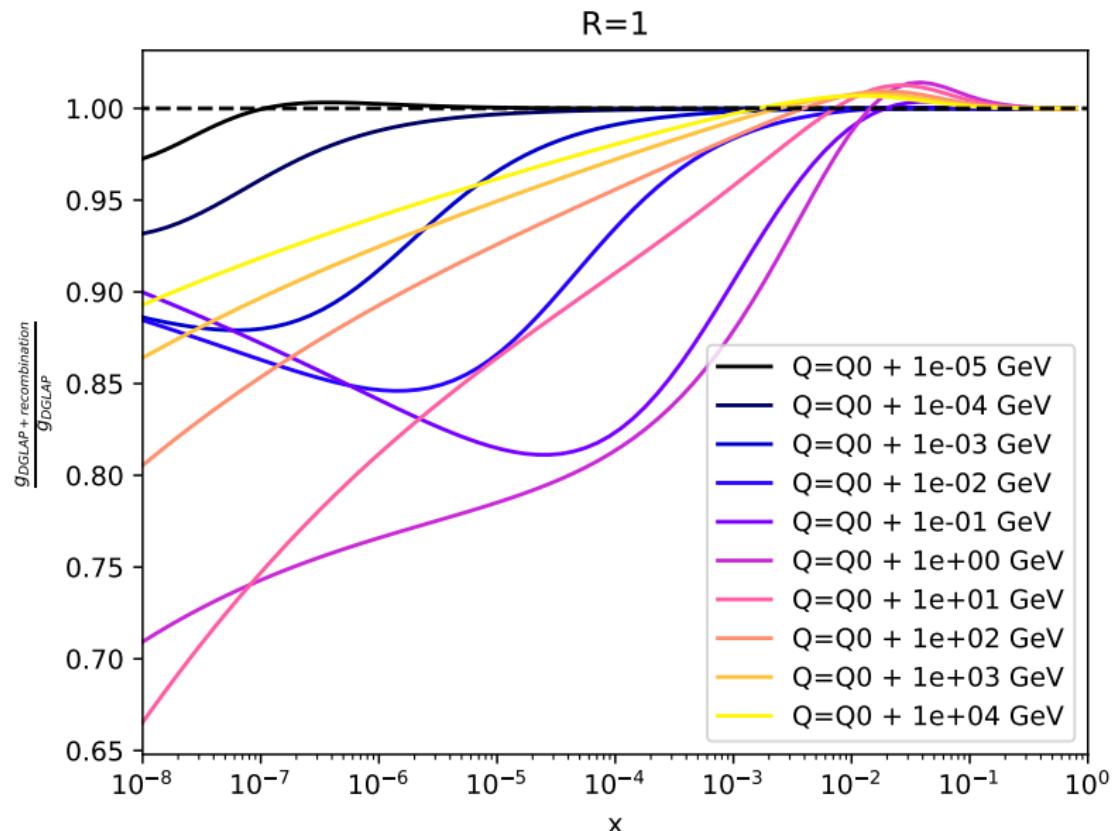
- ▶ Convolution codes like HOPPET are highly optimized to solve the linear DGLAP evolutions quickly

Solution: Treat $G^2(x, Q^2)$ as a separate flavour → calculation just 20% slower than regular DGLAP

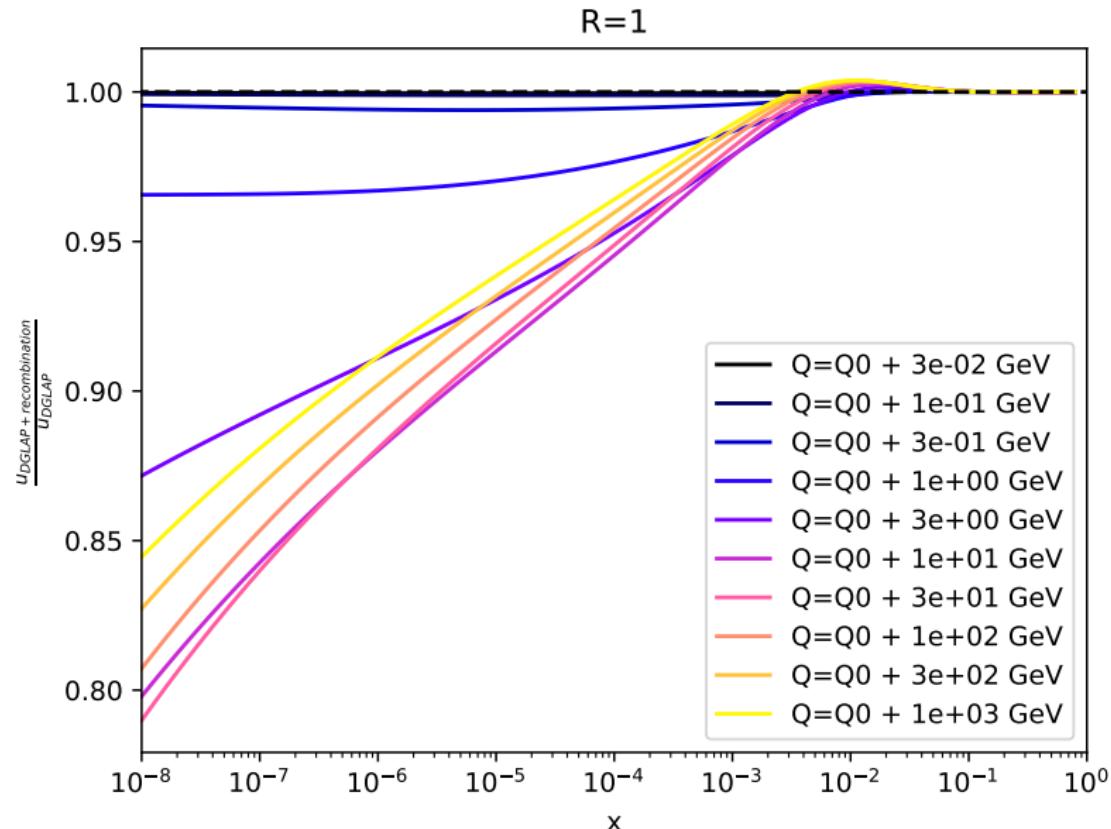
- ▶ Can be integrated into xFitter to allow PDF fitting [Eur.Phys.J.C 75 (2015) 7, 304]

HOPPET and xFitter patches available upon request

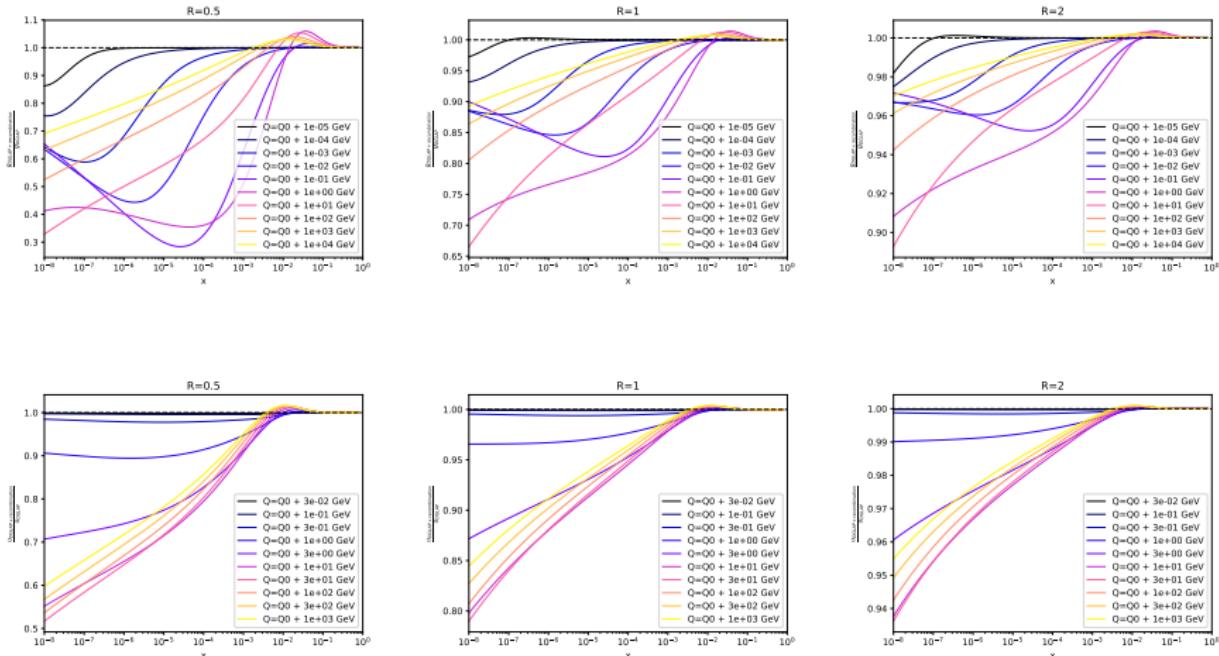
Q -DEPENDENCE OF NONLINEAR CORRECTIONS



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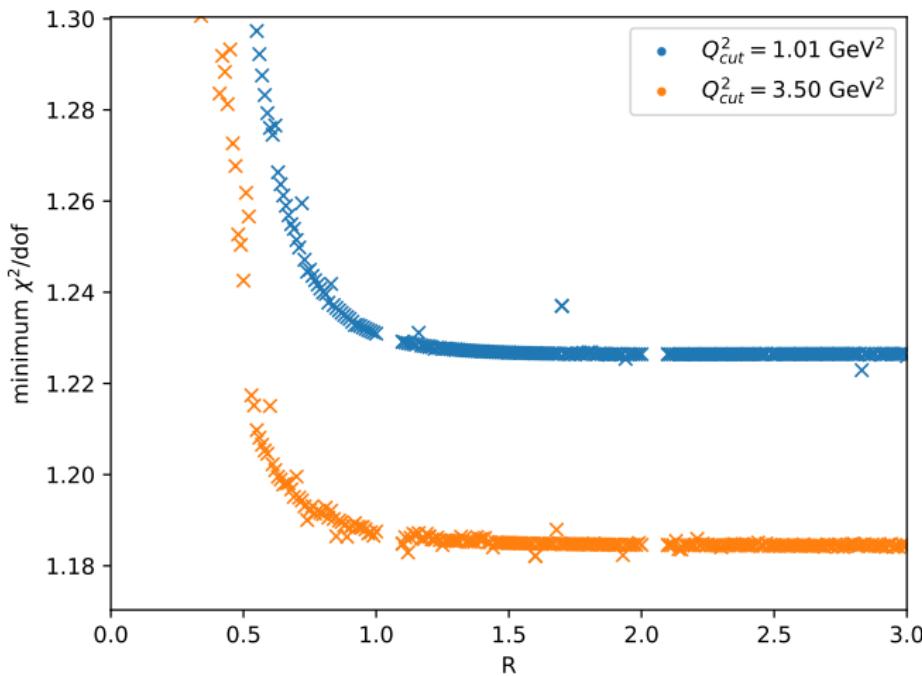
R -DEPENDENCE OF NONLINEAR CORRECTIONS



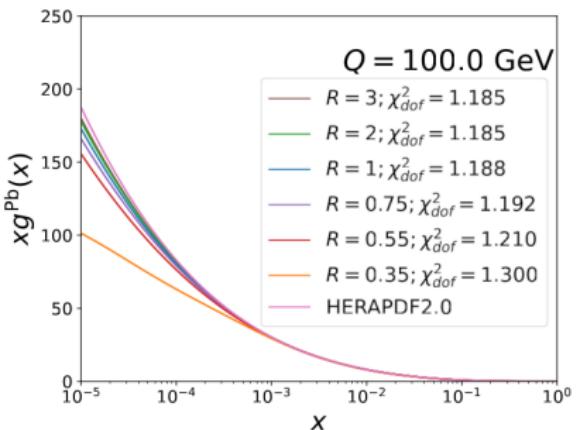
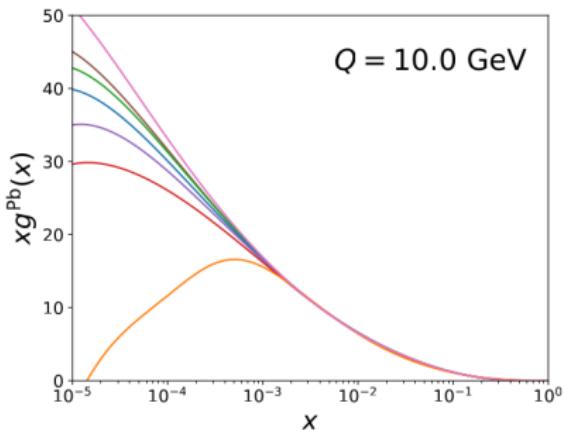
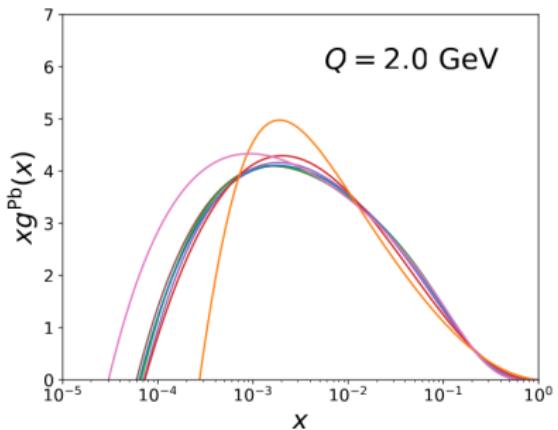
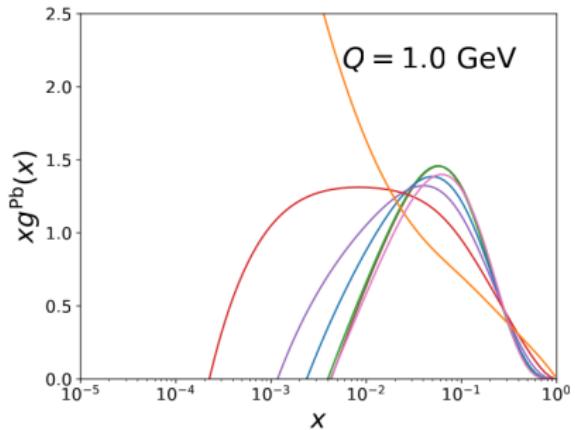
- ▶ Very minor qualitative differences
- ▶ Numerically instable for $R \lesssim 0.1$

R -DEPENDENT PDF FITS

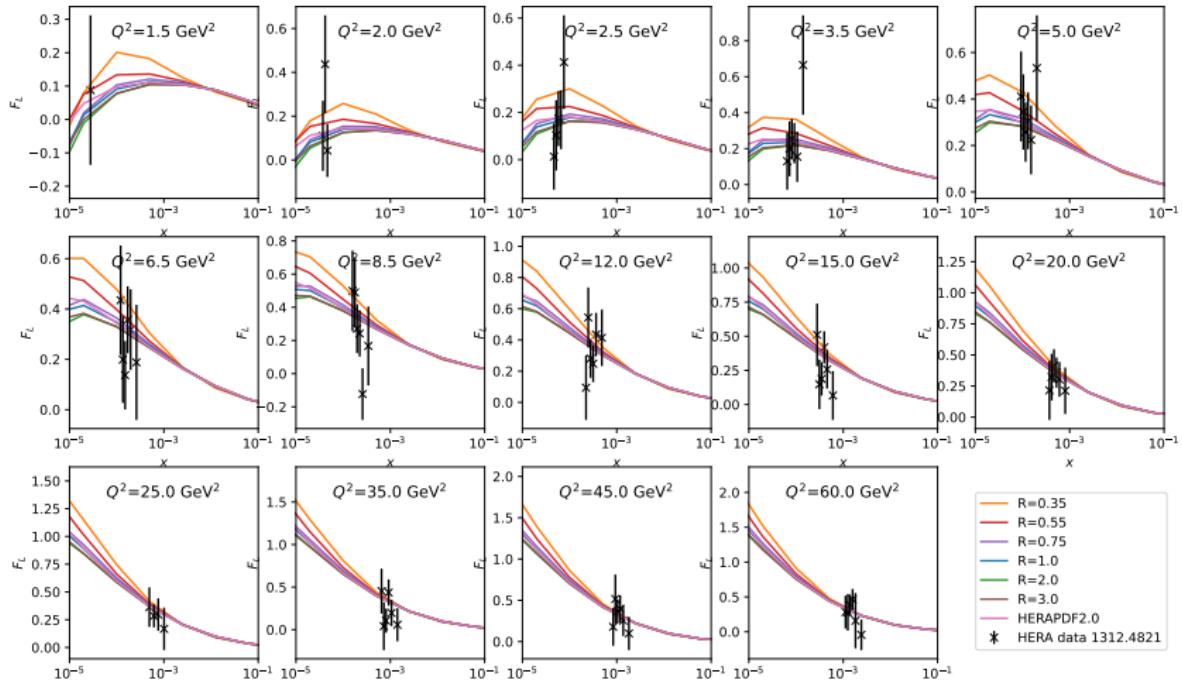
- ▶ HERAPDF2.0 parameterization, NNLO
- ▶ 1130 / 1198 data points (HERA DIS only)



R -DEPENDENT PDF COMPARISONS

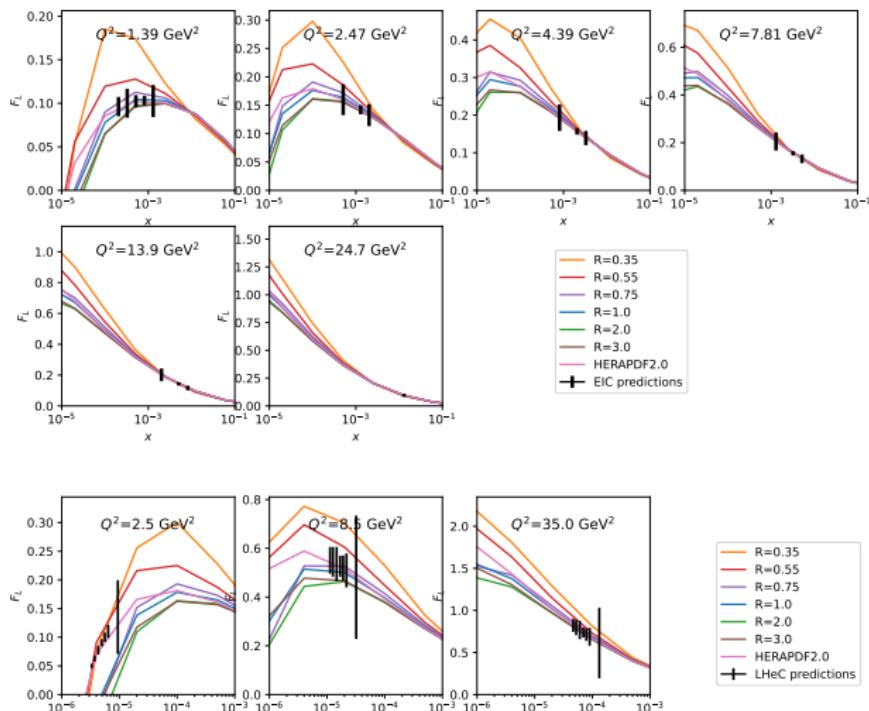


COMPARING TO HERA F_L DATA



- ▶ HERA F_L data cannot distinguish different R values

F_L DATA FROM FUTURE EXPERIMENTS



[E. Aschenauer et al.,
[https://www.phenix.bnl.gov/
WWW/publish/elke/EIC/EIC-R&D-Tracking/Meetings/f1.pdf](https://www.phenix.bnl.gov/WWW/publish/elke/EIC/EIC-R&D-Tracking/Meetings/f1.pdf)]

- ▶ Future data will be significantly more sensitive to recombination effects

[P. Agostini et al., J.Phys.G 48 (2021) 11, 110501]

CONCLUSIONS AND OUTLOOK

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- ▶ Gluon recombination might explain saturation
- ▶ GLR equations violate momentum conservation → calculation by Zhu + Ruan avoids this problem
- ▶ Implemented in HOPPET + xFitter to run fits
- ▶ Current DIS data shows no signs of gluon recombination

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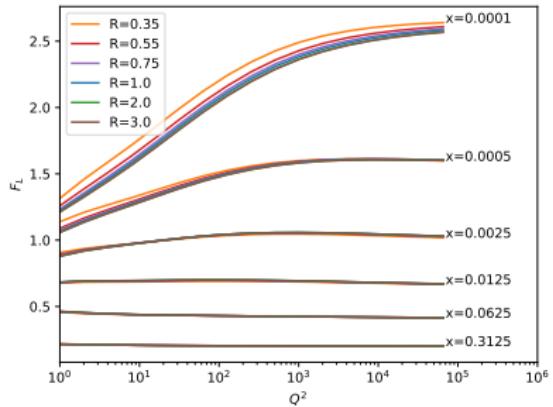
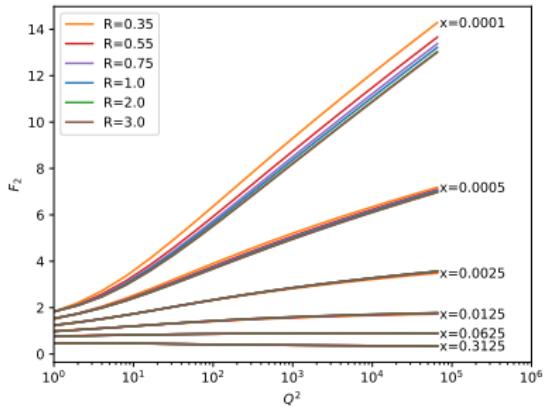
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Outlook

- ▶ Tools and results (LHAPDFs) will become available soon
- ▶ Certainly worth revisiting once EIC / LHeC data becomes available

Q -DEPENDENCE OF F_2 AND F_L



- ▶ F_L is more sensitive at low Q
- ▶ F_2 is more sensitive at high Q