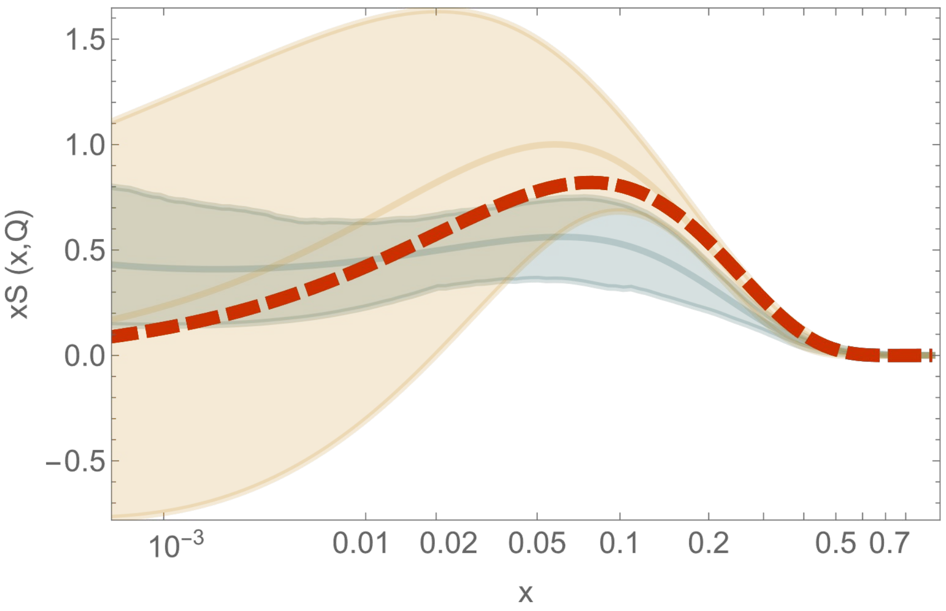
Bézier Curve approximation for Pion PDFs



Lucas Kotz, Southern Methodist University

Collaborators: Prof. Aurore Courtoy (UNAM), Prof. Pavel Nadolsky (SMU), Prof. Fred Olness (SMU), Maximiliano Ponce-Chavez (UNAM)

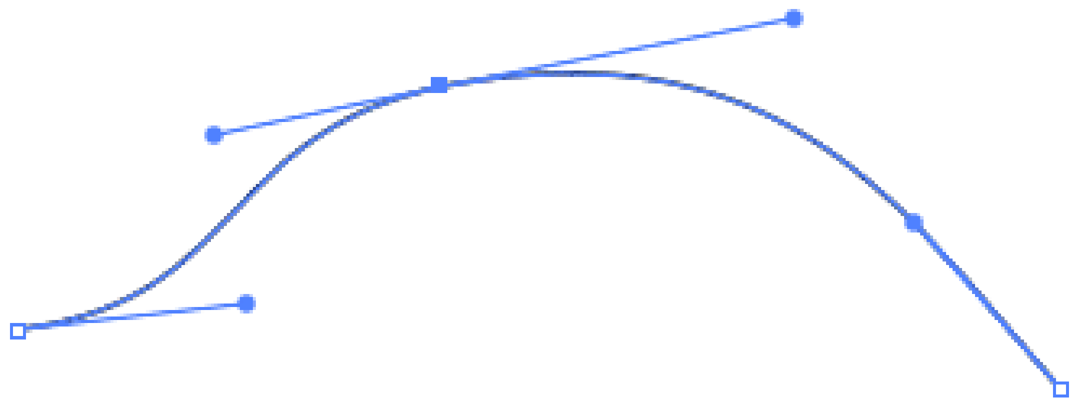
Inter-American Network of Networks of QCD challenges

Fantômas4QCD PDF Parameterization

- **Fantômas4QCD** (Fantômas for short) is a new module implemented into xFitter.
- The parameterization we use to calculate PDFs is called a **metamorph**.
- Metamorphs are polynomial parameterizations that can approximate a variety of functional behaviors typical for PDFs and provide an alternative to neural networks.

Metamorph and Bézier Cruves

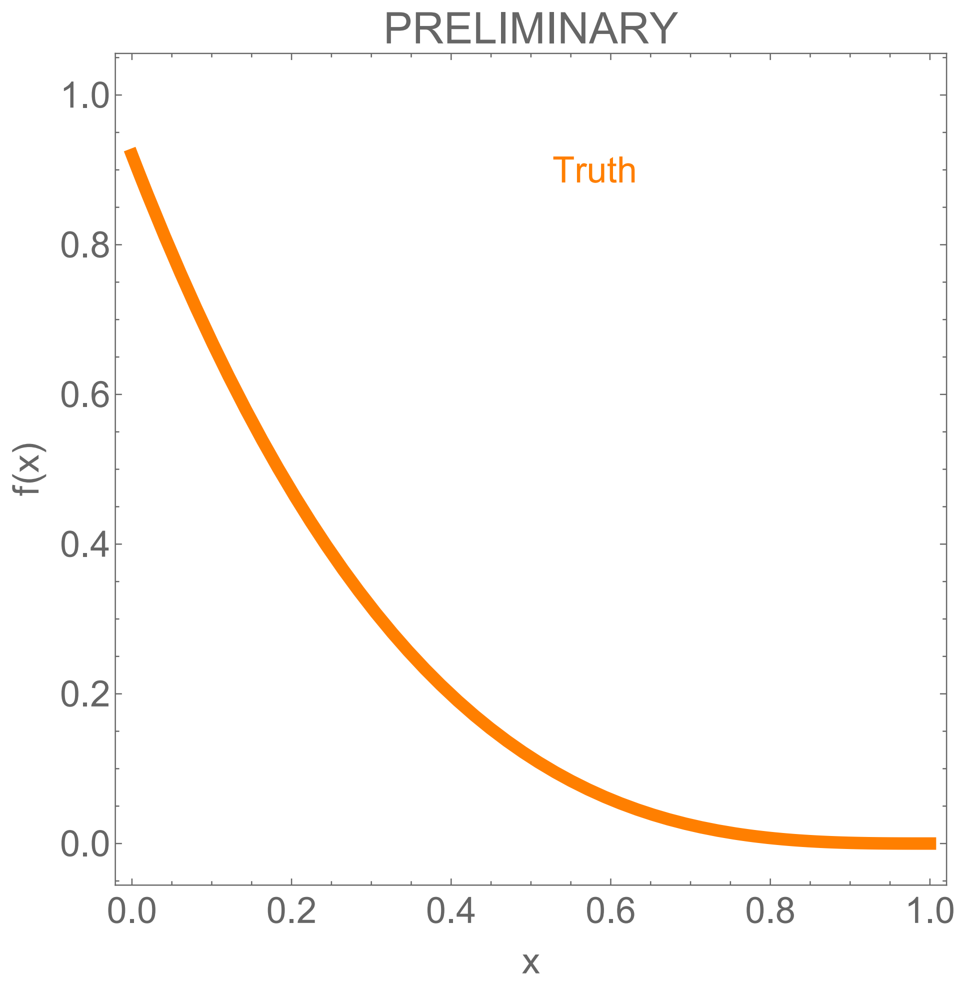
- A metamorph utilizes a Bézier curve, a polynomial of degree N_m computed from its values at **control points.**
- Flexibility of these curves allow a metamorph to approximate many PDF behaviors.



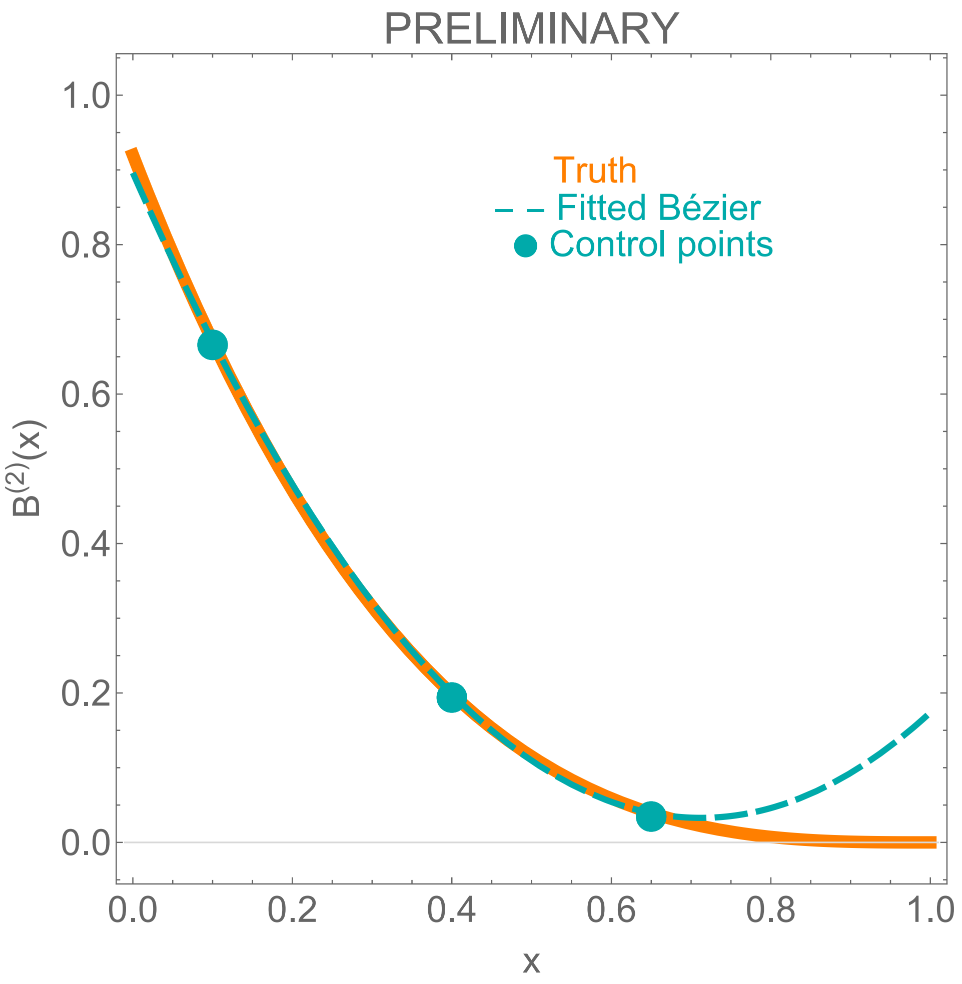
P. Nadolsky

[*A. Courtoy, P. Nadolsky*, arXiv: 2011.10078]

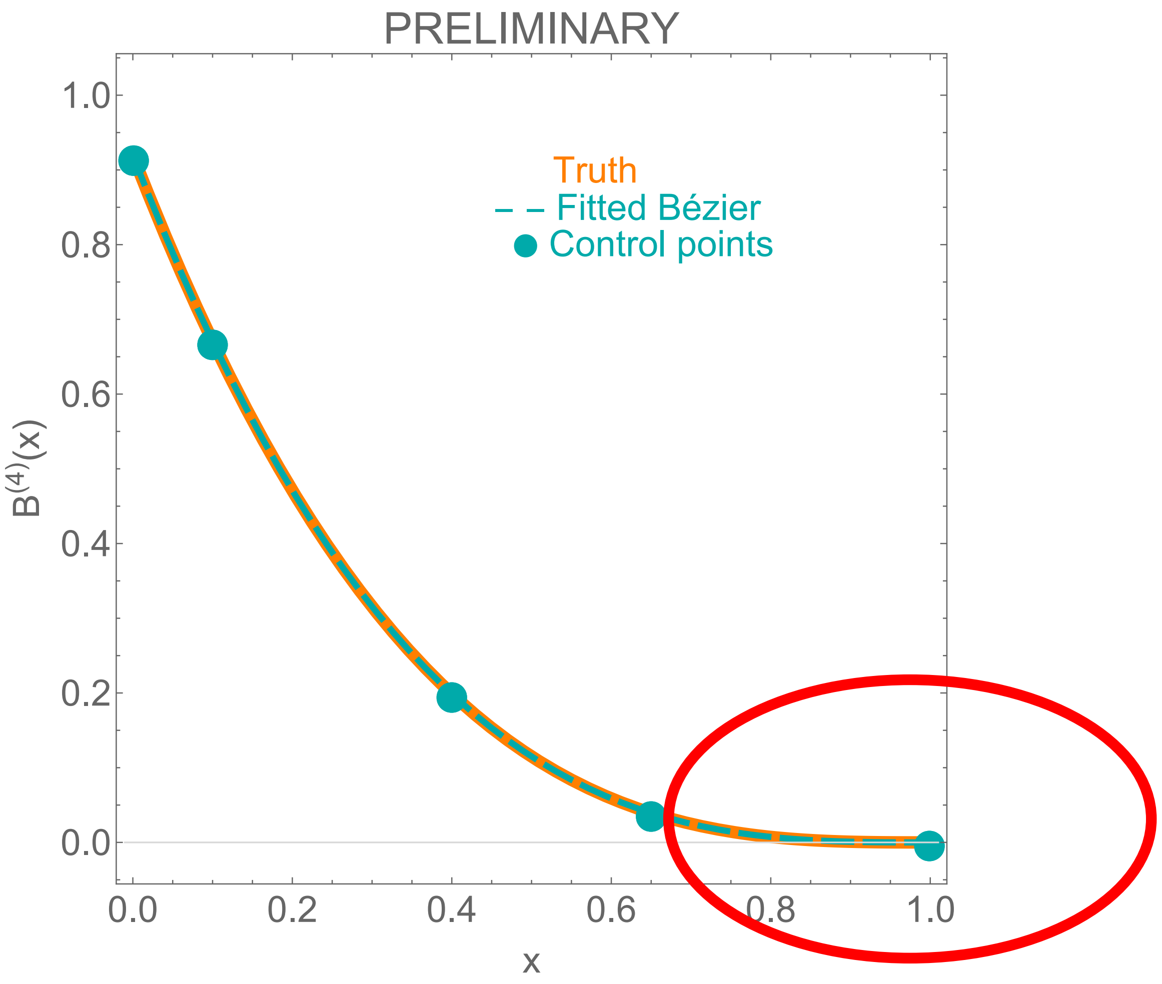
Control Points and Bézier Curves



Control Points and Bézier Curves ($N_m = 2$)



Control Points and Bézier Curves ($N_m = 4$)



Bézier Curve

$$\mathcal{B}^{(N_m)}(y) = \sum_{l=0}^{N_m} C_l B_{N_m,l}(y)$$

$$B_{N_m,l}(y) \equiv \binom{N_m}{l} y^l (1-y)^{N_m - l}$$

$$\Rightarrow \mathcal{B} = \boldsymbol{T} \cdot \boldsymbol{M} \cdot \boldsymbol{C}$$

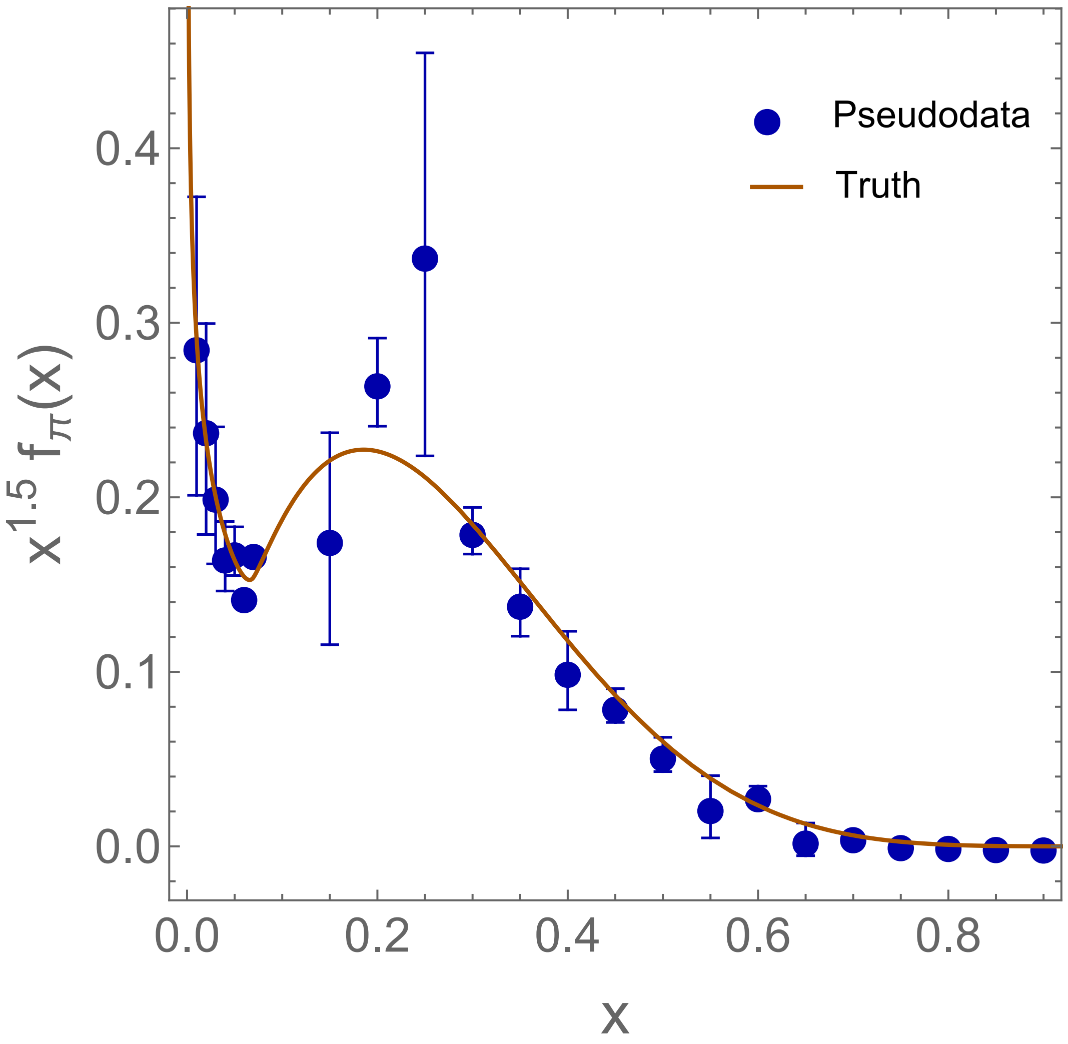
$$\text{or } \boldsymbol{C} = \boldsymbol{M}^{-1} \cdot \boldsymbol{T}^{-1} \cdot \boldsymbol{P}$$

- $\mathcal{B}^{(N_m)}(y)$: Bézier function of N_m^{th}-degree.
- $\boldsymbol{C}$: $N_m + 1$ vector containing Bézier coefficients.
- $B_{N_m,l}(y)$: Bernstein basis polynomial.
- $\boldsymbol{M}$: A fixed $N_m + 1 \times N_m + 1$ matrix containing binomial coefficients. Determined by N_m.
- $\boldsymbol{T}$: A fixed $N_m + 1 \times N_m + 1$ matrix. Determined by the positions of control points.
- $\boldsymbol{P}$: $N_m + 1$ vector containing the values at the control points.

G. Farin (2001)
Kamermans, Mike Pomax: https://pomax.github.io/bezierinfo

Performing Fits with a Metamorph

- Pseudodata are constructed by Gaussian fluctuations around the "truth" function.
- The metamorph function is fitted to the pseudodata.



Performing Fits with a Metamorph

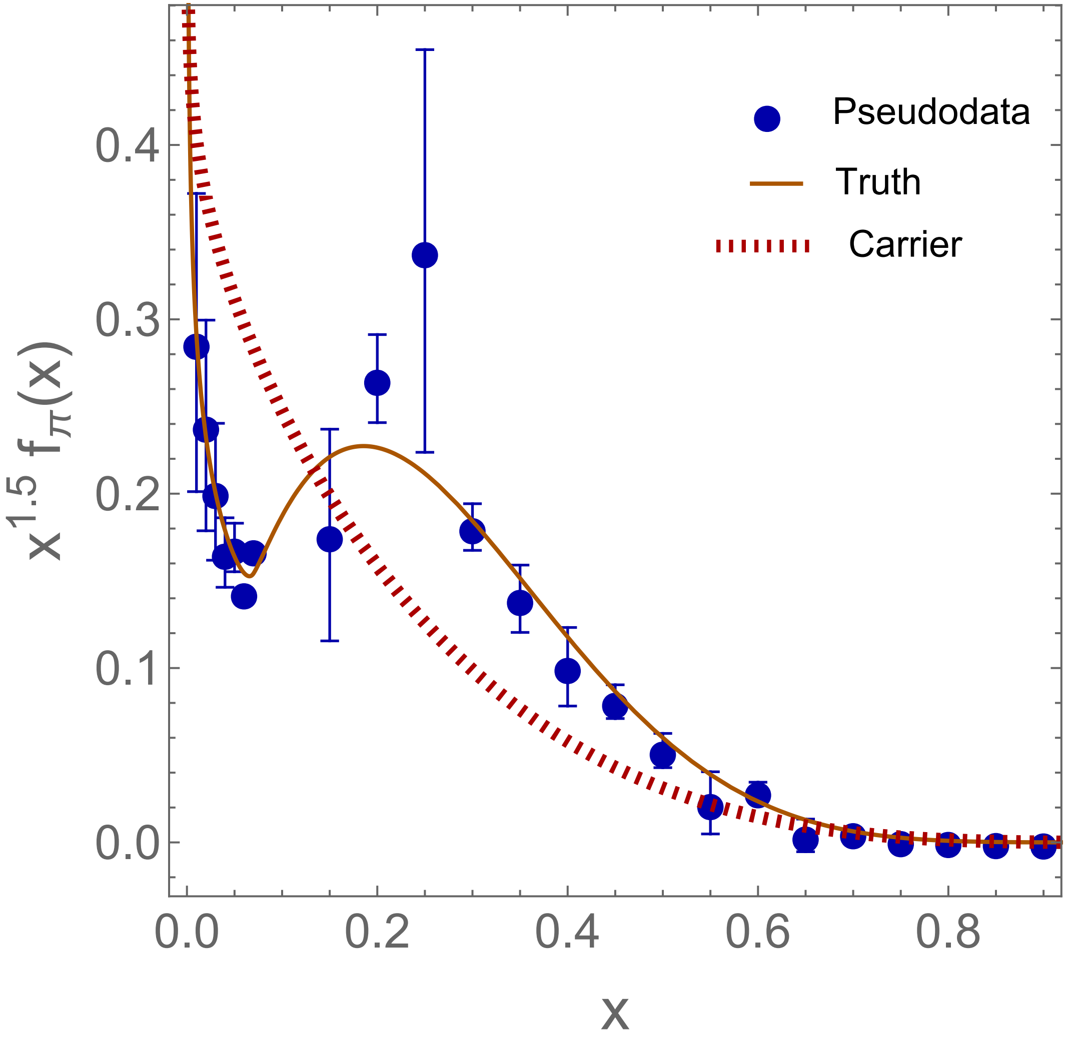
- The functional form of the Fantômas4QCD parameterization is

$$xf(x,Q_0^2) = f_{\text{Carrier}}(x) * f_{\text{Modulator}}(x^{\alpha_x})$$

where

$$f_{\text{Carrier}}(x) \equiv A_f x^{B_f}(1-x)^{C_f}.$$

- The Carrier specifies asymptotic limits of $xf(x,Q_0^2)$ at $x \to 0$ or 1.



Performing Fits with a metamorph

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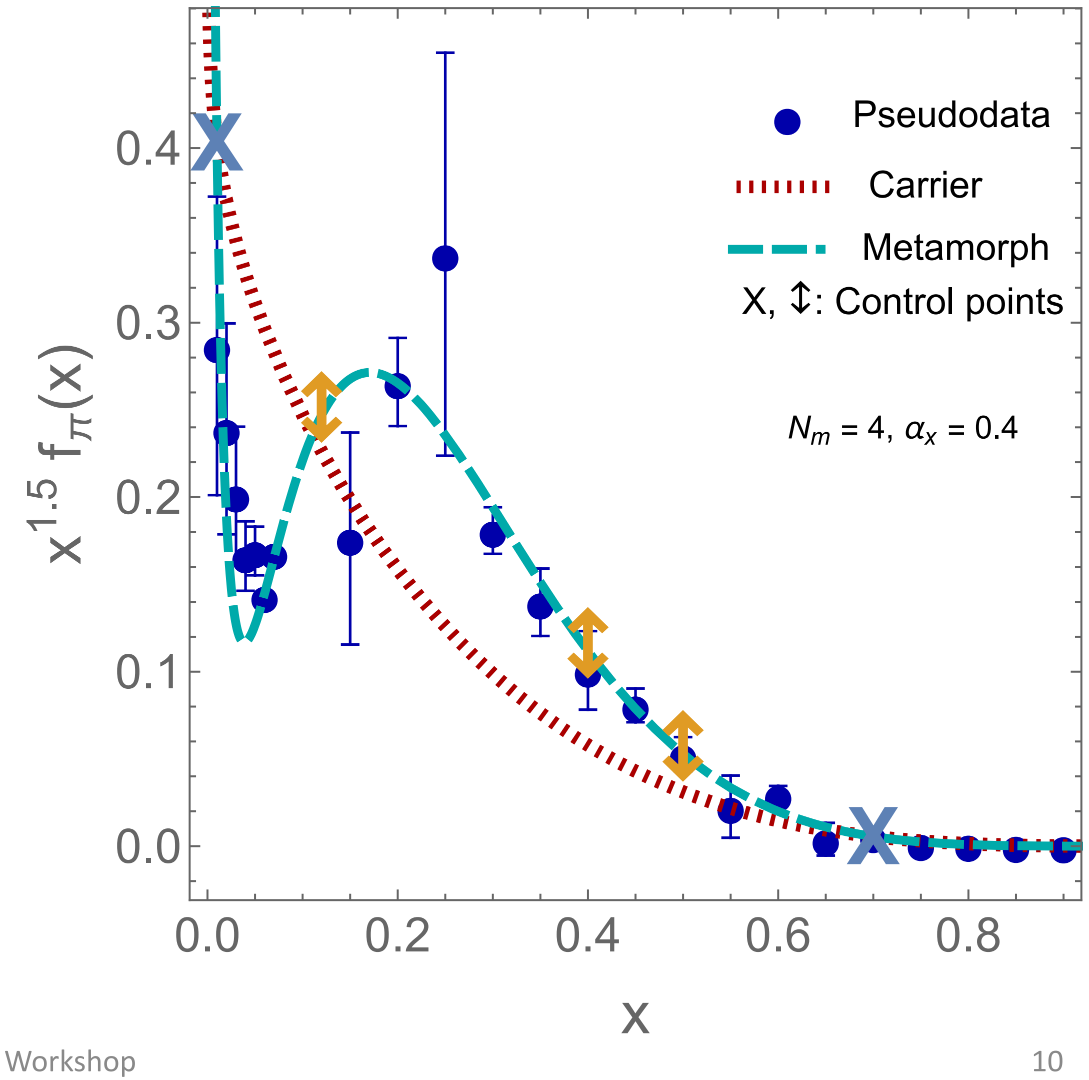
where we choose

$$f_{\text{Modulator}}(x^{\alpha_x}) = \mathcal{B}^{(N_m)}(x^{\alpha_x}).$$

- The Modulator modifies $xf(x, Q_0^2)$ at $0 < x < 1$. α_x is an x-stretching power between 0 and 1.



Performing Fits with a metamorph

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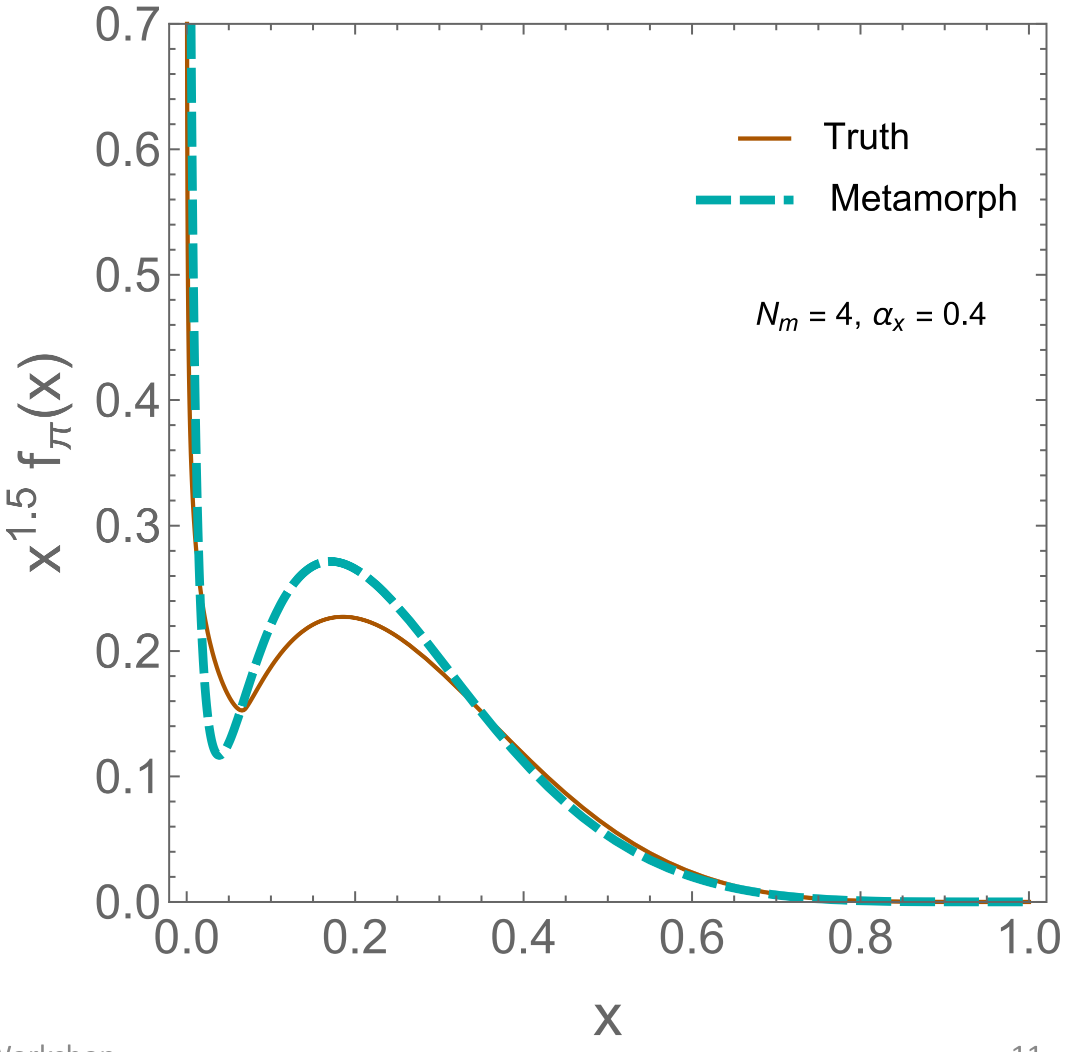


Performing Fits in xFitter

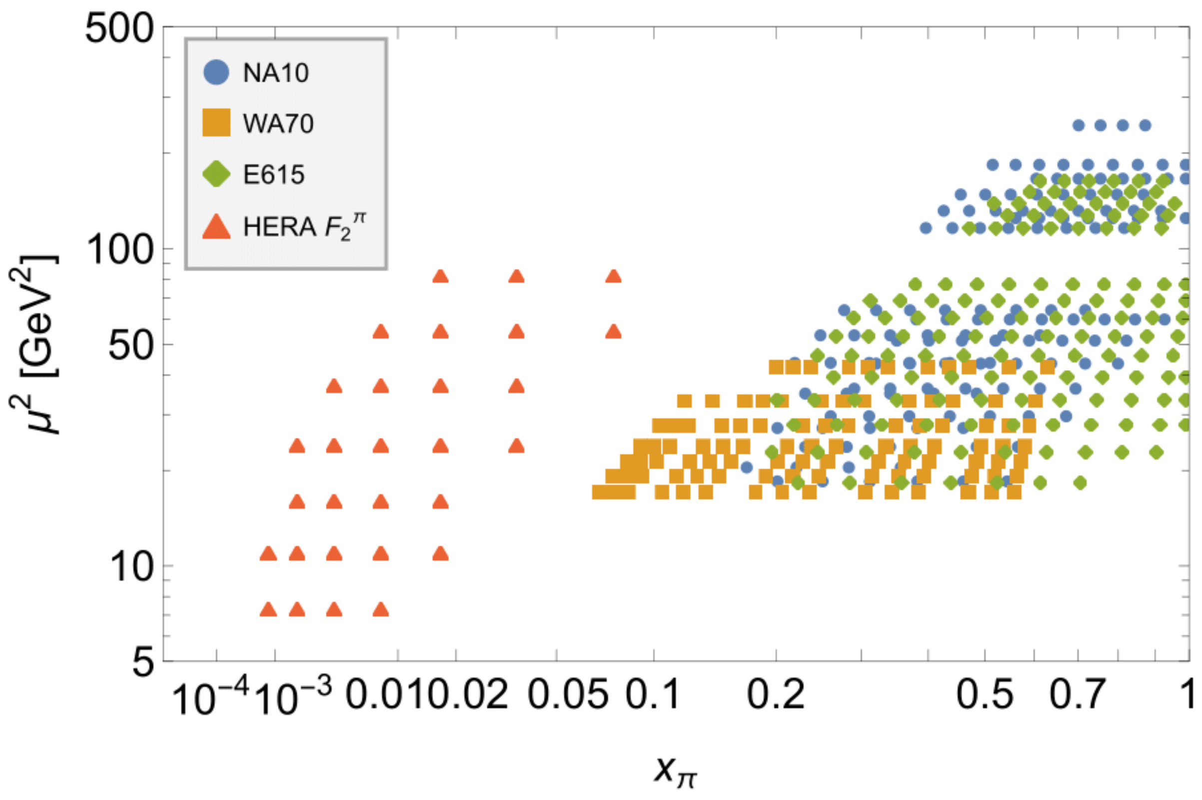
- The carrier can be fixed or free to vary within xFitter.
- Fixing the carrier requires an $N_m = 0$ fit to find the parameters to base the fit.
- A free carrier allows for further flexibility to search for the absolute best fit with no constraints.
- The control points are distributed by the user.
  - At a fixed control point, the modulator is constant. Fixed control points are used e.g. to reproduce the asymptotic power laws at x-> 0 or x->1.

Pion PDF Uncertainties

- The pion structure is related to properties of QCD at low energy. Non-perturbative methods can be used to describe it in terms of quarks and gluons.
- On the phenomenological point of view, the pion PDF has been extracted from data.
- Pion data is already implemented into xFitter.
- The modulator is chosen to be $1 + \mathcal{B}^{(N_m)}(x)$.

Datapoints used in fits

- NA10 & E615: Covers the main kinematic region of $x > 0.2$ and $Q^2 > 10\ \mathrm{eV}\ \ ^2$. Constrains valence very well.
- WA70: Provides some sensitivity to the gluon PDFs that the DY data could not provide.
- HERAF_2^π: Constrains the Sea and gluon PDFs at low-x. Uses the HERA prescription.



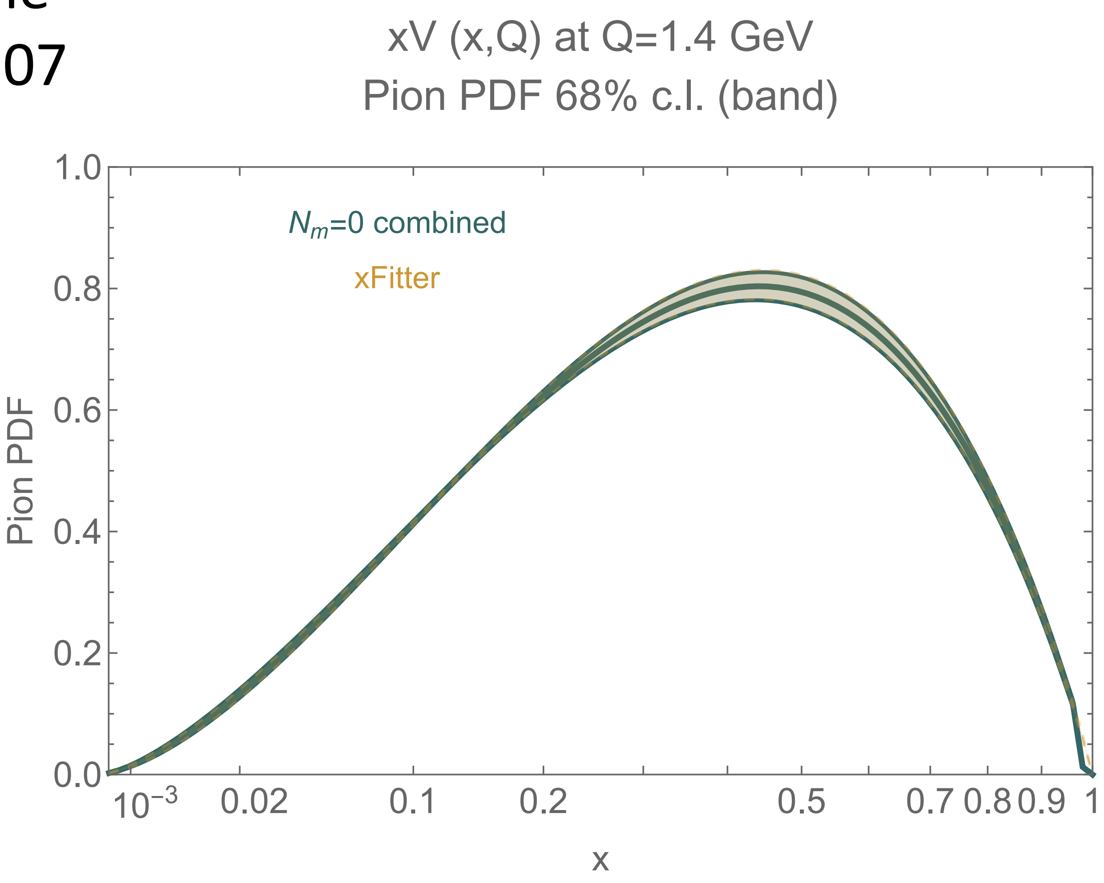
$N_m = 0$ Pion fits

- Multiple error bands were combined using the META PDF method (J. Gao, P. Nadolsky, JHEP 07 (2014) on the range of $-0.27 < B_s < 1.07$.

Central Pion PDF (DY)	$\frac{\chi^2}{\mathrm{d.o.f.}}$	$< xv >$	$< xS >$	$< xg >$
xFitter	1.19	0.56	0.21	0.23
$N_m = 0$, $B_s = -0.27$	1.20	0.55	0.34	0.11
$N_m = 0$, $B_s = 0.47$	1.19	0.56	0.22	0.23
$N_m = 0$, $B_s = 1.07$	1.19	0.56	0.17	0.27

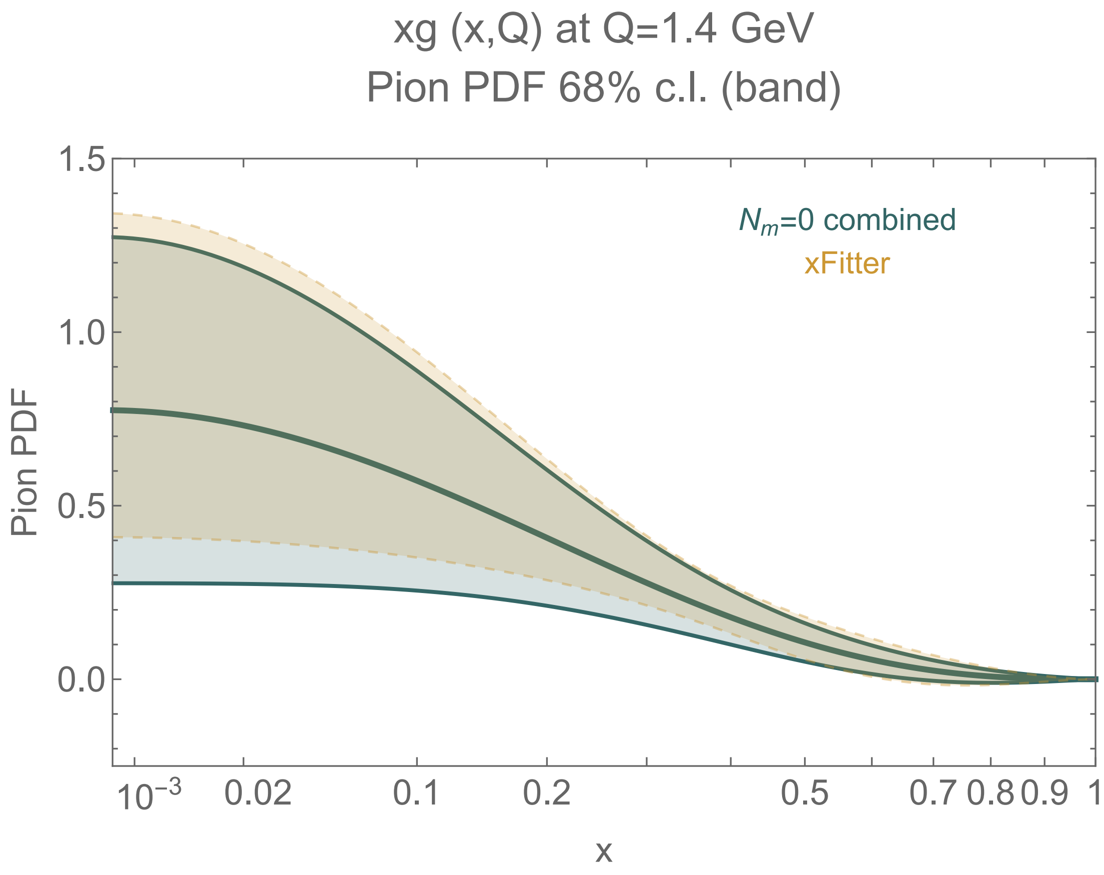
xFitter Developers' team (2020),
arXiv: [2002.02902]

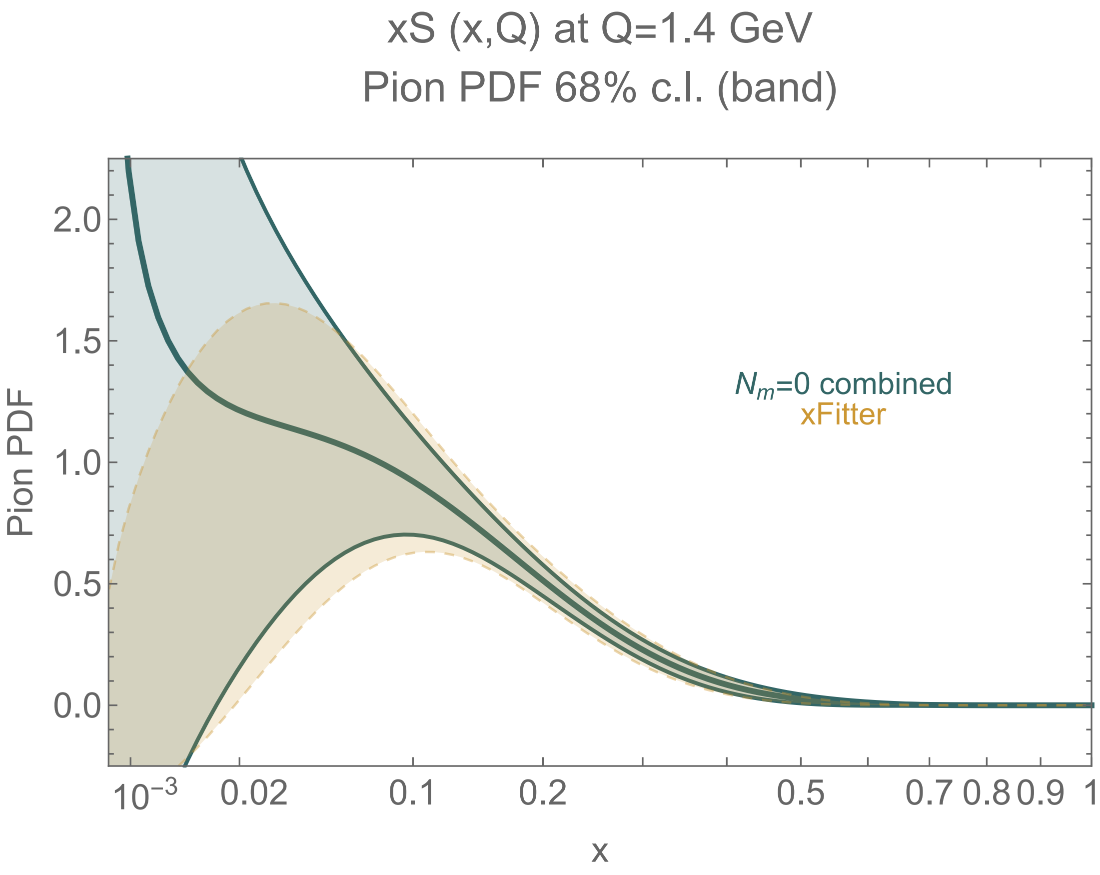
PRELIMINARY



$N_m = 0$ Pion fits

PRELIMINARY





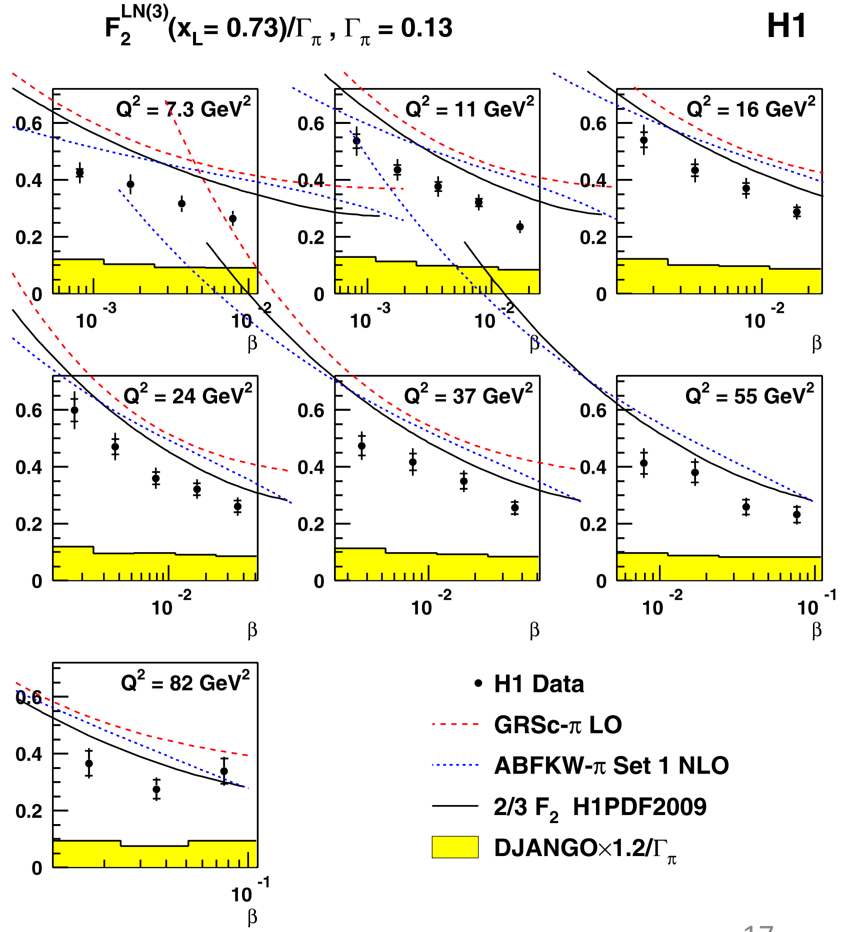
Leading-neutron data in DIS

- We follow the HERA prescription [Aaron et al, Eur. Phys. J. C, 68, 2010]
- H1 analysis identifies the single-pion production to be valid around the range $0.68 < x_L < 0.77$ at low p_T of order $p_T = 0.2\ \mathrm{eV}$ -- LN production could be used to extract the pion PDF in that range.

$$F_2^{LN(3)}(Q^2, x, x_L) \quad = \quad 2\, f_{\pi N}(1 - x_L)\, F_2^{\pi}(x_\pi, Q^2)$$

- We use the flux prescription -- based on the light-cone representation of H.Holtmann et al., Phys.Lett.B338, 363(1994)

$$f_{\pi N}(x_L = 0.73) \simeq 0.13 \pm 0.04$$

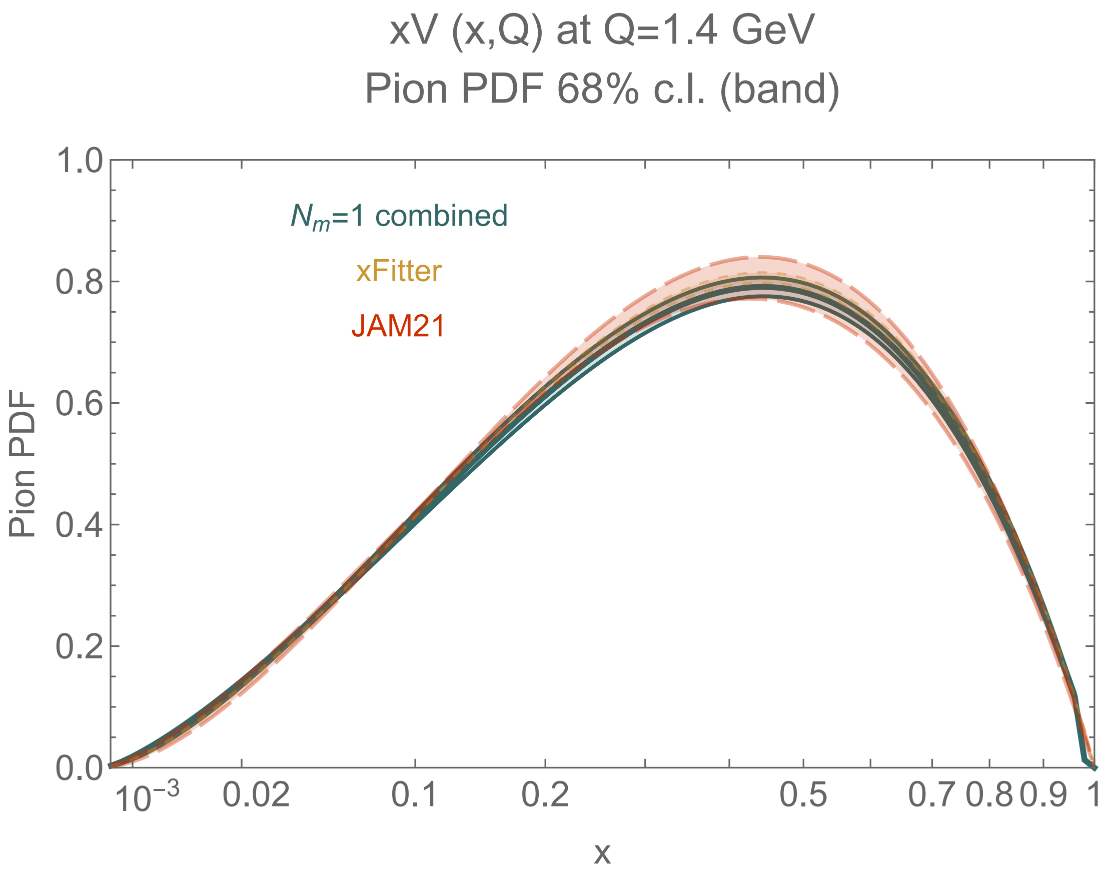


$N_m = 1$ Pion fits

- The combined PDF uses several fits that were performed with $N_m = 1$ and various options.

Central Pion PDF (DY+LN)	$\frac{\chi^2}{\mathrm{d.o.f.}}$	$< xv >$	$< xS >$	$< xg >$
xFitter (DY)	1.19	0.56	0.21	0.23
JAM21nlo	0.81	0.53	0.14	0.34
$N_m = 1$, Fixed Carrier Fixed High CP	1.12	0.55	0.18	0.27
$N_m = 1$, Fixed Carrier Fixed Low CP	1.12	0.55	0.18	0.27
$N_m = 1$, Fixed Carrier No Fixed CP	1.13	0.55	0.18	0.27

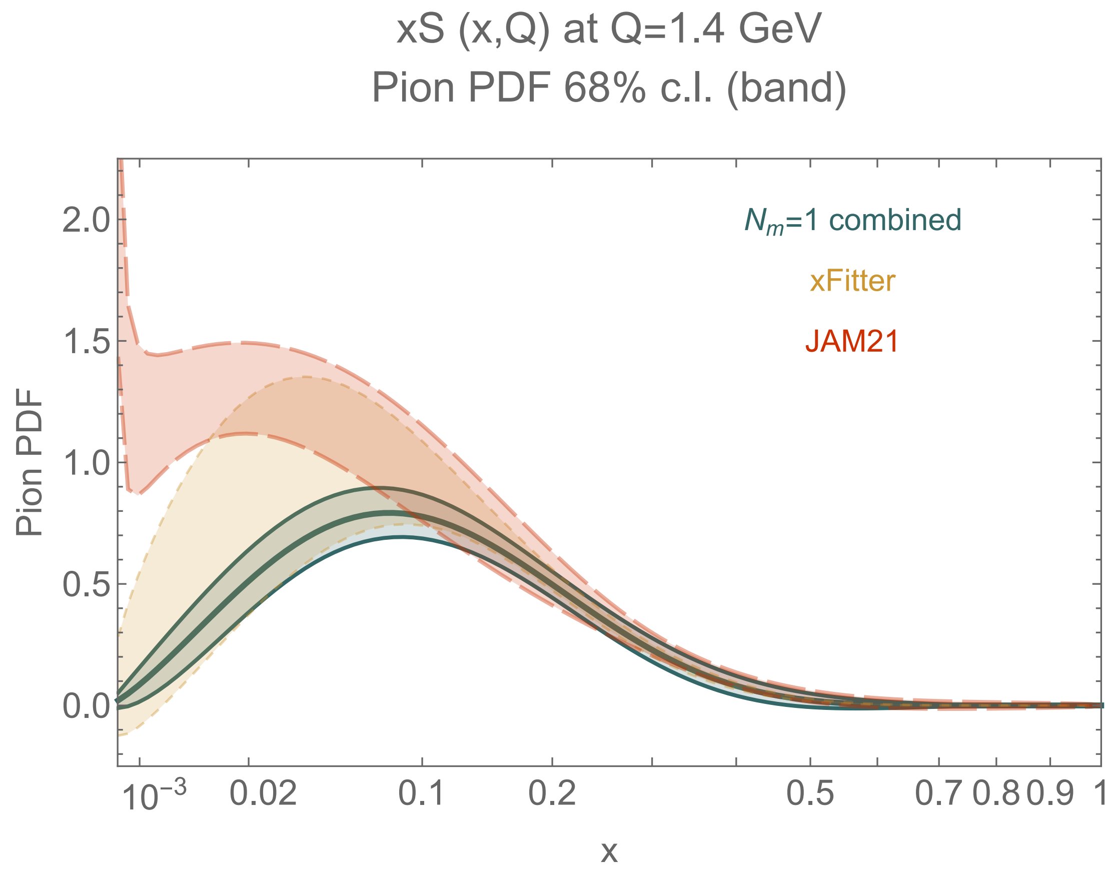
PRELIMINARY

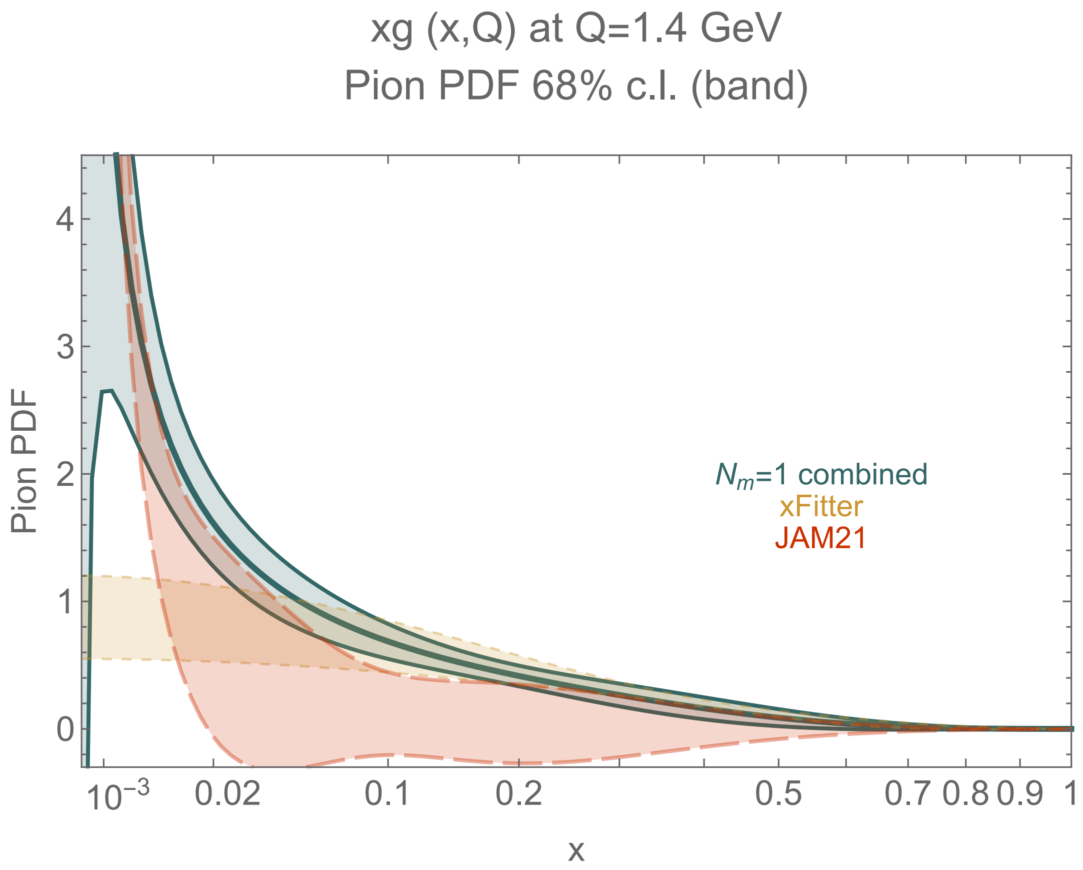


JAM Collaboration (2021),
arXiv: [2108.05822](2108.05822)

$N_m = 1$ Pion fits

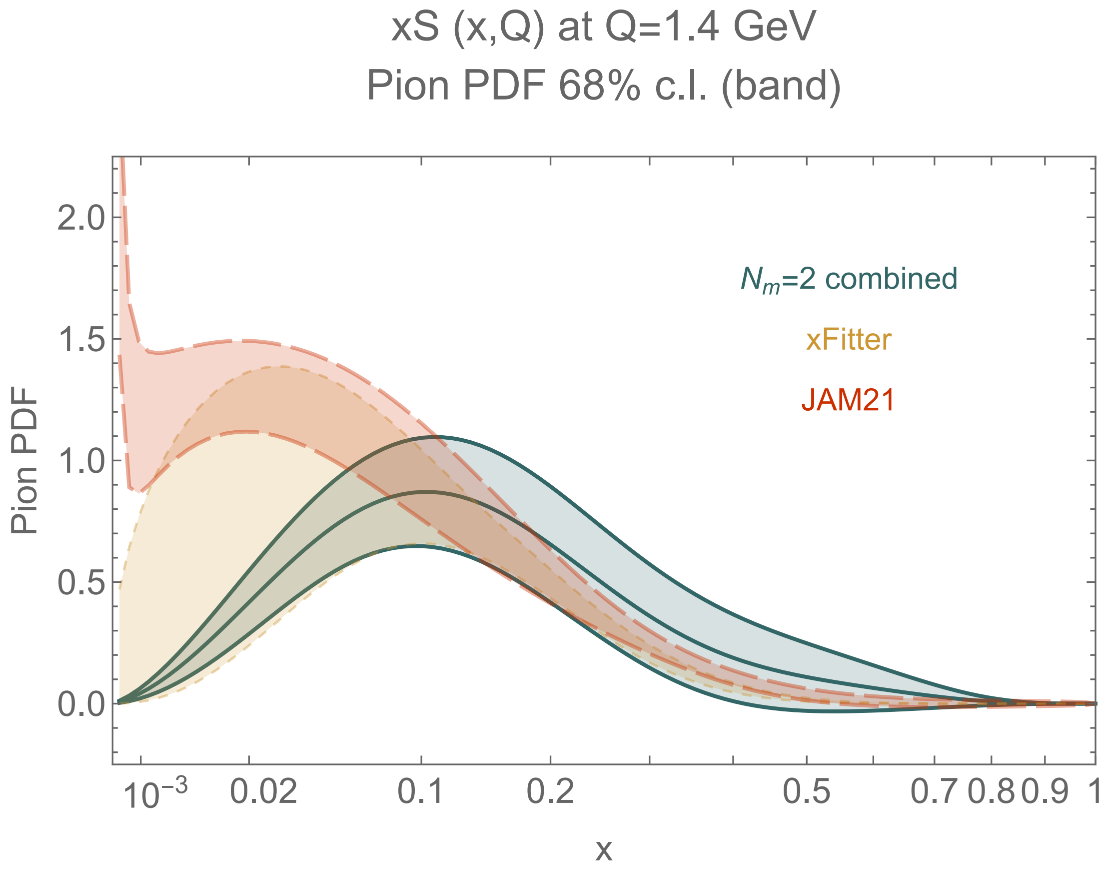
PRELIMINARY

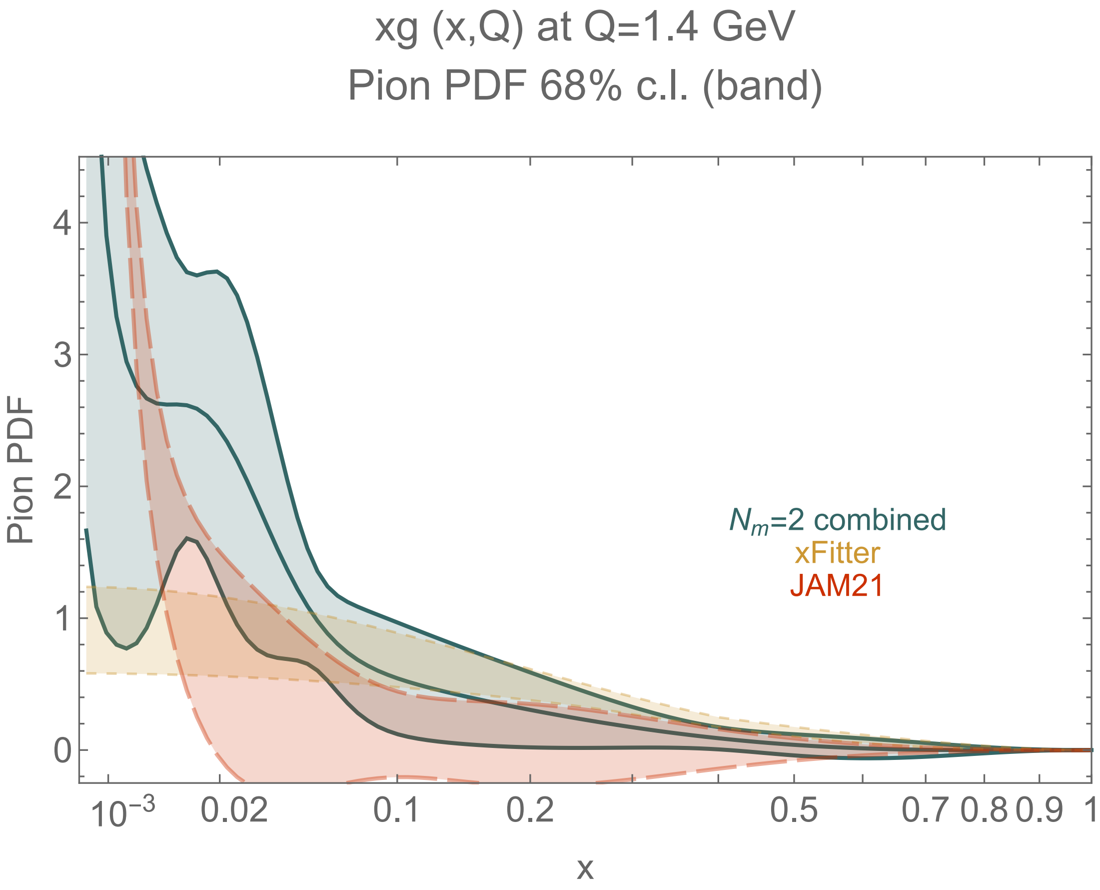




$N_m = 2$ Pion Fits

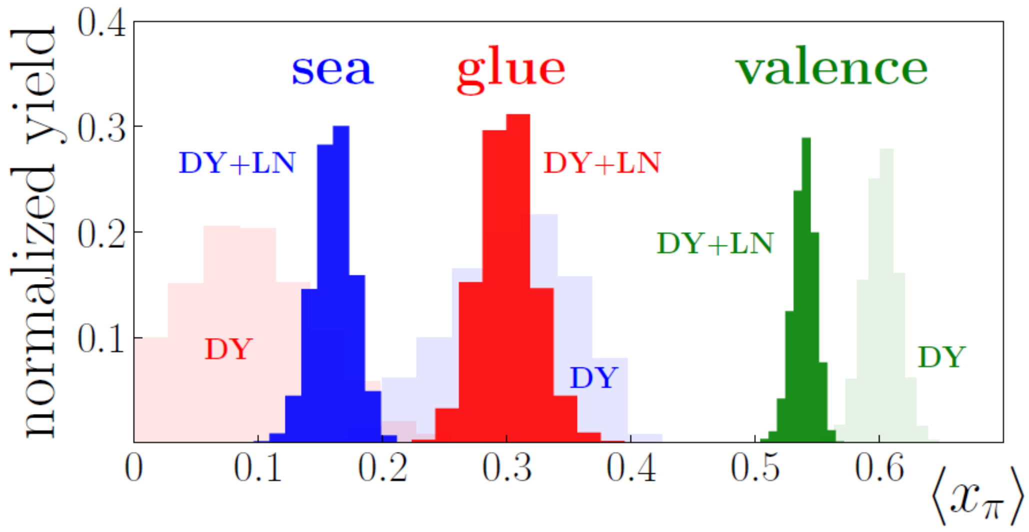
PRELIMINARY





Momentum Fractions ($Q^2 = 5$ ev ^{2})

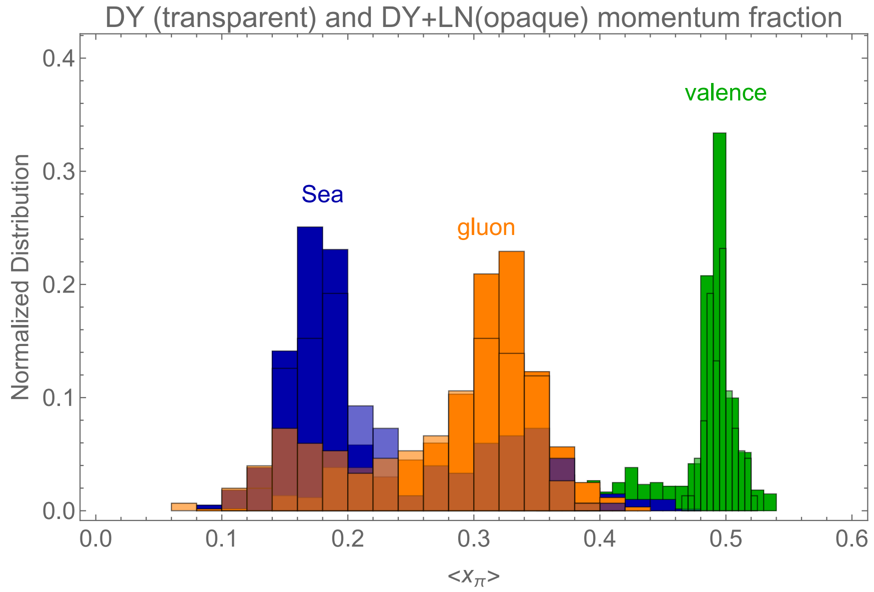
JAM'18



- $\overline{< xS >}_{\mathrm{DY+LN}} = 0.17 \pm 0.01$
- $\overline{< xg >}_{\mathrm{DY+LN}} = 0.35 \pm 0.02$
- $\overline{< xv >}_{\mathrm{DY+LN}} = 0.48 \pm 0.01$

JAM Collaboration (2018),
arXiv: [1804.01965]

Fantomas (PRELIMINARY)



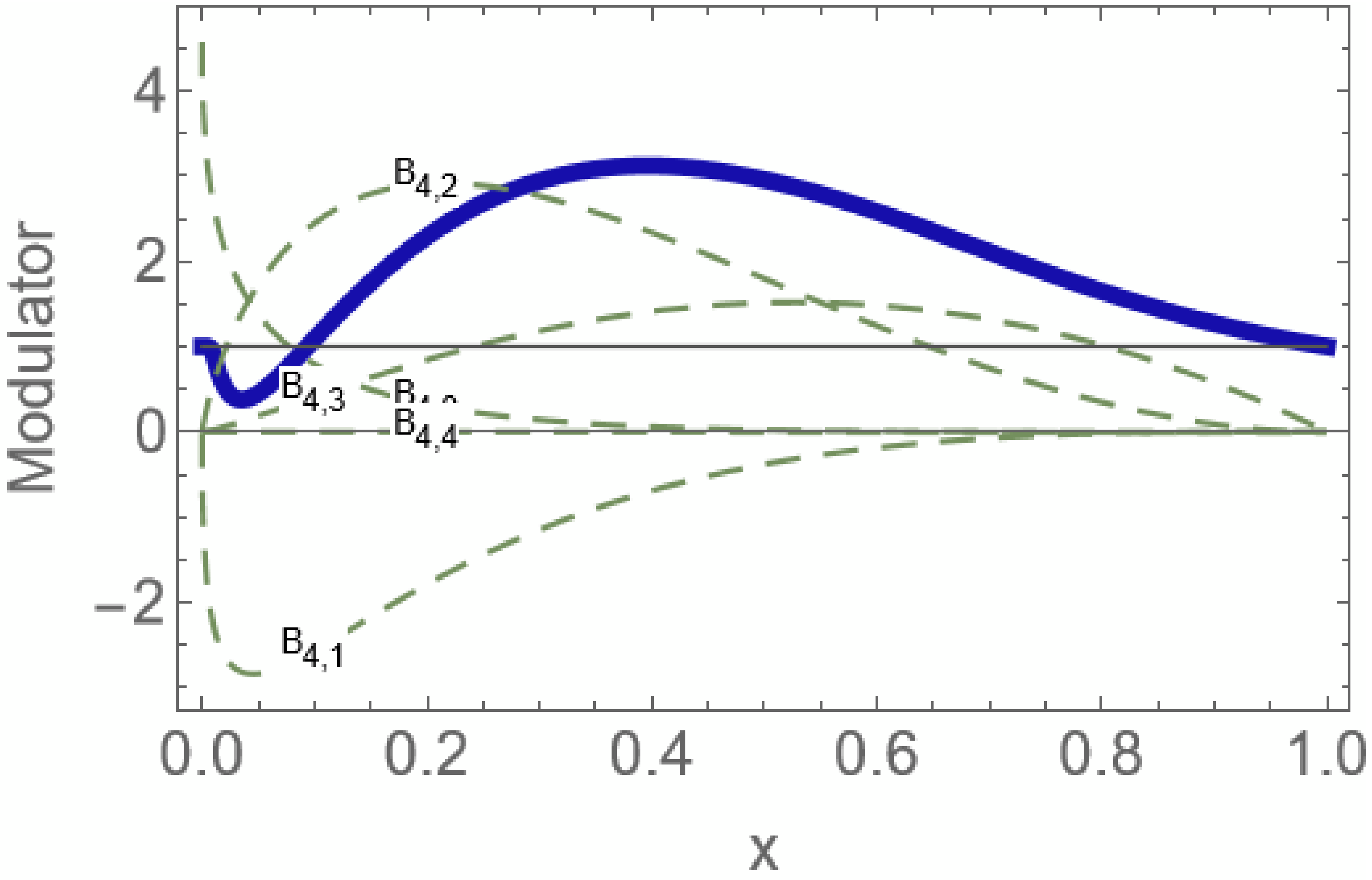
- $\overline{< xS >}_{\mathrm{DY+LN}} = 0.21 \pm 0.08$
- $\overline{< xg >}_{\mathrm{DY+LN}} = 0.30 \pm 0.05$
- $\overline{< xv >}_{\mathrm{DY+LN}} = 0.48 \pm 0.04$

Conclusion

- Bézier curves are polynomial interpolations that approximate a variety of functional behaviors typical for PDFs.
- The inherent flexibility of Bézier curves can give insight into PDF uncertainties at low- and high-x values.
- Metamorphs can take on a variety of functional forms, allowing them to be used to extensively explore not only pion PDFs, but many other PDFs.

Extra Slides

Modulator



Parameter Table for Toy Fit

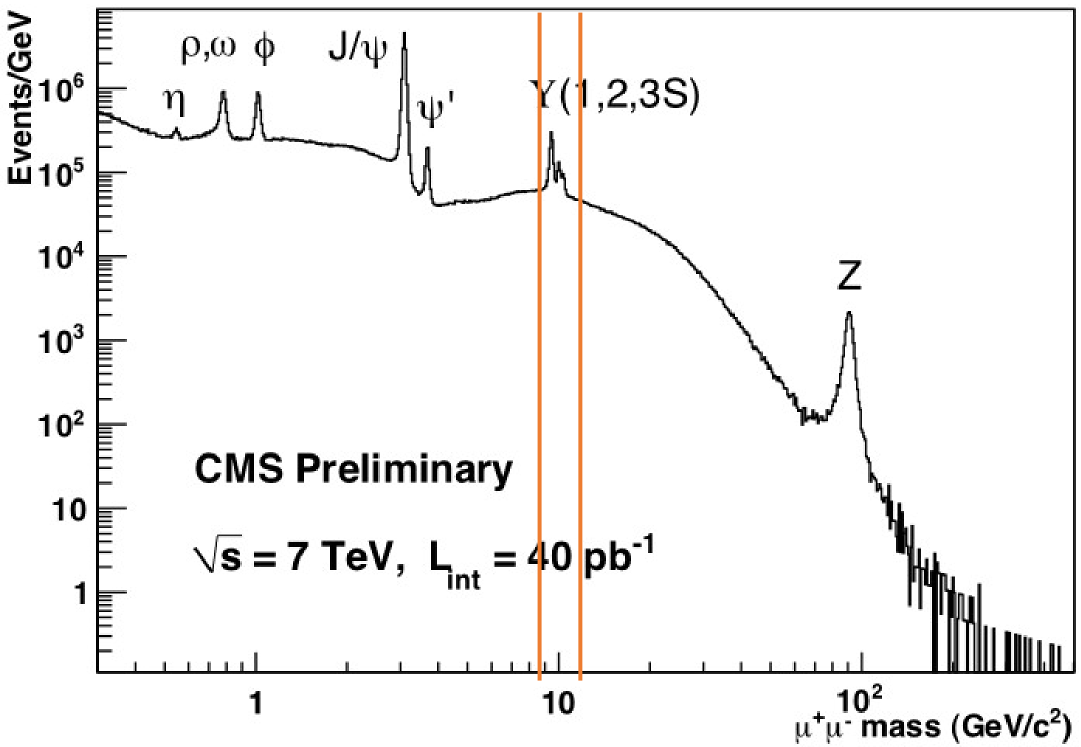
	Estimate	Standard Error	t-Statistic	P-Value
da0	−0.705153	0.156905	−4.49414	0.000319731
da1	−0.575125	0.140292	−4.0995	0.000747529
da2	0.321579	0.439633	0.731472	0.474454
db1	0.092065	0.244706	0.376228	0.711402
db2	0.930947	0.326039	2.85532	0.0109504
db3	0.698893	0.240317	2.90821	0.0097906

$$f_{\mathrm{Carrier}}(x) = (1 + \mathrm{da0})x^{0+\mathrm{da1}}(1 - x)^{3+\mathrm{da2}}$$

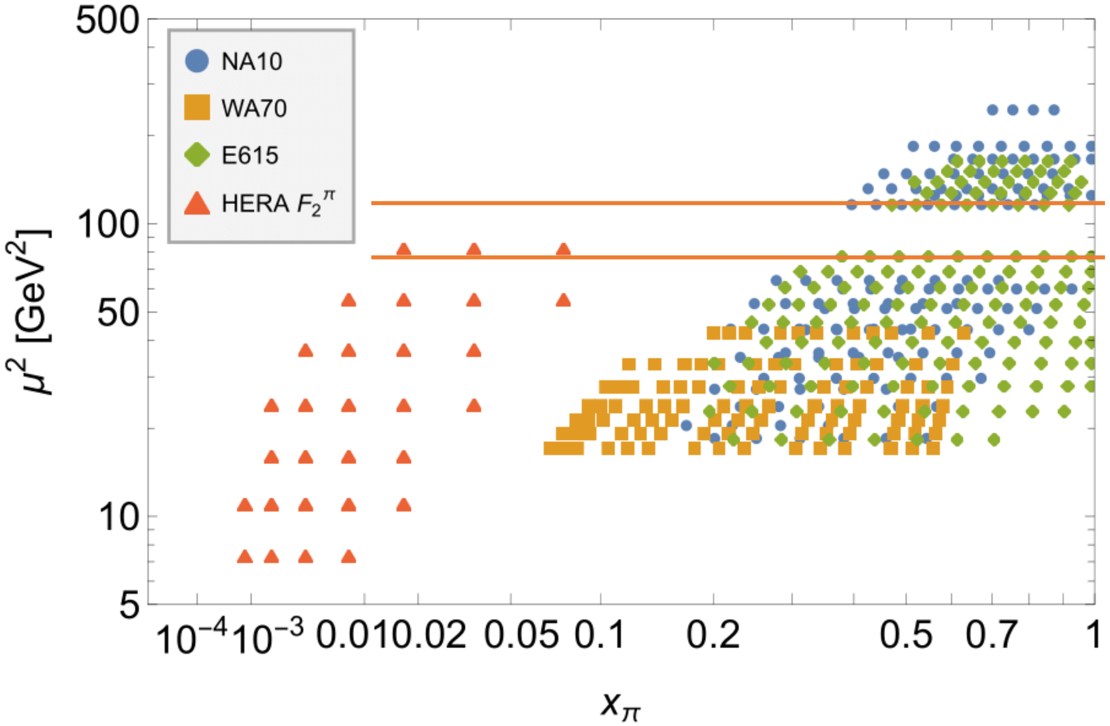
$$P_i = f_{\mathrm{Carrier}}(x_i) + \mathrm{dbi}$$

$$\chi^2 = 36.6$$

Exclusion of DY data

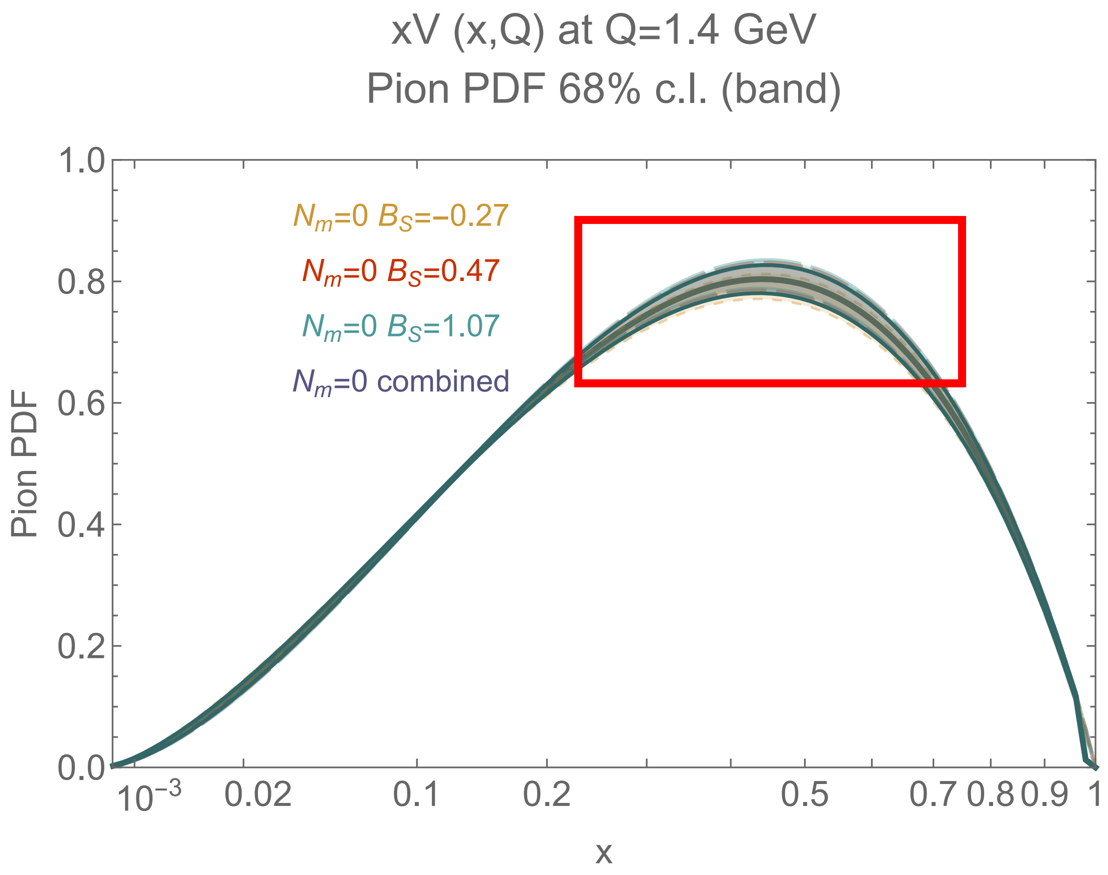


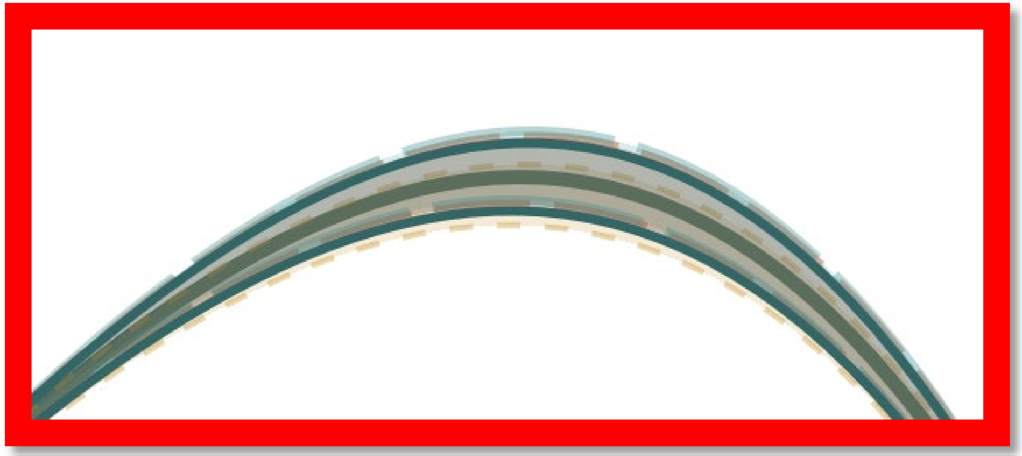
R. Ruiz, Quantum Diaries



Constraint on Sea and gluon (DY)

PRELIMINARY





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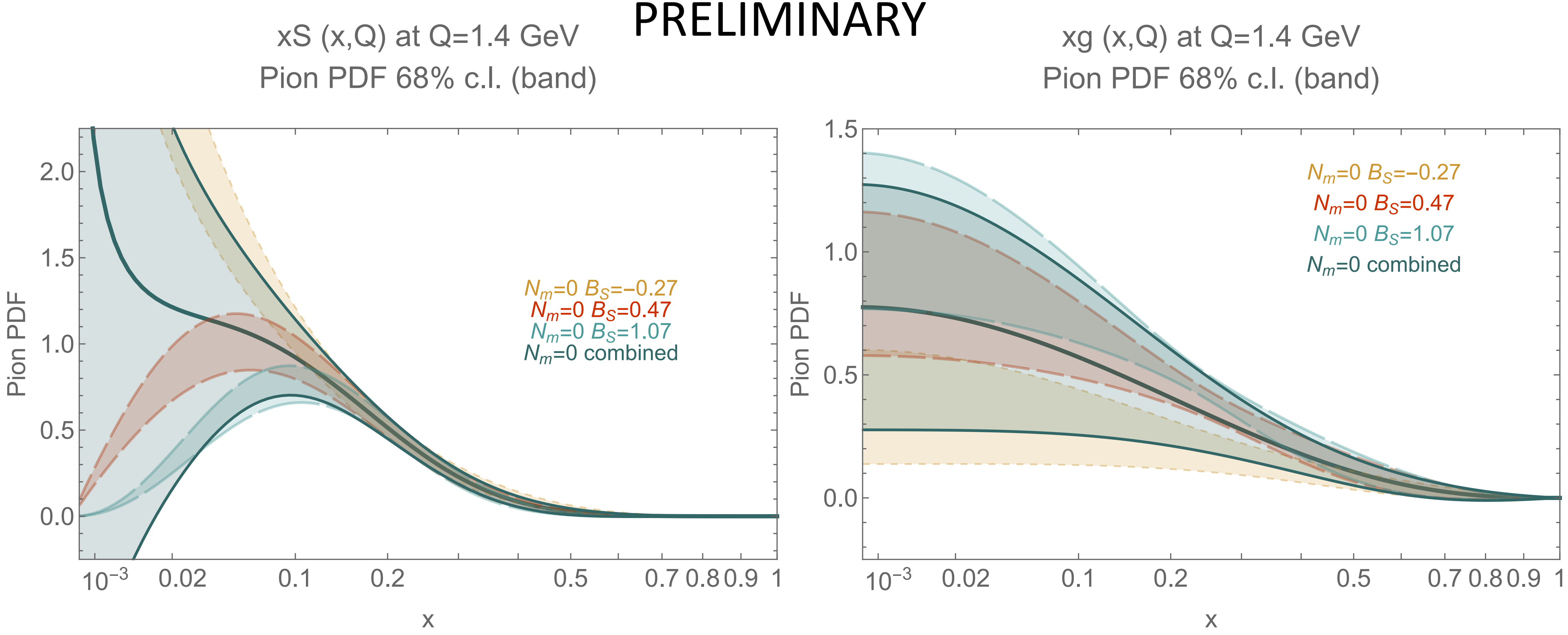
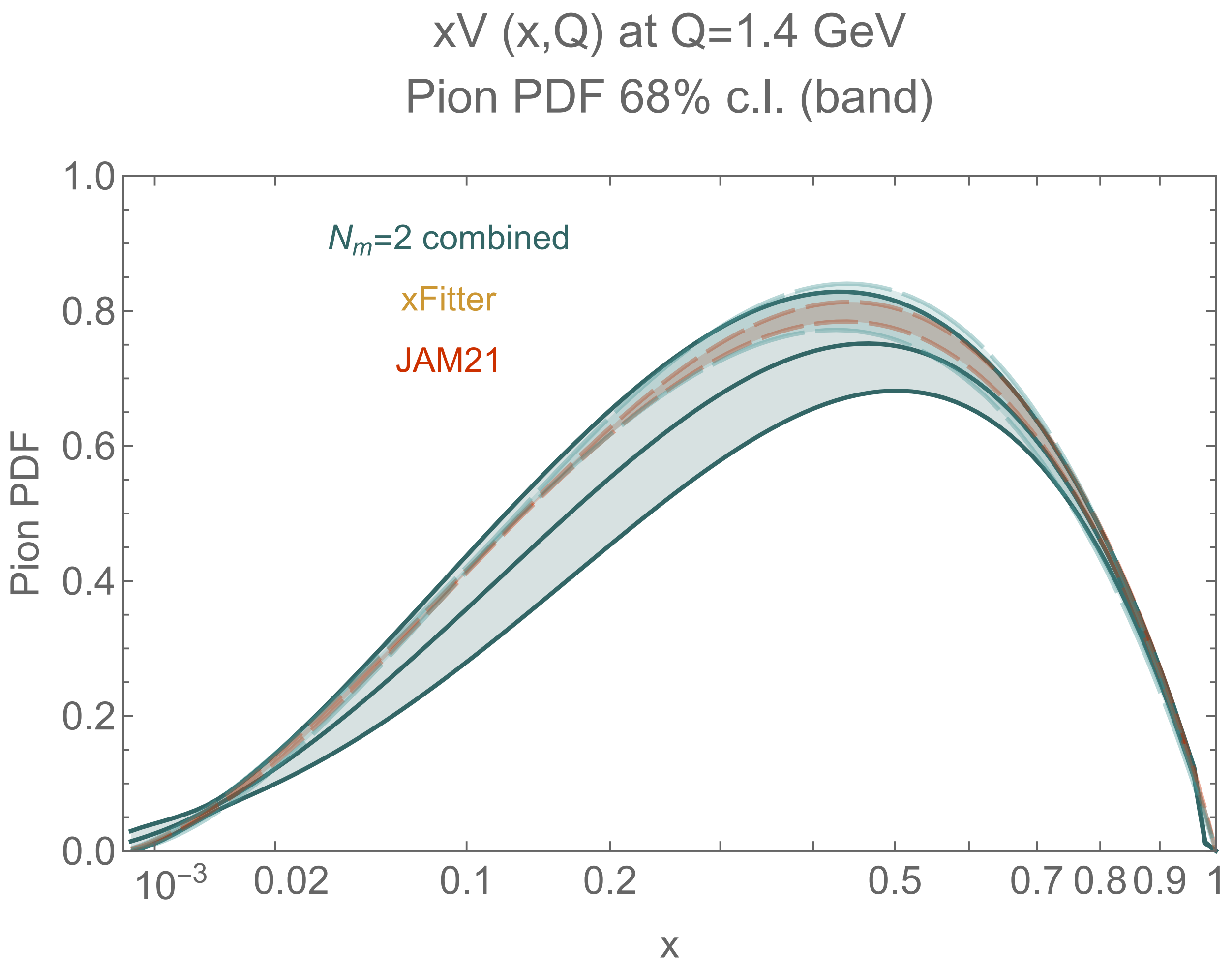
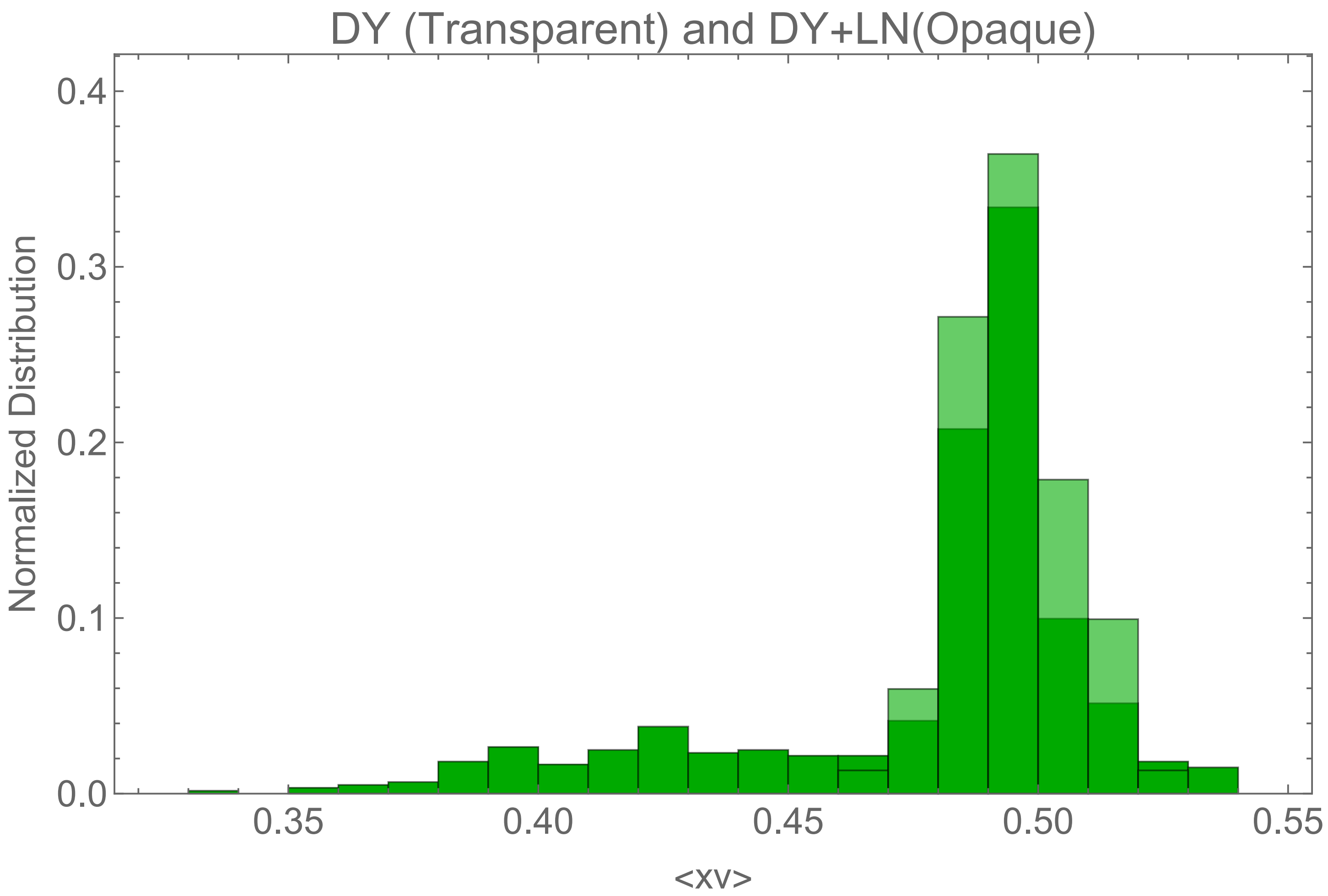


$N_m = 2$ valence

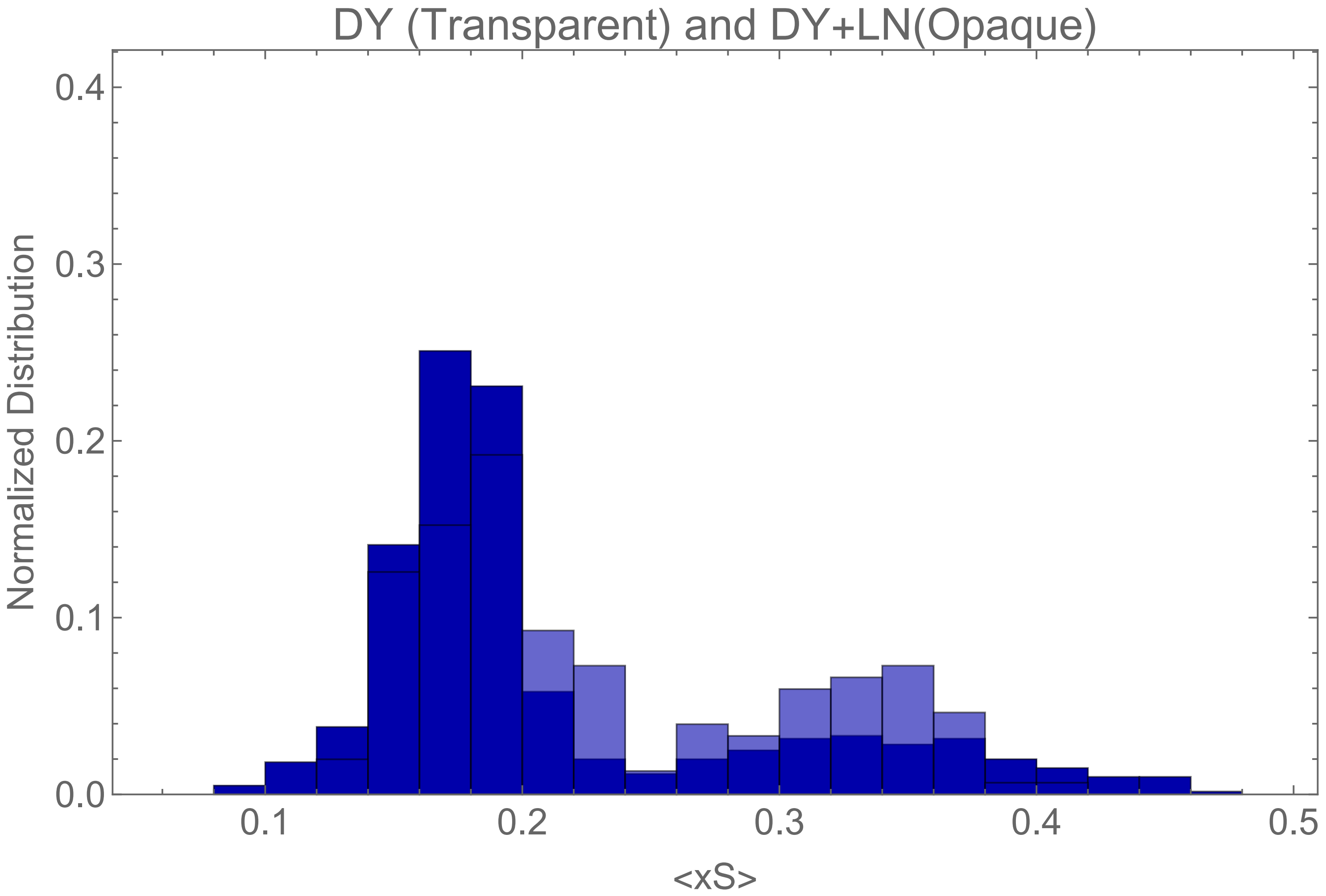


xV (x,Q) at Q=1.4 GeV
Pion PDF 68% c.l. (band)
N_m=2 combined
xFitter
JAM21
Pion PDF
1.0
0.8
0.6
0.4
0.2
0.0
10^{-3}
0.02
0.1
0.2
0.5
0.7
0.8
0.9
1
x

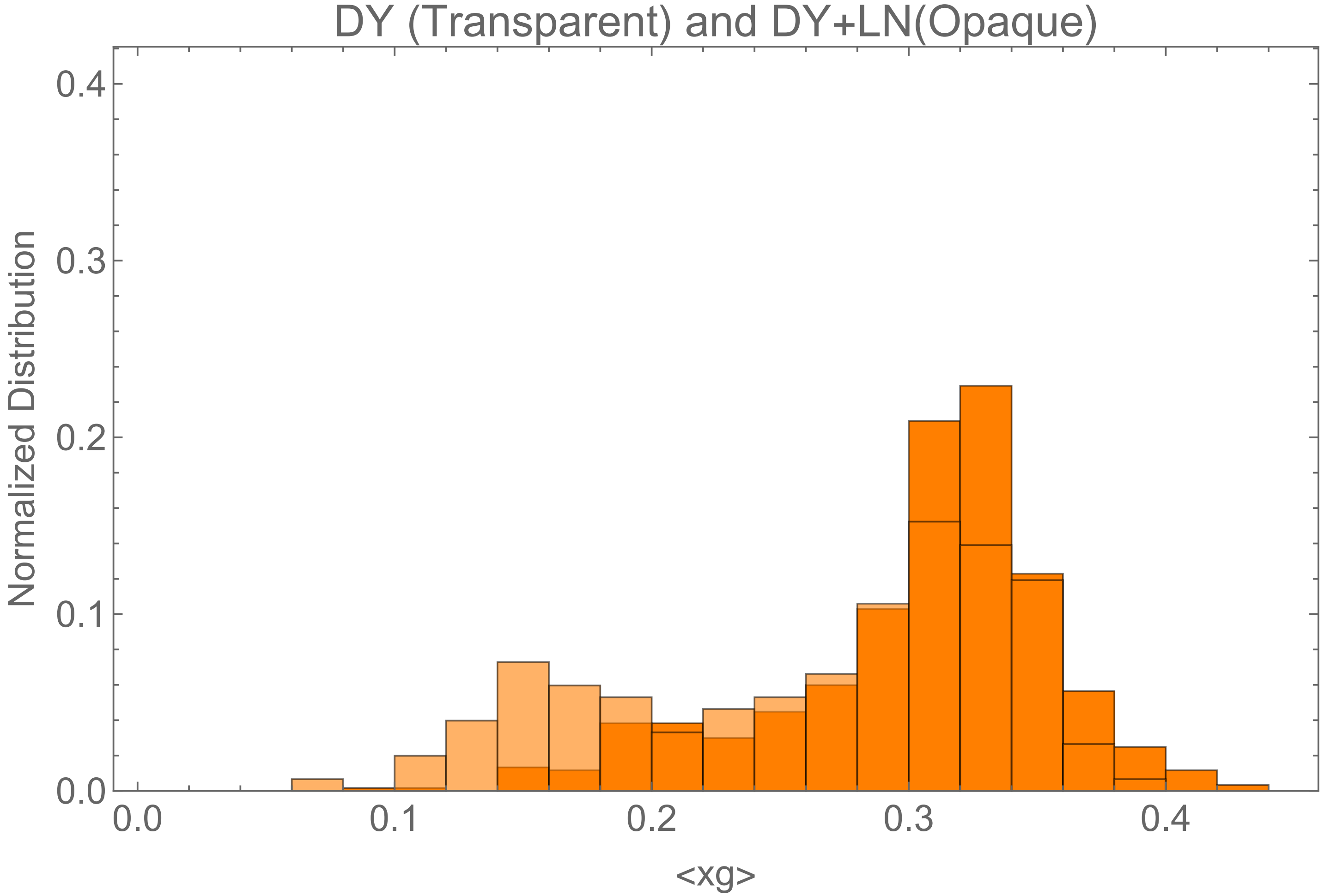
DY and DY+LN Valence momentum distribution (PRELIMINARY)



DY and DY+LN Sea momentum distribution (PRELIMINARY)



DY and DY+LN Gluon momentum distribution (PRELIMINARY)



Enforcing Positivity

- A more general expression for metamorph is

$$xf(x) = f_{\text{Carrier}}(x) * F(\mathcal{B}^{(N_m)}(y)).$$

- where $F(x)$ is some function that is always positive.
  - i.e. $F\left(\mathcal{B}^{(N_m)}(y)\right) = e^{a*\mathcal{B}^{(N_m)}(y)}$ where a is a constant.